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NEAR-RATIONALITY, HETEROGENEITY AND AGGREGATE CONSUMPTION

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ABSTRACT

The simple permanent income model provides a good description of the medium-long run behavior of aggregate non-durables consumption, while it fails in describing its short run behavior. In this paper I present a non-representative agent model with near-rational microeconomic units that simultaneously explains the observed excess smoothness of consumption to wealth innovations, the excess sensitivity of consumption to lagged income changes, as well as small conditional asymmetries found in the data. In spite of the presence of large non-diversifiable idiosyncratic uncertainty, the estimated dollar equivalent utility cost of the microeconomic near-rational strategy required to explain the *aggregate* facts is only 0.26γ percent of consumption per year, where γ is the coefficient of relative risk aversion.

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1 INTRODUCTION

It is well known that, when applied to nondurables consumption, the simplest form of the Permanent Income (PIH) model (Hall 1978) does not survive formal hypothesis testing. Simply put, there is more serial correlation on aggregate consumption than what is implied by the simplest PIH model. Alternatively, in the income space, there is excess smoothness to income innovations and excess sensitivity to lagged income (Deaton 1987, Campbell and Deaton 1989).[2]

Figure 1 plots the actual path of the logarithm of postwar U.S. quarterly aggregate nondurables consumption (dashed line) and the path implied by a simple PIH model (solid line) for the period 1954:1—1989:4; both series are per capita and in deviation from their deterministic trends.[3] This figure suggests that the simple PIH model describes well the medium-long run stochastic behavior of consumption, but that its description of short run dynamics is not so accurate.

Cochrane (1989) takes the point of Figure 1 one step further. Essentially, he feeds the area between the two curves into a representative agent utility function and concludes that the *economic* departure between the two paths is negligible.[4] In his words, "...high frequency deviations like lagged responses or failure to adjust consumption immediately in response to information announcements have especially low utility costs. But it is precisely the exact timing of the use of information and the exact timing of consumption changes that have been the focus of empirical work and the source of rejections since Hall (1978) and Hansen and Singleton (1983)..."

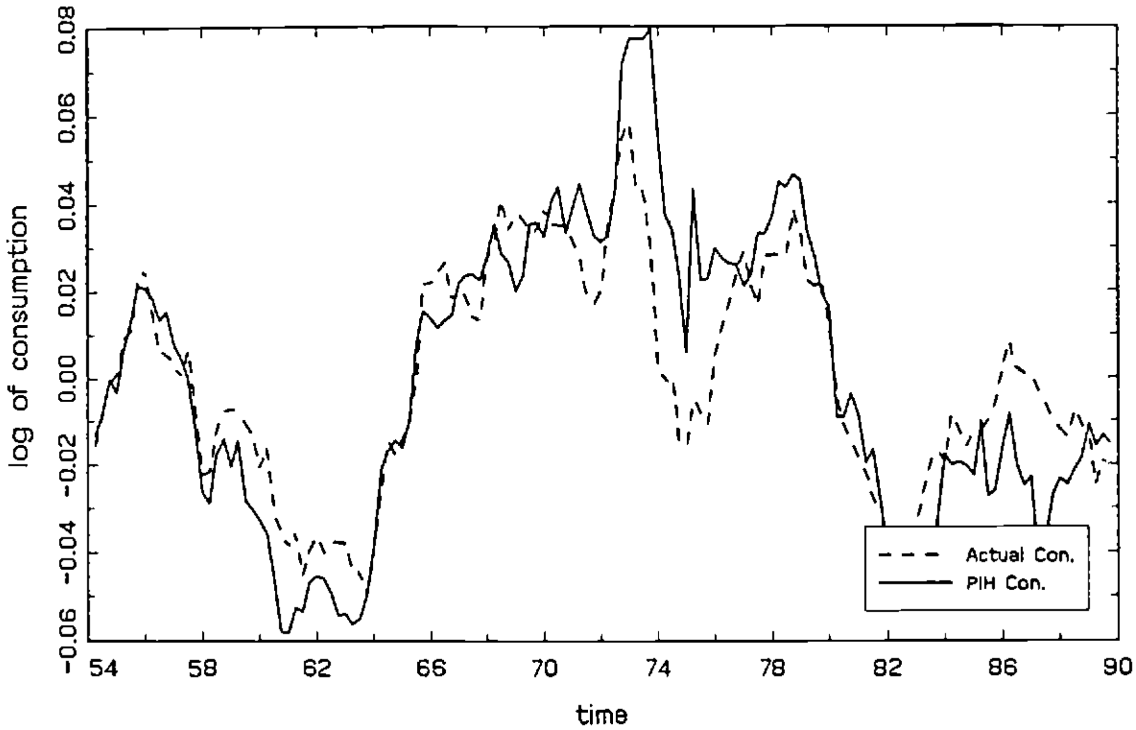
As Cochrane (1989) recognizes, however, his near rationality argument does not necessarily apply to non-representative agent models. One of the key elements in his calculations is that fluctuations in the representative agent's consumption level are small, which does not hold for individual consumers if non-diversifiable idiosyncratic uncer-

[2]In this paper I do not address the asset pricing failure of PIH type models.

[3]The logarithm of per capita PIH consumption corresponds to the logarithm of per/capita disposable income, except for a constant and a deterministic trend, which are left unconstrained (i.e., the figure does not capture deterministic discrepancies). This approximation of wealth is justified by the fact that (detrended) per/capita disposable income follows a process very close to a random walk.

[4]This is an oversimplification of Cochrane's argument. He derives the utility loss under a wide variety of alternatives and shows that the second order nature of the loss stems from the representative agent's first order conditions and the actual volatility of aggregate consumption.

Figure 1
U.S. Aggregate Nondurables Consumption
1954:1-1989:4



tainty is significant. Yet, the idea of near-rationality — now at the microeconomic level — seems realistic. In particular, Akerlof and Yellen's (1985) version of it; which in this context means that an individual does not adjust his consumption level continuously, but waits until the departure between his actual and PIH consumption levels is "large." This defines the purpose of the paper: To study whether plausible combinations of microeconomic near-rationality and non-diversifiable idiosyncratic uncertainty, can generate *aggregate* dynamics consistent with actual U.S. consumption data.

