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THE DEBT BURDEN AND DEBT MATURITY

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ABSTRACT

At low and moderate levels of government debt, there appears to be little relation between the level of debt and its maturity. But at high levels of debt, a strong inverse relation emerges. We start the paper by documenting this inverse relation for those OECD countries which have reached very high levels of debt. We then provide a theory of the joint movements of debt and maturity which can explain both sets of facts. It is based on the idea that, at high levels of debt, the government may need to decrease the maturity of the debt as debt increases, in order to maintain the credibility of its anti-inflation stance.

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Introduction

When one looks at OECD countries over the last 30 years, there appears to have been little systematic relation between the level of debt and its maturity. [1]. One set of countries stands however in clear exception to this general statement, namely those countries which have now reached debt-GNP ratios approaching or exceeding 100%. There, the increase in debt has been associated with a sharp reduction in maturity. Our paper provides a tentative explanation for these two sets of facts.

In section 1, we look at debt and maturity for the three countries, Italy, Belgium and Ireland, which have all reached high levels of debt. For each, we construct an effective maturity series. By effective maturity, we mean the maturity relevant to the effect of inflation on the value of the debt; thus for example, we treat foreign and indexed debt as zero maturity debt (The motivation for such a definition of maturity is given by the model we develop later). We document the strong inverse relation between effective maturity and the debt-GNP ratio.

This leads us to develop in section 2 a simple model based on the now standard idea of a reputation equilibrium. A government which has nominal debt clearly has an incentive to try to inflate it away so as to decrease the debt burden. It will resist the urge if the rewards are small, and the cost of a lost reputation is high. Given that the rewards from unexpected inflation are increasing in both the level of debt and its maturity, the government will keep its no-inflation pledge credible by decreasing maturity as debt increases. Or more precisely, the *maximum*

1. As has been noted by a number of authors (Calvo and Guidotti [1990b]), the US have exhibited a positive relation between maturity and debt, with both maturity and the debt-GNP ratio decreasing until the mid 1970's, and increasing since then. But the US appear to be very much the exception, with most other countries exhibiting little correlation between debt and maturity movements. A detailed review of the evidence for a number of OECD countries is given in Missale [1991].

maturity consistent with a credible no-inflation pledge will decrease with the level of debt.

The model developed in section 2 is a barebone model, with a number of strong simplifying assumptions. In particular it postulates rather than derives a tax rule, assuming taxes to be set so as to yield a constant level of debt in the absence of unexpected inflation. In section 3, we endogenize the timing of taxes by allowing for a tax smoothing motive. We derive the joint dynamics of debt and maturity as a function of an exogenous sequence of government spending, and show that, again, maximum maturity moves inversely with the level of debt.

In section 4, we conclude and relate our results to the recent research on debt and maturity. In doing so, we take up the obvious loose end in our argument, the fact that we have derived a theory of maximum rather than actual maturity. We argue informally that, once the other motives explored in the literature are taken into account, the maximum maturity is likely to be binding only at high levels of debt. This explanation can thus potentially account for the two sets of facts presented at the beginning, the existence of a clear relation between maturity and debt at high levels of debt, and the absence of such a relation at lower levels. A formalization is however left to future work.

1 Evidence from three countries

In 1990, three OECD countries, Belgium, Ireland and Italy, had (gross) debt to GNP ratios around or above 100%, a number roughly twice as high as the OECD-Europe average. We focus in this section on the evolution of debt and maturity in those three countries since 1960 [2]. To do so, we construct two series for each of

2. A fourth country, Greece, is fast on its way to reach those levels. We were unable however to obtain detailed enough information on maturity and on Central Bank holdings, and thus have not included Greece in our sample. The rough evidence is however consistent with the evidence for the

the three countries. (Sources for the series and details of construction are given in the appendix).

The first gives the "market holdings" of debt —that part of the government debt held by the public rather than by the central bank or government agencies—, as a proportion of GNP. Government debt held by the public rather than total debt is the relevant tax base for unexpected inflation. The difference between market holdings and total debt is sometimes substantial; in Italy, central bank holdings were equal to 13% of GNP in 1960, going up to 40% in 1976 (through monetization of the deficit), and back down to 13% in 1989.

The second gives the "effective maturity" of the debt. This effective maturity is different from the conventionally measured maturity. What matters, from the point of view of the theory we develop later, is the effect on the value of the debt of a change in inflation. With respect to fixed rate nominal debt denominated in domestic currency, the official definition of maturity is fine. But European governments have issued substantial amounts of other types of debt over the last three decades. One is price-level indexed debt; we assume all such debt to have zero maturity. Another is foreign currency (or, in the recent past, ECU) denominated debt; in Ireland for example, the share of such debt increased from 13% in 1960 to 50% in 1983 and now stands at 36%. We also assume all such debt to have zero maturity, thus assuming implicitly any inflation to be reflected one for one in currency depreciation [3]. Yet another is "financially" indexed debt, with the interest rate on long term debt indexed to a short term rate. In Italy, the share of such debt has increased from 0% in 1976 to 31% in 1989. For such debt, we use the appropriate short rate as the relevant rate for purposes of computing maturity [4].

