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HETEROGENEITY,
STRATIFICATION, AND GROWTH

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ABSTRACT

We examine how economic stratification affects inequality and growth over time. We study economies where heterogenous agents interact through local public goods or externalities (school funding, neighborhood effects) and economy-wide linkages (complementary skills, knowledge spillovers). We compare growth and welfare when families are stratified into homogeneous local communities and when they remain integrated. Segregation tends to minimize the losses from a given amount of heterogeneity, but integration reduces heterogeneity faster. Society may thus face an intertemporal tradeoff: mixing leads to slower growth in the short run, but to higher output or even productivity growth in the long run. This tradeoff occurs in particular when comparing local and national funding of education, which correspond to special cases of segregation and integration. More generally, we identify the key parameters which determine which structure is more efficient over short and long horizons. Particularly important are the degrees of complementarity in local and in global interactions.

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1 Introduction

In the United States a family's income, assets, education level, ethnic background and lifestyle can be predicted quite accurately from its zip code -and this in spite of the great diversity of the American population.[1] This strong degree of social and economic segregation, epitomized of late by the spread of gated communities, is reflected in wide disparities in the funding and quality of local public services. such as primary and secondary education or law enforcement. It also manifests itself in the increasingly different types of behavior and values to which the young are exposed during their formative years.

The production of goods and services thus brings together on the factory floor and at the office workers on the one hand, managers and professionals on the other, whose upbringing and levels of human capital are becoming increasingly disparate. Could this polarization of educational opportunities and outcomes be a contributing factor not only to the widening inequality in income and wealth observed over the last decade, but also to the slowdown of productivity growth? One reads for instance in the MIT Commission's Report on Industrial Productivity:

> "American and foreign students differ not only in their average scores on standardized tests but also in the dispersion of those scores around the mean. The Japanese aim at bringing all students to a high common level of competence, and they are largely successful: as a result ...new entrants to the Japanese work force are generally literate, numerate. and prepared to learn. In the U.S. work force, employers have discovered high rates of illiteracy and difficulty with basic mathematics and reading in workers with high school diplomas ... Only a tiny fraction of young Americans are technologically literate and have some knowledge of foreign societies.
>
> (Made in America, (1989))

The U.S. workforce is of course much more heterogeneous than the Japanese. But this greater heterogeneity may be in good part endogenous, precisely because any exogenous differences in human capital (due to historical factors, immigration, or just plain luck) are magnified by economic or social stratification In

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[1] Weiss (1989) provides a comprehensive and lively description of the "clusters" used in commercial and political marketing.

fact, a more diverse population may increase the value of integration, as it accelerates the homogenization of the labor force. Motivated by these observations, this paper investigates the relationship between the extent to which an economy is stratified into homogeneous communities, its degree of income inequality, and its growth performance over time.

The central question is the following: given a heterogeneous population, which social structure is more efficient: segregation by income and education, or integration? It includes as a special case the issue of whether public education should be funded locally or nationally. We formalize these issues in a simple but quite general growth model with both economy-wide linkages (such as complementarity in production or knowledge spillovers) and local, community-level externalities or public goods (such as locally funded schools or neighborhood effects). We show that the answer to the question raised above depends on two basic effects, which can give rise to an interesting intertemporal tradeoff.

The first effect measures how efficient each social structure is at *processing heterogeneity*, i.e. at aggregating disparate levels of human capital into the production of goods and, ultimately, new knowledge. We show that when family background and community quality are complements in a child's education, a segregated economy tends to have smaller instantaneous losses, hence faster growth, for any given amount of heterogeneity. The second effect is dynamic: because an integrated society is better at *reducing heterogeneity*, it converges faster to a homogeneous outcome-or in the presence of shocks, converges to a less unequal distribution of skills and income. Integration thus delivers much of its payoff over the course of several generations. Is this effect important enough for mixing to be more efficient in the long run, even when it leads to losses in the short run?

We show that the answer tends to be affirmative, so that integration at first hurts the better off but eventually raises everyone's income. It is then Pareto improving, without need for redistribution, provided agents have a low enough discount rate. Conversely, increased segregation leads to: (i) a widening in the distribution of income; (ii) a short-lived burst of growth, benefiting mostly the better off households; (iii) a decline in the economy's long-run level of output, or even in the long-run growth rate. Remarkably, these results remain even as all global complementarities linking together rich and poor families become

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vanishingly small.

Of course, it need not be the case that mixing is preferable in the long run. For instance. we show that stratification remains more efficient if the *degree of complementarity* between agents' levels of human capital is much stronger in local interactions than in global interactions. Intuitively, this means that disparities in knowledge at the community level (e.g. in schooling) entail *sufficiently* greater losses than at the aggregate level (e.g. in production).

What this paper offers is thus a framework in which the costs and benefits of different degrees of stratification can be spelled out. It also helps to clarify some issues of aggregation which arise when externalities are combined with heterogeneity. In models with a representative agent, it is irrelevant how spillovers are specified. But as we move to models with heterogeneity, the choice of aggregator becomes crucial. Thinking about extreme cases, this may seem obvious; naturally the equilibrium will be different if the spillover is closer to the minimum, to some average, or to the maximum of all agent's actions. But we show that one cannot even identify -as is common practice- the mean of the logs and the log of the mean (the geometric and arithmetic averages) without significantly changing the economy's long run growth rate, as well as the normative conclusions about the efficiency of stratification and integration. One must therefore go beyond rough intuitions and pin down the key parameters which determine how heterogeneity affects growth. The aim is not only to clarify the properties implicitly embodied in the specifications of previous models, but also to provide a guide for future empirical work on spillovers and neighborhood or peer effects.[2] In addition to the overall degree of returns to scale and the relative weights of family. community and economy-wide inputs in the production of human capital, we bring to light the crucial role played by the three *elasticities of substitution* which operate within local interactions. within global interactions, and between all three inputs.

This paper draws on previous work by Bénabou (1991), Tamura (1991a,b) and Glomm and Ravikumar (1992). Bénabou (1991) demonstrated that in a general equilibrium context, agents' incentives to segregate

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[2] In a sense our effort is similar, in a macroeconomic and dynamic context, to Arnott and Rowse's (1987) study of the mapping between various forms of peer effects and optimal school structure.

themselves due to the presence of local externalities or public goods can have important effects on aggregate productivity and welfare, even with perfect capital markets. But the model was static, hence not suited to study growth or the idea that the merits of integration and segregation may look very different in the short and in the long run. Being a model with ex-ante identical agents, it could also not address the issue of inequality, whether due to initial conditions or ongoing shocks; nor could it capture the notion of homogenization over time.

Tamura (1991a) studies endogenous growth when heterogeneous agents are linked by an economy-wide human capital spillover, and shows two main results. First, because individual accumulation is subject to decreasing returns, heterogeneity slows down growth. Second, this effect is only temporary, as in the long run the economy converges to a homogeneous outcome. Tamura uses simulations to demonstrate the vanishing impact of heterogeneity. In this paper we provide analytical solutions for the economy's entire dynamic path, in a general class of models with both local and global spillovers. This allows us to raise the issue of how stratification affects growth, and to answer it by showing how the losses per unit of dispersion, the convergence speed and their interaction differ under segregation and integration.

Glomm and Ravikumar (1992) compare growth under private and public education. In a private system parents buy education for their own children and there are no spillovers. In a public system education is a nationally funded public good, generating again a global spillover. Glomm and Ravikumar show that a private system offers students better incentives to invest in human capital, and thus leads to a higher long-run growth rate. They also provide an example where heterogeneity can cause a public education economy to grow faster for a while; but they do not analyze why this is so. We make clear the role played by heterogeneity in each system's performance. Most importantly, we show that when children's ability or adults' human capital is subject to random shocks, public education may in fact lead to *faster* long-term growth. If there is even a very small amount of complementarity in the production sector, a move to public education can be Pareto improving, provided families' intergenerational discount rate is low enough. In any case, both private and nationally funded education systems dominate locally funded public education, which has neither the incentive properties of the former nor the homogenization properties of the latter.

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Finally, this paper is also closely related to Durlauf (1992) and S. Cooper (1992), through a shared concern about the effects of stratification in a dynamic, stochastic economy. Durlauf shows how community formation and local funding of education can generate path-dependence in lineage income, trapping some families in pockets of poverty while others enjoy growth. S. Cooper (1992) incorporates redistribution into his model, by allowing communities linked through production externalities to vote on cross-subsidies. Our main concern here is how stratification affects the growth performance and efficiency of the whole economy, with particular emphasis on the potential tradeoff between the short and the long run.

Section 2 presents a model of education and production which motivates and sets up the basic problem. It then shows how similar issues arise in several other models, and provides a general framework in which to study them. Section 3 examines how heterogeneity affects short and long run growth in a segregated and in an integrated economy. Section 4 shows how randomness in innate ability magnifies the long-run effects of stratification. Section 5 considers the model's implications for the efficiency of nationally funded public education, locally funded public education, and private education. Section 6 concludes. The functional forms used in the text are generalized in Appendix A; proofs are gathered in Appendix B.

2 Local and Global Interactions in Human Capital

2.1 A First Model: Education and Production

We first consider the most natural channels through which group-specific and economy-wide complementarities arise in the accumulation of human capital: local funding of education, and imperfect substitutability in production. There is a continuum of overlapping generation families $i \in \Omega$, of unit measure. During each period adults work, consume, and spend time rearing their single child. At time zero, the adult member of dynasty i faces the following problem:

$$\text{maximize} \quad U_0^i \equiv E_0\left[\sum_{t=0}^{\infty} \rho^t u(c_t^i)\right], \quad \text{subject to:}$$

$$(1) \qquad c_t^i = (1 - \tau_t^i)\, y_t^i$$

$$(2) \qquad y_t^i = \nu_t^i \cdot w_t^i$$

$$(3) \qquad h_{t+1}^i = \kappa \cdot \zeta_t^i \cdot \left((1 - \nu_t^i)\, h_t^i\right)^\delta (E_t^i)^{1-\delta}$$

and h_0^i given. There are no financial assets, only human capital. At time t, adult i has human capital h_t^i. She spends a fraction ν_t^i of her unit time endowment at work, earning the hourly wage w_t^i, and devotes the rest to helping her child learn. The term h_t^i in (3) could also be due in part to inherited ability; the unpredictable component of the child's innate ability is represented by the i.i.d. shock ζ_t^i. The other input in the production of human capital is a *local public good*, which is financed by taxing the labor income of local residents. Per capita expenditures are therefore:

$$(4) \qquad E_t^i = \tau_t^i \cdot Y_t^i \equiv \tau_t^i \cdot \int_0^\infty y\, d\, m_t^i(y)$$

where m_t^i is the distribution of income and Y_t^i its average, in the community Ω_t^i to which family i belongs at time t: city, suburban town, state, etc. The most obvious skill-enhancing public good is primary and secondary schooling. But law enforcement, libraries, etc., are also relevant.

To simplify the model, we shall take both the fraction of time ν_t^i spent working and the tax rate τ_t^i to be constant over time and independent of community composition. If agents have logarithmic preferences, their decisions and voting choices will lead, in equilibrium, to such invariant rules.[3] But these are really "ceteris paribus" assumptions by which we abstract from the issues explored in the political economy literature, in order to focus on some new effects .

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[3] This is shown in Section 5.2; the values of ν and τ are given in Proposition 6. Log-utility also leads to constant investment and tax rates in Tamura (1991a) and Glomm and Ravikumar (1992) respectively. Voting models of education with variable tax rates are analyzed by Perotti (1990), Saint-Paul and Verdier (1991) and Fernandez and Rogerson (1992), among others.

