#### NBER WORKING PAPERS SERIES

# STRATEGIC TRADE POLICY WITH INCOMPLETELY INFORMED POLICYMAKERS

S. Lael Brainard

David Martimort

Working Paper No. 4069

NATIONAL BUREAU OF ECONOMIC RESEARCH 1050 Massachusetts Avenue Cambridge, MA 02138 May 1992

We are grateful to Jean-Jacques Laffont, Julio Rotemberg, Jean Tirole, and Kala Krishna for helpful discussions, and to the Ecole Polytechnique and the National Science Foundation for research support. This paper is part of NBER's research program in International Trade and Investment. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

# STRATEGIC TRADE POLICY WITH INCOMPLETELY INFORMED POLICYMAKERS

#### **ABSTRACT**

Ever since the inception of research on strategic trade policy, economists have warned that the informational requirements are high, and unlikely to be met in practice. This paper investigates the implications of incomplete information for a simple, rent-shifting trade policy of the type proposed in Brander-Spencer (1985). We find that asymmetric information undermines the precommitment effect of unilateral government intervention. This "screening" effect induces a downward distortion in the optimal subsidy, and it may be so great as to require a tax rather than a subsidy for high levels of uncertainty, given a zero-profit participation constraint. Second, in contrast to the full-information case with strategic substitutes, the introduction of a rival interventionist government reinforces rather than countervails precommitment effect, by reducing the incentive for the domestic firm to misrepresent its private information. Finally, when a nonintervention-profit participation constraint is substituted for the conventional zero-profit participation constraint to take into account the special relationship between firms and policymakers in trade, the government eschews intervention altogether for high levels of uncertainty.

S. Lael Brainard Sloan School of Management M.I.T. Cambridge, MA 02139 and NBER David Martimort
Institut d'Economie Industrielle (IDEI)
Universite des Sciences Sociales (UTI)
Place Anatole France
31042 Toulouse FRANCE

#### I. Introduction

This paper analyses the implications of incomplete information for strategic trade policy. It revisits in an incomplete information context the simple but powerful point made by Brander and Spencer (1985) - that in an oligopolistic industry, unilateral government intervention can shift rents by providing a strategic advantage.

The analysis is motivated by the observation that informational asymmetries between firms and policymakers are often quite acute in trade policy. In the US for example, when firms petition for antidumping or countervailing duty investigations, the investigators rely on proprietary information frequently provided by the petitioners. The potential for informational asymmetries is most obvious in the case of infant industries, where the cost structure of emerging enterprises is evolving, and possibly unique to each enterprise. But it also afflicts mature industries, where efficient targeting may be compromised by long-standing relationships between regulators and managers that serve to obscure information from central authorities.

Ever since the inception of research on strategic trade policy, economists have been warning that the informational requirements are enormous, and unlikely to be met in practice, due to the sensitivity of policy recommendations to the particularities of the market. This warning has been borne out in practice, as even attempts to evaluate the effects of trade policies in oligopolistic industries *ex post* have encountered severe informational obstacles. So far, however, there has been little research on the specific ways that informational failures might affect policy, and on modifying policy recommendations to account for such failures.

This paper is a first step in addressing these issues, taking insights from regulation theory as its starting point. Regulation theory has explored informational

See Grossman (1986) for a clear statement of this concern.

<sup>&</sup>lt;sup>2</sup> See, for example, Dixit (1988), Baldwin and Krugman (1988), and Krugman and Brainard (1988).

asymmetries extensively in recent years.<sup>3</sup> But so far there has been little spillover from the incomplete-information regulation literature into trade theory.<sup>4</sup> This is in part because trade policy differs from regulatory policy in several important ways.

One important difference between trade and regulation is in the government's objective. While in regulation the exercise of market power almost always reduces welfare, in trade the exercise of market power by the domestic firm raises domestic welfare whenever it is at the expense of foreign consumers. In this paper, we highlight this difference by assuming the firms produce purely for export.

An important consideration in trade policy, unlike in regulation, is the potential for countervailing intervention on the part of a foreign government. This calls for analysis of games in contracts among government-firm pairs. In this paper, we compare domestic surplus under unilateral and bilateral intervention, building on recent theoretical research on competition between principal-agent pairs with incomplete information.<sup>5</sup>

A third important distinction involves the firm-government relationship. In regulation, the firm exists at the sufferance of the government. The firm has no recourse; it must accept regulation or exit the industry. In contrast, in trade, firms are generally considered footloose, or capable of redeploying their assets in a more friendly policy environment. Moreover, firms frequently are themselves the instigators of interventionist trade policies.

<sup>&</sup>lt;sup>3</sup> This literature analyses relationships between regulatory authorities and regulated firms, interaction between various interest groups and regulatory agencies, and regulatory hierarchies in the context of imperfect information. In these models, attainment of the social optimum is usually compromised by the potential for allocative distortions due to informational rent-seeking. See, for example, Baron and Myerson (1982) and Laffont and Tirole (1986, 1990, and 1991).

<sup>&</sup>lt;sup>4</sup> Except in the area of transfer pricing within multinationals. See Prusa (1990), Raff (1991), and Gresik and Nelson (1991).

<sup>&</sup>lt;sup>5</sup> See Gal-or (1988), Caillaud, Julien, and Picard (1990), Katz (1991), and Martimort (1991).

We attempt to capture these features in a transparent framework. Our perfect information benchmark is a simple trade policy designed to shift rents, along the lines of Brander-Spencer. We modify this framework in several ways to incorporate imperfect information and highlight the essential features of trade identified above.

First, we analyze the difference in the firm-government relationship between regulation and trade by introducing a novel participation constraint. In the Brander-Spencer analysis, it is not clear whether the government uses the firm as a precommitment device, or the reverse. Here we distinguish between these two cases explicitly by comparing the optimal policy for a standard zero-profit participation constraint with a nonintervention-profit participation constraint, which effectively gives the firm residual rights of control.

Second, following Laffont and Tirole (1986), Gruenspecht (1988), and Neary (1991), we assume there is a cost of raising public funds. This resolves a crucial ambiguity in the Brander-Spencer perfect information framework about the sharing of rents between the government and firms.<sup>6</sup> The government maximizes domestic surplus less the cost of net transfers to the firm. In contrast to the analysis in both Neary and Gruenspecht, we solve for the optimal complete-information policy without restricting the number of instruments.

Lastly, we incorporate asymmetric information in the form of adverse selection; firms are assumed to have private information about their costs.

This paper is only a first step toward an understanding of strategic trade policy under incomplete information. However, even in the simple framework developed here, the presence of informational asymmetries changes the results in several interesting ways. First, we show that the effects of government

<sup>&</sup>lt;sup>6</sup> The formulation of the government's objective function with costly public funds and a nonintervention participation constraint can be interpreted in terms of a Hicksian welfare function, where there is a cost to redistribution. Caillaud et. al. (1985) discuss the relationship between the assumption of costly public funds and a weighted social welfare function.

precise knowledge of the firms' profit functions, the government faces a tradeoff between manipulating the payoffs to the local firm in a way that enhances its strategic advantage, and preventing the firm from deriving distortionary rents from its private information.<sup>7</sup> The informational asymmetry induces a downward distortion of the optimal subsidy, which may be severe enough to force the subsidy below 0 with a zero-profit participation constraint, and to 0 with a nonintervention-profit constraint. We term this the "screening" effect.

Second, a two-dimensional policy is required to target both the distortion associated with strategic precommitment and that associated with informational rent-seeking. With asymmetric information, the range of tools available to policymakers is even more important than under complete information because different tools present the firms with different opportunities to extract distortionary informational rents. Here, the optimal policy can be implemented as a menu of contracts specifying a per-unit subsidy and a lump-sum tax as a function of the firm's reported cost. The publicly observable announcement of a subsidy serves as a credible commitment to expand the firm's output, just as in Brander-Spencer, while the lump-sum transfer serves as a screening device.

Third, the introduction of a rival interventionist government, rather than nullifying the effectiveness of the domestic government's intervention, as in the full-information case, actually enhances it under incomplete information. The foreign agency problem mitigates the domestic agency problem, since the contract between the foreign government and foreign firm reduces the temptation for the domestic firm to misrepresent its private information. This "competing contracts" effect reduces

<sup>&</sup>lt;sup>7</sup> "Local" is used to denote the firm whose production is located within the jurisdiction of the trade authority.

<sup>&</sup>lt;sup>8</sup> Obviously, in a world with unrestricted lump-sum transfers, the government could induce truthful revelation costlessly, by threatening a sufficiently large negative transfer.

the "screening" distortion, and thus reinforces the precommitment effect, rather than countervailing it as under full information.

Lastly, we show that when firms are free to reject intervention, nonintervention may obtain endogenously as the optimal policy, for a sufficiently large spread in the distribution of private information. In this case, the likelihood of intervention is greater for bilateral intervention than for unilateral intervention, because of the countervailing effect from competing contracts.

Section II develops the benchmark case with complete information. It describes the game between the firms, and the government's objective function. It then specifies the firm's participation constraint for both the nonintervention and zero-profit cases, and solves for the optimal unilateral and bilateral policies. Section III introduces an informational asymmetry in the form of adverse selection between each government and its local firm. The firm's incentive compatibility constraint is defined, and the optimal policy is derived for the case of a zero-profit participation constraint for both unilateral and bilateral intervention. Section IV modifies the optimal unilateral and bilateral policies to incorporate the nonintervention-profit participation constraint. Section V discusses the implications of changing several of the central assumptions. It discusses the case where the firms' decision variables are strategic complements, the robustness to changes in the specification of the informational asymmetry, and the connection between contract observability, renegotiation, and precommitment. Section VI concludes.