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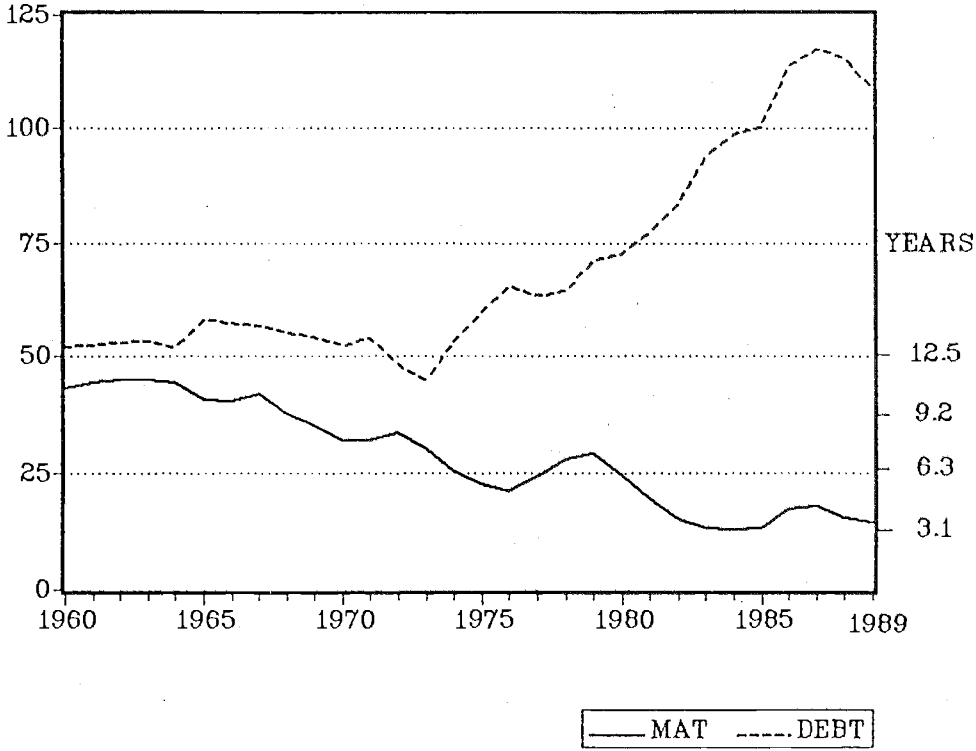
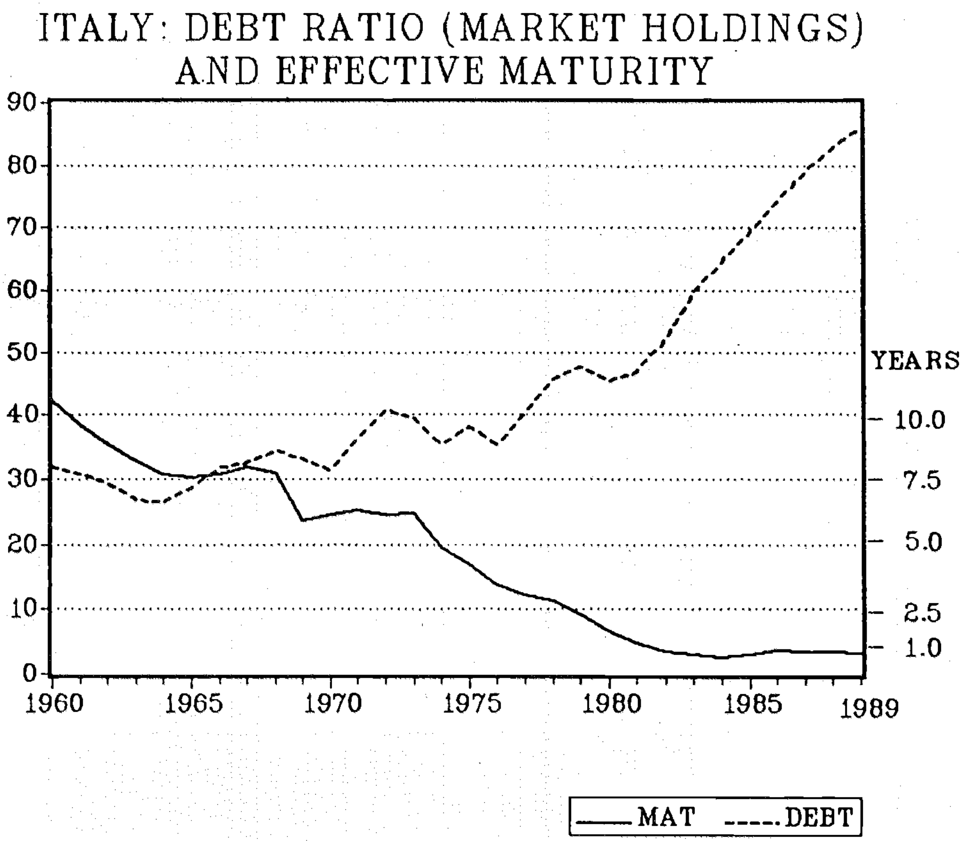
three countries we focus on.

3. We see this assumption as a first approximation, which should be explored further, both theoretically and empirically.

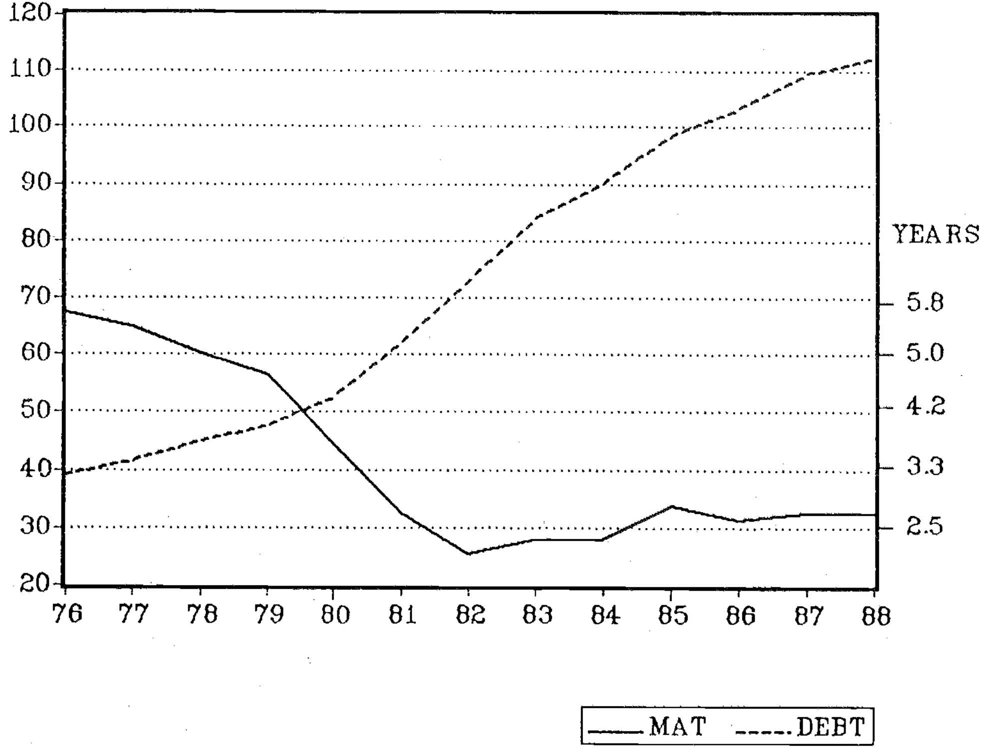
4. This last adjustment makes a substantial difference for Italy. Compare our effective maturity for

