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ABSTRACT

Large long-run swings in the United States stock market over the past century correspond to swings in estimates of fundamental values calculated by using a long moving average of past dividend growth to forecast future growth rates. Such a procedure would have been reasonable if investors were uncertain of the structure of the economy, and had to make forecasts of unknown and possibly-changing long-run dividend growth rates. The parameters of the stochastic process followed by dividends over the twentieth century cannot be precisely estimated even today at the century's end. Investors in the past had even less information about the dividend process. In such a context, it is difficult to see how investors can be faulted for implicitly forecasting future dividends by extrapolating past dividend growth.

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Large long-run swings in the United States stock market over the past century correspond to swings in estimates of fundamental values calculated by using a long moving average of past dividend growth to forecast future growth rates. Such a procedure would have been reasonable if investors were uncertain of the structure of the economy, and had to make forecasts of unknown and possibly-changing long-run dividend growth rates. The parameters of the stochastic process followed by dividends over the twentieth century cannot be precisely estimated even today at the century's end. Investors in the past had even less information about the dividend process. In such a context, it is difficult to see how investors can be faulted for implicitly forecasting future dividends by extrapolating past dividend growth.

I. Introduction

The U.S. stock market has exhibited large fluctuations relative to the baseline of the ex post "perfect-foresight" fundamental—the actual value, discounted at a constant real rate, of the future dividends actually paid. At times the real S&P stock market index plotted in figure 1 has been more than twice, and at times less than half of what its smoothly-growing ex post perfect-foresight value turned out to be.[1]

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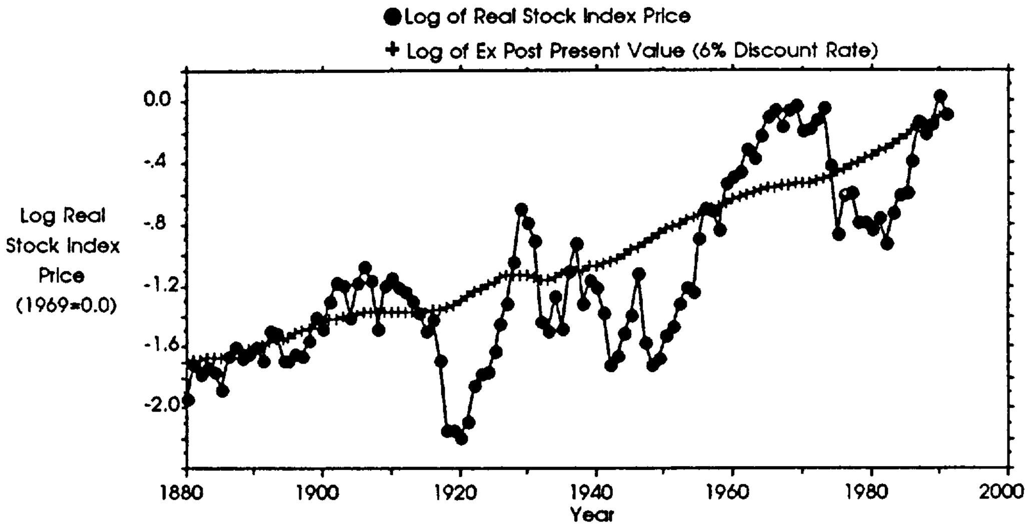
*We would like to thank George Bulkley, Robert Shiller, Andrei Shleifer, Lawrence Summers, Robert Waldmann, and many seminar participants for helpful discussions and comments.

[1]The S&P composite is taken from Standard and Poor's *Securities Price Index Record* and from Cowles *et al.* [1939]. The data series from 1871 up to the late-1980's is printed in Shiller [1989]. Stock prices are real values for January. Dividends are totals for the year divided by the year's average producer price level. In calculating perfect-foresight fundamentals, he present value of post-sample dividends is assumed to be equal to the terminal price.

Figure 1
The Real Value of the S&P Composite Index, and the Ex Post Realized Present Value of Future Dividends, 1880–1991



Shiller [1989, ch. 5; a reprint of Shiller, 1981] and LeRoy and Porter [1981] argued that such high volatility relative to perfect-foresight fundamentals posed severe difficulties for the efficient markets hypothesis.[2] The current price can be seen of a forecast of the perfect-foresight fundamental; one implication of rationality is that forecasts vary less than the realized values of the quantities forecast;[3] yet in figure 1 the implicit forecast given by the market—the real value of the stock index—is much more volatile than the realized perfect-foresight fundamental.[4] Thus

[2]Shiller also stresses other anomalies in asset pricing. See Shiller [1989, chs. 2, 12–18, 24].

[3]If they varied more, a better forecast would simply shrink the original forecast toward its mean.

[4]Allen Kleidon [1986a and b] (see also Merton and Marsh [1987], among many others) argues that tests like Shiller [1989, ch. 5] are significantly biased if dividends contain a quantitatively important "unit root." The considerations adduced by Kleidon. however, appear too small to quantitatively account for excess variance in stock prices (see Shiller [1989, ch. 7, a reprint of Shiller, 1988]; also see Shiller [1986] and Mankiw, Romer, and Shapiro [1985, 1990]). In addition, further studies have shown a case that dividends contain a substantial long-run mean-reverting component. This has further diminished the leverage of the considerations noted by Kleidon (see Shiller [1989, ch. 8; a reprint of Campbell and Shiller, 1988]). "Regression tests" have led to conclusions similar to those of the excess volatility literature (see Fama and French [1988] and Poterba and Summers [1988]). Overviews of the debate through 1988 and 1989 are given by West [1988] and LeRoy [1989].

the market exhibits "excess volatility."

