NBER WORKING PAPERS SERIES

INTERNATIONAL ADJUSTMENT WITH HABIT-FORMING CONSUMPTION: A DIAGRAMMATIC EXPOSITION

Maurice Obstfeld

Working Paper No. 4094

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
June 1992

This paper is part of NBER's research program in International Trade and Investment. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

NBER Working Paper #4094
June 1992

INTERNATIONAL ADJUSTMENT WITH HABIT-FORMING CONSUMPTION: A DIAGRAMMATIC EXPOSITION

ABSTRACT

This paper presents a simple diagrammatic analysis of an open economy's external adjustment process under habit-forming individual preferences. The exposition focuses on the consumption side and aims to make transparent the linkage among wealth, past consumption experience, and current consumption. An extension of the standard representative-agent model to a growing economy of overlapping generations completes the paper. Under habit formation an agent's consumption exhibits a form of hysteresis, in that his current consumption depends on his past consumption experience as well as initial assets. In the overlapping-generations model aggregate hysteresis disappears in the long run.

Maurice Obstfeld
Department of Economics
787 Evans Hall
University of California, Berkeley
Berkeley, CA 94720
and NBER

1. Introduction

The idea that rational consumption behavior may be influenced by habits cultivated over time has been fruitfully applied to a wide range of questions in economics. Thus, Becker and Murphy (1988) model and analyze rational addiction; Sundaresan (1989), Constantinides (1990), and others examine the effects of consumption habits on saving and asset pricing under uncertainty; and Mansoorian (1991) studies the link between saving and the terms of trade in open economies. All of this work represents a revival of interest in the model of intertemporally dependent preferences originally advanced by Ryder and Heal (1973).[1]

In this paper I present a simple diagrammatic exposition of habit-forming consumption behavior, with the aim of making transparent the linkage among wealth, past consumption experience, and current consumption. The setting, closely akin to the one assumed by Mansoorian (1991), is a small open economy facing a given world real interest rate. To obtain the closed-form solutions that give my exposition its simplicity, I follow Becker and Murphy (1988) in endowing individuals with quadratic period utility functions. It is straightforward, however, to apply similar techniques near the stationary positions of more general models.

Going beyond its exposition of the standard representative-agent case, the paper also applies the habit-formation model to an exogenously growing economy populated by overlapping infinitely-lived generations. Under habit formation a representative agent's consumption exhibits a form of hysteresis, in that her current consumption depends on her past consumption experience as well as initial assets. In the overlapping-generations model, however, *aggregate* hysteresis disappears in the long run. The extended model suggests an alternative diagrammatic mode of analysis that encompasses, as a particular case, the nongrowing representative-agent economy.

The balance of the paper is organized as follows. Section 2 describes the individual's optimization problem in a fairly unrestricted competitive market setting. Section 3 specializes by assuming quadratic utility and a constant interest rate. Section 4 introduces the central diagrammatic apparatus used to analyze international adjustment in the representative-agent context. Section 5 extends the model to capture a growing number of overlapping generations of agents. Section 6 analyzes the long-run equilibrium and dynamic stability of this heterogeneous-agent economy. Section 7 summarizes once again the paper's main findings.

2. Individual Optimization

A representative individual maximizes the intertemporal criterion

$$(1) \qquad \int_0^{\infty} U[C(t),S(t)]e^{-\theta t}dt,$$

where C(t) is time-t consumption of a single (composite) good and S(t) is the accumulated stock of consumption "experience."[2] The stock S(t) is predetermined and its motion is described by the differential equation

$$(2) \qquad \dot{S}(t) = \rho[C(t) - S(t)],$$

with S(0) given. Thus S(t) is a distributed lag on past consumption,

$$S(t) = \rho\int_0^t C(\tau)e^{-\rho(t-\tau)}d\tau + e^{-\rho t}S(0),$$

and the preferences represented by (1) are nonseparable over time. The period utility function U(C,S) is assumed to be strictly concave, with $U_C(C,S) > 0$. (This function will be specialized further in the next section.) Consumption must be nonnegative.

Let r(t) denote the real rate of interest and A(t) the real value of wealth (including the present value of labor income, net of taxes paid to the government). Lifetime consumption is limited by the intertemporal budget constraint

$$(3) \qquad \int_0^\infty e^{-R(t)}C(t)dt \leq A(0),$$

where

$$R(t) \equiv \int_0^t r(\tau)d\tau.$$

First-order necessary conditions for an individual optimum are found easily through the Maximum Principle (Arrow and Kurz, 1970). Toward this end, consider the optimal-control problem of maximizing (1) subject to (2) and the wealth-accumulation identity

$$\dot{A}(t) = r(t)A(t) - C(t),$$

which, when supplemented with the terminal condition $\lim_{T\to\infty} e^{-R(T)}A(T) \geq 0$, implies (3). One can write the problem's current-value Hamiltonian in the form

$$\mathcal{H}[C(t),S(t),A(t),\lambda(t),\varphi(t)] = U[C(t),S(t)]$$
$$+ \lambda(t)[rA(t) - C(t)] + \varphi(t)\rho[C(t) - S(t)],$$

where $\lambda(t)$ and $\varphi(t)$ are costate variables. The necessary conditions for optimality include

$$(4)\quad \mathcal{H}_C[C(t),S(t),A(t),\lambda(t),\varphi(t)] = U_C[C(t),S(t)] - \lambda(t) + \varphi(t)\rho = 0$$

and the transition equations for the costates,

$$(5a)\quad \dot{\lambda}(t) = \theta\lambda(t) - \mathcal{H}_A[C(t),S(t),A(t),\lambda(t),\varphi(t)]$$

$$= [\theta - r(t)]\lambda(t)$$

and

$$(5b)\quad \dot{\varphi}(t) = \theta\varphi(t) - \mathcal{H}_S[C(t),S(t),A(t),\lambda(t),\varphi(t)]$$

$$= (\rho + \theta)\varphi(t) - U_S[C(t),S(t)].$$

The relevant solution to differential equation (5b) discloses that

$$\varphi(t) = \int_t^\infty U_S[C(\tau),S(\tau)]e^{-(\rho+\theta)(\tau - t)}d\tau.$$

Thus $\varphi(t)$ is the discounted lifetime utility effect of a small increase in S(t), given that "experience" depreciates at the rate ρ. The costate $\lambda(t)$ is the shadow value of wealth, as usual. Accordingly, (4) has the following interpretation: along an optimal path the marginal current utility of time-t consumption, *plus* the marginal contribution of greater time-t consumption to the utility stream derived from future experience, equals the time-t shadow value of wealth. A similar condition arises in time-nonseparable models based on Uzawa's (1968) brand of endogenous time preference, in which subjective discount factors depend on the prior path of consumption (Obstfeld, 1990; Shi and Epstein, 1991). An optimal consumption path $\{C(t)\}_{t=0}^{\infty}$ is one that satisfies (4) at every moment, given that S(t) evolves according to (2) [with S(0) predetermined], that $\lambda(t)$ evolves according to (5a), and that $\lambda(0)$ is chosen so that budget constraint (3) holds with equality.

