

NBER WORKING PAPERS SERIES

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AN APPLICATION TO SOCIAL SECURITY

David Altig

Steve J. Davis

Working Paper No. 3913

NATIONAL BUREAU OF ECONOMIC RESEARCH  
1050 Massachusetts Avenue  
Cambridge, MA 02138  
November 1991

This paper is part of NBER's research program in Taxation. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

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ABSTRACT

We develop the implications of borrowing constraints and two-sided altruism in an overlapping generations framework with agents who live three periods. Our analysis identifies six equilibrium patterns of intertemporal and intergenerational linkages in the no-loan economy, one of which corresponds to the traditional life-cycle model, and one of which corresponds to Barro's dynastic model. Novel linkage patterns involve parent-to-child transfers early in the life cycle, child-to-parent gifts late in the life cycle, or both. Capital accumulation behavior and the consequences of fiscal policy interventions depend, often critically, on which linkage patterns prevails. We show how unfunded social security interventions can significantly depress aggregate capital accumulation, even when every generation is linked to its successor generation by altruistic transfers. We also derive a non-Ricardian neutrality result for gift-motive economies that holds whether or not borrowing constraints bind and whether or not parent and child are connected by an operative altruism motive at all points in the life cycle.

David Altig  
Research Department  
Federal Reserve Bank of Cleveland  
Cleveland, Ohio 44101

Steve J. Davis  
Graduate School of  
Business  
University of Chicago  
Chicago, IL 60637  
and  
Federal Reserve Bank of  
Chicago and NBER

## 1. Introduction

To many students of macroeconomics, the introduction of intergenerational altruism into models of aggregate capital accumulation has one central implication – the conversion of finitely-lived households into infinitely-lived dynasties. Although this perception stands as a testament to the power of the Ricardian equivalence hypothesis espoused by Robert Barro (1974) nearly two decades ago, a recently emerging literature stresses the consequences of intergenerational altruism in non-Ricardian environments. We contribute to this literature by developing the consequences that follow from the interaction of borrowing constraints and two-sided intergenerational altruism.

Available evidence indicates that private intergenerational transfers are quantitatively important. A 1979 survey of the President's Commission on Pension Policy found that 18.4 percent of U.S. households received some sort of transfer from other households (Cox and Raines, 1985). Although the majority of recipient households received resources from older households, gifts from younger to older generations were also apparent: 7.3 percent of the recipient households reported receiving transfers from younger households. Of the households that reported giving resources to other household units, 18.9 percent indicated that the transfers went to older households. Gifts from younger to older households appear more prevalent in developing countries. For example, Cox and Jimenez (1990) find that nearly 28 percent of reported inter-household transfers in Peru were received from children.

Other empirical evidence indicates that about 20% of households are subject to binding borrowing constraints (Mariger (1986), Zeldes (1989) and Jappelli (1990)). Cox (1990) and Guiso and Jappelli (1991) provide evidence that borrowing constraints are an important determining factor in the incidence of intergenerational transfers. Prompted by these empirical observations, recent papers by Altig and Davis (1989) and Laitner (1990) examine the theoretical implications of binding borrowing constraints in models with operative intergenerational transfer motives. A key insight of this research is that fiscal policy effects in models with interactions between borrowing constraints and intergenerational altruism are distinct from their effects in both the life-cycle and dynastic paradigms.

Motivated by this insight and the important real-world role of intergenerational altruism and borrowing constraints, this paper continues the examination of models in which borrowing constraints and altruism interact. Here, as in Altig-Davis (1989) and Laitner

(1990), we consider capital market imperfections that take the form of an inability to borrow against future wage and transfer income (including public transfers). The model we analyze admits general degrees of two-sided altruism. This model generalizes and nests the model in Altig-Davis, which considers parental altruism only, and complements the model in Laitner, which considers preference structures in which parents and children weight the other's utility equally to own utility. Our work here also extends the work of Abel (1987), who analyzed two-sided altruism in an economy with perfect capital markets.

In the context of an overlapping generations framework populated by three-period lived persons, we identify all equilibrium configurations of intertemporal and intergenerational linkages that can arise, with and without binding borrowing constraints, and with and without various combinations of operative parent-to-child and child-to-parent transfer motives. There turn out to be six equilibrium configurations in the model, one of which corresponds to the standard life-cycle model with perfect capital markets, and one of which corresponds to Barro's dynastic model. Underlying the other possible equilibria are two simple propositions regarding the implications of borrowing constraints for the timing of intergenerational transfers. Specifically, if children face binding borrowing constraints when young, then: (a) any parent-to-child transfers occur early in the life of children; and (b) any child-to-parent transfers occur late in the life of parents.

The timing propositions carry important implications for fiscal policy in economies with altruistic agents. We illustrate this general point by studying the dynamic and steady-state response to funded and unfunded social security interventions in cases where borrowing constraints bind. We analytically characterize the dynamic and steady-state effects of nonneutral social security interventions. We also carry out numerical simulation exercises to gauge the magnitude of the effects that stem from the interaction of borrowing constraints and intergenerational altruism.

Our analysis shows that the interaction between borrowing constraints and parental altruism significantly alters the aggregate savings response to unfunded social security interventions relative to the response in traditional life-cycle models à la Feldstein (1974), Kotlikoff (1979), and Auerbach and Kotlikoff (1987) and relative to the response in models with intergenerational altruism and perfect capital markets à la Barro (1974). Indeed, the capital stock decline caused by an unfunded social security program can be larger in

an environment with parental altruism and borrowing constraints than in environments with (a) non-altruistic agents and perfect capital markets or (b) non-altruistic agents and borrowing constraints.

A very different result emerges from our analysis when child-to-parent altruistic gift motives operate. We prove that an operative child-to-parent gift motive (pre- and post-intervention) implies neutrality of the steady-state interest rate with respect to *all* lump-sum government interventions. This interest-rate neutrality result holds regardless of whether borrowing constraints bind and regardless of whether the young and middle-aged are connected by altruistic linkages. The logic underlying this result also survives the introduction of non-altruistic agents into the economy, provided that the gift motive continues to operate for the altruists. Unlike neutrality results in the tradition of Barro (1974), Becker (1974), and Bernheim and Bagwell (1988), our interest-rate neutrality argument does not rely on direct or indirect altruistic linkages between persons who are taxed and/or subsidized in the government intervention.

## 2. Patterns of Intertemporal and Intergenerational Linkages

It will be helpful to preview the analysis by describing the patterns of intertemporal (capital market) and intergenerational linkages that can emerge as steady-state equilibrium outcomes. The diagrams in Figure 1 illustrate the six distinct linkage patterns that can emerge in our model.<sup>1</sup> Dashed lines in the diagrams depict altruistically motivated intergenerational linkages, and solid lines depict intertemporal linkages operating through the capital market. For convenience, we shall refer to child-to-parent transfers as gifts, and parent-to-child transfers as simply transfers.

Regimes A and B in Figure 1 represent, respectively, the familiar life-cycle and Ricardian cases. In the life-cycle regime, borrowing constraints are nonexistent or nonbinding, so that each individual's consumption profile satisfies an Euler condition at all points in the life cycle. This regime represents an extension of the Diamond (1965) economy to a world with agents who live three periods. In the Ricardian regime, the intertemporal linkages in the Diamond economy are augmented by intergenerational linkages that stem from an operative transfer motive, an operative gift motive, or both.

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<sup>1</sup>We thank Doug Bernheim for suggesting this expositional device.

Regimes C through F represent cases in which borrowing constraints bind, as indicated by the absence of an intertemporal linkage between youth and middle age. Regime C has no operative intergenerational linkages, whereas Regime D has an operative (parental) transfer motive. We prove here the assertion made in Altig-Davis (1989) that, when borrowing constraints bind and transfer motives operate, all transfers to children occur when children are young and parents are middle-aged.

Regime E exhibits binding borrowing constraints and an operative gift motive only. Analogous to the operative transfer case, binding borrowing constraints restrict the equilibrium timing of gifts. Specifically, if borrowing constraints bind and the gift motive operates, all gifts are made when children are middle-aged and unconstrained. To our knowledge, economies with the characteristics of Regime E have not been studied before.

Finally, Regime F represents the case in which borrowing constraints bind and both transfer and gift motives operate. The economy studied by Laitner (1990) is similar in some respects to the Regime F-type economy studied in this paper.