IRELAND: DEBT RATIO (MARKET HOLDINGS) AND EFFECTIVE MATURITY

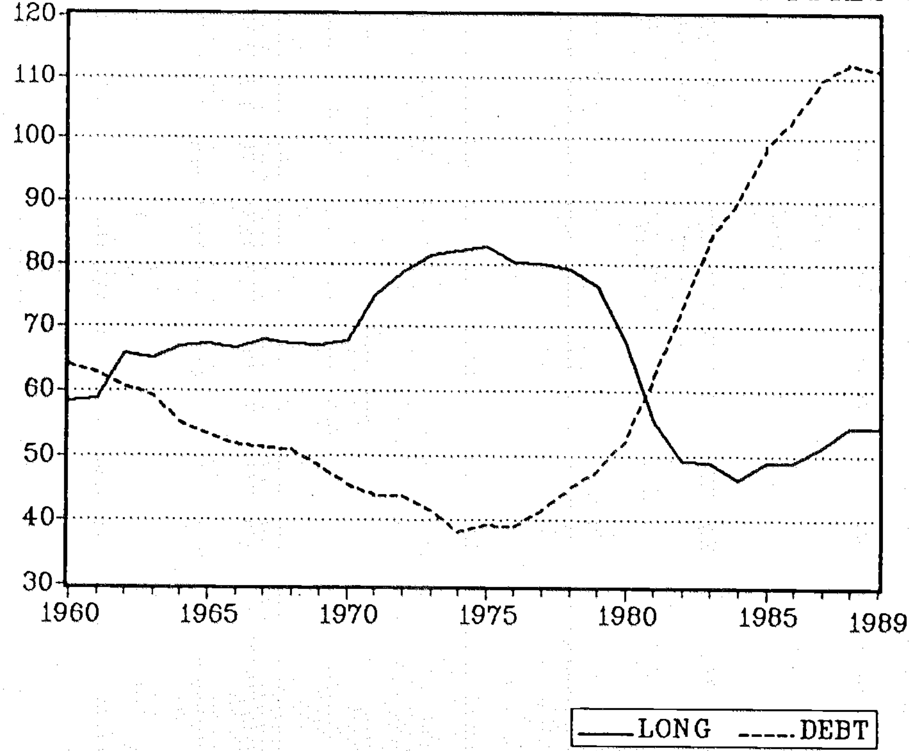



ITALY: DEBT RATIO (MARKET HOLDINGS)
AND EFFECTIVE MATURITY
90
80
70
60
50
40
30
20
10
0
1960
1965
1970
1975
1980
1985
1989
YEARS
10.0
7.5
5.0
2.5
1.0
MAT
DEBT

BELGIUM: DEBT RATIO (MARKET HOLDINGS) AND EFFECTIVE MATURITY



BELGIUM: DEBT RATIO (MARKET HOLDINGS) AND SHARES OF LONG TERM SECURITIES



Ideally, one would like to compute effective maturity for market holdings of debt rather than for total debt; the data on the maturity of the debt held by the central bank are however not easily available, and thus the effective maturity we report is for total debt.

Figures 1 and 2 give the evolution of debt and effective maturity since 1960 for Ireland and Italy respectively. Figure 3 does the same for Belgium since 1976, as detailed data before 1976 are not available. What is available for a longer period of time is the share of long term debt. The behavior of that share since 1960 is given in Figure 4.

All four figures show that the sharp increase in the debt to GNP ratio, which started in all three countries in the mid 70's, has been accompanied by an equally pronounced decrease in effective maturity. In Belgium, the inverse relation between the share of long term debt and the debt-GNP ratio, both on the way up and on the way down is striking. In all three countries, the effective maturity of debt has decreased by half or more since the mid 70's. It is now very low, standing at 3.6 years for Ireland, 0.8 years for Italy and 2.7 years for Belgium compared to, for example, 6.1 years for the US or 7 years for the UK.

Table 1 reports the results from simple regressions of maturity on debt for the period 1960-1989. The dependent variable for both Ireland and Italy is the logarithm of maturity. Given that maturity is not available for Belgium for the whole sample, the dependent variable for Belgium is instead the share of long term debt. Three regressions are presented for each country.

The goal of the first regression is to answer the question of whether the two series move together at low frequencies, of whether the two series are co-integrated. Thus, it regresses log maturity (or the share of long term debt) on the log of the

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Italy given in figure 2 below to that using the official maturity of the debt, given in Alesina et al. [1990].

Table 1. Maturity and Debt ; Ireland, Italy and Belgium

		Dependent variable: log(maturity)					
Country	const.	log(D/Y)	log(π)	Time	R^2	DW	t statistic
Ireland	7.0	-1.23			0.72	0.34	-2.6*
		(-8.9)					
	7.4	-1.24	-.17		0.85	0.82	-2.9*
		(-12.2)	(-5.0)				
	2.5			-0.04	0.86	0.59	
				(-14.5)			
Italy	10.8	-2.59			0.85	0.47	-2.8*
		(-12.8)					
	10.7	-2.40	-0.31		0.91	0.43	-2.7*
		(-14.3)	(-4.2)				
	2.7			-0.10	0.89	0.18	
				(-16.0)			
		Dependent variable: Share of long term debt					
Country	const.	log(D/Y)	log(π)	Time	R^2	DW	t statistic
Belgium	.48	-.31			0.82	0.42	-2.5*
		(-11.6)					
	.48	-0.31	-0.00		0.82	0.42	-2.5*
		(-10.3)	(-0.1)				
				-0.52	0.12	0.13	
				(-2.3)			

Sample period : 1960 to 1989. "t statistic" : t statistic of coefficient on lagged residual, in a regression of the first difference of the residual on the lagged level and the lagged first difference. A star indicates significance at the 10% level.

debt-GNP ratio; the statistic in the last column corresponds to the test of the hypothesis that the two series are not cointegrated. In all three countries, debt is highly significant. This is precisely what the eye saw in figures 1 to 4. In all three countries however, the Durbin Watson statistic is low, indicating that the relation is at most present at low frequencies, and raising the issue of cointegration. In all three countries, the hypothesis of no-cointegration is rejected at levels of confidence between 5% and 10%.

The goal of the second regression is to explore the possibility that the correlation is spurious, and that the decrease in maturity is in fact due to the rise in inflation, itself correlated with debt. It has indeed long been argued that high inflation is associated with higher inflation uncertainty, leading to higher risk premia on long term nominal debt, and thus leading governments to stop issuing long term debt. We have no doubt that this line of explanation is relevant, and that when inflation is very high, long nominal assets, private or public, disappear. But the second regression in table 1, which regresses log maturity (or the share of long term debt) on the log of the debt-GNP ratio and on inflation shows that there is more to the evolution of maturity in those three countries than simply the effect of inflation. In all three, the debt-GNP ratio dominates inflation, both quantitatively and statistically. In Ireland and Italy, inflation significantly decreases maturity. But there is no discernible effect in Belgium. The basic reason why regressions favor the debt-GNP ratio comes from the evidence at the end of the sample. In the late 80's, inflation has slowed down, while the debt-GNP ratio has only stabilized; and, like the debt-GNP ratio, effective maturity has stabilized rather than increased back to earlier levels.

