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HOW LONG DO UNILATERAL TARGET ZONES LAST?

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ABSTRACT

We examine the expected survival time of a unilateral exchange rate target zone, when constraints on monetary policy prevent the central bank from exclusively focusing on defending the target zone. Generally, the width of the target zone has a negligible effect on the expected survival time, and the dominant determinants are reserve levels and the degree of real and monetary divergence between the country in question and the rest of the world. For seemingly realistic parameters, the expected survival time is fairly long: a few decades rather than a few years.

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1. Introduction

In a unilateral exchange rate target zone the central bank defends the target zone alone, without the cooperation of other central banks. The central bank sells reserves when its currency is weak and buys reserves when it is strong. Several countries, for instance Finland, Norway and Sweden, have such unilateral exchange rate target zones.

Any given exchange rate target zone normally lasts a limited time before it is either realigned by a change in the currency's central parity or by a change in the bandwidth, or completely abandoned in favor or some other exchange rate regime, for instance a managed or free float. We will refer to this time to either abandonment or realignment as survival time, and this paper will examine the determinants of the survival time. Many factors clearly determine this survival time; this paper will focus specifically on the role of central bank reserves.

Central bank reserves need not necessarily be a determinant of the survival time. A country with a unilateral target zone, without the benefit of cooperation from other countries and without capital controls, can nevertheless defend its target zone indefinitely if only monetary policy is exclusively focused on that goal. If the country's central bank is loosing reserves, domestic credit can in principle always be reduced so as to stop the capital outflow and even get the lost reserves back. Equivalently, there is always a sufficiently high domestic interest rate that results in a sufficiently high interest rate differential relative to the rest of the world so that investors are compensated for any devaluation risk and become indifferent between retaining their domestic-currency denominated assets and selling them for foreign-currency denominated assets.

However, there may be constraints on monetary policy that preclude an exclusive focus on defending the target zone. Too high interest rates may be politically infeasible, or domestic credit may be forced to expand steadily in order to finance a budget deficit. There may exist a floor level for the money supply below which the assets of the central bank cannot be reduced, etc. In such circumstances the means to prevent reserve loss are constrained, and the survival time of the exchange rate target zone indeed depends on the current level of reserves. In this paper we assume that implicit constraints on monetary policy result in a critical low level of central bank reserves at which the target zone is either abandoned or realigned. In some cases, there may also exists a critical *high* level of reserves, at which the central bank prefers to abandon or realign the target zone.

This paper is hence concerned with the depletion/accumulation of reserves which accompanies intervention while the target zone is in existence. We study the cumulative flow of interventions until the critical stocks of reserve are reached. We examine the determinants of the survival time of the target zone, especially the bandwidth and the drift and variance of underlying fundamentals.[1]

The paper is organized in 7 sections. Section 2 lays out the framework, much of which is familiar from earlier work. Section 3 contains the survival time of the target zone, and shows that it can be interpreted as a first-passage time of a Brownian motion. Sections 4, 5 and 6 present the solution to the expected survival time in three separate cases. Section 7 summarizes and concludes.

2. The target zone model

The target zone is defended by non-sterilized interventions that affect money supply. The (absolute) money supply, M, is the sum of domestic credit, $D > 0$, and foreign exchange reserves, R: $M = D + R > 0$. We assume that the constraints on monetary policy that we referred to in Section 1 can be represented by a minimum level of the money supply, $\bar{M} > 0$, so that the money supply never falls short of this level, $M \geq \bar{M}$. Let us for simplicity assume that all interventions are done with foreign exchange reserves and that domestic credit is held constant at the minimum level of money supply, $D = \bar{M}$.

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1 The properties of the target zone need to be further specified, of course, as will be seen below. The properties of the target zone include the precise intervention rule followed by the central bank. The properties of the target zone also include the precise scenario of what happens when the target zone is abandoned or realigned, since this will determine the critical levels of reserves.