3. Implications of a Constant Interest Rate and Quadratic Utility

Following Becker and Murphy (1988), I specialize in this section to the case of quadratic period utility. To facilitate the diagrammatic approach developed later, I assume that the real rate of interest is constant at $r(t) = r = \theta$. The analysis applies equally well to an individual and to a small open economy facing a given world interest rate.

Because r is constant at θ, (5a) implies that λ is constant as well: $\dot{\lambda} = 0$. Differentiation of equation (4) and substitution of (2), (4), and (5b) into the result leads to the following dynamic equation for consumption:

$$(6) \qquad \dot{C} = \frac{1}{U_{CC}}\left[(\rho + \theta)(U_C - \lambda) + \rho U_S - \rho U_{CS}\cdot(C - S)\right].$$

(I will omit noting time dependence in parentheses from now on when this omission does not risk confusion.)

Equations (2) and (6) together describe the joint dynamics of C and S, conditional on the fixed value of λ. As will become clear below, (2) and (6), together with an assumption on the second partial derivatives of U(C,S), imply a unique convergent path for consumption. The unique equilibrium value of λ, in turn, is pinned down by the requirement that budget constraint (3) hold with equality along that convergent trajectory. Solving for λ thus leads to a complete description of the equilibrium path. The change over time in total assets, A, is determined recursively by the differential equation $\dot{A} = rA - C$.

To provide a closed-form solution for the equilibrium path, I assume that the function U(C,S) in (1) is quadratic,

$$U(C,S) = \gamma_c C + \gamma_s S + \gamma_{cs} CS + \gamma_{cc} C^2/2 + \gamma_{ss} S^2/2,$$

with $\gamma_c > 0$, $\gamma_{cc} < 0$, and $\gamma_{ss} < 0$. Then (6) can be expressed as

$$\text{(7)} \qquad \dot{C} = \kappa + (\rho + \theta)C + \left[(\theta + 2\rho)\frac{\gamma_{cs}}{\gamma_{cc}} + \rho\frac{\gamma_{ss}}{\gamma_{cc}}\right]S,$$

where

$$\text{(8)} \qquad \kappa = [(\rho + \theta)(\gamma_c - \lambda) + \rho\gamma_s]/\gamma_{cc}.$$

Notice that the dynamic system under study has no *extrinsic* sources of motion; there are no exogenous forcing variables that vary over time. It is thus plausible to assume that C and S will converge to steady-state values, $\overline{C}$ and $\overline{S} = \overline{C}$. (The value of $\overline{C}$ will be computed shortly.) Define Ω as the coefficient of S in (7),

$$\Omega \equiv (\theta + 2\rho)\frac{\gamma_{cs}}{\gamma_{cc}} + \rho\frac{\gamma_{ss}}{\gamma_{cc}} .$$

The dynamic system described by (2) and (7) can be expressed in matrix form as

$$(9) \qquad \begin{bmatrix} \dot{C} \\ \dot{S} \end{bmatrix} = \begin{bmatrix} \rho + \theta & \Omega \\ \rho & -\rho \end{bmatrix} \begin{bmatrix} C - \overline{C} \\ S - \overline{S} \end{bmatrix}.$$

The determinant of the matrix above, Δ, is

$$\Delta = -\rho(\rho + \theta) - \rho\Omega.$$

Δ is the product of the characteristic roots of (9), which are real numbers, as in Becker and Murphy (1988). When $\gamma_{cs} < 0$, $\Delta < 0$ and so the system (9) is saddlepath stable--it has one negative root and one positive root. I will restrict the model by assuming that $\Delta < 0$ always. Equilibrium solutions to (9) necessarily depend *exclusively* on the system's negative, stable root, denoted by $-\psi$.[3] Thus, equilibrium solutions lie along the convergent saddlepath.[4]

The saddlepath's slope depends on the eigenvector of (9) associated with the stable root, $-\psi$. That eigenvector is proportional to $\begin{bmatrix}\frac{\rho - \psi}{\rho} & 1\end{bmatrix}'$, so the saddlepath is described by

$$(10)\quad C(t) - \overline{C} = [(\rho - \psi)/\rho][S(t) - \overline{S}],$$

$$S(t) - \overline{S} = [S(0) - \overline{S}]e^{-\psi t}.$$

Clearly, C and S will covary positively along the saddlepath if and only if $\rho - \psi > 0$. It is easy to show that the preceding inequality holds if and only if $\Omega < 0$; a necessary (but not sufficient) condition for this is that $\gamma_{cs} > 0$. Ryder and Heal (1973) define "adjacent complementarity" in preferences by the condition $\Omega < 0$. Becker and Murphy (1988, p. 681) identify this property of preferences with addiction.

As noted above, the multiplier λ has so far been taken as given. But, by (7) and (8), $\overline{C}$ depends on λ:

$$\overline{C} = \frac{[(\rho + \theta)(\gamma_c - \lambda) + \rho\gamma_s]}{-\gamma_{cc}(\rho + \theta + \Omega)}.$$

Only one value of λ will yield a value of $\overline{C}$, and hence a path (10) for C(t), that is consistent with the intertemporal budget constraint (3). By substituting (10) into (3), and remembering that $\overline{C} = \overline{S}$, one finds that

$$(11)\quad \bar{C} = \frac{r\rho(\psi + r)}{\psi(\rho + r)} A(0) - \frac{r(\rho - \psi)}{\psi(\rho + r)} S(0).$$

Thus, the equilibrium λ depends on S(0) as well as A(0). All else the same, higher initial assets raise long-run consumption. Whether greater initial consumption experience raises long-run consumption depends, however, on the sign of $\rho - \psi$. When $\rho - \psi > 0$ (i.e., when $\Omega < 0$, the case of adjacent complementarity), a higher S(0) implies higher initial consumption, C(0), and hence lower *long-run* consumption, $\bar{C}$. When $\rho - \psi < 0$, in contrast, higher initial consumption experience implies higher long-run consumption.

In a model without habit-forming consumption, $\gamma_{cs} = \gamma_{ss} = 0$; it is easy to verify that in this special case $\psi = \rho$, so that, according to (11), $\bar{C} = rA(0)$. When utility is time separable and depends on consumption only, a consumer whose time-preference rate equals a constant interest rate finds it optimal to maintain an unvarying wealth level over time. One way of understanding the model is to ask how habit-persistence induces a departure from this well-known "benchmark" behavior.