We now turn to the production sector. All workers take part in the production of a numeraire good, performing complementary tasks or specializing in imperfectly substitutable intermediate inputs. Thus total output is:

$$(5) \qquad Y_t = \nu \cdot \left(\int_0^\infty h^{\frac{\sigma-1}{\sigma}} \, d\mu_t(h) \right)^{\frac{\sigma}{\sigma-1}} \equiv \nu \cdot H_t, \qquad \sigma > 1$$

where μ_t denotes the distribution of human capital in the whole labor force Ω.[4] This complementarity is meant to capture the idea that poorly educated, insufficiently skilled production or clerical workers will drag down the productivity of engineers, managers, doctors, etc. Conversely, lagging advances in knowledge by scientists, engineers and managers will mean lagging wages for basic workers. Such interdependence seems quite plausible, especially as we allow σ to be arbitrarily large, but finite. Given (5), any worker's wage and labor income depend positively on the economy-wide level of human capital:

$$(6) \qquad y_t^i = \nu \cdot w_t^i = \nu \cdot (H_t)^{\frac{1}{\sigma}} (h_t^i)^{\frac{\sigma-1}{\sigma}},$$

and the same is true for any community's level of per capita income:

$$(7) \qquad Y_t^i = \int_0^\infty y \, d\, m_t^i(y) = \nu \cdot (H_t)^{\frac{1}{\sigma}} \cdot \left(\int_0^\infty h^{\frac{\sigma-1}{\sigma}} \, d\mu_t^i(h) \right) \equiv \nu \cdot (H_t)^{\frac{1}{\sigma}} \cdot (L_t^i)^{\frac{\sigma-1}{\sigma}}.$$

where μ_t^i is the distribution of human capital in community Ω_t^i. Note that $\sigma > 1$ is required for income to increase with the level of skills. Incorporating (4) and (7) into (3), the accumulation of human capital takes the form:

$$(8) \qquad h_{t+1}^i = \Theta \cdot \zeta_t^i \cdot (h_t^i)^\alpha (L_t^i)^\beta (H_t)^\gamma$$

where $\alpha = \delta$, $\beta = (1 - \delta)(\sigma - 1)/\sigma$, $\gamma = (1 - \delta)/\sigma$, with $\alpha + \beta + \gamma = 1$ and $\Theta = \kappa \cdot (1 - \nu)^\delta (\nu \tau)^{1-\delta}$. Equation (8) involves both a local linkage L_t^i, because public goods are funded by community income, and a

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[4] We develop in appendix a variant of Ethier's (1982) model of specialization which leads to (5) and (6) below; see the proof of Proposition 6. Tamura (1991b) offers a model leading to an aggregate production function closely related to (5). Kremer (1992) and R. Cooper (1992) study equilibrium and optimal task assignment in firms or production teams.

global linkage $H_t = Y_t/\nu$, because all workers are complementary in production. Both are CES aggregates, with the same elasticity of substitution. As workers become better substitutes, communities become less interdependent: β rises and γ falls.

This model allows us to ask the following questions. Is it more efficient, from the point of view of maximizing aggregate output and growth, to have the population stratify into homogeneous communities ($L_t^i = h_t^i$) or mix into identical, integrated communities ($L_t^i = H_t$)? Does the answer depend on whether one takes a short-run or long-run perspective? Can integration be Pareto improving even without compensating transfers to richer families? These issues can also be rephrased in a more directly policy-relevant manner. Suppose that households *are* in fact geographically segregated by education and income. Which system of public education funding is then more efficient: local funding, where each community's school budget reflects the income of its residents, or national funding, where expenditures uniformly reflect national income? Before answering these questions for the specific model developed above, we show that similar issues arise very naturally in a wide class of models from the growth and human capital literatures.

2.2 More Models and a Puzzle

There are many potential channels which can give rise to a model with local and economy-wide interactions like (8). First, aggregate income clearly matters if either production or education uses some nationally provided public good, such as defense or infrastructure. Second, technological spillovers a la Romer (1986)-Lucas (1988) may affect workers' productivity, leading to individual production functions of the type $y_t^i = \Theta \cdot (h_t^i)^a (H_t)^b$. As long as the accumulation of knowledge uses produced resources, in schooling or R&D, it will again be affected by H_t. Alternatively, Tamura (1991a) assumes that the aggregate level of knowledge directly affects the generation of new human capital: $h_{t+1}^i = \Theta \cdot (h_t^i)^a (H_t)^b$.

One would expect some knowledge spillovers to be confined to a smaller sphere than the whole economy, if only because geographical distance limits frequency of interaction. Indeed, sociologists have long described, and economists recently modelled, a variety of channels through which a community's human capital makeup affects the educational outcome of its young people. These sources of "social capital"

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(Loury (1977), Coleman (1990)) include: peer effects between students of different ability in the classroom or within the school (Banerjee and Besley (1991)); the fact that neighboring adults provide role models. good or bad, as well as networking contacts for the young (Wilson (1987), Streufert (1991) and Montgommery (1990)); and crime or other activities which interfere with education. There is also a fair amount of empirical evidence of such peer or neighborhood effects; see Bénabou (1991) for a brief review. In contrast to fiscal spillovers, pure neighborhood effects can generally not be remedied by simply improving access to capital markets or by redistributing income across communities.[5]

Of course any combination of the channels discussed here and in the preceding section is possible. even likely. But their common feature is the presence of a local aggregate L_t^i and perhaps an economy-wide aggregate H_t in the production function for new human capital. The question then arises again: is it more efficient —in the sense of increased output and in the Pareto sense— for society to stratify into homogeneous clusters, or for each community to reflect the nation-wide distribution of human capital?

Naturally, one expects the answer to depend on the form which interactions take. To show the surprising extent to which this is true, introduce the idea of local elasticity of substitution, and give some empirical flesh to the discussion of non-fiscal spillovers, let us consider the following example. Borjas (1992a) investigates whether human capital externalities operate within ethnic groups. Using longitudinal data, he estimates the model:

(9) $$\log(h_{t+1}^i) = \alpha \log(h_t^i) + \beta \log(L_t^i) + \text{control variables} + \eta_t^i.$$

where h_{t+1}^i is a son's level of human capital, measured as his hourly wage; h_t^i is his father's level of human capital; and L_t^i is "ethnic capital", defined as the geometric average of human capital levels among adults in the father's ethnic group: $\log(L_t^i) \equiv \int_0^\infty \log(h) \, d\mu_t^i(h)$. Finally, η_t^i is an unpredictable individual shock. Borjas estimates both α and β to be between .25 and 0.30 and statistically significant.[6]

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[5] Bénabou (1991) shows that they can lead to inefficient self-stratification even in a representative agent model with perfect capital markets.

[6] Borjas also uses years of education instead of log-wages. The remarks made below concerning the consequences of

These results are very interesting in and of themselves, adding to the body of evidence that group interactions influence the acquisition of skills. But they also raise the following question. From the (narrow) point of view of maximizing aggregate income, is it more efficient that ethnic groups mix, i.e. live together, study together, etc., or that they remain separate?[7] One might hope to use Borjas' estimates to answer this question. Unfortunately, we shall see that by using a geometric average, (9) constrains the answer *a priori*: for any values of α and β, the path of total labor income is always higher if each child is exposed to his own group's average human capital than if all are exposed to the population average. In the long-run the two paths converge to the same level if $\alpha + \beta < 1$.

This conclusion seems rather distressing. But suppose that instead of assuming that ethnic capital operates through the geometric average, one had used the arithmetic average: $\log(L_t^i) \equiv \log(\int_0^\infty h\, d\mu_t^i(h))$. As we show later on, capital accumulation will now be generally more efficient under mixing than under segregation, especially in the long run. In this instance, the common practice of not distinguishing between the mean of the logs and the log of the mean can be quite misleading: with heterogeneous agents, Jensen's inequality implies that both individual and the economy's growth rates depend on the aggregator through which the externality operates. This point is in fact independent of the stratification issue: if there is heterogeneity *within* each ethnic group, an equation like (9) will be misspecified unless the particular aggregator which it imposes happens to be the correct one.

These remarks demonstrate the importance of the *elasticity of substitution* among individual inputs into the production of a peer effect or neighborhood externality; the underlying intuition is developed below. It will therefore be quite important in empirical work to estimate this parameter, rather than constraining it to either one or infinity as is usually done.[8]

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within-group inequality and inter-group mixing are quite general, and apply to that specification as well.

[7] This question is not purely hypothetical; one suspects, and Borjas' (1992b) later work indeed tends to indicate, that "ethnic capital" really arises from neighborhood effects combined with ethnic segregation.

[8] One could specify L_t^i as a CES index and estimate its elasticity ϵ from a non-linear regression, or more simply include in (9) the group's variance $(\Delta_t^i)^2$ of log-human capital. Its coefficient will provide an estimate of $-1/\epsilon$; see Section 3.1

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2.3 A General Framework

Recognizing in the various examples discussed above a common underlying structure, we shall consider from here on the general model of knowledge accumulation:

$$(10) \qquad h_{t+1}^i = F(\zeta_t^i, h_t^i, L_t^i, H_t)$$

where ζ_t^i is a random shock, h_t^i is parental human capital, and L_t^i, H_t are respectively a local and an economy-wide index of human capital. In general, equation (10) is not a purely technological assumption, but a *reduced form* which already embodies a variety of market and non-market interactions: equilibrium wages, financing of local public goods, technological spillovers, peer effects in schooling, etc. Our earlier examples incorporated at most one local and one global link at a time, but we discuss below how multiple externalities can be reduced to (10), where L_t^i and H_t are appropriate composite indices.

The two levels of interaction in (10) open up the possibility of an *intertemporal tradeoff*. When heterogeneous families share the same school or community, or when some input into education is equalized, the rich lose and the poor gain. The net loss, positive or negative, represents the efficiency cost of local heterogeneity. It must be weighted against the value, positive or negative, of a more homogeneous workforce in the next generation. Intuitively, heterogeneity causes greater losses at the local and global level, the less substitutable are individual human capital inputs in L_t^i and H_t respectively. We therefore specify the external effects as (symmetric) CES averages, with potentially different elasticities of substitution:

$$(11) \qquad L_t^i = \left(\int_0^\infty h^{\frac{\iota-1}{\iota}} \, d\mu_t^i(h) \right)^{\frac{\iota}{\iota-1}}$$

$$(12) \qquad H_t = \left(\int_0^\infty h^{\frac{\tau-1}{\tau}} \, d\mu_t(h) \right)^{\frac{\tau}{\tau-1}}$$

While H_t is computed over the whole population, L_t^i only reflects the composition of the group Ω_t^i to which individual i belongs at time t. Depending on the context this can be a school, a community, a region, even a country.

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We will show that the *costs of heterogeneity* in L_t^i and H_t are indeed measured by $1/\epsilon$ and $1/\sigma$. As indicated on **Figure 1**, we allow them to take any values -even negative ones. In that case agents are substitutes rather than complements, and inequality is a source of gains. [9] As $1/\sigma$ decreases from $+\infty$ to $-\infty$, H_t spans the whole range from a Leontieff technology, $H_t = min\{h_t^i, i \in \Omega\}$, to a "frontier" technology, $H_t = max\{h_t^i, i \in \Omega\}$. The latter case corresponds to the model of Murphy, Shleifer and Vishny (1991), where the best innovation becomes embodied into the next generation of technology or know-how. Similarly at the local level, we allow all cases between peer effects of the type "one bad apple spoils the bunch" to role models where the best individual sets the standard. More generally, the accumulation of human capital may involve several interactions at each level, say:

(10')
$$h_{t+1}^i = F(\zeta_t^i; L_{1,t}^i, \dots L_{K,t}^i; H_{1,t}, \dots H_{N,t})$$

For instance, $H_{1,t}$ could arise from complementarity in the production of goods, which makes heterogeneity costly ($1/\sigma_1 > 0$) and is priced through wages, while $H_{2,t}$ could be associated to the generation of non-rival, non-excludable new ideas, where inequality is efficient ($1/\sigma_2 < 0$). But all local and global spillovers will matter only through two weighted averages: we show in appendix that (10') reduces to (10), where L_t^i and H_t are appropriately defined. Finally, another important specification is that of $F(\cdot)$, the production function for new human capital. The literature almost universally assumes the multiplicative form (8) (with either β or γ equal to zero), constraining the elasticity of substitution *between* h_t^i, L_t^i and H_t to equal one. To simplify the exposition, we retain a Cobb-Douglas specification for most of the paper. In appendix, we generalize $F(\cdot)$ to a CES aggregator, and show how the effects of heterogeneity and social structure also depend importantly on the extent to which parental, local and national inputs in a person's education are substitutes or complements.

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[9] For a CES index with constant returns such as (12), or just $H(x,y) = (\frac{1}{2} x^{\frac{\epsilon-1}{\epsilon}} + \frac{1}{2} y^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}}$, $1/\sigma$ simultaneously measures (in the neighborhood of $x = y$) the complementarity between inputs, the concavity of H and the cost of heterogeneity. The last property -which is relevant here- remains true with any return to scale (when the index is raised to some power γ, as in (8)) whereas the first two do not. Indeed, H_{12} is then proportional to $\gamma - 1 + 1/\sigma$, H_{11} and H_{22} to $\gamma - 1 - 1/\sigma$ and $det(H'') \equiv H_{11} H_{22} - (H_{12})^2$ to $\gamma^2(1 - \gamma)/\sigma$, but $\log((H(\frac{x+y}{2}, \frac{x+y}{2})/H(x,y))$ is simply proportional to γ/σ; see Section 3.1.