Then foreign exchange reserves must be nonnegative to ensure that money supply does not fall short of the given minimum money supply, $R \geq 0$. We choose to measure (log) reserves as

$$r \equiv \ln(M) - \ln(\bar{M}) = \ln(R+\bar{M}) - \ln(\bar{M}). \tag{2.1}$$

These reserves must hence be nonnegative, $r \geq 0$.[2]

We employ the standard target-zone framework in which the logarithm of the exchange rate (measured in units of domestic currency per unit foreign currency) is a function of a fundamental, f. We will actually not need to model the exchange rate determination explicitly, though, and it will be enough to express the model in terms of the fundamental.[3]

The fundamental is defined as the sum of a composite exogenous disturbance term, v, called velocity, and the home country's reserves, r,

$$f \equiv v + r. \tag{2.2}$$

Velocity may, with some abuse of language, include a number of exogenous variables in addition to "true" velocity, for instance the foreign money supply or price level, foreign interest rates, the real exchange rate, domestic and foreign output, etc. Velocity is assumed to follow a Brownian motion with drift μ and variance σ^2,

$$dv \equiv \mu dt + \sigma dz, \tag{2.3}$$

where z is a Wiener process (that is, it has zero drift and unit variance).

The central bank intervenes by adjusting reserves so as to keep the fundamental

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[2] The crucial assumption is that money supply must not fall short of a given minimum level. It is not assumed that the central bank cannot borrow foreign exchange reserves from other central banks. If the central bank can do that, net foreign exchange reserves can be negative and reduce money supply below the level of domestic credit. In that case we can simply measure reserves as the excess of net reserves over the level of net reserves that result in the minimum permissible level of money supply (still holding domestic credit constant). The central bank may also, of course, adjust domestic credit by open market operations. In that case we can simply conduct the analysis in terms of changes in money supply rather than foreign exchange reserves, and still measure (log) reserves as the difference between the log of money supply and the log of the minimum level of money supply.

[3] See Krugman (1991) for the details of the target zone model.

within a specified band with lower and upper boundaries a and b,

$$a \leq f \leq b. \tag{2.4}$$

This way the central bank can keep the exchange rate in a specified band. The interventions are assumed to be marginal, infinitesimal, and reflecting. That is, they occur only at the edges of the fundamental (and exchange rate) band, they do not induce finite jumps in the fundamental, and they result in the fundamental being a reflected Brownian motion. Then reserves r are a so-called finite variation process which has the property that it stays constant when the fundamental is in the interior of its band, increases only when the fundamental is at the lower edge of the band ($f = a$), and decreases only when the fundamental is at the upper edge of the band ($f = b$).[4]

The prospect of the foreign exchange reserve level falling down to zero may trigger a buying attack,[5] the size P of which depends on the precise scenario (the escape clause) for abandonment or realignment of the target zone, as has been shown by Delgado and Dumas (1990).[6] If an attack occurs, foreign exchange reserves drop suddenly from R to $R - P$, and (log) reserves drop from r to $p \equiv \ln(R - P + D) - \ln(D)$. To prevent an attack R must be greater than P, which means that r must be greater than p, the log size of the speculative attack.

We also introduce the possibility that the central bank would not tolerate having more reserves on hand than some level $\bar{R}$. For this reason, we consider a selling attack of absolute size Q. To prevent a selling attack, R must remain below $\bar{R} - Q$, and r must remain below a level $q \equiv \ln(\bar{R} - Q + D) - \ln(D)$.[7] The collapse scenario only determines

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[4] We neglect the interest earned on the assets of the central bank, because including it would force us to take into account the bank's asset mix.

[5] I.e., an attack in which investors buy foreign exchange reserves from the central bank.

[6] To find out how P and Q are determined, see Salant and Henderson (1978), Krugman (1979), Krugman and Rotemberg (1990), and Delgado and Dumas (1990).

[7] We still do not know, since Krugman and Rotemberg (1990), exactly by what mechanism a narrow target zone would collapse directly to free float. See also Delgado and Dumas (1990) and Buiter and Grilli (1989). The definition of collapse we employ here does not accomodate the paradoxical situation discussed in these papers. Hence we are

the critical levels p and q, it does not affect the day-to-day dynamics of reserves when the reserves are away from the critical levels.

