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Stephen J. Turnovsky

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ABSTRACT

This paper investigates the impact of change in the terms of trade on the economic performance of a small economy. Both the effects of unanticipated shocks and changes in the mean and variance of the probability distribution generating these disturbances are discussed. In all cases, the key element determining the response of the economy is the effect on the rate of growth of real wealth, to which all other real quantities are directly tied in equilibrium. Conditions for the Harberger-Laursen-Metzler effect to hold are discussed. The impact of these changes on economic welfare, as measured by expected discounted utility of the representative agent is also addressed.

Stephen J. Turnovsky
Department of Economics
University of Washington
301 Savery Hall
Seattle, WA 98103
and NBER

1. INTRODUCTION

The effects of terms of trade shocks on the economy have been extensively discussed since the 1950s when Harberger (1950) and Laursen and Metzler (1950) identified what is now known as the Harberger–Laursen–Metzler (HLM) effect. According to this proposition, a deterioration in the terms of trade will raise real expenditure out of a given income level, thereby reducing savings, and given the level of investment, cause a deterioration in the current account.

Recently, several authors have revisited this issue, approaching it using the framework of the representative agent, intertemporal optimizing model. Work in this direction originated with Obstfeld (1982), who showed how, if the country's rate of time preference is increasing in utility, a deterioration in its terms of trade will lead it to increase its rate of savings and run a current account surplus, thereby contradicting the HLM proposition. Subsequent authors show how the validity of this proposition depends upon other aspects of the formulation of the model. Using a two period framework, Svensson and Razin (1983) demonstrate that *a priori*, the effect of a rise in the price of importables is ambiguous. Persson and Svensson (1985) consider an overlapping generations model. They show how the effect depends critically upon whether the shock is permanent or temporary, on the one hand, or anticipated or unanticipated, on the other. Bean (1986) emphasizes the role of the labor–leisure choice, as do Sen and Turnovsky (1989). The latter authors also consider the role of investment, as does Matsuyama (1988), although these two studies focus on rather different aspects. Whereas Sen and Turnovsky emphasize the relative importance of the income and substitution effects generated by a terms of trade shock, the Heckscher–Ohlin framework adopted by Matsuyama highlights the role of sectoral capital intensities in the adjustment process.

All of this literature is based on deterministic models. A recent paper by Stulz (1988) analyzes the effects of terms of trade shocks in a situation where they evolve stochastically over time. He considers the relationship between unanticipated changes in the terms of trade, on the one hand, and consumption expenditures, together with the current account balance, on the other. His main result is to show how the effect of an unanticipated change in the terms of trade on consumption and the current account depends upon the differential between the expected real rates of return on foreign and domestic bonds. In the absence of such a differential, the agent will choose a portfolio of assets such that an unanticipated change in the terms of trade has no effect through consumption on the current account.

The present paper employs the model developed by Stulz, though with a somewhat different emphasis, and focusing on a broader range of issues. First, while he was concerned primarily with the effect of the consumer's ability to hedge against terms of trade shocks, on the current account, our focus is oriented towards more general issues pertaining to the impact of terms of trade shocks on real activity and growth.

Secondly, the impact of changes in the terms of trade on real behavior is sensitive to the units in which the real activity is being measured. Previous discussions of the HLM conditions have varied in this regard. The original discussion measured everything in terms of the exportable good.[1] More recent authors, including Stulz, consider real activity in terms of a general price index which reflects the overall cost of living.[2] Our analysis emphasizes how in certain respects, the real effects depend upon the units in which activity is being measured. Thirdly, the measure of the real current account chosen by Stulz is the change in the real quantity of traded bonds held by the domestic economy. This is inclusive of the capital gains or losses arising from the change in the price of the asset. The usual measure of the current account deficit abstracts from this and the behavior of this more conventional (although not necessarily superior) measure is radically different. It is much closer to that of the trade balance, which was in fact the focus of the original Harberger paper. The qualitative responses of these latter measures are also much less sensitive to the units in which real activity is being measured.

Finally, in addition to considering unanticipated shocks, we consider a wider set of relationships pertaining to terms of trade disturbances. Specifically, we analyze the effects of both expected changes in the terms of trade, as well as the variances in these changes, on the expected rates of change of consumption, savings, growth, and the current account, all of which are very closely related in equilibrium. We also examine the relationship between the variance in the terms of trade shock and the variance along the equilibrium growth path. Paradoxically, we shall show how an increased variance in the terms of trade may quite plausibly have a stabilizing effect on the growth path (measured in terms of domestic output), in the sense of reducing its variance.

The analytical framework we employ is based on a continuous–time, representative agent, stochastic optimizing model, in which the stochastic terms of trade is represented by a Brownian motion process. As Stulz (1988) and others have shown in other contexts, this is a very convenient framework for analyzing stochastic behavior in an intertemporal setting. One particular aspect is that the value function, used to solve the representative agent's intertemporal optimization problem provides a natural measure for assessing the effects of the terms of trade shock on the welfare of the representative agent. Despite the fact that welfare is the fundamental issue and implicitly what discussions of the HLM effect have had in mind, it has frequently been proxied by measures such as expenditure and the current account balance. One important exception to this is Svensson and Razin (1983), who using duality theory are able to obtain precise measures of the welfare effects of terms of trade disturbances.[3] The present approach also enables us to address the welfare issue in a direct, and at least for the present model, in a definitive way, thereby circumventing problems associated with the choice of units defining real quantities.

2. STOCHASTIC EQUILIBRIUM IN A SMALL OPEN ECONOMY

We consider a small open economy which is specialized in the production of a single good. The representative agent in the economy consumes both this good, and a second good which it imports from abroad. The relative price P of the imported good is taken as given and is assumed to be generated by the Brownian motion process

$$\frac{dP}{P} = pdt + du \tag{1}$$

where p is the instantaneous expected rate of change in the relative price and du is a temporally independent, normally distributed, random variable with mean zero and variance $\sigma_p^2 dt$. The terms of trade are therefore $\frac{1}{P}$, with an increase in P representing a deterioration in the terms of trade.

The representative individual holds two securities in his portfolio. These include traded bonds and claims (which may also be traded) on physical capital. There is no money, so the model is real. Domestic output Y is produced using domestic capital K, by means of the simple linear production function

$$dY = \alpha K dt \tag{2}$$

Accordingly, assuming that domestic capital can be adjusted instantaneously in the economy, the real rate of return on domestic capital, expressed in terms of domestic output, is

$$dR_S = \frac{dY}{K} = \alpha dt \tag{3a}$$

Traded bonds are assumed to be short bonds, paying a rate of interest i^*. The real rate of return on foreign bonds over the period dt, expressed in terms of the domestic good, is therefore

$$dR_F = (i^* + p)dt + du. \tag{3b}$$

Over the instant of time dt, the representative agent consumes output of the two commodities at the nonstochastic rates $C_1(t)dt, C_2(t)dt$, respectively. His objective is to select these rates of consumption, together with his portfolio of assets, to maximize the expected value of discounted utility

$$E_0 \int_0^{\infty} \frac{1}{\gamma} \left[C_1^{\theta} C_2^{1-\theta}\right]^{\gamma} e^{-\rho t} dt \qquad -\infty < \gamma < 1 \qquad 0 < \theta < 1 \tag{4a}$$

subject to the wealth constraint

$$W = K + PB \tag{4b}$$

and the stochastic accumulation equation

$$dW = W[n_S dR_S + n_F dR_F] - (C_1 + PC_2)dt \tag{4c}$$

where

B = stock of traded bonds, held by domestic residents, denominated in units of foreign output,

K = stock of capital, expressed in units of domestic output,

W = real wealth, expressed in units of domestic output,

$n_S = K/W$ = share of capital in portfolio of domestic agents,

$n_F = PB/W$ = share of traded bonds in portfolio of domestic agents.

