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PRICE RIGIDITIES, ASYMMETRIES, AND OUTPUT FLUCTUATIONS

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ABSTRACT

In this paper we characterize the *average* response of output to aggregate demand shocks in an economy where individual firms follow *state*-dependent pricing rules. We find that: (i) the average response of output to aggregate demand shocks decreases with core inflation and varies non-monotonically with aggregate uncertainty, (ii) there is an asymmetry in the response of output to aggregate demand expansions and contractions, which increases with core inflation and decreases with aggregate uncertainty, and (iii) this asymmetry also rises with the degree of asymmetry of aggregate demand shocks. Using annual data from 37 moderate-low inflation countries for the period 1960-1982, we find support for the basic implications of the model.

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1 INTRODUCTION

Most economists agree that money, in some of its forms, matters.[1] One prominent explanation for the relation between money —here used interchangeably with aggregate demand— and output fluctuations, is the presence of fixed costs of adjusting prices at the firm level. Descriptions of the corresponding microeconomic behavior are well known by now:[2] firms do not adjust their prices continuously but wait until the departure of their actual price from its frictionless counterpart — the *state* — is large enough to justify a price realignment, thereby starting a new cycle. With some abuse of terminology, models of this type are known as *state*-dependent models.

Unlike *time*-dependent models, where the timing of agents' price changes is constant for a given stochastic environment, in state-dependent models the timing of price changes evolves endogenously. How many firms change their prices in any given time interval does not only depend on the stochastic mechanism underlying aggregate and idiosyncratic shocks,[3] but also on the actual realization of these shocks. It follows that, in order to describe the dynamic behavior of the aggregate price level, one has to keep track of the evolution of the cross section distribution of price departures from their optima.[4]

The importance of the endogenous evolution of this cross section distribution is what makes state-dependent models hard to work with, but, at the same time, it is the reason why they can generate realistic and rich dynamic relationships between aggregate demand and output.[5] Furthermore, the economy's "structure" — for us, the stochastic processes describing aggregate demand and idiosyncratic events — affects the "average" shape of the crucial cross section distribution, thus influencing the impact of aggregate demand shocks on output fluctuations.

The main technical contribution of this paper is to circumvent the complex dynamic features of state dependent economies by computing well defined averages over all possible

[1] See, e.g., Rotemberg (1987) and Blanchard (1990).

[2] See, e.g., Barro (1972), and Sheshinski and Weiss (1983).

[3] As is the case in standard time-dependent models.

[4] See Caballero and Engel (1991) and (1992a).

[5] Caplin and Leahy (1991a), Caballero (1991a,b), Bertola and Caballero (1990), and Caballero and Engel (1992b) provide frameworks that describe different aspects of this dynamic relationship: Caplin and Leahy show the existence of a Phillips curve in non-inflationary state-dependent economies affected only by aggregate shocks, while the other papers track down the dynamic behavior of state-dependent economies for given sequences of aggregate shocks. The latter papers also consider idiosyncratic shocks, which have non-trivial and realistic consequences in state-dependent models; see below.

sample paths of aggregate demand. Once this is done, we are able to provide a unifying and simple framework to study the influence of the economy's structure on the *average* aggregate demand/output relationship, in the context of state-dependent models. In particular, we characterize the impact of core inflation, uncertainty and asymmetry of aggregate shocks on the average response of output to aggregate demand shocks, and on the asymmetry in the response of output to aggregate demand expansions and contractions.[6]

We show that the *average* response of output to aggregate demand surprises is decreasing with respect to core inflation, while its relationship with aggregate demand uncertainty is non-monotonic, depending both on idiosyncratic uncertainty and core inflation. The model also implies an asymmetry in the response of output to aggregate demand contractions and expansions, which is increasing with respect to core inflation and decreasing with respect to aggregate and idiosyncratic uncertainty.[7] We also find that asymmetric aggregate demand shocks accentuate the asymmetric response of output to aggregate demand expansions and contractions. Using annual data from 37 moderate-low inflation countries for the period 1960-1982, we find broad support for the basic implications of the model.

The remainder of the paper is organized as follows. Section 2 presents the basic model. The methodology we use to calculate long run averages, which is closely related to the Ergodic Theorem and we therefore call the "ergodic insight," is presented in Section 3. In the following two sections we use the ergodic insight to compute the long run averages of indices that capture how the relation between output fluctuations and aggregate demand depends on the structure of the economy. We also present multi-country evidence supporting the basic implications of state-dependent models. Section 4 studies how the elasticity of output with respect to aggregate demand varies with core inflation and aggregate uncertainty. Asymmetries between output responses to positive and negative shocks, as well as

[6]Our approach allows for direct comparisons with the results in Lucas (1973) and Ball, Mankiw and Romer (1988), who consider the relation between the slope of the short run Phillips curve and the economy's "structure" in economies with imperfect information and time dependent pricing, respectively.

[7]This paper's contributions lies in the computation of proper averages; many of the basic insights leading to our results have been known for quite some time in the state-dependent literature. Most prominently, Tsiddon's (1989) shows that as inflation rises the response of output to aggregate demand shocks declines and the asymmetry between the response of output to positive and negative aggregate demand shocks increases. Even though he considers the effect of a once-and-for-all shock that hits a resting economy, the underlying mechanism is the same as in our case: higher inflation implies that individual units are typically closer to the barrier that triggers price increases than to the one that triggers price reductions. Caballero (1991b) describes a closely related dynamic concept: an abnormally large sequence of positive aggregate shocks leaves firms more prone to increase prices (consumption in his case), thus more (less) responsive to further positive (negative) aggregate shocks. Ball and Mankiw (1992) discuss issues similar to those in Tsiddon's paper but in the context of a two-periods state dependent model; interestingly, the "finite horizon" nature of their model accentuates the asymmetries found by Tsiddon.

the effect of asymmetries in aggregate demand shocks on output fluctuations, are discussed in Section 5. Section 6 includes final remarks; a short appendix follows.

2 The Model

In this section we present a simple framework, rich enough to study how the economy's structure affects the average relation between aggregate demand and output fluctuations. Unless stated otherwise, all variables are in logarithms.

We consider an economy with a large number, n, of industries, each one with a continuum of firms. Firms face non-convex adjustment costs and therefore change their prices only when the difference between their frictionless optimal price and their nominal price reaches S or $-S$.[8] Whenever the difference between the price a firm would charge if there were no adjustment costs and the firm's actual price —a difference we call *"the firm's price deviation"*— is large enough (equal to S), the firm increases its nominal price by a fixed amount. Whenever the price deviation reaches its lower threshold, the firm decreases its price by a fixed amount. For simplicity we assume that price increases and price decreases have the same magnitude, L (where $L < 2S$).[9]

We discretize time and space into intervals of lengths dt and h, respectively, and refer to a time interval of length dt as *"a time period."* We assume that every firm's frictionless optimal price either increases or decreases by h during a time period; the probability of either event taking place is determined by the economy's structure (i.e., core inflation, aggregate uncertainty and idiosyncratic uncertainty), as summarized by Figure 1 and explained below.

Aggregate and idiosyncratic shocks affect a firm's frictionless price one-for-one.[10] Thus, as long as a firm's price deviation remains within the inaction range $(-S, S)$, changes in this variable during a given time interval are equal to the sum of accumulated aggregate and idiosyncratic shocks. Once a firm's price deviation reaches one of its trigger values, the firm resets its price (by either increasing or decreasing it by L) and therefore its price deviation (to either $S - L$ or $L - S$). A new cycle then begins where shocks affect price

[8] Assuming symmetric thresholds removes a constant that is irrelevant for the issues we consider.

[9] This simplifies calculations considerably but does not affect the qualitative aspects of our conclusions.

[10] This requires choosing parameters such that strategic interactions among firms cancel. This can be done in a three-stage, discrete version of the model in Caballero and Engel (1992a). The main results of the paper do not depend on this simplification, however.

deviations one-for-one.

Aggregate demand within a given time period either expands (with probability q) or contracts (with probability $1-q$); in the former case the average increase of frictionless prices is $\Delta a^+ > \bar{\pi} dt$, in the latter case it is $\Delta a^- < \bar{\pi} dt$.[11] The unconditional expected increase of frictionless prices, $q\Delta a^+ + (1-q)\Delta a^-$, is equal to $\bar{\pi} dt$.

We introduce idiosyncratic uncertainty only at the industry level. Thus all firms within a given industry are subject to the same aggregate demand shocks and the same (industry specific) idiosyncratic shocks. The probability that frictionless optimal prices within an industry increase during a time period is larger during aggregate demand expansions than during contractions; we denote these probabilities by q^+ and q^-, respectively.

Conditional on the aggregate demand shock, changes in frictionless prices during a given time period are independent across industries. We also have that the probability that the frictionless optimal price of firms in different industries moves in the same direction grows with the relative importance of aggregate uncertainty.

