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DISINFLATION WITH IMPERFECT CREDIBILITY

Laurence Ball

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ABSTRACT

This paper presents a theory of the real effects of disinflation. As in New Keynesian models, price adjustment is staggered across firms. As in New Classical models, credibility is imperfect: the monetary authority may not complete a promised disinflation. The combination of imperfect credibility and staggering yields more plausible results than either of these assumptions alone. In particular, an announced disinflation reduces expected output if credibility is sufficiently low.

Laurence Ball
Department of Economics
Fischer Hall
Princeton University
Princeton, NJ 08544
and NBER

I. INTRODUCTION

Disinflations are an important cause of recessions in economies like the postwar United States. In explaining the effects of disinflation, New Classical economists stress the inability of the monetary authority to make credible policy announcements. New Keynesians emphasize rigidities in nominal wages and prices -- and especially the staggered timing of price adjustment. This paper argues that both schools are right. I present a model that explains the effects of disinflation through the interaction of imperfect credibility and staggering.

This paper starts from the results in Ball (1990) on disinflation with staggering. That paper assumes full credibility: a disinflation is announced at its outset, and the announcement is believed. The surprising finding is that quick disinflations can cause booms rather than recessions. This result is empirically implausible, and thus suggests that staggering alone does not explain the effects of disinflation. The current paper shows that the results are more appealing if the assumption of full credibility is relaxed. Most important, combining imperfect credibility with staggering yields a better explanation for the effects of disinflation than credibility problems alone.

To clarify the role of staggering, I begin in Section II with a model that lacks it. Firms set prices each period before observing the current money stock. The monetary authority (hereafter, the "Fed") announces a slowdown in money growth. However, credibility is imperfect: with a known probability, the Fed does not keep its promise. The results in this model are rather trivial. Firms base prices on the expected path of money growth given the likelihood that the Fed keeps it promise. A recession occurs if money growth falls more than expected, but a boom occurs if money growth falls less than expected. With rational expectations, the average output response is zero regardless of the level of credibility. I argue that this result is unrealistic. In actual economies, announcements of disinflation lead to recessions on average: output is more likely to fall than to rise.

Sections III and IV consider a continuous-time model in which price adjustment is staggered. In this case, I focus on the following experiment, which is detailed in Section III. As in Ball (1990), money growth is initially positive, but the Fed announces that it will decline linearly to zero. The Fed begins to carry out this promise. However, departing from the previous paper, there is a constant hazard at each instant that the Fed "reneges." If it reneges, it stops disinflating and instead keeps money growth at the current level forever. I interpret a large hazard of reneging as a low level of credibility.

Section IV derives the real effects of disinflation. With full credibility — a zero hazard that the Fed reneges — quick disinflations raise output, as in Ball (1990). With a positive hazard, output depends on whether and when the Fed reneges. However, if the hazard is sufficiently high, the expected output effect is negative. That is, in contrast to the model without staggering, credibility problems can reduce output on average. Indeed, the "expected sacrifice ratio" -- the expected output loss divided by the expected fall in inflation — increases monotonically with the hazard of reneging. Finally, for a sufficiently large hazard there is a stronger result: output falls not only on average, but for all realizations of Fed behavior. There is a small recession if the Fed reneges quickly, and a large recession if it does not.

To gain some intuition for these results, consider first the case of full credibility. When money growth begins to fall, only a fraction of firms can adjust prices quickly, because of staggering. However, the firms that do adjust early in the disinflation know that money growth will fall considerably while their prices are in effect. As a result, they greatly reduce their price increases, causing aggregate inflation to fall quickly even though most prices are fixed. In contrast, if disinflation begins but firms fear that the Fed will soon renege, they set higher prices. Inflation does not fall fast enough to match the fall in money growth that occurs as long as the Fed has not reneged yet. The initial tightening plus the prospect of future easing causes a recession.

Section V considers the robustness of the results to alternative specifications of imperfect credibility. Section VI concludes.

II. DISINFLATION WITH SYNCHRONIZATION

This section considers a simple model without staggered price adjustment. An announced disinflation has no effect on average output, regardless of the level of credibility.

A. The Model

The economy contains a continuum of imperfectly competitive firms indexed by i and distributed uniformly on [0,1]. Firm i's profit-maximizing relative price is increasing in aggregate output:

$$(1) \qquad p_i^* - p = vy , \qquad 0<v<1 , \quad p = \int_{i=0}^{1} p_i di ,$$

where p_i is the firm's nominal price, p is the aggregate price level, and y is aggregate output (all variables are in logs). This equation can be derived from isoelastic cost and demand functions (see Ball, 1990). Intuitively, an increase in aggregate spending raises a firm's desired price by shifting out the demand curve that it faces.

Money enters the model through a quantity equation for money demand:

$$m - p = y, \tag{2}$$

where m is the money stock. Combining (1) and (2) yields a firm's desired nominal price in terms of m and p:

$$p_i^* = vm + (1-v)p. \tag{3}$$

For simplicity, the model is set in discrete time. A firm adjusts its price every period, but it adjusts before observing current money. The firm sets its price for t, p_{it}, equal to $E_{t-1}p_{it}^*$, where E_{t-1} is the expectation conditional on information at t-1. Along with (2) and (3), these assumptions lead to simple solutions for aggregate prices and output[1]:

$$(4) \qquad \begin{aligned} p_t &= E_{t-1}m_t; \\ y_t &= m_t - E_{t-1}m_t. \end{aligned}$$

Aggregate output equals the current monetary surprise.