## II. Complete Information

## i. Game between the Firms

We develop a simple international duopoly model, adopting the device used by Brander and Spencer of assuming that a single home firm and a single foreign firm export to a third market. This could be taken as a (vastly simplified) model of US and EC air frame manufacturers competing for exports in the Canadian market, for instance. The firms take simultaneous quantity decisions in a one-stage game with differentiated products. Demand is assumed linear for the sake of simplicity:

(1) 
$$P_i(q_i,q_j) = a - \frac{b}{2}q_i + dq_j$$
  $i=h$  when  $j=f$  and the reverse

where h denotes home, and f foreign. We restrict attention to the case where d is negative, such that the firms' choice variables are strategic substitutes. We also assume that  $b^2-2d^2>0$ .

Both firms have symmetric marginal costs given by a random variable,  $\sigma$ , distributed according to a uniform density function  $f(\sigma)$  on  $[\sigma, \sigma]$  (alternatively, the model specification permits interpretation of  $\sigma$  as a demand shift variable). The realization of  $\sigma$  is assumed to be publicly observable and verifiable in this section. In the interests of tractability, both firms' profit functions are assumed to be affected symmetrically by the same shock. In addition, firms' profits are affected by perunit subsidies imposed in either market,  $s_i$ , for i=h,f. Firms choose quantities simultaneously, after having observed the realization of  $\sigma$ , and announced subsidies. Quantities are determined as the Nash equilibrium of the Cournot game. These are straightforward to compute as the intersection of the best response schedules:

(2) 
$$\max_{q_i} [P_i(q_i,q_i) - \sigma + s_i]q_i$$
 i=h when j=f and the reverse

associated with the unique equilibrium:

<sup>&</sup>lt;sup>9</sup> Similar results would be obtained by assuming only that the firms' shocks are positively correlated. See Section V for further details.

<sup>&</sup>lt;sup>10</sup> The assumption of Cournot competition and strategic substitutes can be justified as in Kreps and Scheinkmann (1983). If firms first choose their production capacities, and then compete with each other in prices, the reduced form of the profit function in capacities can be expressed as Cournot competition in quantities.

(3) 
$$q_i(s_i, s_j, \sigma) = \frac{1}{b-d} \left[ a - \sigma + \frac{bs_i + ds_j}{b+d} \right]$$

yielding profits of:

(4) 
$$\pi_i(s_i, s_j, \sigma) = \frac{b}{2} q_i(s_i, s_j, \sigma)^2$$

#### ii. Government Objective Function

We next introduce a domestic government, who attempts to maximize the rents of the domestic firm less the budgetary cost of net transfers to the firm. We assume that the social cost of public funds exceeds 1 by an amount c. The assumption of costly public funds is an ad hoc way of capturing the general equilibrium effects of the sectoral intervention, where raising revenues incurs administrative costs or creates distortions in other sectors. In the absence of such a cost, the government would simply impose a per-unit subsidy.

In contrast to the Neary and Gruenspecht models, we assume that the government faces no constraints in its instrument choice. It turns out that the optimal can be obtained with a two-dimensional contract of the form  $\{s(\sigma), t(\sigma)\}$  stipulating a per-unit subsidy, s, and a lump-sum reimbursement, t, as a function of the realized cost parameter,  $\sigma$ . Given the realization of costs and the announced value of the subsidy, each firm chooses its optimal output. After receiving profits, the firm reimburses the government by way of the lump-sum transfer. The level of output does not enter into the contract directly, consistent with its precommitment function. However, the government is assumed rational, so that it correctly anticipates the firms' optimal output responses to the subsidy it imposes. Moreover, it is straightforward to show that if the government were able to contract on the domestic firm's output in addition to the subsidy, it would choose the same level of output as the firm.

Thus, we can express the objective function of the domestic government as

the domestic firm's rents less the budgetary cost of net transfers to the firm:

(5) 
$$Max_{s_k(\sigma),t_k(\sigma)} \quad U_k(\sigma) - (1+c)[s_k(\sigma)q_k(\sigma) - t_k(\sigma)]$$

where U<sub>h</sub>( $\sigma$ ) is the rent earned by the home firm:<sup>11</sup>

$$U_k(\sigma) = \pi_k(s_k(\sigma), s_k(\sigma), \sigma) - t_k(\sigma)$$

## iii. Participation Constraint

There is an ambiguity in the Brander-Spencer, full-information model that turns out to be critical once incomplete information is introduced. Although it is clear in their model that government intervention on behalf of a firm can serve as a precommitment device, it is not clear whether this is achieved by the government taking the firm as a precommitment device, or the reverse. Moreover, in the B-S model, this distinction does not matter.<sup>12</sup> With incomplete information, however, this distinction becomes important, since firm interests conflict with government interests over informational rents.

A number of papers in industrial organization analyze the principal's ability to precommit by "hiring" an agent to play a game on the principal's behalf (Tirole (1988)). These papers effectively assume that the government has complete residual rights of control; it can induce the firm to participate as long as the firm breaks even. <sup>13</sup> This is wholly appropriate in regulation, where the firm has no recourse other than exiting. In the model developed here, this assumption can be incorporated

 $<sup>^{11}</sup>$  We include  $s_f$  as a placeholder whenever we are defining general functional forms in order to avoid repeating equations, with the understanding that this term is 0 under unilateral intervention, and the optimal foreign subsidy under bilateral intervention.

<sup>&</sup>lt;sup>12</sup> However, this distinction is important in a full-information context when firms play Bertrand, as in Gruenspecht (1988) or Carmichael (1987), or when there are fixed costs, as in Brainard (1990). The nonintervention participation constraint is in the spirit of these models, all of which shift the balance of power in favor of firms by assuming that firms move before governments.

<sup>13</sup> In Gal-Or, with asymmetric information, principals must leave a rent to the firm.

in a zero-profit participation constraint (ZPC):

$$(6) U_{\mathbf{k}}(\sigma) \ge 0$$

In trade, however, it may be more accurate to think of firms inducing governments to intervene on their behalf. This assumption more nearly approximates the footloose character of firms in traded goods industries, or the trade policy process in political systems where firms initiate intervention, as described in the introduction. We formalize this by assuming that the firm can refuse to participate if it would earn less under intervention than in the absence of intervention. Define profits when neither government intervenes under unilateral intervention as  $\pi^{ou}(\sigma)$ , and profits when the rival government alone imposes the optimal subsidy under bilateral intervention as  $\pi^{ob}(\sigma)$ :

$$\pi_i^{om}(\sigma) = \frac{b}{2}[q_i(0,s_j,\sigma)]^2$$
  $s_j=0$  for  $m=u$ ;  $s_j=s_j(\sigma)$  for  $m=b$ 

Then the domestic firm's nonintervention-profit participation constraint (NPC) is expressed:

(7) 
$$U_k(\sigma) \ge \pi_k^{om}(\sigma)$$
 for  $m = u, b$ 

This formulation of the government's problem can alternatively be interpreted in more traditional terms as a Hicks-Kaldor social welfare function, where there is a cost to redistribution. Although this is a fairly novel approach in principal-agent theory, it has the additional attraction that it leads to more plausible results, since it limits the size of lump-sum transfer that the government extracts.

## iv. Intervention Equilibrium with Complete Information

We solve for the optimal policy under complete information as a benchmark. In this situation, the government establishes a per-unit subsidy and lump-sum reimbursement to solve (5) subject to participation constraint (6) or (7), after observing the realization of  $\sigma$ .

It is straightforward to prove that the participation constraint must bind for each realization of costs in the complete information case. If, to the contrary, it did not bind for some values of  $\sigma$ , then it would be possible to raise welfare by reducing the firm's rents by a small amount, without violating the participation constraint. But then clearly this would not have been an optimum in the first place.

Define the optimal lump-sum transfer for firm i under intervention regime m in the ZPC case as  $\tilde{t}_{ix}^{m}(\sigma)$ :

(8) 
$$t_{iz}^{*m}(\sigma) = \pi_i(s_i(\sigma), s_i(\sigma), \sigma) \qquad \text{for } m = u, b, \ i = h, f$$

The lump-sum transfer is set just equal to the firm's subsidized profits, leaving the firm exactly indifferent between participating under intervention and shutting down, for all values of r. In the case of the NPC define the optimal lump-sum transfer for firm i under intervention regime m as  $f_m^m(\sigma)$ :

(9) 
$$t_{in}^{*m}(\sigma) = \pi_i(s_i(\sigma), s_j(\sigma), \sigma) - \pi_i^{om}(\sigma) \qquad \text{for } m = u, b; \ i = h, f$$

Here the transfer leaves the firm just indifferent between nonintervention and intervention, for all values of  $\sigma$ . In both cases, the transfer is a decreasing, quadratic function of  $\sigma$ .

Differentiating the objective function with respect to  $s_i(\sigma)$  yields the same necessary and sufficient condition on the optimal subsidy for either participation constraint:

(10) 
$$s_i(\sigma) = \frac{d^2[(a-\sigma)(b+d)+ds_j(\sigma)]}{b(b^2-2d^2)} \quad i=h \text{ when } j=f \text{ and the reverse}$$

Setting  $s_i(\sigma)$  equal to zero in the unilateral case yields the following expression for the optimal home subsidy:

(11) 
$$s_h^{*u}(\sigma) = (a - \sigma) \frac{d^2(b + d)}{b(b^2 - 2d^2)}$$

The optimal unilateral subsidy is a linear, decreasing function of  $\sigma$ . Notice that the subsidy is positive for all values of the cost parameter, implying that targeting is

efficient over the entire interval.<sup>14</sup> The optimal subsidy induces the domestic firm to expand its output to the Stackelberg leadership level, and correspondingly, the foreign firm to contract. This is precisely the result derived in Brander-Spencer. The government is able to give its firm a strategic advantage because it has full precommitment power with full information and no countervailing intervention.