The empirical section proceeds in two steps: It first estimates a non-representative agent version of an Akerlof-Yellen type model, without imposing the constraint that individual consumers' utility losses be small; and asks whether such model can account for the short-run behavior of aggregate consumption. It then goes back to the initial motivation of microeconomic consumption policies, and asks whether the utility losses implied by the estimates are indeed small. The answer to these two questions turn out to be affirmative. First, the model simultaneously explains the observed excess smoothness of consumption to wealth innovations and the excess sensitivity of consumption to lagged income changes. It also explains small conditional asymmetries found in the data: in good times consumers respond more promptly to positive than to negative wealth shocks, while the opposite is true in bad times. And second, the estimated dollar equivalent utility cost of the near-rational microeconomic strategy is only 0.26γ percent of consumption per year, where γ is the coefficient of relative risk aversion.

Section 2 presents the microeconomic model and its connection with aggregate outcomes. The results are presented in Section 3, and Section 4 computes the implied microeconomic utility loss. Final remarks are provided in Section 5.

2 The Model

There is a large number of individuals, approximated by a continuum, and indexed by $i \in [0,1]$. The PIH model determines a consumption function for each individual:

$$n_i^*(t) = \lambda_i(t)' m_i(t),$$

where $n_i^*(t)$ is PIH nondurables consumption and $m_i(t)$ is wealth, for individual i at time

t. Individual i's marginal propensity to consume out of wealth is λ_i', and it may change over time. Equivalently,

$$c_i^*(t) = \lambda_i(t) + h_i(t), \tag{1}$$

where $c_i^* \equiv \ln n_i^*$, $\lambda_i \equiv \ln \lambda_i'$, and $h_i \equiv \ln m_i$.

The logarithm of actual consumption by (near-rational) individual i, $c_i(t)$, on the other hand, remains constant most of the time and is reset only when $z_i(t) \equiv c_i(t) - c_i^*(t)$ reaches a lower trigger point L or an upper trigger point U.[5,6] To simplify the exposition, I assume that $L = -U$ and that when either of the trigger points is reached, $z_i(t)$ is brought back to zero.[7,8]

From the definition of z_i as the log-departure between actual and PIH consumption, it is possible to write the rate of growth of individual i's consumption of nondurables (equal to zero, except at a measure zero set of points in time when it is infinite) as follows:

$$dc_i(t) = dc_i^*(t) + dz_i(t).$$

A simple aggregate counterpart of this equation is easily obtained by (a) multiplying each side of it by $\alpha_i(t)$, the share of individual i's consumption in aggregate consumption, (b) assuming that $\alpha_i(t)$ is not too different from the share of individual i's PIH consumption in PIH aggregate consumption, and (c) integrating each side of this equation with respect

[5]Although the motivation for the (L, C, U) policy in this paper is near-rationality, it is well known that it can be obtained as an optimal response when there are fixed adjustment costs (see e.g. Harrison, Sellke, and Taylor 1983).

[6]Actual consumption being constant between adjustments is just a convenient simplification. It is trivial to extend the model to the case where — when not adjusted discretely — consumption grows at a positive and constant rate, or even at a stochastic rate — as long as this growth rate does not match exactly the (stochastic) rate of growth of PIH consumption.

[7]These symmetry assumptions are harmless for the purpose of this paper; see Caballero (1990b). In the empirical and utility loss computation sections, however, I center the inaction interval around the constant that makes the sample averages of aggregate PIH and actual consumption equal, which is a weak (long run average) budget constraint.

[8]Of course, the specific form of this near-rational microeconomic rule needs not be taken literally. Having fixed barriers is just a mathematical simplification of the idea that as consumers get further away from their PIH consumption level, on average, they are more likely to update their actual consumption level. See Caballero and Engel (1992) for a discussion of this point.

to i:

$$dC(t) = dC^*(t) + \int_0^1 \alpha_i(t) dz_i(t)\, di,$$

where capital letters denote aggregates. This can be written more compactly by assuming that changes in the z_i's are approximately independent of the α_i's. Then:

$$dC(t) = dC^*(t) + dZ(t),$$

where, after exchanging derivatives and integrals,

$$dZ(t) = d\int_0^1 z_i(t)\, di.$$

Thus, dZ represents the change in the average departure of (the log of) actual and PIH consumption across all individuals. Letting $f(z,t)$ represent the *cross sectional* density of z_i's at time t permits us to write dZ as:

$$dZ(t) = d\int_L^U z f(z,t)\, dz,$$

or

$$dZ(t) = \int_L^U z df(z,t)\, dz. \tag{2}$$

This says that the dynamic difference between the *aggregate* rate of consumption growth and its PIH counterpart can be described in terms of the changes in the cross sectional density of the z_i's, which is intuitive. Alternatively, one can describe the path of aggregate consumption directly through the gross flows of microeconomic units upgrading and downgrading their consumption patterns:

$$dC(t) = P(t) - M(t), \tag{3}$$

where $P(t)$ and $M(t)$ are the consumption upgrading and downgrading flows, respectively. The connection between (2) and (3) comes from the fact that the evolution of $P(t)$ and $M(t)$ is closely related to the evolution of the cross sectional density of the z_i's. In order to describe this connection more fully, one needs to make explicit the properties of the driving processes. For this, let each individual's PIH consumption be described by the process:

$$dc_i^*(t) = \theta dt + \sigma dW_i(t), \tag{4}$$

where W_i is a standard Brownian Motion such that $\mathrm{E}[dW_i(t)dW_j(t)] = (\sigma_A^2/\sigma^2)dt$ for $\{j \neq i; j \in [0,1]\}$. The parameters θ, σ_A^2 and σ^2, are the aggregate drift, and aggregate and total (the sum of aggregate and idiosyncratic) variances, respectively.

Since Brownian motions are continuous processes, the upgrading flow in a time-interval dt, starting at t, $P(t)$, must be a function of the number of consumers in the "neighborhood" of the lower trigger barrier, $-U$, at time t. No unit is "at" $-U$ since this is a trigger point, thus the leading term defining the neighborhood is the first (right) derivative of the density at $-U$, $f_z(-U^+, t)$. How deep is the neighborhood (i.e. how many units are "close" to $-U$) and how many of these units reach the trigger point in the time-interval dt is determined by the quadratic variation of Brownian motion, $(\sigma^2/2)dt$: the larger is σ the deeper is the neighborhood, and about half of these units will move in the direction of the barrier in a small time interval. The upgrading flow is then obtained by multiplying the number of upgrading consumers by the size of their adjustment, U. This yields:

$$P(t) = U\frac{\sigma^2}{2}f_z(-U^+, t)dt.$$

A similar derivation shows that:

$$M(t) = -U\frac{\sigma^2}{2}f_z(U^-, t)dt.$$

Thus, the actual rate of growth of aggregate nondurables consumption is:[9]

$$dC(t) = U\frac{\sigma^2}{2}\left\{f_z(-U^+, t) + f_z(U^-, t)\right\}dt, \tag{5}$$

which can be compared with the equation describing the aggregate rate of growth under

[9]See Propositions 2 and 3 in Caballero (1990a) for a formal derivation of a similar equation in the context of durable goods.

the PIH, obtained from integrating equation (4) over i:

$$dC^*(t) = \theta dt + \sigma_A dW_A(t), \tag{6}$$

with $W_A(t)$ a Standard Brownian motion.