B. Disinflation

Assume that money growth is initially constant and expected to remain constant. At some point, the Fed announces that money growth will decline over one or more periods (the precise path that it announces is not important). There is some probability, which price setters know, that the Fed disinflates less than promised. I interpret a high probability as a low level of credibility.

In this setting, the effects of disinflation are obvious. Each period, firms base their prices on expected money growth given the probability that the Fed keeps its promise. If money growth falls faster than expected — that is,

[1]The price-setting rule and equation (3) imply $p_{it}=vE_{t-1}m_t+(1-v)E_{t-1}p_t$. Aggregating yields $p_t=vE_{t-1}m_t+(1-v)E_{t-1}p_t$, which leads to (4).

if the Fed proves unusually tough — then output falls. If money growth falls less than expected, then output rises. With rational expectations, the average output effect is zero, regardless of the level of credibility. Announced disinflations cannot systematically reduce output in this model.[2]

A number of previous authors, such as Fischer (1986) and Cukierman and Meltzer (1986), argue that non-credible disinflations do reduce output. However, these papers and the current one focus on different questions. Previous papers consider episodes in which a tough policymaker reduces money growth even though the public does not expect it. That is, they consider particular realizations of Fed behavior in which money growth is less than its mean. As confirmed in my model, these realizations produce recessions. In this section, my central point is that credibility does not affect average output if expectations are correct on average. Previous models are consistent with this result.

III. DISINFLATION WITH STAGGERING: THE EXPERIMENT

This section and the next derive the effects of disinflation when price adjustment is staggered. In contrast to the synchronized case, the level of credibility influences average output.

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[2]Can this result be changed without introducing staggered adjustment? One approach is to assume that adjustment is synchronized, but that prices are fixed for several periods (or for a discrete interval in continuous time). In this case, disinflation reduces output until the next adjustment date. There are, however, several unappealing features of this approach. First, steady money growth causes fluctuations in output: output falls on price-adjustment dates and rises in between. Second, the Fed can eliminate the expected output loss from disinflation simply by making its announcement on an adjustment date. Finally, as in the basic model of this section, lower credibility cannot reduce output on average. A smaller probability that the Fed keeps its promise implies higher expected output until the next adjustment (expected money growth falls less, and prices are fixed). Credibility has no effect on expected output after the next adjustment.

A. Staggered Adjustment

The specification of staggering follows Ball (1990), and is close in spirit to Taylor (1979, 1980) and Blanchard (1983, 1986). The model is set in continuous time. Aggregate output and firms' desired prices are still given by (1)-(3). Each firm adjusts its price at intervals of length one (a normalization), with adjustments by different firms distributed uniformly over time. Let x(t) denote an individual price set at time t for the interval [t,t+1) (I drop the i subscript because all firms acting at t choose the same price). A firm sets x(t) equal to the average of its desired prices over [t,t+1)[3]:

$$(5)\qquad x(t) = \int_{s=0}^{1} E_t p_i^*(t+s)\,ds \;.$$

Finally, with uniform staggering the price level is the average of prices set over the last unit of time:

$$(6)\qquad p(t) = \int_{s=0}^{1} x(t-s)\,ds \;.$$

B. Disinflation

With staggering, the effects of disinflation are sensitive to the particular path announced by the Fed and the nature of the credibility problem. I focus on one example, and consider robustness in Section V. In the example, money growth is initially steady: at all t<0, money grows at rate one (another normalization):

[3]Equation (5) can be derived by assuming that firms maximize a second-order approximation to profits, with no discounting (Ball, 1990).

$$m(t) = t, \quad t<0;$$
(7)
$$\dot{m}(t) = 1.$$

where m is the level of money and $\dot{m}$ is money growth. At t<0, firms expect money growth to remain constant forever. However, at time zero the Fed announces a disinflation. It promises that money growth will decline linearly, reach zero in k units of time, and then remain at zero permanently:

$$\text{(8)} \quad \dot{m}'(t) = 1 - t/k, \quad 0 \le t < k;$$
$$= 0, \quad t \ge k,$$

where $\dot{m}'$ is announced money growth. Figure 1A plots this path. The parameter k measures the speed of disinflation.

Ball (1990) assumes that (8) is carried out with certainty. In this paper, by contrast, the Fed may not fully keep its promise. Specifically, the Fed starts to carry out (8), but at each instant there is a hazard h that it "reneges." h is a constant known to price setters. If the Fed reneges, it stops reducing money growth and instead keeps money growth at its current level forever. That is, if the Fed reneges at time τ<k, the actual path of money growth is

$$\text{(9)} \quad \dot{m}(t) = 1 - t/k, \quad 0 \le t < \tau;$$
$$= 1 - \tau/k, \quad t \ge \tau.$$

This path is presented in Figure 1B. If the Fed does not renege before time k, actual money growth follows the promised path (8).