The ability of the domestic government to raise domestic surplus depends critically on its ability to precommit. When it intervenes unilaterally, the government enables its firm to commit to an expanded output level. When both governments intervene simultaneously, neither government can give its firm an advantage through precommitment. In this case, the optimal subsidies are symmetric in equilibrium:

(12) 
$$s_i^{*b}(\sigma) = s^{*b}(\sigma) = (a-\sigma)\frac{d^2}{(b^2-db-d^2)} i=hf$$

The bilateral subsidies raise the equilibrium quantities of both firms above the free-market Cournot level, and surplus in both economies falls relative to the free market equilibrium. There is thus a classic prisoner's dilemma: although welfare in both countries is higher when neither government intervenes, both intervene because each government anticipates that its rival will intervene if it does not.

Notice that in both the bilateral and unilateral case, the form of the participation constraint affects only the sharing of surplus between the firm and the government through the transfer. It has no effect on the equilibrium quantities or aggregate surplus.

Next we show that the introduction of incomplete information changes these results significantly.

<sup>14</sup> Assuming that  $a - \sigma > 0$ .

## III. Adverse Selection

Returning to the case of unilateral intervention, we introduce incomplete information about the level of  $\sigma$ . The distribution of  $\sigma$  is common knowledge for both governments and firms, but only firms know the true realization of their costs. While each firm is fully informed about both firms' profit functions, the government is not able to observe the effect of the shock on either firm's profits. This would be the case, for instance, if air frame manufacturers had precise information about the effect of an input price shock or a demand shock on their own profits, and (assuming similar technology) on those of their rivals, but the domestic government had access only to the consolidated financial statements required for tax purposes.

We assume that the government is not able to contract on other informative variables, such as either firm's output or prices. Such contracts are ruled out either because the sales terms are secret, or because such contracts could not be enforced due to verifiability problems. Since the government does not observe or cannot verify the realization of  $\sigma$ , profits, foreign output, or prices, it bases its policy on a report from the firm,  $\hat{\sigma}$ . The government offers a menu of per-unit subsidies and lump-sum transfers, conditional on the firm's reported costs,  $\{s_n(\hat{\sigma}), t_n(\hat{\sigma})\}$ .

We further assume that the contracts are publicly observable, and cannot be renegotiated. With complete information, the potential for renegotiation has no effect. With incomplete information, however, renegotiation through secret contracts can have precommitment value. Here, we rule it out in order to distinguish clearly between precommitment effects and asymmetric information effects. We discuss the implications of allowing renegotiation in Section V.

Such an assumption can be justified on several counts. First, it may be in the government's interest to establish a reputation for nonrenegotiation. Reputation

<sup>&</sup>lt;sup>15</sup> Clearly, if sufficient variables were observable and verifiable, the government could use them in conjunction with knowledge of their relation to the unobserved variable to solve the information problem.

would be important if the government were engaged in a repeated game with a firm in a single industry, <sup>16</sup> or in several one-shot games with firms in a variety of industries. <sup>17</sup> Alternatively, the government and/or the firm might have an interest in avoiding renegotiation if a large fixed investment were incurred for each round of negotiation.

## i. <u>Incentive Compatibility Constraint</u>

With incomplete information, the optimal policy must satisfy an incentive compatibility constraint to induce truthful revelation, in addition to the participation constraint. Before specifying the incentive compatibility constraint, it is useful to examine firm behavior when faced with the optimal full-information policy, given private information. Suppose in the unilateral case that the government assigns the full-information contract  $\{s_h^*(\hat{\sigma}), t_h^*(\hat{\sigma})\}$  for w=n,z, when the firm reports its costs as  $\hat{\sigma}$ . Define the value to the domestic firm with true costs  $\sigma$  of making report  $\hat{\sigma}$ , given policy  $\{s_h(\hat{\sigma}), t_h(\hat{\sigma})\}$ , as  $U_h(\hat{\sigma}, \sigma)$ :

(13) 
$$U_{k}(\hat{\sigma},\sigma) = \pi_{k}(s_{k}(\hat{\sigma}),s_{k}(\sigma),\sigma) - t_{k}(\hat{\sigma})$$

The firm's optimal choice of report is the value of  $\hat{\sigma}$  that maximizes  $U_h(\hat{\sigma}, \sigma)$ .

Then in the ZPC case, the firm's optimal report is:

$$\hat{\sigma} = \frac{a(b^2 - d^2) + d^2\sigma}{b^2} > \sigma$$

while in the NPC case, it is:

<sup>&</sup>lt;sup>16</sup> Ratchet effects from sequential contract offers can be ruled out by assuming that the firm's costs are drawn independently each period. In a dynamic framework of this nature, the government is not credibly able to write long-term contracts because there is some probability implicit in the discount rate that it will not get reelected.

<sup>&</sup>lt;sup>17</sup> Models along these lines assume that the government's time horizon exceeds that of firms, either because the government is longer-lived, as in Fudenberg, Levine (1989), or, as in Schmidt (1991), because it has a lower discount rate, and the game between the government and firms is of "conflicting interest."

$$\hat{\sigma} = \frac{a + \sigma}{2} > \sigma$$

In both cases, the firm optimizes by misrepresenting its costs as higher than the true value. This is because the decrease in the lump-sum tax associated with higher levels of reported costs more than offsets the effect on profits from the decrease in the value of the subsidy, given that true costs are lower. It is straightforward to show that similar results obtain in the case of bilateral intervention. As is common in the imperfect information literature, it will turn out that in the presence of incentive compatibility problems, the transfer plays a critical role as a screening device.

Under imperfect information, any proposed policy has to satisfy an incentive compatibility requirement on the firm's report. The revelation principle enables us to restrict attention to the class of direct mechanisms for which truthful revelation is optimal.<sup>18</sup> Thus:

$$\sigma \in argmax \ _{h} U_{h}(\hat{\sigma}, \sigma)$$

Then  $U^h(\sigma)$  is simply the firm's informational rent when its incentive compatibility constraint is satisfied:

$$U_{\mathbf{k}}(\sigma,\sigma) \equiv U_{\mathbf{k}}(\sigma)$$

Assuming that we can restrict attention to piecewise-differentiable contracts, the first order condition for the firm's revelation problem is:

(14) 
$$-\frac{\partial t_{k}(\sigma)}{\partial \sigma} + \frac{\partial s_{k}(\sigma)}{\partial \sigma} \frac{\partial \pi_{k}(s_{k}(\sigma), s_{j}(\sigma), \sigma)}{\partial s_{k}} = 0 \forall \sigma$$

yielding:

<sup>&</sup>lt;sup>18</sup> Myerson (1982) shows that in a bilateral principal-agent structure, it is a Nash equilibrium in reports for the two agents to reveal their private information truthfully, as long as collusion between the two agents is ruled out in the reporting state of the game, and each agent is associated uniquely with one principal. In the context considered here, this implies that the domestic firm chooses its optimal report knowing that the foreign firm reveals truthfully under bilateral intervention.

(15) 
$$\frac{\partial U(\sigma)}{\partial \sigma} = -q_k(s_k(\sigma), s_j(\sigma), \sigma) \frac{b}{(b-d)} \left( 1 - \frac{d}{(b+d)} \frac{\partial s_j(\sigma)}{\partial \sigma} \right)$$

And the local second order conditions are satisfied for:

(16) 
$$\frac{\partial s_{h}(\sigma)}{\partial \sigma} \left( 1 - \frac{d}{(b+d)} \frac{\partial s_{f}(\sigma)}{\partial \sigma} \right) \leq 0$$

It will turn out that the form of the optimal contract - a subsidy that is piecewise linear in  $\sigma$ , and a transfer that is piecewise quadratic in  $\sigma$  - ensures that the local second order conditions are sufficient for a global maximum. The proof is in Appendix 3.

## ii. Optimal Policy with Unilateral Intervention

The government chooses its policy to maximize domestic surplus, less the cost of net transfers to the firm. Substituting for the transfer yields the expression for the objective function:

$$Max \int_{s_{k}(\sigma),U_{k}(\sigma)} \int_{\alpha}^{\overline{\sigma}} \left[ \pi(s_{k}(\sigma),s_{k}(\sigma),\sigma) - s_{k}(\sigma)q_{k}(s_{k}(\sigma),s_{k}(\sigma),\sigma) - \frac{c}{(1+c)}U_{k}(\sigma) \right] f(\sigma)d\sigma$$

subject to the incentive compatibility conditions in equations (14) and (15), and the ZPC in equation (6). We first solve for the optimal policy ignoring the second order conditions in (16), and then check to ensure that they are indeed satisfied at the optimum.

The government's problem can thus be expressed as the Hamiltonian:

(18) 
$$H(s_{k}, U_{k}, \lambda, \sigma) = \left[\pi(s_{k}(\sigma), s_{f}(\sigma), \sigma) - s_{h}(\sigma)q_{h}(s_{k}(\sigma), s_{f}(\sigma), \sigma) - \frac{c}{(1+c)}U_{h}(\sigma)\right]f(\sigma)$$
$$-\lambda(\sigma)\frac{b}{(b-d)}q_{h}(s_{k}(\sigma), s_{f}(\sigma), \sigma)\left(1 - \frac{d}{b+d}\frac{\partial s_{f}(\sigma)}{\partial \sigma}\right)$$

which is concave in  $\sigma$ , and where  $\lambda(\sigma)$  is the multiplier on the incentive constraint.