Utility is represented by the constant elasticity function, with $r \equiv 1 - \gamma$ measuring the constant coefficient of relative risk aversion. The value $\gamma = 0$ corresponds to the logarithmic function. Dividing (4b) by W and substituting for dR_S, dR_F into (4c), these two equations may be expressed as

$$n_S + n_F = 1 \tag{4b'}$$

$$dW = W[n_S \alpha + n_F(i^* + p)] - (C_1 + PC_2)]dt + W n_F du \tag{4c'}$$

where it should be recalled that P evolves in accordance with (1).

The maximization of (4a). subject to (4b′), (4c′) and (1) is a straightforward problem in stochastic optimization and the details of the solution are outlined in the Appendix. Defining aggregate consumption. expressed in terms of domestic output, by

$$C \equiv C_1 + PC_2$$

the first order conditions are

$$C_1 = \theta C \tag{5a}$$

$$PC_2 = (1-\theta)C \tag{5b}$$

$$\frac{C}{W} = \frac{1}{1-\gamma}\left\{\rho + \gamma[p(1-\theta) - \alpha] - \frac{1}{2}n_F^2\sigma_p^2\gamma(1-\gamma) - \frac{1}{2}\sigma_p^2\gamma(1-\theta)[1+\gamma(1-\theta)]\right\} \tag{5c}$$

$$\alpha = i^* + p + n_F\sigma_p^2(\gamma - 1) - \sigma_p^2\gamma(1-\theta) = \eta \tag{5d}$$

where η is the Lagrange multiplier associated with the wealth constraint (4b′). Equations (5a) and (5b) describe the consumptions of the two goods as fixed fractions of overall consumption expenditure, expressed in terms of domestic output. Equation (5c) is the solution for the aggregate consumption-wealth ratio. As well as depending upon the preference parameters (θ, ρ, γ), and the parameters pertaining to the stochastic shocks (p, σ_p^2), it also depends upon the portfolio share, n_F, the solution to which is obtained from (5d) and given in (6) below. Note that in the logarithmic case, $\gamma = 0$, this reduces to the well known constant ratio, $\frac{C}{W} = \rho$.

Solving (5d) for n_F, we obtain

$$n_F = \frac{i^* + p - \alpha}{\sigma_p^2(1-\gamma)} - \frac{\gamma(1-\theta)}{1-\gamma} \tag{6}$$

which together with (4b′) yields the equilibrium portfolio shares of the two assets. This expression highlights the two factors determining the optimal n_F. The first is the speculative component, which depends upon the differential expected real rate of return $(i^* + p - \alpha)$ (expressed in terms of the domestic good), while the second reflects hedging behavior on the part of the investor. Note that the domestic economy may be either a net creditor $(n_F > 0)$ or a net debtor $(n_F < 0)$ and it is also possible for $n_S < 0$. However, for expository convenience, we rule out this last case, assuming that domestic residents always hold some positive fraction of domestic capital in their portfolios.

Substituting (4b′), (5a), and (5b) into the accumulation equation (4c′), we see that the equilibrium rate of wealth accumulation is

$$\frac{dW}{W} = \left[\alpha + (i^* + p - \alpha)n_F - \frac{C}{W}\right] dt + n_F du \tag{7}$$

where the equilibrium ratio $\frac{C}{W}$ and portfolio share n_F are obtained from (5c), (6). It follows further from the constancy of the equilibrium $\frac{C}{W}$ ratio, as well as the constancy of the portfolio shares $\frac{K}{W}, \frac{PB}{W}$, that in equilibrium, the real quantities, C, K, and PB all evolve proportionately to W. Denoting this common real growth rate by $\frac{dX}{X}$, we thus obtain

$$\frac{dX}{X} \equiv \frac{dC}{C} = \frac{dK}{K} = \frac{d(PB)}{PB} = \frac{dW}{W} = \left[\alpha + (i^* + p - \alpha)n_F - \frac{C}{W}\right] dt + n_F du. \tag{8}$$

Accordingly, the common real growth rate is seen to equal the rate of earnings on assets less the consumption–wealth ratio.

To this point, everything has been expressed in terms of units of domestic output. The fact that in equilibrium all real quantities so defined are proportional is a consequence of the constant elasticity utility function, the linear technology, and the fact that the dynamics in real terms can be expressed by a single state variable. It will be observed that as long as $\alpha + (i^* + p - \alpha)n_F - \frac{C}{W} \neq 0$, the equilibrium is one of steady growth or decline. However, this can be shown to occur at a sufficiently slow rate, so as to be sustainable in the sense of being consistent with the intertemporal budget constraint (transversality condition) facing the economy.[4]

As noted previously, several authors have evaluated the real effects of terms of trade shocks in terms of a general price index. It is well known that the constant elasticity utility function in (4a) implies an exact price index, Q say, of the form

$$Q = NP^{1-\theta} \tag{9}$$

where N is a constant. Deflating expenditures by Q yields real quantities expressed in terms of the domestic consumption bundle. Stochastic differentiation of (9) implies[5]

$$\frac{dQ}{Q} = \left[(1-\theta)p - \frac{1}{2}\theta(1-\theta)\sigma_p^2\right] dt + (1-\theta)du. \tag{10}$$

The real rates of return expressed in terms of the domestic consumption bundle are

$$dR'_S = [\alpha - (1-\theta)p + \frac{1}{2}(1-\theta)(2-\theta)\sigma_p^2]dt - (1-\theta)du \equiv r'_S dt - (1-\theta)du \tag{11a}$$

$$dR'_F = [i^* + \theta p - \frac{1}{2}\theta(1-\theta)\sigma_p^2]dt + \theta du \equiv r'_F du + \theta du. \tag{11b}$$

Using these definitions of the expected returns r'_S, r'_F, the equilibrium portfolio shares of the two assets, determined by (6) and (4b′) can be written in the convenient form

$$n_F = \frac{r'_F - r'_S}{\sigma_p^2(1-\gamma)} + (1-\theta) \tag{12a}$$

$$n_S = \frac{r'_S - r'_F}{\sigma_p^2(1-\gamma)} + \theta. \tag{12b}$$

With the expected real rates of return, upon which the speculative components are based, now being expressed in terms of the domestic consumption bundle, the hedging components become $(1-\theta), \theta$, respectively. That is, the fraction of the foreign and domestic asset to be hedged should equal the fraction of the corresponding commodity in the overall consumption bundle.