To answer this question, recall that steady-state consumption must be linked to steady state wealth, $\bar{A}$, via the relation

$$\bar{C} = r\bar{A}.$$

Thus, equation (11) can be rewritten in the form

$$\bar{A} - A(0) = \left[\frac{r(\rho - \psi)}{\psi(\rho + r)}\right]\left[\frac{rA(0) - S(0)}{r}\right].$$

When by accident it happens that the initial stock of consumption experience equals the annuity value of wealth, the economy can jump immediately to a steady state with $\bar{C} = S(0) = rA(0) = r\bar{A}$. In general, however, $S(0)$ can bear any relationship to initial wealth. If $\rho > \psi$ and $rA(0) > S(0)$, say, wealth rises during the transition to the long run. A stable transition necessarily calls for rising consumption experience; and adjacent complementarity implies that the positive effect of higher future S on the marginal utility of future consumption is strong enough to induce people to accumulate wealth. When instead $\rho < \psi$ with $rA(0) > S(0)$, people decumulate as S rises. Thus $\bar{A}$ ends up below $A(0)$.

Clearly the habit-forming preference model predicts hysteretic consumption behavior, in the sense that the past history of consumption influences current consumption independently of initial asset holdings.

4. Picturing External Adjustment

A simple diagram helps to clarify the different adjustment patterns described in the last section.

The first step in constructing this diagram is to derive the relationship among the initial values of consumption, consumption experience, and wealth implied by (10) and (11). [Recall that

(10) described the convergent consumption path conditional on a given long-run consumption level $\bar{C}$, while (11) indicates the unique $\bar{C}$ consistent with the budget constraint (3), given the dynamic path specified in (10).] By setting $t = 0$ in (10) and using (11) to eliminate $\bar{C} = \bar{S}$, one finds that

$$(12) \quad C(0) = \frac{\psi + r}{\rho + r} [rA(0)] + \frac{\rho - \psi}{\rho + r} S(0).$$

The optimal consumption choice is a weighted average of the annuity value of initial wealth and the initial consumption-experience stock. When $\rho > \psi$ experience receives a positive weight in the consumption decision (and "permanent" income, therefore, a weight less than one). When $\rho < \psi$ experience receives a negative weight (and permanent income a weight greater than one).

Figure 1 graphs relation (12) in the (S,C) plane in the case $\rho > \psi$. Superimposed on the figure is the locus of points such that $\dot{S} = 0$, a 45° line the unit slope of which is indicated [recall (2)]. Note that the vertical intercept of equation (12) lies strictly below rA(0), and that the equation's slope is positive. In the alternative case $\rho < \psi$, which is not shown, equation (12) has a negative slope and a vertical intercept higher than rA(0). By construction, the graph of (12) gives the optimal initial consumption level C(0) corresponding to any given initial value of the predetermined stock variable S.

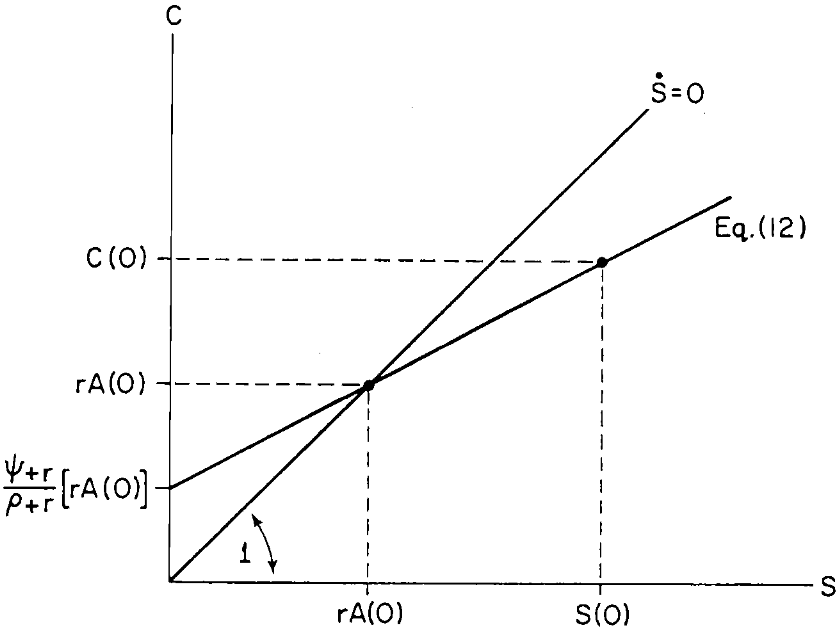

FIGURE 1

Consumption as a Function of Assets A(0) and Experience S(0)

The initial level of income, rA(0), is easily read off of the diagram. In figure 1, rA(0) is shown at the intersection of the consumption function (12) and the $\dot{S} = 0$ locus. We know from the last section that if S(0) happened to equal rA(0), equilibrium consumption would be at a stationary position on the $\dot{S} = 0$ locus.

This coincidence will not obtain in general, however. To elucidate the dynamics that will occur more generally, figures 2 ($\rho > \psi$) and 3 ($\rho < \psi$) add to the figure the stable saddlepath, EP, described by the first equation in (10). (EP stands for "equilibrium path.") After time t = 0, C(t) and S(t) evolve along EP, as was argued above. The stable path necessarily emanates from point E on the graph of equation (12). Its slope, $(\rho - \psi)/\rho$, is algebraically greater than the slope of equation (12), $(\rho - \psi)/(\rho + r)$, when $\rho > \psi$ (as in figure 2), and is algebraically less than the latter slope when $\rho < \psi$ (as in figure 3).

The system's long-run equilibrium is located at the intersection of EP and the $\dot{S} = 0$ locus, point P. This point is the only one that simultaneously satisfies (3), (10), and the steady-state requirement that S = C.

Given A(0), the exact position of the *relevant* saddlepath EP depends on S(0). For example, if S(0) in figure 2 were located to the right of rA(0), as happens to be true in figure 1, the relevant saddlepath EP would intersect $\dot{S} = 0$ at a point between the origin and (rA(0),rA(0)).