Differentiating with respect to Uh yields the first order condition:

(19) 
$$\frac{\partial \lambda(\sigma)}{\partial \sigma} = \frac{c}{1+c} f(\sigma)$$

Recalling that firms are tempted to overreport the value of  $\sigma$  when faced with the full-information optimal policy, in order to save on the transfer to the government, we expect the participation constraint to bind for the least efficient type if it binds for any type,  $U_h(\overline{\sigma})=0$ . This implies that  $\lambda(\underline{\sigma})=0$ , which enables us to solve for the multiplier:

(20) 
$$\lambda(\sigma) = \frac{c}{1+c} \int_{\alpha}^{\sigma} f(\sigma) d\sigma = \frac{c(\sigma - \underline{\sigma})}{(1+c)(\overline{\sigma} - \underline{\sigma})}$$

Setting  $s_t(\sigma)=0$  for the unilateral case, and differentiating with respect to  $s_b(\sigma)$  yields the following expression for the optimal unilateral subsidy:

Thus, the optimal subsidy is linear and decreasing in  $\sigma$ , and is distorted below the full-information subsidy at every value of  $\sigma$  except the most efficient by an amount that increases in the distance from  $\underline{\sigma}$ . With incomplete information, the government distorts the optimal contract in order to induce the domestic firm to produce efficiently consistent with its true costs.

Notice that for a sufficiently great divergence between  $\sigma$  and  $\underline{\sigma}$ , the subsidy actually becomes negative. Call the value at which this takes place  $\sigma^{u}$ :

(22) 
$$\sigma^{\mathbf{u}} = \left(\sigma \mid (\sigma - \underline{\sigma}) = (a - \sigma) \frac{(1 + c)}{c} \frac{d^2}{b^2 (b^2 - 2d^2)}\right)$$

 $\sigma^{u}$  may fall on either side of  $\sigma$ . Define  $\widetilde{\sigma}^{u} = \min\{\sigma^{u}, \overline{\sigma}\}$ . For the range  $\sigma < \widetilde{\sigma}^{u}$ , the effect of the subsidy is to raise the home quantity above the free market level, but by less than the full-information subsidy, and to discourage foreign production relative to the free market equilibrium, but by less than the full-information subsidy:

## $q_f^{*\mathtt{u}}(\sigma) \! \le \! q_f^{\mathtt{u}}(\sigma) \! < \! q^o(\sigma) \! < \! q_h^{\mathtt{u}}(\sigma) \! \le \! q_h^{*\mathtt{u}}(\sigma)$

with equality holding at  $\underline{\sigma}$ . For the range in which the subsidy becomes a tax,  $\overline{\sigma}^{u} < \sigma \leq \overline{\sigma}$ , the effect of intervention is to reduce home output relative to the free market equilibrium, which induces an expansion in foreign output. Thus, for a sufficiently wide range of uncertainty, the screening effect actually dominates the precommitment effect, and optimal policy serves to strategically disadvantage the home firm.

In either case, the government chooses a schedule of transfers that gives higher rents to low cost firms as an inducement to produce efficiently, and leaves other types just indifferent between operating under intervention and shutting down. In effect, the optimal policy trades off the external objective of strengthening the domestic firm's strategic advantage against the internal objective of minimizing allocative distortions associated with the domestic firm's informational rent-seeking.

## iii. Optimal Policy with Bilateral Intervention

In the bilateral case, each government is assumed to choose a menu of transfers and subsidies as a function of the local firm's cost report,  $\sigma$ . The governments announce their trade policies simultaneously. Contracts are assumed reciprocally observable, but nonrenegotiable. The two firms, having observed the realization of  $\sigma$  and both governments' policy announcements, simultaneously choose their report to the government, and make their output decisions. Again, neither government can make the contract contingent on the rival firm's output, profits, or prices.

Both governments are assumed to maximize domestic surplus less the cost of net transfers to the local firm, subject to the incentive compatibility and participation constraints of the local firm. As explained in Footnote 18, the truthful revelation restriction can be applied to both firms. Thus, the first and second order conditions described in Equations (14), (15), and (16) apply to each of the two firms,

and the Hamiltonian defined in Equation (18) applies to each government separately.

Differentiating each government's objective function with respect to  $U_h(\sigma)$ , and conjecturing that the constraint is slack at  $\underline{\sigma}$  yields the condition on the multiplier in Equation (19). Differentiating with respect to  $s_j(\sigma)$ , j=h,f, and solving for a linear, symmetric solution yields a unique solution for the case d>0:<sup>19</sup>

$$s_{i}^{b}(\sigma) = s_{i}^{*b}(\sigma) - \alpha^{b}(\sigma - \underline{\sigma}) \qquad \alpha^{b} = \frac{cb^{3}(b^{2} - 2d^{2})}{(b^{2} - d^{2} - bd)[(1 + c)(b + d)(b^{2} - d^{2} - bd) - cb^{2}d]}$$

As in the unilateral case, we find that the subsidy is decreasing in  $\sigma$ , and distorted downward relative to the complete information optimum by an amount that increases in  $(\sigma - \underline{\sigma})$  for all values of  $\sigma$  on  $[\underline{\sigma}, \overline{\sigma}]$ . And again, the distortion associated with the screening effect may be sufficient to push the subsidy below 0. Define the cutoff value of  $\sigma$  for which the subsidy is 0 in the bilateral case as  $\sigma^b$ :

(24) 
$$\sigma^b = \left(\sigma \mid (\sigma - \underline{\sigma}) = (a - \sigma) \frac{d^2}{\alpha^b (b^2 - d^2 - db)}\right)$$

Again,  $\sigma^b$  may fall on either side of  $\sigma$ , depending on the spread of the distribution. Define  $\widetilde{\sigma}^b = \min\{\sigma^b, \overline{\sigma}\}$ . For  $\sigma < \widetilde{\sigma}^b$ , the symmetric subsidies are positive, and both firms expand their output relative to the free market equilibrium, but by less than under complete information. For  $\widetilde{\sigma}^b < \sigma < \overline{\sigma}$ , the symmetric subsidies are negative, and both firms contract output relative to the free market equilibrium.

However, with bilateral intervention, there is an additional effect that countervails the screening effect (although it does not eliminate it altogether). The competition in contracts between the incompletely informed governments loosens each firm's incentive compatibility constraint, due to the expansionary impact of the

<sup>&</sup>lt;sup>19</sup> In the absence of symmetry, a solution for optimal pairs of policies could be derived based on linearity alone.

rival subsidy on rival output.<sup>20</sup> Comparison of the threshold values of  $\sigma$  under bilateral and unilateral intervention in Equations (22) and (24) reveals that the range of types for which the subsidy is positive is greater under bilateral intervention, or  $\tilde{\sigma}^{u} < \tilde{\sigma}^{b}$  (for  $\sigma^{u} < \tilde{\sigma}$ ). Thus, in sharp contrast to the complete-information case, the introduction of a rival contract between the foreign government and firm reinforces the precommitment effect in the presence of incomplete information. Asymmetric information mitigates the prisoner's dilemma, and may eliminate it on the interval where the subsidy becomes a tax. Indeed, from an expost point of view, it may appear that intervention is not the dominant strategy.

It is straightforward to verify the approach taken here by checking the second order conditions on each firm's problem. In both the unilateral and bilateral cases, the expressions for the optimal subsidies are linear in  $\sigma$ , implying that the optimal transfers are quadratic in  $\sigma$ . Each firm is thus maximizing a quadratic function of  $\hat{\sigma}$ , which ensures that the local second order conditions are sufficient for global optimality. Since the optimal subsidy is decreasing in  $\sigma$  in both the bilateral and unilateral cases, the local second order conditions in equation (18) are satisfied. This is also consistent with the assumption that the rent is decreasing.

<sup>&</sup>lt;sup>20</sup> This result depends on the assumption of strategic substitutes. The case of complements is discussed in Section V.

## IV. Nonintervention-profit Constraint

So far, we have been effectively treating firms as the agents of their respective governments, by adopting the ZPC. The assumption that firms are willing to operate under intervention as long as they make nonnegative rents leads to the conclusion that asymmetric information undermines the government's ability to use the local firm as a precommitment device. Indeed, the effect may be so extreme that intervention creates a strategic disadvantage. In this case, intervention on the part of a rival government can have the beneficial effect of loosening the incentive compatibility constraint of the local firm.

As discussed above, however, in a trade context, it may be more accurate to view the firm as having residual rights of control, or of having additional bargaining power vis-a-vis the government. This consideration can be incorporated by adopting the NPC in equation (7). In effect, with the NPC, we attempt to find an incentive-compatible mechanism that does at least as well as nonintervention (which is itself an incentive-compatible mechanism).<sup>21</sup>

## i. Government Objective

To isolate the part of the firm's rent that accrues to it because of asymmetric

Nonintervention is also characterized by the absence of any need for communication between firms and governments, which would be important if the potential for collusion were admitted, since communication provides scope for collusion.

information, we define  $V_h(\sigma) = U_h(\sigma) - \pi^o(\sigma)$ . This new state variable evolves according to the equation:

(25) 
$$\frac{\partial V_h(\sigma)}{\partial \sigma} = -Ks_h(\sigma) \left[ 1 - \frac{d}{(b+d)} \frac{\partial s_f(\sigma)}{\partial \sigma} \right] \qquad \text{for } K = \frac{b^2}{(b-d)^2(b+d)}$$

The government's program is then:

$$H \left(\sigma_{s}S_{k}(\sigma), V_{k}(\sigma), \mu(\sigma), \lambda(\sigma)\right) =$$

$$(26) \qquad f(\sigma) \left[\pi_{k}(S_{k}(\alpha), S_{k}(\sigma), \sigma) - \pi_{k}(0, S_{k}(\sigma), \sigma) - S_{k}(\sigma)q_{k}(S_{k}(\sigma), S_{k}(\sigma), \sigma)\right] - \frac{c}{1+c}V_{k}(\sigma)\right]$$

$$-\lambda(\sigma)KS_{k}(\sigma) \left[1 - \frac{d}{(b+d)} \frac{\partial S_{k}(\sigma)}{\partial \sigma}\right] - \mu(\sigma)V_{k}(\sigma)$$

where  $\lambda(\sigma)$  is the multiplier on the incentive constraint, and  $\mu(\sigma)$  is the multiplier on the participation constraint

(and the local incentive compatibility conditions are assumed to apply only to those values of  $\sigma$  for which the optimal subsidy is differentiable).