Figure 1 shows a tree diagram that summarizes how aggregate and idiosyncratic shocks take place every time period. The probabilities q, q^+ and q^- and the quantities $\Delta a^+, \Delta a^-$ and h are determined — as functions of dt, core inflation $\bar{\pi}$, aggregate uncertainty σ_A and total uncertainty σ — by imposing that (a) the (unconditional) mean and variance of the total change in frictionless prices be equal to $\bar{\pi} dt$ and $\sigma^2 dt$, respectively; (b) the conditional mean of the total change in frictionless prices be equal to Δa^+ and Δa^- depending on whether aggregate demand growth is high or low; and (c) the mean and variance of aggregate shocks be equal to $\bar{\pi} dt$ and $\sigma_A^2 dt$, respectively. The corresponding expressions are derived in the Appendix.

As is usual in these models, aggregate output is equal to real balances:

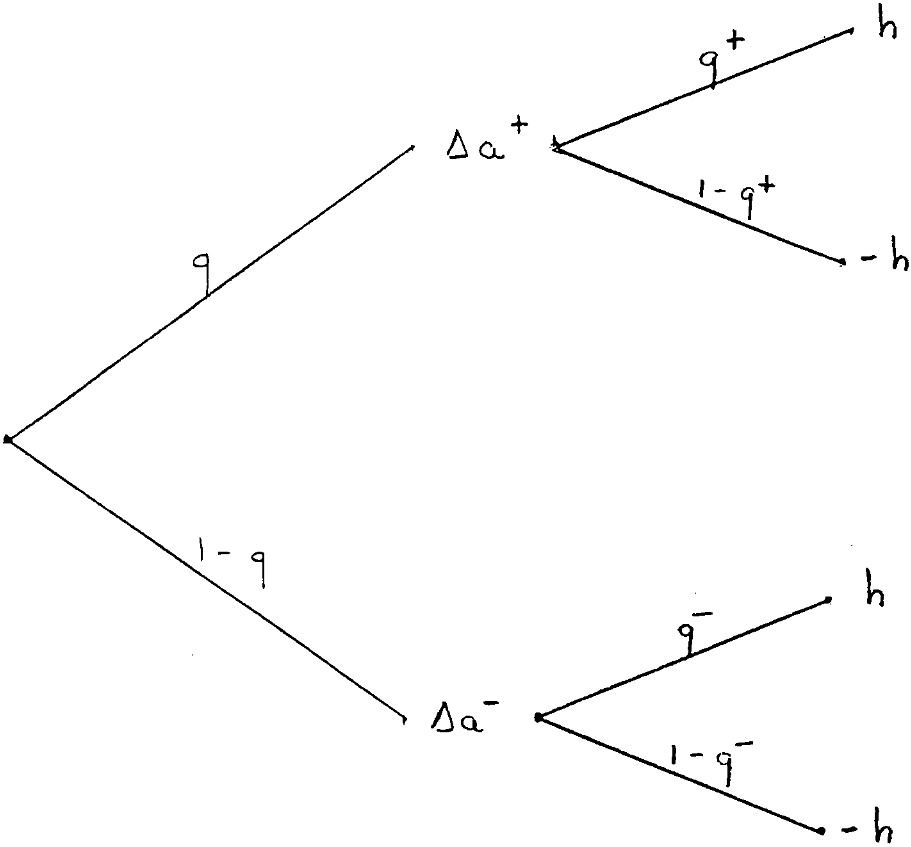
$$y(t) = m(t) - p(t). \tag{1}$$

The aggregate price level is a convex combination of industries' price levels:

$$p(t) = \sum_{i=1}^{n} \alpha_i p_i(t), \tag{2}$$

[11]These averages do not depend on the actual realizations of idiosyncratic shocks because the number of industries is large.

FIGURE 1



where α_i indicates the relative size of the i-th industry and $\sum \alpha_i = 1$.[12] Denoting the price level that would attain in the absence of adjustment costs by $p^*(t)$, we may write:[13]

$$y(t) = (m(t) - p^*(t)) + (p^*(t) - p(t)). \tag{3}$$

Assuming no strategic complementarities, money neutrality in the absence of frictions implies that the first term on the right hand side of (3) is zero,[14] and therefore:

$$y(t) = \sum_i \alpha_i (p_i^*(t) - p_i(t)). \tag{4}$$

The i-th industry's aggregate price deviation, $z_i(t) \equiv p_i^*(t) - p_i(t)$, is equal to the mean of the corresponding cross-section distribution of price deviations. Denoting by $f_{i,t}(z)$ the cross-section density of price deviations within the i-th industry at time t, we have that:

$$y(t) = \sum_i \alpha_i z_i(t) = \sum_i \alpha_i \int z f_{i,t}(z) dz. \tag{5}$$

Aggregate output is therefore equal to a weighted average of mean price deviations across industries. For example, when prices within most sectors in the economy are above their frictionless optimal values, and industries' mean price deviations are therefore near their minimum values, aggregate output is below trend.

To complete our description of the economy, we specify the initial cross-section distribution of price deviations *within* every industry. We assume these distributions are uniform on subintervals of length L included in $[-S, S]$. It then follows from Caplin and Leahy (1991a) that this distribution remains uniform on a subinterval of length L as time passes.[15] Thus the i-th industry's aggregate price deviation, $z_i(t)$, is equal to the midpoint of the corresponding cross-section distribution at time t, and therefore takes values between $-R$ and R, where $R \equiv S - \frac{1}{2}L$.[16]

The simplifying assumptions we have made above (on aggregate and idiosyncratic

[12] The relative sizes could change over time.

[13] Footnote 10 also applies here.

[14] Or equal to a linear time term corresponding to smooth trend-growth.

[15] This is a straightforward generalization of Caplin and Leahy (1991a) who consider the continuous time case with $L = S/2$. Firms' optimal behavior is compatible with $L \neq S/2$ when firms' adjustment costs have both fixed and proportional components (Harrison, Sellke and Taylor, 1983). Caplin and Leahy (1991b) show that strategic interactions may lead to similar microeconomic policies, even when menu costs are only of the lump-sum type.

[16] We assume that both S and R are integer multiples of h.

shocks, the magnitudes of price changes and the initial cross section distributions within industries) imply that our basic unit of analysis, the z_i's, either remain constant or change by $\pm h$; thus, contrary to individual firms, industry-wide deviations never jump.[17] This reduces the complexity of the problem considerably, as we illustrate in the following section.[18]

The presence of idiosyncratic uncertainty is not only realistic but also adds important dimensions to aggregate dynamics. For example, in the presence of idiosyncratic shocks output contractions may occur even when money growth slows down but remains positive. To show this we note that a period of sustained and high monetary expansion typically leads to a situation in which the average price deviation within most industries is near its highest value ($z_i \cong R$); in this case a slowdown in money growth may not be sufficient to prevent positive idiosyncratic shocks from leading to price increases in a large fraction of industries while negative idiosyncratic shocks are unlikely to lead to price reductions in many industries. This "inertia" effect could lead to a reduction in real balances and a consequent fall in aggregate output.[19] Moreover, this effect may be present even when the stock of money decreases, thus creating an environment where recessions are sharper than expansions and larger than the aggregate demand shock itself (see section 5 below).

3 Long Run Averages and the Ergodic Insight

The effect of aggregate demand shocks on output fluctuations in the economy described in Section 2 is highly non-linear and history dependent. For this reason, summarizing this relation via tractable indices is not, a priori, an easy task. In this section we show how to calculate the *average* response of output to aggregate demand shocks, where the averaging procedure weights all possible histories of the economy appropriately. This is the building block we use to construct various indices of average output elasticities in the following sections.

We illustrate the principle underlying our approach by showing how to determine the average response of output when aggregate demand growth is above trend: $\mathrm{E}(\Delta y \,|\, \Delta a^+)$.

[17] Where a jump is a change by more than one unit in state space during a time period.

[18] If we were to incorporate idiosyncratic shocks at the individual firm level, we would be working with processes with jumps at the boundaries. This makes the algebra harder, but does not affect the qualitative aspects of our conclusions.

[19] This inertia effect is similar to the dynamic "attractor" effect described in Caballero (1991a).

This quantity is a function of core inflation and aggregate and idiosyncratic uncertainties. To calculate it, we must (*a*) consider all cross-section distributions of price deviations and all possible combinations of idiosyncratic shocks across industries, (*b*) determine Δy in each case, and (*c*) average over all possible values of Δy weighting them according to their probabilities. We now show that this complex set of calculations reduces to a simple "steady-state-like" experiment once we consider the *linear* nature of the statistics we have chosen — this is one of the main insights in this paper.