3. Collapse and survival time

We shall refer to the abandonment or realignment of the target zone simply as the "collapse" of the target zone. As discussed in Section 2, we specify that there are critical low and high reserve levels, p and q, respectively, that trigger the collapse of the target zone.

While the target zone is operating we must hence have

$$p \leq r_t \leq q \text{ and } a \leq f_t \leq b. \tag{3.1}$$

A collapse then occurs if and only if *either* $r_t = p$ and $f_t = b$, *or* $r_t = q$ and $f_t = a$. That is, a collapse occurs when either the reserves have reached a critical low level and the central bank is called upon to defend the currency by selling additional reserves (which only occurs at the upper edge of the fundamental band) or the reserves have reached a critical high level and the central bank is called upon to buy additional reserves (which only occurs at the lower edge of the fundamental band).

Let τ denote the date of collapse of the target zone, that is, the first date at which either of the above conditions are fulfilled:

$$\tau \equiv \min\{t \mid r_t = p \text{ and } f_t = b, \text{ or } r_t = q \text{ and } f_t = a\}. \tag{3.2}$$

We denote by $T \equiv \tau - t$, where t is the current date, the survival time of the target zone.

We want to compute the probability distribution and the expected value of the survival time, conditional upon initial levels of f and r. In order to do this, we make the crucial observation that *a collapse occurs if and only if velocity v_t ($\equiv f_t - r_t$) equals either $a - q$ or $b - p$.*

In order to understand this observation, we first note that the "only if" part is trivial:

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implicitly assuming a collapse scenario or a band specification such that the Krugman paradox does not arise.

$f = b$ and $r = p$ implies $f - r = b - p$. Similarly, $f = a$ and $r = q$ implies $f - r = a - q$. The "if" part is almost as simple and is easy to see from *Figure 1*. Inequalities (3.1) imply that the process (r,f) is confined to a rectangle, as shown in the figure. If $f - r = b - p$, then (r,f) is at point A, at which $f = b$ and $r = p$. If $f - r = a - q$, then (r,f) is at point B, at which $f = a$ and $r = q$.

It follows that while the target zone is operating, $v_t \equiv f_t - r_t$ must fulfill

$$a - q \leq v_t \leq b - p. \tag{3.3}$$

Furthermore, the time at which collapse occurs is the first time at which v_t hits either $b - p$ or $a - q$. As far as the time to collapse is concerned, the band (2.4) on the fundamental is replaced by the band (3.3) on velocity.

We have already assumed above in (2.3) that velocity v_t is an unregulated Brownian motion. Hence the survival time is simply a first-passage time of a Brownian motion, the problem of which has a well-known solution.[8] Formulas for the probability distribution and expected value of first-passage times are readily available in most textbooks on stochastic processes.

In the next section we examine the implications of this finding in a special case. We return to the general case in the Section 5.

4. No critical high level of reserves

As a special case, suppose now that only the lower critical level for r, p, matters and that the target zone remains in place even if reserves become very large. The central bank still replenishes its reserves at $f = a$ but collapse occurs only on the occasion of $r = p$ and $f = b$. In that case, while the target zone is operating it must be the case that

$$v_t \leq b - p, \tag{4.1}$$

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[8] We do not give an alternative first–passage time interpretation in terms of two shadow exchange rates reaching the upper or lower edge of the exchange rate band, as has been done for fixed–exchange rate systems [see Flood and Garber (1983) and Buiter (1989)]. Such an interpretation is certainly possible, but it would depend on the assumed collapse scenario.

and collapse occurs when v_t equals $b - p$ for the first time.

The mean of the first-passage time of a Brownian motion may be found in, for instance, Cox and Miller (1965, page 221). Using this, we can directly write down the mean of the survival time, conditional upon the initial value of $v \equiv f - r$, or as we shall prefer, conditional upon initial values of r and f. The mean is well-defined only if the drift μ is strictly positive positive. It is

(4.2) $$E[T|r, f] = [(b-p) + (f-r)]/\mu \equiv [(r-p) + (b-f)]/\mu.$$

The expected survival time can be neatly decomposed into $(b - f)/\mu$, which is the expected time to reach the boundary of the fundamental band, plus $(r - p)/\mu$, which is the expected survival time "bought" by the existing reserves. The variance plays no role in this calculation.