It is common in principal-agent theory to assume that the participation constraint is exogenous - the reservation value is assumed either constant, or, less commonly, inversely related to the rent. Here, however, as is evident from the NPC in Equation (7), both the rent and the reservation profits are decreasing in  $\sigma$ , so that it is not obvious where the participation constraint binds (or equivalently, which

subset of firms is just indifferent).<sup>22</sup> Intuitively, we expect that efficient firms are tempted to claim higher costs in order to reduce the transfer payment. Thus, similar to the ZPC case, the government should reduce the reward to cheating by distorting the subsidy downward for each value of  $\sigma$ . However, in contrast to the ZPC case, with the NPC it turns out that the government chooses to refrain from intervening rather than imposing a per unit tax for sufficiently high levels of uncertainty, leaving the least efficient firms without any protection.

## ii. <u>Unilateral Policy</u>

Solving (26) for the optimal policy in the case of unilateral intervention subject to the NPC yields the following proposition:

## Proposition 1

Define  $s^{u}(\sigma)$ ,  $\sigma^{u}$ , and  $\widetilde{\sigma}^{u}$  as in Equations (21) and (22). Then the optimal subsidy that satisfies the nonintervention-profit constraint under unilateral intervention is given by:

(27) 
$$\tilde{s}^{u}(\sigma) = \begin{pmatrix} s^{u}(\sigma) & on \left[\underline{\sigma}, \tilde{\sigma}^{u}\right] \\ 0 & on \left(\tilde{\sigma}^{u}, \overline{\sigma}\right] \end{pmatrix}$$

Lewis and Sappington (1989) were the first to analyze this type of problem in their study of countervailing incentives. In work more closely related to the model presented here, Faggart (1991) modifies the Baron-Myerson model to incorporate a constraint that regulation must leave the firm no worse off than under monopoly. Although the results are broadly consistent with ours (unfortunately we were unaware of them until after our analysis was completed), Faggart finds that there are successive regimes of intervention and nonintervention, while in the model above these regimes are distinct.

## Proof Appendix 1

Even with a stronger participation constraint, asymmetric information distorts subsidies downward, and thus weakens the precommitment effect of intervention. However, when firms are given residual rights of control with the NPC, it is never optimal to tax the output of even the least efficient firms. A per-unit tax, which would necessitate a positive lump-sum transfer in order to satisfy the participation constraint, would violate incentive compatibility, encouraging efficient firms to report costs above their true value. Thus, it is optimal for the government to limit the range in which it intervenes, leaving the least efficient firms to the mercy of the market.

#### iii. Bilateral Policy

Bilateral intervention introduces additional effects from the competition between contracts with the NPC, similar to the ZPC. First, the competition in contracts weakens the participation constraint, which affects the level of the lump-sum transfer, but not the size of the subsidy. Second, it loosens the incentive constraint, by reducing the level of the asymmetric information distortion (again, assuming strategic substitutes and the foreign subsidy is decreasing in  $\sigma$ ). Similar to the ZPC case, the effect of competing contracts reinforces the precommitment effect, leading to higher output, but does not fully offset the asymmetric information distortion. And similar to the unilateral case with a NPC, there is a range of nonintervention under bilateral intervention, such that the per-unit subsidy is nonnegative.

## Proposition 2

Define  $s^b(\sigma)$ ,  $\widetilde{\sigma}^b$ , and  $\sigma^b$  as in equations (23) and (24). Then the optimal bilateral subsidy that satisfies the NPC is given by:

(28) 
$$\tilde{s}^{b}(\sigma) = \begin{pmatrix} s^{b}(\sigma) & on \left[\sigma, \tilde{\sigma}^{b}\right] \\ 0 & on \left(\tilde{\sigma}^{b}, \overline{\sigma}\right] \end{pmatrix}$$

## Proof Appendix 2

An important corollary is that the introduction of the rival contract broadens the set of efficient firms that are targeted in equilibrium, because the competition in contracts loosens the incentive compatibility constraint.

## Corollary

$$\widetilde{\sigma}^{\mathrm{u}} < \widetilde{\sigma}^{\mathrm{b}}$$
 for  $\sigma^{\mathrm{u}} < \overline{\sigma}$ 

## Proof Appendix 2

Recall in the full-information case that there is a prisoner's dilemma structure to the bilateral game, such that the dominant strategy is always intervention. In sharp contrast, under incomplete information with a strong participation constraint, bilateral nonintervention obtains endogenously as the Nash equilibrium of the game in contracts for a sufficient range of uncertainty. For a narrower range of uncertainty, both governments intervene with positive subsidies in equilibrium, similar to the complete information case, but at a lower level of subsidies.

Similar to the analysis with the ZPC in both the unilateral and bilateral cases it is straightforward to verify that the local second order conditions are satisfied at the optimum. Global optimality is more complicated to verify with the NPC, so this is relegated to Appendix 3.

## V. Alternative Assumptions

## i. Renegotiation

Throughout, we have assumed that renegotiation is impossible. This assumption is by no means innocuous. As long as renegotiation is ruled out, observable contracts can do no better than unobservable initial contracts. As a result, the government's ability to precommit by delegating action to the firm is compromised by the informational asymmetry. However, if secret renegotiation between the government and the firm were permitted after the private information had been realized, it would be possible for the government to capture some precommitment power above that obtained by delegation.

Dewatripont (1988) considers renegotiation in a unilateral game with a contracting stage preceding the realization of uncertainty. He shows that when secret renegotiation is possible after the agent has learned its private information, it is possible and optimal for the principal to commit ex ante to contracts with inefficient ex post outcomes where no pareto improvement is possible. For instance, the principal could sign a public contract committing the domestic firm to sufficiently

high output to discourage foreign production, as long as there is a potential for secret renegotiation after the private information is realized; such a contract is credible because of the asymmetry in information.

Caillaud, Julien, and Picard (1990) make a similar point in a bilateral principal-agent framework. With secret ex post renegotiation, and strategic substitutes, the observed initial contracts serve to alter each agent's reservation utility level in such a way that it is committed to a more aggressive output level. Each principal publicly commits to the initial contract, knowing that it will be renegotiated after the private information has been realized such that each firm is at least as well off as under the observed initial contract, and surplus net of informational rents is higher.

## ii. Alternative Information Structure

The results presented here are robust to other specifications of the information structure. For instance, distributions other than the uniform would yield similar results, as would a formulation in which costs were imperfectly correlated across firms (as, for example, in Gal-Or (1988)). Alternatively, the uncertainty could affect each firm's costs differently. For instance, domestic costs could differ from foreign by a factor of (1+t), where t might represent a transport cost.<sup>23</sup>

An interesting extension would be to model each government as having access to a different type of information about the local firm, which would correspond to different political frameworks, or different mechanisms of control.

## iii. Consumption at Home

We have imposed the artificial convention that both firms are exporting to a third market, to isolate the interaction between precommitment effects and asymmetric information from complications associated with consumer surplus. An obvious extension would incorporate production for the home market as well as for export. In this case, if the informational asymmetry applied to both types of production, the government would distort downward subsidies on the firm's total production in order to minimize allocative distortions from informational rent-seeking, while using the reimbursement as a screening device. Alternatively, it is possible that screening could be accomplished in these circumstances by using two distinct subsidies. We leave the investigation of policies comprised of two marginal tools in the context of incomplete information for future research, due to its complexity.

## iv. Strategic Complements

Eaton and Grossman (1986) show that with full information, the optimal policy is an export tax when the firm's decision variables are strategic complements, whereas it is a subsidy in the case of strategic substitutes. The difference in policy prescriptions is attributable to the difference in sign of the cross-partial second derivative. This difference in the sign of the optimal policy in the case of complements carries over to incomplete information, but here it operates through the firms' incentive compatibility constraints.

With strategic complements (d>0) in the model developed here, there is a continuum of symmetric separating contracts, which makes it difficult to compare equilibrium outcomes under unilateral and bilateral intervention. However, there is a unique optimal linear tax for both unilateral and bilateral intervention. Comparing these linear policies to the optimal policies under full information establishes that the optimal tax is distorted upwards under incomplete information, because of the incentive compatibility constraint. Thus, the optimal policy is overly restrictive relative to the full-information case.

In contrast to the substitute case with incomplete information, under bilateral intervention the governments' contracts reinforce rather than countervail each other with complements, by raising the rival firm's temptation to cheat. Each government attempts to reduce the local firm's output, but the resulting contraction exacerbates the rival firm's incentive constraint. As a result, in the linear equilibrium, production is restricted more severely under bilateral than unilateral intervention. The intuition for this is the mirror image of that in the case of substitutes; with complements, each firm's contraction is met by reciprocal contraction, thus exacerbating rather than mitigating the incentive compatibility constraints.

Martimort (1991) derives this result for strategic complements in a general framework with competing principal-agent pairs.

<sup>&</sup>lt;sup>25</sup> The lack of uniqueness is attributable to the assumption of symmetry.

