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ABSTRACT

The effect of uninsured labor income risk on the joint saving/portfolio composition decision is analyzed using new techniques from the theory of multiple risk-bearing. Applying this analysis, the effect of labor income taxes on the demand for risky securities is considered. It is well known that when private insurance markets are incomplete, the insurance afforded by labor income taxes can reduce overall saving. This paper establishes that - given plausible restrictions on preferences - the insurance afforded by labor income taxes increases the demand for risky securities, even when labor income is statistically independent of the returns to risky securities.

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What are the effects on saving of a decrease in current taxes combined with an offsetting increase in future taxes? If the offsetting increase occurs within an individual's lifetime, then in a world of perfect capital markets and no uncertainty, the individual will save the entire amount of the tax cut. However, in reality, there is significant uncertainty about future labor income, and because proportional (or progressive) labor income taxes reduce the variance of income, they provide insurance against this uncertainty. An increase in future taxes increases this insurance and, through the precautionary saving motive, reduces an individual's saving relative to the Ricardian benchmark just stated. Moreover, the increased insurance may change the composition of that saving, because the reduction in labor income risk can affect the amount of financial risk that an individual chooses to bear. In this paper we explore the effect of labor income taxes on the willingness to bear financial risk.

We study a simple two-period life-cycle model in which individuals make two choices: how much to save in total, and how to divide that saving between a risky asset and a risk-free asset. We find that, given plausible restrictions on individuals' preferences, any change in taxes that lowers labor income risk and does not make an individual worse off will lead that individual to invest more in the risky security. This result applies even when labor income is statistically independent of the return to the risky asset, although not if the risky asset actually provides insurance for labor income risk.

In particular, consider once more deferring labor income taxes with no change in their expected present value. In a Ricardian world with riskless labor income, deferring labor income taxes leaves national saving unchanged (by raising total private saving by the same amount as the increase in public dissaving) and has no effect on private investment in the risky asset. All of the extra private saving is in the riskless asset since the future tax liability involves no risk and individuals want to offset that future liability. In a Ricardian world with risky labor income, however, our analysis shows that deferring labor income taxes raises private investment in the risky asset. In essence, individuals respond to a reduction in one risk by increasing their exposure to another independent risk.[1]

Surprisingly, the effect of deferring labor income taxes on total national saving is unclear. If the uncertainty of labor income were the only risk an individual faced, the analysis of Chan (1983),

[1] This result is closely related to the findings of Pratt and Zeckhauser (1987) and Kimball (1991) that, for a wide class of utility functions, when an agent is forced to accept one risk, the agent will be less willing to accept other independent risks. Kimball's (1991) analysis is the more closely related to the analysis in this paper because (a) Kimball (1991) links the interaction between risks to the effect of the risks on expected marginal utility, which also governs consumption decisions, and (b) Kimball (1991) deals with differential changes in risk as well as with the discrete introductions of risk treated by Pratt and Zeckhauser (1987).

Barsky, Mankiw and Zeldes (1986), and Kimball and Mankiw (1989) would apply: the individual consumes more in response to the tax rescheduling and in the aggregate, national saving falls. When the individual faces two or more risks, though, we cannot rule out the possibility that the interaction between the risks that is our main subject might allow a deferral of labor income taxes to cause an *increase* in consumption and a *reduction* in national saving.

Given our main result that deferring labor income taxes leads to increased investment in risky assets, two natural questions arise. First, is the effect on financial risk-taking of a reduction in idiosyncratic labor income risk due to tax reschedulings big enough to be worth studying? Second, is an increase in financial risk-taking desirable? We believe that the answer to both questions should be "yes." We argue in Section IV that additional financial risk-taking induced by a reduction in idiosyncratic labor income risk would increase social welfare. We argue here that the effects on financial risk-taking of idiosyncratic labor income risk, and of possible changes in the level of that risk, are important.

First, the ability to earn labor income in the future is the largest asset held by many people, and carries with it a large and mostly undiversifiable and unmarketable risk. Davies and Whalley state that "human capital is substantially larger in aggregate value than the physical capital stock" (1991, pp. 191-192). MacDonald writes: "That the extent to which success is achieved ... is highly variable even within narrowly defined groups is a basic fact of empirical labor economics" (1988, p. 155). And Barsky, Mankiw and Zeldes (1987) conclude that "an examination of the degree of income uncertainty suggests that this uncertainty is substantial, indicating that the risk-sharing effect may be important" (p. 688).