We now define real consumption, capital, stock of bonds, and wealth in terms of the domestic consumption bundle by:

$$c \equiv \frac{C}{Q} \quad k \equiv \frac{K}{Q} \quad b \equiv \frac{BP}{Q} \quad w \equiv \frac{W}{Q}.$$

By stochastic differentiation of these quantities, it can be easily shown that c, k, b, and w now all follow the identical stochastic process

$$\begin{aligned} \frac{dc}{c} &= \frac{dk}{k} = \frac{db}{b} = \frac{dw}{w} \\ &= \left[\alpha - (1-\theta)p - \frac{C}{W} + n_F[(i^* + p + \alpha) - (1-\theta)\sigma_p^2] + \frac{1}{2}(1-\theta)(2-\theta)\sigma_p^2\right] dt \\ &\quad + [n_F - (1-\theta)]du. \end{aligned} \tag{13}$$

Denoting this common growth rate by $\frac{dx}{x}$, and using the definitions of r'_S and r'_F, this equation may be written as

$$\frac{dx}{x} = \left[r'_S + (r'_F - r'_S)n_F - \frac{C}{W}\right] dt + [n_F - (1-\theta)]du. \tag{13'}$$

Expressed in this way, the parallel between (13′) and (8) becomes immediate. In terms of the chosen unit of measurement, all real variables must grow at the same steady rate.

3. TERMS OF TRADE SHOCKS AND GROWTH

Equations (8) and (13) provide the information pertaining to how changes in the terms of trade impact on the real growth of the economy. A comparison of these two relationships indicates the importance of the choice of units and both shall be discussed in turn. Critical elements in understanding the behavior of the growth rate are the response of the portfolio share n_F and the consumption–wealth ratio $\frac{C}{W}$, and we therefore begin with a consideration of these.

Differentiating (5c) and (6) we can establish:[7]

$$\frac{\partial n_F}{\partial p} = \frac{1}{\sigma_p^2(1-\gamma)} > 0; \qquad \frac{\partial n_F}{\partial \sigma_p^2} = -\frac{(i^* + p - \alpha)}{\sigma_p^4(1-\gamma)}$$

$$\frac{\partial [C/W]}{\partial p} = \frac{\gamma}{1-\gamma}[(1-\theta) - n_F] = \frac{\gamma}{(1-\gamma)^2}\left(\frac{r'_S - r'_F}{\sigma_p^2}\right)$$

$$\frac{\partial [C/W]}{\partial \sigma_p^2} = \frac{\gamma}{2(1-\gamma)^2}\left[\frac{(i^* + p - \alpha)^2}{\sigma_p^4} - (1-\theta)(1-\gamma\theta)\right].$$

These effects are essentially modifications of early results obtained by Levhari and Srinivasan (1969), Sandmo (1970), and others. analyzing the effects of uncertainty and rates of return on savings and consumption. An increase in p will raise the expected return on foreign bonds, leading to an increase in n_F. It also increases the rate at which the relative price of the imported good is expected to increase. Whether an increase in p results in an overall positive or negative net real return to the domestic agent depends, upon whether $n_F \gtrless (1-\theta)$. If, for example, the net effect is positive, it follows from the results of Levhari and Srinivasan that the consumption–wealth ratio will fall if the elasticity $\gamma > 0$, and rise otherwise.[8]

An increase in the variance σ_p^2 will lead the agent to reduce his speculative position in the foreign bond; i.e., to decrease n_F if $i^* + p > \alpha$ and to increase n_F if $i^* + p < \alpha$. As shown by Levhari and Srinivasan the increase in the uncertainty of returns will lead to an increase in the $\frac{C}{W}$ ratio if $\gamma > 0$, and a reduction if $\gamma < 0$. In addition, the higher variance will raise the uncertainty associated with future changes in the terms of trade and this tends to have the opposite effect on the $\frac{C}{W}$ ratio.

A. Growth in Terms of Domestic Output

From equations (7) and (8), the following responses can be derived

$$cov\left(\frac{dX}{X}, du\right) = n_F \sigma_p^2 dt \tag{14a}$$

$$\frac{\partial}{\partial p}\left(\frac{E(dX)}{X}\right) = \left[(i^* + p - \alpha)\frac{\partial n_F}{\partial p} + n_F - \frac{\partial [C/W]}{\partial p}\right] dt = \frac{(2-\gamma)}{1-\gamma} n_F dt \tag{14b}$$

$$\begin{aligned}\frac{\partial}{\partial \sigma_p^2}\left(\frac{E(dX)}{X}\right) &= \left[(i^* + p - \alpha)\frac{\partial n_F}{\partial \sigma_p^2} - \frac{\partial [C/W]}{\partial \sigma_p^2}\right] dt \\ &= -\frac{1}{2(1-\gamma)^2}\left[\frac{(2-\gamma)(i^* + p - \alpha)^2}{\sigma_p^4} - \gamma(1-\theta)(1-\theta\gamma)\right] dt\end{aligned} \tag{14c}$$

$$\frac{\partial}{\partial \sigma_p^2}\left(var\left(\frac{dX}{X}\right)\right) = \left(n_F^2 + 2n_F\sigma_p^2\frac{\partial n_F}{\partial \sigma_p^2}\right) dt = -n_F\left(n_F + \frac{2\gamma(1-\theta)}{1-\gamma}\right) dt. \tag{14d}$$

Equation (14a) indicates that the effect of an unanticipated deterioration in the terms of trade, i.e., a positive shock du on the current rate of growth, depends upon the sign of n_F, i.e., whether the economy is a net creditor or a net debtor. In the former case, when $n_F > 0$, a stochastic deterioration in the terms of trade raises the real rate of return on traded bonds, expressed in terms of domestic output, leading to an increase in the rate of accumulation of wealth, $\frac{dW}{W}$ and therefore in the ratio of savings to wealth. With the equilibrium consumption–wealth ratio and the portfolio shares being constant over time, it immediately follows that the rates of growth of real consumption, capital, and domestic output, and the rate of accumulation of traded bonds, all expressed in terms of domestic output, also increase at the same rate as wealth. The additional savings are therefore distributed across the assets in proportion to their respective portfolio shares. These effects are reversed—in which case the HML effect holds—if and only if the country is a net debtor.

The effect of an increase in the anticipated rate of appreciation of the relative price p on the expected real growth rate is reported in (14b). It is equal to the difference between its impact on the total expected rate of earnings and the consumption–wealth ratio. This net effect depends upon n_F in much the same way, and for much the same reason, as does the effect of an unanticipated shock du.