In the next three sections we formally characterize the equilibria underlying the diagrams in Figure 1. The key results are contained in section 5, where we prove our assertions about the timing of intergenerational transfers and gifts in the no-loan economy. Sections 6 and 7 analyze government interventions that redistribute resources across generations and show how fiscal policy effects depend on the prevailing linkage regime.

### 3. The Overlapping Generations Framework With Two-sided Altruism

We describe an overlapping generations framework with three-period lived persons and no government, postponing the discussion of fiscal policy variables to section 6. Within this framework we consider an economy with perfect capital markets and an economy with no consumption-loans market. Each person in these economies inelastically supplies homogeneous labor services according to a lifetime productivity profile,  $(\alpha_1, \alpha_2, \alpha_3)$ . Parents choose the timing and magnitude of altruistically motivated transfers to children. Children choose the timing and magnitude of altruistically motivated gifts to parents.

We assume that an individual's productivity profile slopes up over at least the first two periods of life, so that  $\alpha_2 > \alpha_1$ . We have shown elsewhere (Altig and Davis, 1989) that a life-cycle income profile that slopes up over the first two periods of life greatly reduces

the degree of altruism necessary to generate transfers from parents to children. To make our discussion of borrowing restrictions nontrivial, we further assume that  $\alpha_2$  is sufficiently greater than  $\alpha_1$  so that the consumption-loans market influences the equilibrium capital stock and consumption profile. In other words, we focus on parameter configurations in which the equilibrium capital stock and consumption profile differ between the loan and no-loan economies.

#### A. Preferences and the Consumer Choice Problem

In the consumption-loans economy with no government a representative member of generation  $t$  chooses  $(C_{1t}, C_{2t}, C_{3t}, x_{1t}, x_{2t}, g_{1,t-1}, g_{2,t-1}, b_{1,t+1}, b_{2,t+1}, b_{3,t+1})$  to maximize:

$$U_t = \sum_{i=1}^3 \beta^{i-1} u(C_{it}) + \beta \gamma U_{t+1}^* + \frac{\rho}{\beta} U_{t-1}^* \quad (1)$$

subject to:

$$C_{1t} + x_{1t} + g_{2,t-1} = \alpha_1 W_t + b_{1t} \quad (2)$$

$$C_{2t} + (1+n)b_{1,t+1} + x_{2t} + g_{3,t-1} = (1+r_{t+1})x_{1t} + \alpha_2 W_{t+1} + b_{2t} + (1+n)g_{2t} \quad (3)$$

$$C_{3t} + (1+n)b_{3,t+1} + (1+n)b_{2,t+1} = (1+r_{t+2})x_{2t} + (1+r_{t+2})b_{3t} + \alpha_3 W_{t+2} + (1+n)g_{3t} \quad (4)$$

$$C_{1t}, C_{2t}, C_{3t}, g_{2,t-1}, g_{3,t-1}, b_{1,t+1}, b_{2,t+1}, b_{3,t+1} \geq 0 \quad (5)$$

where:

$C_{it}$  = consumption by generation  $t$  in the  $i$ th period of life,

$x_{it}$  = capital purchases (i.e., savings) by generation  $t$  in the  $i$ th period of life,

$b_{i,t+1}$  = transfer made by a generation- $t$  parent to each  $(1+n)$  offspring in the children's  $i$ th period of life (an inter vivos transfer for  $i = 1, 2$ , a bequest for  $i = 3$ ),

$g_{i,t-1}$  = gift made by a generation- $t$  child during the parent's  $i$ th period of life,

$n$  = the population growth rate,

$\beta$  = intertemporal discount factor,  $0 < \beta < 1$ ,

$\gamma$  = interpersonal discount factor on children's utility,  $0 < \gamma \leq (1+n)/\beta$ , which insures a positive steady-state interest rate when the transfer motive operates in the loans economy,

$\rho$  = interpersonal discount factor on parents' utility,  $\rho\gamma \leq 1 + n$ , which is a necessary condition for the existence of a steady-state equilibrium,<sup>2</sup>

$u(\cdot)$  = period utility function, satisfying  $u'(\cdot) > 0$ ,  $u''(\cdot) < 0$ ,  $\lim_{C \rightarrow 0} u'(C) = \infty$ , and  $\lim_{C \rightarrow \infty} u'(C) = 0$ ,

$U_{t+i}^*$  = maximum utility attainable by a generation  $t + i$  agent as a function of the transfer or gift received, where  $i = 1$  for children and  $i = -1$  for parents,

$W_t$  = the period- $t$  wage in units of the good, and

$r_{t+1}$  = the one-period rate of return on physical capital (or consumption loans) held from  $t$  to  $t + 1$ .

We follow Abel (1987) in (1) and assume that the gift decision is made taking the gifts of siblings as given.<sup>3</sup> The absence of nonnegativity constraints on savings by the young and middle-aged reflects the availability of a costless consumption-loans market.

In the no-loan economy a representative consumer of generation  $t$  maximizes (1) subject to (2) through (5) and

$$x_{1t}, x_{2t} \geq 0. \quad (6)$$

This additional constraint reflects the absence of a viable enforcement mechanism to support the operation of a consumption-loans market.<sup>4</sup> We show below that, if the young choose to dissave in the consumption-loans economy, the constraint  $x_{1t} \geq 0$  always binds in the corresponding no-loan economy.

Our specification of the consumer's problem sidesteps some complicated, and potentially important, issues involving strategic aspects of altruistically motivated interpersonal

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<sup>2</sup>If this condition fails to hold, the transfer motive and gift motive first-order conditions contradict each other; see Abel (1987) for elaboration on this point.

<sup>3</sup>In an environment with perfect capital markets and operative intergenerational linkages—i.e., an environment with dynastic families—living persons' treatment of deceased ancestors' utility calculations bears on both the existence and form of a solution (Kimball, 1988). As we will show, the dynastic character of the representative person's problem is destroyed when borrowing constraints bind, so that the treatment of deceased ancestors' utility has no bearing on the solution.

<sup>4</sup>The constraint (6) has more than one interpretation. First, borrowing constraints can arise from high costs of enforcing loan repayment, due partly to bankruptcy laws and other legal protections afforded to debtors. Second, the asymmetric tax treatment of interest income and interest payments on consumption-loans can lead consumers to choose a corner outcome with respect to their borrowing and saving decision (Altig and Davis, 1991). Third, and somewhat further removed from our framework, sufficiently severe adverse selection effects can prevent the operation of a consumption-loans market.



transfers. Of particular consequence is the so-called “Samaritan’s dilemma”—the possibility that gift and transfer recipients pursue consumption plans that exploit the altruism of donors.<sup>5</sup> The Samaritan’s dilemma cannot arise in our framework with binding borrowing constraints and parental altruism only (Regime D). Since parents want borrowing-constrained children to consume the entire transfer, over-consumption by the young is not an issue. There is a potential for the Samaritan’s dilemma to arise in Regimes E and F, because parents might over-consume during middle age to elicit larger gifts from children during old age. We preclude this possibility by assuming that middle-aged parents take the size of children’s gifts as given when they optimize. Likewise, in Regime B we assume that children take the size of parental transfers as given when they optimize. Laitner (1990) obviates the potential difficulties associated with the Samaritan’s dilemma by assuming strong reciprocal altruism on the part of parents and children.

Although our treatment of the Samaritan’s dilemma problem can be criticized, we note that the same treatment is implicit in most analyses of intergenerational altruism in dynamic general equilibrium models. Our treatment allows us to easily compare Regime D-F economies to the perfect-capital market economies studied by Barro (1974), Abel (1987), and many others.