Second, calculations like those in Barsky, Mankiw and Zeldes (1987) and Kimball and Mankiw (1989) indicate large effects of risk-sharing taxes on consumption behavior. Barsky, Mankiw and Zeldes (1987) write: "The marginal propensity to consume out of a tax cut, coupled with a future income tax increase, appears closer to the Keynesian value that ignores the future taxes than to the Ricardian value that treats the future taxes as if they were lump sum" (p. 688). One of our main results (Proposition 2 below) makes these calculations relevant to the question at hand by affirming that the effect of idiosyncratic labor income risk on financial risk-taking is even stronger than its effect on consumption, where the strength of each effect is measured by the change in initial wealth that would be needed to completely offset it. In other words, government bonds matched by extra risk-sharing taxes in the future represent more "net wealth" for the purpose of portfolio composition decisions than for the purpose of consumption decisions.[2]

Third, our results about the effect of labor income taxes on risk-taking apply with equal force to a range of social insurance programs. Almost all research on the effect of Social Security on saving, for example, considers only total saving, even though many of the same economists worry about risky saving in other contexts. For example, Feldstein (1983) writes that "these [tax-induced] distortions in portfolio behavior imply that capital is misallocated among industries and firms, ... and [alter] the mix of financial assets that firms supply and the allocation of risk-bearing in the economy" (pp. 12-13). But when looking at the effects of Social Security, Feldstein (1985) writes that "the primary cost of providing social security benefits is the welfare loss that results from reductions in private saving," (p. 305) with no mention of the effects on risk-taking. Our analysis shows that these concerns must be integrated, and that if Social Security does lower overall saving, it must increase investment in risky assets. One should therefore be more cautious than Feldstein (1985) in concluding that the direct risk-sharing benefits of Social Security come at a serious indirect cost because of an adverse effect on saving. It is at least possible that social insurance has a positive effect on the dynamism of the economy by making individuals more willing to take entrepreneurial risks.

The rest of the paper is organized as follows. Section I discusses its relationship to previous research. Section II describes the model we use, and Section III presents the main results about the effect of labor income uncertainty on an individual's joint saving/portfolio composition decision. Section IV, as mentioned above, considers the importance of risk-taking to social welfare. Section V compares the effects of government deficit finance suggested by our analysis to the previous literature on financial crowding in. Section VI offers a brief conclusion.

I. Relation to Previous Research

In its analysis of the effect of labor income taxes on the demand for risky assets, this paper bridges two lines of research. The first is concerned with the role of progressive labor income taxes in providing insurance for risky labor income and their resulting effect on the consumption/saving decision. It includes research about the aggregate demand effects of tax cuts (for example, the papers mentioned above) and about the merits of redistributive taxation (for example, Eaton and Rosen, 1980, and Varian, 1980), but little about the possible effects of labor income taxes on portfolio composition.

Analysis of the consumption/saving decision under uncertainty began in earnest with Leland

[2] This way of thinking about the problem obviously owes something to Barro (1974).

(1968), Sandmo (1970), and Drèze and Modigliani (1972), and is continued recently by papers mentioned above and by Skinner (1988), Zeldes (1989), Kimball (1990a,b), Caballero (1990), Weil (1991), and Kimball and Weil (1991). Drèze and Modigliani (1972) discuss portfolio choices but only determine the conditions for their separability from saving decisions and conclude that perfect insurance markets for labor income are essential for separation to hold. Varian (1980) studies the effect of uncertainty and uncertainty-reducing taxes in an analysis of the merits of redistributive taxation. He determines the optimal tax schedule by balancing two effects: the effect of social insurance on the amount of saving and the direct effect of social insurance on individuals' expected utility.[3] If it is appropriate to encourage investment in risky assets, as we argue in Section IV, then the implicit insurance provided by labor income taxes has an additional benefit neglected by Varian (1980).

The second line of research is concerned with the role of capital taxes in providing insurance for financial risks and thus affecting the overall amount of financial risk-taking in the economy. Domar and Musgrave (1944), and many following them, analyze optimal portfolio selection among a collection of fully marketable securities. Some of this research (summarized by Sandmo, 1985) includes the consumption/saving decision (Sandmo, 1969 and Ahsan, 1976) but does not allow for risky labor income. Friend and Blume (1975) discuss the role of human capital in their empirical study of the "market price of risk," but they assume a fixed amount of savings and do not discuss the role of taxes as insurance. Feldstein (1969) notes that "the optimal portfolio behavior for an individual is not independent of the uncertainty of his other income sources" (p. 762) but does not pursue the idea very far. Davies and Whalley analyze the effects of taxes on human and physical capital formation, but do not allow for uncertainty.

The previous analyses of financial capital do not suffice as analyses of human capital because of fundamental differences between the two assets. Human capital can be acquired but not resold, and its return depends on both effort expended (which is unobservable) and a large random element which is mostly idiosyncratic. Thus, the key characteristics of human capital are: (1) irreversibility of investment (except for a small amount of depreciation), so that the timing of investment decisions is different from that for most financial assets; (2) risk that is undiversifiable (to the individual) and privately uninsurable; and (3) a relatively low correlation of returns with the market portfolio due to the large idiosyncratic element in human capital risk.[4] All of these characteristics justify a

[3] He mentions the equity effect of changing the distribution of income, but does not pursue it. He holds labor supply fixed, as we do.

[4] There is no contradiction in believing that the value of aggregate human capital has a high correlation with

separate treatment of human capital.