The goal of the third regression is to give some perspective on the strength of the results by running the simplest possible horse race, a comparison of the first regression with a regression of the dependent variable on a time trend. The results vary across the three countries. In Ireland and Italy, the time trend does as well as the debt-GNP ratio. In Belgium in which effective maturity and debt have behaved very differently from smooth trends, the time trend does poorly in comparison to the debt-GNP ratio.

Overall, the regression results confirm the visual impression given in the figures. In all three countries, debt and maturity have moved in opposite directions over the last 30 years; the relation is not there from year to year, but appears to be present at low frequencies. This is the stylized fact which motivates the model presented in the next two sections [5]. In those sections, we ignore the fact that the relation disappears at low levels of debt; we return to it in the last section.

2 Maturity, credibility and reputation

Our tentative explanation for the facts of the previous section is the following. A government which has nominal debt clearly has an incentive to try to inflate it away so as to decrease the debt burden. It will resist the urge if the rewards are small, and the cost of a lost reputation is high. Given that the rewards from unexpected inflation are increasing in the level of debt and increasing in maturity, the government will keep its non inflation pledge credible by decreasing maturity as debt increases.

The argument would seem to be a straightforward application of earlier models of inflation, such as Barro and Gordon [1983]. It turns out to be more, and, in the process, to be more interesting. This is for two reasons, and both have long been known to policy makers. The first is that a successful expropriation of debtholders through unexpected inflation decreases the need for revenues in the future and thus reduces the incentive to inflate in the future. The second is that, once

5. The model below implies that other no-inflation commitment devices, such as the credible commitment to a fixed exchange rate, allow for higher maturity at a given level of debt. We have thus experimented for Italy with different EMS timing dummies, trying to capture such a commitment effect. Our efforts were unsuccessful.

the government has had recourse to inflation, it can from then on drastically decrease the maturity of the debt, again decreasing its temptation to have recourse to inflation in the future. In other words, a strong burst of unexpected inflation today removes most of the incentives to inflate tomorrow, and the government can further reduce whatever incentives are left by decreasing the maturity of its debt. But by reducing the need and the incentives to inflate, this in turn decreases the punishment incurred by the government if it inflates today, and makes a reputation equilibrium harder to sustain. Our model shows the mechanisms at work, and the role of maturity.

2.1 Debt accumulation, debt maturity and unexpected inflation

We want to capture two aspects of the problem faced by the government. The first is that, other things equal, a higher level of nominal debt leads to a stronger temptation to inflate. The second is that, the higher the maturity of the debt, the larger is the decrease in the market value of the debt associated with a given unexpected increase in inflation. We formalize the relation between debt, maturity and inflation by the following accumulation equation:

$$D' = (1+r)(1-m(\pi - \mathrm{E}\,\pi))D + G - T \tag{2.1}$$

D denotes the real value of debt at the beginning of period t, G and T denote government spending and taxes during period t, r is the real interest rate, which is assumed constant. Next period values are denoted by primes. The important assumption is in the formalization of the relation between maturity, unexpected inflation and the value of debt. We formalize maturity by an index m, which gives the effect of a given unexpected rate of inflation on the value of the debt.

A strict interpretation of this assumption is that the government can choose to issue a combination of indexed debt —zero maturity nominal debt— and one-period maturity nominal debt. If the government issues only zero-maturity debt,

m is then equal to zero: there is no effect of unexpected inflation on the value of the debt. If the government issues only one-period nominal debt, then m is equal to one. [6]

We shall feel free however to use informally a more general interpretation, in which m stands for the average maturity of debt, conceptually allowing debt to be of maturity longer than one period. The reason why that interpretation is more questionable is that, when the maturity of the debt exceeds one period, the sequence of unexpected inflation over the life of the bonds should appear in equation (2.1). This can be introduced, but at some cost in simplicity; we return to the issue below.

In the rest of this section, we assume that government spending is constant, and –this purely for notational convenience– equal to zero. We assume that taxes are set so that debt remains constant in the absence of unexpected inflation: $T = r(1 - m(\pi - \mathrm{E}\,\pi))D + G = r(1 - m(\pi - \mathrm{E}\,\pi))D$. We shall allow for variations in government spending and derive the optimal timing of taxation in the next section; we shall show that this apparently ad-hoc rule is indeed the optimal rule for the case of constant government spending. Replacing this expression for taxes in (2.1) gives:

$$D' = (1 - m(\pi - \mathrm{E}\,\pi))D \tag{2.2}$$

Thus, under this tax rule, debt remains unchanged in the absence of unexpected inflation. The effect of unexpected inflation is to decrease the value of the debt, and the strength of the effect depends on m.

6. Note that, even under that assumption, (2.1) is only a linear approximation. In particular, it does not exclude that $1 - m(\pi - \mathrm{E}\,\pi)$ is negative, clearly an absurd outcome, as the most unexpected inflation can do is reduce the value of the debt to zero.

2.2 The objective function of the government

Our specification of the objective function of the government follows tradition. The government minimizes the expected present discounted value, V, of current and future values of the one-period loss function L, discounted at rate δ. The one-period loss function is the sum of three terms:

$$L = (1/2)\pi^2 - b(\pi - \mathrm{E}\,\pi) + cT \tag{2.3}$$

The first two terms are familiar. The first reflects the costs of inflation, the second the benefits of unexpected inflation, presumably through output effects which need not be made explicit here. The third reflects the cost of taxation. The assumption that the loss is linear rather than quadratic in taxes is theoretically unappealing –as, in the small, deadweight losses are quadratic in taxes–, and is made for convenience. Allowing for a quadratic term complicates the algebra but does not affect the qualitative results. We return to this issue in the next section. Using $T = r(1 - m(\pi - \mathrm{E}\,\pi))D$, and equation (2.2), and replacing in (2.3) gives:

$$L = (1/2)\pi^2 - b(\pi - \mathrm{E}\,\pi) + cr(1 - m(\pi - \mathrm{E}\,\pi))D \tag{2.4}$$

The timing of decisions is the following. At time t, the government inherits D, whose equation of motion is given by (2.2). The government decides on the maturity of the debt, m, for period t. This maturity is known to people when they form their rational expectations. The government then chooses the rate of inflation for period t.