In the special case of non-habit-forming consumption, we have

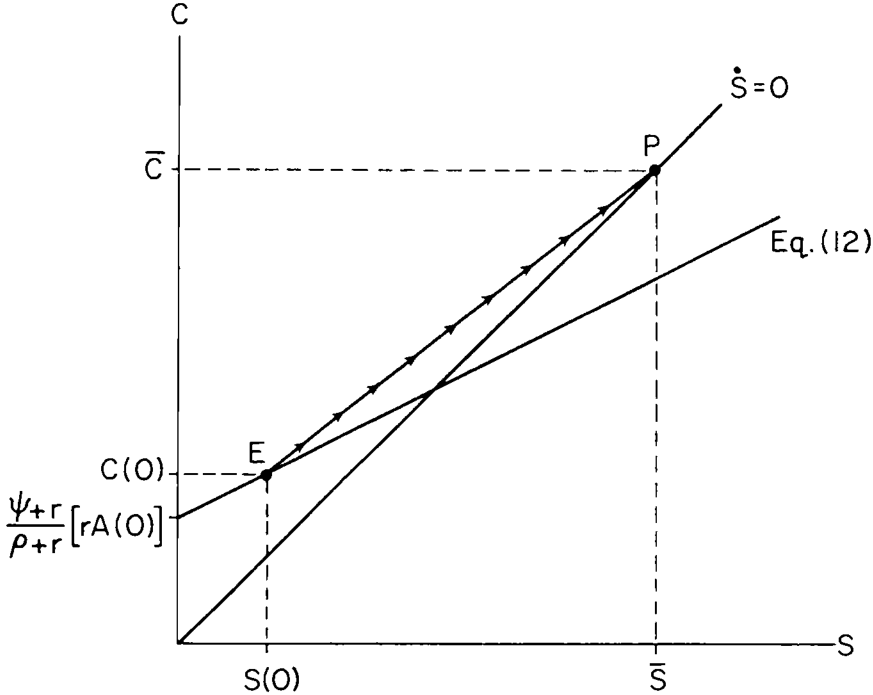

FIGURE 2

Dynamic Adjustment for Given A(0) and S(0) when $\rho > \psi$

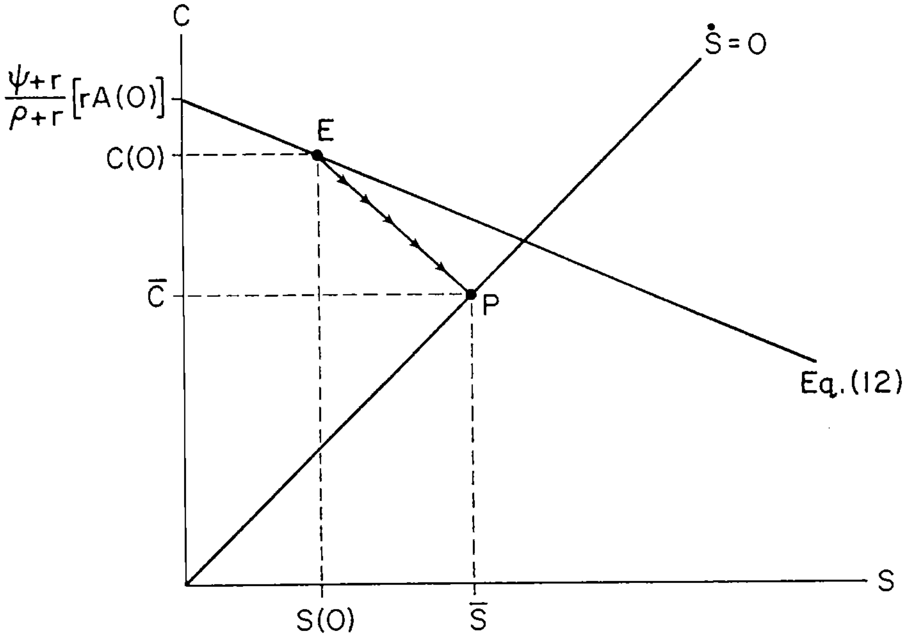

FIGURE 3

Dynamic Adjustment for Given A(0) and S(0) when $\rho < \psi$

seen that $\rho = \psi$. In this case equation (12) is a horizontal line with vertical intercept rA(0), and this line coincides with the saddlepath. If S(0) > rA(0), for example, consumption will jump to, and remain at, rA(0). S(t) will then fall toward rA(0) without affecting consumption behavior.

Notice that each point on the saddlepath is associated with a distinct level of wealth, A(t), which can be read off the horizontal axis simply by extending from (S(t),C(t)) a straight line of slope $(\rho - \psi)/(\rho + r)$--the slope of (12)--and multiplying its intercept by $(\rho + r)/(\psi + r)$. In figure 2 wealth rises over time along EP, while in figure 3 wealth falls. Of course, if (S(0),C(0)) in figure 2 were located to the right of $\dot{S} = 0$, wealth would subsequently fall; in figure 3, this alternative initial configuration would imply a rising trajectory for wealth.

Figures 2 and 3 lead to the following interpretation of how wealth accumulation depends on the relative sizes of ρ and ψ. Consumers with $\rho < \psi$ (figure 3) always try to return *part* of the way to their habitual standard of living. In particular, when the stock of consumption experience exceeds the annuity value of wealth, such consumers build up wealth, and they run it down in the opposite circumstance. To see this last point in algebraic terms, note that the amount consumers add to wealth is rA − C and that (12) therefore gives the rate of wealth accumulation as

$$rA - C = \frac{\rho - \psi}{\rho + r}(rA - S).$$

In contrast, the consumption and wealth of consumers with $\rho > \psi$ does not display this type of mean-reverting behavior (figure 2). Instead, accumulation behavior under adjacent complementarity tends to amplify the long-run effects of wealth shocks on consumption. When $\rho > \psi$, consumers whose consumption experience is below the annuity value of wealth will be adding to wealth--they expect to become accustomed to a higher standard of living in the future. At the same time, consumers who are unexpectedly impoverished are decumulating their way into deeper poverty. This process need not be stable in general, as pointed out by Becker and Murphy (1988), although I have assumed stability here.

Figures 2 and 3 can be used to investigate the effects of disturbances to an initial equilibrium.

Consider, for example, an unanticipated transfer of wealth to the home country, an event that raises A(0) to A(0)′ but leaves S(0) unchanged in the short run. In figure 4 (depicting the case $\rho > \psi$), we see an immediate upward jump in consumption (point E′), followed by a rising consumption path in the transition to point P′. Wealth also rises during the transition. The interesting point to note, however, is that $\overline{A}' - A(0)'$, the difference between initial and ultimate wealth, is greater than $\overline{A} - A(0)$, the total wealth accumulation that would have occurred along the original unperturbed transition path. Thus, the positive shock to wealth raises the accumulation rate under the adjacent complementarity assumed in figure 4. People raise their accumulation rate after a