In general, the effect of an increased variance in the terms of trade on the expected growth rate depends to an important degree upon the elasticity γ. To consider this, it is convenient to begin with the benchmark logarithmic case, $\gamma = 0$, when the higher variance is seen from (14c) to have an unambiguously adverse effect on the expected growth rate. To see why, consider the case where $n_F > 0$, so that the expected return on foreign bonds exceeds that on capital. The increase in σ_p^2 will cause a substitution away from the higher to the lower earning asset, causing a decline in the overall expected real rate of return, and therefore in the expected rate of growth. The case where $n_F < 0$ can be reasoned similarly.[9] If $\gamma < 0$, so that the coefficient of relative aversion $r > 1$, an increase in σ_p^2 continues to have an adverse effect on growth. This is because the reduction in earnings more than offsets any possible rise in $\frac{C}{W}$. However, when $\gamma > 0$, it is possible for an increase in σ_p^2 to be growth enhancing. For example, this will be so if $i^* + p - \alpha = 0$, when the only effect of the higher variance is to reduce the $\frac{C}{W}$ ratio, thereby increasing the growth rate.

Equation (14d) describes the impact of an increased variance in the terms of trade on the variance along the real growth path. This consists of two offsetting effects. On the one hand, for a given portfolio share n_F, an increase in σ_p^2 will increase the variance along the real growth path. But at the same time, the higher σ_p^2 will induce a reduction in the country's position in traded bonds, thereby reducing the impact of a given variance σ_p^2 on the variance of the growth path. Which effect prevails depends upon n_F and γ. For the logarithmic utility function, the portfolio adjustment is the dominant effect, so that on balance, and rather paradoxically, more variability in the terms of trade will actually stabilize the real growth path. As a second example, consider the case where $r'_S = r'_F$, so that $n_F = (1-\theta)$; see (12a). In this case, the higher variance will be stabilizing if and only if $\gamma > -1$ or equivalently $r < 2$.

B. Growth in Terms of Domestic Consumption Bundle

Analogous kinds of propositions can be derived from a consideration of equation (13), which expresses the rate of growth (denoted by $\frac{dx}{x}$) in terms of the consumption bundle. Parallel to (14), we can establish the following:

$$cov\left(\frac{dx}{x}, du\right) = [n_F - (1-\theta)]\sigma_p^2 dt \tag{15a}$$

$$\begin{aligned}\frac{\partial}{\partial p}\left(\frac{E(dx)}{x}\right) &= \left[\frac{\partial r'_S}{\partial p} + (r'_F - r'_S)\frac{\partial n_F}{\partial p} + \left(\frac{\partial r'_F}{\partial p} - \frac{\partial r'_S}{\partial p}\right)n_F - \frac{\partial [C/W]}{\partial p}\right]dt \\ &= \left(\frac{2-\gamma}{1-\gamma}\right)(n_F - (1-\theta))dt\end{aligned} \tag{15b}$$

$$\begin{aligned}\frac{\partial}{\partial \sigma_p^2}\left(\frac{E(dx)}{x}\right) &= \left[\frac{\partial r'_S}{\partial \sigma_p^2} + (r'_F - r'_S)\frac{\partial n_F}{\partial \sigma_p^2} + \left(\frac{\partial r'_F}{\partial \sigma_p^2} - \frac{\partial r'_S}{\partial \sigma_p^2}\right)n_F - \frac{\partial [C/W]}{\partial \sigma_p^2}\right]dt \\ &= -\frac{1}{2(1-\gamma)^2}\left[\frac{(2-\gamma)(i^* + p - \alpha)^2}{\sigma_p^4} - (1-\theta)[(1-\theta) + (1-\gamma)]\right]dt\end{aligned} \tag{15c}$$

$$\frac{\partial}{\partial \sigma_p^2}\left[var\left(\frac{dx}{x}\right)\right] = [n_F - (1-\theta)]\left[n_F - (1-\theta) + 2\sigma_p^2 \frac{\partial n_F}{\partial \sigma_p^2}\right] dt$$

$$= -[n_F - (1-\theta)]\left[n_F + (1-\theta)\left(\frac{1+\gamma}{1-\gamma}\right)\right] dt \tag{15d}$$

From equation (15a) it is seen that the effects of an unanticipated adverse terms of trade disturbance on growth in terms of the consumption bundle depends upon $n_F-(1-\theta)$. While a positive disturbance du will raise the rate of return and the accumulation of wealth, to the extent that $n_F > 0$, the effect on real wealth, now measured in terms of the consumption bundle, is offset by the higher price of the proportion of goods imported, $(1-\theta)$. As in Section 3.A, everything is driven by real wealth (now redefined), and for the same reasons as before. Thus, an adverse terms of trade shock will increase the rates of growth of real consumption expenditure, capital and output, and the real accumulation of foreign bonds, with all real quantities now being measured in terms of cost of the consumption bundle, if and only if $n_F > (1-\theta)$. Alternatively, the HLM proposition will hold, in the sense of an adverse terms of trade shock being associated with a decline in savings and the real accumulation of traded assets, if and only if $n_F < (1-\theta)$; i.e., if and only if the share of traded bonds in the portfolio of domestic investors is less than the share of foreign output in the overall consumption bundle. In the case where these these two shares are equal, the growth of the real domestic economy, so defined, is independent of the foreign price disturbance.[10]

The result in (15a) is identical to that obtained by Stulz, although he emphasizes it in a somewhat different form. He establishes that real consumption covaries positively with the terms of trade $1/P$ (negatively with P) if and only if

$$r'_S > r'_F.$$

Recalling (11a), (11b) and (12a), we can easily show that

$$r'_S - r'_F = [\alpha - i^* - p + (1-\theta)\sigma_p^2]dt = [(1-\theta) - n_F]\sigma_p^2(1-\gamma)dt$$

implying that

$$sgn(r'_S - r'_F) = sgn[(1-\theta) - n_F] = -sgn \; cov\left(\frac{dx}{x}.du\right), \tag{15}$$

which is precisely the Stulz proposition.

Equation (15b) describes the effect of an increase in the expected rate of change of the terms of trade on the expected real growth rate. It is qualitatively the same as for the unanticipated shock, as before. It thus follows that the criterion derived by Stulz for determining the nature of the covariance between current terms of trade shocks and current growth, extends to the relationship between the expected terms of trade shocks and the expected growth rate.

Comparing (15c) with (14c), one can see that an increase in the variance of the terms of trade has a more positive (or less negative) effect on the expected real growth rate, defined in terms of the consumption bundle, than on the expected growth defined in terms of the domestic good. This is because the increase in σ_p^2 reduces the expected growth rate of the cost of living index, thereby providing a stimulus to growth in the economy. It is possible for two growth rates to respond in opposite ways. This possibility can be illustrated quite simply with the logarithmic utility function. In this case, an increase in σ_p^2 will always *lower* the expected growth rate $E(dX)/X$, but it will *raise* the expected growth rate $E(dx)/x$ as long as $2(i^* + p - \alpha)^2 < (1-\theta)(2-\theta)\sigma_p^4$.