We first note that, given the cross-section distribution of price deviations within the i-th industry and the sign of the shock it was subject to, we can determine the change in the industry price deviation, Δz_i. For example, consider the case where the shock increases the industry's frictionless optimal price by h. If the shock does not trigger any price increase, then all firms' frictionless optimal prices increase by h and so does the industry price deviation ($\Delta z_i = h$). Alternatively, if the shock forces a fraction of firms to increase their prices, the average price deviation across the industry remains constant because the cross-section distribution of price deviations does not change (even though individual firms' price deviations do) and in this case $\Delta z_i = 0$.[20]

We let s_i denote a random variable that is equal to one if the i-th industry's frictionless price increases (by h) in a given time period (of length dt), and zero otherwise; that is, s_i is equal to one when the i-th industry's idiosyncratic shock is positive and zero when it is negative. Conditional on the size of the aggregate shock, the s_i's are independent across industries and over time. Since aggregate output is a weighted sum of mean price deviations across industries and we have assumed that there are no strategic interactions between firms' pricing decisions,[21] we have that:

$$(6) \qquad (\Delta y \,|\, \Delta a^+; z_1, z_2, \ldots, z_n; s_1, \ldots, s_n) = \sum_{i=1}^{n} \alpha_i (\Delta z_i \,|\, \Delta a^+, z_i, s_i),$$

where Δy denotes the change in aggregate output and the conditional notation is self-explanatory.

[20] Remember that the discrete nature of the state space determines that no firm lies at a distance less than h from the trigger barriers.

[21] See Section 2.

Taking expectations with respect to the s_i's in (6) now leads to:[22]

$$(7) \qquad \mathrm{E}(\Delta y \mid \Delta a^+; z_1, \ldots, z_n) = \sum_{i=1}^{n} \alpha_i \mathrm{E}(\Delta z_i \mid \Delta a^+, z_i).$$

This equation shows that, since our indices are linear in output, we can calculate them without knowing the joint distribution of the mean price deviations of different industries. All we need are the (conditional) first moments for every industry. Taking expectations with respect to all possible values of industries' mean price deviations on both sides of (7) yields:[23]

$$(8) \qquad \mathrm{E}(\Delta y \mid \Delta a^+) = \sum_{i=1}^{n} \alpha_i \mathrm{E}(\Delta z_i \mid \Delta a^+).$$

We calculate the right hand side of (8) by weighting different values of Δz_i according to the ergodic distribution of the beginning-of-period z_i's, that is, the distribution that would emerge if we plotted a histogram with the z_i's over time.[24,25] The expectations on the right hand side of equation (8) are all identical, because trigger and target levels are the same across industries. It follows that, letting f^- and f^+ denote the probabilities the ergodic distribution assigns to the largest and smallest values of the price deviation, we have:

$$(9) \qquad \mathrm{E}(\Delta y \mid \Delta a^+) = \Delta a^+ - q^+ h f^+ + (1 - q^+) h f^-,$$

or, using the relation $\Delta a^+ = q^+ h - (1 - q^+) h$,

$$(10) \qquad \mathrm{E}(\Delta y \mid \Delta a^+) = q^+ h (1 - f^+) - (1 - q^+) h (1 - f^-).$$

This expression can be interpreted in terms of the following abstract "steady-state-like" experiment:

1. Suppose the cross-section distribution of mean price deviations within industries is

[22] At this step we use the fact that the s_i's are independent conditional on the size of the aggregate demand shock.

[23] This step assumes that Δz_i and z_j are independent, conditional on z_i; this follows from there being no strategic interactions.

[24] The relevant variance when determining the ergodic distribution is the total uncertainty facing an individual firm: σ^2.

[25] We show in the Appendix that this distribution is a truncated geometric distribution. This is not surprising, since its continuous counterpart is a truncated exponential (Harrison, 1985).

the corresponding ergodic distribution (where the corresponding variance is the total variance facing an industry).

2. Calculate the change that takes place in the mean of the preceding distribution after an aggregate demand expansion (like the one depicted in the upper half of Figure 1). This quantity is equal to $\mathrm{E}(\Delta y \mid \Delta a^{+})$.

We call the argument we used above to derive expression (10) "the ergodic insight," since it builds strongly on the Ergodic Theorem.[26] In spite of the similarity of our procedure with the "once-and-for-all" type experiments which have typically been used in this literature — thus the expression "steady-state-like" — it is important to highlight the fundamental difference with such experiments. The standard once-and-for-all experiments assume that the economy experiences only idiosyncratic shocks, therefore the *cross-section* distribution converges to an invariant state. Once the economy is at this resting state, idiosyncratic shocks are suppressed for an instant, the economy is hit with one fully unexpected once-and-for-all common shock, and idiosyncratic shocks reappear bringing the economy back to its steady-state. In our case, on the other hand, we have continuous and simultaneous aggregate and idiosyncratic shocks, thus the cross-section distribution *never* converges to an invariant state. More importantly, aggregate shocks of the same sign and magnitude have very different impact on aggregate output depending upon the history of aggregate shocks up to that time. Our results take the proper *average* over all these possible paths.

Somewhat surprisingly, however, the average over all possible sample paths ends up being a very simple expression that differs from the once-and-for-all shortcut only because of the treatment of idiosyncratic shocks at the time the aggregate shock occurs.[27,28] When ongoing simultaneous aggregate and idiosyncratic shocks are considered, there exist firms that adjust their price in the opposite direction than that implied by the aggregate shock; the bias introduced by ignoring these firms grows with the degree of idiosyncratic uncertainty and can be misleading.

The "ergodic insight" can be used in a similar way to determine other expressions of

[26]The insight can be applied in the continuous space–continuous time case as well, since it is based only on the linearity (in mean price deviations) of the quantity we consider and the assumption of no strategic interactions. Yet, obtaining analytic expressions for the corresponding averages over all trajectories of the aggregate shock process — in the presence of idiosyncratic uncertainty — is more cumbersome in this case.

[27]In a sense, the once-and-for-all shortcut treats the aggregate shock as a finite shock in a model where all sample paths of idiosyncratic shocks are continuous.

[28]The simplicity of this result is due to the linearity of the statistics considered and is lost when calculating higher moments.

interest. For example, a steady-state-like experiment analogous to the one described above shows that $E(\Delta y \mid \Delta a^-)$ is equal to the change that occurs in the mean price deviation when the invariant distribution is subject to an aggregate demand shock of size Δa^- (see the lower half of Figure 1). This leads to:

$$E(\Delta y \mid \Delta a^-) = q^- h(1 - f^+) - (1 - q^-)h(1 - f^-). \tag{11}$$

4 Output Elasticities and Fundamentals

In this section we study how fundamentals — in our model core inflation, and idiosyncratic and aggregate uncertainties — affect the average short run impact of aggregate demand on output.

4.1 Basic Relations

We follow a minimalist strategy and start by describing the expected response of output to a single aggregate demand shock. Despite its simplicity, this statistic is useful to illustrate several qualitative aspects of the average relation between output and aggregate demand in state dependent models. We extend the framework to allow for more than one shock when needed (see Section 5).

We summarize the average effect of aggregate demand shocks on output by the corresponding average elasticity:

$$\tau = E\left[\frac{\Delta y}{\Delta s}\right], \tag{12}$$

where $\Delta s \equiv \Delta a - \overline{\pi} dt$, is the deviation of the aggregate shock, Δa, from its mean, and the expectation is taken over all possible sample paths of the economy as discussed in Section 3. We call Δs "the surprise" since it corresponds to the deviation from expected aggregate demand growth. It is important to notice, however, that the effects we obtain have little to do with expectations: in a sense, for a given set of fixed band policies, the average shape of the cross section distribution replaces rationally formed expectations.

Our objective is to characterize the parameter τ and determine how it varies with fundamentals. In the notation of Section 3 we have that:

$$\tau = \frac{q}{\Delta s^+}\mathrm{E}(\Delta y|\Delta a^+) + \frac{(1-q)}{\Delta s^-}\mathrm{E}(\Delta y|\Delta a^-),$$

where $\Delta s^+ = \Delta a^+ - \bar{\pi}dt$ and $\Delta s^- = \Delta a^- - \bar{\pi}dt$. It then follows from the ergodic insight that:[29]

$$\tau = 1 - \tfrac{1}{2}(f^+ + f^-), \tag{13}$$

where f^+ and f^- denote the fraction of industries which, on average, are nearest to their upper and lower trigger levels, respectively.

Equation (13) can be interpreted as follows: What determines the responsiveness of output to aggregate demand shocks in an economy where pricing is state-dependent is the fraction of industries adjusting their prices.[30] The larger the fraction of firms that adjust their prices, the smaller the average short run effect of aggregate demand shocks on output.