The expected survival time bought by reserves is strictly proportional to the excess of reserves over the critical low level, $r - p$, the proportionality factor being $1/\mu$. The width of the fundamental band matters only via the expected time to reach the upper boundary of the band; it does not influence the expected survival time bought by reserves. Except when collapse is imminent, the effect of the bandwidth will be dominated by the effect of the reserves, since the bandwidth is relatively small.

The expected time to collapse is inversely proportional to the parameter μ, the drift of the "velocity". We may interpret the drift parameter as an indicator of the combined monetary and real divergence between the domestic country and the rest of the world.[9] We can then conclude that this divergence is a much more important determinant of the expected survival time than the bandwidth. We can also conclude that the expected survival time can be rather long also for large divergences. A 100 (log) percent excess of (log) reserves over the (log) critical level does not seem unrealistic (100 log percent

[9] In the monetary model, for instance, velocity can be written $v = k - p^* + q - y + \alpha i^*$ or $v = -m^* + k - k^* + q + y^* - y$, where k the "true" velocity, p^* is the foreign price level, q is the real exchange rate, y is the domestic output, i^* is the foreign interest rate, m^* is the foreign money supply, k^* is the true foreign velocity, and y^* is the foreign output, and where all variables except the foreign interest rate are logs.

corresponds to multiplication by the factor $e = 2.718..$), and a 10 percent per year divergence seems rather large. Nevertheless this results in an expected survival time of 10 years, disregarding the effect of the bandwidth. The effect of typical bandwidths is in this case small: A Swedish band of ±1.5 percent would only buy at most an additional 1.4 year in expected survival time (if the fundamental and exchange rate initially are at the strong edge of the band). To arrive at this number we use the result that an exchange rate band of size ±1.5 percent corresponds to a fundamental band of close to 14 percents width (with a variance σ^2 for the fundamental equal 1 percent per year, and the "α"-coefficient, the elasticity of the exchange rate with respect to expected exchange rate depreciation, equal to 3).[10]

The probability density function for the first-passage time of a Brownian motion is also available in, for instance, Cox and Miller (1965, page 221). Using this function, we can directly write the probability density function of the survival time T, conditional upon initial levels of r and f, as

$$(4.3)\quad \varphi(T|r,f) = \{[(b-p)-(f-r)]/\sigma(2\pi t^3)^{1/2}\}\exp\{-[(b-p)-(f-r)-\mu t]^2/[2\sigma^2 t]\},$$

for $\mu > 0$. The probability distribution is plotted in *Figure 2*, for the parameters $a =$ -8.99 percent, $b =$ 4.91 percent, $p = 0$, $r =$ 100 percent, $f = (a+b)/2 =$ -2.04 percent, $\mu =$ 10 percent per year and $\sigma^2 =$ 1 percent per year (to give the Swedish target zone, ±1.5 percent). With these parameters in (4.2), the mean is 10.7 years. We see that the distribution is somewhat skewed to the right. The probability of a collapse within the first 5 years is very small.

5. Both low and high critical reserve levels: Zero drift

Let us now return to the case with both a low and a high critical level of reserves. Let us first consider the case when the drift μ is zero. The probability density and mean of

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[10] See Svensson (1991) for similar calculations. That the fundamental band is wider than the exchange rate band is Krugman's (1991) "honeymoon" effect. A smaller α would result in a smaller fundamental band for a given exchange rate band.

the first-passage time to either of two barriers can be found in Karatsas and Shreve (1988, page 99). From this the expected survival time, conditional upon initial levels of r and f, can directly be written as

$$\mathrm{E}[T|\,r,\,f] = [(f-r) - (a-q)]\cdot[(b-p) - (f-r)]/\sigma^2. \tag{5.1}$$