#### VI. Conclusion

This paper is far from conclusive; understanding strategic trade policy in the context of asymmetric information is likely to be a complex task, requiring a variety of models to address a number of different questions. Nonetheless, the simple model developed above suggests several interesting insights.

First, the effect of the informational asymmetry is to lessen the precommitment effect of unilateral government intervention. Second, in contrast to the full-information case, the introduction of a rival interventionist government reinforces rather than countervails the precommitment effect, although not sufficiently to completely offset the distortion from the informational asymmetry. This mitigates the prisoner's dilemma associated with fully-informed bilateral intervention, and may eliminate it altogether for sufficiently high levels of Third, the informational asymmetry distorts the optimal subsidy uncertainty. downward in all cases, and when the range of uncertainty is high, this distortion may be sufficient to require a tax rather than a subsidy, given a zero-profit constraint. Fourth, when the participation constraint gives the firm residual rights of control, the government eschews intervention for the least efficient firms, and subsidies are always nonnegative in equilibrium.

At minimum, these results caution that profit-shifting trade policies may have undesirable consequences for social welfare when governments are even partially ignorant about firms' profit functions. Attainment of the informationallyconstrained social optimum requires a complicated menu of contracts combining perunit subsidies and lump-sum transfers, which may be impracticable in actual trade policy contexts.

#### References

Baldwin, R. and P.R. Krugman, (1988): "Market Access and International Competition: A Simulation Study of 16K Random Access Memories," in R. Feenstra (ed.), Empirical Methods for International Trade, Cambridge, MIT Press.

Baron, D. and R. Myerson, (1982): "Regulating a Monopolist with Unknown Costs," <u>Econometrica</u>, 50: pp. 911-930.

Brainard, S.L., (1990): "Last One Out Wins: Trade Policy in an International Exit Game," NBER Working Paper No. 3553.

Brander, J.A. and B.J. Spencer, (1985): "Export Subsidies and International Market Share Rivalry," <u>Journal of International Economics</u>, 18, pp. 83-100.

Caillaud, B., R. Guesnerie, P. Rey, and J. Tirole, (1985): "The Normative Economics of Government Intervention in Production in the Light of Incentive Theory: A Review of Recent Contributions," Stanford University IMSSS Technical Report No. 473.

Caillaud, B., B. Jullien and P. Picard, (1990): "Competing Vertical Structures: Precommitment and Renegotiation," Mimeo.

Carmichael, C.M., (1987): "The Control of Export Credit Subsidies and its Welfare Consequences," <u>Journal of International Economics</u>, 23, pp. 1-19.

Dewatripont, M., (1988): "Commitment Through Renegotiation-Proof Contracts with Third Parties," Review of Economic Studies, IV, pp. 377-390.

Dixit, A.K., (1988): "Optimal Trade and Industrial Policy for the U.S. Automobile Industry," in R. Feenstra (ed.), <u>Empirical Methods for International Trade</u>, Cambridge, MIT Press.

Eaton, J. and G.M. Grossman, (1986): "Optimal Trade and Industrial Policy under Oligopoly," Quarterly Journal of Economics, 51: May, pp. 383.

Faggart, M.C., (1990): "Concurrence, Asymétrie d'Information, Théorie des Incitations: Ouelques Développements du Modèle Principal Agent", Mimeo.

Fudenberg, D. and Levine, (1989): "Reputation and Equilibrium Selection in Games with a Patient Player," <u>Econometrica</u>, 57, pp. 755-778.

- Gal-Or, E., (1988): "Vertical Separation in Oligopolistic Markets with Incomplete Information," University of Pittsburgh Working Paper.
- Gresik, T.A. and D.R. Nelson, (1991): "Incentive Compatible Regulation of a Foreign-owned Subsidiary," Mimeo.
- Grossman, G.M., (1986): "Strategic Export Promotion: a Critique," in P.R. Krugman (ed.), <u>Strategic Trade Policy and the New International Economics</u>, pp. 47-68.
- Gruenspecht, H.K. (1988): "Export Subsidies for Differentiated Products," Journal of International Economics, 24, pp. 331-344.
- Katz, M., (1991): "Game Playing Agents: Unobservable Contracts as Precommitments," <u>Rand Journal of Economics</u>, 22, pp. 307-327. Kreps and Scheinkman, (1983): "Quantity Precommitments and Bertrand Competition Yields Cournot Outcomes," <u>Bell Journal of Economics</u>, 14, pp. 326-337.
- Krugman, P.R. and S.L. Brainard, (1988): "Problems in Modelling Competition in the International Aircraft Industry," Mimeo.
- Laffont, J.J. and J. Tirole, (1986): "Using Cost Observation to Regulate Firms", Journal of Political Economy, 94, pp. 614-641.
- Laffont, J.J. and J. Tirole, (1991): "A Theory of Incentives," manuscript.
- Laffont, J.J. and J. Tirole, (1990a): "The Politics of Government Decision-Making: A Theory of Regulatory Capture," Mimeo.
- Laffont, J.J. and J. Tirole, (1990b): "The Politics of Government Decision Making: Regulatory Institutions," Journal of Law, Economics and Organization.
- Lewis, T. and D. Sappington, (1989): "Countervailing Incentives in Agency Problems," <u>Journal of Economic Theory</u>, 49, pp. 294-313.
- Martimort, D., (1991): "Optimal Contracts and Games Between Principal-Agent Pairs," Mimeo.
- Myerson, R., (1982): "Optimal Coordination Mechanisms in Generalized Principal-Agent Problems," <u>Journal of Mathematical Economics</u>, 10, pp. 67-81.
- Neary, J.P., (1991): "Export Subsidies and Price Competition," in E. Helpman and A. Razin (eds.), International Trade and Trade Policy, MIT Press.

Prusa, T., (1990): "An Incentive-Compatible Approach to the Transfer Pricing Problem", <u>Journal of International Economics</u>, 28, pp. 155-172.

Raff, H., (1991): "Intrafirm Exports and Optimal Host Country Commercial Policy under Asymmetric Information," Mimeo.

Schmidt, (1991): "Reputation and Equilibrium Characterization in Repeated Games of Conflicting Interest," Mimeo.

Tirole, J. (1988): The Theory of Industrial Organisation, MIT Press, Cambridge.

#### APPENDIX 1

## **Proof of Proposition 1**

We start by defining the Hamiltonian for the government's program:

$$H (\sigma, s_{k}(\sigma), V_{k}(\sigma), \mu(\sigma), \lambda(\sigma)) =$$

$$f(\sigma) \left[ \pi_{k}(s_{k}(\alpha), s_{j}(\sigma), \sigma) - \pi_{k}(0, s_{j}(\sigma), \sigma) - s_{k}(\sigma) q_{k}(s_{k}(\sigma), s_{j}(\sigma), \sigma) \right] - \frac{c}{1+c} V_{k}(\sigma)$$

$$-\lambda(\sigma) K s_{k}(\sigma) \left[ 1 - \frac{d}{(b+d)} \frac{\partial s_{j}(\sigma)}{\partial \sigma} \right] - \mu(\sigma) V_{k}(\sigma)$$

where:

$$* \qquad K = \frac{b^2}{(b-d)^2(b+d)}$$

- \*  $\lambda(\sigma)$  is the multiplier on the incentive constraint
- \*  $\mu(\sigma)$  is the multiplier on the participation constraint

The necessary conditions for optimization are:

- \*  $\lambda(\sigma)$  is piecewise continuous on subsets  $[\underline{\sigma}, \sigma_k], \dots, [\sigma_k, \sigma_{k+1}], \dots, [\sigma_n, \overline{\sigma}]$
- \*  $\mu(\sigma)$  is continuous on subsets  $[\underline{\sigma}, \sigma_k], \dots, [\sigma_k, \sigma_{k+1}], \dots, [\sigma_n, \overline{\sigma}]$

In addition:

(2) 
$$\frac{\partial V_k(\sigma)}{\partial \sigma} = -Ks_k(\sigma)$$

(3) 
$$\frac{\partial \lambda(\sigma)}{\partial \sigma} = \frac{c}{1+c} f(\sigma) - \mu(\sigma)$$

for all intervals on which  $s_k(\sigma)$  is continuous

$$\mu(\sigma) \le 0$$

$$\mu(\sigma)V_{\mathbf{i}}(\sigma) = 0$$

(5) 
$$H(\sigma_{,s_{h}}(\sigma),V_{h}(\sigma),\lambda(\sigma),0) \geq H(\sigma,\omega(\sigma),V_{h}(\sigma),\lambda(\sigma),0)$$

$$\forall \text{ subsidies } \omega(\sigma)$$

The boundary conditions are:

(6) if 
$$V_{h}(\underline{\sigma})$$
 is free,  $\lambda(\underline{\sigma})=0$  if  $V_{h}(\overline{\sigma})$  is free,  $\lambda(\overline{\sigma})=0$ 

We next determine  $\Omega$ , the subset of values of  $\sigma$  on which  $V_h(\sigma)=0$ . For any value  $\sigma_k$  at which  $\lambda(\sigma)$  is discontinuous, there exists  $\beta_k \leq 0$  such that:

(7) 
$$\lambda(\sigma_k^{\dagger}) - \lambda(\sigma_k^{\bar{}}) = \beta_k$$

Start by supposing that  $\Omega$  does not contain  $\overline{\sigma}$ . Then  $V_h(\overline{\sigma})$  is free, with  $V_h(\overline{\sigma}) > 0$ . The boundary condition is then  $\lambda(\overline{\sigma}) = 0$ , and, in addition,  $\mu(\overline{\sigma}) = 0$ . Call  $\sigma_1 = \sup_{\Omega \in \Omega} \Omega$ . Then, using (3),  $\lambda(\sigma)$  can be computed on  $[\sigma_1, \overline{\sigma}]$  as:

(8) 
$$\lambda(\sigma) = \frac{c}{1+c} \left[ \frac{\overline{\sigma} - \sigma}{\overline{\sigma} - \underline{\sigma}} \right]$$

Applying Equation () to this subset yields the following subsidy:

$$s_{k}^{\mu}(\sigma) = s_{k}^{*\mu}(\sigma) + \alpha^{\mu}[\overline{\sigma} - \sigma]$$

which is decreasing. This subsidy exceeds  $s_h^*(\sigma)$ , which is positive on  $(\sigma, \overline{\sigma}]$ . This implies that the rent  $V_h(\sigma)$  is decreasing on the subset  $[\sigma_1, \overline{\sigma}]$ , and that  $V_h(\sigma_1) = 0$ . But this contradicts the condition that  $V_h(\sigma) > 0$  on  $[\sigma_1, \overline{\sigma}]$ . So we can conclude that  $\sigma_1 = \overline{\sigma}$ .