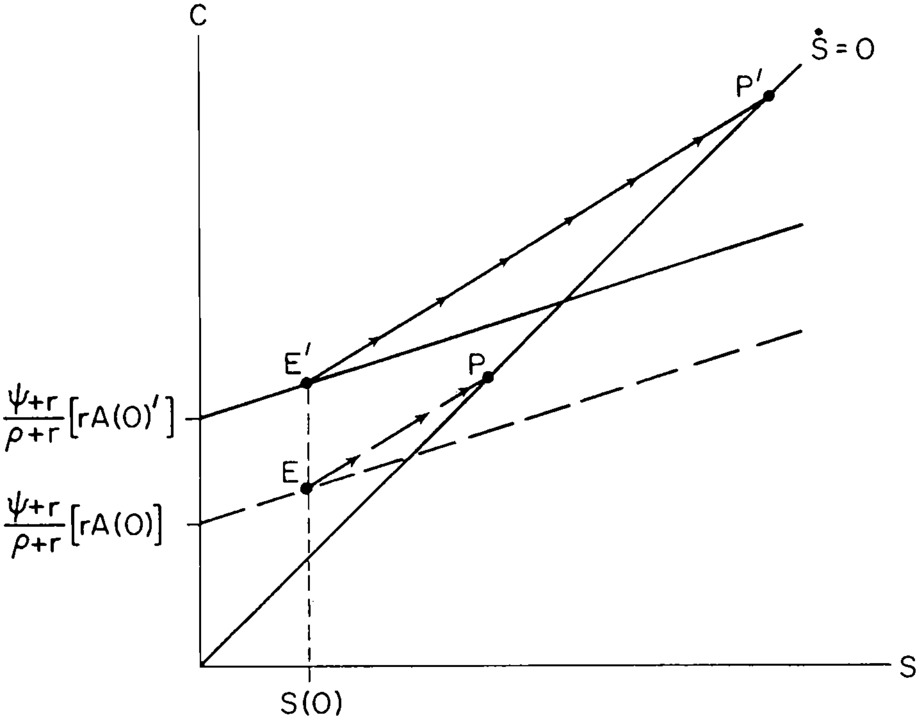

FIGURE 4

A Positive Wealth Transfer when $\rho > \psi$

positive wealth shock to finance the high-consumption "addiction" (to use Becker and Murphy's terminology) that they know they will develop.

If the economy starts out in long-run equilibrium, with no ongoing accumulation, a wealth transfer will lead it to accumulate further wealth. With time-separable utility the economy would jump right away to its new long-run consumption level; with Uzawa preferences it would spend wealth down to its pre-shock level.

This last result is reversed in the case $\rho < \psi$, shown in figure 5. In the figure, the economy is initially accumulating wealth when $A(0)'$ unexpectedly rises. After an initial upward jump of consumption, the accumulation surpluses continue (provided the rise in wealth is not too large).[5] Unlike in figure 4, however, $\bar{A} - A(0)$ now *exceeds* $\bar{A}' - A(0)'$: an initial increase in wealth results in a perturbed path along which accumulation is lower than it would otherwise have been.

In the special case that the economy starts out in long-run equilibrium, its ultimate asset level $\bar{A}'$ will be below $A(0)'$, but higher than $A(0) = \bar{A}$. In contrast, with Uzawa preferences (as noted above), the economy would spend down *all* of its windfall, ending up at its pre-shock wealth level.

Notice that the arguments developed above apply equally to permanent and transitory income transfers alike. All that matters is the transfer's effect on the present value of wealth.

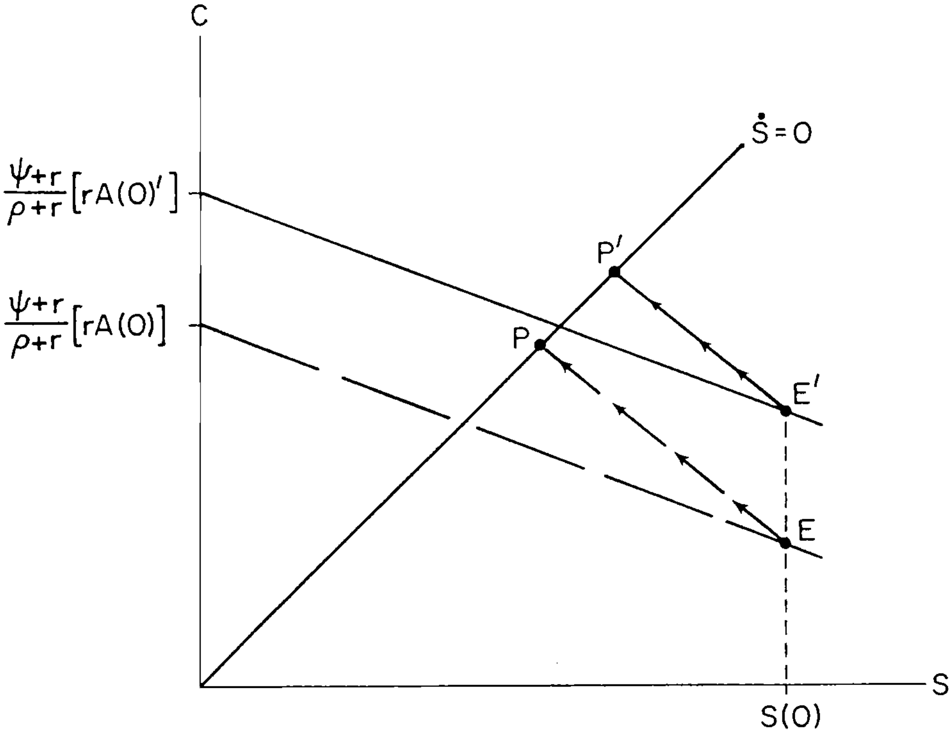

FIGURE 5

A Positive Wealth Transfer when $\rho < \psi$

5. An Economy of Overlapping Generations

This section extends the preceding analysis to a different demographic environment: the population now consists of a continuum of infinitely-lived agents with different birth dates, rather than a single infinitely-lived agent. The model, which is due to Weil (1989), makes the central assumption that individuals born on different dates are not altruistically linked. Open-economy versions of Weil's model have been explored previously by Obstfeld (1989) and by Alogoskoufis and van der Ploeg (1991), among others.

The economy starts at $t = 0$ with a population of $N(0)$ agents. By choice of units, $N(0) = 1$ is assumed. Population grows at the nonnegative proportional rate $n < r$, so that $N(t) = e^{nt}$. An individual born on date v (the individual's "vintage") maximizes the objective function

$$\int_{v}^{\infty} U[C(v,t),S(v,t)]e^{-\theta(t-v)}dt,$$

where $C(v,t)$ is a vintage-v agent's time-t consumption and $S(v,t)$ is a vintage-v agent's time-t stock of experience. Lifetime utility is maximized subject to $\dot{S}(v,t) = \rho[C(v,t) - S(v,t)]$, with $S(v,v)$ given, and the intertemporal budget constraint,

$$\int_v^\infty C(v,t)e^{-r(t-v)}dt \le F(v,v) + Y/r.$$

Above, F(v,t) denotes vintage v's time-t holdings of financial wealth and Y is a fixed endowment of the consumption good (think of it as labor income).[6] If the newly born enter the world owning only the latter endowment, F(v,v) = 0 for all v. Note that I have implicitly continued to assume that the real interest rate, r, is constant through time.

The first-order necessary conditions for an individual optimum are the same as those discussed in section 2. Thus, equations (4), (5a), and (5b) still apply, with all quantities and shadow prices indexed by vintage. Assume once again that the period utility function is of the quadratic form specified in section 2. Repeating the arguments made before, one finds that when $\theta = r$ the consumption function is given by (12) (which is reproduced below in the present model's notation):

$$(13) \quad C(v,t) = \frac{\psi + r}{\rho + r}[Y + rF(v,t)] + \frac{\rho - \psi}{\rho + r}S(v,t).$$

In (13), the root ψ has the same value as in section 3.