The symmetric hump-shaped curve in *Figure 3* is a graph of the expected survival time for the case $\mu = 0$. It is apparent from (5.1) that this quantity does not depend on r and f separately but simply on their difference $f - r$ ($\equiv v$). Hence we have used a representation with $v \equiv f - r$ on the horizontal axis. The parameters are: $\sigma^2 = 1$ percent/year, $a = -b = -6.74$ percent (which with $\mu = 0$ results in the Swedish exchange rate band, ±1.5 percent), $p = 0$ and $q = 200$ percent. The segments which are drawn more thickly on the $\mu = 0$ curve represent the variation of the expected survival time as a function of f for a given level of r ; the leftmost segment corresponds to $r = q$, the rightmost to $r = p$ and the center segment to a level in-between: $r = (p+q)/2 = 100$ percent. As reserves are gained or lost the segment moves to the left or to the right on the thinly drawn curve.

The parameters in this numerical example are somewhat arbitrary but far from unreasonable. The variance of the velocity is of course important. A variance of 1 percent per year corresponds to a standard deviation of 10 percent for yearly changes,[11] which seems rather large. For this variance, and a "Swedish" target zone of ±1.5%, it is striking to find that a reserve level equal to about 2.7 times the speculative attack size ($r - p = 100$ log percent, which correspond to a multiplicative factor of $e = 2.718$) suffices to produce an expected survival time equal to 120 years! A smaller variance implies an even longer expected survival time.

As is illustrated by Figure 3, reserves do not necessarily postpone the collapse time, because excessive reserves also trigger a collapse when $r = q$.

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[11] Note that the instantaneous standard deviation has the dimension $1/\sqrt{\text{year}}$, since the instantaneous variance has the dimension 1/year.

Except when reserves are halfway between the low and high critical levels and the curve in Figure 3 is flat, the bandwidth has now a rather large effect. When the reserves have reached the minimum level ($r = p$), it still takes an expected time of almost 30 years to collapse if the fundamental and exchange rate start at the strong edge of the band.

6. Both low and high critical reserve levels: Non-zero drift

For the case in which $\mu \neq 0$, the expected survival time is

$$E[T| r, f] = \{-(f-r) + (a-q) + [(b-p) - (a-q)] \frac{1 - \exp\{-\theta[(f-r)-(a-q)]\}}{1 - \exp\{-\theta[(b-p)-(a-q)]\}}\} / \mu, \quad (6.1)$$

where $\theta = 2\mu/\sigma^2$. Here again, the expected survival time depends not on r and f separately but on their difference $v \equiv f - r$. *Figure 3* also shows a graph of $E[T| r, f]$ as a function of f - r for the case when $\mu = 10$ percent per year. The parameters a and b are now again -8.99 percent and 4.91 percent, respectively (to result in the Swedish target zone with $\mu = 10$ percent per year), $p = 0$ and $q = 200$ percent.

Figure 3 now shows that with strong positive drift the effect of the bandwidth is rather small. The effect of the bandwidth is large only when collapse is imminent with reserves being at their high critical level. For other levels of reserves, the expected survival time is a linear function of $f - r$, and the variability of f inside the fundamental band has little effect on the expected survival time, as shown by the thick portions of the thin curve for $\mu = 10$ percent per year. Indeed, for reserve levels sufficiently far below the critical high level, that is, on the linear portion of the curve, the expected survival time is effectively given by (4.2), as in the case with no critical high level of reserves.

Equation (6.1) shows the effect of real and monetary divergence as measured by μ. Here again such divergence shortens the life of the target zone. The extent of this shortening is indicated by a comparison between the two curves of Figure 3, one drawn for $\mu = 0$ and one drawn for $\mu = 10$ percent per year. While keeping in mind that the choice of parameters can be questioned, one cannot help being struck by the result that a 100 (log) percent buffer stock of reserves brings about an expected survival time of the target zone of 10 to 20 years, even though 10 percent per year is a very high level of divergence. A unilateral target zones seems to be a robust construct.