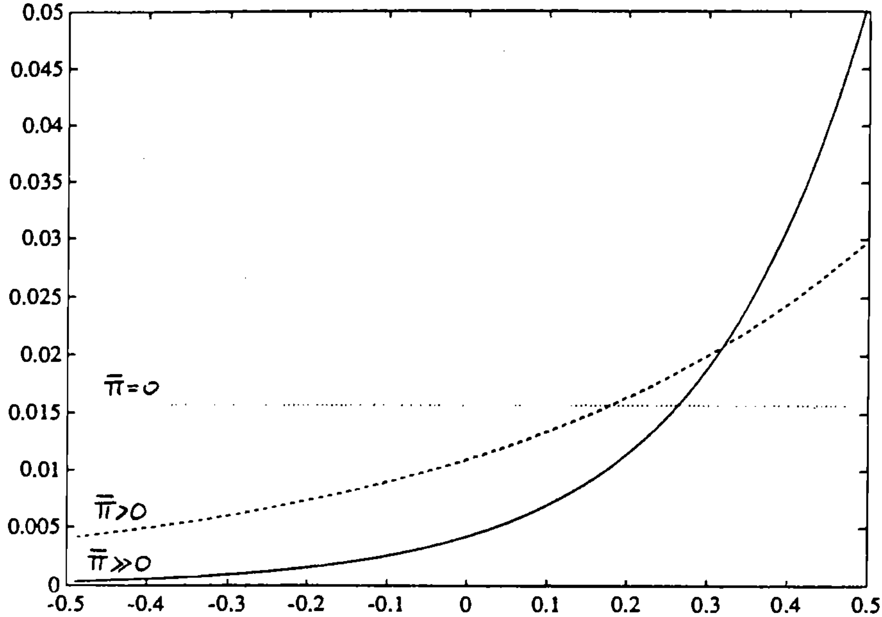
Figure 2 shows the ergodic density of price deviations for various values of $\bar{\pi}/\sigma^2$. When core inflation is positive, industries are closer (on average) to the point where they increase their price than to the point that triggers a price decrease. As core inflation becomes larger, the fraction of firms that increases prices grows and the fraction lowering prices becomes smaller. Figure 2 shows that the number of firms increasing prices grows faster with core inflation than the fraction lowering prices decreases; it follows that τ decreases with core inflation. We call the effect underlying this result the "convexity effect," since it stems from the fact that the ergodic density becomes more convex as core inflation becomes larger.

Next we look at how changes in aggregate uncertainty, σ_A, affect the average elasticity index τ. Equation (13) shows that an increase in σ_A has an effect on τ only if it is accompanied by an increase in σ, which we assume is the case. There are two opposing effects that set in as σ_A varies. First, since the ergodic density becomes flatter as σ becomes larger, the convexity effect described above implies that an increase in aggregate uncertainty leads to a larger value of τ. This effect is absent when core inflation is zero (because in this case the ergodic density is uniform for all values of σ_A); its magnitude grows with core inflation. Yet there is a second effect which may revert the preceding conclusion: as

[29] See the appendix for details.

[30] The magnitudes of price adjustments and how these vary with fundamentals also matter; yet this effect only dampens the magnitude of the derivatives of τ with respect to fundamentals, it does not change their sign.

Figure 2: Ergodic densities



uncertainty grows, firms that change their prices may be further away from their trigger levels and therefore their number may increase. The relation between aggregate uncertainty and the expected response of output to aggregate demand shocks is more cumbersome when more than one shock is considered; however the mechanisms we have stressed here are still predominant.

Figures 3a and 3b plot τ as functions of $\overline{\pi}$ and σ_A for a wide range of parameter values and different levels of idiosyncratic uncertainty; where the latter corresponds to $\sigma_I \equiv \sqrt{\sigma^2 - \sigma_A^2}$.[31] It can be seen that τ decreases with core inflation and that the relation with σ_A is non-monotonic. The convexity effect dominates when core inflation is relatively large and total uncertainty small; in this case τ grows with the level of uncertainty in the economy. The opposite happens when core inflation is small and aggregate uncertainty is sufficiently large.

4.2 Empirical Assessment

Following Ball, Mankiw and Romer (1988) (henceforth B-M-R), our data consist of annual real and nominal GNP or GDP (as available) for 37 unplanned economies with at least 10 percent of output in manufacturing and less than 30 percent in agriculture, for the period 1960 to 1982. We also exclude extreme —in terms of inflation and variance of aggregate demand— observations (Argentina, Bolivia, Brazil, Israel, Peru and Zaire). We split the observations in each country into pre-1973 and 1973 onwards; and treat each half as separate countries.[32]

We estimate τ as a function of $\bar{\pi}$ and σ_A (where the latter are in percentage points) by running the panel-regression:

$$\widetilde{\Delta y_{ij}}(t) = \tau(\overline{\pi}_{ij}, \sigma_{A_{ij}})\, \widetilde{\Delta x_{ij}}(t) + e_{ij}(t), \tag{14}$$

[31] In these and the following panels we assume a time interval of one month and a value of R which varies with the level of total uncertainty. We let $R = 0.1$ for the average case in Figure 3a, and adjust the value of the others by a factor $(\sigma^2/\bar{\sigma}_{3a}^2)^{1/4}$; where σ^2 is the total variance of the particular case and $\bar{\sigma}_{3a}^2$ is the value of the variance for the average case in Figure 3a. We should point out, however, that the endogenous behavior of bands has only second order effects on the diagrams. For this reason we do not discuss the effect due to change in bandwidth in our experiments.

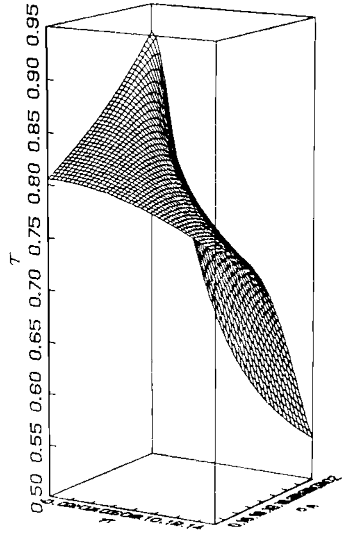
[32] This integrates the two separate dimensions of the data exploited by B-M-R into a single procedure, which enhances the power of our tests. B-M-R show that the hypothesis of equal pre- and post-1973 τ's can be rejected in about 63% of the countries.

FIGURE 3

3a

τ

σ_r=0.02,dt=1/12



3b

τ

σ_r=0.20,dt=1/12

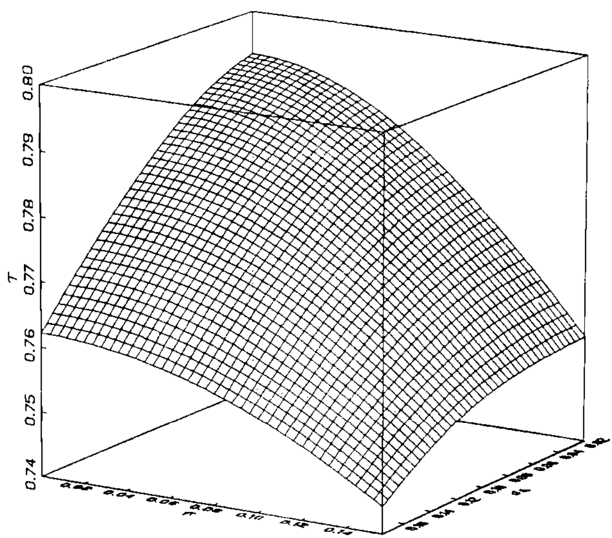


Table 1: Average short run elasticities

	const.	$\bar{\pi}$	σ_A
τ	0.456	−0.029	0.025
	(0.053)	(0.003)	(0.006)

where $\Delta w(t) \equiv w(t) - w(t-1)$, $\widetilde{\Delta y_{ij}}(t) \equiv \Delta y_{ij}(t) - \overline{\Delta y_{ij}}$, $\widetilde{\Delta x_{ij}}(t) \equiv \Delta x_{ij}(t) - \overline{\Delta y_{ij}} - \bar{\pi}_{ij}$, country and sample period (pre- and post-1973) are indexed by i and j, respectively, Δy and Δx denote growth rates of real and nominal GNP/GDP, and bars denote the averages of the corresponding variable over the sample period.[33] We tried linear and log-linear functional forms for τ; since this did not matter much, below we only report the results with the linear specification.[34]

Table 1 shows the results from the regression described above (standard deviations are in parenthesis). The sign of the coefficient for core inflation is that predicted by the model and the coefficient is statistically significant. The coefficient for σ_A is positive and significant, which is consistent with our model (when core inflation is relatively large and total uncertainty relatively small).

The quantitative impact of changes in $\bar{\pi}$ and σ_A are relevant, as can be seen from Table 2. For example, the increase in average inflation of a U.S.-like country between the periods 1961-72 and 1973-82 from 1.5% to 6.5% (accompanied by almost no change in aggregate uncertainty) leads to a decrease of τ from 0.46 to 0.32.