It is well known that in Weil's model, even values of θ different from r can be consistent with the existence of a steady state for the small open economy. To preserve algebraic simplicity, I nonetheless discuss in detail only the case in which

θ and r are equal.[7]

The key novel element in formulating the model is the link between the individual and aggregate per capita stocks of consumption experience. The total population at time t, $N(t) = e^{nt}$, can be expressed as

$$N(t) = N(0) + \int_0^t \dot{N}(v)\, dv = 1 + \int_0^t ne^{nv} dv.$$

The time-t per capita average level of a variable X(v,t), denoted X(t), is simply the population weighted average of X(v,t),

$$X(t) = \left[X(0,t) + \int_0^t ne^{nv} X(v,t) dv \right] \div e^{nt}.$$

In particular, the average per capita stock of consumption experience, S(t), is given by

$$(14) \quad S(t) = \left[S(0,t) + \int_0^t ne^{nv} S(v,t) dv \right] \div e^{nt}.$$

Notice, however, that S(v,t) is a distributed lag on generation v's past consumption:

$$S(v,t) = \rho \int_v^t C(v,\tau) e^{-\rho(t-\tau)} d\tau + e^{-\rho(t-v)} S(v,v).$$

Replacing S(v,t) in (14) with the above expression leads, after a change in the order of integration, to

$$S(t) = \rho\int_0^t C(\tau)e^{-(\rho+n)(t-\tau)}d\tau$$

$$+ e^{-(\rho+n)t}S(0,0) + \int_0^t nS(v,v)e^{-(\rho+n)(t-v)}dv.$$

Differentiation of this expression reveals the law of motion for S(t):

(15) $\dot{S}(t) = \rho C(t) - (\rho + n)S(t) + nS(t,t).$

To close the model, we need to specify S(t,t), the beginning stock of consumption experience with which agents born at time t are endowed. A natural assumption is that each newborn is a *tabula rasa*, with S(t,t) = 0. In this case aggregate per capita consumption experience follows

(16) $\dot{S}(t) = \rho C(t) - (\rho + n)S(t),$

reflecting the fact that new entrants to the population arrive at rate n with zero consumption "capital." Equally plausible is the view that newborn individuals are endowed at birth with societal norms that in effect set S(t,t) equal to S(t), society's average consumption-experience stock. In this case (2) describes the evolution of S(t), with all variables interpreted as per capita aggregates. More generally, S(t,t) could be dynamically related to C(t) in a complex manner; and this relationship would feed back into the dynamics of S(t) described by (15).

Let's turn next to the dynamics of aggregate wealth, which are the same as in Weil (1989). Denote by F(t) the economy's aggregate per capita stock of financial assets at time t:

$$F(t) = \left[F(0,t) + \int_0^t ne^{nv}F(v,t)dv\right] \div e^{nt}.$$

This variable can be interpreted as the country's net external assets at t = 0.[8] The individual's wealth-accumulation identity states that $\dot{F}(v,t) = Y + rF(v,t) - C(v,t)$. One thus finds that the time derivative of F(t) is

$$\dot{F}(t) = Y + (r - n)F(t) - C(t) + nF(t,t).$$

On the assumption that F(t,t) = 0, this equation becomes

(17) $\dot{F}(t) = Y + (r - n)F(t) - C(t).$

The next step is to examine the dynamic implications of these relationships.

6. Convergence and the Steady State

This section explores the joint dynamics of F(t) and S(t). These are summarized through a simple diagram that encompasses, as a special case, the representative-agent economy studied earlier.

When averaged over the extant population, equation (13) becomes the aggregate consumption function

$$(18)\quad C(t) = \frac{\psi + r}{\rho + r}\,[Y + rF(t)] + \frac{\rho - \psi}{\rho + r}\,S(t).$$

Substitute (18) into (17) to get our first aggregate dynamic equation,

$$(19)\quad \dot{F} = \left(\frac{\rho-\psi}{\rho+r}\right)Y + \left[r\left(\frac{\rho-\psi}{\rho+r}\right) - n\right]F - \left(\frac{\rho-\psi}{\rho+r}\right)S.$$

To be concrete, assume now that S(t,t) = 0 so that (16) holds true. Substitution of (18) into (16) yields

$$(20)\quad \dot{S} = \rho\left(\frac{\psi+r}{\rho+r}\right)Y + \rho r\left(\frac{\psi+r}{\rho+r}\right)F - \left[\rho\left(\frac{\psi+r}{\rho+r}\right) + n\right]S,$$

our second aggregate dynamic equation.[9] The key features behind this very simple account of aggregate dynamics were the assumptions that all agents have identical quadratic utility functions and identical experience-accumulation technologies

The system consisting of (19) and (20) can be written in the matrix form

$$(21)\quad \begin{bmatrix} \dot{F} \\ \dot{S} \end{bmatrix} = \begin{bmatrix} r\left(\frac{\rho - \psi}{\rho + r}\right) - n & -\frac{\rho - \psi}{\rho + r} \\ \rho r\left(\frac{\psi + r}{\rho + r}\right) & -\rho\left(\frac{\psi + r}{\rho + r}\right) - n \end{bmatrix} \begin{bmatrix} F - \overline{F} \\ S - \overline{S} \end{bmatrix},$$

where $\overline{F}$ and $\overline{S}$ are steady-state values such that $\dot{F} = \dot{S} = 0$. The characteristic roots of the matrix in (21) are $-n$ and $-(\psi + n)$, both of which are negative and real. Thus the system will display a nonoscillatory convergence to long-run equilibrium.

The steady-state values of F, S, and C are given by

$$(22)\quad \overline{F} = \frac{(\rho - \psi)Y}{(\psi + n)(\rho + r)}, \quad \overline{S} = \frac{\rho\overline{C}}{\rho + n}, \quad \overline{C} = Y + (r - n)\overline{F}.$$

Several aspects of this long-run equilibrium warrant discussion:

1. Unlike in the representative-agent economy of sections 2 through 4, the aggregate long-run equilibrium does *not* depend on the initial level of S(0): the long-run hysteresis that characterizes individual consumption behavior [see equation (11)] disappears at the aggregate level. Also in contrast to the representative-agent case, the long-run equilibrium is independent of F(0) as well. In this respect the aggregate economy reproduces a trait characteristic of Uzawa (1968) consumers. These features of the economy are due to the continuous entry of new agents,

which causes the influence of history to decay over time.