Equations (5) and (6) show that the rates of growth of actual and PIH consumption — dC and dC^*, respectively — are described by very different mechanisms. The latter results from aggregating the infinitesimal changes of all units in the system, while the former corresponds to the sum of large changes in the consumption patterns of an infinitesimal fraction of the population. The key elements to determine in equation (5) are the derivatives of the cross sectional density at its boundaries, $f_z(-U^+, t)$ and $f_z(U^-, t)$. I postpone the formal description of these terms until the appendix. In what follows I provide an informal discussion of the behavior of such derivatives, which I use to summarize the main empirical implications of the model.

2.1 The Mechanism

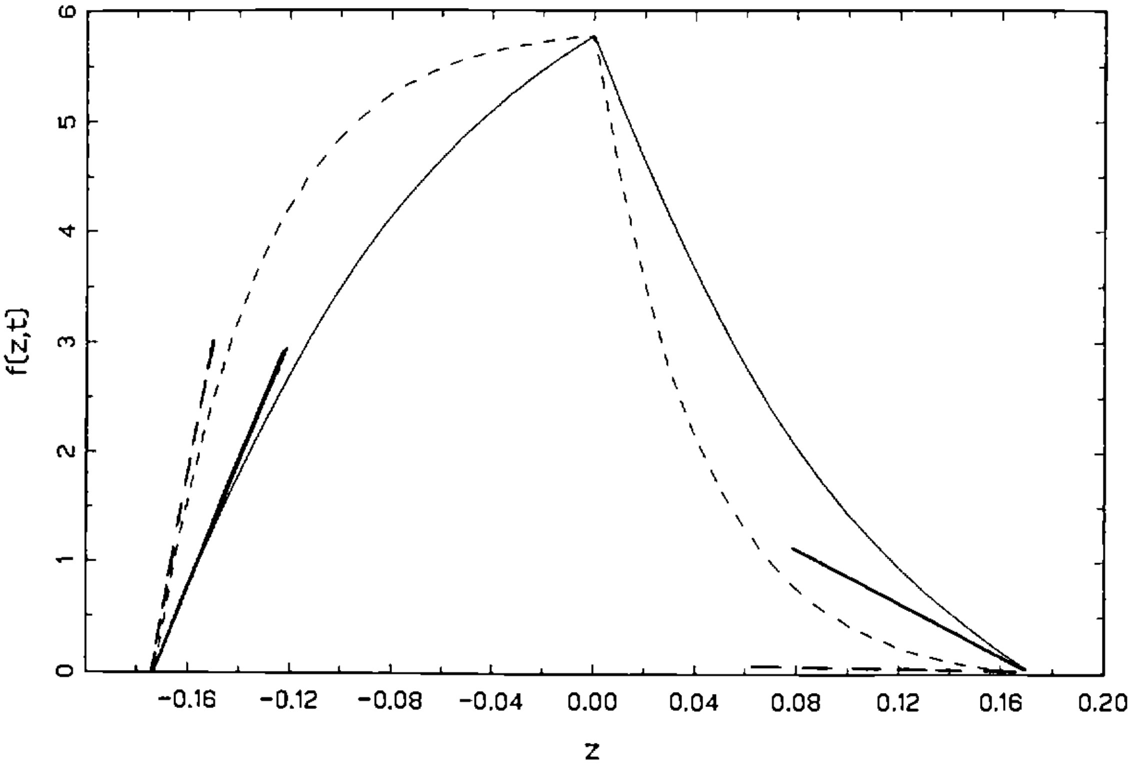
In order to clarify the mechanisms underlying the basic results, let me use a formally implausible but useful example.[10] Imagine that the economy has not had an aggregate surprise for a long time, so $f(z,t)$ has converged to a density like the one depicted by the solid curve in Figure 2, where the skewness is due to the presence of a positive drift in consumers' wealth ($\theta > 0$).[11] In this steady state $dZ(t) = 0$, $dC(t) = dC^*(t) = \theta\, dt$, and $f_z(-U^+) > f_z(U^-)$ (solid tangents), which says that the positive steady state rate of consumption growth is supported by a larger fraction of consumers upgrading their consumption patterns than consumers downgrading theirs (i.e., $f_z(-U^+) = |f_z(U^-)| + 2\theta/(U\sigma^2)$).

Now assume that this economy is followed by a sequence of positive and constant aggregate shocks: $dW_A(t) = \omega(dt) > 0$. This causes an immediate jump of PIH consumption growth to the new rate $dC^*(t) = \theta dt + \omega$. The rate of growth of actual consumption,

[10] It is formally implausible in the sense that the path described can not be generated by a Brownian motion.

[11] If the no-action microeconomic policy is to let consumption grow at the rate θ' instead of zero, the relevant drift for the density is $(\theta - \theta')$.

Figure 2
Cross Sectional Densities



on the other hand, only picks up slowly as more and more units approach the barrier that triggers upward changes, and fewer approach the downgrading barrier. In terms of equation (5), the slopes of the cross sectional density at the boundaries —indexing the number of consumers altering their consumption patterns— change slowly over time. In this process — i.e. while the slopes change sufficiently to match the PIH rate of consumption growth — part of the "force" of the new driving force is absorbed by the shift in the cross sectional density (and slopes at the boundaries), which induces *excess smoothness* of aggregate consumption to wealth innovations.

The other prominent fact about consumption, *excess sensitivity*, is best understood by terminating the expansion;[12] in this case dC^* falls immediately back to $\theta\, dt$, while dC returns more slowly as the "abnormally" large (small) number of units close to the upgrading (downgrading) barrier introduce inertia. This is illustrated by the return of the slopes of the cross sectional density at the boundaries back to those of the solid line in Figure 2. That is, excess sensitivity results from the slow use of the "force" absorbed (stored) by the cross sectional density during the expansion.