#### *B. The Production Technology and the Market-Clearing Conditions*

Turning to the production side of the economy, and normalizing so that generation 0 has one member, the aggregate period- $t$  labor supply is

$$L_t = \left[ \alpha_1 + \frac{\alpha_2}{1+n} + \frac{\alpha_3}{(1+n)^2} \right] (1+n)^t \equiv \alpha(1+n)^t, \quad (7)$$

where  $\alpha$  is per capita labor supply. Defining  $k = K/L$  as the capital-labor ratio, we write the aggregate production function as

$$Y_t = \alpha(1+n)^t f(k_t), \quad (8)$$

where  $f'(\cdot) > 0$ ,  $f''(\cdot) < 0$ ,  $\lim_{k \rightarrow 0} f'(k) = \infty$ , and  $\lim_{k \rightarrow \infty} f'(k) = 0$ . The representative firm’s competitive profit maximization conditions are

$$W_t = f(k_t) - k_t f'(k_t), \text{ and} \quad (9)$$

<sup>5</sup>See Buchanan (1975), Bernheim and Stark (1988), Lindbeck and Weibull (1988), and Bruce and Waldman (1990).

$$r_t = f'(k_t). \quad (10)$$

Assuming that the government claims no share of the economy's output, we obtain the goods market-clearing condition:

$$\alpha(1+n)k_{t+1} - \alpha k_t + C_{1t} + \frac{C_{2,t-1}}{1+n} + \frac{C_{3,t-2}}{(1+n)^2} = \alpha f(k_t), \quad (11)$$

and the capital market-clearing condition:

$$k_t = \frac{(1+n)x_{1,t-1} + x_{2,t-2} + b_{3,t-2}}{(1+n)^2\alpha}. \quad (12)$$

This completes the description of the economy with no government. To introduce the government, one need only add the government budget constraint and make appropriate modifications to the consumer budget constraints.

### C. The Consumer's Optimization Conditions

The consumer's intertemporal first-order conditions for own consumption are

$$u'(C_{1t}) \geq \beta(1+r_{t+1})u'(C_{2t}) \quad \text{and} \quad (13)$$

$$u'(C_{2t}) \geq \beta(1+r_{t+2})u'(C_{3t}). \quad (14)$$

Given a desire to borrow by the young, the properties of the production technology imply that (14) holds with equality. Equation (13) holds with equality in the loan economy and in the no-loan economy when (6) fails to bind. In these cases, equations (13) and (14) represent the familiar condition that the marginal rate of substitution between own current consumption and own future consumption equals the time-discounted gross rate of return to savings.

Using the envelope theorem, the first-order conditions governing intergenerational transfers are

$$u'(C_{it}) \geq \frac{\gamma}{1+n} u'(C_{i-1,t+1}) \quad i = 2, 3 \quad (15)$$

for inter vivos transfers and

$$u'(C_{3t}) \geq \frac{\gamma\beta}{1+n} (1+r_{t+3})u'(C_{3,t+1}) \quad (16)$$

for bequests. Equations (15) and (16) state that when a transfer motive is operative, the discounted marginal rate of substitution of the parent's consumption for children's consumption equals the population deflated interpersonal discount factor  $\gamma$ .

Finally, again using the envelope theorem, the first-order conditions governing intergenerational gifts are

$$u'(C_{it}) \geq \rho u'(C_{i+1,t-1}), \quad i = 1, 2. \quad (17)$$

The conditions represented by equation (17) state that when a gift motive is operative, the discounted marginal rate of substitution of the child's consumption for parents' consumption equals the interpersonal discount factor  $\rho$ .

#### D. Equilibrium

An equilibrium in the consumption-loans economy is a sequence  $\{C_{1t}, C_{2,t-1}, C_{3,t-2}, x_{1t}, x_{2,t-1}, g_{2,t-1}, g_{3,t-1}, b_{1t}, b_{2,t-1}, b_{3,t-2}, W_t, r_{t+1}, k_t, Y_t\}_{t=0}^{\infty}$  that satisfies (1)-(5) and (7)-(17) for all  $t$ , given the initial condition  $(b_{3,-2}, x_{1,-1}, x_{2,-2}, k_0)$ . Similarly, an equilibrium in the no-loan economy is a sequence that satisfies (1)-(17). A steady-state equilibrium is characterized by constancy of the per capita capital stock and life-cycle consumption profiles.

### 4. Transfers, Gifts, and Interest Rates in the Loan Economy

In this section we briefly characterize steady-state equilibria in the perfect capital market regimes (A and B) depicted in Figure 1. Results in this section are not new – see Abel (1987) – but they serve as an important point of comparison for results in section 5.

Steady-state versions of the intertemporal and interpersonal first-order conditions for consumers lead directly to

*Proposition 1:* The steady-state interest rate in the loan economy,  $\bar{r}$ , satisfies

$$\frac{\rho}{\beta} - 1 \leq \bar{r} \leq \frac{(1+n)}{\gamma\beta} - 1 \equiv r^*.$$

with equality on the left if gifts are positive, and on the right if transfers are positive.

Because the Inada conditions require a positive interest rate, this proposition implies that  $\rho > \beta$  is a necessary condition for the existence of a steady-state equilibrium with an operative gift motive. Likewise,  $\gamma\beta < 1 + n$  is a necessary condition for the existence of a

steady-state equilibrium with an operative transfer motive. Furthermore, the proposition implies that both gift and transfer motive first-order conditions hold with equality in the loan economy if, and only if,  $\rho = \frac{(1+n)}{\gamma}$ .

Without further assumptions, we are unable to state general sufficient conditions for operative transfer or gift motives in terms of underlying preference and technology parameters. An alternative approach, following Weil (1987) and Abel (1987), would be to derive sufficient conditions in terms of the steady-state interest rate in the corresponding Diamond economy. This approach, too, is difficult to implement in our framework because of the analytical unwieldiness of Diamond-type economies when agents live more than two periods. We therefore address the existence question numerically for standard specifications of preferences and technology.

Our results are summarized in Figure 2, which indicates the steady-state equilibrium linkage regime for various combinations of  $\gamma$  and  $\rho$ , given fixed values of the other parameters. To construct Figure 2, we assumed isoelastic utility with intertemporal substitution elasticity equal to .4, Cobb-Douglas production with capital's share equal to .25,  $(\alpha_1, \alpha_2, \alpha_3) = (1.5, 6.0, 2.5)$ ,  $n = 0$ , and  $\beta = (.973)^{25} = .5$ . The qualitative properties of the figure do not depend on this particular parametrization provided that the corresponding Diamond economy is dynamically efficient – see Abel (1987) on this point.

In Figure 2,  $\gamma^*$  is defined as the degree of altruism for which transfers just equal zero at an interior solution. Transfer motives are inoperative when parents “love their children” less than  $\gamma^*$ . Likewise,  $\rho^*$  is the degree of altruism for which gifts just equal zero at an interior solution. Gift motives are inoperative when children love their parents less than  $\rho^*$ . From Proposition 1, the  $\rho\gamma = 1 + n$  locus depicts all points in the parameter space for which both gift and transfer motives first-order conditions hold with equality in the loan economy. Equilibria with parameter values such that  $\rho\gamma > 1 + n$  do not exist.

## 5. Transfers, Gifts, and Interest Rates in the No-Loan Economy

We now show which linkage patterns can emerge in equilibrium when borrowing constraints bind and one or both of the altruism motives operate. In deriving these linkage patterns, the following result will prove useful. If borrowing constraints bind in the no-loan economy, then  $\bar{r} < r^*$ , where  $\bar{r}$  is the steady-state equilibrium interest rate in the no-loan

economy, and  $r^*$  is as defined in Proposition 1.<sup>6</sup> This result has a straightforward intuition when parental altruism operates; namely, that borrowing constraints eliminate dissaving by the young and thereby drive down the interest rate below the modified golden rule rate.

We have

*Proposition 2:* If borrowing restrictions bind in a steady-state equilibrium, then any inter-generational transfers occur from middle-aged parents to young children, and any inter-generational gifts occur from middle-aged children to old parents.

*Proof:* Consider the case where borrowing constraints bind, parental altruism operates, and parents make bequests or transfers late in life. Then equations (14)-(16) imply

$$\bar{r} = \frac{(1+n)}{\beta\gamma} - 1 = r^*,$$

a condition which violates the requirement that  $\bar{r} < r^*$ . From this contradiction, it follows that parents never make bequests or transfers late in life to children who were constrained at an earlier point in the life cycle.

Next, consider the case where borrowing constraints bind, child altruism operates, and children make gifts when young (and borrowing constrained). Then equations (13) and (17) imply

$$\rho > \beta(1 + \bar{r}).$$

But, because equation (14) holds with equality, this implies

$$u'(C_2) < \rho u'(C_3),$$

a condition that violates the interpersonal first-order condition in equation (17). From this contradiction it follows that children never make gifts to parents when they are borrowing constrained.