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$$L = \min\{h^i\}$$

$$\frac{1}{\varepsilon}$$

local complements,
global substitutes

local and
global complements

$$H = \max\{h^i\}$$

$$H = \min\{h^i\}$$

$$\frac{1}{\sigma}$$

local and
global substitutes

local substitutes,
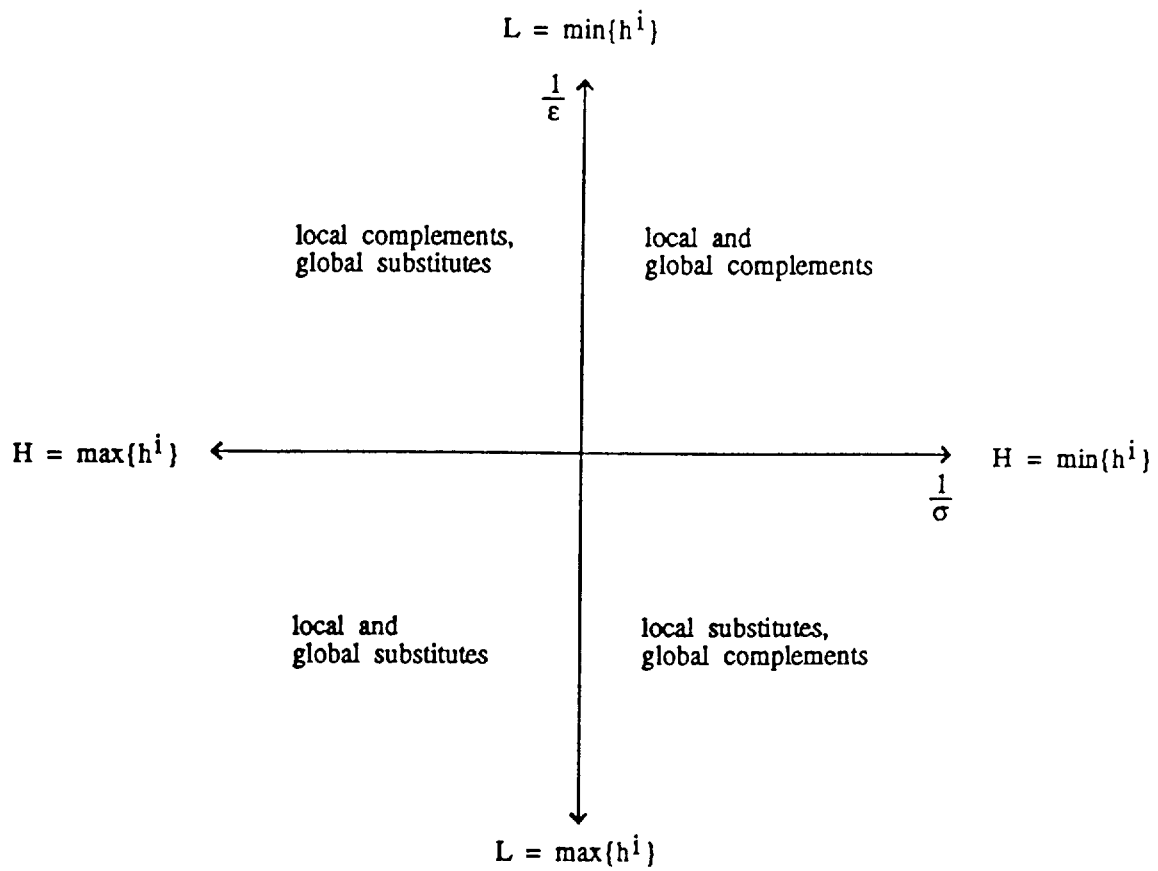global complements

$$L = \max\{h^i\}$$

Figure 1: The costs of heterogeneity: local and global
degrees of complementarity $1/\varepsilon$ and $1/\sigma$.

2.4 Community Composition

Given the general model described by (10)-(12), we shall compare the dynamics of human capital accumulation and welfare under two regimes of interest. The first is perfect stratification, where each type of agent lives in a separate, homogeneous community. The second is perfect integration, where each community's composition is the same as that of the population at large. Intermediate cases of partial segregation could easily be considered using the same methods; but we do not seek in this paper to offer a theory of endogenous community formation. This is for three reasons.

First, Bénabou (1991) already provides such a model, where a differential sensitivity to the quality of their environment leads high and low-skill workers to segregate as much as technology and institutions permit. The same basic force is at work here: with $\partial^2 F/\partial h\,\partial L > 0$, parental human capital h_t^i and local public goods or externalities L_t^i are strategic complements in the production of h_{t+1}^i. This tends to make more educated parents willing to outbid less educated ones for land or housing in a "better" community. One could thus obtain once again segregation as the only stable equilibrium, sustained by land rent differentials. Alternatively, one could follow Durlauf (1992) and allow each community's residents to vote on zoning or minimum income requirements. In the absence of significant fixed costs, the rich have no desire to let in the poor, so this would again lead to stratification. Implementing either approach, however, would require tying oneself to a specific choice of preferences and of the "technology" of segregation: price-elasticity of housing supply in each location, school district boundaries, feasibility of zoning, size of setup costs for a school or a community, mobility costs, etc.

The second reason is that the mixing and sorting regimes correspond to alternative policies: local or national funding of schools, tracking or busing, mixed income housing, etc. The last reason is that there are many sources of stratification which are unrelated to parents' concern for their children's education: differences in income, tastes, racial segregation, etc. We therefore choose to be agnostic about the causes of stratification and focus on the growth performances of two "pure" cases which deliver the main insights: complete segregation and complete integration.

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3 Stratification and Growth: the Short and the Long Run

Let us start with the simplest possible case. There are two types of agents, A and B, with measure 1/2 each. They differ only by their initial endowments of human capital: $\Delta \equiv \frac{1}{2} \log(h_0^A / h_0^B) > 0$. Thus Δ^2 is the initial variance of log-human capital.[10] We abstract from all uncertainty: $\zeta_t^i \equiv 1$.

3.1 Dynamics and Losses from Heterogeneity

In a stratified economy, the local environment compounds family differences:

$$(13) \qquad h_{t+1}^i = \Theta \cdot (h_t^i)^{\alpha + \beta} (H_t)^\gamma, \qquad i = A, B.$$

In an integrated economy, all agents share in the same level of local externality or public good. $L_t^A = L_t^B \equiv \hat{L}_t$. Denoting all variables in the integrated economy with a hat, we have:

$$(14) \qquad \hat{h}_{t+1}^i = \Theta \cdot (\hat{h}_t^i)^\alpha (\hat{L}_t)^\beta (\hat{H}_t)^\gamma, \qquad i = A, B.$$

Mixing agents at the local level thus has two effects. First, it decreases the return to scale on parental human capital from $\alpha + \beta$ to α, and correspondingly raises the return to scale on the local aggregate from 0 to β; the effect of H_t remains unchanged. These changes in the effective technology of human capital accumulation will alter the impact of any *given* amount of heterogeneity on the economy's growth rate, in a way which we make precise below. In other words, one of the two social structures will be more efficient at aggregating heterogeneous levels of knowledge than the other.

The second effect of mixing is to accelerate convergence to a homogeneous society. Denoting $\Delta_t =$

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[10] As L_t^i depends on group composition but not group size, any proportions $(a, 1 - a)$ of A and B families will lead to the same results, with $\Delta \equiv \sqrt{a(1 - a)} \cdot \log(h_0^A / h_0^B)$. More generally, the Taylor approximations used below apply to any distribution with small enough dispersion.

$\frac{1}{2} \cdot \log(h_t^A/h_t^B)$ in the segregated economy and $\dot{\Delta}_t = \frac{1}{2} \cdot \log(\dot{h}_t^A/\dot{h}_t^B)$ in the integrated one, we have

(15) $$\Delta_t = (\alpha + \beta)^t \Delta > \alpha^t \Delta = \dot{\Delta}_t.$$

Intuition suggests that these two effects may lead to an intertemporal tradeoff: mixing may for instance be less efficient for any given distribution of human capital, but still more efficient in the long run because it reduces heterogeneity faster.[11] This issue is investigated below; but first we must determine exactly how heterogeneity affects growth. Consider for instance the segregated economy. The distribution of human capital at time t is fully described by the degree of inequality Δ_t and any aggregate index of human capital. We focus on the per capita stock of knowledge $A_t \equiv (h_t^A + h_t^B)/2$, which brings out the role played by heterogeneity most clearly.[12] From (13) we get:

$$A_{t+1} = \Theta \cdot \left(\frac{(h_t^A)^{\alpha+\beta} + (h_t^B)^{\alpha+\beta}}{2} \right) \left(\frac{(h_t^A)^{\frac{\sigma-1}{\sigma}} + (h_t^B)^{\frac{\sigma-1}{\sigma}}}{2} \right)^{\frac{\gamma\sigma}{\sigma-1}}$$

In a representative agent economy where everyone had the average level of human capital, the right hand side would be $\Theta \cdot A_t^{\alpha+\beta} \cdot A_t^\gamma$ and the growth rate $\log(A_{t+1}/A_t) = \theta + (\alpha + \beta + \gamma - 1)\log(A_t)$, with $\theta \equiv \log(\Theta)$. When levels of knowledge are unequal, however, the two bracketed terms differ from $A_t^{\alpha+\beta}$ and A_t^γ due to Jensen's inequality. The differences represent the losses caused by heterogeneity when communities face decreasing returns and agents are complements in the production of the aggregate H_t. Conversely, heterogeneity is a source of gain if $\alpha + \beta > 1$ or $1/\sigma < 0$. We shall often focus the *exposition* on the first case, but it should be kept in mind throughout the paper that the model allows for any configuration of parameters: we do *not* impose that inequality be bad for growth.

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[11] In the general model (10) we define greater efficiency as increased total human capital. In specific models, i.e. given a production technology and preferences (e.g. Section 2.1) we shall also consider aggregate output and individual welfare.

[12] It will be clear how to go from this arithmetic average to any other aggregate index, such as H_t. A_t is a logical choice since it is unaffected by dispersion, and therefore not biased toward segregation or integration: as $\Delta_t > \dot{\Delta}_t$, $A_t = \dot{A}_t$ implies $H_t > \dot{H}_t$ for any $\sigma > 0$, and vice-versa for $\sigma < 0$.

We can simplify the expression for the growth rate under heterogeneity by using Taylor approximations. For any λ and x, y such that $\Delta \equiv \frac{1}{2}\log(x/y)$ is not too large, the loss function $\Psi_\lambda(\Delta) \equiv \log\left(\left(\frac{x+y}{2}\right)^\lambda \Big/ \left(\frac{x^\lambda + y^\lambda}{2}\right)\right)$ can be approximated as $\Psi_\lambda(\Delta) \approx \lambda(1-\lambda)\cdot\Delta^2/2$. Thus $\log(H_t/A_t) \approx -\Delta_t^2/2\sigma$ and:

$$(16) \qquad \log\left(\frac{A_{t+1}}{A_t}\right) \approx \theta + (\alpha + \beta + \gamma - 1)\log(A_t) - \left((\alpha + \beta)(1 - \alpha - \beta) + \frac{\gamma}{\sigma}\right)\frac{\Delta^2}{2} \cdot (\alpha + \beta)^{2t}$$

For the integrated economy, similar derivations lead to:

$$(17) \qquad \log\left(\frac{\hat{A}_{t+1}}{\hat{A}_t}\right) \approx \theta + (\alpha + \beta + \gamma - 1)\log(\hat{A}_t) - \left(\alpha(1 - \alpha) + \frac{\beta}{\epsilon} + \frac{\gamma}{\sigma}\right)\frac{\Delta^2}{2} \cdot \alpha^{2t}$$

We see that the drag on each economy's growth is the product of two factors. The first is the economy's efficiency loss *per unit* of dispersion, to be discussed below. The second is the current variance Δ_t^2 or $\dot{\Delta}_t^2$ of the human capital distribution.