This specification captures situations in which policymakers launch a disinflation but may be unable to complete it. For example, in 1979 Margaret Thatcher announced a five-year disinflation path and began to tighten policy. However, there were political obstacles to the completion of the plan, which was opposed by Labor and the Social Democrats. At the outset of the program, Sargent (1983) wrote:

> Mrs. Thatcher's party now runs third in the political opinion polls. In addition, throughout her administration, speculation has waxed and waned about whether Mrs. Thatcher herself would be driven to implement a U-turn in macroeconomic policy actions, and whether her stringent monetary policy actions would be reversed by the Conservative Party itself, by choosing a new party leader. [pp. 57-58].

The risk that disinflation ends prematurely with a shift to looser policy is captured in my model by the hazard that the Fed reneges. I interpret a large hazard as a low level of credibility.

IV. THE BEHAVIOR OF OUTPUT

This section derives the behavior of output during disinflation. I begin by reviewing the full credibility case in Ball (1990). Then I derive the expected path of output with imperfect credibility, and the paths for particular realizations of Fed behavior.

A. Full Credibility

The model reduces to the full credibility case when the hazard of reneging is zero. In this case, fairly quick disinflations cause booms: output rises above the natural rate and never falls below it. In particular, Ball (1990) shows that the linear path (8) produces a boom as long as k, the length of disinflation, exceeds .68. Recall that the time unit is the period between price adjustments. If prices are fixed for a year, a boom occurs if disinflation lasts at least .68 years.[4]

To understand this result, consider the early stages of disinflation. Money

[4]A year is the median interval between price adjustments in Blinder's (1991) survey of firms.

growth begins to fall and, with staggering, only a fraction of prices can adjust quickly. With full credibility, however, the firms that do adjust know that money growth will fall considerably while their prices are in effect. As a result, they sharply reduce their price increases, causing aggregate inflation to fall quickly even though most prices are fixed. Inflation falls more quickly than money growth, causing a boom.[5]

B. Expected Output with Imperfect Credibility

When h is positive, the path of output depends on whether and when the Fed reneges. However, the following result lets one calculate the expected output path without considering individual realizations. Let E denote an expectation at time zero, when disinflation has just been announced. Then

Lemma: The expected path of output, Ey(t), is defined by the expected path of money growth, $E\dot{m}(t)$.

That is, expected output depends only on the mean of money growth, not on its distribution. This certainty-equivalence result follows from the log-linearity of the model (see Appendix).

Given the lemma, the first step in finding expected output is to find the expected path of money growth. This path is determined by the announced path (8) and the hazard of switching to (9). The Appendix derives

[5]See Ball (1990) for a more detailed discussion of this result, and a comparison to other models with full credibility. Taylor (1983) and Fischer (1986) find that a credible disinflation must be quite slow to avoid a recession. The source of this result is the assumption that multi-period contracts specify different wages or prices for different periods. My earlier paper argues that this case is less important empirically than the one considered here.

$$(10) \qquad E\dot{m}(t) = 1 + \frac{e^{-ht}-1}{hk}, \qquad 0 \le t < k;$$

$$= 1 + \frac{e^{-hk}-1}{hk}, \qquad t \ge k.$$

For k=1, Figure 2 shows expected money growth for various values of h. When h is positive, expected money growth declines sharply at first but then flattens, because it becomes increasingly likely that the Fed has reneged. $E\dot{m}$ is constant after time k, because disinflation ends at k if not before.

Equation (10) and the equations defining prices yield an implicit solution for the expected output path (see Appendix). Explicit paths are derived numerically. For k=1 and v=1/4, Figure 3 presents expected output for several values of h. When h=0, there is a boom (note that k=1 exceeds the bound of .68 discussed above). For positive but small values of h, such as h=1, there is an expected boom. However, large h's, such as 5 or 10, produce expected recessions. These results illustrate the central point of the paper: with staggering, the level of credibility influences average output.[6]

C. The Expected Sacrifice Ratio

The relation between credibility and average output is non-monotonic. An increase in h can turn a boom into a recession, but the recession disappears as $h \to \infty$. Intuitively, $h \to \infty$ means the Fed reneges at time zero, so money growth and output remain constant.

There is, however, a monotonic relation between h and the "expected sacrifice ratio," defined as

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[6]For any h, expected output rises at time zero. Output rises because the announcement causes inflation to jump down, and money growth is continuous. For a large h, however, the initial boom is negligible compared to the recession that follows.

$$R = \frac{-\int_{t=0}^{\infty} Ey(t)\,dt}{1 - E\dot{m}(k)} .$$

The numerator of R is the expected output loss summed over time. The denominator is the expected long-run change in inflation, which equals initial money growth minus expected money growth at k. R is a natural extension of the sacrifice ratio in non-stochastic models (for example, Fischer [1986]). As illustrated in Table I, R is increasing in h. For $v=1/4$ and $k=1$, R is $-.10$ when $h=0$, zero when $h=3.2$, and .09 when $h=10$. As $h\to\infty$, R approaches a positive asymptote.[7]

Why does low credibility produce expected recessions? With full credibility, inflation falls quickly when money growth starts to slow, because firms reduce their price increases in anticipation of further disinflation. With low credibility, by contrast, firms expect money growth to level off (see Figure 2). They set higher prices, so inflation falls slowly even though, on average, money growth falls quickly at first. That is, an initial monetary tightening is accompanied by the expectation of future easing, which keeps inflation high. Thus average output falls.