The proof is concluded by noting that no such contradictions are implied by transfers from middle-aged parents to young, borrowing-constrained children or gifts from middle-aged, unconstrained children to old parents. Q.E.D.

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<sup>6</sup>Altig and Davis (1989) prove this claim in an economy with no gift motive. If the gift motive operates, then the claim follows directly from steady-state versions of (14) and (17).

Drawing on the timing results in Proposition 2, the next proposition further characterizes the no-loan economy and compares it to the loan economy.

*Proposition 3:*

- (a)  $\beta < \rho < \frac{(1+n)}{\gamma}$  is necessary for the existence of a steady-state equilibrium with binding borrowing constraints and operative gift and transfer motives (Regime F).
- (b) In a steady-state equilibrium with an operative gift motive and binding borrowing constraints (Regimes E and F)

$$\tilde{r} = \frac{\rho}{\beta} - 1 \equiv r_g^* < r^*.$$

- (c) Let  $\tilde{r}_{ng}$  denote the steady-state interest rate in the economy with binding borrowing constraints but an inoperative gift motive (Regimes C and D). Then  $r_g^* < \tilde{r}_{ng} < r^*$ .

*Proof:*

Part (a): Follows directly from equations (13)-(17) and proposition 2.

Part (b): Recognizing that the Inada conditions imply positive saving by the middle-aged, the proof follows directly from equations (14) and (17) and Proposition 2.

Part (c): Using part (b), if the gift motive does not operate in the no-loan economy, then

$$u'(C_2) = \beta(1 + \tilde{r}_{ng})u'(C_3) > \rho u'(C_3) = \beta(1 + r_g^*)u'(C_3).$$

This proves the first inequality. The second inequality is Proposition 2, parts (iii) and (iv), in Altig and Davis (1989).

**Q.E.D.**

Part (a) of Proposition 2 indicates that strong altruism is necessary for an operative gift motive. As a referee points out, introducing restrictions on the viability of the savings technology in this economy would expand the scope for operative gift motives. To the extent that developing economies offer more limited opportunities for retirement savings than developed economies, this observation provides an explanation for the greater frequency of child-to-parent gifts in developing economies.

Parts (b) and (c) of the proposition inform us that binding borrowing constraints on the young have no effect on steady-state capital accumulation when the gift motive operates (Regimes E and F). This result points to the force of the modified golden rule

condition in part (b) of the proposition. Operative gift motives push the steady-state interest rate down to  $r_g^*$ , regardless of whether the young dissave.

When the gift motive is inoperative, the effects of binding borrowing constraints on capital accumulation behavior is not so straightforward. Unlike the Regime E and F economies, the steady-state capital stock does not satisfy a modified golden rule condition in Regimes C and D. Thus, the capital accumulation effects of borrowing constraints depend on the properties of technology and preferences (including the strength of the transfer motive) in ways that are not important for the regimes with operative gift motives.

Alternative equilibria in the no-loan economy are illustrated in Figure 3, which is constructed using the same parameterization as Figure 2. We have drawn Figure 3 for the case in which borrowing constraints reduce both the minimal weight that parents must place on children's utility to generate positive transfers and the minimal weight that children must place on the utility of parents to generate positive gifts.<sup>7</sup> A key result illustrated by Figure 3 is that both transfer and gift motives operate in the no-loan economy for an open region of the parameter space.

We have now established that Regimes A-F exhaust the set of equilibrium linkage regimes in the no-loan economy. Regimes A and B emerge when borrowing constraints fail to bind, and Regimes C-F emerge when borrowing constraints bind. The next two sections investigate capital accumulation behavior in the regimes with binding borrowing constraints.

## 6. The Capital Accumulation Effects of Social Security in No-Gift Regimes

In this section we analytically and numerically characterize the capital accumulation effects of social security interventions when borrowing constraints bind and gift motives are inoperative (Regimes C and D). The capital accumulation effects identified here are also present under any other intervention or shock that alters the distribution of resources among generations. As in Diamond (1965), the key ingredients of the analysis are an aggregate savings function and a stability condition that characterizes the dynamic behavior

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<sup>7</sup>The effect of borrowing constraints on the minimal value of  $\gamma$  for which transfer motives operate is studied extensively in Altig-Davis (1989). Although there appears to be no general proof that borrowing constraints weaken the degree of altruism required to generate positive transfers, we showed that this result holds for standard preference specifications.

of the economy along the transition path to a steady-state equilibrium.

#### A. Social Security Interventions

Let  $T_{i,t}$  denote lump-sum taxes (subsidies, if negative) levied on members of generation  $t$  during the  $i$ th period of life. Let  $d_t$  denote the time- $t$  issue of one-period government debt per middle-aged person. The government budget constraint is

$$\frac{(1+r_t)}{1+n}d_{t-1} = (1+n)T_{1t} + T_{2,t-1} + \frac{T_{3,t-2}}{1+n} + d_t.$$

We shall typically assume  $T_3 \leq 0$  and  $T_1, T_2 \geq 0$ .

We define a funded social security intervention as a forced savings program that pays a market rate of return. That is, a funded social security program obeys

$$-T_{3,t} = (1+r_{t+1})(1+r_{t+2})T_{1,t} + (1+r_{t+2})T_{2,t}. \quad (18)$$

Note that the government runs a budget surplus under a funded social security program.

We define an unfunded or pay-as-you-go social security intervention as a forced inter-generational transfer program that satisfies

$$-T_{3,t} = (1+n)^2T_{1,t+2} + (1+n)T_{2,t+1}. \quad (19)$$

Note that, in a steady state, unfunded social security programs offer the individual a rate of return equal to the population growth rate.

#### B. Stability Analysis

In Appendix 1 we derive saving functions for the middle-aged in Regimes C and D when the government engages in social security interventions of the type described above:

$$s_{t+1} = s[\alpha_2 W_{t+1} - T_{2t}, \alpha_3 W_{t+2} - T_{3t}, r_{t+2}], \quad (20)$$

$$s_{t+1} = s\{[\alpha_1(1+n) + \alpha_2]W_{t+1} - (1+n)T_{1,t+1} - T_{2t}, \alpha_3 W_{t+2} - T_{3t}, r_{t+2}\}, \quad (21)$$

with partial derivatives satisfying

$$0 < s_1 < 1, \quad -1 < s_2 < 0, \quad s_1 - (1+r_{t+2})s_2 = 1, \quad \text{and} \quad s_3 > 0. \quad (22)$$



Equation (20) is the saving function for the middle-aged in an economy with no transfers (Regime C). Equation (21) is the saving function for an economy with transfers (Regime D). In the no-transfer regime, savings by the middle-aged is an increasing function of after-tax labor income during middle-age and a decreasing function of after-tax labor income during old age. Savings by the middle-aged increases (decreases) in the interest rate if the substitution (income) effect dominates. The saving function in the transfer regime differs from that of the no-transfer regime only in its first argument. In the transfer regime, savings by the middle-aged is an increasing function of own after-tax labor income during middle-age and the after-tax labor income of children when young.

We now combine the private-sector savings function, the government budget constraint, and the capital market-clearing condition to characterize the dynamic behavior of the aggregate capital stock. The evolution of the aggregate capital stock between  $t + 1$  and  $t + 2$  obeys

$$\alpha(1+n)^2 k_{t+2} = s_{t+1}(\cdot, \cdot, \cdot) - d_{t+1} \equiv S_{t+1}, \quad (23)$$

where  $S_{t+1}$  denotes the aggregate savings function at  $t + 1$ .  $s_{t+1}(\cdot, \cdot, \cdot)$  is given by (20) in the no-transfer regime and (21) in the transfer regime.

Equation (23) implies a relationship between  $k_{t+2}$  and  $k_{t+1}$  which, following Diamond, we refer to as the savings locus. Differentiate (23) to obtain the slope of the savings locus,

$$\frac{dk_{t+2}}{dk_{t+1}} = \begin{cases} \frac{-\alpha_2 s_1 k_{t+1} f''(k_{t+1})}{\alpha(1+n)^2 + \alpha_3 s_2 k_{t+2} f''(k_{t+2}) - s_3 f''(k_{t+2})}, & \text{in the no-transfer economy;} \\ \frac{-[\alpha_1(1+n) + \alpha_2] s_1 k_{t+1} f''(k_{t+1})}{\alpha(1+n)^2 + \alpha_3 s_2 k_{t+2} f''(k_{t+2}) - s_3 f''(k_{t+2})}, & \text{in the transfer economy.} \end{cases} \quad (24)$$

The numerator is unambiguously positive, but the denominator can be positive or negative. If  $\alpha_3 = 0$ , so that the old supply no labor services, the middle term in the denominator vanishes, and the expression for the slope of the savings locus has the same basic form as in Diamond's model. (See Blanchard and Fischer (1989), p. 96.)