This paper sets forth a theory of the mechanism underlying excess volatility. The high volatility of the U.S. stock market could be in large part accounted for if investors formed their valuations of the stock market by extrapolating past dividend growth into the future. Investors would naturally do so if they believed that the dividend process was subject to both transitory and permanent shocks to dividend growth rates, or if they believed that there was a chance that the process was subject to permanent growth rate shocks. The past century's worth of observed dividend growth would not lead an investor to reject a prior belief in such permanent growth rate shocks.

The interpretation advanced here can be seen as a positive rational-expectations model of low-frequency stock market swings, focusing on the limited information known to investors ex ante and on the process by which they learn about the possibly-changing structure of the economy. Or it can be seen as a positive model of the less-than-rational investor heuristic—extrapolation—which creates excess volatility.

The argument proceeds in several steps. Following this first introductory section, the first part of section II shows that excess volatility springs predominantly from a more than proportional response of prices to long swings in dividends. When dividend growth over the preceding generation has been rapid, the price-dividend ratio is high—not low, as would be the case if prices were rational-expectations forecasts of the present value of a stationary dividend process. The second part of section II argues that this pattern is consistent with the hypothesis that marginal investors form their expectations of future dividends by extrapolating past dividend growth rates. They take a long moving average of past dividend growth rates and project it into the future.

The discussion in section II is framed by writing warranted stock market values in the form of what Shiller [1989] calls the Gordon equation:[5]

[5] From Gordon [1962].

$$(1) \qquad P_t = \frac{D_t}{r - g_t}$$

In equation (1), P_t is the value of the stock market index, D_t is the current dividend paid on the index. r and g_t are the appropriate long-run rates of discount, and of expected dividend growth. The variable g_t is the "permanent" dividend growth rate in the sense used in the definition of "permanent" income.

The key insight is that the denominator $r-g_t$ is a small number. Information about the dividend process is scarce. The expected dividend growth rate g_t is uncertain. Investors will inevitably revise their estimates of the growth rate g_t. Small shifts—a percentage point or so—in g_t produce large proportional shifts—twenty-five percent or so—in the denominator of equation (1), $r-g_t$. These shifts carry with them similar large percentage changes in the level of stock prices P_t. And the actual path of stock prices is closely tracked by the valuations of an investor who forms his expectation of g_t by extrapolating past dividend growth into the future.

Section III analyzes why investors might extrapolate. It argues that the long-run twentieth century growth rate of dividends was not known to investors at the century's beginning. Investors had to form and update their estimates of the underlying long-run growth rate. Moreover, they had to guard against the possibility that this long-run growth rate might shift.

In such an environment—where the parameters and perhaps the structure of the dividend process are unknown—it might be rational, and is certainly natural, to form forecasts of future dividend growth by extrapolation from a moving average of past dividend growth rates. Investors' lack of information about the economy in which they are embedded, and the slow process by which they learn about the prospective future, combine to make extrapolation of growth an intelligible and reasonable forecasting strategy.

Section III can be read as an argument that investors who adopt the procedure of extrapolating past dividend growth into the future are forming rational-expectations estimates of present values given their limited information and the process by which they learn about the possibly-changing structure of the economy. Or the section can be read as an account of the psychological processes by which investors might adopt the less than rational heuristic of extrapolation which would lead to the failure of the efficient markets hypothesis, and to excess volatility. We are not sure that the difference between these two interpretations is testable.

The concluding section IV provides a brief summary of the argument.

II. The Price-Dividend Ratio and Expected Growth Rates

A. Dividends and Earnings

Figure 2 plots real prices and dividends for the U.S. stock market from 1880 to the present. It shows that the large long swings in stock prices are roughly in phase with and somewhat larger than long swings in real dividends. From 1920 to 1929 log stock index prices rise by 1.45 while the log of dividends rises by 1.08. From 1949 to 1969 log stock index prices rise by 1.63 while the log of the real dividends paid on the index rises by only 0.72. From 1969 to 1982 log stock prices fall by 0.91, while log dividends paid fall by only 0.26.

Note that year-to-year dividend changes become substantially less volatile after World War II. Large year-to-year changes in dividends become rare, as if the amount of dividend smoothing has substantially increased. The standard deviation of annual log changes falls from 0.147 over 1880–1939 to .081 over 1940–1981.[6]

Figure 3 plots real dividends and real accounting earnings of the S&P index, showing that earnings measures also show a substantial shift toward year-to-year smoothness in the second half of the sample. Such shifts in the processes of earnings and dividend payouts makes investors' task of determining the average dividend growth rate more difficult and data from the distant past less relevant. If the variance of dividend changes shifts over time, the mean dividend growth rate might well be shifting over time also.

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[6]Under the assumption that annual changes are independent, this difference produces an $F(59, 51)$ statistic of 3.29. The .01 level is 1.90.

