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ARBITRAGE CHAINS

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ABSTRACT

In efficient markets the price should reflect the arrival of private information. The mechanism by which this is accomplished is arbitrage. A privately informed trader will engage in costly arbitrage, that is, trade on his knowledge that the price of an asset is different from the fundamental value if: (1) his order does not move the price immediately to reflect the information; (2) he can hold the asset until the date when the information is reflected in the price. We study a general equilibrium model in which all agents optimize. In each period, there may be a trader with a limited horizon who has private information about a distant event. Whether he acts on his information, and whether subsequent informed traders act, is shown to depend on the possibility of a sequence or *chain* of future informed traders spanning the event date. An arbitrageur who receives good news will buy only if it is likely that, at the end of his trading horizon, a subsequent arbitrageur's buying will have pushed up the expected price. We show that limited trading horizons result in inefficient prices because informed traders do not act on their information until the event date is sufficiently close.

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I. Introduction

The efficiency of security prices depends upon arbitrage, that is, trading based upon knowledge that the price of an asset is different from its fundamental value. For example, suppose an agent has private information about a high future dividend to be paid by a firm. If the current stock price does not reflect this information, then the agent can profit by buying the stock, if his purchase does not instantly raise the price, and holding it until the dividend is paid.

The argument depends on the arbitrageur's information not being instantly reflected in the price upon submission of the buy order (in the example) and on the arbitrageur being able to hold the security until the dividend is paid (in order to realize the profit). The first assumption has been widely studied in models which include "noise" traders, that is, agents who are willing to trade even when they lose money to the informed agents. In most models, following Kyle (1985), noise traders are exogenously motivated and it is not clear how they would behave if they were allowed to react to the presence of informed traders. The assumption of exogenous noise or liquidity trade seems harder to maintain when the expected rate of return the informed earn increases over time (at the expense of the uninformed), as will happen in the model of this paper.

The second assumption, that the arbitrageur's horizon span the event date (i.e., the date at which the dividend arrives in the example) has received less attention (the relevant literature is discussed below). But this second assumption is also crucial for the argument. In the example, suppose the informed trader's trading horizon does not span the event date. For instance, a portfolio manager may have to liquidate his portfolio to make large distributions to pensioners or he may need to show good performance over a short horizon at the end of which his performance is assessed. He will not trade on his information (if arbitrage is costly) if he believes that tomorrow's price, when he must sell the stock, will not reflect the information. In that case he will not buy the stock to start with and the price cannot reflect his information. On the other hand, he may believe that tomorrow another informed trader will arrive and buy the stock, pushing the price up so as to make the arbitrage profitable. Of course, the complication is that if the newly arrived informed trader pushes the price up too much *he* will not buy and the chain of arbitrages will unravel.

With limited trading horizons arbitrageurs' decisions to trade will be influenced by their beliefs about the distribution of the private information across other (future) arbitrageurs with different, perhaps overlapping, trading horizons. If there is the possibility of a sequence of arbitrageurs with overlapping trading horizons which span the event date, then the decision of the first arbitrageur in the sequence will depend on his beliefs about the subsequent chain of arbitrageurs and their decisions. The question we address is whether the actions of arbitrageurs in the chain can replicate the behavior of a single long-lived arbitrageur, or whether it is possible that there is private information which is not acted upon.

In this paper we study a general equilibrium model where informed traders have limited horizons and in which there are no exogenous "noise" traders. The model is one of overlapping generations where prices are formed by a marketmaker. Arbitrage is costly (either because of a brokerage fee or because of a borrowing cost). In the model risk-averse uninformed traders choose an amount to trade based on hedging motives. This causes them to play the role of "liquidity" traders. Each period there is a probability of an informed trader arriving who may choose to trade. Informed traders have private information about a future dividend which, for the first such trader, will arrive beyond his lifetime. Since informed traders profit at the expense of the uninformed, the amount the uninformed trade can change depending on whether they believe there are informed traders operating or not.

The main result of the paper is that informed agents will *not* engage in arbitrage a long time in advance of the event. In particular, only if the probability of another informed trader arriving next period is high enough will an informed agent trade this period. Consequently, information can arrive privately which has no chance of being impounded in prices because the informed trader finds it too costly to trade. The fact that arbitrage can only be accomplished via a chain of traders dramatically reduces arbitrage profitability, compared to the case where arbitrage does not require the asset to be resold because the arbitrageur's lifetime spans the event date. The reason is that for arbitrage to be profitable, the market must be relatively "deep" so that the arbitrageur can buy without simultaneously pushing up the price. With a chain, however, profitable arbitrage also requires selling the asset when a subsequent arbitrageur pushes up the price next period. This requires that the market cannot be too deep. Profitability also depends on the likelihood that another trader will soon receive the same information.

We have assumed that there is a transaction cost for trading the risky security. In any model, transaction costs are likely to cause price inefficencies. In our model, however, the effect of the transaction cost is multiplied by the two factors discussed above: the need to carry out arbitrage via a chain, and the possibility that the chain may be broken. Rather than interpreting the transaction cost as a brokerage fee, the transaction cost may equally be interpreted as a borrowing cost or "cost of carry."

The model does not explain why trading horizons should be limited. The setting we have in mind is one in which institutional investors hire professional money managers under compensation contracts which allow them to be fired if there is poor performance.

The paper proceeds as follows. In Section II we explain the basics of the model. Section III discusses the possible order flows, while Section IV discusses price formation. Section V describes the trading strategies of the informed agents and proves a series of propositions about the behavior of prices in equilibrium. Section VI describes the quantities traded by the uninformed; Section VII discusses out-of-equilibrium beliefs. Section VIII discusses the results and the related literature.

II. The Model

A. Definitions and Notation

We consider an economy with an infinite sequence of periods $t = -\infty,\ldots,\infty$. There is a stock which pays a dividend of 1 or 0 every period, with probabilities π and $1-\pi$ respectively.

Dividend realizations are serially independent. The other available asset is a riskless asset which earns r. We will focus on the periods preceding the period distinguished as $t=T$. The dividend realization at T may become known in advance to some agents, as described below.

Agents live for only two periods. Thus there are overlapping generations of young and old agents. Consumption occurs in old age. In each generation there are 0, 1, or 2 people. Agents may or may not receive a private piece of information. The probability that there is one uninformed agent is ½; the probability that there is no uninformed agent is ½. The probability that there is one informed agent in generation t is δ_t; with probability $1-\delta_t$ there is no uninformed agent. These realizations are independent of each other, and serially. Thus, a generation may contain nobody (probability $\frac{1}{2}(1-\delta_t)$), one informed agent and no uninformed agent (probability $\frac{1}{2}\delta_t$), one uninformed agent and no informed agent (probability $\frac{1}{2}(1-\delta_t)$), or one informed agent and one uninformed agent (probability $\frac{1}{2}\delta_t$).

Uninformed agents are born with endowments of W each. They also receive a wage income in old age of either 0 or 1 which is perfectly negatively correlated with that period's dividend. Thus the uninformed start life by buying a portfolio which they will liquidate in old age. However, we emphasize that they are not forced to participate in the stock market at all. If they want, they can simply invest at the riskless return r.