A more systematic quantification of the effect of real and monetary divergence is afforded by *Figure 4*, which plots the expected time to collapse, computed for $r = (p+q)/2$ (= 100 percent) at the center of the fundamental band, $f = (a+b)/2$ (= -2.04 percent).

7. Summary and conclusion

We have considered a situation in which implicit constraints prevent a central bank from focusing monetary policy exclusively on defending a specified exchange rate target zone. As a result, the central bank's reserve holdings play a role in determining the survival time of the target zone, the time to either a realignment or an abandonment of the target zone for another exchange rate regime.

We have examined how the expected survival time in a standard target zone model with marginal interventions is determined by reserve levels, bandwidths, and drift and variance of underlying fundamentals. By interpreting the survival time as the first-passage time of a Brownian motion we can rely on standard results of theory of stochastic processes.

We have found that the bandwidth has a very small effect on the expected survival time, except in the case when the drift of underlying fundamentals is small. In the case when very large reserves is not a reason for collapse of the target zone, the effect of the bandwidth on the survival time is simply the expected time for the exchange rate to reach the weak edge of the exchange rate band.

The drift of underlying fundamentals has the most important effect on expected survival time. The drift may be interpreted as an indicator of real and monetary divergence between the country in question and the rest of the world. Increasing the bandwidth has a very small effect on survival time when the real and monetary divergence is large. With regard to survival time, the bandwidth might as well be set at zero.

We have also found that with seemingly realistic parameters, the expected survival time of a unilateral target zone is long, perhaps surprisingly long: it is counted in decades rather than in years.

We have assumed throughout the paper that interventions by the central bank are marginal, that is, they occur only at the edges of the band. Interventions inside exchange rate bands, intra-marginal interventions, are quite common in the real world, and recently Lindberg and Söderlind (1991) have demonstrated that a target zone model with mean-reverting intra-marginal interventions fit the Swedish data very well. Intra-marginal interventions introduce special problems in the analysis of a target zone's survival time. Among these are that collapse can occur inside the band and not only at the edges of the band, and that the size of a speculative attack will depend on the exchange rate's position in the band. An analysis of these problems should be suitable for future research.

References

Buiter, W.H., "A Viable Gold Standard Requires Flexible Monetary and Fiscal Policy," *Review of Economic Studies* 56 (1989), 101-118.

Buiter, W.H., and V. Grilli, "The 'Gold Standard Paradox' and Its Resolution," Working Paper, Yale University, (1989).

Cox, D.R., and H.D. Miller, *The Theory of Stochastic Processes*, London: Chapman and Hall, (1965).

Delgado, F., and B. Dumas, "Monetary Contracting between Central Banks and the Design of Sustainable Exchange Rate Zones," NBER Working Paper No. 3440, (1990).

Flood, R.P., and P.M. Garber, "A Model of Stochastic Process Switching," *Econometrica* 51 (1983), 537-551.

Karatzas, I., and S.E. Shreve, *Brownian Motion and Stochastic Calculus*, New York: Springer Verlag, (1988).

Krugman, P., "A Model of Balance-of-Payments Crisis," *Journal of Money Credit and Banking* 11 (1979), 311-325.

Krugman, P., "Target Zones and Exchange Rate Dynamics," *Quarterly Journal of Economics* 106 (1991), 669-682.

Krugman, P., and J. Rotemberg, "Target Zones With Limited Reserves," Working Paper, MIT, (1990).

Lindberg, H., and P. Söderlind, "Target Zone Models and the Intervention Policy: The Swedish Case," Working Paper, IIES, (1991).

Salant, S.W., and D.W. Henderson, "Market Anticipation of Government Policy and the Price of Gold," *Journal of Political Economy* 86 (1978), 280-304.

Svensson, L.E.O., "Target Zones and Interest Rate Variability," *Journal of International Economics* 31 (1991), 27-54.