Figure 2
Real Stock Index Prices and Dividends[7]
1880–1991

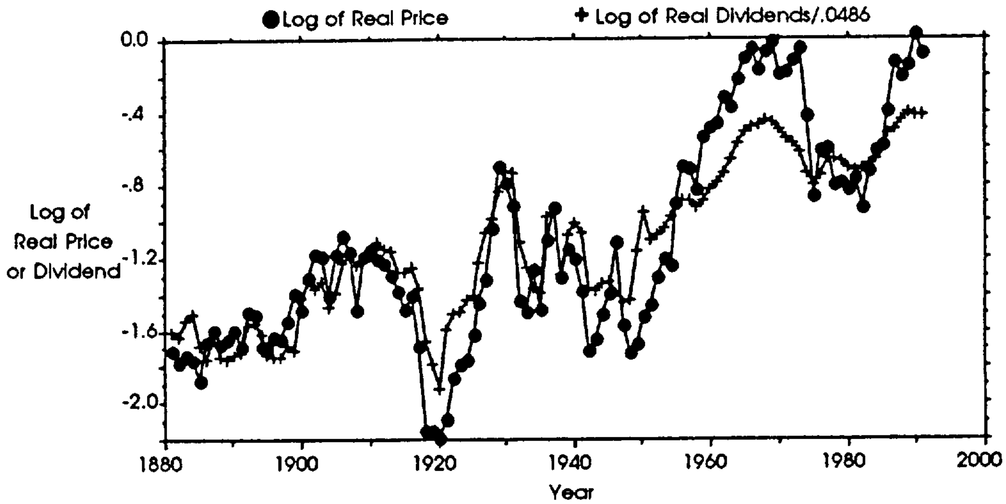
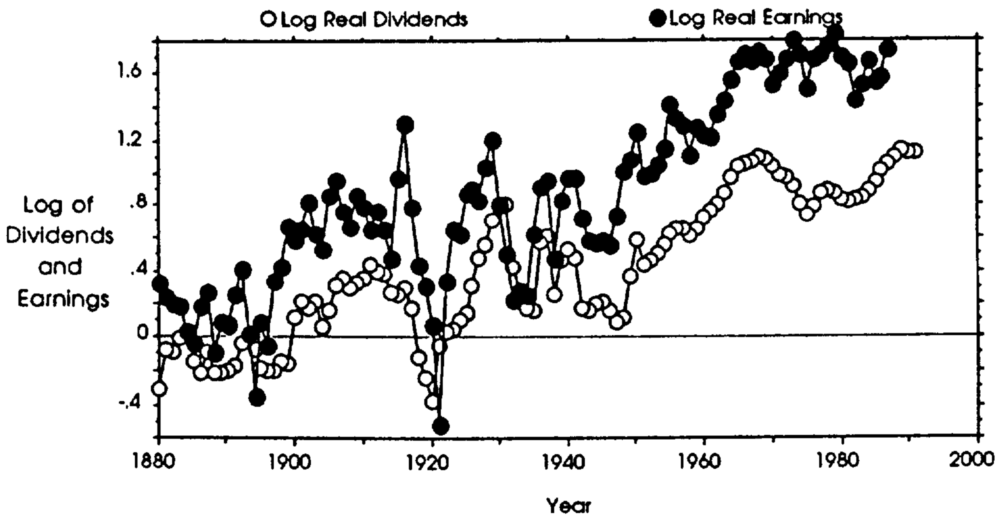


Figure 3
Real Stock Index Earnings and Dividends
1880–1991



[7] The dividend series has been divided by the average yield to place it in the same range as prices.

Figure 3 shows that decades that see rapidly rising dividends are times of rising real earnings as well. The relationship of long swings in dividends and earnings can be quantified by examining the association of multi-year changes in log dividends and earnings. A regression of the twenty-year change in log earnings on log dividends yields a coefficient of 1.33 over the entire sample. And the correlation of twenty-year changes is 0.70.[8] The bulk of long swings in dividends reflect long swings in trend earnings, not shifts in payout ratios.

B. Excess Volatility and the Price-Dividend Ratio

Figure 4 quantifies the relationship between long swings in prices and dividends by regressing twenty-year changes in log real stock index prices against twenty-year swings in the log real dividends paid on the index. The correlation is high: 0.84. More important, the elasticity of stock price with respect to dividend changes is high. The regression slope is 1.61. Over a twenty-year horizon, each one percent increase in dividends is accompanied by an additional 0.61 percent change in the same direction in the price/dividend ratio.[9]

This high elasticity cannot easily be attributed to an endogenous reaction of dividends to factors unconnected with profitability making for high stock prices. Twenty-year changes in log prices are equally closely associated with the components of twenty-year changes in log dividends collinear with and orthogonal to twenty-year changes in earnings.

This high elasticity imposes restrictions on the pattern of growth rate expectations implicit in market prices.[10] A greater than unit elasticity is inconsistent with rational expectations and models of dividends that contain a long-run component mean-reverting in levels, as implicitly

[8]With a standard error of 0.38, adjusting for the overlapping nature of the data.

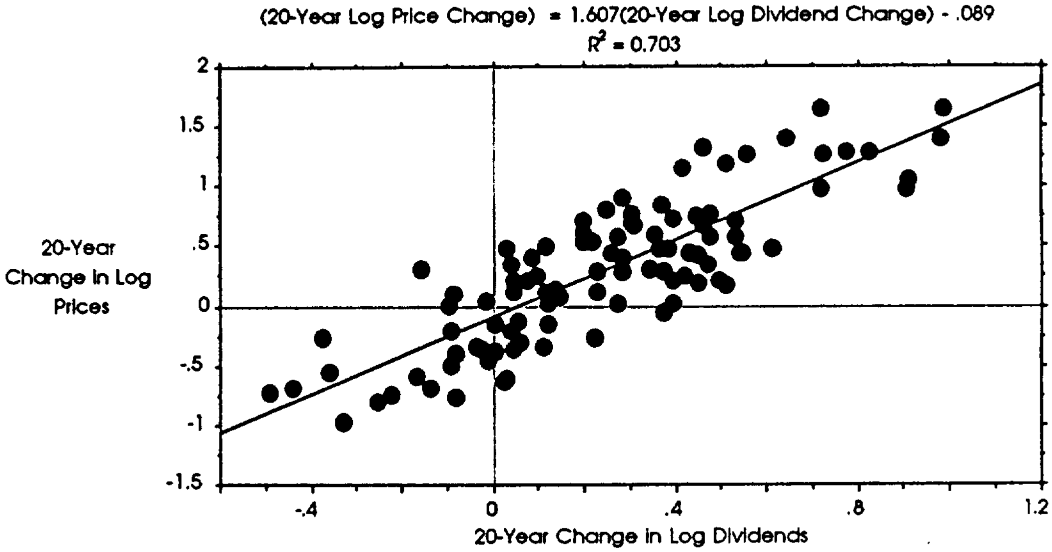
[9]The standard error allowing for overlapping data is 0.216. Thus the null hypothesis that there is not a more than proportional response—that the true slope coefficient is 1.0—can be rejected at the .005 level.