2. Whether the economy is a net foreign creditor or debtor in the long run depends on whether ρ exceeds or falls short of ψ. Under adjacent complementarity, for example, individuals with low stocks of consumption experience relative to assets tend to save more [see (13)]. By the assumption that S(t,t) = 0, newly born agents therefore are relatively high savers when $\rho > \psi$, and this saving continues, at a declining rate, throughout life. The opposite is true when $\rho < \psi$.[10] The average foreign-asset stock of the economy therefore is positive when $\rho > \psi$, negative when $\rho < \psi$. Correspondingly, an economy in which $\rho > \psi$ runs a steady-state per capita current-account surplus of $n\overline{F}$; one in which $\rho < \psi$ runs a deficit of $-n\overline{F}$. These strong results follows from the assumption that θ, the time-preference rate, equals r, the real interest rate. In general any gap between the two also plays a role in determining the long-run external position.[11]

3. Steady-state foreign assets decline with the growth rate n when $\rho > \psi$, and rise with the growth rate when $\rho < \psi$. Faster growth means a younger population, so that individuals on average have had less time to build up foreign assets ($\rho > \psi$) or foreign debts ($\rho < \psi$). Long-run consumption falls or rises with n depending on whether $\rho - \psi$ is positive or negative, respectively.

4. The steady state in (22) is sensitive to the determination of S(t,t). Under the alternative hypothesis that

$S(t,t) = S(t)$, so that the newly born immediately acquire society's consumption norms, the economy also converges to a unique steady state that does not depend on $S(0)$ and $F(0)$. This long-run position is different, however, from the one I have described. It is easy to check that in the alternative case $\overline{F} = 0$ and $\overline{C} = \overline{S} = Y$, regardless of the size of ρ compared with ψ.

5. In the special case $n = 0$ the matrix in (21) becomes singular and it thus is no longer possible to solve (19) and (20) for unique steady-state levels $\overline{F}$ and $\overline{S}$ that are independent of $F(0)$ and $S(0)$. This is the case that was discussed in sections 2 through 4. When $n = 0$, the only information about long-run equilibrium contained in (19) and (20) is that $\overline{S} = Y + r\overline{F}$, a result that, by itself, ties down neither $\overline{F}$ nor $\overline{S}$.

Turn next to the dynamic adjustment implied by (21). There are three configurations of the system, corresponding to the three cases in which (a) $\rho < \psi$; (b) $\rho > \psi$ but $r(\rho - \psi) < n(\rho + r)$, meaning that the northwest entry of the matrix in (21) is negative; and (c) $\rho > \psi$ and $r(\rho - \psi) > n(\rho + r)$. Figure 6 shows case (b). Cases (a) and (c) are left for the curious reader.

Displayed in the figure are the eigenvectors of (21), V_1 and V_2, which are defined by

$$V_1 \propto \begin{bmatrix} \frac{1}{r} \\ 1 \end{bmatrix}, \quad V_2 \propto \begin{bmatrix} \frac{(\rho-\psi)}{\rho(\psi+r)} \\ 1 \end{bmatrix}.$$

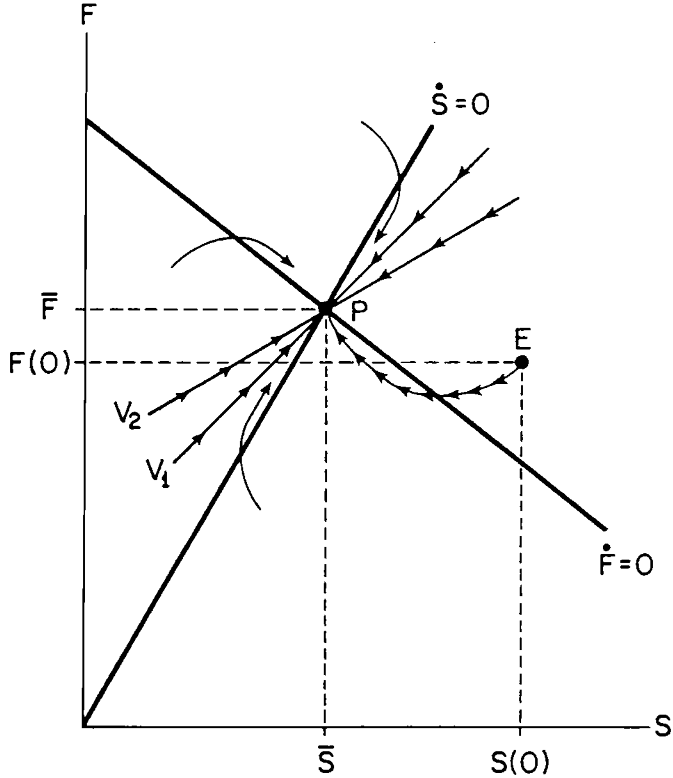

FIGURE 6

Aggregate Dynamics with Overlapping Generations and Population Growth
($\rho > \psi$ but $r(\rho - \psi) < n(\rho + r)$)

Along V_1, only the stable root $-n$ influences the economy's motion. Along V_2, only the stable root $-(\psi + n)$ is at work. Other paths reflect the influence of both roots in conjunction with different initial conditions (S(0),F(0)). Point P is the economy's steady state.

Consider, for example, the adjustment process that would follow from an initial position at point E in figure 6. At E, S(0) is so high relative to F(0) that average per capita asset growth initially is negative, and S is falling. [Remember, $\rho > \psi$, and this would be the transition path implied by figure 2 were S(0) sufficiently high relative to A(0).] In the case shown population growth kicks in quickly to turn average saving positive, and the economy then moves directly to the steady state.

When $n = 0$, an alternative representation of the adjustment process depicted in figures 2 and 3 is obtained. In the representative-agent economy, the $\dot{S} = 0$ and $\dot{F} = 0$ schedules both coincide with the locus along which $S = Y + rF$. Equations (19) and (20) also predict that from any point in the plane, the economy will travel toward this locus of potential steady states.[12] The hysteretic nature of this system is apparent from the influence of the economy's starting position on its asymptotic rest point.

7. Summary

This paper offers an easy diagrammatic approach to the analysis of economies in which consumption preferences are subject to habit formation. Two cases were studied, that of a small economy with a single representative agent and that of a growing overlapping-generations economy. The former case naturally corresponds to that of an individual price-taking consumer.

One way to summarize the main results is to recall the individual's long-run consumption response to an exogenous permanent increase in income, denoted by dY, given an initial long-run equilibrium position. Below I report this response, $d\overline{C}/dY$, under time-nonseparable Uzawa (1968) preferences; under habit-forming consumption without adjacent complementarity (and with $\theta = r$); under standard time-separable preferences with $\theta = r$; and under habit formation and adjacent complementarity:

$d\overline{C}/dY$ for Alternative Preference Assumptions

Uzawa	Habit Formation ($\rho < \psi$)	Time Separable	Habit Formation ($\rho > \psi$)
0	$0 < d\overline{C}/dY < 1$	1	$d\overline{C}/dY > 1$

An economy with a growing population of infinitely-lived unconnected families displays quite different asymptotic behavior at the aggregate level. The steady-state average stocks of net foreign assets and consumption experience do not depend on their initial levels. They depend only on the economy's human wealth endowment and on the parameters of the utility function its residents have in common.