D. Individual Realizations of Output

In addition to computing expected output, one can derive output paths for particular realizations of τ, the date when the Fed reneges. For a given τ, the path of money growth is given by (9). Prices set at each instant depend on expected money growth given the Fed's behavior so far, which is determined by

[7] I cannot show analytically that R is monotonic in h, but this result is confirmed by extensive numerical calculations. (The result can be proved analytically in the special case of $v=1$.) One can show that R remains finite as $h\to\infty$, because the numerator and denominator approach zero at the same rate. It is difficult to derive the limiting value of R, because the numerical results are imprecise when h is very large.

(8), (9), and the hazard h. The Appendix uses these expressions to derive output paths numerically. Figure 4 presents the paths for $k=1$, $v=1/4$, and two values of h: $h=5$ and $h=10$. For each h, the Figure presents output paths for various realizations of τ. Finally, Table II considers additional combinations of h and τ. The Table summarizes the results with the total deviation of output from zero (i.e., the integral of y(t)).

Not surprisingly, Table II shows that total output is decreasing in h for a given τ. That is, holding constant the Fed's actual behavior, output falls with lower credibility. A more striking result is that large values of h produce output losses for <u>all</u> values of τ. For a large h, disinflation reduces output not only on average (as shown above), but for every realization of Fed behavior. Figure 4 illustrates this result. For $h=5$ or $h=10$, disinflation causes a large recession when τ is large (e.g. $\tau=1$), and a small recession when τ is small (e.g. $\tau=.05$).[8]

For a large h, it makes sense that there is a deep recession when τ is large: in this case, money growth falls more than expected. It is more surprising that output falls even if τ is small, so that money growth falls <u>less</u> than expected. In this case, the unrealized expectation of disinflation has little effect on prices, because few firms adjust between time zero and time τ. And prices set before time zero are too high, because the fall in money growth was not anticipated before the announcement. Thus there is a recession (albeit a small one, because the fall in money growth is small).

[8]For small values of h, output rises for all realizations of τ. For intermediate h's, output rises for small τ's and falls for large τ's.

V. ROBUSTNESS

So far I have focused on a specific example of a credibility problem. This section asks whether the results are robust. I focus on the effects of imperfect credibility on expected output.

The essential feature of the example is that expected money growth falls steeply at first, and then flattens as it becomes likely that the Fed has reneged. In other words, it is essential that imperfect credibility makes expected money growth convex. Modifications of the model do not change the qualitative results unless they eliminate this convexity. I now illustrate this point with several additional examples.

As a first example, let the hazard h of reneging vary over time. (In particular, h might decrease over time, because the Fed gains credibility if it has not reneged so far.) This generalization of the model does not affect the qualitative results. Expected money growth is still convex as long as $h(t)>0$ over $[0,k]$. One can show that a larger hazard at any point in time raises the expected sacrifice ratio.

As a second example, assume that h is constant but modify the Fed's behavior when it "reneges." Reneging now means that the Fed returns money growth to its initial level of one, not that money growth stays constant. That is, if the Fed reneges at τ, then

$$(11) \qquad \dot{m}(t) = 1 - t/k \,, \qquad 0 \le t < \tau \,;$$
$$= 1 \,, \qquad t \ge \tau \,.$$

Money growth still follows the announced path (8) if the Fed does not renege before time k. The path (11), in which initial gains against inflation are given up, is one interpretation of the "U-turn" that Sargent predicted for Thatcher.

This modification of the model <u>strengthens</u> the results. The possibility that money growth jumps up makes expected money growth more convex for a given h. (The path flattens more quickly and, for large h, eventually rises.) As a result, disinflation is more likely to reduce output. Figure 5 shows expected output paths when k=1 and v=1/4. In this experiment, the expected sacrifice ratio is negative for all h>1.1. In the basic model, R is negative only if h>3.2.

Finally, I present an example in which lower credibility does not reduce the expected sacrifice ratio. Suppose that the Fed announces the path (8), in which money growth declines at rate 1/k over [0,k]. In fact, the Fed switches between reducing money growth at the promised rate and keeping money growth constant. Switches are a Markov process, with the initial state chosen randomly based on unconditional probabilities. In this case, the expected path of money growth is

$$(12) \qquad E\dot{m}(t) = 1 - qt/k\,, \qquad 0 \leq t < k\,;$$
$$= 1 - q\,, \qquad t \geq k\,.$$

where q is the unconditional probability of the disinflation state. A lower q means less credibility: firms expect less of the promised disinflation to occur. Nonetheless, one can show that the expected sacrifice ratio is independent of q. The explanation is that (12) is always linear over [0,k]. Reducing q affects the slope of (12), but does not introduce the convexity that lowers expected output.

This example shows that the robustness of my results is limited. However, the example is not especially appealing. The crucial convexity in my model appears plausible: in actual disinflations, it is likely that policy is initially tight but then loosens (recall Sargent's discussion of Thatcher).

VI. CONCLUSION

This paper presents a theory of the real effects of disinflation. I assume that price adjustment is staggered, and that credibility is imperfect: the Fed may not complete a promised disinflation. The central result is that low credibility worsens the output-inflation tradeoff: the expected sacrifice ratio rises with the hazard that the Fed reneges. In addition, when the hazard is large, output falls for all realizations of Fed behavior. These results fit the experience of the postwar United States, where disinflations systematically reduce output.