Our finding that in models with state-dependent pricing an increase in aggregate uncertainty may lead to more output responsiveness, which is in agreement with the results in Table 1, is in clear contrast with the results in Lucas' (1973) imperfect information model and Ball, Mankiw and Romer's (1988) time-dependent pricing model: both these models

[33] We have subtracted the average growth rate of real GDP/GNP from Δy_{ij} and Δx_{ij} in (14) because our model has no implications for long run growth.

[34] Our approach is similar to that followed by B-M-R and Lucas (1973), among others, who estimate individual countries' τ's by running the regression $y(t) = \tau\Delta x(t) + \lambda y(t-1) + \gamma t$, and then use the estimated values of τ in a second stage regression against the arguments of main interest.

Table 2: Quantitative Impact

	$(\bar{\pi}, \sigma_A)$	τ
1961 − 72	(4.2 , 3.2)	0.41
Mean		
1973 − 82	(12.5 , 5.3)	0.23
1961 − 72	(1.5 , 1.7)	0.46
U.S.-like		
1973 − 82	(6.5 , 2.0)	0.32

Calculations are based on the results from Table 1.

predict that an increase in aggregate uncertainty leads to less rigidities in the economy.[35] What happens in state-dependent models is that, provided that core inflation is different from zero, an increase in aggregate uncertainty may actually lower the number of firms changing prices through the dampening of the convexity effect, thereby making output more responsive to aggregate demand shocks.

5 Asymmetries

State-dependent models are well known sources of asymmetries: since positive core inflation makes price reductions less likely than price increases, the number of firms that change prices when aggregate demand accelerates is larger than those adjusting when aggregate demand slows down. It follows that prices exhibit more rigidity during aggregate demand expansions than during contractions.

In this section we apply the ergodic insight to calculate well defined average asymmetry

[35] Taking idiosyncratic uncertainty as given, in Lucas' case the degree of confusion faced by agents decreases when σ_A becomes larger, while in Ball, Mankiw and Romer's case the frequency of price adjustments increases.

indices, and study how these indices vary with core inflation and aggregate uncertainty. We also explore the effect of asymmetric aggregate demand shocks on these indices.[36]

5.1 BASIC RELATIONS

Before describing asymmetries of average output elasticities with respect to aggregate demand *surprises*, it is instructive to consider asymmetries between the average elasticities of output with respect to positive and negative aggregate demand *shocks* (Δa^+ and Δa^-, respectively). For this purpose we define:

$$\gamma^+ \equiv \mathrm{E}\left[\frac{\Delta y}{\Delta a^+}|\Delta a^+\right], \qquad \gamma^- \equiv \mathrm{E}\left[\frac{\Delta y}{\Delta a^-}|\Delta a^-\right].$$

Using the conditions $\mathrm{E}[\Delta y] = 0$ and $\Delta a^- = -\Delta a^+$,[37] we have that:

$$\frac{\gamma^-}{\gamma^+} = \frac{q}{(1-q)},$$

which means that the asymmetry in the average elasticity of output with respect to positive and negative aggregate demand shocks is proportional to the inverse of the relative frequency with which these shocks occur. That is, in an economy with positive core inflation, negative surprises are less likely to occur and, precisely for this reason, when they occur their effect on output is larger. Obviously, then, the ratio γ^-/γ^+ is increasing with respect to core inflation and decreasing with respect to aggregate uncertainty.

The same reasoning shows that τ^+ and τ^-, defined as $\mathrm{E}\left[\Delta y|\Delta a^+\right]/\Delta s^+$ and $\mathrm{E}\left[\Delta y|\Delta a^-\right]/\Delta s^-$, must be equal. Indeed,

$$\frac{\tau^-}{\tau^+} = \frac{q}{(1-q)}\left|\frac{\Delta s^+}{\Delta s^-}\right|;$$

but the relative size of the positive and negative surprises is exactly the inverse of the relative probability of experiencing these surprises.[38] Long run neutrality of aggregate demand shocks then rules out any asymmetry with respect to aggregate demand surprises when considering the effect of only one shock.

[36] See Cover (1988) and De Long and Summers (1988) for empirical evidence addressing some of the issues discussed in this section.

[37] The first condition is very weak and says that monetary policy cannot affect the long run rate of output growth; the second condition follows from imposing that the variance of idiosyncratic shocks, conditional on the aggregate surprise being positive and negative, be the same. This condition, which was not imposed in Section 4, is important when considering asymmetries; see the Appendix for more details.

[38] With the notation of Section 3 we have that $\mathrm{E}[\Delta a] = \bar{\pi}dt$ implies that $|\Delta s^+/\Delta s^-| = (1-q)/q$.

In order to capture asymmetries with respect to surprises in aggregate demand we need to look at more than one period, so more than two states of nature can be observed. Continuing with our minimalist approach, Figure 4 shows the corresponding asymmetry index when *two* periods are considered (i.e., there are four possible outcomes). This index compares the cumulative elasticity of output with respect to two consecutive negative aggregate demand surprises, τ_2^-, to the corresponding elasticity with respect to two consecutive positive surprises, τ_2^+.[39] Figure 4 shows results consistent with those obtained above for the asymmetry between γ^- and γ^+: the degree of asymmetry grows with core inflation and decreases with aggregate uncertainty.[40] Because of the convexity effect, a positive shock increases the number of units near the trigger boundaries by more than a negative shock reduces it (relative to the size of the surprises). This implies that a positive shock following a positive shock has a proportionally larger effect on prices than a negative shock following a negative shock; of course the opposite is true for output, explaining the asymmetry.

5.2 Asymmetric Aggregate Demand Shocks

Another natural source of asymmetry in the response of aggregate output to aggregate demand shocks is asymmetries in aggregate demand surprises: e.g., negative surprises are larger (and less frequent) than positive surprises. It is therefore interesting to determine whether asymmetries in aggregate demand shocks further amplify output fluctuations, thereby increasing the asymmetries in output elasticities discussed above. We incorporate asymmetries in shocks by scaling Δs^- by α and Δs^+ by $1/\alpha$.[41]

Figures 5 shows how the average elasticity of output with respect to aggregate demand surprises varies with the asymmetry parameter α.[42] This figure shows that output responsiveness to large (defined as consecutive shocks of the same sign) aggregate demand shocks grows as negative surprises become larger than positive surprises, although these effects are small. Also the asymmetry between output responses to (large) positive and nega-

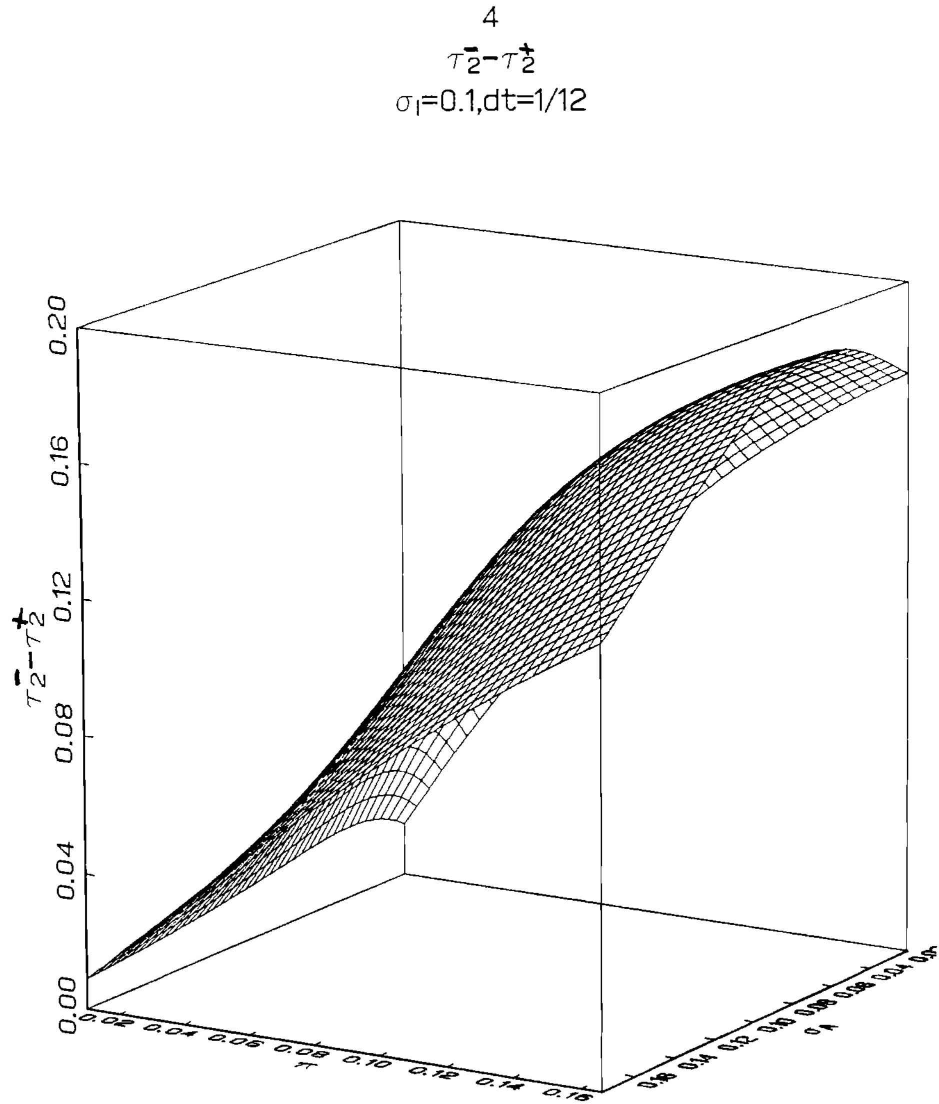
[39]The reason why the condition $E[\Delta y] = 0$ does not constrain these statistics to be equal is that any difference between them is compensated by the difference between the responses of output to a positive-negative and a negative-positive sequences of aggregate demand shocks.