The same example can be used for the case in which there is an initial contraction, showing that excess smoothness and excess sensitivity occur in both directions.

2.1.1 FURTHER IMPLICATIONS

Besides the excess smoothness and sensitivity features, the model has more subtle implications arising from the rich dynamics generated by the endogenous evolution of the cross sectional density. The magnitude and timing of the response of consumption to wealth innovations depend on the shape of the cross sectional density at each point in time, which depends on the stochastic environment faced by consumers and on the path of aggregate shocks in particular.

For example, if the economy has been experiencing a sequence of positive shocks, most consumers are likely to be grouped on the upgrading half of their state space.[13] This translates into a cross sectional density with shape as depicted by the solid curve in

[12] For a clear distinction between the excess smoothness and sensitivity findings, see Campbell and Deaton (1989).

[13] See the discussion in terms of the slopes at the trigger barriers in the previous section.

Figure 3, where the value of $Z(t)$, denoted by Z_1 in the figure, is very low. At this point, a further reduction in $Z(t)$ is very difficult, not only because of the stationary nature of $Z(t)$ (as it would happen in a partial adjustment model) but also because of the closeness of the cross sectional density to the invariant (to positive aggregate shocks) uniform one. This limit uniform distribution has the property that the fraction of consumers upgrading their consumption patterns after a positive (continuous) aggregate shock ΔH — which leads to a change in PIH consumption equal to ΔH — is equal to $\Delta H/U$, and since the size of their change is U, the product of these two quantities is approximately ΔH, precisely the PIH response. This limit is never literally reached; however it suggests that when $Z(t)$ is low, consumption —satisfying $C(t) = C^*(t) + Z(t)$— is unlikely to exhibit much excess smoothness with respect to a new *positive* wealth surprise. Conversely, actual consumption should respond very little to a *negative* innovation in wealth, since most of this would be absorbed by the increase in $Z(t)$ owing to the change in the shape of the cross sectional density. Exactly the opposite happens if the economy has been experiencing a sequence of negative wealth shocks, so that the initial cross sectional density looks like the dashed curve in Figure 3 (with mean Z_2).

Figure 4 illustrates the response of consumption to changes in PIH consumption (due to wealth shocks) for different histories of aggregate shocks. The 45^o line depicts the PIH responses, while the dashed and solid lines portray the responses as indicated by a near-rational model simulated with the parameters found in the empirical section and shown in Table 1 below. The solid line corresponds to a case in which consumption has been growing very fast (4 percent per quarter) for some time. The increasing slope of this curve shows that in "good times" there is more excess smoothness to negative than to positive wealth shocks. Exactly the opposite happens in a case in which consumption has been declining for a long time at the rate of 4 percent per quarter. This is illustrated by the short-dashes line. Finally, the long-dashes line represents an intermediate case where the responses are fairly symmetrical.

It is also apparent from this figure — which has a very large range of values for ΔC^* and ΔC — that the nonlinearities are not very pronounced. Of course this conclusion depends on the value of the parameters chosen, but, as said before, the figure was constructed with parameters obtained from actual U.S. data (see the next section). I will

Figure 3
Cross Sectional Densities

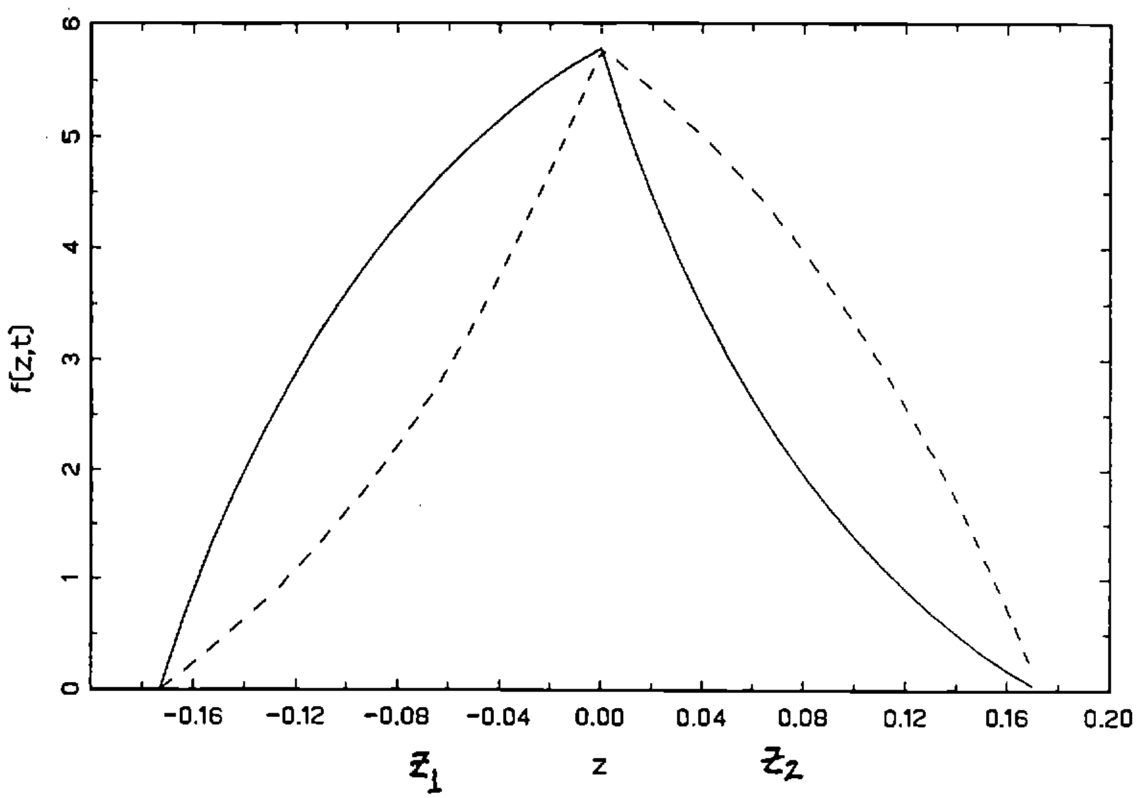
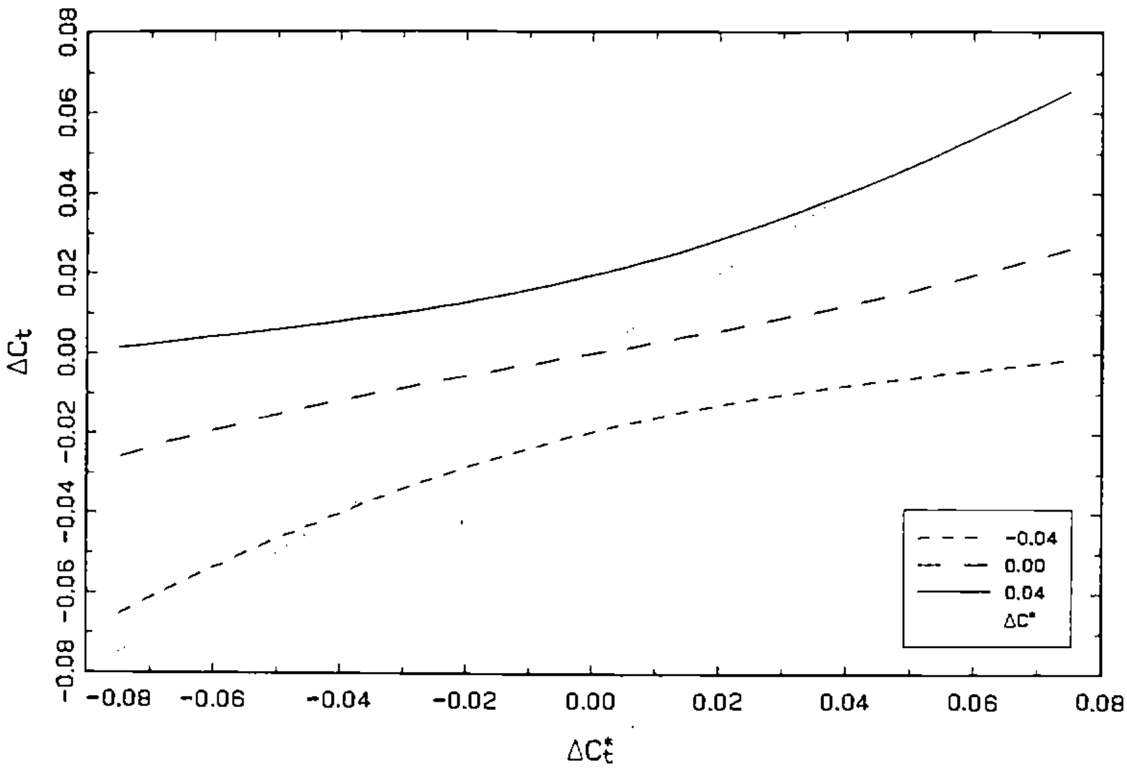