Finally, analogous to the previous case, an increase in the variance of the terms of trade may either stabilize or destabilize the variance of the growth path, now measured in terms of the consumption bundle. A constrast with the previous case does arise for the logarithmic utility function. While an increase in σ_p^2 reduces the variance of the growth rate of domestic output, it also incrases the variance of the domestic cost of living. The net effect on the variance of the growth rate of the consumption bundle depends upon which effect dominates.

At this point, it is instructive to reflect briefly on these results from the perspective of the existing deterministic literature. First, whether described in terms of (8) or (13), the equilibrium is one of steady stochastic growth. This is a consequence of the linear technology and the constant elasticity utility function. Consequently, our discussion of the terms of trade shocks pertain to the effects on growth rates. While this is natural here (as in Stulz), it does contrast with much of the literature which focuses on levels. Secondly, the counterpart to the price shock considered in the deterministic literature is P itself. In the present case a nonstochastic increase in P would have no effect on the equilibrium described by (8) or (13). All it would do would be to lead to a corresponding reduction in C_2, leaving the expenditure share $\frac{PC_2}{C}$, and everything else, unchanged. This is perfectly consistent with say the Sen–Turnovsky (1989) model, which being based on the intertemporal optimization of an infinitely lived agent with capital accumulation, is probably the closest deterministic analogue to the present analysis. The reason why a change in P does have an effect in that model is through the endogeneity of labor. As the authors observe, if labor were fixed, as it is here, the capital stock which is the driving force of the dynamics, would no longer respond to a change in P, and the adjustment would degenerate precisely as it would here; see Sen and Turnovsky (1989, p. 246). The fact that there is no exact counterpart in the deterministic literature to the shocks considered here, underscores the significance of addressing this issue within a stochastic framework.

4. THE CURRENT ACCOUNT AND TRADE BALANCE

The two measures pertaining to external activity, considered in Section 3, namely $d(BP)/BP, db/b$, describe the real growth rates of foreign bond holdings expressed in terms of domestic output and the domestic consumption bundle, respectively. While each represents a perfectly valid measure of the external account, neither corresponds to the usual measure of the current account. Nor does it exactly reflect the balance of trade, which was the concern of the original Harberger paper.

The balance of trade expressed in terms of the domestic good is defined by

$$dZ = dY - Cdt - dK. \tag{16}$$

The conventional definition of the current account, expressed in terms of the domestic good. is thus[11]

$$(P + dP)dB = dZ + i^* PBdt. \tag{17}$$

The left hand side of this equation may be written as,

$$BP\left[\frac{d(BP)}{BP} - \frac{dP}{P}\right]$$

which differs from the real growth rate, dX/X, by the netting out of the capital gains component dP/P.

Substituting for $d(BP)/BP$ from (8) and dP/P from (1), we obtain the following expressions for the current account and trade balance

$$(P + dP)dB = BP\left[\left(\alpha - p + (i^* + p - \alpha)n_F - \frac{C}{W}\right)dt - n_S du\right] \tag{18a}$$

$$dZ = -BP\left[\left(\frac{C}{W} + (i^* + p - \alpha)n_S\right)dt + n_S du\right]. \tag{18b}$$

From these two relationships, we obtain

$$cov[(P + dP)dB, du] = cov[dZ, du] = -Kn_F\sigma_p^2 dt. \tag{19}$$

Thus we see that a positive shock in the relative price of the import good (i.e., an unanticipated deterioration in the terms of trade), will lead to a deterioration in both the current account balance, as measured by (17), and the trade balance if and only if the country is a *net creditor* ($n_F > 0$). In this case the HLM effect holds. This is in direct contrast to the effect on the savings rate and the real growth rate of foreign assets which were both shown to respond negatively to an adverse shock in the terms of trade if and only if the country is a *net debtor* ($n_F < 0$). The difference is accounted for by the exclusion of the capital gains dP/P, which contributed positively to the rate of growth as defined in terms of domestic output.[12]

Likewise, the current account expressed in terms of the cost of living units Q is

$$\frac{(P+dP)dB}{Q+dQ} = \frac{dZ}{Q+dQ} + \frac{i^{*}PB}{Q+dQ}dt \tag{20}$$

the left hand side of which can be approximated by

$$\frac{BP}{Q}\left[\frac{db}{b} + \left(\frac{dQ}{Q} - \frac{dP}{P}\right)\left(1 - \frac{dQ}{Q}\right)\right].$$

Substituting for $db/b, dQ/Q$, and dP/P, from (13), (10), and (1), respectively, the real current account and trade balance, now measured in terms of the consumption bundle, are

$$\frac{(P+dP)dB}{Q+dQ} = b\left[\left(\alpha - p - \frac{C}{W} + (i^{*} + p - \alpha)n_F + n_S(1-\theta)\sigma_p^2\right)dt - n_S du\right] \tag{21a}$$

$$\frac{dZ}{Q+dQ} = b\left[\left(-\frac{C}{W} + n_S[(1-\theta)\sigma_p^2 - (i^{*} + p - \alpha)]\right)dt - n_S du\right]. \tag{21b}$$

The covariances between these measures and a stochastic shock in the terms of trade are now

$$cov\left[\frac{(P+dP)dB}{Q+dQ},du\right]=cov\left[\frac{dZ}{Q+dQ},du\right]=-\frac{K}{Q}n_F\sigma_p^2dt. \tag{22}$$

Comparing this expression to (19), we see that the proposition that an unanticipated deterioration in the terms of trade will have an adverse effect on the current account or trade balance if and only if the country is a net creditor, holds irrespective of the choice of units. This is in contrast to the measures of real growth discussed earlier.

It is straightforward to analyze the effects of changes in the parameters of the probability distribution characterizing the terms of trade on the expected current account and balance of trade, as well as their variances. However, we do not pursue these issues here.

5. TERMS OF TRADE SHOCKS AND WELFARE

The results discussed so far have been describing the effects of the terms of trade disturbance, and characteristics of the probability distribution generating it, on various measures of economic performance, pertaining to growth, real expenditures, and external balance. But what ultimately is of concern, and what these measures are presumably attempting to proxy, is economic welfare. We now address this question directly, by considering the welfare of the representative agent, as specified by the intertemporal utility function (4a), evaluated at the optimum. By definition, this is equal to the value function used to solve the intertemporal optimization problem.

As shown in the Appendix, for the constant elasticity utility function, the optimized level of expected utility (welfare) starting from an initial real stock of wealth (in terms of the domestic good) W_0, and relative price P_0, is[13]

$$\Omega(W_0,P_0)=\beta W_0^{\gamma}P_0^{-\gamma(1-\theta)} \tag{23a}$$

where

$$\beta \equiv \frac{1}{\gamma}\theta^{\gamma\theta}(1-\theta)^{\gamma(1-\theta)}\left(\frac{\hat{C}}{W}\right)^{\gamma-1} \qquad 23b)$$

and $\hat{\frac{C}{W}}$ is evaluated at the optimum determined by (5c).[14] The expressions in (23a) and (23b) can now be used to assess the effects of the various parameters pertaining to the terms of trade shock on the welfare of the representative agent. In considering the results we shall obtain below, it is important to bear in mind that (23a) and (23b) imply that $\Omega\gamma > 0$.