Except for the dynamic complications introduced by the dynamics of debt and the ability to choose maturity, the problem we have set up is standard. Clearly the best outcome is the no-inflation outcome. But, in the absence of reputational effects, the no inflation outcome is time inconsistent, and the outcome is positive rather than zero inflation. In what follows, we focus on the existence and characteristics of a reputational equilibria. So long as the government does not use inflation, people assume that it will not do so in the future. If the government relies on unexpected inflation, people then assume that it will act opportunistically every period, choosing inflation every period so as to minimize V given people's expectations.

2.3 Reputation and the maturity of the debt

Solving for the reputational equilibrium requires the derivation of the value of the loss function under no cheating and thus zero inflation, and under cheating and the subsequent loss of reputation. We derive them in turn.

If the government does not cheat –i.e does not attempt to inflate away the debt, both inflation and expected inflation are equal to zero and the value of the loss function, V_R (R for reputation) is given by:

$$V_R = (1 + (1/\delta))L_R = (1 + (1/\delta))c\tau D \tag{2.5}$$

The loss comes from the taxation required to service the debt inherited from the past, D. As debt is constant over time, V_R is also constant over time.

If, instead, the government inflates in the current period, it loses its reputation for all future periods. To solve for the rate of inflation in the current period, we solve for the equilibrium backward in time.

Once a government has lost its reputation, it will want to choose a level of maturity equal to zero. The reason is simple: the higher the maturity of the debt, the higher the incentive to inflate. Given the loss of reputation, this only leads people to anticipate higher inflation, leading in turn to higher actual and expected inflation and an increased value of the loss function. When the maturity of debt is equal to zero, debt is unaffected by inflation and, under our assumptions about taxes, remains constant forever. The minimization problem faced by the government is therefore the same every period. For example, for period "prime", the period following the loss of reputation, the problem faced by the government is that of minimizing :

$$L_C' = (1/2)\pi'^2 - b(\pi' - \mathrm{E}\,\pi') + crD'$$

as maturity, m', is set equal to zero. The rate of inflation —actual and expected— is thus given by

$$\pi' = \mathrm{E}\,\pi' = b$$

implying that the present value of the loss function from next period on is given by:

$$V_C' = (1 + (1/\delta))L_C' = (1 + (1/\delta))((1/2)b^2 + crD') \qquad (2.6)$$

Equation (2.6) is interesting in two respects. First, to the extent that inflation in the current period reduces the real value of debt –i.e reduces D' below D–, the burden of taxation and thus the value of the loss function is reduced *for all future periods.* The returns to cheating can therefore be substantial. Second, because the government can put maturity equal to zero, it can substantially reduce the equilibrium rate of inflation. Indeed, if the only incentive to inflate came from the presence of nominal debt, i.e if b were equal to zero, putting maturity equal to zero would remove all incentives to inflate, leading to a zero equilibrium rate of inflation in all future periods.

Consider now the minimization problem faced by a government who decides to inflate in the current period. For the moment, take the decision about maturity, m, as given. Given that people's expectations of inflation are equal to zero, the government minimizes:

$$\begin{aligned} V_C &= (1/2)\pi^2 - b\pi + cr(1 - m\pi)D + (1/(1+\delta))V_C' \\ &= (1/2)\pi^2 - b\pi + (1 + (1/\delta))(1 - m\pi)crD + (1/2)(b^2/\delta) \end{aligned}$$

where, in the second line, we have replaced V_C' by its value from (2.6) and D' by its value from (2.2). Solving for the inflation rate gives:

$$\pi = b + (1 + (1/\delta))crmD \qquad (2.7)$$

The inflation rate is an increasing function of b, the effect of unexpected inflation on output and of c, the weight given to the burden of the debt in the loss function. More interestingly for our purposes, the inflation rate is an increasing function of maturity and of the level of the debt. Replacing π by its value from (2.7) in the expression above gives the present value of the loss function under cheating and the attending loss in reputation:

$$V_C = -(1/2)[b + (1 + (1/\delta))crmD]^2 + (1 + (1/\delta))crD + (1/2)(b^2/\delta) \qquad (2.8)$$

We can now solve for the conditions under which the government will prefer not to inflate. This requires that the value of the loss from not cheating V_R be no greater than V_C, the value under cheating. Using (2.5) and (2.8), and rearranging gives the following condition:

$$mDcr \leq b(\sqrt{\delta} - \delta)/(1 + \delta)$$

This condition can in turn be solved for the maximum maturity, call it m^*, consistent with zero inflation:

$$m^* = [b(\sqrt{\delta} - \delta)]/[cr(1 + \delta)D] \qquad (2.9)$$

The maximum maturity is a decreasing function of the debt level [7]. Indeed, in

7. This assumes that δ is less than one. Note that $(1/\delta)$ is the present value of "one unit" from next

our model, the incentive to inflate is proportional to the product of m and D. For reputation to remain an equilibrium when D increases, m has to decrease to leave the product of the two constant. As long as the coefficient b, which reflects the incentive to inflate for other reasons than debt reduction, is positive, m^* is positive: there is always a maturity short enough to sustain the zero inflation equilibrium. Note also that if b is equal to zero, so that the only incentive to inflate is to reduce debt, then there exists no positive maturity which can sustain the reputation equilibrium; this is because in that case, the government can, by choosing zero maturity after having cheated, fully avoid being punished in the future.

2.4 An assessment

We have shown that, when a government wants to keep its zero inflation stance credible, the maximum maturity of the debt will be a decreasing function of the level of debt. We can think of three ways in which our initial model should be extended.

The first is the relaxation of the assumption that only current unexpected inflation affects the value of the debt. This assumption is correct, we indicated, only if the maturity of the debt is between zero and one period. But it is difficult to decide what the unit period of this model stands for (time between "policy decisions" ?), and thus whether the assumption is reasonable or not. It probably is not. A more attractive assumption is that the value of the debt depends on revisions of not only current but also future inflation, with the effect of future inflation a function of the maturity structure of the debt. This is a conceptually straightforward –if substantially more complicated– extension, which does not appear to affect the

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period on, so that the condition $\delta < 1$ can be stated, somewhat loosely, as the condition that the present value of the future matters more than the present. We assume this condition to be satisfied in our discussion.

qualitative results derived above nor to give particular insights [8]. Thus we do not develop it further.