3.2   The Short Run

We first ask which economy grows faster, for any *given* amount of heterogeneity. In other words, suppose that at time $t = 0$ previously segregated populations become integrated: will human capital at $t = 1$ be higher or lower? From (16), the efficiency loss in the segregated case is $\mathcal{L} \cdot \Delta^2/2$, with:

$$(18) \qquad \mathcal{L} \equiv (\alpha + \beta)(1 - \alpha - \beta) + \gamma/\sigma$$

The intuition is clear: losses reflect the concavity of the function $h^{\alpha+\beta}$ and the complementarity $1/\sigma$ of agents' inputs in the aggregate H, which has weight γ. In an integrated economy, the corresponding reduction in growth is $\hat{\mathcal{L}} \cdot \Delta^2/2$, where:

$$(19) \qquad \hat{\mathcal{L}} \equiv \alpha(1 - \alpha) + \beta/\epsilon + \gamma/\sigma,$$

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with a similar interpretation involving returns to scale at the family rather than the community level, and both local and global elasticities of substitution.

Proposition 1 *The mixed economy has higher growth in the short run, i.e. for any given amount of heterogeneity, if and only if $\mathcal{L} > \hat{\mathcal{L}}$, or:*

$$(20) \qquad \phi \equiv \beta(1 - 2\alpha - \beta - 1/\epsilon) > 0.$$

When $2\alpha + \beta < 1$, the education production function is less concave in the previous generation's human capital under integration than under segregation. Mixing will then accelerate growth even in the short run, provided there is enough local substitutability so that the poor do not drag $\hat{L}_t$ too far below the per capita endowment $\hat{A}_t$. Such is clearly the case if the local spillover operates through the arithmetic average ($\epsilon = \infty$), as in Glomm and Ravikumar (1992).On the other hand if $\hat{L}_t$ is a geometric average ($\epsilon = 1$) as in Borjas (1992a), the mixed economy is more vulnerable to heterogeneity than the segregated one. Finally when $2\alpha + \beta > 1$, mixing tends to reduce human capital accumulation in the short run.[13] This case is quite plausible since under constant returns, $\alpha + \beta + \gamma = 1$, it means that parental human capital is more important to a child's education than the economy-wide aggregate H_t: $\alpha > \gamma$.

The intuition for Proposition 1 is best understood by showing how ϕ embodies the effects at work in standard, static models of matching. By definition, mixing is inefficient if the losses of the rich exceed the gains of the poor, meaning that:

$$(21) \quad (A_1 - \tilde{A}_1)/(H_0^\gamma/2) = (h_0^A)^\alpha \cdot \left[(h_0^A)^\beta - (L_0)^\beta \right] - (h_0^B)^\alpha \cdot \left[(L_0)^\beta - (h_0^B)^\beta \right]$$

$$= \quad \left[(h_0^A)^\alpha - (h_0^B)^\alpha \right] \cdot \left[(h_0^A)^\alpha - A_0^\alpha \right] + (h_0^B)^\alpha \cdot \left[(h_0^A)^\beta + (h_0^B)^\beta - 2 A_0^\beta \right] + \left[(h_0^A)^\alpha + (h_0^B)^\alpha \right] \cdot \left[A_0^\beta - L_0^\beta \right]$$

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[13] The same is true for $\log(\tilde{H}_1/H_1) \approx \beta \cdot (\Delta^2/2) \cdot ((1 - 2\alpha - \beta)(1 - 1/\sigma) + 1/\sigma - 1/\epsilon)$, unless $1/\sigma - 1/\epsilon$ is sufficiently large.

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is positive. The first term arises from the complementarity of parental capital and local inputs ($F_{12} > 0$). It is positive since children from better backgrounds lose more from a given decline in L_0, such as from h_0^A to the per capita average A_0. For small dispersion this term is approximately equal to $2\alpha\beta \cdot A_0^{\alpha+\beta} \cdot \Delta^2/2$. The second term comes from the decreasing impact of marginal improvements in local conditions ($F_{22} < 0$). It is negative since an extra unit of human capital has a larger impact in a poor community than in a rich one. For small dispersion this term is close to $-\beta(1-\beta) \cdot A_0^{\alpha+\beta} \cdot \Delta^2/2$. The final term incorporates the pure losses from heterogeneity in generating the local spillover L_0 : if $1/\epsilon > 0$, poorly educated agents drag down L_0 more than well educated agents pull it up, so $L_0 < A_0$. For small dispersion this last term is approximately equal to $(\beta/\epsilon) \cdot A^{\alpha+\beta} \cdot \Delta^2/2$. The first and last effects tend to make sorting more efficient than mixing; the second one goes in the opposite direction. Summing all three yields a net impact of stratification proportional to $2\alpha\beta - \beta(1-\beta) + \beta/\epsilon = -\phi$.

3.3 The Long Run

Because mixing equalizes knowledge faster, the drag on growth due to dispersion eventually becomes smaller in the integrated than in the segregated economy, as shown by the last terms in (16)-(17). But what really matters is whether this effect is sufficient for $\hat{A}_t$ to make up its initial handicap and overtake A_t. To answer this question, we solve the difference equations (16) and (17). Denoting $R \equiv \alpha + \beta + \gamma$ and $C_t \equiv \theta \cdot (1 - R^t)/(1 - R)$, we have:

$$\log\left(\frac{A_t}{A_0 R^t}\right) \approx C_t - \mathcal{L} \cdot \frac{\Delta^2}{2} \sum_{k=0}^{t-1} R^{t-1-k}(\alpha + \beta)^{2k} = C_t - \mathcal{L} \cdot \frac{\Delta^2}{2} \cdot \frac{R^t - (\alpha + \beta)^{2t}}{R - (\alpha + \beta)^2}$$

$$\log\left(\frac{\hat{A}_t}{A_0 R^t}\right) \approx C_t - \mathcal{L} \cdot \frac{\Delta^2}{2} \sum_{k=0}^{t-1} R^{t-1-k}\alpha^{2k} = C_t - \hat{\mathcal{L}} \cdot \frac{\Delta^2}{2} \cdot \frac{R^t - \alpha^{2t}}{R - \alpha^2}.$$

Therefore:

Proposition 2 *For any $\gamma > 0$, the gap between the integrated and segregated economies is, for t large enough:*

(22)
$$\log\left(\frac{\dot{A}_t}{A_t}\right) \approx \Phi \cdot \frac{\Delta^2}{2}(\alpha + \beta + \gamma)^t$$

where:

(23)
$$\Phi \equiv \frac{(\alpha + \beta)(1 - \alpha - \beta) + \gamma/\sigma}{\alpha + \beta + \gamma - (\alpha + \beta)^2} - \frac{\alpha(1 - \alpha) + \beta/\epsilon + \gamma/\sigma}{\alpha + \beta + \gamma - \alpha^2}$$

In the long run, the gap shrinks to zero if total returns to scale $R \equiv \alpha + \beta + \gamma$ are less than one, tends to a finite limit if $R = 1$, and explodes if $R > 1$. Equation (23) embodies the main insights of the paper. The two numerators represent each economy's *instantaneous losses per unit of heterogeneity*. The two denominators reflect the different *speeds of convergence* to a homogeneous society. The tradeoff between incurring the costs of local heterogeneity and reducing the losses from aggregate heterogeneity at a faster rate is apparent in the fact that Φ is decreasing in β/ϵ and increasing in γ/σ. But an additional, less obvious factor is involved: the difference between the concavity of the technologies faced by a community and by a family, adjusted by the appropriate convergence speeds. This value of Φ for $1/\epsilon = 1/\sigma = 0$ is generally positive, as will be seen below.

We are now ready to answer the question: when is the long-run human capital stock larger under mixing than under segregation? Let us start with a useful *benchmark case*, assuming: (a) $R = 1$, constant returns; (b) $\epsilon = \sigma$: heterogeneity is equally costly or beneficial at the local and economy-wide levels, a "neutrality" assumption; (c) $1/\sigma < 1$: there is more substitutability *within* the composite inputs L_t and H_t than *between* the three inputs h_t^i, L_t, H_t.[14] Note that the model of Section 2.1 imposed all three restrictions; in particular, (c) was required for a worker's income to rise with her human capital. Assumption (c) also seems plausible for most alternative interpretations. Parental background, peer group quality and society's general level of knowledge or income are likely to be poorer substitutes in a child's education than workers' different skill levels in the production of output or know-how. In the benchmark

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[14] When F(h,L,H) is a CES function with elasticity λ, the relevant comparison is $1/\sigma < 1/\lambda$; see the appendix.

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case equation (23) becomes:

(24)
$$\Phi = \frac{\beta}{(1+\alpha)(1+\alpha+\beta)} \left(\frac{\sigma-1}{\sigma}\right) > 0,$$

so that integration raises the long-run level of human capital by $\Phi \cdot \Delta^2/2$.[15]

Figure 2 illustrates what happens when a previously integrated economy becomes stratified at time t_0, given $\phi < 0 < \Phi$ and $R = 1$. The common trend $\theta \cdot t$ is factored out from all variables, making them stationary. Initially, the richer A agents benefit, while the poorer B's lose. The distribution of income worsens, but the overall impact is favorable and growth accelerates. Over time, however, it slows down due to the fact that society remains more heterogeneous. Eventually, even the A's accumulation is reduced, and all dynasties' capital stocks converge to a common level which is lower than what it would have been had society remained integrated. To get a feel for orders of magnitude, let $\alpha = .5$, $\beta = .3$, $\gamma = .2$ and $\epsilon = \sigma = \infty$. Let $\Delta = 1$, or $h_0^A/h_0^B = 7.4$; this corresponds to a coefficient of variation of 0.76. The secession of the rich at first raises growth by 4.5%, but eventually lowers the steady-state path of the economy by 5.6 %. The initial boom is erased two generations later. Integrating a previously segregated society leads to the converse scenario, with an initial growth slowdown but higher steady-state output.

Of course, it need not be the case that mixing is preferable in the long run. We now examine more generally the relative performance of the two social structures, by varying key parameters. First, note that the steady-state gap (24) remains positive even when the degree of economy-wide linkage γ tends to zero. The impact of stratification on the economy's long-run performance is thus surprisingly different when rich and poor are completely independent from one another and when their fates are linked to an arbitrarily small extent.[16] We shall come back to this feature when evaluating each dynasty's welfare later on. Second, we show in appendix that decreasing total returns $R < 1$ raise the benefits of integration; the converse is true for $R > 1$. Third, and most importantly, segregation remains preferable in the long run if

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[15]Note that (24) allows for inequality to be beneficial, i.e. for $1/\sigma = 1/\epsilon < 0$. When all externalities operate through geometric averages (Tamura (1991a), Borjas (1992a)), (24) shows that mixing and segregation lead to the same steady-state.

[16]On the other hand as $\alpha + \beta = 1 - \gamma$ approaches one, the speed at which the segregated economy tends toward its lower asymptote approaches zero; see the expression for $\log(A_t/A_0 R^t)$ in appendix.
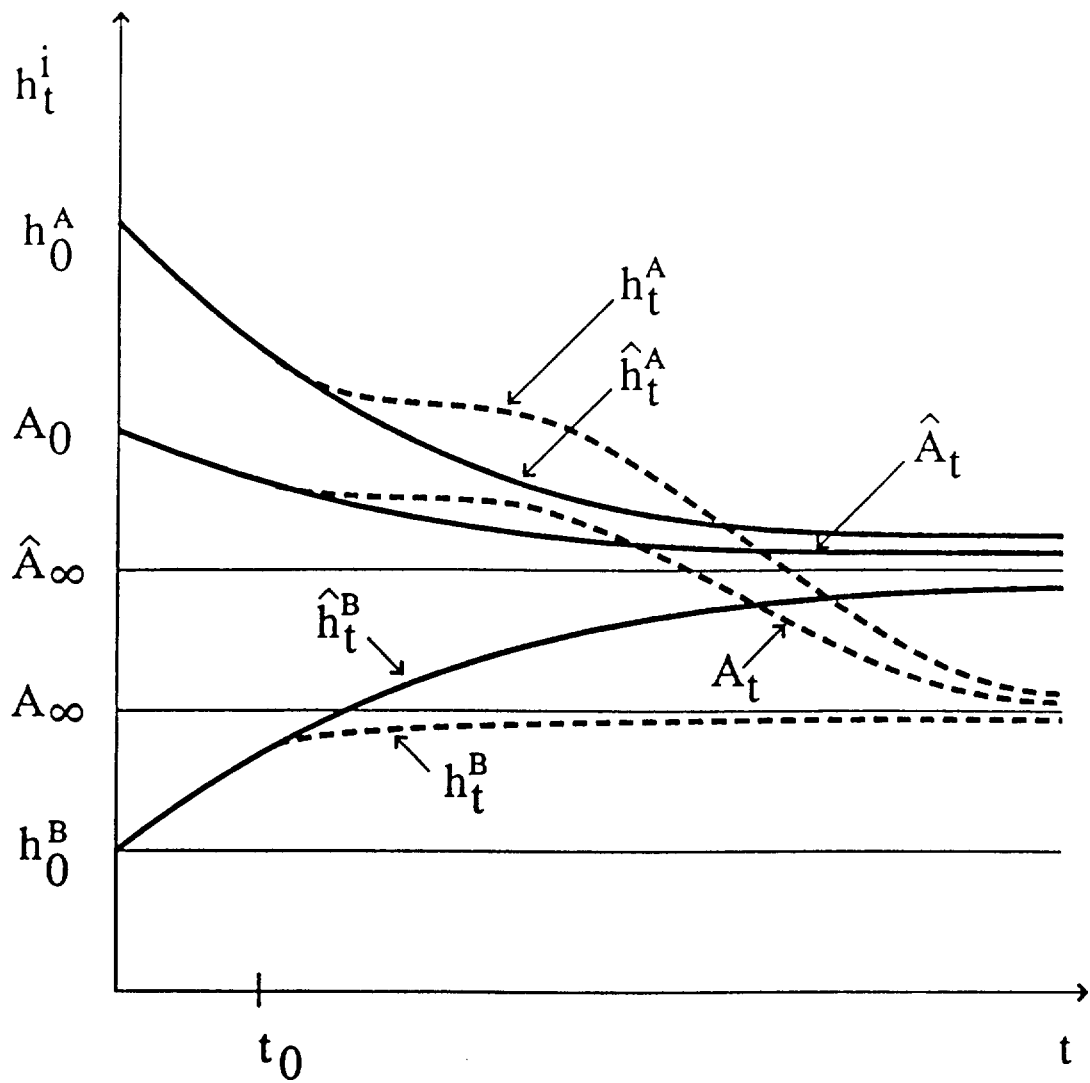
Figure 2: The short and long run effects of stratification

disparities in knowledge entail *sufficiently* greater losses at the community level, e.g. in schooling, than at the aggregate level, e.g. in production. Rearranging (23), with $R = 1$:

(25) $$\Phi \gtrless 0 \qquad \text{as} \qquad \frac{1}{\epsilon} - \frac{1}{\sigma} \lessgtr \left(1 - \frac{1}{\sigma}\right)\left(\frac{1-\alpha}{1+\alpha+\beta}\right)$$

The two regions, illustrated on **Figure 3**, are quite intuitive. Perhaps most noteworthy is the triangular area between the boundary and the two axes; since $1/\sigma < 0 < 1/\epsilon$, heterogeneity creates negative spillovers at the local level but positive ones at the aggregate level. Nonetheless, mixed communities lead to a superior long-run outcome due to the differential combination of returns to scale in accumulation and convergence speeds discussed earlier. In order to overcome this effect and make $\Phi < 0$, $1/\sigma$ must be sufficiently larger than $1/\epsilon$. For instance, it will be efficient for the managerial and working classes to live separately if it is much easier for good managers to make up for poorly qualified workers in the production of output, than for students from favorable backgrounds to offset the effect of weaker schoolmates in peer interactions. Similarly, it will be optimal to sort successive generations of Ph.D. students into departments of differential qualities when complementarities are stronger during graduate studies than they are during research careers.

3.4 Welfare and Pareto Optimality

It is easy to go from the aggregates A_t or $\hat{A}_t$ to each group's path of human capital. For instance, under segregation: $\log(h_t^A) = \log(A_t) + \log(2/(1 + e^{-2\Delta_t})) \approx \log(A_t) + \Delta_t - \Delta_t^2/2$. Given a specification of preferences and of the relationship between skills h_t^i and income y_t^i, for instance as in Section 2.1, we could compute present values of each family's utility or of any planner's social welfare function. But the main insights are clear even without doing so. First, if integration is more efficient even in the short run $(0 < \phi < \Phi)$, it will lead to a higher value in each period, not only of total human capital, but also of output and of any social welfare function which is a CES aggregate of individual human capital or consumption levels. In the more interesting case where $\phi < 0 < \Phi$, social welfare will be higher under mixing if the planner has a low enough discount rate. Moreover, if individual agents' discount rate ρ is high enough,
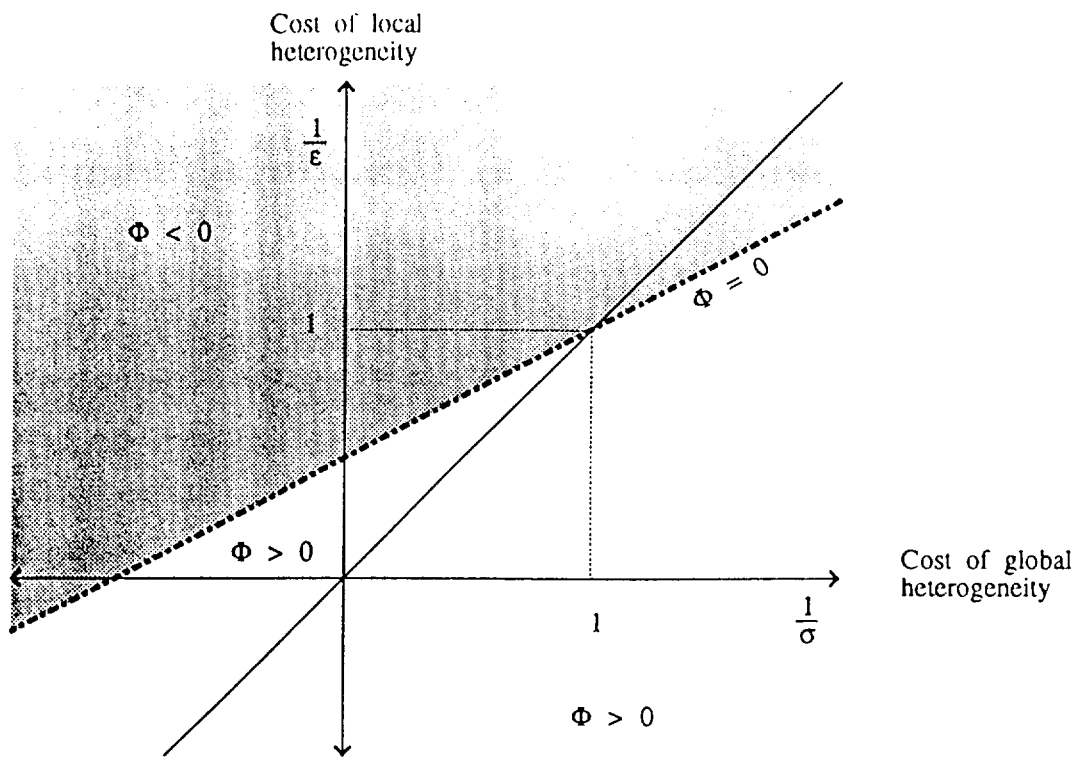
Figure 3: Stratification versus integration in the long run

integration will be *Pareto improving* even without any redistribution, because in the long-run all agents are identical and share the same level of human capital A_∞ or $\dot{A}_\infty$.

4 Random Ability, Stratification and Long Run Growth

The previous model allowed us to draw the essential distinction between the short and the long run effects of stratification, and to identify the main parameters which determine how heterogeneity affects growth. But it had two drawbacks. First, the long-run distribution of income was always degenerate –unless $a + \beta \geq 1$, but then $R > 1$ implies explosive growth. This is clearly contrary to the evidence. Second, the way in which the economy was stratified had no effect on the long run *growth rate*, except again in the case of increasing returns; see (22).

In this section we solve both problems by incorporating random shocks to children's ability or uncertain returns to education, as in Loury (1981). Such random draws of luck constitute a permanent source of inequality, but also of social *mobility*: the relative rankings of any two dynasties will no longer be preserved forever, but will change with positive probability. Because a mixed society "undoes" inequality faster, it will have a less dispersed asymptotic distribution of human capital and income than a segregated one. On the other hand, it may still be less efficient at processing any given amount of heterogeneity. Under constant returns, the balance of these two effects will determine which of the integrated or segregated economies has the higher long-run *growth rate*.

4.1 Dynamics

Let the accumulation of human capital be given by (8), where L_t^i and H_t are defined as in (11)-(12) and the shocks ζ_t^i are i.i.d. with $\log(\zeta_t^i) \sim \mathcal{N}(0, s^2)$. Assuming independent shocks involves little loss of generality, since intergenerational correlation of ability is already captured by the h_t^i term. We also take the initial distribution of human capital to be log-normal: $\log(h_0^i) \sim \mathcal{N}(m, \Delta^2)$. The advantage of this specification, which builds on Glomm and Ravikumar's (1992) deterministic model, is that h_t^i

remains log-normally distributed. This allows CES aggregates and loss functions to be computed exactly: if $\log(h_t^i) \sim \mathcal{N}(m_t, \Delta_t^2)$, then: $\Psi_\lambda(\Delta_t) \equiv \log\left(\left(\int_0^\infty h\, d\mu_t(h)\right)^\lambda / \int_0^\infty h^\lambda\, d\mu_t(h)\right) = \lambda(1-\lambda)\cdot\Delta_t^2/2$. Setting $\lambda = (\sigma-1)/\sigma$ yields:

$$(26) \qquad H_t = \left(\int_0^\infty h^{\frac{\sigma-1}{\sigma}}\, d\mu_t(h)\right)^{\frac{\sigma}{\sigma-1}} = exp\left(m_t + \left(\frac{\sigma-1}{\sigma}\right)\frac{\Delta_t^2}{2}\right) = A_t\cdot exp\left(-\frac{\Delta_t^2}{2\sigma}\right).$$

Now consider human capital at time $t+1$. Taking logarithms in (8) with $L_t^i = h_t^i$ (segregation):

$$(27) \qquad \log(h_{t+1}^i) = \theta + \log(\zeta_t^i) + (\alpha+\beta)\log(h_t^i) + \gamma(m_t + \frac{\sigma-1}{\sigma}\frac{\Delta_t^2}{2})$$

Human capital at time $t+1$ is therefore also log-normally distributed: $\log(h_{t+1}^i) \sim \mathcal{N}(m_{t+1}, \Delta_{t+1}^2)$. with:

$$(28) \qquad \begin{cases} m_{t+1} &= \theta + R\cdot m_t + \gamma\left(\frac{\sigma-1}{\sigma}\right)\frac{\Delta_t^2}{2} \\ \Delta_{t+1}^2 &= (\alpha+\beta)^2\Delta_t^2 + s^2 \end{cases}$$

Integration yields similar expressions, with $\alpha+\beta$ replaced by α and $\beta(\epsilon-1)/\epsilon$ added to $\gamma(\sigma-1)/\sigma$. The steady-state variance of human capital is then $\hat\Delta_\infty^2 \equiv \frac{s^2}{1-\alpha^2}$, which is lower than $\Delta_\infty^2 \equiv \frac{s^2}{1-(\alpha+\beta)^2}$. as expected. We could solve (28) for the mean of log-human capital m_t at any point in time. But in order to make the losses from heterogeneity appear most clearly, it is better to track once again the behavior of total human capital $A_t \equiv \int_0^\infty h\, d\mu_t(h)$. Using (26) to (28), we obtain the growth rates of the two economies:

$$(29) \qquad \log\left(\frac{A_{t+1}}{A_t}\right) = \theta + \frac{s^2}{2} + (R-1)\log(A_t) - \frac{\Delta_t^2}{2}\left((\alpha+\beta)(1-\alpha-\beta) + \frac{\gamma}{\sigma}\right)$$

$$(30) \qquad \log\left(\frac{\hat A_{t+1}}{\hat A_t}\right) = \theta + \frac{s^2}{2} + (R-1)\log(\hat A_t) - \frac{\hat\Delta_t^2}{2}\left(\alpha(1-\alpha) + \frac{\beta}{\epsilon} + \frac{\gamma}{\sigma}\right)$$

These expressions are identical to (16) and (17), up to a constant. In particular, we recognize the *loss factors* $\mathcal{L} \equiv (\alpha+\beta)(1-\alpha-\beta) + \gamma/\sigma$ and $\hat{\mathcal{L}} \equiv \alpha(1-\alpha) + \beta/\epsilon + \gamma/\sigma$ for each economy, multiplied by their respective variances. The comparison between the two social structures' efficiency at aggregating levels of

knowledge, i.e. between their capital stocks one period after starting from the same initial conditions, is therefore unchanged: $\log(\hat{A}_1/A_1) = \beta(1 - 2\alpha - \beta - 1/\epsilon) \cdot \Delta^2/2 = \phi \cdot \Delta^2/2$.

To examine the impact of stratification on long-term output and growth, we solve (28)-(30) for (Δ_t, A_t) and $(\hat{\Delta}_t, \hat{A}_t)$; see the appendix. We first consider the case where initial endowments are the only source of inequality.