An unanticipated permanent terms of trade shock du, which occurs at time 0 say, will impact on both the initial level of real wealth W_0 and on the relative price P_0. The effect on expected welfare, obtained by differentiating (23), is therefore

$$\frac{d\Omega(W_0,P_0)}{du} = \Omega\gamma\left[\frac{dW_0/du}{W_0} - (1-\theta)\frac{dP_0/du}{P_0}\right].$$

Substituting for the percentage changes in wealth and price from (8) and (1) respectively, yields

$$\frac{d\Omega(W_0,P_0)}{du} = \Omega\gamma\left[n_F - (1-\theta)\right] \equiv \Omega\gamma\frac{dx/du}{x}. \qquad (25)$$

Thus it is seen that the effect of an unanticipated positive shock in the terms of trade on welfare is proportional to its effect on the real growth rate $\frac{dx}{x}$, measured in terms of the consumption bundle. While the positive shock will increase welfare through the higher rate of return, this is offset by the adverse effect on the cost of living. In the limiting case of the logarithmic utility function, ($\gamma = 0$) it can be easily verified that $\Omega\gamma \to \frac{1}{\rho}$, in which case (25) becomes[15]

$$\frac{d\Omega(W_0,P_0)}{du} = \frac{1}{\rho}[n_F - (1-\theta)]. \qquad (25')$$

In this case, the effect on the real growth rate $\frac{dx}{x}$, capitalized at the discount rate ρ provides an accurate measure of the welfare effect of the unanticipated shock.

The effect of an increase in p on welfare can be shown to be

$$\frac{d\Omega(W_0, P_0)}{dp} = \Omega(\gamma - 1)\frac{d(C/W)/dp}{C/W} = \frac{\Omega\gamma}{C/W}[n_F - (1-\theta)] \tag{26}$$

and analogous to the unanticipated shock this also depends upon the effect on the real growth measure dx/x. Of greater interest is the effect of a higher variance in the terms of trade on welfare. This is given by[16]

$$\begin{aligned}\frac{d\Omega(W_0, P_0)}{d\sigma_p^2} &= \Omega(\gamma - 1)\frac{d(C/W)/d\sigma_p^2}{C/W} \\ &= \frac{\Omega\gamma}{2(1-\gamma)(C/W)}\left[(1-\theta)(1-\gamma\theta) - \frac{(i^* + p - \alpha)^2}{\sigma_p^4}\right]\end{aligned} \tag{27}$$

Again, there are two effects. On the one hand, the higher variance in the relative price will, for a given portfolio, raise welfare. This is because the value function in (23) is convex in P. At the same time, the higher variance will reduce the rate of wealth accumulation. The net effect thus depends upon which of these two effects dominates.

The result that a risk averse country may be made better off with an increase in the variability in its terms of trade, is actually not new, although the present intertemporal framework within which it is established, is significantly different. This result was first obtained in the case of an individual consumer facing stochastic prices by Waugh (1944), using an analysis based on linear demand functions and consumer surplus welfare measures. It was found to hold in more general cases for risk averse consumers by several authors including Hanoch (1977), Turnovsky, Shalit and Schmitz (1980). The finding is also a manifestation of the result, obtained in the 1970's, that the gains from trade may increase when there is uncertainty; see e.g., Turnovsky (1974), Anderson and Riley (1976). But in contrast to the present approach, these analyses were all based on static welfare measures.

6. CONCLUSIONS

This paper has investigated the impact of changes in the terms of trade on the economic performance of a small economy. Both the effects of unanticipated shocks and changes in the probability distribution generating these disturbances have been discussed. In all cases, the key element determining the response of the economy is the effect on the rate of growth of real wealth, to which in our analysis, all other real quantities are tied directly in equilibrium. The response of the common real growth rate depends to an important degree upon the choice of unit of measurement. Thus, an unanticipated deterioration in the terms of trade will raise the rates of growth of real wealth. savings, consumption expenditures, and stock of traded bonds, all measured in terms of the domestic good. if and only if the country is a net creditor. Conversely, these rates of growth will all be adversely affected and the Harberger–Laursen–Metzler effect hold, if and only if the country is a net debtor.

In the case where real quantities are defined by deflating by the exact cost of living index, an unanticipated deterioration in the terms of trade will generate positive or negative real growth, depending upon whether the fraction of foreign bonds held in the portfolio of domestic investors is greater than, or less than, the share of foreign goods in its consumption basket. The conditions for the HLM effect to hold may therefore be substantially less stringent than in the previous case.

But even when the HLM condition does prevail, the mechanism is very different from that characterizing the original discussion of the issue. In the early analyses, the result is driven largely by the fact that in order to maintain consumption in the face of an adverse terms of trade shock, savings must be reduced, thereby generating a current account deficit. By contrast, in the present analysis, it is generated by the adverse effect the terms of trade shock may have on the rate of return on savings, thereby reducing the rate of wealth accumulation and the corresponding equilibrium rate of consumption.

The effects of anticipated changes in the terms of trade and its variance have also been considered. Sharpest results pertaining to the effects of the latter are obtained for the logarithmic utility function. In particular, in this case a larger variance in the terms of trade will have an adverse effect on the expected growth rate, in terms of domestic output. But at the same time it reduces the expected growth in the cost of living, thereby having a positive impact on the expected growth rate in terms of the overall consumption bundle. Somewhat paradoxically, a higher variance will stabilize the real growth path, as measured in terms of domestic output. However, the variance of the real growth path measured in terms of the consumption bundle may either increase or decrease. These results are subject to some modification for more general forms of the constant elasticity utility function.

In discussing the effects of the terms of trade shock on the external account, we have distinguished between the real growth rate of foreign asset holdings, and the more usual measure of the current account which abstracts from the capital gains on existing bonds. Irrespective of the units in which real quantities are being measured, this more conventional measure of the current account will be adversely affected by an unanticipated deterioration in the terms of trade, if and only if the country is a net creditor. The same applies to the trade balance, which was the focus of the original Harberger analysis. Overall, whether the HLM effect holds, depends upon what precisely the measure of real external activity is, as well as the units in which it is being measured.

Finally, and most importantly, we have addressed the question of how terms of trade shocks impact on economic welfare, as measured by the expected discounted utility of the representative agent. We have found that an unanticipated terms of trade shock will impact on welfare in precisely the same way as it does on the growth rate measured in terms of the consumption bundle. It is also possible for a higher variance in the terms of trade shocks to be welfare improving. This result parallels that of previous analyses assessing the welfare effects of price uncertainty on consumers and small economies.