Uninformed agents are risk averse and so will have an incentive to buy stock to hedge their income. The reader may wish to read the first part of the paper assuming that they are *infinitely* risk averse to simplify the analysis: in this case, they hedge perfectly by buying 1 unit of stock. In section VI, we explain how the quantity traded is determined for the case of finite risk-aversion; this is quite straightforward and separate from the rest of model. In particular, we stress that the equilibrium prices and the period at which information starts to be revealed in equilibrium are not affected. For technical reasons, an uninformed agent should be viewed as representing a mass of infinitely small, identical, uninformed agents. This assumption will be needed in the derivation of the equilibrium quantities traded by the uninformed in section VI, and plays no role in the rest of the paper. To avoid lengthy circumlocutions, elsewhere in paper we will simply refer to "an uninformed agent" rather than "a mass of uninformed agents," etc.

Informed agents are risk neutral. Like the uninformed, they receive an endowment W when young (since they are risk neutral, we assume for simplicity that they receive no wage income as this will not affect their decisions). An informed agent, if there is one, receives private information about the dividend to be received at a fixed date T. "Good news" means that he learns it will be high (equal to 1); "bad news" means it will be zero. As time approaches date T, the probability δ_t there is an informed agent is increasing. The reader may find it convenient to think of δ_t as following the relation:

$$\delta_t = \epsilon\delta_{t+1} = \epsilon^{(T-t)},$$

which converges smoothly to 1 at date t, though the analysis does not require us to use any specific process.

Apart from these agents, there is an institution for pricing and trading stocks. In the stock market, prices are set by a risk-neutral marketmaker who faces Bertrand competition and who has an inventory of stocks and cash (à la Kyle (1985)). He observes all the (market) orders for the stock, and then posts a price and meets the net order out of his inventory. Because of competition, the price in each period is equal to his expected value of the asset. (His inventory is discussed further below.) We assume that the marketmaker observes all buy and sell orders separately (as will be seen below, in this model it would be equivalent to assume the marketmaker could only observe the net aggregate order).

Notice that when we refer to "agents," we do not include the marketmaker in this terminology.

There is a per-share transaction cost, c, to trading the stock. For notational simplicity c is the cost for a round trip transaction (since all stocks bought when young will be sold next period in old age).

The timing of events within each period may be summarized thus:

(Start of period t)

1. Dividends, old wage income, young endowment arrive.
2. Information arrives (if there is an informed trader).
3. Orders are submitted.
4. Marketmaker sets price.
5. Trades executed.
6. Consumption occurs.

(End of period t)

B. Comments on assumptions

Here we comment briefly on several aspects of the assumptions that may be questioned by the reader.

1. Why are informed agents risk-neutral and the uninformed risk averse?

Uninformed agents' risk aversion makes them want to trade so as to hedge their wage income shock. If they were risk-neutral they would simply invest at the risk-free rate and there would be no "liquidity" trades for the informed to hide behind. Informed traders are risk-neutral for simplicity (it will be clear that the results do not depend on this).

2. Why not have a lump-sum, rather than proportional, transactions cost? Why is the proportional transactions cost per-share, not per-dollar?

A lump-sum transaction cost seems less realistic and would complicate matters. The per-share assumption is made for analytical tractability. We have solved the model with proportional costs per dollar, but the closed-form solutions of the model much more complex. The results are unchanged except that the formula in Proposition 3 is modified appropriately.

Although a proportional cost per dollar might seem more plausible a priori, there is empirical evidence to the contrary. Dimson and Marsh (1989) found that the bid-ask spread on the London Stock Exchange is approximately six pence regardless of the share price. Brennan and Hughes (1991) show that brokerage commissions in the USA, expressed as a percentage of the value of the transaction, fall for shares with higher prices ("big" shares).

In Section VIII we show that the cost may be interpreted as a borrowing cost, since the per dollar version of the transaction cost is exactly equivalent to an interest charge on a loan.

3. Why is δ_t increasing over time?

Our model does not require that δ_t increase over time, but it is more plausible and simplifies the exposition to assume that it does. If it is not increasing, it is extremely easy to derive equilibria where long-term information is ignored. For example, if there is always an informed agent at T-2 but never in any other period, there will be such a non-revealing equilibrium. The harder question is whether information may be ignored even when it is increasingly likely to arrive as we approach the event. Not only is this question harder, but it is more important since increasing δ_t seems more plausible.

Two further points are worth noting about the model.

4. While both the uninformed and the informed agents have limited horizons, the marketmaker is infinitely-lived. Thus, as we discuss in Section VIII, the results cannot be due to all agents having short horizons. What matters is that the privately informed agents have short horizons.

5. The model endogenizes the "liquidity" trade in an attractive way. In our multi-period framework, agents start without assets and do not have to participate in the stock market if they

do not want to. If they do participate, however, in order to hedge, then in the second period of their lives they must unwind their positions, i.e., sell their holdings. This distinguishes our model from Biais and Hillion (1992). They interpret liquidity trade as coming from rational agents who hedge shocks to non-tradable assets, similar to our wage-income shocks. However, theirs is a one-period model where agents start with an endowment of assets, trade once, then the assets pay liquidating dividends and the agents consume.

III. Order flow in equilibrium

We will now describe the equilibrium order flow and stock prices. The equilibrium has the property that informed agents who receive good news act on the information starting a fixed number of periods from the end, but not before. Date K is the date at which informed agents start to act. Subsequently we will show that this is indeed an equilibrium, determine K, and show that this is the unique equilibrium of the model.

It is possible that after K, the informed trader's actions will completely reveal the news to the market-maker and the price will immediately jump to the full information price from then on. In that case, subsequent informed agents will not act. But unless this happens, in our model informed traders with good news will always act after K for all other price histories.

Let x_t be the amount of stock bought in equilibrium by an uninformed (risk-averse) agent born at date t. Because of the transactions cost, the agent will only choose to hedge fully ($x_t = 1$) if he is infinitely risk-averse. The determination of x_t is discussed in Section VI below. However, the prices and other properties of equilibrium, including the determination of date K, will not be affected by x_t.

What are the possible orders after period K?

If no agent is born in period t, no orders will be submitted. If there is an uninformed agent, he will buy x_t. We will show in Section VII below that, in order to disguise himself, an informed trader who receives good news will also buy x_t. Thus the possible buy orders are 0, x_t, or $2x_t$. In other words, at all dates orders will be in multiples 0, 1, or 2 of x_t.

Note that informed traders who receive bad news do not act, because they would immediately be identified by the marketmaker. They do not sell the asset short. This is because in equilibrium the sell orders at date t will be the same as the buy orders from date t−1. If an informed agent with bad news did sell short, sell orders at date t would exceed buys in t−1, so the information would be completely revealed and he would have incurred the transactions cost. (The model could easily be extended slightly to allow for short sales by informed agents, but we do not pursue that.)