Next, notice that if  $\Omega$  contains a subset with a non-empty interior, Equation

(2) implies that  $s_h(\sigma)=0$  on this subset. Suppose that  $\Omega$  contains two values  $\sigma_a$  and  $\sigma_d$  for which  $V_h(\sigma_a)=V_h(\sigma_d)=0$ , and  $V_h(\sigma)>0$  on  $(\sigma_a,\sigma_d)$ . We could then find two values of  $\sigma$  in  $(\sigma_a,\sigma_d)$ ,  $\sigma_b$  and  $\sigma_c$ , such that:

i) 
$$\sigma_{\rm b} < \sigma_{\rm c}$$

ii) 
$$V'_h(\sigma_h) > 0 > V'_h(\sigma_c)$$

which from (1) implies that  $s_h(\sigma_b) < 0 < s_h(\sigma_c)$ . But this would contradict the incentive compatibility condition that  $s_h(\sigma)$  is decreasing. We conclude that  $\Omega$  is an interval of  $[\sigma, \overline{\sigma}]$ .

This in turn implies that  $V_h(\underline{\sigma})$  is free. If instead,  $\Omega = [\underline{\sigma}, \overline{\sigma}]$ , the government could do better by giving firms whose costs lie on  $[\underline{\sigma}, \underline{\sigma} + \epsilon]$  the subsidy  $s^u_h(\sigma)$  associated with the ZPC, which exceeds that associated with the NPC. Thus, we can conclude that  $V_h(\underline{\sigma})$  is free, and that the participation constraint binds on some subset  $[\sigma_0, \overline{\sigma}]$ .

Next we integrate over  $[\underline{\sigma}, \sigma_1]$  to find:

(9) 
$$\lambda(\sigma) = \frac{c}{1+c} \frac{(\sigma - \underline{\sigma})}{(\overline{\sigma} - \underline{\sigma})}$$

The optimal subsidy equals  $s_h^u(\sigma)$  on this interval,  $\tilde{s}_h^u(\sigma) = s_h^u(\sigma)$ . Allowing for the possible discontinuity of  $\lambda(\sigma)$  at the value  $\sigma_1$ , define  $\beta_1$  as in Equation (7) above. For  $\sigma$  above  $\sigma_1$  we have:

(10) 
$$\lambda(\sigma) = \frac{c}{1+c} \frac{(\sigma - \underline{\sigma})}{(\overline{\sigma} - \underline{\sigma})} + \beta_1 + M(\sigma)$$

where:

$$\mathbf{M}(\sigma) = \int_{\sigma}^{\sigma} \mu(t) dt$$

is such that  $M(\sigma) = 0$  on  $[\underline{\sigma}, \sigma_1]$ , and  $M(\sigma)$  is decreasing above  $\sigma_1$ .

Optimization of the subsidy on  $[\sigma_1, \overline{\sigma}]$  leads to:

(11) 
$$K[\beta_1 + M(\sigma)] = \frac{1}{(\overline{\sigma} - \underline{\sigma})} \left[ \frac{\partial \pi(\overline{s}_k(\sigma), 0, \sigma)}{\partial s_k} - \frac{c}{1 + c} K(\sigma - \underline{\sigma}) \right]$$

Using linearity, we obtain:

(12) 
$$K[\beta_1 + M(\sigma)] = \frac{1}{(\overline{\sigma} - \underline{\sigma})} \left[ \frac{2(b^2 - 2d^2)}{(b + d)^2} \right] [s_h^{\mu}(\sigma) - \overline{s}_h^{\mu}(\sigma)]$$

With  $\beta_1 = 0$  and  $\widetilde{s}_h^u(\sigma) = 0$  on  $[\sigma_1, \overline{\sigma}]$ ,  $M(\sigma)$  will be negative and decreasing above  $\sigma_1 = \widetilde{\sigma}^u(\sigma)$ , because  $s_h^u(\sigma)$  is negative and decreasing above this value. Straightforward differentiation of  $M(\sigma)$  yields the corresponding value of  $\mu(\sigma)$ , and confirms the switch of regimes at  $\sigma_1 = \widetilde{\sigma}^u$ .

#### **APPENDIX 2**

## **Proof of Proposition 2**

We look for a Nash equilibrium in contracts. Thus, the home government takes as given the subsidy schedule announced by the foreign government:

(13) 
$$\tilde{s}_{f}^{b}(\sigma) = \begin{bmatrix} s_{f}^{b}(\sigma) & on \ [\underline{\sigma}, \tilde{\sigma}^{b}] \\ 0 & on \ [\tilde{\sigma}^{b}, \overline{\sigma}] \end{bmatrix}$$

We start by verifying:

(i) 
$$1 - \frac{d}{b+d} \frac{\partial s_f^b(\sigma)}{\partial \sigma} > 0$$

on  $[\underline{\sigma}, \overline{\sigma}^{b}]$ , so the necessary condition for implementability (Equation (16) in the text) is that  $s_{h}^{b}(\sigma)$  is decreasing.

(ii)  $\widetilde{s}_{i}(\sigma) = 0$  on  $[\widetilde{\sigma}_{i}, \overline{\sigma}]$ . The same condition as above implies that  $s_{i}(\sigma)$  is decreasing.

Again call  $\Omega$  the subset of values of  $\sigma$  for which  $V_h(\sigma)=0$ , and the domestic government does not intervene. As before, we establish that  $\sup \Omega = \overline{\sigma}$ . The proof is the same as for Proposition 1, because  $\widetilde{s}_h^b(\sigma)=0$  close to  $\overline{\sigma}$ .

Second, we establish that  $\Omega$  is an interval,  $\Omega = [\sigma_2, \overline{\sigma}]$ , because the subsidy must be decreasing; the proof uses the incentive compatibility condition that the subsidy must be nonincreasing, just as in Proposition 1.

Proceeding as in Proposition 1, we then verify as before that  $V(\underline{\sigma})$  is free, such that the regime on  $[\underline{\sigma}, \sigma_2]$  is one of intervention:

(14) 
$$\lambda(\sigma) = \frac{c}{1+c} \left( \frac{\sigma - \underline{\sigma}}{\overline{\sigma} - \underline{\sigma}} \right)$$

It is straightforward to check that  $\tilde{s}_h^b(\sigma) = s_h^b(\sigma)$ . Again, call the possible discontinuity of  $\lambda(\sigma)$  at  $\sigma_2$ ,

 $\beta_2 = \lambda(\sigma_2^+) - \lambda(\sigma_2^-)$ . As in the previous proof, we write for  $\sigma > \sigma_2$ :

(15) 
$$\lambda(\sigma) = \frac{c}{1+c} \frac{(\sigma - \sigma)}{(\overline{\sigma} - \sigma)} + M(\sigma) + \beta_2$$

defining  $M(\sigma)$  as in the proof of Proposition 1.

Optimizing the Hamiltonian with respect to the subsidy for  $\sigma > \sigma_2$  leads to an expression that we rewrite by using the definition of  $s^b(\sigma)$ :

$$K[\mathbf{M}(\sigma) + \beta_{2}] \left[ 1 - \frac{d}{b+d} \frac{\partial \tilde{s}_{f}^{b}(\sigma)}{\partial \sigma} \right] =$$

$$\frac{1}{(\overline{\sigma} - \underline{\sigma})} \frac{b}{(b^{2} - d^{2})^{2}} \left[ (b^{2} - 2d^{2})[s^{b}(\sigma) - \tilde{s}_{h}^{b}(\sigma)] + \frac{d^{3}}{b} [\tilde{s}_{f}^{b}(\sigma) - s^{b}(\sigma)] \right]$$

$$-K \frac{c}{1+c} \frac{(\sigma - \underline{\sigma})}{(\overline{\sigma} - \underline{\sigma})} \left[ 1 - \frac{d}{b+d} \left[ \frac{\partial \tilde{s}_{f}^{b}(\sigma)}{\partial \sigma} - \frac{\partial s^{b}(\sigma)}{\partial \sigma} \right] \right]$$

Notice if we set  $\sigma_2 = \tilde{\sigma}^b$ , we find:

(17) 
$$\beta_2 = -\frac{c}{1+c} \left( \frac{\tilde{\sigma}^b - \underline{\sigma}}{\overline{\sigma} - \underline{\sigma}} \right) \left( 1 + \frac{d}{b+d} k \right) < 0 \quad \text{for } k = \frac{\partial s^b(\sigma)}{\partial \sigma}$$

(where k is a negative constant because  $s^b(\sigma)$  is linear and decreasing). Thus, for  $\sigma > \widetilde{\sigma}^b$ .