What does (24) imply about the transition path to the steady-state equilibrium? Restricting attention to stable steady states, there are two cases to consider. If  $0 < dk_{t+2}/dk_{t+1} < 1$  (in the neighborhood of the steady-state equilibrium), then the capital stock converges monotonically to its steady-state value. Alternatively, if  $0 > dk_{t+2}/dk_{t+1} > -1$ , then the capital stock oscillates around the steady-state value along the transition path.

Equation (24) also determines the steady-state capital stock response to nonneutral social security interventions, as in Samuelson's (1947) correspondence principle. As we show in Appendix 2, when the denominator in (20) is positive, the partial equilibrium response of aggregate savings to social security interventions carries over, in qualitative terms, to the general equilibrium response. In contrast, when the denominator in (24) is negative, the partial equilibrium effect of social security on aggregate savings is reversed in general equilibrium. Hence, we refer to steady-state equilibria that satisfy  $0 < dk_{t+2}/dk_{t+1} < 1$  as stable and regular.

### *C. The Effects of Social Security In the No-Gift Regimes*

We are now prepared to characterize the effects of social security interventions on capital accumulation in Regimes C and D. We first describe the steady-state effects.

*Proposition 4:* Consider the overlapping generations framework with binding borrowing constraints on the young and an inoperative gift motive (Regimes C and D). Assume that the steady-state equilibrium is stable, regular and unique (pre- and post-intervention).

- (a) A funded social security system financed by taxes on the middle-aged has no effect on capital accumulation.
- (b) A funded social security system financed by taxes on the young increases the steady-state (per capita) capital stock.
- (c) An unfunded social security system decreases the steady-state capital stock.
- (d) If the transfer motive operates, the generational incidence of the taxes used to finance old-age benefits under an unfunded system is irrelevant to the determination of the capital stock. If the transfer motive is inoperative, a shift in taxes from the middle-aged to the young increases the capital stock.

*Proof:* See Appendix 2.

If we drop the uniqueness assumption in Proposition 4, then the results apply in some neighborhood of the initial steady-state equilibrium. If we drop the regularity assumption, then the qualitative responses to nonneutral interventions are reversed.

Returning to the linkage diagrams in Figure 1, the intuition behind Proposition 4 can be understood as follows. The neutrality result in part (a) reflects the intertemporal link between the middle-aged and the old in Regimes C and D. Since the middle-aged are

already trading-off own current consumption for own future consumption at the rate  $(1+r)$ , they fully offset the funded social security intervention. In this respect, the borrowing-constrained economies mirror the behavior of the standard life-cycle economy depicted in the diagram by Regime A.

Likewise, the irrelevance result in part (d) of the proposition for the transfer economy reflects the intergenerational link between the middle-aged and young as illustrated in the diagram for Regime D. When the transfer motive operates, the young and middle-aged are trading-off consumption at the rate  $(1+n)$ , which is identical to the trade-off implied by shifts in the generational incidence of taxes under an unfunded social security system. This logic holds regardless of whether the young are borrowing constrained.

Turning to the nonneutral interventions, consider a funded social security program financed by a one dollar tax on each young person. There are distinct impact and secondary effects here, both of which lead to an increase in the capital stock. First, aggregate savings rises because the government forces each of the  $(1+n)$  young persons to save one dollar. This impact effect is mitigated, but not reversed, when the transfer motive operates, because middle-aged parents adjust transfers to partially compensate the young for their disposable income loss. Hence, when the transfer motive operates, the partial equilibrium impact effect on aggregate savings is  $(1+n)(1-s_1)$ . Second, after the funded program has been in operation for more than one period, each middle-aged person experiences a  $(1+r)$  dollar increase in own wealth over the last two periods of life. This effect leads to a further increase in aggregate savings in the amount of  $(1+r)$  times the marginal propensity to save out of middle-aged income. Thus, in the no-transfer economy, the partial equilibrium effect is to increase aggregate savings by  $(1+n) + s_1(1+r)$ . In the transfer economy, the partial equilibrium effect is to increase aggregate savings by only  $(1+n)(1-s_1) + s_1(1+r)$ . The regularity condition,  $0 < dk_{t+2}/dk_{t+1} < 1$ , insures that these partial equilibrium effects carry over to the general equilibrium.

Now, consider the effects of an unfunded social security program. An unfunded social security program weakens the life-cycle motive for saving by shifting the timing of income receipt to a later period of life. The increase in after-tax income during old age leads to a partial equilibrium reduction in aggregate savings. This is the only effect when taxes fall entirely on the borrowing-constrained young and the transfer motive is inoperative.

If taxes fall on the young and the transfer motive operates, then altruistic transfers from the middle-aged to the young rise. Hence, the net-of-transfer income of the middle-aged falls, and there is a further depressive effect on aggregate savings. If the tax falls on the middle-aged, then the decline in the after-tax income of the middle-aged is an additional effect contributing to the reduction in savings. Under all of these scenarios, an unfunded social security program depresses savings.

We can use Proposition 4 to draw a sharp distinction between our no-loan economy with operative transfers and that of Laitner. In our no-loan economy, the nonneutrality of unfunded social security programs entirely reflects the effects of government-mandated transfers between persons who are members of the same family line. Furthermore, in regime D nonneutrality holds despite altruistic linkages that connect each person to his parent and children at some stage of the life cycle. In Laitner's model, government-mandated transfers between persons who are members of the same family line are neutral. Neutrality of these transfers holds in Laitner's model, because each person weights his parent's and child's utility as heavily as his own. It follows that the nonneutrality of unfunded social security in Laitner's model entirely reflects the effects of government-mandated transfers between persons who are members of different family lines. Presumably, a sufficiently rich model would capture both the between-generation intra-family effects of our analysis and the between-generation inter-family effects of Laitner's analysis.

To conclude this section, we note that our stability analysis can be used to characterize the dynamic capital accumulation response to nonneutral social security interventions. For example, consider a one-time, permanent social security intervention in the economy with binding borrowing constraints and an inoperative gift motive. Assume that the initial and new steady-state equilibria are stable, regular and unique. If the intervention is nonneutral, then it can be shown that (per capita) capital accumulation is monotonic along the transition path from the initial to the new steady-state equilibrium.<sup>8</sup>

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<sup>8</sup>The proof of this claim is implicit in the preceding discussion. Our stability analysis shows that, under the assumed conditions, the transition path to a steady-state equilibrium is monotonic. It remains only to check that any secondary effects of a social security intervention shift the savings locus in the same direction as the impact effect. For interventions involving changes in an unfunded program, there are no secondary effects. For interven-

#### *D. The Magnitude of the Response to Social Security Interventions*

We have carried out several numerical simulation experiments to gauge the magnitude of the capital accumulation responses to nonneutral social security interventions. Our experiments trace out the transition path from the initial steady state to the new steady state in Regimes A, C and D in response to one-time, permanent social security interventions. In the interest of brevity, we discuss here only the main messages of the experiments. Altig and Davis (1990) describe the numerical simulation technique and provide a full discussion of the results.

Our baseline parametrization assumes: capital's share equal to .25 in a Cobb-Douglas production function; a lifetime productivity profile  $(\alpha_1, \alpha_2, \alpha_3) = (1.5, 6.0, 2.5)$ ; no government taxes or subsidies at the initial steady state; an intervention that introduces an old-age benefit payment equal to 6% of the old's wage income in the initial steady state; an intertemporal elasticity of substitution in consumption equal to .4; a population growth rate,  $n$ , equal to  $(1 + .01)^{25} - 1$ ; and an intertemporal discount factor,  $\beta$ , equal to .99<sup>25</sup>. Here, we interpret a period in the model as corresponding to twenty-five years. In Regime D we set the interpersonal discount factor,  $\gamma$ , equal to .10. Our sensitivity analysis varies the intertemporal substitution elasticity within the range  $[1/3, 1]$  while varying  $\gamma$  within the range  $[\cdot 1, .52]$ . The main messages of our simulation results are unaffected by parameter variation within these ranges.