[40]This index is calculated using the ergodic insight, see the appendix. We also assume that an increase in σ_A leads to an increase in σ.

[41]This preserves the condition $|\Delta s^+ \Delta s^-| = \sigma_A^2 dt$, which constrains the sizes of surprises; see the Appendix.

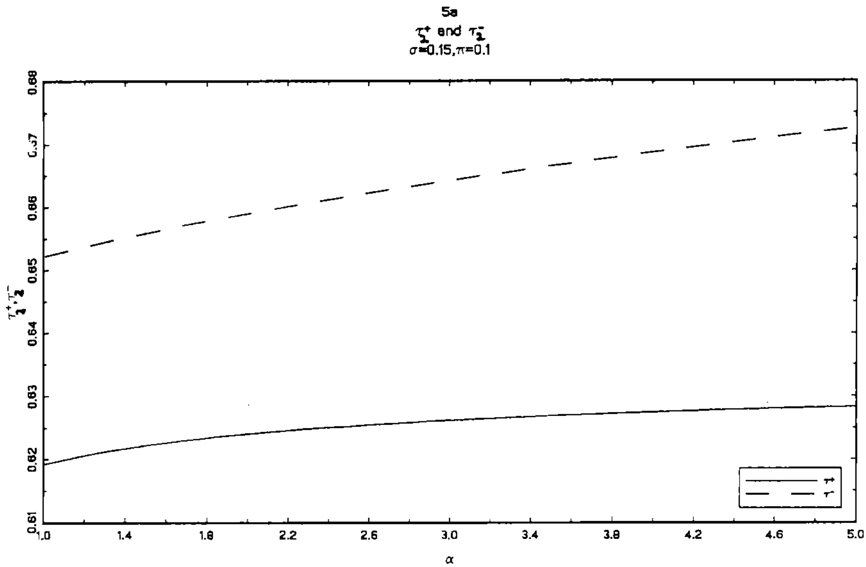
[42]We consider two time periods. An argument similar to the one we gave in Section 4 shows that we need to consider more than one time period when constructing this index; i.e., the index τ considered in Section 4 and the indices τ^+ and τ^- constructed in this section do not depend on α.

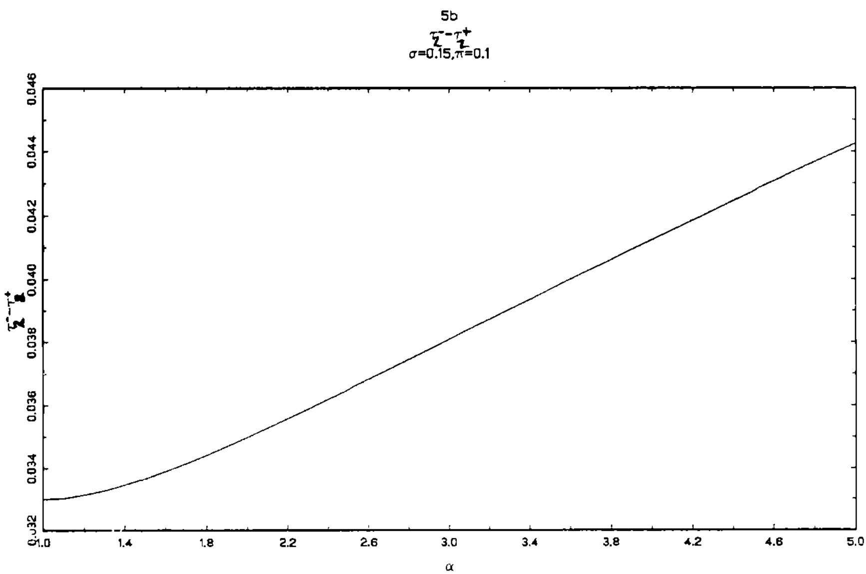
FIGURE 4



4
$\tau_2^- - \tau_2^+$
σ_1=0.1,dt=1/12
0.20
0.16
0.12
0.08
0.04
0.00
$\tau_2^- - \tau_2^+$
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
π
σ_A

FIGURE 5


5a
τ_2^+ and τ_2^-
σ=0.15, π=0.1
τ_2^+, τ_2^-
0.61
0.62
0.63
0.64
0.65
0.66
0.67
0.68
1.0
1.4
1.8
2.2
2.6
3.0
3.4
3.8
4.2
4.6
5.0
α
τ^+
τ^-


5b
$\tau_2^- - \tau_2^+$
σ=0.15, π=0.1
$\tau_2^- - \tau_2^+$
0.032
0.034
0.036
0.038
0.040
0.042
0.044
0.046
1.0
1.4
1.8
2.2
2.6
3.0
3.4
3.8
4.2
4.6
5.0
α

tive aggregate demand surprises grows with the relative importance of negative surprises. The explanation for this is similar to that of the asymmetry discussed above. As negative surprises become larger, they necessarily become less frequent, thus when they take place their effect on output is larger. This argument, which directly explains asymmetries in γ-like statistics, can be converted into an argument about τ_2 statistics by means of the convexity effect described when discussing Figure 4.

5.3 Empirical Assessment

We estimate τ_2^+, τ_2^-, γ^+ and γ^- as functions of core inflation, aggregate uncertainty and an index of asymmetry of aggregate demand shocks using the data described in Section 4.2.

We estimate τ_2^+ and τ_2^- from the equation:

$$\widetilde{\Delta y_{ij}}(t) = \tau_2^+(\bar{\pi}_{ij}, \sigma_{Aij}, \alpha_{ij}) I\{\widetilde{\Delta x_{ij}}(t) > \sigma_{Aij}\}\widetilde{\Delta x_{ij}}(t) + \tau_2^-(\bar{\pi}_{ij}, \sigma_{Aij}, \alpha_{ij}) I\{\widetilde{\Delta x_{ij}}(t) < -\sigma_{Aij}\}\widetilde{\Delta x_{ij}}(t) + e_{ij}(t), \tag{15}$$

where α_{ij} is an index of asymmetry in aggregate demand shocks (equal to the ratio of the average of the squares of shocks below and above the mean), and $I\{A\}$ denotes the indicator function of the set A.[43]

The equation where γ^+ and γ^- are estimated is similar to (15), except for the cutoff points and the treatment of core inflation (here added to the right hand variables):[44]

$$\widetilde{\Delta y_{ij}}(t) = \gamma^+(\bar{\pi}_{ij}, \sigma_{Aij}, \alpha_{ij}) I\{\widetilde{\Delta x_{ij}}(t) > 0\} + \gamma^-(\bar{\pi}_{ij}, \sigma_{Aij}, \alpha_{ij}) I\{\widetilde{\Delta x_{ij}}(t) < -\bar{\pi}_{ij}\}(\widetilde{\Delta x_{ij}}(t) + \bar{\pi}_{ij}) + e_{ij}(t). \tag{16}$$

The results in Table 3 clearly accord with the basic implications of the state dependent model described here: among the six differences between coefficients measuring responses to aggregate demand expansions and contractions (two for each one of the parameters $\bar{\pi}$, σ_A and α) five are significant (four at the 5% level and one at the 10% level) and all of

[43] Observations for which both indicators functions on the right hand side of (15) are zero are excluded from the panel regression.

[44] Footnote 43 also holds for equation (15).