Both the New Keynesian assumption of staggered adjustment and the New Classical assumption of imperfect credibility are essential for the results. With staggering but full credibility, quick disinflations raise output. With imperfect credibility but synchronization, disinflations are neutral on average. Neither of these cases appears realistic.

To be clear, imperfect credibility alone can explain output losses in certain episodes, such as the Volcker and Thatcher disinflations. These can be interpreted as cases when policymakers proved tougher than expected. However, the model with synchronization implies that such episodes are balanced by booms when tough-talking policymakers disinflate <u>less</u> than expected. In practice, policymakers often make promises of disinflation that they fail to keep, but this behavior does not appear to raise output significantly. Volcker caused a recession by disinflating, but William Miller did not produce a boom by failing to disinflate.[9]

I conclude with a suggestion for future research. In this paper, the

[9] Of course such anecdotal evidence is not conclusive. In future work, it would be desirable to construct a sample of disinflation announcements and systematically examine the behavior of output.

behavior of the monetary authority is exogenous. It would be desirable to make this behavior endogenous — to explain why disinflations sometimes end prematurely. In the spirit of Alesina (1988), imagine a model with two political parties, Conservatives who favor disinflation and Liberals who do not. The parties alternate in power stochastically, with a higher probability of a switch when the economy is performing poorly. In this framework, a disinflation launched by Conservatives is not fully credible, because the Liberals might gain power and reverse it.

I conjecture that such a model has multiple equilibria. If the Conservatives announce a disinflation and credibility is high, then output rises. With rising output, the Conservatives are unlikely to lose office; thus they succeed in disinflating, justifying the public's confidence. On the other hand, if credibility is low, then disinflation reduces output. In this case the Liberals are likely to gain power and end the disinflation; thus low credibility is also self-fulfilling. Sargent argues that disinflation should be accompanied by a "once-and-for-all, widely understood and widely agreed upon, change in the monetary and fiscal policy regime" (1983, p. 57). This regime change can perhaps be interpreted as a shift from a low-credibility equilibrium to a high-credibility equilibrium.[10]

[10]Hoshi (1991) and Blanchard and Fischer (1989, ch. 11) contain related discussions of multiple equilibria.

APPENDIX

A. The Price Level and Certainty Equivalence

Here I derive the general behavior of the price level and prove the lemma in Section IV. At $t<0$, steady inflation is expected to continue, so $E_t m(t+r)=E_t p(t+r)=t+r$. Substituting these results into (5) yields $x(t)=t+1/2$ for $t<0$. $x(t)$ is given by (5) for $t\geq 0$. Substituting the expressions for $x(t)$ into (6) yields an equation for the price level:

$$(A1) \qquad p(t) = \int_{s=0}^{t}\int_{r=0}^{1} E_{t-s}p_i^*(t-s+r)\,dr\,ds + \int_{s=t}^{1}(t-s+1/2)\,ds\,, \qquad 0\leq t<1$$

$$= \int_{s=0}^{1}\int_{r=0}^{1} E_{t-s}p_i^*(t-s+r)\,dr\,ds\,, \qquad t\geq 1\,,$$

where again $p_i^*=vm+(1-v)p$. Taking the expectation of (A1) at time zero yields:

$$(A2) \qquad Ep(t) = \int_{s=0}^{t}\int_{r=0}^{1} Ep_i^*(t-s+r)\,dr\,ds + \int_{s=t}^{1}(t-s+1/2)\,ds\,, \qquad 0\leq t<1$$

$$= \int_{s=0}^{1}\int_{r=0}^{1} Ep_i^*(t-s+r)\,dr\,ds\,, \qquad t\geq 1\,,$$

where E is the expectation at time zero, and I use the law of iterated expectations. Equation (A2) simplifies to

$$(A3) \qquad Ep(t) = \int_{s=0}^{t} sEp_i^*(s)\,ds + \int_{s=t}^{1} tEp_i^*(s)\,ds$$

$$+ \int_{s=1}^{t+1} (1+t-s)\,Ep_i^*(s)\,ds + \frac{1}{2}(t-t^2)\,, \quad 0\le t<1\,;$$

$$= \int_{s=0}^{1} (1-s)\,Ep_i^*(t-s)\,ds + \int_{s=0}^{1} (1-s)\,Ep_i^*(t+s)\,ds\,, \quad t\ge 1\,.$$

This equation implicitly defines the expected path of prices in terms of the expected path of money. The equation also defines expected output in terms of expected money, since $Ey(t)=Em(t)-Ep(t)$. This establishes the lemma in Section IV.