[10]With the possible exception of the 1916–21 World War I period, this high elasticity of low-frequency movements in stock prices relative to dividends cannot be attributed to independent shifts in the price deflator. Twenty-year swings in nominal prices are even more highly correlated with twenty-year swings in nominal dividends.

assumed in Shiller [1989, ch. 5; a reprint of Shiller, 1981]. Indeed, Shiller [1989, ch. 1; a reprint of Shiller, 1984] regards this high responsiveness of prices to dividends as an alternative way of stating the "excess volatility" puzzle.

Figure 4

Long 20-Year Swings in Log Prices Regressed on 20-Year Swings in Log Dividends, 1890–1991



The implications of such a high elasticity for investors' expectations of dividend growth rates are easily calculated. Holding discount rates constant in the framework of equation (1), using ∂ to denote a partial derivative, and writing lower case "p" and "d" for the logs of prices and dividends, the elasticity of price changes with respect to dividend changes is approximately:[11]

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[11] Where

$$g_t = (r-g_0)\sum_{i=0}^{\infty}(1-(r-g_0))^i E_t\{\Delta d_{t+i}\}$$

is the anticipated "permanent" dividend growth rate, the present value of all future dividend growth.

(2) $$\frac{\partial p_t}{\partial d_t} = 1 + \left[\frac{1}{r - g_t}\right]\frac{\partial g_t}{\partial d_t}$$

For $\partial p_t/\partial d_t$ to be greater than one, expected future growth rates g_t implicit in current market prices must be positively correlated with past shifts in dividends.[12] Moreover, the relationship between expected future growth rates and past dividend changes must be strong. The r-g_t term in the denominator of equation (2) is on the order of 0.04. To fit the 1.6 regression coefficient of twenty-year log price changes on dividend changes, each extra ten percent increase in dividends over twenty years—each increase of 0.5% per year in average dividend growth over a twenty-year period—must carry with it a shift from the beginning to the end of the twenty-year period in the value of the estimated g_t of 0.24%/year. To account for the actual correlations, half of any shift in average dividend growth rates over a twenty-year period must be expected to persist indefinitely into the future.

C. Long Swings and Extrapolated Growth Rates

Taking logarithms of both sides of equation (1) gives an expression for the log stock price:

(3) $$p_t = d_t - \ln(r - g_t)$$

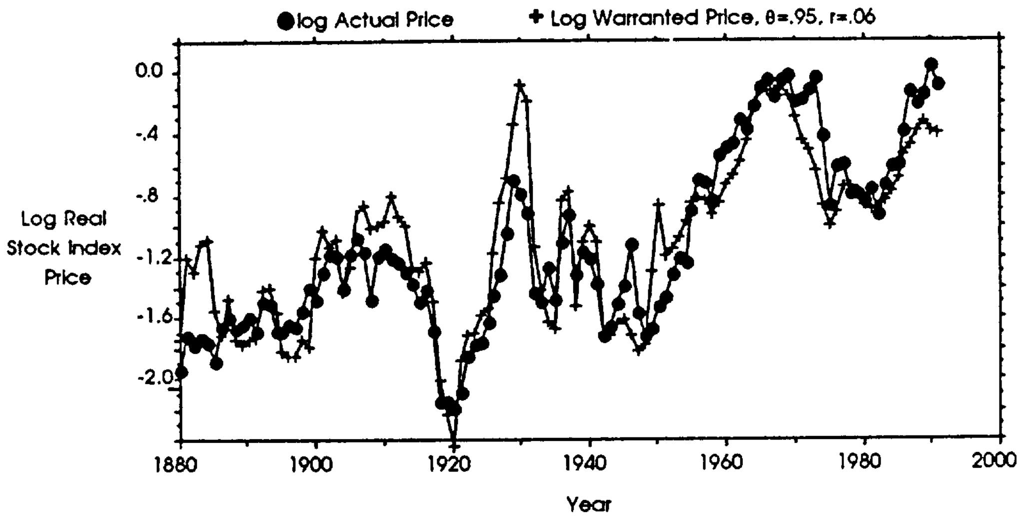
Suppose representative investors form their expectations of the permanent dividend growth rate g_t by extrapolating past dividend growth, using a simple geometric lag specification:

(4) $$g_t = (1-\theta)\sum_{i=0}^{\infty}\theta^i \Delta d_{t-i}$$

[12]Alternatively the rate of discount r could vary. We think that the line of research undertaken here is more promising than explanations based on changes in discount rates. It appears implausible to us that investors in the late 1920's or in the 1960's anticipated lower than average real returns on their investments. For an explanation based on variation in discount rates rather than anticipated growth rates, see Cecchetti *et al.* [1990].

Equation (4) is a parsimonious forecasting rule that leads to the required positive correlation between past changes in dividends Δd_{t-i} and expected future dividend growth rates g_t.

Figure 5
Actual and Warranted Real Stock Index Prices for θ=.95, 6% Real Discount Rate 1880–1991



This simple forecasting rule fits the low-frequency variation in real stock index prices over the twentieth century. Moreover, it does so for values of the parameter θ that are close to one. Figures 5 and 6 plot actual stock prices and "warranted" values constructed according to equations (3) and (4) for the parameter values $\theta = .95$ and $\theta = .97$, respectively.[13] Because θ is near one, only a negligible part of year-to-year variance in dividend growth is the result of revisions in the long-run growth rate of dividends g_t. Shifts in g_t account for 1/400 of the variance of dividend changes in the case shown in figure 5, and for approximately 1/1000 in the case shown in figure 6.