Figure 4
Endogenous Smoothness



return to this point later when presenting the empirical evidence on nonlinearities.

3 Results

The model presented up to now has the potential to account for the short-run behavior of aggregate consumption. The purpose of this section is to find out whether it can actually do it and to estimate the basic parameters of the model. The latter will be used in the next section to compute the implicit utility loss for near-rational agents.

This section is divided into two parts. The first part reproduces the basic consumption facts and characterizes the PIH part of the model, i.e. C^*. The second part focuses on the dynamic part of the model, i.e. on Z, and provides estimates of the inaction index, U, and of the amount of microeconomic level uncertainty, σ.

The data are per capita for the U.S. for the period 53:1-89:4 (CITIBASE, quarterly).

3.1 The PIH Model and Basic Facts

Aggregating the first difference version of (1), yields:

$$dC^*(t) = d\lambda(t) + \int_0^1 \alpha_i(t) dh_i(t)\, di,$$

whith $d\lambda(t) \equiv \int_0^1 \alpha_i(t) d\lambda_i(t)\, dt$, or

$$dC^*(t) = d\lambda(t) + dH(t), \tag{8}$$

where dH is the rate of growth of aggregate wealth.[14] I let $\lambda(t)$ be a linear function of time that is estimated from the cointegrating relationship between $C(t)$ and $C^*(t)$.[15]

The first two columns in Table 1 summarize the basic facts. The coefficients $\beta_{\Delta H}$ show the average response of current consumption and PIH consumption, respectively, to (unexpected) changes in wealth. These are obtained from simple univariate OLS

[14]Which corresponds to the rate of growth of NIPA's measure of disposable income. This is justified by the fact that detrended disposable income is appropriately described by a random walk process.

[15]Thus, I run the regression $(C(t) - H(t)) = \beta_0 + \beta_1 t$ and set $\widehat{d\lambda}(t) = \widehat{\beta}_1$.

regressions of the rate of growth of actual and PIH consumption on the rate of wealth growth. A comparison of the coefficients for actual and PIH consumption yields a measure of the *excess smoothness* of consumption to unanticipated wealth (income) changes.[16] The coefficients $\beta_{\Delta Y(-1)}$ show the other well known fact about consumption: its *excess sensitivity* to lagged (therefore anticipated) income changes. Actual consumption growth is positively correlated with lagged disposable income growth, while PIH consumption growth is uncorrelated with lagged income growth.

Finally, a measure of symmetry in the response of consumption to (contemporaneous) wealth shocks can be constructed by splitting wealth growth into positive and negative surprises. The last two rows of Table 1 show, first, that the excess smoothness finding applies both to positive and negative innovations in wealth, and second, that there is no strong evidence of an asymmetric response of consumption to wealth changes; and the weak evidence suggests more excess smoothness when wealth surprises are positive than when they are negative.

3.2 DYNAMICS

The next step is to estimate equation (5). The key ingredients of this equation are U, σ and the path of the slopes of the cross sectional density at the boundaries. The latter is the most difficult and time consuming part of the problem, since it requires to track down the path of the cross sectional density; this amounts to solving the following stochastic partial differential equation:

$$df(z,t) = f_z(z,t)\,dC^*(t) + \frac{\sigma^2}{2} f_{zz}(z,t)\,dt,$$

subject to the boundary conditions: $f(-U,t) = f(U,t) = 0$, $f(0^+,t) = f(0^-,t)$ and $f_z(0^+,t) - f_z(0^-,t) = f_z(U^-,t) - f_z(-U^+,t)$, for each combination of parameters, U and

[16]This is a somewhat stronger concept of excess smoothness than the one used in the literature, where it is used to denote the fact that the variance of actual changes in consumption is less than the variance of changes in PIH consumption. Here, the ratio of actual to PIH consumption growth variances is 0.64. This is larger than the number obtained by Campbell and Deaton (1989), who used labor income instead of disposable income to construct PIH consumption. The qualitative result is the same, however.