B. Expected Money and Output During Disinflation

Here I derive the expected path of output for the basic example in Section IV. The first step is to find the expected path of money. At time $t<k$, the probability that the Fed has not yet reneged is e^{-ht}. It has reneged at time $s<t$ with density he^{-hs}. Using (8) and (9), expected money growth is

$$(A4) \qquad E\dot{m}(t) = e^{-ht}(1 - t/k) + \int_{s=0}^{t} he^{-hs}(1 - s/k)\,ds\,, \qquad 0\le t<k\,;$$

$$= \int_{s=0}^{k} he^{-hs}(1 - s/k)\,ds\,, \qquad t\ge k\,.$$

This equation simplifies to equation (10) in the text. Finally, integrating (10) yields the expected path of the level of money:

$$(A5) \qquad Em(t) = t - \frac{t}{hk} + \frac{1}{kh^2} - \frac{1}{kh^2}e^{-ht}\,, \qquad 0\le t<k\,;$$

$$= \frac{1}{kh^2} + t - \frac{t}{hk} + e^{-hk}\left[-\frac{1}{kh^2}-\frac{1}{h}+\frac{t}{hk}\right]\,, \quad t\ge k\,.$$

The expected path of prices is defined by substituting (A5) into (A3). For given v, h, and k, this path can be derived numerically by making an initial guess, substituting it into (A3), and iterating (see Ball, 1990, note 5). Subtracting the expected path of prices from (A5) yields the expected path of output.

C. Individual Realizations

Here I describe how to calculate the output path for a particular realization of Fed behavior. For simplicity, I consider only the case of k=1. The major step is to calculate the path of individual prices, x(t). Using (3), (5), and (6), one can derive the following expression for x(t):

$$(A6) \qquad x(t) = v\int_{s=0}^{1} E_t m(t+s)\,ds + (1-v)\int_{s=0}^{1}(1-s)\,x(t-s)\,ds + (1-v)\int_{s=0}^{1}(1-s)\,E_t x(t+s)\,ds .$$

To find x(t) numerically, I discretize the path with step size Δ (Δ=.005 in the reported results). I assume that $x(t)=\int_0^1 m(t+s)ds$ for t>T (T=10); this is equivalent to assuming that y(t)=0 for t>T+1. I then use separate approaches to derive x(t) for $t\in[0,\tau]$ and for $t\in[\tau,T]$. (If the Fed never reneges, τ is set to one.)

To find x(t) for $t\in[0,\tau]$, I derive x(0), x(Δ),..., X(τ) sequentially. To find x(t) for a given t, I first use (8), (9), and the hazard h to derive the path of money expected at t:

$$(A7) \qquad E_t m(t+s) = t + s - st - \frac{t^2}{2} - \frac{s}{h} + \frac{1-e^{-hs}}{h^2}, \qquad 0\le t+s<1 ;$$

$$= t+s-st-\frac{t^2}{2}-\frac{s}{h}+\frac{1}{h^2}+e^{-h(1-t)}\left[\frac{t+s-1}{h}-\frac{1}{h^2}\right], \qquad t+s\ge 1 .$$

This result determines the first integral on the right side of (A6). Since I derive x(t) sequentially, the second integral is determined by previous steps. (For $t-s<0$, $x(t-s)=t-s+1/2$.) It remains to find the third integral, which involves expected future x's. Equation (A6) implies

$$(A8)\qquad E_t x(q) = v\int_{s=0}^{1} E_t m(q+s)\,ds + (1-v)\int_{s=0}^{1} (1-s)\,E_t x(q-s)\,ds + (1-v)\int_{s=0}^{1} (1-s)\,E_t x(q+s)\,ds \ .$$

For $q\in[t,T]$, I guess a path for $E_t x(q)$, substitute it into (A8) to produce a new path, and iterate to convergence. This procedure yields the expected x's in the final term of (A6).

For $t\in[\tau,T]$, there is perfect foresight. Thus the expectations operator can be dropped from equation (A6). Using (8) and (9), the path of money is

$$(A9)\qquad m(t) = t - t\tau + \tau^2/2 \ , \qquad t\geq\tau \ .$$

I substitute this result into the perfect foresight version of (A6). Then I determine the path of x(t) over $[\tau,T]$ by substituting an initial guess into (A6) and iterating.

Once the path of x(t) is determined, I use (6) to find p(t). Subtracting p(t) from the actual path of money yields y(t).

D. The Alternative Disinflation Path

Finally, I consider the alternative disinflation experiment defined by equation (11). Using (8) and (11), one can show that expected money growth is

$$(A10)\qquad E\dot{m}(t) = 1 - \frac{t}{k}e^{-ht} \ , \qquad 0\leq t<k \ ;$$
$$\qquad\qquad = 1 - e^{-hk} \ , \qquad t\geq k \ .$$

The expected path for the level of money is

$$(A11) \qquad Em(t) = t - \frac{1}{kh^2} + e^{-ht}\left(\frac{t}{kh} + \frac{1}{kh^2}\right), \qquad 0 \le t < k ;$$

$$= t - \frac{1}{kh^2} + e^{-hk}\left(\frac{1}{h} + \frac{1}{kh^2} - t + k\right), \qquad t \ge k .$$

As in the basic experiment, Ep(t) and Ey(t) are derived numerically after substituting Em(t) into (A3).