Proposition 3 : *The effect of initial inequality. If $s^2 = 0$ then for any $\gamma > 0$ and t large enough:*

$$\log(\frac{\hat{A}_t}{A_t}) \approx \left(\frac{\mathcal{L}}{R - (\alpha + \beta)^2} - \frac{\hat{\mathcal{L}}}{R - \alpha^2} \right) \cdot \frac{\Delta^2}{2} \cdot R^t = \Phi \cdot \frac{\Delta^2}{2} \cdot R^t$$

This is exactly the same expression as in the two-group case, so all the results derived previously extend to this model.

Proposition 4 : *The effect of ongoing inequality. If $s^2 > 0$, then for large enough t:*

$$\log(\frac{\hat{A}_t}{A_t}) \approx \frac{1}{2} \left(\mathcal{L} \cdot \Delta^2_\infty - \hat{\mathcal{L}} \cdot \hat{\Delta}^2_\infty \right) \left(\frac{1 - R^t}{1 - R} \right) = \left(\frac{\mathcal{L}}{1 - (\alpha + \beta)^2} - \frac{\hat{\mathcal{L}}}{1 - \alpha^2} \right) \cdot \frac{s^2}{2} \cdot \left(\frac{1 - R^t}{1 - R} \right)$$

Under constant returns, the integrated economy's long run growth rate exceeds that of the segregated economy by $\Phi \cdot s^2/2$.

As seen earlier, $\Phi > 0$ unless the cost of local heterogeneity exceeds that of aggregate heterogeneity by a sufficient margin. Proposition 4 shows, quite intuitively, that recurrent inequality impacts the two economies one level higher than initial dispersion does. When $R = 1$, s^2 affects long-run growth rates in the same way as Δ^2 affects long-run levels. When $R < 1$, s^2 impacts long-run levels whereas the effect of Δ^2 vanishes asymptotically.

4.2 Discounting and Welfare

Let us now examine the welfare of individual families, asking in particular whether integration can bring about a Pareto improvement without redistribution. To derive simple, closed form expressions, we assume that agents have logarithmic utility and compute the expected present value of log-human capital for each family. Note that if labor income depends not only on own human capital but also on aggregate productivity, as in (6), this will understate the relative benefit of integration with respect to segregation.[17] Finally, we use the benchmark specification $1/\epsilon = 1/\sigma < 1$ and $R = 1$. It should be clear from Section 3.3 in which direction deviations from this case will pull the results.

Straightforward but tedious derivations allow us to derive family i's distribution of human capital at any time t, and ultimately its expected intertemporal welfare, conditional on its initial endowment. Taking differences between the mixed and stratified cases yields:

$$(31) \qquad \hat{U}_0^i - U_0^i \;\equiv\; E_0\left[\sum_{t=0}^{\infty}\rho^t \log(\hat{h}_t^i/h_t^i)\mid h_0^i\right] \;=\; \frac{\rho\beta(m - \log(h_0^i))}{(1 - \rho\alpha)(1 - \rho(\alpha + \beta))}$$
$$+ \;\left(\frac{\rho}{1 - \rho}\cdot\frac{\sigma - 1}{\sigma}\cdot\frac{\rho s^2 + (1 - \rho)\Delta^2}{2(1 - \rho)}\cdot\left(\frac{\beta + \gamma}{1 - \rho\alpha^2} - \frac{\gamma}{1 - \rho(\alpha + \beta)^2}\right)\right)$$

The first term reflects the impact of the initial endowment; ceteris paribus, dynasties which start above the mean lose from integration, while those which start below the mean, gain. The second term is always positive, reflecting the fact that ceteris do not remain paribus: integration raises the level (through Δ^2) and possibly the growth rate (through s^2) of the unconditional average of (log) human capital. Since this is the expectation of the asymptotic distribution facing each dynasty, it makes all better off to an extent which reflects their degree of patience or intergenerational altruism. As ρ tends to one, so does the fraction of dynasties made better off by integration. Since in practice the distribution of human wealth at any point in time has finite support, integration is for all practical purposes *Pareto improving* if agents are *sufficiently patient*. Note that this has nothing to do with any insurance effect: in (31), dynasty i's own

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[17] Assuming $1/\sigma > 0$; the bias is reversed when $1/\sigma < 0$.

shocks $\log(\zeta_t^i)$ are set to their expected value of zero under both integration and segregation.

Let us next examine the role played in this result by the global externality H_t^γ which ties together the accumulation paths of rich and poor (due for instance to complementarity in production, as in Section 2.1). Suppose that this link becomes very weak: γ tends to zero and, maintaining $R = 1$, $\alpha + \beta$ tends to 1. Recall from (24) that the asymptotic difference in growth rates $\Phi \cdot s^2/2$ or in levels $\Phi \cdot \Delta^2/2$ between the two economies remains positive in the limit; on the other hand, the speed $(\alpha + \beta)^t$ at which the stratified economy converges to its inferior trajectory slows down to zero. Equation (31) incorporates these two opposing effects into discounted present values; when γ goes to zero, it becomes:

$$(32) \qquad \hat{U}_0^i - U_0^i = \frac{\rho(1-\alpha)}{1-\rho} \left[\frac{m - \log(h_0^i)}{1-\rho\alpha} + \left(\frac{\sigma - 1}{\sigma} \right) \left(\frac{1-\alpha}{1-\rho\alpha^2} \right) \left(\frac{\rho s^2 + (1-\rho)\Delta^2}{2(1-\rho)} \right) \right]$$

In the limit, integration's long run effect on the unconditional mean of (log) human capital still contributes positively to each dynasty's net welfare. When initial endowments are the only source of inequality, $s^2 = 0$, this is only a level effect, so only a bounded fraction of the population has a positive net gain $\hat{U}_0^i - U_0^i$, for any value of the discount factor. Integration is not Pareto improving unless richer families receive compensating transfers. When $s^2 > 0$, however, mixing has a growth rate effect. This dominates any level effect from initial conditions, provided the discount factor is high enough. Thus we see that once again the fraction of net gainers becomes arbitrarily close to one as ρ tends to one. If agents are patient enough, integration makes almost all families better off, even with an arbitrarily small degree of complementarity between rich and poor.[18]

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[18] The formal statement corresponding to (31) is: $\forall h_0$, $\forall \gamma > 0$, $\exists \rho(h_0, \gamma)$ such that if $\rho \geq \rho(h_0, \gamma)$ all dynasties starting with $h_0^i \leq h_0$ are better off under integration. The stronger statement corresponding to (32) is: $\forall h_0$, $\exists \rho^*(h_0)$ such that: $\forall \gamma > 0$, if $\rho \geq \rho^*(h_0)$ then all dynasties starting with $h_0^i \leq h_0$ are better off under integration.

5 Applications of the General Model

5.1 Local versus National Funding of Public Education

What do the results of the general model imply for the relative efficiency of locally and nationally funded public schooling? Recall from the model of Section 2.1 that if δ and $1 - \delta$ are the weights of parental education and school expenditures in a child's human capital, local funding ($E_t^i = \tau \cdot Y_t^i$) is equivalent to income segregation in the general model (8), and national funding ($E_t^i = \tau \cdot Y_t$) equivalent to integration, with: $\epsilon = \sigma > 1$, $\alpha = \delta$, $\beta = (1 - \delta)(\sigma - 1)/\sigma$, $\gamma = (1 - \delta)/\sigma$ and $R = 1$. Therefore, by (20) and (24):

Proposition 5 *(1) National funding of education leads to slower human capital accumulation than local funding in the short run, since:*

$$\phi = -\delta(1 - \delta)\left(1 - \frac{1}{\sigma^2}\right) < 0.$$

The same is true for output growth if $\delta(1 + \sigma) > 1$.[19]

(2) National funding, however, is more efficient in the long run, since:

$$\Phi = \left(\frac{\sigma}{2\sigma + \delta - 1}\right)\left(\frac{1 - \delta}{1 + \delta}\right)\left(\frac{\sigma - 1}{\sigma}\right)^2 > 0$$

When initial endowments are the only source of inequality, national funding raises the long run levels of human capital A_t and output Y_t by $\Phi \cdot \Delta^2/2$. When children's ability or returns to education are uncertain, it raises the long run growth rate of human capital and output by $\Phi \cdot s^2/2$.

The higher is agents' intergenerational discount factor, the larger the proportion of them who would vote for a national rather than a local system; for a high enough ρ, there would be unanimity. Being derived from a very simple model, Proposition 5 should of course be taken with caution. In particular, one should keep in mind two important maintained assumptions.

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[19]Using (26) yields: $\log(\tilde{Y}_1/Y_1) = (1 - \delta(1 + \sigma)) \cdot (1 - \delta) \cdot (\sigma - 1)^2 \cdot (\Delta^2/2\sigma^2)$.

First, tax rates are kept constant across the two regimes. In practice, richer families may respond to the redistribution inherent in national funding by voting for lower taxes; on the other hand, agents are more likely to internalize economy-wide spillovers when voting on a national education budget rather than that of their school district. Richer families could also withdraw their children from the public school system, thus preserving or restoring stratification. Variations in tax rates or a switch to private education will essentially be reflected in different values of the constant factor Θ. This is examined in the next section.

Second, the underlying source of inefficiency in local funding is the absence of a capital market where poor communities can borrow from richer ones to finance schools. National equalization of expenditures amounts to a partial and gradual redistribution of human capital, with some payback to the rich in the form of a higher H_t. Given decreasing returns to dynastic accumulation $(\alpha + \beta = 1 - (1 - \delta)/\sigma)$, it is intuitive that it should increase aggregate efficiency. So what is perhaps most surprising is that this is only true in the long run: early on, output and human capital accumulation may actually be reduced, as rich families lose more than poor ones gain. Also unexpected is the result that the steady-state gain from a national scheme remains finite even when dynasties face returns arbitrarily close to one: as workers become almost perfect substitutes, $lim_{\sigma \to \infty}(\Phi) = \frac{1}{2} \cdot (1 - \delta)/(1 + \delta) > 0$.

5.2   Public versus Private Education

Let us now consider privately purchased education, along the lines of Glomm and Ravikumar (1992). In the model of Section 2.1, we simply replace equation (4) by:

(4')
$$e_t^i = \tau_t^i \cdot y_t^i$$

where τ_t^i now represents the fraction of her income which adult i spends on her child's education, and the corresponding input e_t^i takes the place of the per capita school budget E_t^i in the accumulation equation (3). We again keep the preferences side of the model as simple as possible by assuming logarithmic utility. We also assume log-normally distributed initial conditions and shocks, as in Section 4, or a small enough

dispersion that the Taylor approximations of Section 3 are legitimate. With these conditions, we prove in appendix:

Proposition 6 *Under private education, the fractions $1 - \nu$ and τ of their time and income which adults devote to their children are given by:*

$$\nu \equiv 1 - \rho\delta, \qquad\qquad \tau \equiv \frac{\rho(1 - \delta)}{1 - \rho\delta} \cdot \left(\frac{\sigma - 1}{\sigma}\right)$$

Under national public education, adults' time allocation is unchanged. The tax rate unanimously preferred by voters is τ^* or $\bar{\tau}$, depending on whether or not they internalize the complementarity of human capital levels in output $Y_t = \nu \cdot H_t$:

$$\bar{\tau} \equiv \frac{\rho(1 - \delta)}{1 - \rho\delta + \rho(1 - \delta)(\sigma - 1)/\sigma} \cdot \left(\frac{\sigma - 1}{\sigma}\right) \;<\; \tau \;<\; \frac{\rho(1 - \delta)}{1 - \rho\delta} \equiv \tau^*.$$

The reason why $\bar{\tau} < \tau$ is that the *private* marginal value of human capital, hence also the return on savings, is higher under private than under public education. This is because in the first case, an extra unit of human capital enables the adult to not only consume more, but also to buy more education for her offspring. This is very similar to the effect identified by Glomm and Ravikumar (1992), who allow the young a choice between leisure and effort at studying, and show that they work harder when education is privately purchased. In our model, time is allocated not between studying and leisure but between production and at-home education, both of which allow adults to pass on more instruction to their offspring. The difference in marginal values of human capital therefore shows up not in different values of ν, but in different preferred savings rates. More importantly, this implies that private education *need not* lead to higher investment in human capital. Whether the accumulation factor Θ under private education is higher or lower than its counterpart $\hat{\Theta}$ in a publicly funded system depends on whether $\hat{\tau} = \bar{\tau}$ or $\hat{\tau} = \tau^*$.