Table 1: Basic Facts and Results

	Facts		Dynamic Model
	ΔC	ΔC^*	$\widehat{\Delta C}$
U	–	–	17.3
	–	–	(0.3)
σ_A	–	–	1.8
σ	–	–	9.0
	–	–	(2.0)
% Expl.(ΔZ)	–	–	60.0
$\beta_{\Delta H}$	0.398	1.000	0.385
	(0.060)	—	(0.025)
$\beta_{\Delta Y(-1)}$	0.160	0.069	0.209
	(0.067)	(0.084)	(0.035)
$\beta_{\Delta H^+}$	0.333	1.000	0.373
	(0.108)	—	(0.043)
$\beta_{\Delta H^-}$	0.466	1.000	0.417
	(0.110)	—	(0.043)

Notes: Standard errors are in parentheses. Entries in the upper part of the table are in percents. % Expl.(ΔZ) is the percentage departure between the actual rate of consumption growth and the PIH consumption growth explained by the model. U: maximum departure from PIH allowed by consumers. σ_A: aggregate uncertainty (annualized). σ_I: idiosyncratic uncertainty (annualized). The coefficients in the bottom panel were obtained from the following regressions (all of them with a constant): (1) $\Delta X = \beta_{\Delta H}\Delta H$, (2) $\Delta X = \beta_{\Delta Y(-1)}\Delta Y(-1)$, and (3) $\Delta X = \beta_{\Delta H^+}\Delta H^+ + \beta_{\Delta H^-}\Delta H^-$. With ΔX equal to ΔC, ΔC^*, and $\widehat{\Delta C}$; the rates of growth of actual, PIH, and estimated consumption, respectively. ΔH is the rate of growth of wealth, and ΔH^+ and ΔH^- denote changes above and below the mean, respectively. $\Delta Y(-1)$ is the rate of growth of disposable income, lagged once.

Table 2: Nonlinearities

	ΔC	ΔC^*	$\widehat{\Delta C}$
β_{flex}	0.510	1.000	0.446
	(0.086)	—	(0.035)
β_{rigi}	0.359	1.000	0.333
	(0.080)	—	(0.033)

Notes: Standard errors in parentheses. All regressions include a constant. The sample is 54:1-89:4. $\Delta X(t+1) = \beta_{flex} 1[flex]\Delta H(t+1) + \beta_{rigi} 1[rigi]\Delta H(t+1)$, where ΔX is equal to ΔC, ΔC^* and $\widehat{\Delta C}$, $1[flex] \equiv [(\Delta H(t+1) > \Delta \bar{H} | Z(t) < \bar{Z})$ or $(\Delta H(t+1) < \Delta \bar{H} | Z(t) > \bar{Z})]$ and $1[rigi] \equiv 1[(\Delta H(t+1) > \Delta \bar{H} | Z(t) > \bar{Z})$ or $(\Delta H(t+1) < \Delta \bar{H} | Z(t) < \bar{Z})]$.

σ.[17] The appendix describes the procedure in detail.

The results are presented in the last column of Table 1. The first row shows that, on average, individual agents let their consumption pattern depart from the level of consumption indicated by the PIH by up to 18 percent before updating their patterns. The second and third rows show that the standard deviation of common or aggregate shocks is about 2 percent per year, while the standard deviation of total uncertainty faced by individuals is about 9 percent per year, yielding a ratio of aggregate to total uncertainty of about 0.2. These estimates of uncertainty do not seem at odds with previous aggregate and microeconomic evidence (see e.g. Hall and Mishkin 1982). These parameters in conjunction imply that, on average, individual consumers change their consumption patterns approximately every four years.

The next rows illustrate the fit of the model and its conformity with the basic consumption facts. The fourth row shows that the model accounts for approximately 60 percent of the discrepancy between the actual and PIH rates of consumption growth while the next four rows show that the model mimics well the excess smoothness and sensitivity features, including the slight asymmetry with respect to positive and negative wealth changes.[18]

Table 2 illustrates some of the aspects of the nonlinearities described in Section 2. For

[17] The path of C^* provides estimates of θ and σ_A.

[18] Adding a drift term to the consumption pattern, whereby average consumption growth is fully accounted for by planned consumption growth, leaves the results of Table 1 virtually unchanged.

this, I have split wealth surprises into two groups according to the slopes in Figure 4: The first one, denoted *flex*, contains the observations for which the slope is likely to be large: positive innovations in wealth given that the beginning of period value of Z is below its mean (i.e. given that previous times where favorable), and negative changes in wealth given that the beginning of period value of Z is above its mean (i.e. given that previous times where unfavorable). The second one, denoted *rigi*, contains the complement.[19] This yields the equation:

$$\Delta X(t+1) = \beta_{flex} 1[flex] \Delta H(t+1) + \beta_{rigi} 1[b] \Delta H(t+1),$$

where

$$1[flex] \equiv 1[(\Delta H(t+1) > \bar{\Delta H} | Z(t) < \bar{Z}) \text{or} (\Delta H(t+1) < \bar{\Delta H} | Z(t) > \bar{Z})],$$

$$1[rigi] \equiv 1[(\Delta H(t+1) > \bar{\Delta H} | Z(t) > \bar{Z}) \text{or} (\Delta H(t+1) < \bar{\Delta H} | Z(t) < \bar{Z})],$$

and ΔX denotes ΔC, ΔC^* and $\widehat{\Delta C}$. According to the above discussion, we would expect to observe less excess smoothness — that is, a larger β — during *flex* than during *rigi* periods. The first column in Table 2 shows that this is indeed the case, although as usual with non-linearities, this proposition cannot be supported at high significance levels. Moreover, Figure 4 shows that for the parameters reported in Table 1, one ought to expect nonlinearities to be small. Column 3 shows that if the estimated consumption path is used instead of the actual one, the results are similar to those in column 1, though the estimates are more precise (and the asymmetry smaller), which suggests that the lack of precision of the asymmetry estimates in column 1 may be partly due to nonsystematic noise.[20]

Of course one cannot take the mild non-linearity alone as a proof of the validity of the model here proposed; however taken in conjunction with the simultaneous account-

[19] The asymmetry in ΔC cannot be explained by standard mean reversion arguments. If lagged consumption growth is added to the regressors β_{flex} and β_{rigi} become 0.494 (0.087) and 0.335 (0.083), respectively (standard errors in parentheses).