The second is that while our facts have shown a strong inverse time series relation between the level of debt and its maturity, our model generates in equilibrium a constant level of debt. Our result is that of an inverse relation across steady states, not across time. This raises the issue of whether we can generate the time series relation between debt and maturity which is observed in the data. This is the extension we take up in the next section.

The third is that what we have developed is a theory of maximum rather than actual maturity. To turn it into a theory of actual maturity requires the assumption that, at least over some range, the government prefers longer to shorter maturity nominal debt. We take up the issue in the last section.

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8. We have explored the case where the government issues a combination of indexed, one-period and two-period nominal bonds. Then the equation of motion for debt can be written as: $D' = (1+r)(1 - m_1(\pi - \mathrm{E}\,\pi) - m_2(\mathrm{E}'\,\pi' - \mathrm{E}\,\pi'))D + G - T$, with m_1 and m_2 two parameters capturing the maturity structure of the debt. The change in the value of debt from the beginning of the current period to the beginning of the next depends both on unexpected inflation in the current period, which affects the real value of both one-period and two-period bonds, and on the revisions of inflation for the next period, which affect the value of the two-period bonds issued this period (which become one-period bonds at the beginning of the next period).
In the equilibrium we characterized in the text, it was clear that once the government had cheated, it had an incentive to reduce the maturity of debt to zero. This is in general no longer the case here. Once the government has inflated and lost reputation, all newly issued debt should, for the same reasons as in the text, be of zero maturity. But, after cheating, the government still has some one-period debt (two period bonds issued in the previous period) outstanding. The value of that part of the debt can be further reduced by inflation. Thus, in the period following cheating, the maturity of the debt usually remains positive, and inflation is higher than in the case studied in the text.

3 Dynamics of debt and maturity

3.1 Introducing tax smoothing

In the previous section, we assumed a loss function linear in taxes together with an ad-hoc tax rule, relating taxes to interest payments on the debt. Given the linearity of the loss function, had we allowed instead for endogeneity of the timing of taxes, the outcome would have been either indeterminate or pathological, with all taxes all raised in the first period or indefinitely postponed. To get a non trivial determination of the timing of taxation, we thus modify the one-period loss function, which becomes:

$$L = (1/2)\pi^2 + (b/2)(k - (\pi - \mathrm{E}\,\pi))^2 + (c/2)T^2 \qquad (3.1)$$

This differs from the loss function in the previous section in two ways. The first and important one is that the loss is quadratic in taxes. This is the assumption which will deliver tax smoothing and non trivial debt dynamics [9]. The second and less important modification is that the second term is also quadratic —rather than linear— in unexpected inflation. We make this assumption (which is standard in the literature) mostly for symmetry; assuming the loss to be linear in unexpected inflation does not affect the qualitative results below. The government minimizes the present discounted value of L, at rate δ. To avoid any other motive than tax smoothing for the timing of taxes, we assume that the discount and the interest rates are equal, that $\delta = r$.

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9. We would not want to argue that the increase in debt in the countries we studied earlier is fully explained by tax smoothing. Part of it probably is; throughout the 1970's, most countries probably expected the future to be brighter than it turned out to be, and thought of deficits as largely cyclical. Part of it probably comes from the inability in most political systems to quickly adjust to more difficult times. See for example Roubini and Sachs [1987] for some empirical evidence. The overwhelming reason to use tax smoothing as a theory of debt dynamics here is its convenience.

Debt accumulation is characterized in the same way as before:

$$D' = (1 + r)(1 - m(\pi - \mathrm{E}\,\pi))D + G - T \tag{3.2}$$

Government spending is however no longer assumed constant. G varies over time, following an arbitrary but deterministic process. [10]. It is convenient for the analysis below to introduce permanent government spending, G_p, defined as the annuity value of current and future spending, discounted at the interest rate r:

$$G_p \equiv (r/(1 + r))[\sum_{0}^{\infty}(1 + r)^{-s}G_s] \tag{3.3}$$

We assume that the behavior of G is such that the infinite sum above is always finite. We shall use below the relation between G_p and G which follows from the definition of G_p:

$$(G_p{}' - G_p) = r(G_p - G) \tag{3.4}$$

We are now ready to derive the joint behavior of maturity and debt consistent with a zero inflation equilibrium. We proceed as before, first deriving inflation and taxes under reputation and under cheating, then characterizing the maximum maturity consistent with the reputation equilibrium, and its relation to the level of debt.

3.2 Inflation and taxes under reputation

In the reputational equilibrium, the government does not attempt to inflate the debt away, and both actual and expected inflation are equal to zero. The one-period loss function is thus given by $L = (b/2)k^2 + (c/2)T^2$, and the debt accumulation is given by $D' = (1 + r)D + G - T$. The only decision left to the

10. Allowing for a stochastic process, as desirable as it may be, substantially complicates the analysis. We have been unable to make progress in that direction.

government is that of the timing of taxes, and, under the assumption that the discount and interest rates are equal, the solution takes the simple form:

$$T = \bar{T}_R = rD + G_p \tag{3.5}$$

Not surprisingly, tax smoothing and the absence of uncertainty imply that taxes are *constant* over time, at level $\bar{T}_R$. Equivalently, they are set equal to interest payments on the debt plus permanent government spending. While taxes remain constant, permanent spending and debt change over time. Equation (3.4) gave the behavior of permanent spending, and together with equation (3.5) implies that debt in turn follows:

$$D' - D = (G - G_p) = (G + rD - \bar{T}_R) \tag{3.6}$$

Debt increases when permanent spending exceeds current spending, that is when current spending is unusually high in comparison to spending in the future. [11]. Replacing taxes by their value from equation (3.5) gives the (constant) value of the loss function under reputation, V_R

$$V_R = (1 + (1/r))L_R = (1 + (1/r))[(b/2)k^2 + (c/2)(\bar{T}_R)^2] \tag{3.7}$$

3.3 Inflation and taxes under cheating

To characterize the value of the loss function under cheating, we again solve backwards, starting in period "prime", after the government has used unexpected inflation to reduce the debt burden.