In both cases, because θ is near one the forecasts of future dividend growth rates implicit in the warranted price series are a very long moving average of past dividend changes. In figure 5,

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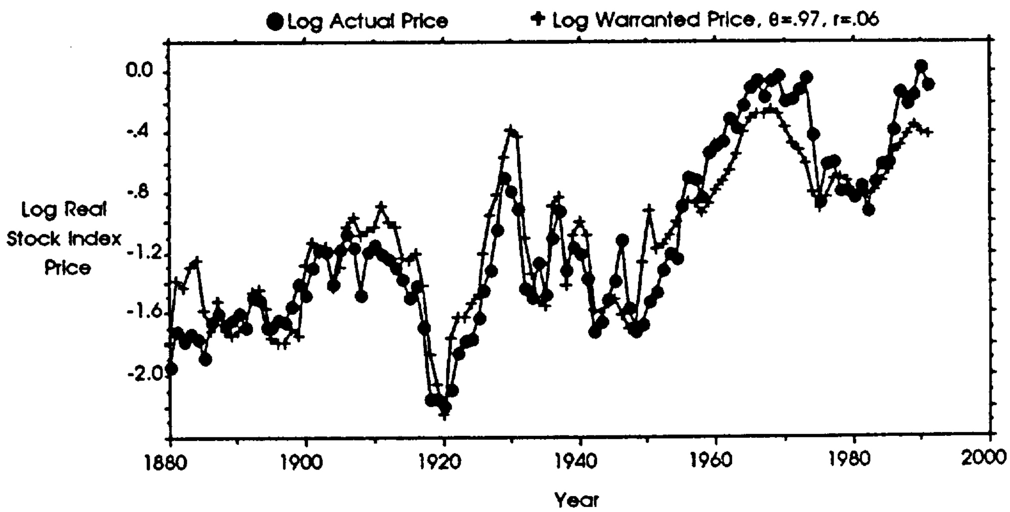
[13]Figures 5 and 6 assume a constant real discount rate of 6 percent per year.

35 percent of the weight in the forecast of future dividend growth is placed on dividend growth more than twenty years in the past. In figure 6, fully 40 percent of the weight is placed on dividend growth more than thirty years in the past. Thus neither figure contains a warranted price series that places high weight on the very recent dividend growth experience. Yet in figure 5 long swings in warranted prices are substantially greater than, and in figure 6 long swings are about the same magnitude as, actual low-frequency long swings in the stock market.

Figure 6
Actual and Warranted Real Stock Index Prices for θ=.97, 6% Real Discount Rate
1880–1991



Regressing twenty-year log changes in actual stock prices on twenty-year log changes in the warranted prices plotted in figure 6 (with θ=.97 and r = .06) produces a slope of 1.00 and an R^2 of 0.73. The variance of twenty-year price changes is 0.353, but the variance of twenty-year price changes relative to shifts in the "warranted" price series of figure 6 is only 0.102.[14] This fit between actual low-frequency movements in stock index prices and movements in warranted prices calculated by extrapolating past dividend growth should come as no surprise. For θ=.97

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[14] The standard error of the slope estimate correcting for the overlapping nature of the data is 0.130.

and r = .06, the regression slope of twenty-year changes in calculated log warranted prices on changes in log dividends is 1.62—almost exactly the 1.61 slope plotted in figure 4 for the regression of actual twenty-year changes in log stock prices on dividends.

Note that a rule of thumb that took the future dividend growth rate (and also the required rate of discount) to be a constant, and marked real dividends up by a constant multiple, would not do badly in accounting for long-run stock price movements over the past century.[15] The amount of "excess volatility" in stock prices is an order of magnitude smaller when assessed in terms of the variability of the price-dividend ratio than when assessed in terms of the variability of prices. The variance of twenty-year log changes in the price/dividend ratio is 0.140—only 40% more than the variance of twenty-year changes in stock prices relative to the "warranted" price series of figure 6. But the regression coefficient of twenty-year log price changes on dividend changes is not 1.00. Instead, it is 1.61. Prices react more than proportionately to long swings in dividends, and this more than proportional reaction is both economically and statistically significant.[16]

III. Why Might Investors Extrapolate?

The previous section has shown that the bulk of the long swings in U.S. stock prices could be accounted for if investors formed their expectations of future dividend growth by extrapolating past dividend growth into the future. But for what reason might investors adopt such an extrapolative procedure? The first two parts of this section advance two reasons: first, extrapolation might be the correct strategy if the dividend process is subject to both transitory

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[15]This was one of the major points of Mankiw, Romer, and Shapiro [1985].

[16]The hypothesis advanced in this paper about the causes of low frequency movements in stock prices is similar in structure to an interpretation proposed for the nineteenth century Gibson paradox. Investors determine warranted prices by marking up dividends by a multiple that depends on a growth rate, which is estimated from a long moving average of past growth. Long run moving averages of past growth are highly correlated with present levels. Movements in warranted prices appear an amplified version of long movements in dividends, just as in the Gibson paradox nominal interest rates appeared correlated not with the inflation rate but with the price level. See Barsky and Summers [1988], and Shiller [1989, ch. 14; a reprint of Shiller and Siegel, 1977].

and permanent growth rate shocks. Second, extrapolation might be a risk-minimizing strategy for an investor uncertain of the nature of the dividend process who fears that there might be such permanent growth rate shocks: the past century's data on dividend growth does not contain enough evidence to dispel such fears.

The third part of this section argues that the presence of a short-run component in the dividend process that is mean-reverting in levels is not a reason for investors to assume away the possibility of permanent shocks to dividend growth rates. The two issues—short-run mean reversion and long-run shifts in growth rates—are largely separate, and the second has by far the more important implications for warranted valuations.

Moreover, economists today cannot precisely estimate the dividend process. Thus it seems unreasonable to find investors in the past culpable for failing to know then, with less data at their disposal, features of the dividend process that economists dispute today.

A. Forecasting Dividend Growth

Investors might well value the stock market by using a moving average of past dividend growth to forecast future growth if they had to estimate the underlying long-run dividend growth rate. Of the quantities on the right-hand side of equation (1):

$$(1) \qquad P_t = \frac{D_t}{r - g_t}$$

all except the growth rate g_t are easily observed. Current dividends can be read in the *Wall Street Journal* or, earlier, the *Commercial and Financial Chronicle*. The rate of discount can be assessed through introspection. However, information about the remaining variable g_t—the permanent growth rate of dividends—is scarce and unreliable.