Table 3: Average short run elasticities

	const.	$\bar{\pi}$	σ_A	α
τ_2^+	0.516 (0.082)	−0.024 (0.005)	0.010 (0.008)	0.047 (0.018)
τ_2^-	0.642 (0.088)	−0.008 (0.006)	0.012 (0.009)	0.135 (0.033)
$\tau_2^- - \tau_2^+$	0.126 (0.120)	0.016 (0.008)	0.002 (0.012)	0.088 (0.038)
γ^+	0.175 (0.030)	−0.007 (0.002)	0.009 (0.003)	0.023 (0.008)
γ^-	1.181 (0.309)	0.110 (0.027)	−0.059 (0.036)	0.342 (0.123)
$\gamma^- - \gamma^+$	1.006 (0.319)	0.117 (0.027)	−0.068 (0.036)	0.319 (0.123)

Table 4: Quantitative Impact

	$(\bar{\pi}, \sigma_A, \alpha)$	τ_2^+	τ_2^-	γ^+	γ^-
1961 – 72	(4.2 , 3.2 , 0.76)	0.39	0.47	0.14	1.00
Mean					
1973 – 82	(12.5 , 5.3 , 0.50)	0.18	0.24	0.09	1.56
1961 – 72	(1.5 , 1.7 , 1.19)	0.46	0.54	0.16	0.94
U.S.-like					
1973 – 82	(6.5 , 2.0 , 1.17)	0.33	0.50	0.13	1.49

Calculations are based on the results from Table 3.

these differences have the right sign.[45]

Most interestingly, Table 3 shows that an increase in core inflation clearly lowers the average response of the economy to aggregate demand shocks and raises the asymmetry in the response to aggregate demand expansions and contractions, regardless of how the latter are measured (around zero or large around core inflation). An increase in the degree of asymmetry in aggregate demand shocks enhances the effect of aggregate demand on output and sharpens the asymmetry between negative and positive elasticities.

Table 4 below reports the quantitative impact of our estimates. The first row shows the different statistics for a country that has core inflation, standard deviation of aggregate demand, and index of aggregate demand asymmetry, equal to the respective sample averages (across countries) for the period 1961–72: 4.2 and 3.2 percent, and 0.76. The second row corresponds to the same concept for the period 1973–82. The bottom two rows illustrate the implied values for a country with average inflation, aggregate demand uncertainty, and index of aggregate demand asymmetry like those in the U.S. in the two subsamples. The table shows that the short run impact of aggregate demand on real output is on average large and that the asymmetry is sharp.

The table also shows that the interaction between core inflation, aggregate demand

[45]The lack of precision in the estimates of the function τ^- is likely to be due to the fact that only 230 out of the 814 observations are used.

uncertainty and the index of aggregate demand asymmetry is important. Countries with very low inflation (e.g. U.S.-like 61-72) can still have large asymmetries if accompanied by low levels of uncertainty and large aggregate demand asymmetry indices.

Finally, it is important to notice the large values of γ^-, implying that the "inertia" effect is important; this highlights the relevance of considering idiosyncrasies when describing aggregate implications of adjustment costs models.

6 Conclusion

In this paper we characterize the implications of state-dependent models for the impact of aggregate demand shocks on real activity. A distinctive feature of these models is a strong and cumbersome form of history dependence; the main feature of our approach is that we consider a well defined *average* over all possible histories.

Within this framework we show that in general output becomes less responsive to aggregate demand surprises as core inflation rises. The effect of aggregate uncertainty on this statistic, on the other hand, is cumbersome and depends on core inflation and idiosyncratic uncertainty. We also show that (*i*) the asymmetry in the response of output to aggregate demand contractions and expansions is increasing with respect to core inflation and generally decreasing with respect to aggregate uncertainty, and that (ii) the impact of asymmetric aggregate demand shocks is amplified by the state dependent mechanism.

We find evidence from a sample of low-moderate inflation countries to be broadly supportive for the implications of our model. This is not only qualitatively true but also quantitatively important. These results are suggestive but certainly much remains to be done. In particular, our model assumes that the likelihood of positive and negative aggregate demand shocks is independent of the position of the cross-section distribution of price deviations, which is undoubtedly unrealistic if stabilization policies are present at all. We intend to pursue this in future research.

APPENDIX

A1. THE MODEL'S PARAMETERS AND THE STRUCTURE OF THE ECONOMY

In this section we derive the expressions for h, q, q^+, q^- and Δs^+ in terms of the parameters that characterize the economy described in Section 2: $\bar{\pi}$, σ, σ_A, dt and Δs^-.[46,47]

We denote the change in the frictionless optimal price of firms within a given industry during a time period of length dt by Δp_i^* and the size of the corresponding aggregate demand shock and surprise by Δa and Δs, respectively.

PROPOSITION 1 *If the economy satisfies:*

$$\text{(17)} \quad \mathrm{E}[\Delta a] = \bar{\pi} dt,$$

$$\text{(18)} \quad \mathrm{Var}[\Delta a] = \sigma_A^2 dt,$$

$$\text{(19)} \quad \mathrm{E}[\Delta p_i^* | \Delta a^+] = \Delta a^+,$$

$$\text{(20)} \quad \mathrm{E}[\Delta p_i^* | \Delta a^-] = \Delta a^-,$$

$$\text{(21)} \quad \mathrm{E}[\Delta p_i^*] = \bar{\pi} dt,$$

$$\text{(22)} \quad \mathrm{Var}[\Delta p_i^*] = \sigma^2 dt,$$

$$\text{(23)} \quad (\Delta p_i^*)^2 = h^2;$$

then:

$$\text{(24)} \quad h = \sqrt{\sigma^2 dt + \bar{\pi}^2 dt^2},$$

$$\text{(25)} \quad |\Delta s^+ \Delta s^-| = \sigma_A^2 dt,$$

$$\text{(26)} \quad q^+ = \tfrac{1}{2}\left(1 + \tfrac{\Delta s^+ + \bar{\pi} dt}{h}\right),$$

$$\text{(27)} \quad q^- = \tfrac{1}{2}\left(1 + \tfrac{\Delta s^- + \bar{\pi} dt}{h}\right),$$

$$\text{(28)} \quad q = \frac{|\Delta s^-|}{\Delta s^+ + |\Delta s^-|}.$$

PROOF: From (21) and (22) we have that $\mathrm{E}[(\Delta p_i^*)^2] = \sigma^2 dt + \bar{\pi}^2 dt^2$, which combined with (23) leads to the expression for h.

[46] See Section 2 for the definitions of these expressions.
[47] Recall that $\Delta s^{\pm} = \Delta a^{\pm} - \bar{\pi} dt$.

Condition (19) implies that $q^+h-(1-q^+)h = \bar{\pi}dt + \Delta s^+$, which leads to the expression for q^+. A similar argument yields the expression for q^-.

Condition (17) implies that:

$$\bar{\pi}dt = q(\bar{\pi}dt + \Delta s^+) + (1-q)(\bar{\pi}dt + \Delta s^-),$$

which leads to the expression for q.

Conditions (17) and (18) lead to the following expression for $\mathrm{E}[(\Delta a)^2]$:

$$\sigma_A dt + \bar{\pi}dt^2 = q(\bar{\pi}dt + \Delta s^+)^2 + (1-q)(\bar{\pi}dt + \Delta s^-)^2,$$

which, after some algebra (that uses (27)), yields:

$$|\Delta s^+ \Delta s^-| = \sigma_A^2 dt. \tag{29}$$

A straightforward calculation can be used to show that *all* conditions (17 – 23) are satisfied by the expressions in (24 – 28), thereby completing the proof. ∎

It is interesting to note that the economy's structure does not determine the posible sizes of aggregate demand surprises, it only fixes their product (see (29)). The following examples illustrate two cases of particular interest.

EXAMPLES

1. If we impose that both aggregate demand *surprises* have the same magnitude ($\Delta s^- = -\Delta s^+$) then $q = \frac{1}{2}$ and
$$q^{\pm} = \frac{1}{2}\left(1 \pm \frac{\sigma_A\sqrt{dt} \pm \bar{\pi}dt}{h}\right).$$

2. If we impose that both aggregate demand *shocks* have the same magnitude ($\Delta a^- = -\Delta a^+$) then

$$q = \frac{1}{2}\left(1+\frac{\bar{\pi}dt}{h_A}\right), \tag{30}$$
$$q^+ = \frac{1}{2}\left(1+\frac{h_A}{h}\right), \tag{31}$$
$$q^- = \frac{1}{2}\left(1-\frac{h_A}{h}\right), \tag{32}$$

where $h_A = \sqrt{\sigma_A^2 dt + \bar{\pi}^2 dt^2}$. ∎

The conditions that determine the parameters of the economy in Proposition 1 do not consider the conditional idiosyncratic variances. The following proposition shows that if we impose that these variances be the same, then Δs^- is no longer a "free" parameter and is determined by $\bar{\pi}$, σ_A, σ and dt instead.

PROPOSITION 2 *Suppose the conditions in Proposition 1 hold. Then a necessary and sufficient condition for having* $\text{Var}[\Delta p_i^* | \Delta a^+] = \text{Var}[\Delta p_i^* | \Delta a^-]$ *is that* $\Delta a^- = -\Delta a^+$. *In this case* q, q^+ *and* q^- *are given by the expressions derived in Example 2 above and both conditional idiosyncratic variances are equal to* $\sigma^2 - \sigma_A^2$.