Once the government has inflated, and lost reputation, it has, just as in the previ-

11. See Barro [1979] for further analysis of the dynamic implications of tax smoothing.

ous section, an incentive to reduce maturity to zero, so as to reduce the actual and the expected inflation rates. Having done so, it must choose taxes and inflation. Given the nature of the maximization problem, those are chosen to be the same in period prime and all future periods, and they are given by:

$$\bar{T}'_C = rD' + G_p' \tag{3.8}$$

$$\pi' = bk \tag{3.9}$$

Tax smoothing implies constant taxes at level $\bar{T}'_C$ from period prime on. The level depends on the value of permanent spending and government debt after the government has cheated. Because of the zero debt maturity, inflation does not depend on the level of debt, and is positive only to the extent that the government has other motives than debt repudiation for wanting to generate unexpected inflation, i.e. to the extent that both b and k are different from zero. Replacing the constant inflation rate and the constant taxes in the loss function at time prime gives:

$$V_C' = (1/2)(1 + (1/r))[b(1 + b)k^2 + c(rD' + G_p')^2] \tag{3.10}$$

Working back to the current period, the government which decides to inflate minimizes :

$$V_C = L_C + (1/(1 + r))V_C'$$

Using (3.10) and (3.2), V_C can be expressed as a function of, in particular, current maturity, current taxes and current inflation. For the moment, we take the decision about maturity, m, as given; the government has only two decisions left, taxes and inflation, which, from the first order conditions, are characterized by:

$$T = \bar{T}'_C = r(1 - m\pi)D + G_p \tag{3.11}$$

$$\pi = b(k - \pi) + cT(1 + r)mD \tag{3.12}$$

The government sets taxes at the *same* level as the level it intends to set them in future periods, $\bar{T}'_C$; this in turn implies that taxes are set according to the second equality in (3.11). Inflation is set so as to equalize the marginal cost of inflation on the left in (3.12) to the two marginal benefits of unexpected inflation on the right. The first is the benefit in increased output. The second is the marginal effect of inflation on debt, $(1 + r)mD$ times the marginal resource cost of taxation, $c\bar{T}'_C = cT$. Solving those two equations for inflation gives:

$$\pi = \frac{bk + c(1 + r)mD\bar{T}_R}{1 + b + c(1 + r)r(mD)^2} \tag{3.13}$$

where $\bar{T}_R$ is the constant level of taxes that the government would levy in the reputation equilibrium, which we derived earlier. Note that the effect of maturity on inflation is now ambiguous. To understand why, return to equations (3.11) and (3.12). For a given permanent level of taxes, higher maturity implies a stronger incentive to inflate and higher inflation. But, for a given rate of inflation, higher maturity decreases the level of permanent taxes, decreasing the marginal cost of taxation and the incentive to inflate. For maturities high enough, the second effect may dominate, leading to a decrease in the rate of inflation with an increase in maturity. The value of the loss function under cheating, V_C can be obtained by replacing π by its value from (3.13) and T by its value from (3.11). It is not particularly nice or intuitive, and we do not report it.

3.4 Debt and maximum maturity

We can now derive the maximum value of maturity consistent with reputation. We first consider the difference between V_R and V_C for a given value of m. From the results above and some manipulation, this difference is proportional to:

$$\begin{aligned} V_C - V_R \;\propto\; & (bk)^2(1 - r + b) \\ & -2rc(1 + r)bk\bar{T}_R(mD) \end{aligned} \tag{3.14}$$

$$-rc(1+r)[c(1+r)\bar{T}_R^2-(bk)^2](mD)^2$$

Note that in this expression, only two variables have time indices, m and D. The level of taxes under reputation, $\bar{T}_R$, depends on the initial level of debt and the initial value of permanent spending, but is constant over time. All the other elements are parameters. We define the *maximum maturity*, m^*, as the maximum value of m consistent with reputation being an equilibrium, with the right hand side of (3.14) being non negative. Noting that the expression is a second degree polynomial in the product mD, it is easy to show the following:

For low enough values of permanent taxes, reputation is an equilibrium independent of the level of debt –the initial level of which however affects the value of permanent taxes in the first place–, and of the initial level of maturity. When this is the case, maturity must be determined by other considerations than those considered in this model. The condition for this to happen is that the determinant of the second degree polynomial on the right hand side of (3.14) be negative, in which case $V_C - V_R$ is positive for all values of m:

$$\bar{T}_R^2 \leq \frac{(1-r+b)(bk)^2}{c(1+r)(1+b)} \tag{3.15}$$

For higher values of permanent taxes, there is a maximum maturity consistent with reputation [12].

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12. The algebra is as follows. For higher values of permanent taxes, condition (3.15) does not hold, and the determinant of the polynomial is positive. There are therefore two roots, and two cases to consider depending on the sign of the coefficient of the term in m^2, $-rc(1+r)D^2[c(1+r)\bar{T}_R^2 - (bk)^2]$.

If $[c(1+r)\bar{T}_R^2 - (bk)^2] > 0$, which is a stronger condition than for the determinant to be positive, the polynomial is concave in m, and has one negative and one positive root, say m^*_1 and m^*_2 respectively. Hence the polynomial is positive for values of m between m^*_1 and m^*_2, and the

The maximum maturity is decreasing in the level of debt. This follows from the observation that m and D enter (3.14) always as a product, and that starting from given values of m and D which are such that the right hand side is equal to zero, the product must remain constant for reputation to remain an equilibrium. Thus, using this result and equation (3.6), our model implies the following dynamics of debt and maximum maturity:

$$\begin{aligned} D' - D &= (G - G_p) = (G + rD - \bar{T}_R) \\ m^* &= \lambda(\bar{T}_R)/D \end{aligned}$$

where λ can be shown to be a decreasing function of the permanent level of taxes. Thus, a sustained period of unusually high spending indeed leads to a sustained increase in debt and a sustained decrease in maximum maturity.

4 From maximum to actual maturity

We started this paper by documenting the striking inverse relation between debt and maturity in those countries which have reached high debt-GNP ratios over

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maximum maturity consistent with reputation is given by m^*_2.

If $[c(1+r)\bar{T}_R^2 - (bk)^2] < 0$, which may hold even if the determinant is positive, the polynomial is convex and has two positive real roots, say m^*_1 and $m^*_2 \geq m^*_1$. As the polynomial is positive for values of m less than m^*_1 and greater than m^*_2, reputation would appear to hold both for values of m less than m^*_1 and for values greater than m^*_2. Using (3.13) however, it can be shown that values of m equal to or greater than m^*_2 imply a negative terminal value of debt, i.e. a negative value of $(1 - m\pi)D$, and thus are unacceptable. Thus, the relevant root is m^*_1 and values of m less than m^*_1 are required to maintain reputation.

the last two decades. We then provided, in two steps, a theory of the joint movements of debt and the maximum feasible maturity, m^*, based on the idea that maturity of the debt can be used by the government to maintain its anti-inflation credibility.