How could these multiples 0, 1, or 2 occur, and what are their probabilities?

Buy = 0: This can occur in two ways: either no agent is born (probability $\frac{1}{2}(1-\delta_t)$), or an informed agent is born and receives bad news ($\frac{1}{2}\delta_t(1-\pi)$):

$$\tfrac{1}{2}(1-\delta_t) + \tfrac{1}{2}\delta_t(1-\pi) = \tfrac{1}{2}(1 - \delta_t\pi).$$

Buy = 1: This can occur in three ways: only an uninformed agent is born ($\frac{1}{2}(1-\delta_t)$); only an informed agent is born and he receives good news ($\frac{1}{2}\delta_t\pi$); both an uninformed and an informed agent are born, and the informed agent receives bad news ($\frac{1}{2}\delta_t(1-\pi)$):

$$\tfrac{1}{2}(1-\delta) + \tfrac{1}{2}\delta_t\pi + \tfrac{1}{2}\delta_t(1-\pi) = \tfrac{1}{2}.$$

Buy = 2: This occurs if both an informed agent and an uninformed agent are born, and the informed agent receives good news:

$$\tfrac{1}{2}\delta_t\pi.$$

The total stockholding of the traders in equilibrium is therefore either 0, 1, or 2 shares. Hence the number of shares in existence could be as few as 2, in which case the marketmaker would hold 2, 1 or 0 shares (respectively) in inventory. In particular, it is not necessary for the marketmaker to hold an infinite inventory, as in Kyle (1985) and Glosten and Milgrom (1985). A more important difference is that in those models the marketmaker's inventory has no mean reversion because it follows a random walk. The reason for this difference is that our uninformed traders "unwind" their positions, whereas in the above models "liquidity" trade each period is an independent random variable.

IV. Stock Prices.

If we are more than T−K periods from T, by hypothesis, informed agents do *not* act on their information. In that case there is no information in a buy order (there will be either 0 or 1 orders) and the price is given by:

$$p = \pi/r$$

which is the marketmaker's expected valuation.

We now consider prices less than T−K periods from T. Suppose first that the marketmaker knows for sure that the dividend at time T will be 1. Then the price is:

$$p_t = \pi/r + (1/(1+r))^{T-t}(1-\pi)$$

If the dividend is known to be zero, the price is

$$p_t = \pi/r - (1/(1+r))^{T-t}\pi.$$

The former case will happen in equilibrium if the marketmaker observes two buy orders - which can only arise when both an uninformed trader and an informed trader with good news are born. The latter case will not arise in equilibrium, since informed traders do not act on bad news.

If the marketmaker observes 0 or 1 buy orders, he does not know the date T dividend for sure. However, the order flow is informative and will be used to update the marketmaker's belief. For example, an order flow of 0 could arise if there is an informed trader with bad news, or no informed trader, but not if there is an informed trader with good news. So 0 orders will cause the belief to be revised downwards.

Let β_t be the marketmaker's belief at date t that the date T dividend will be 1. This belief is formed at date t, having observed the order flow at date t and is used to set price p_t. The stock price will be a weighted average of the above prices,

$$p_t = \beta_t[\pi/r + (1/(1+r))^{T-t}(1-\pi)] + (1-\beta_t)[\pi/r - (1/(1+r))^{T-t}\pi]$$

$$= \pi/r + (1/(1+r))^{T-t}(\beta_t-\pi)$$

for $t \geq T-K$. Note that $\beta_{T-K-1} = \pi$. We now derive the updating rule for subsequent beliefs.

Note that many of formulas for the probabilities derived below are similar to those in Section III above, but those were marginal probabilities (i.e. not conditional on the realized history of orders in previous periods), while these are the marketmaker's beliefs. So these formulas have β_t where the previous ones had π.

If there are 0 buy orders: As explained in section III above, this can occur either if no agents are born, or if only an informed agent is born and he receives bad news. The probability that no agents are born is $\frac{1}{2}(1-\delta_t)$. The probability that only an informed agent is born and he receives bad news is $\frac{1}{2}\delta_t(1-\beta_{t-1})$. So the probability that the dividend is high and there is a buy order of zero is $\frac{1}{2}(1-\delta_t)\beta_{t-1}$. The probability of zero buy orders is

$$\tfrac{1}{2}(1-\delta_t) + \tfrac{1}{2}\delta_t(1-\beta_{t-1}) = \tfrac{1}{2}(1-\delta_t\beta_{t-1}).$$

So

$$\beta_t = [\tfrac{1}{2}(1-\delta_t)\beta_{t-1}]/[\tfrac{1}{2}(1-\delta_t\beta_{t-1})]$$

$$= (1-\delta_t)\beta_{t-1}/(1-\delta_t\beta_{t-1}).$$

If there is 1 buy order: This can occur in three ways: if only an uninformed agent is born (probability $\tfrac{1}{2}(1-\delta_t)$), if only an informed agent is born and he receives good news (probability $\tfrac{1}{2}\delta_t\beta_{t-1}$), or both an informed and an uninformed agent are born, but the informed agent receives bad news (probability $\tfrac{1}{2}\delta_t(1-\beta_{t-1})$). The probability that there is 1 buy order and the dividend is high is

$$\tfrac{1}{2}(1-\delta_t)\beta_{t-1} + \tfrac{1}{2}\delta_t\beta_{t-1} = \tfrac{1}{2}\beta_{t-1}.$$

The probability of 1 buy order is

$$\tfrac{1}{2}(1-\delta_t) + \tfrac{1}{2}\delta_t\beta_{t-1} + \tfrac{1}{2}\delta_t(1-\beta_{t-1}) = \tfrac{1}{2}.$$

So

$$\beta_t = (\tfrac{1}{2}\beta_{t-1})/\tfrac{1}{2}$$

$$= \beta_{t-1}.$$

If there are 2 buy orders: As discussed above, this can only occur if there are both an uninformed trader and an informed trader with good news. This therefore reveals that the date T dividend is high and so $\beta_t = 1$.

This completes the description of beliefs, hence describes the evolution of stock prices as a function of the history of information arrival. It remains to determine K.

V. Informed Agents' Trading Strategies.

We now consider the decision problem of an informed agent.

First, if next period an informed agent (if there is one) would not act, then an informed agent this period will not act. Because next period's price cannot reflect more information than today's, he would simply incur the transaction cost without any benefit. This is Proposition 1.

On the other hand, if next period an informed *would* act, then an informed agent this period may or may not act, depending on the likelihood of next period's price reflecting more information than today's and on the size of the capital gain in that event, balanced against the transaction cost.

It may be the price already completely reflects the information. Clearly in this case the informed agent will not trade. But if the price does not already completely reflect the information, then we show in Proposition 2 that if an informed agent *last* period would have decided to act (because the expected capital gain outweighed the transaction cost), then an informed agent this period will also decide to act. In other words, the expected capital gain increasingly outweighs the transaction cost.

Combining Propositions 1 and 2, the equilibrium must have the property that there exists a critical date K before which informed agents never act and after which they always act, except in the event that the price is fully revealing. It remains to shows in Proposition 3 how date K is determined.