The most important message to emerge from our simulation experiments is that large crowding-out ratios are fully consistent with altruistic intergenerational linkages. In our baseline parametrization of Regime D, the introduction of unfunded social security causes a long-run capital stock decline equal to 58% of the old-age benefit payment. During the first period of operation, the intervention reduces the capital stock by 36% of the old-age benefit payment. Convergence to the new steady-state equilibrium is largely complete after four periods. Our sensitivity analysis reveals closely similar results for a wide range of alternative values of the altruism parameter  $\gamma$ . Thus, borrowing constraints imply a quantitatively significant departure from the Ricardian benchmark.

A second important message involves the capital accumulation response to shifts in the conditions involving changes in a funded program, the wealth effect on the savings behavior of the middle-aged reinforces the impact effect.

generational incidence of social security taxes when borrowing constraints bind. Viewed from the perspective of either regimes C or D, standard life-cycle models provide highly misleading implications about the capital accumulation effects of shifts in the generational incidence of social security taxes. Under an unfunded intervention in our baseline parametrization, a shift from taxes on the young to taxes on the middle-aged reduces the crowding-out ratio from .43 to .05 in the life-cycle regime A. In Regime D, the shift has no effect (Proposition 4(d)). In Regime C, the shift *increases* the crowding-out ratio from .28 to .64. Under a funded intervention, social security is neutral in all regimes when the middle-aged pay the taxes. But a shift in taxes to the young causes a modest increase in the capital stock when borrowing constraints bind, as in Regimes C and D, whereas the shift has no effect in the life-cycle Regime A.

## 7. The Effects of Social Security In Gift Regimes

We turn now to the effects of social security in regimes E and F. We begin with the following neutrality proposition, where a “small” intervention means one that does not shift the economy between regimes.

*Proposition 5:* Assume that borrowing constraints bind, and that the gift motive operates.

- (a) Any (small) social security intervention that fails to impinge on the budget constraint of the young is neutral in its impact on capital accumulation, the consumption profile and welfare.
- (b) If the transfer motive operates, any (small) unfunded social security intervention is neutral in its impact on capital accumulation, the consumption profile and welfare.

*Proof:* Part (a) follows from the linkage diagrams for Regimes E and F. Part (b) follows from the linkage diagram for Regime F.

**Q.E.D.**

Proposition 5, and its proof, is entirely in the spirit of standard Ricardian neutrality results in environments with operative gift or transfer motives and perfect capital markets. That is, the proposition rests on the implications of intergenerational altruism for the interconnectedness of budget constraints. Despite this apparent parallel between the no-loan and loan economies, operative gift motives turn out to carry much stronger implications for capital accumulation behavior than operative transfer motives, when borrowing

constraints bind. As a corollary to the modified golden rule condition in Proposition 3(b), we have the following *non-Ricardian* neutrality result.

*Proposition 6:* Assume that the gift motive operates. Then all (small) social security interventions are neutral in their impact on the steady-state interest rate and capital stock.

An operative gift motive does not imply full neutrality when borrowing constraints bind and parent-to-child altruistic transfer motives are inoperative. In this case (Regime E), social security interventions that impinge on the budget constraint of the young affect the shape of the lifetime consumption profile.

Possible effects on the consumption profile notwithstanding, Proposition 6 is a surprisingly robust neutrality result. It applies regardless of whether parent-to-child transfer motives operate early in the life cycle. It applies regardless of whether young persons are borrowing constrained. Provided that the gift motive remains operative for the altruists, Proposition 6 survives the introduction of non-altruistic agents into the economy.

To place this non-Ricardian neutrality result in perspective, two comments are in order. First, Proposition 6 differs in an essential way from the neutrality results that appear in Barro (1974), Becker (1974), Bernheim and Bagwell (1988), Altig and Davis (1989), and the many related papers in the literature. The neutrality results in the Barro-Becker-Bernheim/Bagwell tradition rest upon an extensive interconnected network of budget constraints. Hence, these neutrality results break down, partially or completely, if operative altruistic linkages are insufficiently pervasive to maintain the fully interconnected network of budget constraints. In contrast, our interest-rate neutrality result follows immediately from the intertemporal and gift-motive first-order conditions of the middle-aged.<sup>9</sup> Thus, Proposition 6 directly exploits the properties of altruistic preferences, unlike neutrality results in the Barro-Becker-Bernheim/Bagwell tradition, which exploit the implications of altruistic preferences for connections among budget constraints.

Second, Proposition 6 fails if we sufficiently relax the separability assumptions em-

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<sup>9</sup>To the best of our knowledge, Summers (1982) and Altig and Davis (1991) are the only other writers to exploit the first-order conditions in this way to obtain steady-state neutrality results. Neither of these papers derives a neutrality result in the presence of borrowing constraints.

bodied in (1). Consider the the general form for preferences

$$U_t = u(C_{1t}, C_{2t}, C_{3t}, U_{t-1}^*),$$

where we ignore parental altruism for simplicity. By combining the steady-state versions of (14) and (17), assuming an operative gift motive, we obtain

$$1 + r = u_4(C_1, C_2, C_3, U^*).$$

Now, in the context of regime E, consider a social security intervention that impinges on the budget constraint of the young. If  $u_{41} \neq 0$ , then interest-rate neutrality fails to hold. But, note that either intertemporal *or* interpersonal separability implies  $u_{41} = 0$ . Even if  $u_{41}$  is nonzero, interest-rate determination in Regime E is radically different than in Ricardian and life-cycle regimes.<sup>10</sup>

## 8. Concluding Remarks

The interaction between capital market imperfections and intergenerational altruism carries important implications for the life-cycle timing of intergenerational transfers and for aggregate capital accumulation behavior. We characterize these implications when capital market imperfections take the form of borrowing constraints on the young and altruistic preferences do not engender strategic behavior. Our analysis complements the work of Laitner (1990) who adopts a different approach to simplifying the potentially complex set of interactions between borrowing constraints and altruistically motivated intergenerational linkages.

Several important questions remain open.

First, it is natural to ask whether our key results survive in environments with milder imperfections in the consumption-loans market. In Altig and Davis (1991), we consider environments with intergenerational altruism and small imperfections in the capital market. The imperfections take the form of a wedge between borrowing and lending rates that stems from the asymmetric tax treatment of interest income and interest payments on

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<sup>10</sup>We thank Jim Davies for directing our attention to the separability assumption that underlies Proposition 6.



consumption loans. Our timing proposition survives in this environment, and the interest-rate neutrality proposition emerges in an even more powerful form. Surprisingly, however, a dichotomy arises between the short-run and long-run capital accumulation responses to social security when parental altruism operates and households face a kink in their intertemporal budget constraint. In the short-run, an unfunded social security program crowds-out capital just as in the no-loan economy of this paper, but eventually the economy returns to the initial equilibrium capital intensity.

The analysis in Altig and Davis (1991, Appendix C) also shows that non-Ricardian interest-rate neutrality propositions survive the introduction of intra-generational heterogeneity. This result suggests that the analysis here captures important aspects of capital accumulation behavior even when segments of the population are not linked by operative altruism motives, or when some segments face other constraints or costs like the absence of a viable saving technology.

Second, we abstracted from individual uncertainty about lifetime earnings and longevity. Coupled with less than perfect insurance and annuity markets, these factors imply incentives for altruistic parents to defer transfers to children, even borrowing-constrained children, as they await the resolution of uncertainty. Thus, uncertainty about earnings and longevity mitigates against the proposition we derived about the optimal timing of parent-to-child transfers. Furthermore, to the extent that social security influences the magnitude of precautionary savings in an uncertain environment, the argument underlying our interest-rate neutrality proposition may be undercut. While we have yet to formally address these issues, straightforward modifications of our analytical framework provide a useful vehicle for doing so. Issues associated with annuity market imperfections, for example, are easily introduced into our framework by assuming that persons face uncertainty about whether they live for two or three periods. In future research, we hope to determine how the interaction among borrowing constraints, imperfect annuity markets, and altruistically motivated intergenerational linkages shapes the aggregate savings and welfare response to social security and other government interventions.