In this final section, we turn to the obvious missing part of the argument. What we have derived is a theory of *maximum maturity*, not of maturity itself. Indeed, in our model, the government is indifferent to choosing any maturity below m^*, and thus could well choose zero effective maturity debt all the time, either in the form of very short maturity nominal debt or in the form of indexed bonds –of any maturity. To turn it into a theory of actual maturity, we need to argue that the government prefers longer to shorter maturity debt, with the implication that the government will always choose the longest feasible maturity, thus will always choose m^*. Or we need to argue that the government, in the absence of reputation considerations, has a preference for a specific finite maturity. Then, as long as debt is not too high, actual maturity is equal to that prefered maturity, but at higher levels of debt, the maximum maturity consistent with reputation becomes the binding constraint. This line of explanation can potentially explain both the lack of a relation at low levels of debt as well as the emergence of an inverse relation at higher levels. But are there plausible arguments for why the government may prefer long to short maturity debt, or has a prefered finite maturity ? Two lines of research on the maturity of government debt have been recently explored and are directly relevant.

First, a number of authors have emphasized that short maturity debt must be refinanced often; this is not only costly, but also leads to a heightened risk of crisis. This idea has been recently formalized by Giavazzi and Pagano [1990] and Alesina et al. [1990], and leads to the conclusion that governments should issue long maturity debt. The notion of maturity implicit in those models is however different from that used in this paper. "Financially indexed debt" for example, i.e. long term debt paying an interest rate tied to the short rate –such as has been issued in Italy over the last 10 years– has a long maturity from the point of view of confidence crises: it only needs to be refinanced infrequently. But it has a short maturity from the point of view of the effect of inflation on its value. The same is true of long indexed or foreign currency bonds. In other words, governments can –and do– use debt instruments which reduce the risk of confidence crises but are sufficiently immune to unexpected inflation to allow the government to maintain a credible anti-inflation stance. Thus, this line of explanation does not provide a convincing argument for why a government would, other things equal, prefer longer maturity –in the sense of this paper– to shorter maturity debt.

A second approach has been explored by Fischer [1983], Bohn [1988], Calvo and Guidotti [1990a], Calvo and Guidotti [1990b]. Quoting Fischer [1983]: "The best of all possible worlds, if governments acted optimally, might be one in which the governments had the option of imposing a capital levy (by inflating) in emergencies like wars". That approach suggests that in the absence of explicitly contingent debt, there will be an optimal elasticity of debt to unexpected inflation, thus a prefered effective maturity, achieved through a combination of the maturity of nominal debt, and a mix of nominal, indexed and foreign currency denominated debt. This prefered maturity is likely to also vary with the level of debt. If it decreases more slowly with the level of debt than does the maximum maturity above, the maximum maturity will be binding only at high levels of debt. We find this line of reasoning attractive, and do not see conceptual difficulties in integrating contingent contract and reputation aspects. We have however been unable at this stage to construct a model which achieves such an integration in a tractable way.

Appendix : Data sources and Data construction.

For all three countries, data for GDP was obtained from *National Accounts*, OECD, and data for CPI inflation was obtained from *Main Economic Indicators*. Data on debt were constructed as follows.

.1 Italy

Data on market holdings of debt for the period 1983-1989 were obtained from *Bollettino Statistico, Banca d'Italia, Servizio Studi*, various issues. For the period 1960-1983, the source was Morcaldo and Salvemini [1984]. Data on the maturity composition of debt were kindly provided by the Banca d'Italia, Servizio Studi.

"Debt" refers to Central Government debt and does not include guaranteed debt. It includes only marketable debt, thus excluding Post Office deposits. Even though those deposits usually have a specified maturity, they are redeemable on demand, at a penalty rate, thus making difficult the computation of average maturity.

The effective maturity was computed as follows:

(1) Foreign currency denominated debt, inclusive of ECU denominated bonds and bills (Certificati de Tesoro in Euroscudi and Buoni del Tesoro in Euroscudi respectively), and price level indexed bonds (Certificati del Tesoro Reali; one issue in 1983), was assigned zero maturity.

(2) Financially indexed debt, namely floating rate bonds (Certificati del Tesoro a Tasso Variabile) was assigned the maturity corresponding to the time remaining before the adjustment of their coupons. Floating rate bonds bear annual or semi-annual coupons. To take into account the imperfection in the indexation mechanism, the maturity computed above was augmented by the lag –2.25 months– between the determination of the reference rate and the beginning of the entitlement.

(3) The earliest redemption date was used to compute effective maturity for bonds

with put options (Certificati del Tesoro con Opzione).

.2 Ireland

Data on public debt and its maturity were obtained from *Finance Accounts* and from *Statistical Yearbook*, Department of finance, Stationery Office, Dublin, various issues. Data on Central Bank's holdings were obtained from *Central Bank of Ireland Quarterly Bulletin*, various issues.

"Debt" refers to Central Government debt. The conventional definition of National Debt suffers from double counting, in that it includes liabilities of the Exchequer to itself. Thus, deductions were made to eliminate such double counting. Market holdings of debt were obtained by deducting Central Bank and Government Holdings from the corrected National debt series.

The effective maturity was computed as follows. In general, bonds for which the maturity date within the year was not known were given a maturity date of July 1 for that year. In addition:

(1) The latest redemption date was used to compute effective maturity for bonds with call options

(2) Foreign currency denominated debt was assigned zero maturity.

.3 Belgium

Data on public debt and its maturity were obtained from *Situation Generale du Tresor Public*, Chambre des Representants, various years, and from *Annuaire Statistique de la Belgique*, Institut National de Statistique, Ministere des Affaires Economiques, various years. Data on Central bank holdings were obtained from *Bulletin de la Banque Nationale de Belgique*. Market holdings were obtained by deducting government debt held by the Central Bank or by the "Fonds des Rentes", the institution performing open market operations.

Effective maturity was constructed as follows. In general, bonds for which the maturity date within the year was not known (Emprunt Special, Emprunts Prives,...) were given a maturity date of July 1 for that year. In addition:

(1) Foreign currency denominated debt was assigned zero maturity.

(2) The earliest redemption date was used for bonds with a put option. The latest redemption date was used for bonds with a call option.

(3) Variable interest rate certificates were excluded from the computation of effective maturity, as we could not find what instrument was used for indexation purposes. (Those bonds represented less than 3% of market holdings in 1985, less than 2% in 1989).

(4) The share of long term debt was defined as the share of fixed rate securities denominated in domestic currency with maturity at the time of issue of 4 years or longer.

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