Proposition 1: Suppose that the probability of an informed agent arriving and acting on his good news next period (t+1) is zero. Then an informed agent will not act on his good news this period (t).

Proof: Since, by hypothesis, an informed agent at date t+1 will not act, $E\beta_{t+1} = \beta_t$. Note that β_t may or may not be updated from date $t - 1$. So

$$p_t = \pi/r + (1/(1+r))^{T-t}(\beta_t - \pi)$$

and p_{t+1} is non-random:

$$p_{t+1} = \pi/r + (1/(1+r))^{T-(t+1)}(\beta_t - \pi)$$

It follows that

$$p_t(1+r) = (1+r)\pi/r + (1/(1+r))^{T-(t+1)}(\beta_t - \pi)$$

$$= \pi + p_{t+1},$$

so if the agent acts, his expected wealth at t+1 is

$$\pi\,[(W - p_t x_t)(1 + r) + x_t(p_{t+1} - c + 1)] + (1 - \pi)\,[(W - p_t x_t)(1 + r) + x_t(p_{t+1} - c)]$$

$$= W(1 + r) - x_t c.$$

This is less than his wealth if he simply invests in the riskless asset, $W(1 + r)$. In other words the expected return on the share (including the expected dividend) is the same as on the riskless asset (r), but ignoring the transactions cost. When the transaction cost is included the return on the share is less. Q.E.D.

We next consider the informed agent's wealth in case he acts or does not act at time t, assuming that an informed agent next period would act. If he does not act his wealth is simply W(1+r). If he acts and buys x_t shares, his wealth depends on whether there is also an uninformed agent present at time t. With probability ½, there is an uninformed agent present at time t. In this case, the informed agent's order will reveal the information and the marketmaker's belief will jump to 1 and remain there, so the informed agent will earn the safe rate of return r, but will also incur a transaction cost $x_t c$. His wealth will be

$$W(1+r) - x_t c.$$

With probability ½ there is no uninformed agent present. The price at time t will not reveal the information. We will use the notation "*" to denote conditioning on the event that there is no uninformed agent present at time t. The marketmaker's current belief, and the corresponding price, are not random conditional on this event. We denote them β_t^* and p_t^* (as shown in Section IV, the marketmaker's belief in this event is the same as last period's belief β_{t-1}). The marketmaker's belief next period, and the corresponding price, conditional on this event, are random and we denote their expectations $E^*\beta_{t+1}$ and E^*p_{t+1}.

With this notation we can now write the informed agent's expected wealth (in the event there is no uninformed agent present at t) as:

$$(W-p_t^*x_t)(1+r) + x_t[E^*p_{t+1} - c + \pi]$$

$$= W(1+r) + x_t[E^*p_{t+1} - p_t^*(1+r) - c + \pi].$$

Averaging over the two events, the informed agent's expected wealth if he acts is:

$$\tfrac{1}{2}[W(1+r) - x_t c] + \tfrac{1}{2}[W(1+r) + x_t[E^*p_{t+1} - p_t^*(1+r) - c + \pi].$$

If he does not act on his information he receives W(1+r). Comparing these, the informed agent will act if

$$E^{*}p_{t+1} + \pi - p_{t}^{*}(1+r) > 2c.$$

This decision rule will enable us to characterize the equilibrium.

Proposition 2: Suppose that there is an initial date K at which an informed agent will act on good news. Then an informed agent will act on good news at all subsequent dates, if the price is not already fully revealing.

Proof: An informed agent with good news at date $t \geq K$ will act if

$$E^{*}p_{t+1} + \pi - p_{t}^{*}(1+r) > 2c$$

i.e.,

$$(\pi/r)(1+r) + (1/(1+r))^{T-(t+1)}(E^{*}\beta_{t+1}-\pi) - (1+r)[(\pi/r) +$$

$$(1/(1+r))^{T-t}(\beta_{t}^{*}-\pi)] > 2c$$

or,

$$(1/(1+r))^{T-(t+1)}(E^{*}\beta_{t+1}-\pi-\beta_{t}^{*}+\pi) > 2c$$

$$E^{*}\beta_{t+1}-\beta_{t}^{*} > 2c(1+r)^{T-(t+1)}.$$

By definition of K this expression holds for t = K. Note that the right-hand-side of this inequality falls with time. We will show that the left-hand-side increases over time regardless of how β_t evolves (so long as it does not reach 1).

Suppose first that an informed agent acting at date K is followed by an uninterrupted sequence of single buy orders until date t > K. We now consider the decision problem of an informed agent at date t. The updating rule for β_t implies that $\beta_t^* = \beta_K = \pi$. Note that from the expression given above for the value of $E^*\beta_{t+1}$,

$$E^*\beta_{t+1} - \beta_t^* = \delta_{t+1}(1 - \beta_t^*)^2/[2(1 - \delta_{t+1}\beta_t^*)].$$

Thus,

$$\partial[E^*\beta_{t+1} - \beta_t^*]/\partial\delta = (1-\beta_t^*)^2/[2(1 - \delta_{t+1}\beta_t^*)^2] > 0.$$

It follows immediately that

$$E^*\beta_{t+1} - \beta_t^* > E^*\beta_{K+1} - \beta_K > 2c(1+r)^{T-(K+1)} > 2c(1+r)^{T-(t+1)},$$

and therefore an informed agent at date t will act.

If the sequence of buy orders between date K and date t include some dates at which there were no buys, then $\beta_t^* < \beta_K$. The reason is that a buy of 0 causes the marketmaker to revise his beliefs downward, while a buy of 1 causes beliefs to remain unchanged. (Note that a buy of 2 reveals the information, so beliefs reach 1, but we are not describing this case here.) However, when beliefs are revised downwards this simply increases $E^*\beta_{t+1} - \beta_t^*$ and the result remains true:

$$\partial[E^*\beta_{t+1} - \beta_t^*]/\partial\beta_t^* = \delta_{t+1}(1 - \beta_t^*)(\delta_{t+1} + \delta_{t+1}\beta_t^* - 2)/[2(1-\delta_{t+1}\beta_t^*)^2] < 0,$$

since $\delta_{t+1} + \delta_{t+1}\beta_t^* < 2$.

Q.E.D.

By Proposition 1 informed agents will not act on good news before date K, and from Proposition 2 they will act afterwards except if the price is fully revealing. Note that Proposition 2 implies there are no mixed strategy equilibria (except possibly in the negligible case that the agent is indifferent at date K).

We will now find the largest δ_t (and hence the last date t) at which an informed agent who receives good news is unwilling to act, assuming that at date t+1 an informed agent will act. This determines date K.

Proposition 3: K is the first date t for which:

$$\tfrac{1}{4}\delta_{t+1}(1-\pi)^2/(1-\delta_{t+1}\pi) > c\ (1+r)^{T-(t+1)}.$$

Proof: Since by hypothesis the informed agent is not supposed to act on his information in t−1, the marketmaker will not update his beliefs when he sees a buy order of 1, unless there is also an uninformed agent at t−1. So long as there is no uninformed agent at t−1, the informed agent will therefore purchase the share at price:

$$p_{t-1} = \pi/r.$$

(Of course, if there is an uninformed agent at t−1 then there will be two buy orders and the good news will be revealed.)