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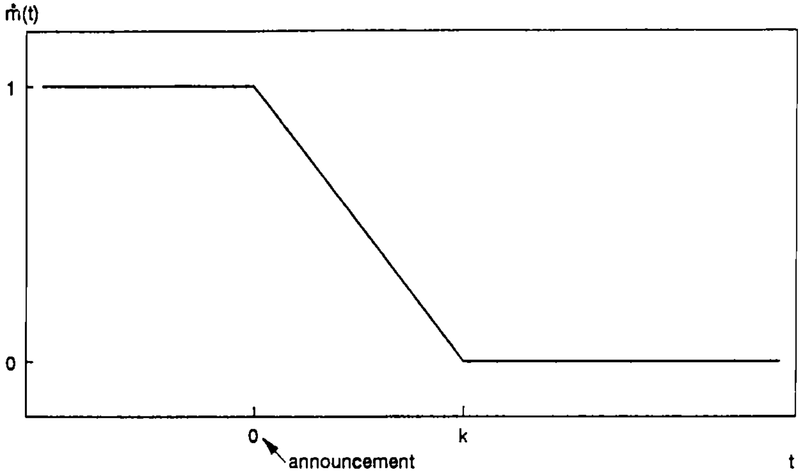
Figure 1
Announced and Actual Money Growth

The Announced Path



The Actual Path If the Fed Reneges at τ

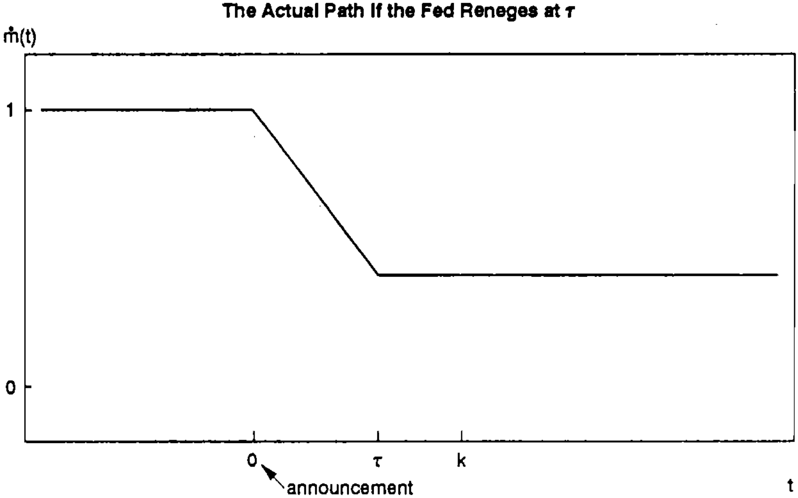


Figure 2
Expected Money Growth
(k=1)

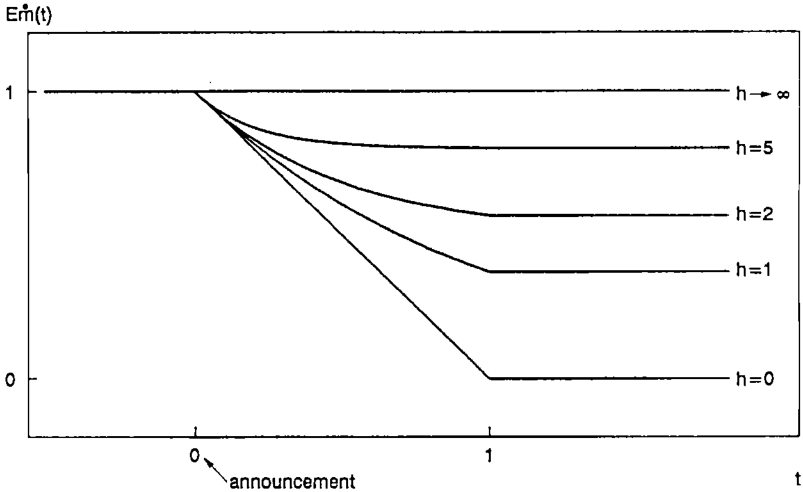


Figure 3
Expected Output
(v=¼, k=1)

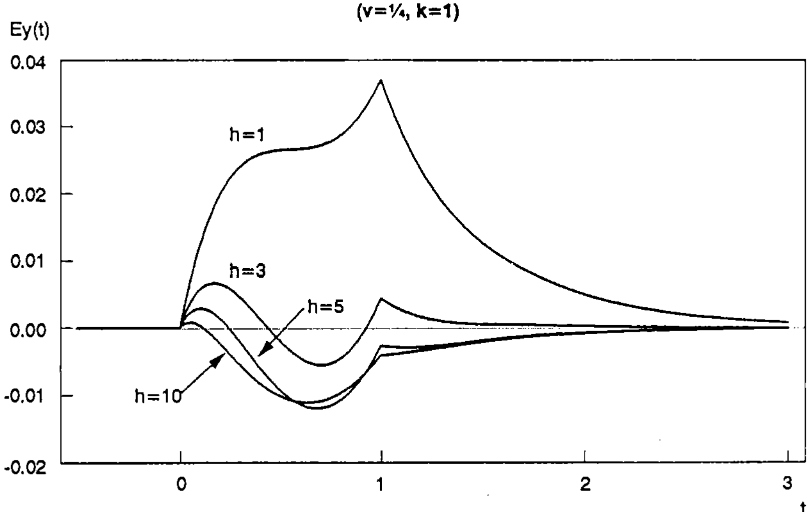


Figure 4
Output for Various τ's
(v=¼, k=1)