APPENDIX

The consumers stochastic optimization problem is to choose consumption and portfolio shares to

$$Max\ E_0 \int_0^\infty \frac{1}{\gamma} \left[C_1^\theta C_2^{1-\theta}\right]^\gamma e^{-\rho t} dt \tag{A.1a}$$

subject to the stochastic wealth accumulation equation and price evolution equation

$$\frac{dW}{W} = \psi dt + n_F du \tag{A.1b}$$

$$\frac{dP}{P} = pdt + du \tag{A.1c}$$

and portfolio shares adding up condition

$$n_S + n_F = 1 \tag{A.1d}$$

where for notational convenience

$$\psi \equiv n_S \alpha + n_F(i^* + p) - \frac{C_1}{W} - \frac{PC_2}{W}.$$

We define the differential generator of the value function $V(W, P, t)$ by

$$\mathcal{L}V(W, P, t) \equiv \frac{\partial V}{\partial t} + \psi W \frac{\partial V}{\partial W} + pP\frac{\partial V}{\partial P} + \frac{1}{2} n_F^2 \sigma_p^2 W^2 \frac{\partial^2 V}{\partial W^2} + \frac{1}{2}\sigma_p^2 P^2 \frac{\partial^2 V}{\partial P^2} + n_F \sigma_p^2 PW \frac{\partial^2 V}{\partial P \partial W} \tag{A.2}$$

Given the exponential time discounting, V can be taken to be of the time separable form

$$V(W, P, t) = \Omega(W, P)e^{-\rho t}.$$

The formal optimization problem is now to choose C_1, C_2, n_S , and n_F to maximize the Lagrangean expression

$$e^{-\rho t}\frac{1}{\gamma}\left[C_1^\theta C_2^{1-\theta}\right]^\gamma + \mathcal{L}[e^{-\rho t}\Omega(W, P)] + \frac{\eta}{\rho}e^{-\rho t}[1 - n_S - n_F]. \tag{A.3}$$

Taking partial derivatives of this expression and cancelling $e^{-\rho t}$ yields

$$\theta \left[C_1^{\theta} C_2^{1-\theta}\right]^{\gamma-1} C_1^{\theta-1} C_2^{1-\theta} = \Omega_W \tag{A.4a}$$

$$(1-\theta) \left[C_1^{\theta} C_2^{1-\theta}\right]^{\gamma-1} C_1^{\theta} C_2^{-\theta} = P\Omega_W \tag{A.4b}$$

$$\alpha W \Omega_W = \eta \tag{A.4c}$$

$$(i^* + p) W\Omega_W + n_F \sigma_p^2 W^2 \Omega_{WW} + \sigma_p^2 PW\Omega_{PW} = \eta \tag{A.4d}$$

$$n_S + n_F = 1. \tag{A.4e}$$

These equations determine the optimal values for C_1, C_2, n_S, n_F, η, as functions of the partial derivatives $\Omega_W, \Omega_{WW}, \Omega_{PW}$, of the value function. In addition, this function must satisfy the stochastic Bellman equation

$$\max_{C_1, C_2, n_S, n_F} \left[\frac{1}{\gamma}\left[C_1^{\theta} C_2^{1-\theta}\right]^{\gamma} e^{-\rho t} + \mathcal{L}\left[e^{-\rho t}\Omega(W, P)\right]\right] = 0. \tag{A.5}$$

This involves substituting for the optimized values obtained from (A.4) and solving the resulting partial differential equations for $\Omega(W, P)$, namely

$$\begin{aligned}\frac{1}{\gamma}\left[\hat{C}_1^{\theta} \hat{C}_2^{1-\theta}\right]^{\gamma} - \rho\Omega(W,P) + \hat{\psi} W\Omega_W + pP\Omega_P \\ + \frac{1}{2}\hat{n}_F^2 \sigma_p^2 W^2 \Omega_{WW} + \frac{1}{2}\sigma_p^2 P^2 \Omega_{PP} + \hat{n}_F \sigma_p^2 PW\Omega_{PW} = 0\end{aligned} \tag{A.6}$$

where ˆ denotes optimized values.

The solution strategy is by trial and error, finding a solution $\Omega(W, P)$ that satisfies both the optimality condition and the Bellman equation. We postulate a solution of the form

$$\Omega(W, P) = \beta W^{\gamma} P^{x} \tag{A.7}$$

where β, x, are to be determined. This equation immediately implies

$$\Omega_W = \beta\gamma W^{\gamma-1}P^x; \;\; \Omega_P = \beta x W^{\gamma} P^{x-1}; \;\; \Omega_{WW} = \beta\gamma(\gamma-1)W^{\gamma-2}P^x$$

$$\Omega_{PP} = \beta x(x-1)W^{\gamma}P^{x-2}; \;\; \Omega_{PW} = \beta\gamma x W^{\gamma-1}P^{x-1} \tag{A.8}$$

To solve, we begin by dividing (A.4a) by (A.4b). Combining the resulting expression with the definition of expenditure, $C \equiv C_1 + PC_2$ yields the expenditure shares

$$C_1 = \theta C$$

$$PC_2 = (1-\theta)C$$

so that

$$C_1^{\theta}C_2^{1-\theta} = \theta^{\theta}(1-\theta)^{1-\theta}CP^{-(1-\theta)}. \tag{A.9}$$

Substituting the expressions for C_1, C_2, and Ω_W back into (A.4a) leads to

$$C = \left[\beta\gamma\theta^{-\gamma\theta}(1-\theta)^{-\gamma(1-\theta)}P^{x+\gamma(1-\theta)}\right]^{\frac{1}{\gamma-1}} W \tag{A.10}$$

Next, substituting from (A.8), (A.9) and (A.10) into the Bellman equation (A.6) we obtain

$$\frac{1}{\gamma}\theta^{-\frac{\theta\gamma}{\gamma-1}}(1-\theta)^{-\frac{(1-\theta)\gamma}{\gamma-1}}(\beta\gamma)^{\frac{\gamma}{\gamma-1}}W^{\gamma}P^{[x+(1-\theta)]\frac{\gamma}{\gamma-1}} - \rho\beta W^{\gamma}P^x$$

$$+ \hat{\psi}\beta\gamma W^{\gamma}P^x + \beta x p W^{\gamma}P^x + \frac{1}{2}\hat{n}_F^2\sigma_p^2(\beta\gamma)(\gamma-1)W^{\gamma}P^x \tag{A.11}$$

$$+ \frac{1}{2}\sigma_p^2\beta x(x-1)W^{\gamma}P^x + \hat{n}_F\sigma_p^2\beta\gamma x W^{\gamma}P^x = 0.$$

This consists of terms involving W and P raised to constant powers.