An informed agent will choose not to act if:

$$E^{*}p_{t+1} + \pi - p_t^{*}(1+r) < 2c,$$

or, substituting for $E^{*}p_{t+1}$ using the formula for p_{t+1},

$$(1/(1+r))^{T-(t+1)}[E^{*}\beta_{t+1} - \pi] < 2c$$

since prices are linear in beliefs. We now compute $E^*\beta_{t+1}$.

Period t+1 is, by hypothesis, the first date at which an informed agent with good news will act. Therefore, depending on whether trading volume at t+1 is 0, 1 or 2 (times x_{t+1}), the marketmaker's belief will be:

$$\beta_{t+1} = (1-\delta_{t+1})\pi/(1-\delta_{t+1}\pi),$$

$$\beta_{t+1} = \pi,$$

or

$$\beta_{t+1} = 1,$$

respectively. Since the agent is informed (he knows the dividend at date T will be 1) the probabilities he attaches to each of these three beliefs (and corresponding prices) occurring are different from the probabilities attached to these events by the uninformed marketmaker. From the point of view of the informed agent their chances of occurrence are as follows:

Buy = 0: No agents born, $\frac{1}{2}(1-\delta_{t+1})$.

Buy = 1: Only one agent and he is informed, $\frac{1}{2}\delta_{t+1}$, or only one agent and he is uninformed, $\frac{1}{2}(1-\delta_{t+1})$. Total probability = ½.

Buy = 2: Two agents, one uninformed and one informed, $\frac{1}{2}\delta_{t+1}$.

Thus

$$E^*\beta_{t+1} = \tfrac{1}{2}(1-\delta_{t+1})[(1-\delta_{t+1})\pi/(1-\delta_{t+1}\pi)] + \tfrac{1}{2}\pi + \tfrac{1}{2}\delta_{t+1}$$

So

$$E^{*}\beta_{t+1} - \pi = \tfrac{1}{2}(1-\delta_{t+1})[(1-\delta_{t+1})\pi/(1-\delta_{t+1}\pi)] - \tfrac{1}{2}\pi + \tfrac{1}{2}\delta_{t+1}$$

$$= \tfrac{1}{2}\delta_{t+1}(1-\pi)^2/(1-\delta_{t+1}\pi).$$

K is therefore defined as the first time index t for which:

$$(1/(1+r))^{T-(t+1)}\,[\tfrac{1}{2}\delta_{t+1}(1-\pi)^2/(1-\delta_{t+1}\pi)] > 2c.$$

Q.E.D.

Figure 1 illustrates the determination of K. Viewing time as a continuous variable, it graphs the left-hand side and right-hand side of the equation defining K:

$$\tfrac{1}{4}\delta_{t+1}(1-\pi)^2/(1-\delta_{t+1}\pi) > c\,(1+r)^{T-(t+1)},$$

for $c = 0.001$, $\pi = 0.01$, $r = 0.01$, and $\delta_t = 0.5^{T-t}$. Note that the (uninformative) price for the asset is $\pi/r = 1$, so c is approximately 0.1% of the asset price. The information cannot be revealed more than 6 periods from the event date T.

Date K is essentially determined by the trade-off between the chance of an informed trader arriving next period (the left-hand side) and the transaction cost (the right-hand side). To see this, note that the left-hand side is almost equal to a constant times δ_{t+1} (because the denominator rapidly approaches 1). On the other hand, the solution is not very sensitive to the interest growth term on the right-hand side. By ignoring this term we get a lower bound for K (the true date K happens *later*).

VI. Uninformed Agents' Trading Strategies: Determination of x_t

To this point, the amount traded by the uninformed agents, x_t, has been taken as given. In this section the amount these risk-averse agents will trade each period will be determined. As was seen above, x_t does not affect the equilibrium prices and strategies so long as it is positive. But if the uninformed are only slightly risk-averse, they might choose not to hedge at all. Therefore, we also provide conditions on their utility function that guarantee that they are sufficiently risk averse to hedge.

In periods before $K-1$, uninformed agents buy and sell at the same price, π/r. Their expected return on the risky asset (ignoring the transaction cost) is r, the same as on the bond. Thus, an uninformed agent who is infinitely risk-averse will choose a portfolio which completely insures him against risk, which in this case means buying one unit of the asset (since the dividend exactly offsets the wage income). If the uninformed agent is not infinitely risk averse he will choose to hedge only partly, because of the transaction cost.

From date $K-1$ to K, an uninformed agent will get an expected return equal to r, but this time the return is uncertain. However, the price variability is independent of the wage income shock.

From date K onwards informed agents will act on their information so that the price an uninformed agent buys at will not be fair. Unless they are infinitely risk-averse, they will not fully hedge but will buy some number of shares $x_t < 1$ even if the transaction cost is zero.

In summary, there are three cases:

1. Before date $K-1$.
2. At date $K-1$.
3. From date K onwards.

The details of the derivations of x_t in the three cases, and the conditions for x_t to be strictly positive, are given in the appendix.

VII. Out-of-Equilibrium Beliefs

To this point we have only considered the possibility that all agents trade a multiple 0, 1, or 2 of x_t. To complete the construction of the equilibrium, it remains to verify that no agent has an incentive to deviate by trading other quantities. Recall that the belief maps β_{t-1} to β_t, as a function of the total quantity traded at time t. We proceed as usual by specifying beliefs for the marketmaker at other quantities, as follows:

1. For trading volume less than or equal to 0, the marketmaker has the same beliefs as he does at 0: $\beta_t = (1 - \delta_t)\beta_{t-1}/(1 - \delta_t\beta_{t-1})$.

2. For trading volume greater than 0, but less than or equal to x_t, the marketmaker has the same beliefs as he does at x_t, so $\beta_t = \beta_{t-1}$.

3. For trading volume greater than x_t, the marketmaker believes the asset is of high value, $\beta_t = 1$.

There are two possible deviations an informed trader can make. He can trade more than x_t, in which case the price will immediately become fully revealing and he will earn the riskless return r less the transactions cost. This is clearly not a profitable deviation.

He can trade less than x_t, in which case the price will be unchanged and he will earn the same percentage return on a smaller quantity. Again, this is clearly suboptimal.

Finally, the uninformed have no incentive to deviate by definition of x_t. Since the uninformed are a continuum of infinitesimal agents, an individual cannot affect the aggregate trading volume, and so cannot change the market-maker's beliefs. The quantity x_t was derived under precisely this assumption.

VIII. Discussion of the Results

In this section we first discuss the factors of the model which are important for the result that prices prior to date K are inefficient. The main ingredients of our model are limited horizons, a transaction cost for trading, and the nature of the information event. Finally, we discuss other related research.