In principle, voters should realize that a marginal increase in any agent's human capital allows an increase not only in her own consumption (as in the case leading to $\bar{\tau}$), or even in that of all her dynasty

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(as under private education, leading to r), but in that of all dynasties. If they do, they will base their vote on the social rather than the private return to educational investment, leading to $r^* > r$. On the other hand, private underinvestment in education could also be addressed by uniformly taxing consumption and subsidizing education. This would be equivalent to raising r without going to publicly, i.e. uniformly funded education. For this reason, but also to highlight the effects of heterogeneity, the case $\Theta > \dot{\Theta}$ might still be considered the most likely one, as we turn to comparing growth rates. Proposition 6 shows that private education is equivalent to locally funded public education in a segregated economy ($L_t^i = h_t^i$). except for the value of Θ. Therefore we have, with $\phi < 0$ and $\Phi > 0$ still given by Proposition 5:

Proposition 7 *Let Θ and $\hat{\Theta}$ denote the trend factors reflecting the differential incentive effects of private and national public education. Public education is less efficient in the short run if $\Theta - \dot{\Theta} > \phi \cdot \Delta^2/2$, but leads to faster growth in human capital and output in the long run if $\Phi \cdot (s^2/2) > \Theta - \hat{\Theta}$.*

This result formalizes some of the main arguments in the debate about public versus private education or related voucher schemes, at least where efficiency is concerned. The key issue is the relative importance of *incentive* and *stratification* effects, and perhaps also the relevant time horizon. Our results also suggest that in the long run, both private and national systems dominate locally funded public education, which has neither the incentive properties of the former nor the homogenization properties of the latter. Of course these conclusions are based on a simple, stylized model and should be taken with caution. But the model is indicative of the major forces at play in each case.

It is also interesting to relate our results to those of Glomm and Ravikumar (1992), whose work we have been building on. Their model leads to the reduced form $h_{t+1}^i = \Theta \cdot (h_t^i)^{\alpha+\beta}$ under private education. and $\hat{h}_{t+1}^i = \hat{\Theta} \cdot (h_t^i)^\alpha (\hat{A}_t)^\beta$ under public education. This is another special case of our general model. with $\gamma = 0$ (no global interaction), $\epsilon = \infty$ (perfect substitutability) and $s^2 = 0$ (no shocks). Glomm and Ravikumar observe that if there is enough inequality, the economy can *temporarily* experience faster growth under public education, provided $2\alpha + \beta < 1$; but they do not explain why. The analysis of the short run effects of segregation in (22) provides the answer: when $2\alpha\beta < \beta(1-\beta)$, the complementarity between

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parental background and school quality is dominated by the decreasing returns to quality. This makes the accumulation equation under public schooling (integration) less concave in parents' human capital than its counterpart under private schooling (segregation). Turning to the long run, the asymptotic growth rate in Glomm and Ravikumar's model reflects incentives only, as the effects of initial inequality Δ^2 vanish over time (for $\alpha + \beta \leq 1$); it therefore always higher under private education. Proposition 7 shows that the situation is quite different when one allows for ongoing sources of heterogeneity such as random of ability or parental altruism, unpredictable obsolescence of specialized skills etc., and for some degree of complementarity in production, no matter how small.

5.3 Immigration

For many countries, the immigration of workers and their families with lower levels of education than the resident population constitutes another periodic or ongoing source of heterogeneity. The unification of East and West Germany provides another example. What the results of this paper tend to show in this context is the following. The economy's long run performance is likely to be superior, benefiting all family lines, if immigrants and their descendants are integrated rapidly, meaning that they share neighborhoods, schools and other public goods with the richer local population, than if they remain isolated in homogeneous "ghettos".[20] However, integration may well have a negative impact on the first few generations of established residents. Their individual incentives will always push society toward segregation. Whether they will collectively (through the political process) recognize and seize the long-run benefits of integration will depend on how they discount the welfare of future generations.

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[20] See Burda and Wyplosz (1991) for a model of migration flows with human capital spillovers. One possible motive for the richer country to accept immigration or unification may be the standard gain from increasing specialization achievable with a larger population; see Tamura (1991b).

6 Conclusion

The model developed in this paper is extremely simple. The acquisition of human capital reflects family, community and economy-wide effects. The accumulation equation is general enough to encompass the reduced forms of most previous models. The degrees of complementarity or substitutability in local and economy-wide interactions capture the direct costs or benefits of heterogeneity at each level.

In spite of its simplicity, this framework allowed us to study several important issues and derive many results. We examined how economic stratification affects growth and welfare, and showed in particular that integration may slow down growth in the short run but promote it in the long run. We also compared the performance of locally and nationally funded public education systems, which exhibited the same intertemporal tradeoff. Introducing private education lead to an additional tradeoff between incentive and stratification effects.

The model could be extended in a number of directions. For instance, it would be interesting to refine the production and labor market side of the model, by distinguishing occupations which play asymmetric roles in the production process, such as managers and workers. This would introduce an optimal degree of human capital inequality in the labor force, which could then be related to those generated by stratification, integration and alternative education systems.[21] The interplay between local complementarities, clustering and global interactions also seems like a promising direction for future research. In Bénabou (1991) this idea allowed us to study the social structure and productivity of cities. In this paper we extended it to a dynamic framework with heterogeneous agents, and to the study of education funding. It should be applicable to a variety of other problems.

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[21] Recall that under symmetry, the optimal degre of inequality is either zero, when agents are complements (the more heterogeneity, the lower is H_t), or infinity, when they are substitutes (the more heterogeneity, the higher is H_t).

Appendix A: More General Education Technologies

A.1 Complementarity Within and Between Inputs

The Cobb Douglas specification (8) is convenient but has no special theoretical or empirical justification. Moreover, it allows stratification to have long-run effects only when $1/\sigma$ or $1/\epsilon$ differ from one. Intuitively, what should matter are the relative values of the elasticities of substitution operating *within* the aggregates L_t^i and H_t, and *between* the three inputs entering into $h_{t+1}^i = \zeta_t^i \cdot F(h_t^i, L_t^i, H_t)$. We show here that such is the case. Let:

$$(A.1) \qquad h_{t+1}^i = \Theta \cdot \zeta_t^i \cdot \left[\alpha'(h_t^i)^{\frac{\lambda-1}{\lambda}} + \beta'(L_t^i)^{\frac{\lambda-1}{\lambda}} + \gamma'(H_t)^{\frac{\lambda-1}{\lambda}} \right]^{\frac{R\lambda}{\lambda-1}}$$

where the weights α', β', and γ' sum to one, $R \geq 0$ is any degree of returns to scale and λ can take any sign. When λ tends to one we obtain (8) in the limit, with $\alpha \equiv R\alpha'$, $\beta \equiv R\beta'$ and $\gamma \equiv R\gamma'$. Using Taylor approximations similar to those of Section 3, one can show that the loss factors for the segregated and integrated economies are:

$$(A.2) \qquad \mathcal{L} \equiv R\left(\frac{(\alpha'+\beta')(1-\alpha'-\beta')}{\lambda} + (\alpha'+\beta')^2(1-R) + \frac{\gamma'}{\sigma} \right)$$

$$(A.3) \qquad \hat{\mathcal{L}} \equiv R\left(\frac{\alpha'(1-\alpha')}{\lambda} + \alpha'^2(1-R) + \frac{\beta'}{\epsilon} + \frac{\gamma'}{\sigma} \right)$$

For instance in $\hat{\mathcal{L}}$, heterogeneity is costly because of: (a) concavity of a child's human capital in that of her parent, due to $\alpha' < 1$ and to the complementarity of the three inputs into her education, $1/\lambda > 0$; if h, L and H are substitutes, on the other hand, heterogeneity is beneficial; (b) decreasing total returns, which make $F(\cdot)$ more concave; $R > 1$ has the opposite effect; (c) complementarity within L_t; (d) complementarity within H_t.

In the *short run*, mixing raises the growth rate if $\phi > 0$, where:

$$(A.4) \qquad \phi \equiv \mathcal{L} - \hat{\mathcal{L}} = R\beta'\left(\frac{1-2\alpha'-\beta'}{\lambda} + (2\alpha'+\beta')(1-R) - \frac{\beta'}{\epsilon} \right)$$

The interpretation is similar to that of (20), to which (A.4) reduces when $\lambda = 1$. Equation (A.4) also shows that decreasing total returns make mixing more efficient. We now turn to the long run. To a second-order approximation, the law of motion (28) for Δ_t^2 remains unchanged, and similarly for $\hat{\Delta}_t^2$. Thus:

Proposition 8 *The gap between the integrated and segregated economies is given by the same expressions as in Propositions 2, 3 and 4, with $\mathcal{L}$ and $\hat{\mathcal{L}}$ now given by (A.2) and (A.3), and:*

$$(A.5) \qquad \Phi \equiv \frac{\frac{(\alpha+\beta)(1-\alpha-\beta)}{\lambda} + (\alpha+\beta)^2 \left(\frac{\lambda-1}{\lambda}\right)\left(\frac{1-R}{R}\right) + \frac{1}{\sigma}}{\alpha + \beta + \gamma - (\alpha+\beta)^2} - \frac{\frac{\alpha(1-\alpha)}{\lambda} + \alpha^2 \left(\frac{\lambda-1}{\lambda}\right)\left(\frac{1-R}{R}\right) + \frac{\beta}{\epsilon} + \frac{1}{\sigma}}{\alpha + \beta + \gamma - \alpha^2}$$

We see that $\Phi = 0$ when $\lambda = \epsilon = \sigma$ and either $\lambda = 1$ or $R = 1$. This is the only case where the manner in which agents are partitioned has no steady-state effect. Intuitively, aggregation then causes similar losses whether it occurs at the level of a child's education (within $F(h, L, H)$), of a community (within L) or of the whole economy (within H). We can also rewrite Φ as:

$$(A.6) \quad \Phi = \frac{\beta}{R - \alpha^2}\left[\frac{1}{\sigma} - \frac{1}{\epsilon} + \left(\frac{1}{\lambda} - \frac{1}{\sigma}\right) \cdot \frac{\gamma(1-\alpha) + (\alpha+\beta)(1-R)}{R - (\alpha+\beta)^2} + \left(1 - \frac{1}{\lambda}\right) \cdot \frac{(1-R)(2\alpha+\beta)}{R - (\alpha+\beta)^2}\right]$$

As we saw in the case $\lambda = 1$, greater complementarity at the aggregate than at the local level ($1/\sigma > 1/\epsilon$) tends to make integration superior in the long run. We now see that greater complementarity (i.e. a higher cost of disparity) between (h_t^i, L_t^i, H_t) than within L_t or H_t ($1/\lambda > 1/\sigma$) has a similar effect. In particular. this is why $\Phi > 0$ when $\lambda = 1$ and $1/\epsilon = 1/\sigma = 0$, as shown on **Figure 3**. Finally, non-increasing returns to scale, $R \le 1$ also tend to make integration beneficial, except when $1/\lambda$ is sufficiently greater than one.

A.2   Multiple Spillovers

Denote by ϵ_k and σ_n respectively the elasticities of substitution of the $L_{k,t}^i$'s and $H_{n,t}$'s entering (10'), and by β_k, γ_n their weights (partial elasticities) in F. Then, whether $F(\cdot)$ is Cobb-Douglas as in Section 3 or CES as above, equations (18)-(19) or (A.2)-(A.3) show that (10') is equivalent to (10) with $\beta \equiv$

$\sum_{k=1}^{K} \beta_k$, $\gamma \equiv \sum_{n=1}^{N} \gamma_n$ and:

$$(A.7) \qquad \beta/\epsilon \equiv \sum_{k=1}^{K} \beta_k/\epsilon_k, \qquad\qquad \gamma/\sigma \equiv \sum_{n=1}^{N} \gamma_n/\sigma_n.$$

Appendix B: Proofs

Proof of Propositions (3) and (4).