[20] Moreover, adding independent white noise errors to the estimated consumption path helps matching the variance of actual consumption and its serial correlation properties.

ing for excess smoothness and sensitivity, it does contribute to supporting the model's explanation of short run consumption behavior. Still, to transform the model's explanation into an actual Akerlof-Yellen near-rationality argument, one must show that the *microeconomic* utility loss of not updating consumption continuously is small. I turn into this in the next section.

4 NEAR-RATIONALITY?

The (present value) utility cost of following the near-rational policy for an individual that has current departure $z(0)$ is equal to (the subindex i is suppressed for simplicity):

$$\int_0^\infty \int_{T-U}^{T+U} \left(\mathcal{U}\left(e^{c^*(t)}\right) - \mathcal{U}\left(e^{c^*(t)+z(t)}\right)\right) e^{-\delta t} g(z,t)\, dz\, dt, \tag{9}$$

where T is the return point,[21] chosen to set the first order term of the departure equal to zero,[22] $\mathcal{U}(.)$ is the consumer's instantaneous utility function, δ is the discount rate, and $g(z,t)$ is the density of z at time t, conditional on $z = z(0)$ at $t = 0$.

I follow Cochrane (1989) and divide the integrand in equation (9) by the corresponding marginal utility evaluated at the PIH consumption level. This transforms the utility value expression into a dollar equivalent expression. Dividing the integrand again, now by PIH consumption, one transforms expression (9) into a weighted (by the discount factor) average of flow costs expressed in term of percentage of PIH consumption sacrificed. Finally, multiplying by δ one gets the annuity value of this present value cost. I denote this expression by $F(z(0))$.

$$F(z(0)) \equiv \delta \int_0^\infty \int_{T-U}^{T+U} \frac{\left(\mathcal{U}\left(e^{c^*(t)}\right) - \mathcal{U}\left(e^{c^*(t)+z(t)}\right)\right)}{\mathcal{U}'\left(e^{c^*(t)}\right) e^{c^*(t)}} e^{-\delta t} g(z,t)\, dz\, dt. \tag{10}$$

Preserving the first two terms of the Taylor expansion of $\mathcal{U}\left(e^{c^*(t)+z(t)}\right)$ around $z(t) = 0$, and taking the average of $F(z)$ over the ergodic density of z, $g(z)$, yields an expression

[21] Which was set to zero before for expository simplicity.

[22] This plays the role of the feasibility constraint in Cochrane's (1989) analysis.

for the average yearly cost — in terms of percentages of PIH consumption — due to the near-rational policy (see the appendix):

$$G \equiv \int_{T-U}^{T+U} F(z)g(z)\,dz = \int_{T-U}^{T+U} \left((1-e^z) + \frac{\gamma}{2}(e^z-1)^2\right) g(z)\,dz,$$

where γ is the coefficient of relative risk aversion. And since T is chosen so as to satisfy the budget constraint (weak form), $\int(e^z-1)g(z)\,dz = 0$, G reduces to:

$$G = \frac{\gamma}{2}\int_{T-U}^{T+U}(e^z-1)^2 g(z)\,dz. \tag{11}$$

Evaluating this expression at the parameters found in the empirical section of the paper yields:

$$G = 0.0026\gamma,$$

a very small number for reasonable values of γ. This closes the argument of the paper: Not only a non-representative agent/discontinuous action microeconomic policy can generate aggregate dynamics consistent with the behavior of U.S. nondurables consumption, but also the degree of inaction required to do so imposes very small costs on individual consumers.

5 FINAL REMARKS

The model presented in the paper provides a structural interpretation of the main features of aggregate consumption. Excess smoothness and sensitivity arise naturally from the endogenous evolution of the cross sectional density of individuals' short-run deviations from the PIH. The endogenous nature of the cross section distribution also determines that the aggregate departure from the PIH varies over the business cycle, enriching the characterization of postwar U.S. data.

In the framework discussed in the paper, heterogeneity plays a key role. Idiosyncratic shocks do not wash away because microeconomic consumption policies are nonlinear. Thus, in this context information about the cross section distribution of consumers' departures (the z_i's) helps explaining the path of aggregate consumption. In the absence of microeconomic data, however, one needs to make a "guess" on the path of the cross sectional distribution. This defines a "distribution" extraction problem, which is what I have done when estimating the model. The estimates suggest that *on average* consumers keep their consumption levels within 17 percent of their PIH consumption level, and that they face uncertainty about the driving forces of their PIH consumption of about 9 percent per year (80 percent of which can be attributed to idiosyncratic uncertainty).

Despite the fairly large occasional microeconomic departures from PIH consumption (when z is close to the barriers), the implied average cost of the *microeconomic* policy is fairly small: 0.26γ percent of PIH consumption. In short, *near-rational* microeconomic consumers — in the Akerlof-Yellen (1985) sense — generate aggregate dynamics consistent with U.S. postwar nondurables consumption data.

APPENDIX

A. DERIVATION OF EQUATION (5)

This section of the appendix starts with the observation that the diffusion forward Kolmogorov equation associated to the controlled Brownian motion z_i, with driving process described by equation (4), is:

$$dh(z,t) = \frac{\sigma^2}{2} h_{zz}(z,t)\,dt + \theta h_z(z,t)\,dt, \tag{A.1}$$

subject to the initial condition:

$$h(z,0) = \bar{h}(z),$$

and the boundary conditions:

$$h(-U,t) = h(U,t) = 0,$$

$$h(0^+,t) = h(0^-,t)$$

and

$$h_z(0^+,t) - h_z(0^-,t) = h_z(U^-,t) - h_z(-U^+,t),$$

where $h(z,t)$ is the probability density of z_i at time t, conditional on the information available at time zero.