A. Limited Horizons

With respect to limited horizons we wish to stress that the risk-neutral marketmaker does not have a limited horizon; he is infinitely-lived. The marketmaker may be viewed as a "sea" of uninformed, risk-neutral traders. The price is determined by the information set of the marketmaker. Thus, our analysis only requires that the privately informed agents have limited horizons.

The short horizons of the informed traders makes the chain of future informed agents important for arbitrage. An additional factor concerns the likelihood of future informed agents arriving. Each of these factors is important for the result that prices are inefficient prior to K. Moreover, each factor is important in delaying trading by arbitrageurs with short horizons, relative to the decision that would be made by an arbitrageur facing the same transaction cost, but without a short horizon.

We analyze these issues by modifying the above model so that informed agents are infinitely lived. We compare this to the model analyzed above, but where the probability of an informed agent arriving next period is 1 (i.e., $\delta_t = 1$, for all t). This allows us to isolate the effect of the chain on price efficiency.

Rather than informed agents living for two periods, suppose that informed agents live forever. Each period there is a probability δ_t of an informed agent arriving (as before--so the marketmaker's beliefs are the same). If the informed agent arrives he will pool with the liquidity trader by buying x_t shares. However, since he has no incentive to sell the shares next period δ_t has no effect on his decision.

Consider the informed agent's decision problem. If he buys at time t, then with probability ½ he submits an order at the same time that an uninformed trader submits a buy order. In this case, he loses cx_t; the value of his wealth at t+1 is $W(1+r) - cx_t$. With probability ½ he is the only trader and he buys at price p_t. Then he never sells the shares; the value of his wealth at t+1 is:

$$W(1+r) + x_t(\pi/r + 1/(1+r)^{(T-(t+1))}(1-\pi) - P_t(1+r) - c + \pi).$$

Therefore, he buys if:

$$\tfrac{1}{2}[W(1+r) - cx_t] + \tfrac{1}{2}[W(1+r) + x_t(\pi/r + 1/(1+r)^{(T-(t+1))}(1-\pi) - P_t(1+r) - c + \pi)] > W(1+r)$$

or,

$$\pi/r + 1/(1+r)^{(T-(t+1))}(1-\pi) - P_t(1+r) + \pi > 2c.$$

We will denote by K^* the date at which the informed agent starts to act in this variant of the model. K^* is determined by the previous expression with $p_t = \pi/r$; it is the first period t for which:

$$\tfrac{1}{2}(1-\pi) > c(1+r)^{T-(t+1)}.$$

Note that the left-hand side, representing the benefit of the arbitrage, has the term ½ in it because half the time the arbitrageur reveals the information when he buys the shares at the same time as an uninformed agent.

By contrast, consider the decision rule of an informed agent in the model of this paper when $\delta_t = 1$, for all t. In other words, assume that an informed agent will always arrive next period, but the informed agents have short horizons and their profits depend on reselling the asset. K is determined by setting $\delta_{t+1}=1$ in the expression in Proposition 3; it is the first period t for which:

$$\tfrac{1}{4}(1-\pi) > c(1+r)^{T-(t+1)}.$$

Comparing the two inequalities, we can see the effect of the arbitrageurs' short horizons which make profit depend on the chain of future arbitrageurs. Since we are examining the case where $\delta_t=1$ (for all t), the chain is certain to be unbroken. However, the profitability of the arbitrage depends not only on buying the asset cheaply at time t (which happens half the time), but on selling it at a high price at time t+1 (which happens only half the time, conditional on having bought cheaply at t). This will only happen if the arbitrageur next period coincides with the arrival of an uninformed buyer (in which case the information is revealed). Thus the dependence on the chain reduces the profitability of arbitrage by a further 50%. This is why ¼ appears in the determination of K, rather than ½ in the determination of K^*.

A different specification of the model might have a greater likelihood of revealing the information at t+1. We have taken the probability of an uninformed trader arriving to be ½. But if this probability were higher, increasing the likelihood of next period's price being fully revealing, then the likelihood of the arbitrageur being able to buy the asset cheaply at time t would be reduced. In other words, the presence of short horizons, in itself, has the effect of reducing arbitrage profit and, thereby, multiplying the effect of the transaction cost on the revelation of information.

When $\delta_t < 1$ the date K at which information may first be revealed is delayed even further.

B. Interpretation of the Transaction Cost

In the model we interpreted the transaction cost essentially as a brokerage fee for trading, but this is not essential. The cost could also be interpreted as a borrowing cost, i.e., c is the interest charge on a loan. In other words, an informed agent who borrowed $x_i p_t$ would repay $x_i p_t(1+r+c)$. This would be equivalent to making the transaction cost proportional to the value of the shares rather than the number of shares. As discussed above, we have solved the model for this case and the results are qualitatively unchanged, although the algebra is considerably more complicated. Thus, our trading cost can be equally viewed as a borrowing cost.

C. Interpretation of the Information Event

We took the information to be knowledge of the dividend at date T. The results would not be significantly changed if we assumed any form of uncertainty in which "news" became public at the event date T. For example, it could be that the probability of the dividend being 1 could increase at every date starting with date T. Alternatively, the dividend growth rate could change at date T.

In our model, the uncertainty relates to the dividend at a single fixed date, T. One could imagine a variant of the model in which information could arrive about dividends in different periods. This scenario is more realistic, but agents' inferences in such a model would be considerably more complex. However, there is no to believe that the properties of the model would be fundamentally altered.

D. Related Literature

Papers in the literature which have studied the effects of limited horizons include De Long et. al. (1990), Shleifer and Vishny (1990), and Froot et. al. (1992). The first two papers emphasize the effects of the presence of irrational traders interacting with rational traders who have short horizons; the third paper focuses on the information production decision of agents with short horizons.

In the model of De Long et. al., prices can deviate from fundamentals because irrational traders trade on incorrect beliefs. There is no asymmetric information. In one equilibrium of the model two identical assets trade at different prices. Rational traders, who are risk averse, do not arbitrage because the irrational traders may drive the price even further from the fundamentals within their horizons. This would be impossible in our setting where prices are set by a risk-neutral, long-lived, marketmaker.

Shleifer and Vishny (1990) consider a three date model with a short-term asset (for which any mispricing disappears at the interim date) and a long-term asset (which realizes its liquidation value at the final date). Noise traders drive initial prices away from fundamental values because they are either "optimistic" or "pessimistic." Arbitrageurs, who borrow in order to trade, prefer the short-term asset because the loan can be paid off sooner (both because the interest cost of carrying the arbitrage position is lower, and because they face rationing in their borrowing of funds for arbitrage). As in De Long et. al. (1990), the argument relies heavily on the noise traders' beliefs being exogenously wrong.

Froot, et. al., (1992) study the behavior of informed agents with short horizons in a Kyle (1985)-type model. The liquidation value of the asset is composed of the sum of two pieces of information. If the informed traders have short horizons then they will coordinate on producing the same piece of information so that it will be impounded in the price when they unwind their position. Presumably a suitable variant of our model would have the same property.