The function (A.7) will be a viable solution if and only if

$$x = -\gamma(1-\theta)$$

in which case (A.10) reduces to

$$C = \left[\beta\gamma\theta^{-\gamma\theta}(1-\theta)^{-\gamma(1-\theta)}\right]^{\frac{1}{\gamma-1}} W \tag{A.10'}$$

Cancelling the terms $W^{\gamma}P^{x}$ (recalling $x = -\gamma(1-\theta)$) in (A.11), and noting the definition of $\hat{\upsilon}$ we find that β is the solution to

$$(1-\gamma)(\beta\gamma)^{\frac{1}{\gamma-1}}\theta^{\frac{-\gamma\theta}{1-\gamma}}(1-\theta)^{\frac{\gamma(1-\theta)}{1-\gamma}} + \gamma[\hat{n}_S\alpha + \hat{n}_F(i^* + p)] - \rho$$

$$- p\gamma(1-\theta) + \frac{1}{2}\hat{n}_F^2\sigma_p^2\gamma(\gamma-1) + \frac{1}{2}\sigma_p^2\gamma(1-\theta)[\gamma(1-\theta)+1]$$

$$+ \hat{n}_F\sigma_p^2\gamma^2(1-\theta) = 0.$$

Using (A.4c) and (A.4d), in conjunction with the relevant terms in (A.8), enables the solution for $\frac{C}{W}$ and the constant β to be expressed in the following form

$$\frac{C}{W} = (\beta\gamma)^{\frac{1}{\gamma-1}}\theta^{\frac{-\gamma\theta}{1-\gamma}}(1-\theta)^{\frac{\gamma(1-\theta)}{1-\gamma}}$$

$$= \frac{1}{1-\gamma}\left\{\rho + \gamma[p(1-\theta)-\alpha] - \frac{1}{2}\hat{n}_F^2\sigma_p^2(1-\gamma) - \frac{1}{2}\sigma_p^2\gamma(1-\theta)[1+\gamma(1-\theta)]\right\}. \tag{A.12}$$

The solution for the value function is therefore

$$\Omega(W,P) = \beta W^{\gamma}P^{-\gamma(1-\theta)} \tag{A.13}$$

where β, obtained from the first equation in (A.12), can be written as

$$\beta = \frac{1}{\gamma}\theta^{\gamma\theta}(1-\theta)^{\gamma(1-\theta)}\left(\widehat{\frac{C}{W}}\right)^{\gamma-1} \tag{A.14}$$

and the optimal consumption-wealth ratio is obtained from the second equation in (A.12). Note the fact that equilibrium $\frac{C}{W} > 0$ implies $\beta\gamma > 0$. Equation (23) in the text corresponds to welfare starting from initial values $W = W_0, P = P_0$.

Finally, substituting for $\Omega_W, \Omega_{WW}, \Omega_{WP}$, into (A.4) yields the optimality condition (5). One can further establish that the transversality condition

$$\lim_{t\to\infty} E\left[e^{-\rho t}\Omega(W,P)\right] = 0$$

is satisfied.

FOOTNOTES

*I am pleased to acknowledge the constructive comments of two referees.

[1] More recent authors measuring real quantities in terms of the exportable or domestic good include Obstfeld (1982), Sen and Turnovsky (1989).

[2] See also, Svensson and Razin (1983), Persson and Svensson (1985), and Bean (1986).

[3] Reference should also be made to the related literature which analyzes the welfare effects of oil price shocks; see e.g., Svensson (1984).

[4] The solution to the stochastic differential equation (7) is

$$W(t) = W_0 e^{[\alpha + (i^* + p - \alpha) n_F - \frac{C}{W}]t} + n_F[u(t) - u_0]$$

with the transversality condition being

$$\lim_{t \to \infty} E W e^{-\int dR_F} = 0.$$

The point about the slow rate of growth can be seen most simply by considering the nonstochastic case, when $i^* + p = \alpha$. The solution for W in this case is

$$W(t) = W_0 e^{[\alpha - \frac{C}{W}]t}$$

so that the transversality condition

$$\lim_{t \to \infty} W(t) e^{-\alpha t} = 0$$

is obviously met.

[5] See e.g., Malliaris and Brock (1982) for a lucid discussion of the methods of stochastic calculus.

[6] Equations (11a), (11b) imply

$$r'_F - r'_S = i^* + p - \alpha - (1 - \theta)\sigma_p^2$$

from which (12a) immediately follows by substitution from (6).

[7] We should emphasize that even though we choose to express some of these derivatives, as well as some of the effects reported in (14) and (15) in terms of n_F, the latter is endogenously determined in accordance with the optimality condition (6).

[8]See, in particular equation (33) of Levhari and Srinivasan (1969). Some economic intuition of the economic significance of the inequalities $\gamma \gtrless 0$ is provided by Sandmo (1970). He shows how an increase in capital risk gives rise to an income effect and a substitution effect. The inequality $\gamma < 0$ corresponds to the case where the former dominates, while $\gamma > 0$ means that the latter dominates.

[9]If $n_F < 0$, then $\alpha > i^* + p$. The increase in σ_p^2 reduces the net indebtedness of the country and the net stock of capital, and hence its rate of growth.

[10]As noted by Stulz, by investing in the portfoio shares $n_S = \theta, n_F = 1 - \theta$ the agent can create a perfectly safe asset in terms of the real consumption bundle.

[11]Note that we are assuming that bonds are bought at the price $P + dP$, rather than P. The difference between these two involves terms of the second order which are unimportant in a world of certainty. But with the variances of the stochastic terms being of the first order, this specification can be shown to be necessary if the consistency of ex ante and ex post wealth accumulation is to be preserved.

[12]Formally, the relationship is as follows:

$$\begin{aligned} cov\left[\frac{d(PB)}{PB}, du\right] &= \frac{1}{PB}\ cov[(P + dP)dB, du] + cov\left[\frac{dP}{P}, du\right] \\ &= -n_S + 1 = n_F. \end{aligned}$$

[13]In the case of the logarithmic utility function, one can establish

$$\Omega(W_0, P_0) = b_0 + \frac{1}{\rho}\ lnW_0 - \frac{(1-\theta)}{\rho}\ lnP_0$$

where

$$\begin{aligned} b_0 \equiv \frac{1}{\rho}\left\{\theta\ ln\ \theta + (1-\theta)(1-\theta) + ln\rho - 1 + \frac{1}{\rho}(\hat{n}_S\alpha + \hat{n}_F(i^* + p))\right\} \\ - \frac{p}{\rho}(1-\theta) - \frac{1}{2\rho}\hat{n}_F^2\sigma_p^2 + \frac{1}{2}\rho(1-\theta)\sigma_p^2. \end{aligned}$$

[14]We shall assume that the equilibrium C/W ratio is positive.

[15]From (23a), (23b) one can establish

$$\begin{aligned} \lim_{\gamma\to 0}\Omega\gamma &= \lim_{\gamma\to 0} W_0^\gamma P_0^{-\gamma(1-\theta)}\theta^{\gamma\theta}(1-\theta)^{\gamma(1-\theta)}\left(\widehat{\frac{C}{W}}\right)^{\gamma-1} \\ &= \lim_{\gamma\to 0}\left(\widehat{\frac{C}{W}}\right) = \frac{1}{\rho}. \end{aligned}$$

[16]In the limiting case of the logarithmic utility function we obtain

$$\frac{d\Omega(W_o, P_0)}{du} = \frac{1}{\rho^2}[n_F - (1-\theta)]$$

$$\frac{d\Omega(W_0, P_0)}{d\sigma_p^2} = \frac{1}{2\rho^2}[(1-\theta) - n_F^2].$$

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