Moreover, there is no reason to suppose that the permanent growth rate of dividends is a constant. No one would claim that estimated growth rates of profits and dividends derived from the years of rapid expansion of the railroad industry after the Civil War era have much relevance for forecasting profit and dividend growth into the 21st century. Economic growth rates can and do change over generations. Since 1950 the growth rate of GDP per capita in West Germany has been more than 3 percent per year, while the growth rate in Argentina has been less than 1.5 percent per year. Yet the two countries were equally rich in 1950, and had seen their GDP per capita levels increase in step since before 1900.[17] Examples could be multiplied: a prudent investor trying to assess "warranted" values should consider that his country might be like Argentina, where growth stalls, or become a Germany or a Spain, where growth accelerates.

A very simple time series model of log dividend growth that captures these considerations is:

$$\Delta d_t = \varepsilon_t + \sum_{i=1}^{t-1}(1-\theta)\varepsilon_{t-i} + g_0 \tag{5}$$

In equation (5), g_0 is the permanent growth rate of dividends as of time 0. The ε_t's are stochastic shocks to dividend growth that have not only a permanent effect on the level (the lead e_t term in equation (5)) but also permanent, albeit attenuated, effects on the growth rate of dividends (the $(1-\theta)\varepsilon_{t-i}$ terms under the summation sign in equation (5)). The growth rate of log dividends is thus an IMA(1,1). At every period in the future the dividend growth rate expected as of time t is the same:

$$E_t\{\Delta d_{t+i}\} = g_t = \sum_{j=1}^{t-1}(1-\theta)\varepsilon_{t-j} + g_0 \tag{6}$$

Thus the permanent dividend growth rate is just this same constant.

The log dividend process generated by equation (5) is, for θ near one, close to a random walk. But it has a rate of drift that is itself slowly time varying. As a result, information gathered in the distant past will over time become less and less relevant to determining the current

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[17]See Barsky and De Long [1990], De Long and Eichengreen [1991]. Germany's rapid post-1950 rate of growth is not due in any large part to recovery from and rebuilding after World War II. Such recovery and rebuilding had been substantially completed by 1950. By 1960 German national product per capita is above not only its pre-World War II but also its pre-1929 trend line. The gaps have continued to widen over the past three decades.

underlying permanent rate of dividend growth. Equation (5) thus captures the intuition that forecasts of the future should not pay much attention to the very distant past.

Neglecting higher-order variance-generated terms, the present value of future dividends that are expected to grow at a constant rate g_t is:[18]

(7) $$p_t = d_t - \ln(r - g_t)$$

Equation (6) can be solved for the expected permanent dividend growth rate in terms of past dividend changes:

(8) $$g_t = (1-\theta)\sum_{i=0}^{\infty}\theta^i \Delta d_{t-i}$$

Equations (7) and (8) are identical to equations (3) and (4). Thus, under the dividend process (5), the warranted price series calculated according to equations (3) and (4), and exhibited in figures 3 and 4, *are* the rational-expectations forecasts of the present value of future dividends for given values of θ. The basic point comes from Muth [1960]. If a variable—in this case

[18] As (5) is written, it is not completely correct to neglect such higher order terms. Taking them into account leads to the conclusion that the log dividend process (5) has an infinite expected present value of future dividends. With finite probability the growth rate becomes, and stays, larger than the discount rate.

Equation (5) can be rationalized as an approximation to a continuous time stochastic process in which the log level of dividends follows a Brownian motion about a mean rate of drift that itself changes over time. There is such a process, with small "nuisance" terms in its specification, that has a well-defined expected present value equal to that given by equation (7): $p_t = d_t - \ln(r - g_t)$. The growth rate g_t must evolve according to:

$$g_t = g_0 + (1-\theta)\sigma_\varepsilon W_t - \int_0^t \frac{(1-\theta)^2\sigma_\varepsilon^2}{r - g_t}\,dt$$

The log level of dividends d_t must evolve according to:

$$d_t = d_0 + \int_0^t g_t\,dt + \sigma_\varepsilon W_t - \int_0^t \frac{\sigma_\varepsilon^2}{2}\,dt$$

where W_t is a standard unit Brownian motion. These "nuisance" terms—the last integral in each equation—are quantitatively insignificant. They are on the order of 0.5% per year in the level equation and 0.01% per year in the growth rate equation.

dividend growth—is an IMA(1,1), its optimal forecast will be a long geometrically-declining weighted average of past values.

In this sense, extrapolation could be accounted for by investors' lack of information. The underlying dividend growth rate is not a known parameter read in each morning's *Wall Street Journal*. It is an unknown, plausibly time-varying, that has to be estimated. Therefore rational investors might well extrapolate past dividend growth into the future.

B. The Magnitude of Growth Rate Shocks

If our argument depended on the existence of a "large" unit root in the dividend growth process—a unit root that made significant contributions to the year-to-year variance of dividend growth—it would be easily refuted. Many have found a random walk with constant drift to be a good first approximation to the U.S. dividend process (see Mankiw, Romer, and Shapiro [1985]. Kleidon [1986). However, the values of θ equal to 0.95 and 0.97 required to generate figures 5 and 6 correspond to a unit root in dividend growth that contributes only a very small share of year-to-year dividend growth volatility, and that is very hard to estimate empirically.

Information about the form and the parameters of the dividend process was limited back at the beginning of this century. Information about the parameters of the dividend process is still limited today. Estimation of the IMA(1,1) of equation (5) produces a maximum likelihood estimate of θ equal to 0.989, with an (asymptotic) estimated standard error of 0.023. However, the most important point is not that the likelihood is maximized for θ=0.989, with a lower bound to the (asymptotic) .95 confidence interval of 0.943. It is that the data do not speak strongly about the value of θ.