Appendix

This appendix gives the details of the derivation of x_t and the condition for x_t to be strictly positive, as described in Section VI.

Case 1: The agent buys and sells a quantity x at $p = \pi/r$. There is no uncertainty in the price. His wealth is:

If the dividend is high:

$$(W-px)(1+r) + x + px - cx = W(1+r) + x - pxr - cx = W(1+r) + x - x\pi - cx$$

If the dividend is low:

$$(W - px)(1+r) + 1 + px - cx = W(1+r) + 1 - pxr - cx = W(1+r) + 1 - x\pi - cx$$

Thus his expected utility is:

$$\pi\, U[W(1+r) + x - x(\pi + c)] + (1-\pi)\, U[W(1+r) + 1 - x(\pi + c)].$$

So for t prior to $K - 1$, x_t is defined to be the maximizer of this function. Taking the derivative with respect to x:

$$\pi\, U'[W(1+r)+x-x(\pi+c)]\,(1 - \pi - c) + (1 - \pi)\, U'[W(1+r)+1-x(\pi+c)]\,(- \pi - c)$$

Note that the transaction cost will prevent the agent from hedging completely. If $c = 0$ it is easy to verify that the derivative is zero when wealth is equal in the two states, hence $x = 1$. This is the usual local risk neutrality argument.

On the other hand, if $c > 0$, the derivative at $x = 1$ is:

$$\pi\,(1-\pi-c)\;U'[W(1+r)+1-(\pi+c)] + (1-\pi)\,(-\pi-c)\;U'[W(1+r)+1-(\pi+c)]$$

$$= [\pi(1-\pi-c) - (1-\pi)(\pi+c)]\;U'[W(1+r)+1-(\pi+c)]$$

$$= -\,c\;U'[W(1+r)+1-(\pi+c)] < 0$$

showing that the agent does not completely hedge.

Next we derive the condition for the agent to hedge when $c > 0$. The derivative at $x = 0$ will be positive and the agent will hedge if:

$$\pi\,(1-\pi-c)\;U'[W(1+r)] > (1-\pi)\,(\pi+c)\;U'[W(1+r)+1]$$

i.e.,

$$U'[W(1+r)]\;/\;U'[W(1+r)+1] > [(1-\pi)(\pi+c)]\;/\;[\pi\,(1-\pi-c)].$$

Our analysis therefore requires that agents be sufficiently risk-averse for this condition to hold. Otherwise there will no trading.

Case 2: The agent buys at price $p_{K-1} = \pi/r$ and sells at a price which will depend on the number of traders in period K.

In every period, the return earned by agents in aggregate is r. Since the marketmaker earns an expected return of r, so do the other agents combined. Since there are no informed agents active from $K-1$ to K, the uninformed agents earn an expected return of r over this period. Recall also that the variation in the expected price at period K is independent of the dividend and the uninformed agent's wage income shock.

In period K, the trading volume may be a multiple 0, 1, or 2 of x_K. In each case there will be a different price which we denote p_K^i, for i = 0, 1, 2. The corresponding probabilities are denoted Pr_K^i. (The formulae for these prices and probabilities were computed in sections III and IV.)

The expected utility of an uninformed agent is therefore

$$\pi \Sigma_i Pr_K^i U[W(1+r) + x_{K-1}(p_K^i - \pi/r - \pi - c + 1)] + (1-\pi)\Sigma_i Pr_K^i U[W(1+r) + x_{K-1}(p_K^i - \pi/r - \pi - c) + 1]$$

x_{K-1} is therefore defined to be the maximizer of this function. The derivative is:

$$\pi \Sigma_i Pr_K^i U'[W(1+r)+x_{K-1}(p_K^i-\pi/r-\pi-c+1)] (p_K^i - \pi/r - \pi - c + 1) + (1-\pi)\Sigma_i Pr_K^i U'[W(1+r)+x_{K-1}(p_K^i-\pi/r-\pi-c)+1] (p_K^i - \pi/r - \pi - c).$$

By evaluating at $x_{K-1} = 0$, we obtain that the agent will hedge a positive amount if

$$\pi U'[W(1+r)] (Ep_K^i - \pi/r - \pi - c + 1) > (1-\pi) U'[W(1+r)+1] (c + \pi + \pi/r - Ep_K^i),$$

because of the independence of dividends and trader arrivals. Furthermore since (as explained above) $Ep_K^i = \pi/r$, this condition becomes

$$\pi (1-\pi-c) U'[W(1+r)] > (1-\pi) (\pi+c) U'[W(1+r)+1]$$

as in case 1.

Case 3: There are two sub-cases. First, when the uninformed agent buys at time t, an informed agent may also submit an order. In this case, the purchase is made at the fully revealing price and next period the shares are sold at the fully revealing price. The second sub-case occurs when only the uninformed agent submits an order at time t. Then when he sells the share at time t+1 there are three possible prices.

We start by computing expected utility conditional on each of these sub-cases.

If good news is revealed at time t, the asset earns an expected return of r thereafter. The expected utility conditional on this sub-case is, as in case 1 above:

$$\pi\, U[W(1+r) + x_t - x_t(\pi + c)] + (1-\pi)\, U[W(1+r) + 1 - x_t(\pi + c)],$$

where we have used the fact that, since the return on the stock is r, $p_t(1+r) = p_{t+1} + \pi$. In this sub-case, x_t will therefore be the same as in periods before K−1.

If there is no other trade at time t (the other sub-case), then the uninformed agent buys at price p_t and sells at one of three prices at time t+1. As before we use p^i_{t+1}, i = 0, 1, 2, to denote the three prices corresponding to the trading volume multiples of 0, 1, or 2 of x_t. The corresponding probabilities are denoted Pr^i_{t+1}. The expected utility conditional on this sub-case is:

$$\pi\, \Sigma_i\, Pr^i_{t+1}\, U[W(1+r) + x_t(p^i_{t+1} - p_t(1+r) - c + 1)] + (1-\pi)\, \Sigma_i\, Pr^i_{t+1}\, U[W(1+r) + x_t(p^i_{t+1} - p_t(1+r) - c) + 1]$$

The first sub-case, where there is an informed agent submitting an order in addition to the uninformed, occurs with probability $\delta_t\beta_t$. The second sub-case, where there is no other agent submitting an order, therefore has probability $1 - \delta_t\beta_t$. The overall expected utility is therefore the expectation, using these two probabilities, of the above conditional expected utilities. x_t is defined to be the maximizer of this expected utility; note that the actual quantity x_t will be sample-path dependent because Pr^i_{t+1} and p^i_{t+1} depend on the marketmaker's beliefs (we give the formulae for these below).