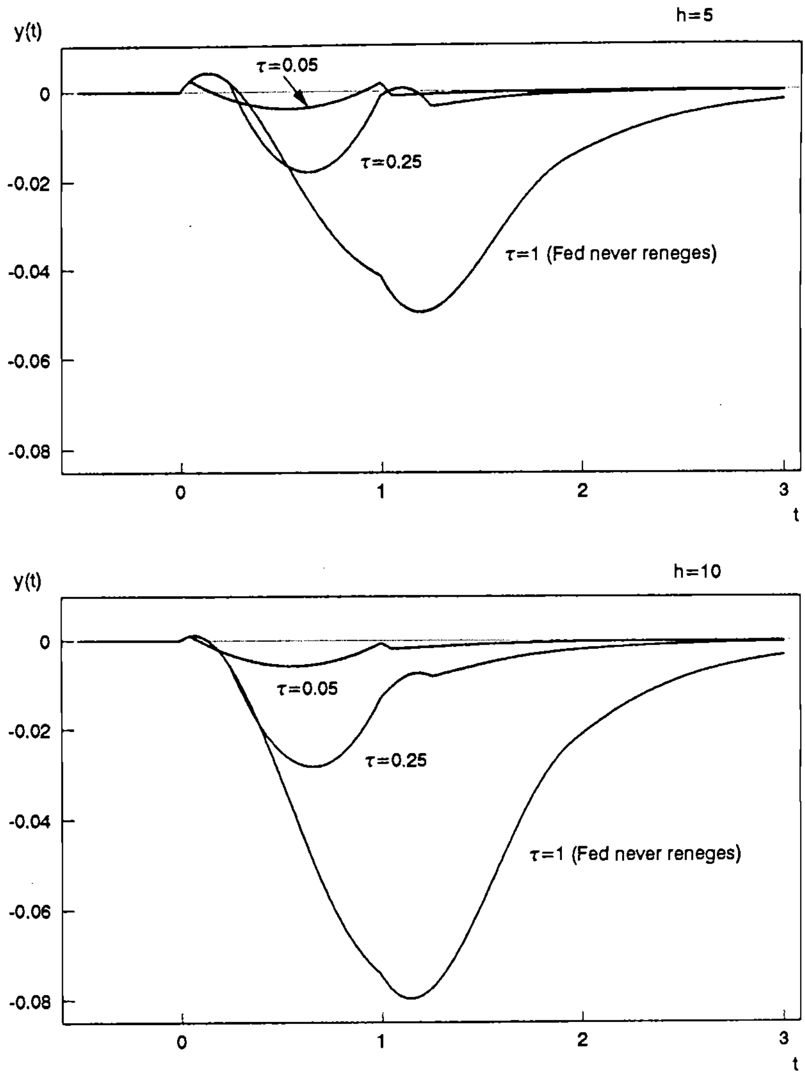


Figure 5
Expected Output – Alternative Specification
(v=¼, k=1)

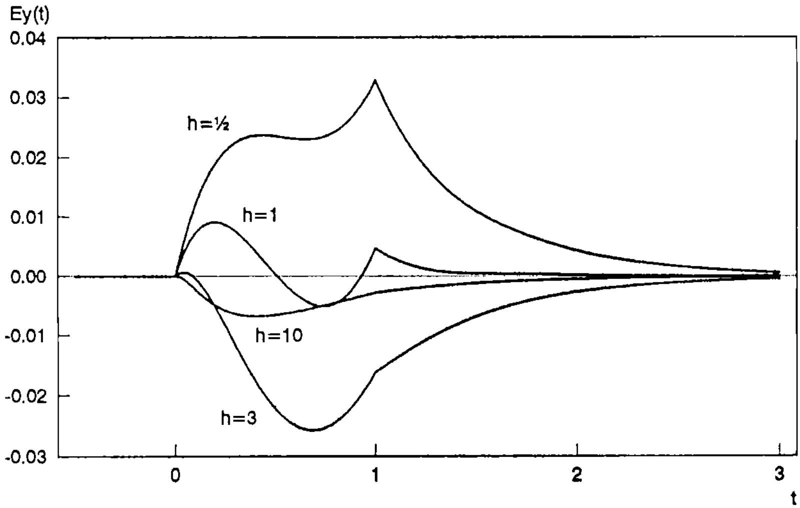


Table 1

Expected Sacrifice Ratio

(v=1/4)

h =	0	1	3	5	10
k = 0.5	0.000	0.012	0.036	0.057	0.095
1.0	-0.101	-0.067	-0.005	0.039	0.093
2.0	-0.199	-0.127	-0.018	0.037	0.093
5.0	-0.278	-0.153	-0.019	0.037	0.093

Table 2

$\int_{t=0}^{\infty} y(t)dt$ - Various τ's

(v=1/4,k=1)

h =	0.1	1	3	5	10
τ = 0.01	0.002	0.001	0.000	-0.000	-0.001
0.05	0.012	0.007	0.001	-0.002	-0.004
0.10	0.023	0.013	0.002	-0.004	-0.009
0.25	0.055	0.032	0.004	-0.009	-0.023
0.50	0.095	0.056	0.006	-0.019	-0.046
0.75	0.112	0.065	0.002	-0.032	-0.070
0.90	0.106	0.057	-0.008	-0.044	-0.086
1.00	0.093	0.044	-0.021	-0.057	-0.099