PROOF: A calculation from first principles shows that

$$\begin{aligned}
\text{Var}[\Delta p_i^* | \Delta a^+] &= \sigma^2 dt - (\Delta s^+)^2 - 2\Delta s^+ \bar{\pi} dt, \\
\text{Var}[\Delta p_i^* | \Delta a^-] &= \sigma^2 dt - (\Delta s^-)^2 - 2\Delta s^- \bar{\pi} dt,
\end{aligned}$$

which implies, among other things, that the latter is larger than the former when both aggregate surprises have the same magnitude and core inflation is positive.

Sufficiency and necessity follow from the preceding expressions and some straightforward (yet tedious) calculations. ∎

A2. THE ERGODIC DISTRIBUTION

The evolution of the cross-section distribution of price deviations *within* a given industry only depends on total uncertainty as measured by σ and not on aggregate uncertainty, σ_A. What determines this evolution, and therefore the invariant distribution of its mean, is the probability that frictionless optimal prices increase by h in any given time period. Denoting this probability by $\bar{q}$, Figure 1 shows that $\bar{q} = q \cdot q^+ + (1-q)q^-$, and Proposition 1 then implies that:

$$\bar{q} = \tfrac{1}{2}\left(1 + \tfrac{\bar{\pi} dt}{h}\right). \tag{33}$$

Let n denote the number of values the average price deviation within a given industry can take: in the notation of Section 2, $n = 2R/h$, which we assume is an integer. Let $f_1, f_2, \ldots, f_n$ denote the probabilities the corresponding invariant distribution assigns to these possible values: in the notation of Section 3 we have $f^- = f_1$ and $f^+ = f_n$. Then the ergodic distribution is characterized by the following set of equations:

$$\begin{array}{rclcl}
f_1 & = & (1-\overline{q})f_1 & + & (1-\overline{q})f_2, \\
f_2 & = & \overline{q}f_1 & + & (1-\overline{q})f_3, \\
f_3 & = & \overline{q}f_2 & + & (1-\overline{q})f_4, \\
f_4 & = & \overline{q}f_3 & + & (1-\overline{q})f_5, \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
f_{n-1} & = & \overline{q}f_{n-2} & + & (1-\overline{q})f_n, \\
f_n & = & \overline{q}f_{n-1} & + & \overline{q}f_n.
\end{array}$$

If initial mean price deviations differ from their largest and smallest possible values, shocks of size h (respectively $-h$) increase (respectively decrease) the average by h. The equations for $f_2, \ldots, f_{n-1}$ follow from this observation. On the other hand, the mean price deviation remains unchanged when it takes its largest (respectively smallest) possible value and a positive (respectively negative) shock takes place. The latter observation leads to the expressions for f_1 and f_n.

PROPOSITION 3 *With the notation and definitions introduced above, the invariant distribution of a particular industry's mean price deviation is such that:*

$$f_k = \frac{\beta - 1}{\beta^n - 1}\beta^{k-1}; \qquad\qquad k = 1, \ldots, n;$$

where $\beta = \overline{q}/(1-\overline{q})$ *and* $\overline{q}$ *is defined in (33).*

PROOF; The existence of the invariant distribution follows from elementary results for Markov chains, since all states can be reached from each other. Solving the set of equations that characterize $f_1, \ldots, f_n$ above and using the fact that $\sum f_k = 1$ the leads to the desired expression for f_k. ∎

A3. EXPRESSIONS FOR THE SUMMARY STATISTICS

The expressions for the summary statistics introduced in Sections 4 and 5 follow directly from the ergodic insight (see Section 3), some straightforward calculations, and the results of the preceding sections of this appendix.

PROPOSITION 4 *With the notation introduced in Section 3 and the assumptions of Proposition 1:*

$$\tau = 1 - \tfrac{1}{2}(f^+ + f^-).$$

PROOF; The ergodic insight (Section 3) implies that:

$$\tau = \frac{qh}{\Delta s^+}[q^+(1-f^+) - (1-q^+)(1-f^-)] + \frac{(1-q)h}{\Delta s^-}[q^-(1-f^+) - (1-q^-)(1-f^-)].$$

Tedious substitutions, using the preceding results from this appendix, then yield:

$$\tau = 1 - \tfrac{1}{2}(f^+ + f^-) - \tfrac{h(\Delta s^+ + \Delta s^-)}{\sigma_A^2 dt}\left[\tfrac{1}{2}(2 - f^+ - f^-)\left(1 + \tfrac{\pi dt}{h}\right) - (1 - f^-)\right].$$

A calculation from first principles shows that the term in square brackets in the expression above is equal to zero, thereby completing the proof. ∎

PROPOSITION 5 *With the assumptions of Proposition 1 and the notation introduced above and in Section 5 we have that:*

$$(34) \qquad \tau_2^+ = \tau - \frac{h}{2\Delta s^+}\left[q^+(f_E^+ - f^+) + (1-q^+)(f^- - f_E^-)\right],$$

$$(35) \qquad \tau_2^- = \tau + \frac{h}{2\Delta s^-}\left[q^-(f^+ - f_C^+) + (1-q^-)(f_C^- - f^-)\right],$$

where $f_E^+ = q^+(f^+ + f_{n-1})$, $f_E^- = (1-q^+)(f^- + f_2)$, $f_C^+ = q^-(f^+ + f_{n-1})$ *and* $f_C^- = (1-q^-)(f^- + f_2)$.[48]

PROOF; Using the ergodic insight to find the cross-section distribution that attains on average *conditional on the sign of the last aggregate surprise*, and then applying an argument similar to the one used to derive τ above, leads to the desired expressions. ∎

[48]The quantities f_E^+, f_E^-, f_C^+ and f_C^- correspond to the fraction of industries near their upper and lower trigger levels, conditional on the last demand shock being expansionary (E) and contractionary (C), respectively.

REFERENCES

1. Ball, L., N.G. Mankiw and D. Romer (1988): "The New Keynesian Economics and the Output-Inflation Trade-off," *Brookings Papers on Economic Activity,* 1, 1–65.

2. Ball, L. and N.G. Mankiw (1992): "Asymmetric Price Adjustment and Economic Fluctuations," mimeo.

3. Barro, R.J. (1972): "A Theory of Monopolistic Price Adjustment," *Review of Economic Studies*, **39**, 17–26.

4. Bertola, G. and R.J. Caballero (1990): "Kinked Adjustment Costs and Aggregate Dynamics" in O.J. Blanchard and S. Fischer (eds.) *NBER Macroeconomic Annual 1990*, MIT Press.

5. Blanchard, O.J. (1990): "Why Does Money Affect Output? A Survey" in B.M. Friedman and F.H. Hahn (eds.) *Handbook of Monetary Economics, Vol. II*, Elsevier Science Publishers B.V.

6. Caballero, R.J. (1991a): "Durable Goods: An Explanation for their Slow Adjustment," mimeo, Columbia University.

7. Caballero, R.J. (1991b): "Fuzzy Preferences and Aggregate Consumption," mimeo, Columbia University.

8. Caballero, R.J. and E.M.R.A. Engel (1991): "Dynamic (S,s) Economies," *Econometrica*, November, 1659–1686.

9. Caballero, R.J. and E.M.R.A. Engel (1992a): "Heterogeneity and Output Fluctuations in a Dynamic Menu Cost Economy," *Review of Economic Studies*, forthcoming.

10. Caballero, R.J. and E.M.R.A. Engel (1992b): "Microeconomic Adjustment Hazards and Aggregate Dynamics," mimeo, MIT.

11. Caplin, A. and J. Leahy (1991a): "State-Dependent Pricing and the Dynamics of Money and Output," *Quarterly Journal of Economics* **CVI-3**, August.

12. Caplin, A. and J. Leahy (1991b): "Aggregation and Optimization in a Model of State-dependent Pricing," mimeo.

13. Cover, J.P. (1988): "Asymmetric Effects of Positive and Negative Money-Supply Shocks," mimeo, University of Alabama.

14. De Long, J.B. and L.H. Summers (1988): "How Does Monetary Policy Affect Output?" *Brookings Paper of Economic Activity,* Fall, 433–480.

15. Harrison, J.M. (1985): *Brownian Motion and Storage Flow Systems,* John Wiley and Sons, New York.

16. Harrison, J.M., T.M. Sellke and A.J. Taylor (1983): "Impulse Control of Brownian Motion," *Mathematics of Operations Research* **8-3**, August.

17. Lucas Jr., R.E. (1973): "Some International Evidence on Output-Inflation Tradeoffs," *American Economic Review* **63**, June, 326-334.

18. Rotemberg, J.J. (1987): "The New Keynesian Microfoundations" in O.J. Blanchard and S. Fischer (eds.) *NBER Macroeconomic Annual 1987*, MIT Press.

19. Sheshinski,E., and Y. Weiss (1983): "Optimum Pricing Policy under Stochastic Inflation," *Review of Economic Studies* **50**, 513–529.

20. Tsiddon, D. (1989): "The (Mis)Behavior of the Aggregate Price Level," mimeo.