The derivative with respect to x_t is:

$$\delta_t\beta_t \{\pi\, U'[W(1+r)+x_t-x_t(\pi+c)]\,(1-\pi-c) + (1-\pi)\, U'[W(1+r)+1-x_t(\pi+c)]\,(-\pi-c)\} + (1-\delta_t\beta_t)\,\{\pi\, \Sigma_i\, Pr^i_{t+1}\, U'[W(1+r)+x_t(p^i_{t+1}-p_t(1+r)-c+1)]\,(p^i_{t+1}-p_t(1+r)-c+1) + (1-\pi)\,\Sigma_i\, Pr^i_{t+1}\, U'[W(1+r)+x_t(p^i_{t+1}-p_t(1+r)-c)+1]\,(p^i_{t+1}-p_t(1+r)-c)\}.$$

We now derive the condition for x_t to be positive. Evaluated at $x_t = 0$, the derivative becomes:

$$\delta_t\beta_t \{\pi\, U'[W(1+r)]\,(1-\pi-c) + (1-\pi)\, U'[W(1+r)+1]\,(-\pi-c)\} + (1-\delta_t\beta_t)\,\{\pi\, U'[W(1+r)](E^{\#}p_{t+1}-p_t(1+r)-c+1) + (1-\pi)\, U'[W(1+r)+1]\,(E^{\#}p_{t+1}-p_t(1+r)-c)$$

$$= [(E^{\#}p_{t+1}-p_t(1+r))(1-\delta_t\beta_t) - c - \pi\delta_t\beta_t]\,[\pi\, U'[W(1+r)] + (1-\pi)\, U'[W(1+r)+1]] + \pi\, U'[W(1+r)],$$

where

$$E^{\#}p_{t+1} = \Sigma_i\, Pr^i_{t+1}\, p^i_{t+1}$$

denotes the expectation of p_{t+1}, conditional on the event that there was no informed agent acting at t. This expectation is analogous to $E^{*}p_{t+1}$ discussed above but here expectations are from the point of view of the uninformed trader.

Therefore, the uninformed trader will choose to hedge a positive amount if:

$$\pi\delta_t\beta_t - (1-\delta_t\beta_t)(E^{\#}p_{t+1} - p_t(1+r)) < \pi\, U'[W(1+r)/\,[(1-\pi)\, U'[W(1+r)+1] + \pi\, U'[W(1+r)]] - c.$$

We can show that this is stronger than the corresponding condition for dates prior to K (case 1):

$$\pi < \pi\, U'[W(1+r)/\, [(1-\pi)\, U'[W(1+r)+1] + \pi\, U'[W(1+r)]] - c,$$

since the expected return to the uninformed after date K is less than the market return, 1+r:

$$(E^{\#}p_{t+1} - p_t + \pi)/p_t < r,$$

or, equivalently:

$$E^{\#}p_{t+1} - p_t(1+r) < -\pi.$$

Thus,

$$\pi\delta_t\beta_t - (1 - \delta_t\beta_t)(E^{\#}p_{t+1} - p_t(1+r)) > \pi.$$

We can derive an explicit expression for the derivative by computing an expression for $E^{\#}p_{t+1} - p_t(1+r)$ in terms of $E^{\#}\beta_{t+1}$:

$$p_{t+1} = \pi/r + (1/(1+r))^{T-(t+1)}(\beta_{t+1} - \pi)$$

$$p_t(1+r) = \pi/r + (1/(1+r))^{T-(t+1)}(\beta_t - \pi)$$

So:

$$E^{\#}p_{t+1} - p_t(1+r) = (1/(1+r))^{T-(t+1)}(E^{\#}\beta_{t+1} - \beta_t).$$

We now compute $E^{\#}\beta_{t+1}$. At time t+1, there may be 0, 1, or 2 buy orders and corresponding values of β_{t+1}. The conditional probabilities of these events are:

Buy = 0: $\quad Pr^0_{t+1} = ½\{(1 - \delta_{t+1}) + \delta_{t+1}[1 - \beta_t(1 - \delta_t)/[(1 - \delta_t) + \delta_t(1 - \beta_t)]]\}$

$$= \tfrac{1}{2}[1 - \delta_{t+1}\beta_t(1 - \delta_t)/(1 - \delta_t\beta_t).$$

Buy = 1: $Pr^1_{t+1} = \tfrac{1}{2}(1 - \delta_{t+1}) + \tfrac{1}{2}\{\delta_{t+1}[1 - \beta_t(1 - \delta_t)/[1 - \delta_t) + \delta_t(1 - \beta_t)]]\} + \tfrac{1}{2}\delta_{t+1}\beta_t(1 - \delta_t)/[1 - \delta_t) + \delta_t(1 - \beta_t)]$

$$= \tfrac{1}{2}.$$

Buy = 2: $Pr^2_{t+1} = \tfrac{1}{2}\delta_{t+1}\beta_t(1 - \delta_t)/[1 - \delta_t) + \delta_t(1 - \beta_t)]$

$$= \tfrac{1}{2}\delta_{t+1}\beta_t(1 - \delta_t)/(1 - \delta_t\beta_t).$$

The marketmaker's beliefs in these three events are:

Buy = 0: $\beta_{t+1} = (1 - \delta_{t+1})\beta_t/(1 - \delta_{t+1}\beta_t).$

Buy = 1: $\beta_{t+1} = \beta_t.$

Buy = 2: $\beta_{t+1} = 1.$

So the expected belief from the uninformed trader's viewpoint is:

$$E^{\#}\beta_{t+1} = [(1 - \delta_{t+1})\beta_t/(1 - \delta_{t+1}\beta_t)] \times \tfrac{1}{2}[1 - \delta_{t+1}\beta_t(1 - \delta_t)/(1 - \delta_t\beta_t)$$

$$+ \beta_t \times \tfrac{1}{2} + 1 \times \tfrac{1}{2}[1 - \delta_{t+1}\beta_t(1 - \delta_t)/(1 - \delta_t\beta_t).$$

$$= \tfrac{1}{2}\beta_t[1 + (1 - \delta_{t+1})\beta_t/(1 - \delta_{t+1}\beta_t)]\,[1 + \delta_{t+1}(1 - \delta_t)/(1 - \delta_t\beta_t)].$$

Using this expression, $E^{\#}p_{t+1} - p_t(1+r)$ may be evaluated to verify the condition that uninformed agents choose to hedge a positive amount. We therefore require that the uninformed agents are sufficiently risk averse that this condition holds for all $t > K$ for all possible non-revealing price paths. Of course if the information has already been revealed prior to t, the prices are set to ensure the riskless return r and the hedging condition for the uninformed is ensured by the analysis in case 1.

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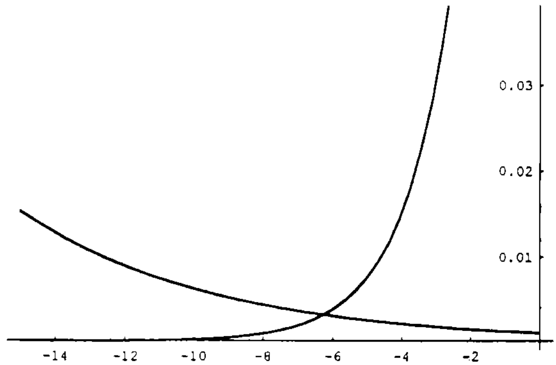
Figure 1: Determination of K



Time to T