Further complicating inference is the lack of rapid convergence to the asymptotic distribution for θ near one. Shephard and Harvey [1990] investigate the small sample behavior of estimates of the Muth-type IMA(1,1) process considered here. They find that there is a disturbingly large probability of calculating a maximum likelihood estimate of θ equal to 1.00 even when θ is less than one and there are permanent shocks. Thus even a finding of a maximum likelihood estimate of θ equal to one would not be evidence that there are no permanent growth rate shocks. Shephard and Harvey [1990] report that for a sample size of 50 and for a true θ of 0.90, there is one chance in three that the maximum likelihood estimate will be at θ=1.00.[19] Our model has more than twice the number of observations available as does Shephard and Harvey's monte carlo study, but it requires an underlying θ only one-third as far from 1.00, so our model possesses less power to resolve differences of θ from 1.00 as does Shephard and Harvey's. Our own monte carlo simulations with a sample size of 120 and a true underlying θ of 0.97 find that 36 out of 100 times the maximum of the likelihood is at 1.00.

Thus the sample size and the relative magnitude of permanent growth rate shocks are too small for estimation of equation (5) to be informative about values of θ in the range needed to produce figures like 5 and 6. Thus an individual with a point belief that θ was .95, or .97, or .99, or 1.0 in 1871, who decided then to hold that belief until evidence forced a statistically significant rejection, would today still hold to his original prior opinion.

Little hinges on the null/alternative framework. Rephrased in Bayesian terms, the likelihood function for the IMA(1,1) process with normal innovations is sufficiently flat over the parameter θ that an individual who in 1871 held a uniform prior for θ over [0, 1], would today hold a posterior with a relatively large variance. Since with a uniform prior the posterior is proportional to the likelihood, such an investor would hold a subjective distribution for θ with a mean of .959. and a standard deviation of .048.[20]

Whether an investor is a Bayesian or follows hypothesis-testing methodology, the parameters of equation (5) are not precisely estimated. An investor could find reason believe that permanent shifts in the rate of mean dividend growth are relatively large, and that the "true" θ=0.95—in which case, as figure 5 showed, the warranted fluctuations in the stock market were significantly

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[19]Under the procedure providing the least chance of incorrectly estimating θ=1.00, which starts with a diffuse prior on the initial state of the system.

[20]He would think that there was a 90% chance that θ was in [.91,1], a 60% chance that θ was in [.96,1], and a 15% chance that θ was in [.99,1].

more volatile than the actual fluctuations of stock indices. An investor could perhaps believe there are no permanent shifts in the rate of mean dividend growth—that the "true" $\theta=1.00$. In this case the stock market has been too volatile: it has been anticipating permanent shifts in dividend growth that have never occurred, and will never occur.

We have shown that an econometrician attempting in 1992 to detect a small unit root in dividend growth could not obtain precise estimates, even with 120 years of data. *A fortiori* an investor in the past—in 1929, 1933, or 1963—operating with the smaller sample of data then available could not determine whether θ was really 1.00 or 0.97. Moreover, a risk averse investor would have allowed for the possibility of some permanent shocks to the dividend growth rate even if he thought that the most likely value for θ was one, and thus that the permanent dividend growth rate was a fixed constant.

C. Mean Reversion

Some have argued that dividends over the twentieth century may have in fact followed a more complicated process, or succession of processes, than the simple random walk with varying drift of equation (5). The autocorrelations of dividend growth suggest that the dividend process may contain a short-run mean reverting component (Shiller [1989]; ch. 8).[21]

Especially at longer horizons, the univariate impulse response of dividends to a given shock is not precisely estimated. A null hypothesis that the cumulative impulse response over twenty-five years is 1.5 could not be rejected. At any horizon, the upper bound to the 95 percent confidence interval for the cumulative impulse response includes one. It is not possible to conclude from the univariate autocorrelations of the dividend process that it definitely contains a mean reverting component. Nevertheless, an investor who relied on the point estimates of the cumulative impulse response would expect recent movements in dividends to be partially

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[21]Univariate estimates of impulse response functions produce point estimates that after nine years 41 percent of an initial shock to the log level of dividends has been eroded away by the decay of a mean-reverting component. Estimates of impulse response functions at horizons of fifteen to twenty-five years produce point estimates that also suggest that two-fifths of an initial shock has been eroded away.

reversed over the next eight years or so, and would place negative weight on recent dividend growth in estimating warranted values.

However, such a mean reverting component at a relatively short horizon could have little effect on major swings of the stock market. Since it is mean reverting, it has a long-run impulse response that dies out. It cannot make any contribution to the variance at some sufficiently long horizon. The impulse responses of non-stationary components—permanent shifts in either levels or growth rates—will dominate the behavior of the dividend process at sufficiently long horizons.

To demonstrate this, replace equation (5) with a more general process:

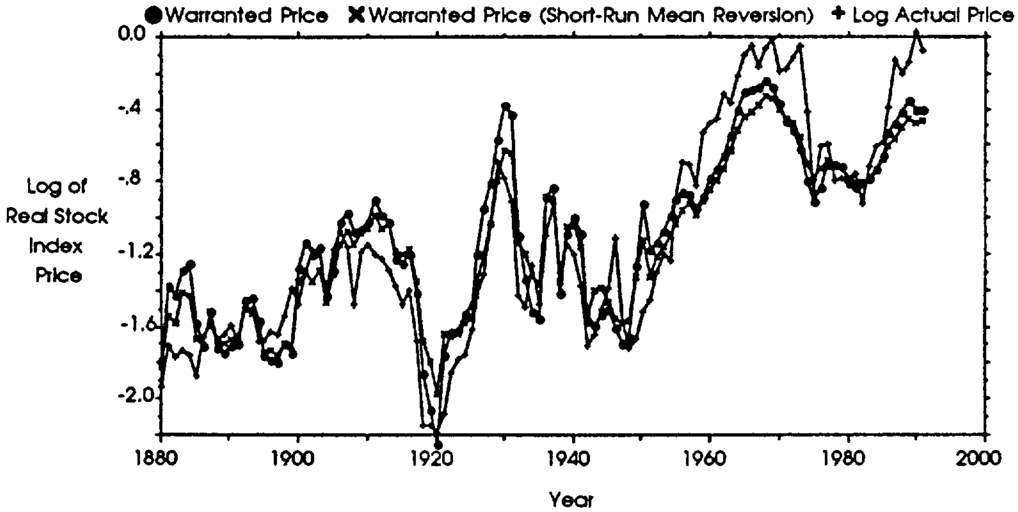
$$(9) \qquad \Delta d_t = \varepsilon_t + \sum_{j=1}^{m} \phi_j \varepsilon_{t-j} + \sum_{i=1}^{t-1} (1-\theta)\varepsilon_{t-i} + g_0$$

Equation (9) includes in the dividend process a short-run mean-reverting component covering the first m periods, for values of the ϕ_j coefficients less than zero.

Figure 7 plots actual stock prices and two sets of "warranted" prices—one from figure 6 with θ=.97, and one with θ=.97, m=8, and the ϕ_j parameters set equal to the first eight autocorrelations of dividend growth. With these parameters, more than 40 percent of shocks to the level of dividends are eroded away by the long-run disappearance of the mean-reverting component. Yet the effect on the long swings in "warranted" prices is small. The two "warranted" series plotted are much closer together than are the warranted series for different values of θ in figures 5 and 6.

From the standpoint of the effect on the pattern of long swings in stock prices, small changes in θ—in the magnitude of permanent shocks to long-run dividend growth—overwhelm large shifts in the degree of short-run mean reversion. The presence of short-run mean-reversion in levels is not cause to neglect the possibility of long-run permanent shocks to growth rates.

Figure 7
Warranted Prices with and without Short-Run Mean Reversion



D. Implications

A possible conclusion to draw is that to ask if stock prices have been rational forecasts of fundamentals is to ask an unanswerable question. There is a view of the process generating dividends—the view set out in Shiller [1989, chs. 5 and 8]—which would have led to stock prices that conformed much more closely to what the actual ex post realized values turned out to be. There are also defensible views (θ=.97) justifying the long swings seen as the best the market could do ex ante, given its lack of knowledge about and need to estimate possibly time-varying long run rates of dividend growth. There are even defensible views (θ=.95) that much wider swings would have been perfectly reasonable given investors' lack of timely information about the dividend process.[22]

Given the uncertainty today about how to model the evolution of dividends over the past

[22]Not to mention the views that argue that there was no single generating process—instead, a unique catastrophe (the Great Depression) and at least two different structural régimes. See Kim, Nelson, and Startz [1990].

century, it seems rash to find investors in some sense culpable when the implicit forecasts reflected in market prices do not correspond to a particular favorite model. It is perhaps better to argue that information was so scarce that it would be surprising if investors had been able to construct good forecasts.

IV. Conclusion

Over the past century the stock market has been at times twice, and at times half, of what its ex post perfect-foresight fundamental turned out to be. Long swings in stock prices are associated with and proportionately larger than long swings in dividends. For twenty-year changes, each one percent shift in dividends is associated with a 1.6 percent shift in actual stock prices—and with almost no shift at all in the perfect-foresight fundamental.

This paper has proposed a model in which these large long-run swings in the stock market arise because investors extrapolate past dividend growth into the future. Such a procedure would be reasonable. Investors are uncertain of the structure of the economy, and they have to form their own forecasts of the possibly changing long-run dividend growth rate. In a context in which the long-run rate of dividend growth is an uncertain and possibly changing parameter that investors must estimate—not a known constant—investors *should* estimate warranted values by forecasting dividend growth from a moving average of past dividend changes. Such an extrapolative estimation procedure would have led to fluctuations in warranted values as large as and in phase with actual bull and bear swings of the past century.[23]

An alternative interpretation of such possible extrapolation of past dividend growth into the future is that the resulting large swings in stock prices are driven not by fundamentals but by

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[23]Some analogous conclusions have been reached by economists examining stock markets in other countries. Bulkley and Tonks [1989] argue that once one takes account of investors' need to estimate the parameters of the dividend process, the post-World War I U.K. stock market does not appear excessively volatile. De Long and Becht [1991] note that post-World War II German prices and price/dividend ratios reach a peak at the beginning of the 1960's—just after the end of the rapid growth decade of the 1950's, and just before two decades of relatively slow dividend growth—also suggesting that investors followed an extrapolation procedure.

"fads and fashions." Over the past century dividends have exhibited long-run mean reversion. Investors have not taken this into account. Under this interpretation this paper has given an example of what Shiller [1990] terms the "popular model" that describes the not necessarily rational expectations-based rules of thumb that investors have used over the past century to value the market.

We see no immediate way to distinguish these interpretations using this particular data set. It is hardly reasonable to require that investors in the past place confidence in some particular favorite model of a present economist. After all, the economist has chosen this model ex post, with the benefit of hindsight.

Uncertainty about the structure of the economy is substantial. Economists today squabble over the proper characterization of the dividend process. Many analysts in year like 1929 and 1962, examining the past track of dividend growth and the state of the economy, did believe that the economy had entered a new régime of accumulation in which economic and dividend growth would be more rapid. These judgments were shared by prominent monetary economists at the end of the 1920's (see Fisher [1930]) and by prominent Keynesians in the 1960's (see Tobin and Weidenbaum [1988]). This suggests that if today's economists chose models without using their hindsight, the expectations in their models might well appear as grossly inconsistent with rational expectations as those implicit in actual stock index prices.

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