Finally, as noted in section 3, we have assumed away the potential difficulties that arise when parents and children behave strategically. We conjecture, however, that this assumption is inessential to the derivation of steady-state interest rate neutrality in the

gift-motive economy. In a noncooperative environment, only the exact form—and not the essential nature—of the intertemporal and interpersonal first-order conditions underlying interest rate neutrality seems to depend on whether parents engage in this type of strategic behavior.

In contrast, strategic behavior in a framework of cooperative bargaining between altruistic parents and children is likely to undercut the interest rate neutrality proposition. This conjecture is based on the observation, stressed by Kotlikoff, Razin and Rosenthal (1988), that government redistributions alter the strategic postures (i.e., threat points) of parents and children in a cooperative bargaining framework, and that strategic postures in turn influence the magnitude of net transfers. Whether the optimal timing propositions carry over directly to a cooperative bargaining framework is not clear to us, but the factors underlying the timing results in our noncooperative environment would seem to be present in a cooperative environment as well.

*Appendix 1-Derivation of Saving Functions*

We first derive the savings function of the middle-aged in Regime C. Defining  $x_{2t} + d_{t+1} \equiv s_{t+1}$ , use the budget constraints (3) and (4) to write (14) as

$$u'[\alpha_2 W_{t+1} - T_{2t} - s_{t+1}] = \beta(1 + r_{t+2})u'[\alpha_3 W_{t+2} + (1 + r_{t+2})s_{t+1} - T_{3t}].$$

This equation implies the existence of a savings function for the middle-aged of the form given by equation (20).

In Regime D the savings function has similar properties, but its derivation is more complicated. Combining the transfer-motive first-order condition (15) and the intertemporal first-order condition (14), we have

$$u'(C_{1,t+1}) = \frac{\beta(1+n)}{\gamma}(1 + r_{t+2})u'(C_{3t}).$$

Thus,

$$\begin{aligned} C_{1,t+1} &= \phi \left[ \frac{\beta(1+n)}{\gamma}(1 + r_{t+2})u'(C_{3t}) \right] \equiv \psi[C_{3t}, r_{t+2}] \\ &= \psi[\alpha_3 W_{t+2} - T_{3t} + (1 + r_{t+2})s_{t+1}, r_{t+2}] \end{aligned}$$

with  $\psi_1 > 0$  by the concavity of  $u(\cdot)$ .

Using this expression for  $C_{1,t+1}$  and the household budget constraints, write (14) as

$$\begin{aligned} u'[\alpha_2 W_{t+1} - T_{2t} - s_{t+1} - (1+n)\psi(\cdot) + \alpha_1(1+n)W_{t+1} - (1+n)T_{1,t+1}] = \\ \beta(1 + r_{t+2})u'[\alpha_3 W_{t+2} + (1 + r_{t+2})s_{t+1} - T_{3t}]. \end{aligned}$$

This equation implies a savings function for the middle-aged given in equation (21).

*Appendix 2-Proof To Proposition 4*

*Part (a):* In this intervention,  $T_{3t} = -(1 + r_{t+2})T_{2t}$ . Using (20)-(22), the time  $t + 1$  partial equilibrium response of savings by the middle-aged is  $-s_1T_{2t} + (1 + r_{t+2})s_2T_{2t} = T_{2t}[(1 + r_{t+2})s_2 - s_1] = -T_{2t}$ . But from (23) and the government budget constraint, government savings rises by  $T_{2t}$ . Hence, the net effect on aggregate savings is nil.

*Part (b):* Consider a shift in the financing of a funded social security system from taxes on the middle-aged to taxes on the young. Since we want to deduce the steady-state effect of this intervention, assume that it has been in operation for more than one period as of  $t + 1$ . Using the government budget constraint, and the steady-state condition  $T_{1t} = T_{1,t+1}$ , the accumulation of capital between  $t + 1$  and  $t + 2$  obeys

$$s[\alpha_2 W_{t+1} + (1 + r_{t+1})T_{1,t+1}, \alpha_3 W_{t+2}, r_{t+2}] + T_{1,t+1} = \alpha(1 + n)^2 k_{t+2}$$

in the no-transfer economy,

$$s[(\alpha_1(1 + n) + \alpha_2)W_{t+1} + (r_{t+1} - n)T_{1,t+1}, \alpha_3 W_{t+2}, r_{t+2}] + T_{1,t+1} = \alpha(1 + n)^2 k_{t+2}$$

in the transfer economy.

Now, calculate the partial equilibrium effect of the increase in  $T_{1,t+1}$  on aggregate savings at  $t + 1$ :

$$\frac{\partial S_{t+1}}{\partial T_{1,t+1}} = \begin{cases} (1 + r_{t+1})s_1 + (1 + n) > 0, & \text{in the no-transfer economy;} \\ s_1 r_{t+1} + 1 + (1 - s_1)n > 0, & \text{in the transfer economy.} \end{cases}$$

We can use this result to determine how the savings locus shifts. Differentiate the savings locus, holding  $k_{t+1}$  constant, to obtain

$$\frac{dk_{t+2}}{dT_{1,t+1}} = \frac{\partial S_{t+1}/\partial T_{1,t+1}}{\alpha(1 + n)^2 + \alpha_3 s_2 k_{t+2} f''(k_{t+2}) - s_3 f''(k_{t+2})}$$

By the regularity assumption, this expression exceeds zero. Hence, the intervention shifts the savings locus A upwards in Figure 4, and the steady-state capital stock rises. Combining this result with the neutrality result in part (a) proves part (b).

*Part c:* Consider an unfunded intervention financed by taxes on the middle-aged. That is,  $T_{1t} = 0$  and  $T_{3t} = -(1 + n)T_{2,t+1}$ . Using the steady-state condition  $T_{2,t+1} = T_{2t}$

and the aggregate savings function, we obtain the partial equilibrium effect on savings in both economies:

$$\frac{\partial S_{t+1}}{\partial T_{2,t+1}} = -s_1 + (1+n)s_2 < 0,$$

using (21). Differentiating the aggregate savings locus for a fixed  $k_{t+1}$ , yields

$$dk_{t+2}/dT_{2,t+1} < 0,$$

using the regularity assumption.

When the unfunded intervention is financed by taxes on the young, the partial equilibrium response of aggregate savings is given by

$$\frac{\partial S_{t+1}}{\partial T_{1,t+2}} = \begin{cases} (1+n)^2 s_2 < 0, & \text{in the no-transfer economy;} \\ -(1+n)s_1 + (1+n)^2 s_2 < 0, & \text{in the transfer economy.} \end{cases}$$

Differentiating the aggregate savings locus as before, and using the regularity assumption, yields  $dk_{t+2}/dT_{1,t+2} < 0$ . This proves part (c).

*Part d:* Compare the partial equilibrium savings responses for the two different methods of financing an unfunded system. In the no-loan economy,  $(1+n)\partial S_{t+1}/\partial T_{2,t+1} < \partial S_{t+1}/\partial T_{1,t+2}$ , so that a shift to taxes on the young, for a fixed old-age benefit, increases the capital stock. In the loan economy,  $(1+n)\partial S_{t+1}/\partial T_{2,t+1} = \partial S_{t+1}/\partial T_{1,t+2}$ , so that the generational incidence of the tax is irrelevant.

#### ACKNOWLEDGEMENTS

We thank three anonymous referees for helpful comments. We also thank Andy Abel, Doug Bernheim, Don Cox, Jim Davies, participants in an NBER conference on Social Insurance, and workshop participants at the Federal Reserve Bank of St. Louis, the Hoover Institution, UCLA, UC-Irvine, and the University of Chicago for helpful comments on an earlier draft of this paper titled, "Altruism, Borrowing Constraints, and Social Security." The earlier paper was written while Altig was at the Graduate School of Business, Indiana University, and while Davis was at the Hoover Institution, Stanford University. We gratefully acknowledge financial support from the Summer Research Grant Program at Indiana and from the Hoover Institution. Davis also thanks the National Science Foundation for its support through a grant to the National Fellows Program at the Hoover Institution. Jim Klopfenstein provided excellent programming assistance.