Figure 1

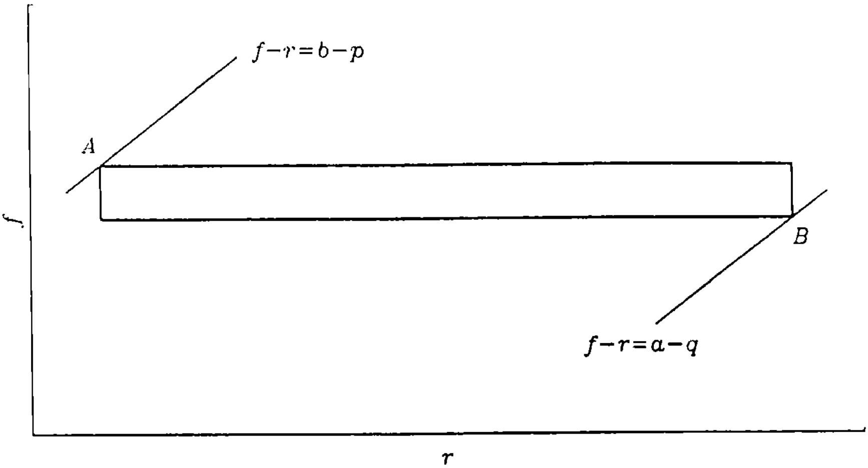

Figure 2. $\varphi(T|r=100\%,\ f=(a+b)/2)$

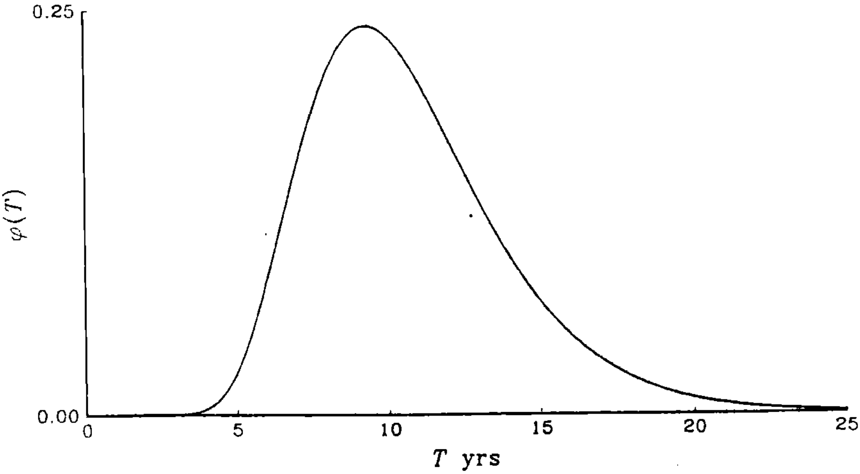

Figure 3. E[T|r, f]

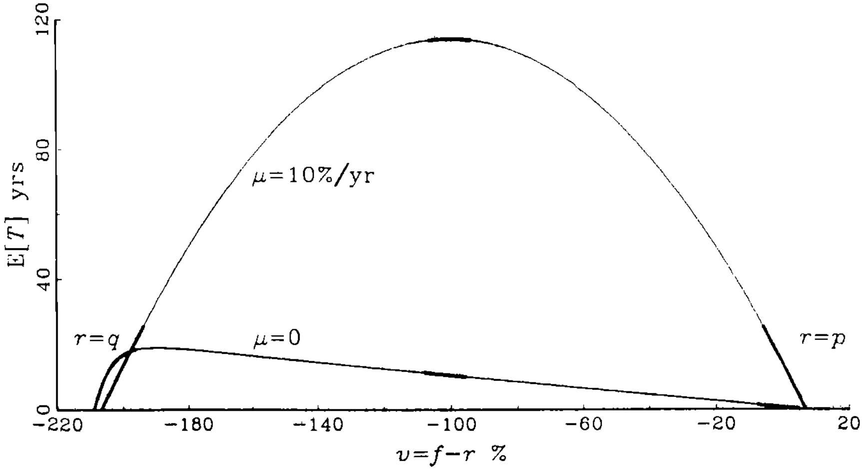

Figure 4. E[T|r=(p+q)/2, f=(a+b)/2]