If $f(z,0) = \bar{h}(z)$ and there are no aggregate shocks, then a direct application of the Glivenko-Cantelli theorem determines that $(A.1)$ and its boundary conditions also describe the path of the cross sectional density $f(z,t)$. Although this step is not directly applicable in the current paper because there are aggregate or common shocks ($\sigma_A > 0$), Proposition 1 in Caballero (1990a) shows that similar argument holds *conditional* on the realization of aggregate shocks. In this case the boundary conditions remain unchanged but the partial differential equation (A.1) is replaced by the *stochastic* partial differential equation:

$$df(z,t) = \frac{\sigma^2}{2} f_{zz}(z,t)\,dt + f_z(z,t)\,dC^*(t). \tag{A.2}$$

LEMMA A1: Let $f(z,t)$ denote the cross sectional density at time t, satisfying the boundary conditions described above for $h(z,t)$, and evolving according to $(A.2)$, then:

$$\int_{-U}^{U} f_z(z,t)\,dz = 0, \tag{a}$$

$$\int_{-U}^{U} f_{zz}(z,t)\,dz = 0, \tag{b}$$

$$\int_{-U}^{U} z f_z(z,t)\,dz = -1, \tag{c}$$

$$\int_{-U}^{U} z f_{zz}(z,t)\,dz = U\left\{f_z(-U^+,t) + f_z(U^-,t)\right\}. \tag{d}$$

PROOF: Parts (a) and (b) are proved by integrating $(A.2)$ with respect to z between $-U$ and U, noticing that the integral of the left hand side is zero for all t and that the diffusion term in dC^* cannot be offset by any other term in the equation. Parts (c) and (d) follow directly from integration by parts, and using the boundary conditions and parts (a) and (b) of this lemma. **Q.E.D.**

It is now straight forward to obtain equation (5). For this note that:

$$dC(t) = dC^*(t) + dZ(t) = dC^*(t) + \int_{-U}^{U} z\,df(z,t)\,dz.$$

Replacing $(A.2)$ in the last expression, yields:

$$dC(t) = dC^*(t) + \frac{\sigma^2}{2}dt\int_{-U}^{U} z f_{zz}(z,t)\,dz + dC^*(t)\int_{-U}^{U} z f_z(z,t)\,dz. \tag{A.3}$$

Equation (5) is obtained by using Lemmas $(1c)$ and $(1d)$ in $(A.3)$:

$$dC(t) = U\frac{\sigma^2}{2}\left\{f_z(-U^+,t) + f_z(U^-,t)\right\}dt.$$

B. Estimation of equation (5)

The difficulty of estimating equation (5) is due to the presence of the slopes $f(-U^+,t)$ and $f(U^-,t)$. The value of these slopes at t, however, depends not only on the realization of aggregate and idiosyncratic shocks but also on the shape of the cross sectional density *inside* the interval $(-U,U)$ in previous periods. In other words, in order to characterize the boundaries of the cross sectional density over time, one needs to track down the path of the entire density. This is the strategy followed in the paper.

For each pair (U, σ), and the realization of the aggregate path $\{C_t^*\}_{t\geq 0}$, equation $(A.2)$ determines a path of a simulated cross sectional density, $f(z,t)$, where $f(z,0)$ is taken as given and equal to the corresponding "steady state" density (defined as the density that solves $(A.1)$ with $dh(z,t) = 0$). The estimation procedure consists in searching over U and σ until finding the pair (U, σ) that minimizes the sum of squared departures between the rate of growth of actual and PIH consumption.

The realization of $\{C^*\}_{t\geq 0}$ is not observed (estimated) continuously but only at quarterly frequency. Instead of solving an extremely cumbersome filtering problem, I take the change in C^* in a quarter to be homogeneously distributed within the quarter. In this case the Fourier representation of the density at time t is (see Caballero 1990a):

$$f(z,t) = g(z;\theta_t) + \sum_{n>0} e^{-\frac{\sigma^2}{2}\left(\frac{\pi^2 n^2}{U^2} + \frac{\theta_t^2}{\sigma^4}\right)} A_n(t)\omega_n(z,t),$$

where the time unit is a quarter, $\theta_t \equiv \Delta C_t^*$, $g(z;\theta_t)$ is the "steady state" density achieved if θ_t remains constant forever:

$$g(z;\theta_t) = \frac{1}{U}\begin{cases} \frac{e^{-\eta_t z} - e^{\eta_t U}}{1 - e^{\eta_t U}} & \text{if} \quad -U \leq z \leq 0 \\ \frac{e^{-\eta_t z} - e^{-\eta_t U}}{1 - e^{-\eta_t U}} & \text{if} \quad 0 < z \leq U, \end{cases}$$

with $\eta_t \equiv 2\theta_t/\sigma^2$,

$$\omega_n(z,t) = \sin\left(\frac{n\pi}{U}z\right) \cdot \begin{cases} e^{-\frac{\eta_t}{2}z} & \text{if} \quad -U \leq z \leq 0 \\ e^{-\frac{\eta_t}{2}(z-U)}(-1)^{n+1} & \text{if} \quad 0 < z \leq U, \end{cases}$$

and, finally,

$$A_n(t) = \frac{2}{U(1+e^{\eta_t U})} \int_{-U}^{U} e^{\eta_t z}\omega_n(z,t) f(z,t-1)\, dz.$$

C. DERIVATION OF G

A simple application of the Law of Iterated Expectations determines that:

$$G \equiv \int_{T-U}^{T+U} F(z)g(z)\,dz = \delta \int_0^\infty \int_{T-U}^{T+U} \frac{\left(\mathcal{U}\left(e^{c^*(t)}\right) - \mathcal{U}\left(e^{c^*(t)+z(t)}\right)\right)}{\mathcal{U}'\left(e^{c^*(t)}\right)e^{c^*(t)}} e^{-\delta t} g(z)\,dz\,dt. \quad (A.4)$$

Keeping the first two terms of a Taylor expansion of $\mathcal{U}\left(e^{c^*(t)+z(t)}\right)$ around $z(t) = 0$, yields:

$$\frac{\left(\mathcal{U}\left(e^{c^*(t)}\right) - \mathcal{U}\left(e^{c^*(t)+z(t)}\right)\right)}{\mathcal{U}'\left(e^{c^*(t)}\right)e^{c^*(t)}} = \left((1-e^z) + \frac{\gamma}{2}(e^z - 1)^2\right) \equiv Y(z). \quad (A.5)$$

Replacing $(A.5)$ in $(A.4)$ and rearranging, yields:

$$G = \delta \int_0^\infty e^{-\delta t}\,dt \int_{T-U}^{T+U} Y(z)g(z)\,dz.$$

Equation (11) in the paper follows immediately from this expression after imposing the feasibility constraint, $\int(1-e^z)g(z)\,dz = 0$.

The explicit expression for $g(z)$ is obtained from the solution to the Kolmogorov equation $(A.1)$ (with its boundary conditions) when $dh(z,t) = 0$, and it corresponds to $g(z;\theta)$.

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