## References

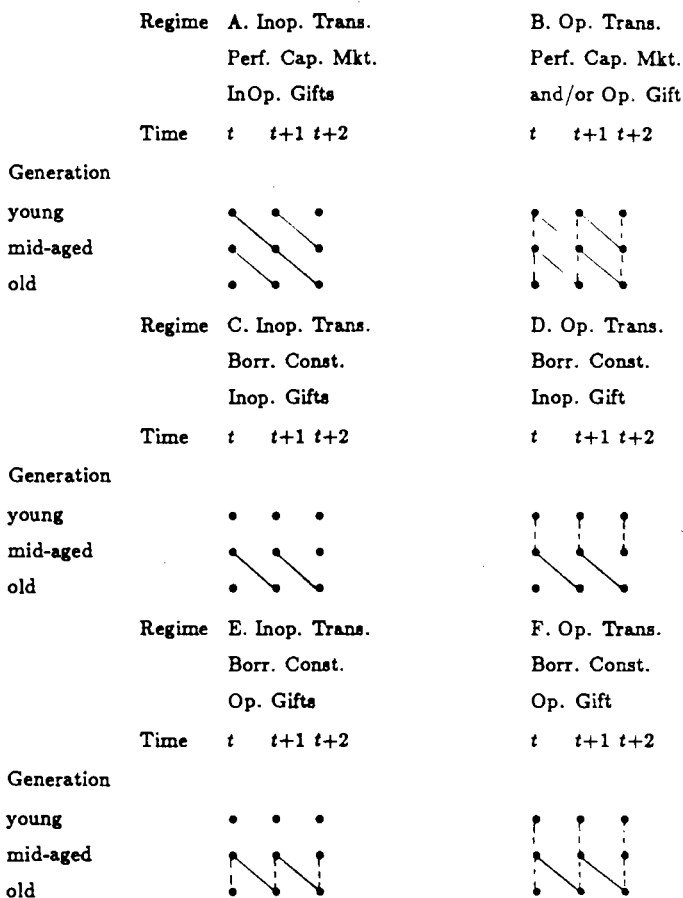
- Abel, A., 1987, Operative gift and bequest motives, *American Economic Review*, 77, 1037-1047.
- Altig, D. and S.J. Davis, 1989, Government debt, redistributive fiscal policies, and the interaction between borrowing constraints and intergenerational altruism, *Journal of Monetary Economics*, 24, 3-29.
- Altig, D. and S.J. Davis, 1990, Altruism, borrowing constraints, and social security, manuscript, Indiana University and University of Chicago.
- Altig, D. and S.J. Davis, 1991, The timing of intergenerational transfers, tax policy, and aggregate savings, NBER Working Paper No. 3753, National Bureau of Economic Research, Cambridge, MA.
- Auerbach, A.J. and L.J. Kotlikoff, 1987, *Dynamic Fiscal Policy*, Cambridge University Press.
- Barro, R.J., 1974, Are government bonds net wealth? *Journal of Political Economy*, 82, 1095-1117.
- Becker, G. S., 1974, A theory of social interactions, *Journal of Political Economy*, 82, 1063-1093.
- Bernheim, B.D. and K. Bagwell, 1988, Is everything neutral? *Journal of Political Economy*, 96, 308-338.
- Bernheim, B.D. and O. Stark, 1988, Altruism within the family reconsidered: Do nice guys finish last?, *American Economic Review*, 78, no. 5, 1034-1045.
- Blanchard, O. and S. Fischer, 1989, *Lectures on Macroeconomics*, MIT Press: Cambridge, MA.
- Bruce, N. and M. Waldman, 1990, The rotten-kid theorem meets the Samaritan's dilemma, *Quarterly Journal of Economics*, 105, no. 1, 155-165.
- Buchanan, J., 1975, The Samaritan's dilemma, in E.S. Phelps, ed., *Altruism, morality, and economic theory*, Russell Sage Foundation: New York.
- Cox, D., 1990, Intergenerational transfers and liquidity constraints, *Quarterly Journal of Economics*, 105, 187-218.
- Cox, D. and E. Jimenez, 1990, Motives for private transfers over the life cycle: An analytical framework and evidence from Peru, manuscript, Boston College.
- Cox, D. and F. Raines, 1985, Interfamily transfers and income redistribution, in M. David and T. Smeeding, eds., *Horizontal equity, uncertainty, and economic well-being*, University of Chicago Press: Chicago.
- Diamond, P. A., 1965, National debt in a neoclassical growth model, *American Economic*

*Review*, 55, 1126-1150.

- Feldstein, M., 1974, Social security, induced retirement, and aggregate capital accumulation, *Journal of Political Economy*, 82, 905-926.
- Guiso, L. and T. Jappelli, 1991, Intergenerational transfers and capital market imperfections: Evidence from a cross-section of Italian households, *European Economic review*, 35, 103-120.
- Jappelli, T., 1990, Who is liquidity constrained in the U.S. economy?, *Quarterly Journal of Economics*, 105, 219-234.
- Kimball, M., 1988, Making sense of two-sided altruism, *Journal of Monetary Economics*, 20, 301-326.
- Kotlikoff, L.J., A. Razin, and R.W. Rosenthal, 1988, A strategic altruism model in which Ricardian equivalence does not hold, working paper, Boston University.
- Laitner, J., 1990, Long-run equilibria with borrowing constraints and altruism, manuscript, University of Michigan, Ann Arbor, MI.
- Lindbeck, A. and J. W. Weibull, 1988, Altruism and time consistency: the economics of fait accompli, *Journal of Political Economy*, 96, no. 6, 1165-1182.
- Mariger, R. P. 1986, *Consumption Behavior and the Effects of Government Fiscal Policy*, Cambridge, Mass.: Harvard University Press.
- Samuelson, P. A., 1947, *Foundations of Economic Analysis*, Harvard University Press: Cambridge, MA.
- Summers, Lawrence, 1982, Tax Policy, the Rate of Return, and Savings, NBER w.p. no. 995.
- Weil, Phillipe, 1987, Love thy children: reflections on the Barro debt neutrality theorem, *Journal of Monetary Economics*, 20, 369-391.
- Zeldes, Stephen, 1989, Consumption and Liquidity Constraints: An Empirical Investigation, *Journal of Political Economy*, 97, no. 2, 305-346.



**Figure 1: Patterns of Intertemporal and Intergenerational Linkages**



**Note:** Solid lines represent capital market (intertemporal) linkages; Dashed lines represent altruistically motivated intergenerational linkages.

Figure 2: Loan Economy

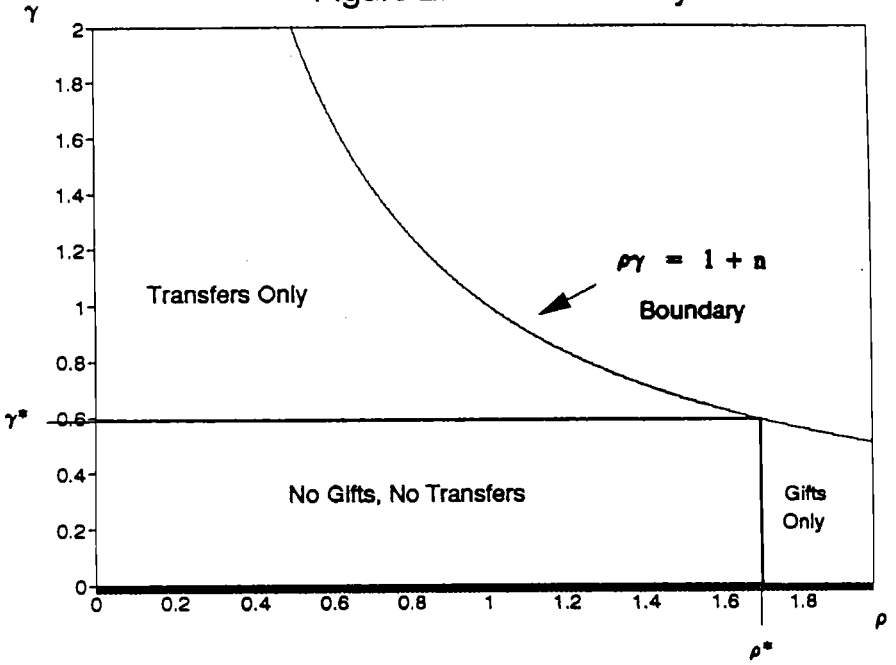


Figure 3: No-Loan Economy