References

Alogoskoufis, George S. and Frederick van der Ploeg, "On Budgetary Policies, Growth, and External Deficits in an Interdependent World," *Journal of the Japanese and International Economies* 4 (1991): 305-324.

Arrow, Kenneth J. and Mordecai Kurz, *Public Investment, the Rate of Return, and Optimal Fiscal Policy*, Baltimore: The Johns Hopkins Press, 1970.

Beals, Richard and Tjalling C. Koopmans, "Maximizing Stationary Utility in a Constant Technology," *SIAM Journal of Applied Mathematics* 17 (1969): 1001-1015.

Becker, Gary S. and Kevin M. Murphy, "A Theory of Rational Addiction," *Journal of Political Economy* 96 (1988): 675-700.

Constantinides, George M., "Habit Formation: A Resolution of the Equity Premium Puzzle," *Journal of Political Economy* 98 (1990): 519-543.

Mansoorian, Arman, "Habit Persistence and the Harberger-Laursen-Metzler Effect in an Infinite Horizon Model," Mimeo, Dalhousie University, 1991.

Obstfeld, Maurice, "Fiscal Deficits and Relative Prices in a Growing World Economy," *Journal of Monetary Economics* 23 (1989): 461-484.

__________, "Intertemporal Dependence, Impatience, and Dynamics," *Journal of Monetary Economics* 26 (1990): 45-75.

Ryder, Harl E., Jr. and Geoffrey M. Heal, "Optimum Growth with Intertemporally Dependent Preferences," *Review of Economic Studies* 40 (1973): 1-33.

Shi, Shouyong and Larry G. Epstein, "Habits and Time Preference," Mimeo, University of Windsor and University of Toronto, 1991.

Sundaresan, Suresh M., "Intertemporally Dependent Preferences and the Volatility of Consumption and Wealth," *Review of Financial Studies* 2 (1989): 73-89.

Uzawa, Hirofumi, "Time Preference, the Consumption Function, and Optimum Asset Holdings," in *Capital and Growth: Papers in Honour of Sir John Hicks*, edited by J. N. Wolfe, Chicago: Aldine, 1968.

Wan, Henry Y., Jr., "Optimal Saving Programs under Intertemporally Dependent Preferences," *International Economic Review* 11 (1970): 521-547.

Weil, Philippe, "Overlapping Families of Infinitely-Lived Agents," *Journal of Public Economics* 38 (1989): 183-198.

Endnotes

*An anonymous referee provided some very helpful suggestions. All remaining errors are my responsibility. Financial support from the National Science Foundation is acknowledged with thanks.

[1]Earlier models with intertemporally dependent tastes include those of Uzawa (1968), Beals and Koopmans (1969), and Wan (1970).

[2]The extension to several consumption goods is straightforward in principle. In general, however, each good will correspond to a good-specific stock of consumption experience, and there may be no simple way to aggregate these stocks into a single measure of *total* consumption experience. For certain utility functions, however, aggregation is possible; see Mansoorian (1991).

[3]The roots of system (9) are $[\theta \pm (\theta^2 - 4\Delta)^{1/2}]/2$.

[4]Since consumption must be nonnegative, any solution involving the system's positive, unstable root, denoted ζ, calls for consumption to grow at a proportional rate that approaches ζ ever more closely over time. From (9), $\zeta - \psi = \theta = r$, so $\zeta = r + \psi > r$. Observe, however, that

$$e^{-rT}A(T) = A(0) - \int_0^T e^{-rt}C(t)dt.$$

If consumption asymptotically grows at a rate exceeding r, $\lim_{T\to\infty} e^{-rT}A(T) < 0$ and therefore (3) cannot hold.

[5]In the case portrayed, a sufficiently large increase in A(0) would result in subsequent wealth decumulation.

[6]The distinction between "human" wealth Y/r and "nonhuman" wealth F(v,t) was unimportant in the representative-agent economy of the preceding sections. It plays a critical role, however, in the present economy's macrodynamics. While aggregate human wealth automatically grows at the rate of labor-force growth, aggregate financial assets grow only through saving.

[7]The assumption $\theta > r$ would be untenable in any case because it implies that an individual's consumption hits zero in finite time. To see why, let $\lambda(v,v) > 0$ be the initial shadow value of wealth for a vintage-v person. It can be shown that the saddlepath value of C(v,t) (in the absence of a nonnegativity constraint on consumption) is given by

$$C(v,t) = -\frac{(\rho+\theta)\gamma_c + \rho\gamma_s}{\gamma_{cc}(\rho+\theta+\Omega)} + \left[\frac{(\rho+\theta-r)(\rho+r)}{(\rho+\theta-r)(\rho+r) + \rho\Omega}\right]\frac{\lambda(v,v)e^{(\theta-r)(t-v)}}{\gamma_{cc}}$$

$$+ \left(\frac{\rho-\psi}{\rho}\right)\left\{S(v,v) - \frac{1}{\gamma_{cc}}\left[\frac{\lambda(v,v)\rho(\rho+r)}{(\rho+\theta-r)(\rho+r) + \rho\Omega}\right] + \frac{(\rho+\theta)\gamma_c + \rho\gamma_s}{\gamma_{cc}(\rho+\theta+\Omega)}\right\} e^{-\psi(t-v)}.$$

The second term on the right-hand side of this equality is negative and grows over time in absolute value when $\theta > r$. The other terms either shrink or remain constant in absolute value. Thus, $C(v,t) < 0$ for t sufficiently large.

When $\lambda(v,v)$ is computed by imposing intertemporal budget balance, one finds that that the preceding equation implies

$$C(v,t) = \frac{(\psi + r)(2r - \theta)}{(\rho + r)}\left[\frac{Y}{r} + F(v,t)\right] + \left(\frac{\rho - \psi}{\rho + r}\right)\left(\frac{\rho + \theta - r}{\rho}\right)S(v,t)$$

$$+ \frac{\psi(r - \theta)[(\rho + \theta)\gamma_c + \rho\gamma_s]}{\gamma_{cc} r\rho(\rho + \theta + \Omega)}.$$

For $\theta < r$, the entire analysis below goes through with this linear relationship used in place of (13) provided $C(v,t) \geq 0$.

[8] Absent by assumption are other types of outside asset, such as physical capital or domestic government debt.

[9] Under the alternative assumption that $S(t,t) = S(t)$ (rather than zero), n would still appear in (19) but would not enter (20).

[10] In general, saving at birth is $[(\rho - \psi)/(\rho + r)]Y$.

[11] See footnote 7 above.

[12] Add (19) to (20). The resulting differential equation has the solution

$$Y + rF(t) - S(t) = [Y + rF(0) - S(0)]e^{-\psi t}$$

when $n = 0$. So even when $\rho > \psi$, $\lim_{T\to\infty}[Y + rF(T) - S(T)] = 0$.