Solving (28) to (29) leads to: $\Delta_t^2 = \Delta_\infty^2 + (\alpha + \beta)^{2t}(\Delta^2 - \Delta_\infty^2)$, $\dot{\Delta}_t^2 = \dot{\Delta}_\infty^2 + \alpha^{2t}(\Delta^2 - \dot{\Delta}_\infty^2)$ and:

$$(B.1) \quad \log\left(\frac{A_t}{A_0 R^t}\right) = \left(\theta + \frac{s^2}{2} - \mathcal{L} \cdot \frac{\Delta_\infty^2}{2}\right)\left(\frac{1 - R^t}{1 - R}\right) - \frac{\mathcal{L}}{2} \cdot (\Delta^2 - \Delta_\infty^2)\left(\frac{R^t - (\alpha + \beta)^{2t}}{R - (\alpha + \beta)^2}\right)$$

$$(B.2) \quad \log\left(\frac{\dot{A}_t}{A_0 R^t}\right) = \left(\theta + \frac{s^2}{2} - \dot{\mathcal{L}} \cdot \frac{\dot{\Delta}_\infty^2}{2}\right)\left(\frac{1 - R^t}{1 - R}\right) - \frac{\dot{\mathcal{L}}}{2} \cdot \left(\Delta^2 - \dot{\Delta}_\infty^2\right)\left(\frac{R^t - \alpha^{2t}}{R - \alpha^2}\right)$$

hence the results as $t \to \infty$, for any $\gamma > 0$.

Proof of Proposition (6).

1. Technology. We first fill in a few details with respect to the production sector of Section 2, so as to compute labor income for any choice of hours worked ν_t^i. We drop time subscripts for simplicity. Output is produced by competitive firms with constant returns, according to the technology:

$$(B.3) \qquad Y_t = \left(\int_0^1 (x_s)^{\frac{\epsilon-1}{\epsilon}} ds\right)^{\frac{\epsilon}{\epsilon-1}}$$

where x_s is the quantity of intermediate input s. The output sector's inverse demand curve for s is then $p_s = (x_s/Y)^{-\frac{1}{\epsilon}}$, yielding revenue $y_s = p_s \cdot x_s = (x_s)^{\frac{\epsilon-1}{\epsilon}} \cdot (Y)^{\frac{1}{\epsilon}}$. We assume that workers must specialize in a single input; each then chooses a different one, and we can replace the index s by $i \in \Omega$. An agent with human capital h^i working $\nu^i \le 1$ hours produces $x^i = \nu^i h^i$ units, and earns $y^i = (\nu^i h^i)^{\frac{\epsilon-1}{\epsilon}} \cdot (Y)^{\frac{1}{\epsilon}}$. This is her *net* income, as production entails only the opportunity cost of time (e.g. the value of leisure, or, in our case, parenting). Labor income also takes the form $y_t^i = \nu^i w^i$, where $w^i = p^i h^i = \partial Y/\partial \nu^i$ can be called the hourly wage; hence (2). When $\nu_t^i = \nu$ for all $i \in \Omega$, we obtain equations (5) (given $N = 1$

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agents in Ω; otherwise, Y should be multiplied by $N^{\frac{\tau}{\tau-1}}$), and (6).

1. Private education. To simplify the exposition, we first assume that all agents except i choose invariant participation and savings rates: $\nu_t^j = \bar{\nu}$ and $\tau_t^j = \bar{\tau}$. We later show that these restrictions are not binding. The Bellman equation for agent i is then:

(B.4)
$$W_t(h_t^i) = max_{(\nu,\tau)} \left\{ \log\left((1-\tau)\cdot(\nu\,h_t^i)^{\frac{\sigma-1}{\sigma}}(Y_t)^{\frac{1}{\sigma}}\right) \right.$$
$$\left. + \rho\cdot E\,W_{t+1}\left(\kappa\cdot\zeta_t^i\cdot((1-\nu)\,h_t^i)^{\delta}\cdot(\tau\,(\nu\,h_t^i)^{\frac{\sigma-1}{\sigma}}(Y_t)^{\frac{1}{\sigma}})^{1-\delta}\right) \right\}$$

The first order conditions are:

(B.5)
$$\frac{1}{\nu_t^i}\left(\frac{\sigma-1}{\sigma}\right) = \rho\left(\frac{\delta}{1-\nu_t^i} - \frac{1-\delta}{\nu_t^i}\left(\frac{\sigma-1}{\sigma}\right)\right)\cdot E\left[h_{t+1}^i\cdot\frac{\partial W_{t+1}}{\partial h}(h_{t+1}^i)\right]$$

(B.6)
$$\frac{1}{1-\tau_t^i} = \frac{\rho(1-\delta)}{\tau_t^i}\cdot E\left[h_{t+1}^i\cdot\frac{\partial W_{t+1}}{\partial h}(h_{t+1}^i)\right]$$

Since the state of the economy is characterized by the Markov process (A_t,Δ_t^2), we guess the form of the value function as: $W_t(h) = a\cdot\log(h) + b\cdot\log(A_t) - c\cdot\Delta_t^2 + d$. This leads to $\nu_t^i = \nu$, $\tau_t^i = \tau$, with:

(B.7)
$$\nu = \frac{1+\rho(1-\delta)a}{1+\rho(1-\delta)a+\rho\delta a\sigma/(\sigma-1)} \quad , \qquad \tau = \frac{\rho(1-\delta)a}{1+\rho(1-\delta)a} \quad .$$

Equilibrium then requires that $\bar{\nu} = \nu$ and $\bar{\tau} = \tau$. Replacing in (B.4) and using the recursion equations (28) and (29) for A_{t+1} and Δ_{t+1}^2 identifies the constants. In particular:

(B.8)
$$a = \frac{(\sigma-1)/\sigma}{1-\rho\delta-\rho(1-\delta)(\sigma-1)/\sigma} \quad , \qquad a+b = \frac{1}{1-\rho} \quad , \qquad c = \frac{b}{2\sigma^2}\cdot\frac{\rho\mathcal{L}+(1-\rho)/\sigma}{1-\rho(\alpha+\beta)^2} \quad ,$$

where $\alpha = \delta$, $\beta = (1-\delta)(\sigma-1)/\sigma$, $\gamma = (1-\delta)/\sigma$ and $\mathcal{L}$, $\hat{\mathcal{L}}$ are given by (18)-(19). Replacing a in (B.7) yields the desired results.

In this equilibrium, all agents choose the same constant values ν and τ. This is in fact the only solution, at least in the following sense. Consider any finite horizon $(T < \infty)$ version of the model, without the

restrictions $\nu_t^j = \bar{\nu}$, , $\tau_t^j = \bar{\tau}$. A backwards induction similar to the derivations above shows that in each period: (a) $\nu_t^i = \nu_t$ and $\tau_t^i = \tau_t$, ensuring $Y_t = \nu_t \cdot H_t$; (b) $W_t(h) = a_t \cdot \log(h) + b_t \cdot \log(A_t) - c_t \cdot \Delta_t^2 + d_t$, where the (a_t, b_t, c_t, d_t)'s satisfy linear difference equations whose fixed points are (a, b, c, d) calculated above. The finite game thus has a unique Markov perfect equilibrium, and it involves symmetric strategies. As $T \to \infty$, it tends to the equilibrium derived earlier.

2. Public education. We start again with some useful simplifications. First, let $\nu_t^j = \bar{\nu}$ for all $j \neq i$, so that the Markov process $(\hat{A}_t, \hat{\Delta}_t^2)$ fully describes the state of economy. We relax this restriction later on. Note from (17) and the analog of (28) under integration that the tax rate $\hat{\tau}_t$ implemented at t influences $\hat{A}_{t+1}$ through $\Theta_t = \kappa \cdot (1 - \bar{\nu})^\delta \cdot (\bar{\nu} \hat{\tau}_t)^{1-\delta}$, but does not affect $\hat{\Delta}_t^2$. Second, let agent i choose τ_t^i as if she expected to be the decisive voter (or a dictator) not just at t but in all future periods. Because this leads to the same preferred tax rate for all agents, any other voting game (e.g., agent i chooses τ_t^i as if she were decisive at t but expected the median voter to prevail at $t' > t$) will lead to same outcome. With these assumptions, agent i's Bellman equation and first order conditions are therefore:

(B.9)
$$V(h_t^i, \hat{A}_t, \hat{\Delta}_t^2) = max_{(\nu, \tau)} \left\{ \log \left((1 - \tau) \cdot (\nu h_t^i)^{\frac{\sigma-1}{\sigma}} (\hat{Y}_t)^{\frac{1}{\sigma}} \right) \right.$$
$$\left. + \rho \cdot E V \left(\kappa \cdot \zeta_t^i \cdot ((1 - \nu^i) h_t^i)^\delta \cdot (\tau \hat{Y}_t)^{1-\delta}, \hat{A}_{t+1}, \hat{\Delta}_{t+1}^2 \right) \right\}$$

(B.10)
$$\frac{1}{\nu_t^i} \left(\frac{\sigma - 1}{\sigma} \right) = \frac{\rho \delta}{1 - \nu_t^i} \cdot E \left[h_{t+1}^i \cdot \frac{\partial V}{\partial h}(h_{t+1}^i, \hat{A}_{t+1}, \hat{\Delta}_{t+1}^2) \right]$$

(B.11)
$$\frac{1}{1 - \tau_t^i} = \frac{\rho(1 - \delta)}{\tau_t^i} \cdot E \left[h_{t+1}^i \cdot \frac{\partial V}{\partial h}(h_{t+1}^i, \hat{A}_{t+1}, \hat{\Delta}_{t+1}^2) \right] + \rho \cdot \frac{\partial \hat{A}_{t+1}}{\partial \tau} \cdot E \left[h_{t+1}^i \cdot \frac{\partial V}{\partial A}(h_{t+1}^i, \hat{A}_{t+1}, \hat{\Delta}_{t+1}^2) \right]$$

Equation (B.11), which determines $\tau_t^{i\,*}$, corresponds to voters who internalize the effect of taxes on $\hat{A}_{t+1}$. If they do not, the corresponding rate $\bar{\tau}_t^i$ is obtained by dropping the last term. Again, we guess the form of the value function: $V(h, \hat{A}, \hat{\Delta}^2) = \hat{a} \cdot \log(h) + \hat{b} \cdot \log(\hat{A}) - \hat{c} \cdot \hat{\Delta}^2 + \hat{d}$. Then (B.10)-(B.11) imply $\nu_t^i = \hat{\nu}$, $\bar{\tau}_t^i = \bar{\tau}$, $\tau_t^{i\,*} = \tau^*$, where:

(B.12)
$$\hat{\nu} = \frac{1}{1 + \rho \delta \hat{a} \sigma / (\sigma - 1)}$$

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$$(\text{B.13}) \qquad \bar{\tau} = \frac{\rho(1-\delta)\hat{a}}{1+\rho(1-\delta)\hat{a}} < \frac{\rho(1-\delta)(\hat{a}+\hat{b})}{1+\rho(1-\delta)(\hat{a}+\hat{b})} = \tau^*.$$

While $\bar{\tau}$ is based on the private marginal value of human capital $\hat{a}$, τ^* reflects the full social marginal value $\hat{a}+\hat{b}$. Replacing in (B.9) and using the recursion equations for $(\hat{A}_{t+1}, \hat{\Delta}^2_{t+1})$ leads to:

$$(\text{B.14}) \qquad \hat{a} = \frac{(\sigma-1)/\sigma}{1-\rho\delta}, \qquad \hat{a}+\hat{b} = \frac{1}{1-\rho}, \qquad \hat{c} = \frac{\hat{b}}{2\sigma^2} \cdot \frac{\rho\hat{C}+(1-\rho)/\sigma}{1-\rho\alpha^2}$$

Replacing in (B.12) and (B.13) leads to the desired results. Note that $\hat{a} < a < \hat{a}+\hat{b}$, implying that $\bar{\tau} < \tau < \tau^*$.

To exclude other equilibria, consider again the finite horizon game, without the restriction $\nu_t^i = \bar{\nu}$. An agent's value function and (Markov) strategy depend on $(h_t^i, \hat{\mu}_t)$, where $\hat{\mu}_t$ is the distribution of human capital. Whether voters internalize the effect of τ_t^i on $\hat{\mu}_{t+1}$ or not, backwards induction easily shows that in each period: (a) $\nu_t^i = \hat{\nu}_t$, ensuring in particular that $\hat{\mu}_t$ remains log-normal and that $\hat{Y}_t = \hat{\nu}_t \cdot \hat{H}_t$; (b) $\tau_t^i = \hat{\tau}_t$, i.e. there is *unanimity* over the sequence of tax rates; (c) $V_t(h, \hat{\mu}_t) = \hat{a}_t \cdot \log(h) + \hat{b}_t \cdot \log(\hat{A}_t) - \hat{c}_t \cdot \hat{\Delta}_t^2 + \hat{d}_t$, where the $(\hat{a}_t, \hat{b}_t, \hat{c}_t, \hat{d}_t)$'s satisfy linear difference equations whose fixed points are $(\hat{a}, \hat{b}, \hat{c}, \hat{d})$ calculated above. Letting the horizon tends to infinity concludes the proof.

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