(18) 
$$KM(\sigma) = \frac{1}{(\overline{\sigma} - \underline{\sigma})} \frac{1}{(b - d)^2} s^b(\sigma) \left[ \frac{b^2 - db - d^2}{(b + d)} \right] + \frac{c}{1 + c} \left[ \frac{\tilde{\sigma}^b - \sigma}{\overline{\sigma} - \underline{\sigma}} \right] \left[ 1 + \frac{d}{b + d} k \right]$$

such that  $M(\sigma)$  is decreasing and continuous at  $\widetilde{\sigma}^b$ , and is negative above  $\widetilde{\sigma}^b$  (since  $M(\widetilde{\sigma}^b)=0$ ). From this  $M(\sigma)$  we can find the value  $\mu(\sigma)$  by simple derivation, confirming the switch of regimes for  $\sigma_2=\widetilde{\sigma}^b$ .

#### Proof of Corollary to Proposition 2

Proposition 1 established that the optimal policy is nonintervention over the interval  $[\widetilde{\sigma}^u, \overline{\sigma}]$ , for  $\widetilde{\sigma}^u < \overline{\sigma}$ . Proposition 2 established that the optimal policies result in bilateral nonintervention over the interval  $[\widetilde{\sigma}^b, \overline{\sigma}]$ , for  $\widetilde{\sigma}^b < \overline{\sigma}$ . The Corollary states that the range of nonintervention is greater under unilateral relative than bilateral intervention. This is equivalent to the claim that  $\widetilde{\sigma}^u < \widetilde{\sigma}^b$ .

Using the definition of  $\tilde{\sigma}^{u}$  from equation (24), define  $C^{u}$  as:

(19) 
$$C^{u} = \frac{(\tilde{\sigma}^{u} - \underline{\sigma})}{(a - \tilde{\sigma}^{u})} = \frac{d^{2}}{b^{2}} \frac{(1+c)}{c}$$

Similarly, from equation (26), define C<sup>b</sup> as:

(20) 
$$C^{b} = \frac{(\tilde{\sigma}^{b} - \underline{\sigma})}{(a - \tilde{\sigma}^{b})} = \frac{d^{2}[(1 + c)(b + d)(b^{2} - d^{2} - db) - cb^{2}d]}{cb^{3}(b^{2} - 2d^{2})}$$

Then, it is clear that  $C^u < C^b$  (for the relevant case, where  $\widetilde{\sigma}^u < a$  and  $< \widetilde{\sigma}^b < a$ ), which in turn implies that  $\widetilde{\sigma}^u < \widetilde{\sigma}^b$ .

#### **APPENDIX 3**

## Proof of Global Optimality for the Agent's Problem in the NPC Case

#### I. Unilateral Case

Consider a firm with costs  $\sigma$  in  $[\underline{\sigma}, \widetilde{\sigma}^u]$ . If the firm claims to be  $\sigma \in [\underline{\sigma}, \widetilde{\sigma}^u]$ , it can do no better than by claiming to be  $\sigma$ , the argument being the same as in the ZPC case. If instead the  $\sigma$  firm claims to be  $\widehat{\sigma} \in [\widetilde{\sigma}^u, \overline{\sigma}]$ , it receives no protection, so it gets  $\pi(0,0,\sigma)$ , which is below  $U(\sigma,\sigma)$ , the participation constraint.

Now consider a firm with costs  $\sigma$  in  $[\widetilde{\sigma}^u, \overline{\sigma}]$ . The firm is indifferent between reporting  $\sigma$  and  $\widehat{\sigma} \in [\widetilde{\sigma}^u, \overline{\sigma}]$ , since both result in no protection. Suppose instead that it claims  $\widehat{\sigma} \in [\underline{\sigma}, \sigma^u]$ . We must prove that  $\pi[0,0,\sigma] \geq \pi[s_h^u(\widehat{\sigma}),0,\sigma] = \widehat{t}^u(\widehat{\sigma})$ . Notice that:

(21) 
$$\tilde{t}^{u}(\hat{\sigma}) = \int_{\delta_{u}}^{\delta} \frac{\partial s_{k}^{u}(t)}{\partial t} \frac{\partial \pi \left[s_{k}^{u}(t), 0, t\right]}{\partial s_{k}} dt$$

But

(22) 
$$\pi[s_h^{u}(\tilde{\sigma}^{u}),0,\sigma] - \pi[s_h^{u}(\hat{\sigma}),0,\sigma] = \int_{h}^{\tilde{\sigma}^{u}} \frac{\partial s_h^{u}(t)}{\partial t} \frac{\partial \pi[s_h^{u}(t),0,\sigma]}{\partial s_h} dt$$

so we have only to check that:

(23) 
$$\int_{\delta}^{\delta^{\mu}} \frac{\partial s_{h}^{\mu}(t)}{\partial t} \left[ \int_{t}^{\sigma} \frac{\partial \pi [s_{h}^{\mu}(t), 0, z]}{\partial s_{h} \partial \sigma} dz \right] dt \geq 0$$

But since  $\hat{\sigma} < \tilde{\sigma}^{u} < \sigma$ , and

$$\frac{\partial \pi(.)}{\partial s_{k}, \partial \sigma} < 0$$

this last inequality is true.

#### II. Bilateral Case

Consider a firm with costs  $\sigma$  in  $[\underline{\sigma}, \widetilde{\sigma}^b]$ . If the firm reports  $\sigma \in [\underline{\sigma}, \widetilde{\sigma}^b]$ , it can do no better than by reporting  $\sigma$  for the same reasons as in the ZPC case. If instead the firm reports  $\sigma \in [\widetilde{\sigma}^b, \overline{\sigma}]$ , it receives no protection, yielding  $\pi[0, s_f^b(\sigma), \sigma]$ , which is below  $U(\sigma, \sigma)$ , the participation constraint.

Now consider a firm with costs  $\sigma$  in  $[\widetilde{\sigma}^b, \overline{\sigma}]$ . It is indifferent between reporting  $\sigma$  and  $\widehat{\sigma}$  in  $[\widetilde{\sigma}^b, \overline{\sigma}]$ , as both leave it with no protection. Suppose instead that it reports  $\widehat{\sigma} \in [g, \widetilde{\sigma}^b]$ , and it obtains

 $-\tilde{t}^{b}(\hat{\sigma}) + \pi[s_{h}^{b}(\hat{\sigma}), 0, \sigma]$ . We must establish that this number is no greater than  $\pi[0,0,\sigma]$ . We can write:

(24) 
$$\tilde{t}^{b}(\hat{\sigma}) = + \int_{s^{b}}^{b} \frac{\partial s_{h}^{b}(t)}{\partial t} \frac{\partial \pi[s_{h}^{b}(t), s_{f}^{b}(t), t]}{\partial s_{h}} dt$$

So we want:

(25) 
$$\int_{ab}^{b} \frac{\partial s_{h}^{b}(t)}{\partial t} \frac{\partial \pi[s_{h}^{b}(t), s_{f}^{b}(t), t]}{\partial s_{h}} dt \leq \int_{b}^{b} \frac{\partial s_{h}^{b}(t)}{\partial t} \frac{\partial \pi[s_{h}^{b}(t), 0, t]}{\partial s_{h}} dt$$

But notice that because  $\sigma > \widetilde{\sigma}^b$ ,  $s^b(\widetilde{\sigma} \not \exists \pi(0)$ , and  $\partial s_k, \partial \sigma$ 

(26) 
$$\frac{\partial \pi[s_h^b(t),0,\sigma]}{\partial s_h} < \frac{\partial \pi[s_h^b(t),s_f^b(\tilde{\sigma}^b),\tilde{\sigma}^b]}{\partial s_h}$$

So

(27) 
$$\frac{\partial \pi[s_{h}^{b}(t),0,\sigma]}{\partial s_{h}} - \frac{\partial \pi[s_{h}^{b}(t),s_{f}^{b}(t),t]}{\partial s_{h}} < \frac{\partial \pi[s_{h}^{b}(t),s_{f}^{b}(\tilde{\sigma}^{b}),\tilde{\sigma}^{b}]}{\partial s_{h}} - \frac{\partial \pi[s_{h}^{b}(t),s_{f}^{b}(t),t]}{\partial s_{h}}$$

But the right term of this inequality equals:

(28) 
$$\int_{t}^{\delta_{h}} \left[ \frac{\partial \pi[s_{h}^{b}(t),s_{f}^{b}(z),z]}{\partial s_{h},\partial s_{f}} \frac{\partial s_{h}^{b}(z)}{\partial z} + \frac{\partial \pi[s_{h}^{b}(t),s_{f}^{b}(z),z]}{\partial s_{h}\partial \sigma} \right] dz$$

Since we assumed a quadratic form for  $\pi[s_n, s_f, \sigma]$ , it is easy to check that the expression in the last integrand is the same as that in the local second-order condition of the agent's problem. This integrand is negative, and:

(29) 
$$\frac{\partial s_{h}^{b}(t)}{\partial t} \left[ \frac{\partial \pi [s_{h}^{b}(t), 0, \sigma]}{\partial s_{h}} - \frac{\partial \pi [s_{h}^{b}(t), s_{f}^{b}(t), t]}{\partial s_{h}} \right] \geq \frac{\partial s_{h}^{b}(t)}{\partial t} \int_{t}^{\delta_{h}} \left[ \frac{\partial \pi [s_{h}^{b}(t), s_{f}^{b}(z), z]}{\partial s_{h}\partial s_{f}} \frac{\partial s_{h}^{b}(z)}{\partial t} + \frac{\partial \pi [s_{h}^{b}(t), s_{f}^{b}(z), z]}{\partial s_{h}\partial \sigma} \right] dz \geq 0$$

where the right term is positive. Integrating this last inequality between  $\hat{\sigma}$  and  $\hat{\sigma}^b$  leads to the desired condition.