This paper is also related to research on the possible "crowding in" of investment by government debt. Consider a decrease in taxes today accompanied by an offsetting increase in taxes in the future. Friedman (1978) argues that such a shift in the timing of taxes might reduce the cost of equity capital (though it would raise the cost of debt capital) and thus increase real investment.[5] Auerbach and Kotlikoff (1987) discuss crowding in resulting from deferring capital income taxes. Temporarily lower taxes encourage individuals to save more, and if this effect exceeds the reduction in saving caused by the transfer of some of the tax burden to future generations, crowding in will occur. Our results show that crowding in of risky investment is in fact likely to occur, but for a different reason than those previously discussed. We return to this point in Section V.

II. The Joint Saving/Portfolio Composition Problem in the Face of Labor Income Risk

Setting. Our analysis uses a simple two-period life-cycle model with additively separable utility:

$$U(c, c') = u(c) + \mathrm{E}\, v(c'), \tag{1}$$

where c is first-period consumption and c' is second-period consumption. Both absolute risk aversion and the absolute strength of the precautionary saving motive decrease with wealth. (We discuss these assumptions in more detail below.)

We assume that individuals have a fixed amount of wealth (coming from first-period labor income as well as any initial wealth) to divide between first-period consumption and saving. That saving can be invested in two traded securities—a risk-free bond with a real after-tax gross return of R, and a risky security with a real after-tax excess return of $\tilde{z}$ (i.e., borrowing \$1 at the interest rate R to buy \$1 of the risky security will yield on net, after taxes and repayment of the loan, the random amount \$$\tilde{z}$ at the beginning of the second period).[6] We assume that individuals can freely borrow or lend through the riskless asset and can freely invest in or short the risky asset.

Let α be the *dollar* value of the individual's investment in the risky security at the end of the first period (not to be confused with the share of the portfolio in the risky security). Then at the

the returns of risky securities, but that a typical individual's human capital has a relatively small correlation with market returns. We emphasize that we do not expect the government to reduce aggregate human capital risk. But the government can reduce each individual's human capital risk, at no risk to itself, by pooling idiosyncratic risks across society. This is exactly what a proportional or progressive tax system does.

[5] His discussion of financial crowding out and crowding in abstracts from the real crowding out which would occur in a fully-employed economy.

[6] Implicitly, we are assuming that capital income taxes are linear.

beginning of the second period, the value of all the individual's investments will be $R(w-c)+\alpha\tilde{z}$.

We assume further that individuals hold risky human capital from which they earn income in the second period. The amount of human capital will be considered fixed as an approximation to the difference in timing between decisions about human capital investment (primarily the choice of occupation) and those about financial investment.[7] Labor is supplied inelastically in both the first and second periods.

Following Barsky, Mankiw and Zeldes (1986), Kimball and Mankiw (1987), and Varian (1980), private insurance markets are assumed to be incomplete, leaving some amount of uninsured labor income risk. Some of this risk may be due to the possibility of disability, if disability insurance is imperfect, but probably a more important source of uninsurable income risk is the possibility of doing worse than expected in one's career. This risk is difficult to insure both for moral hazard reasons (one might be tempted to expend less effort in advancing one's career if failure is cushioned by insurance) and adverse selection reasons (those who have private information that they will do poorly in the future will be more likely to buy insurance than those who know they will do well). Providing insurance also entails marketing and administrative costs.[8] For our purposes, the reasons for the absence of such insurance are not important. As long we consider changes in tax parameters small enough that the amount of private insurance for the component of labor income risk at issue remains at zero, we need not explicitly model why such insurance is not available.[9]

Thus, we model an individual's second-period labor income as a random variable $\tilde{y}$ with a fixed distribution. We want the joint distribution of labor income $\tilde{y}$ and the excess return $\tilde{z}$ to reflect both idiosyncratic income risk and the empirically observed positive correlation between aggregate labor income and the return on financial assets.[10] So we assume that $\tilde{y}$ is the sum of three components: a constant $\bar{y}$, a mean-zero random variable ϵ independent of $\tilde{z}$, plus a fraction β of $\tilde{z}$ itself. Formally:

$$\tilde{y}=\bar{y}+\tilde{\epsilon}+\beta\tilde{z}, \tag{2}$$

[7] See Kanbur (1981) and Driffill and Rosen (1983) on the choice of how much human capital to hold, and Eaton and Rosen (1980) on the choice of the riskiness of one's human capital.

[8] If moral hazard were the only problem, then it would pay for private insurers to offer at least some insurance, since the cost of the distortion from the first dollar's worth of insurance is negligible, just as the cost of the distortion from the first dollar of taxes is negligible.

[9] This strategy is shared by the papers cited at the beginning of the paragraph. Kaplow (1991) argues forcefully that this approach is not adequate when the purpose of the study, as in Varian, is to judge the merits of government "insurance." But our goal is not to determine whether taxes as insurance are an efficient solution to private market failure; we simply note the absence of private insurance and the existence of government insurance, and study the effects of this situation on other features of the economic landscape.

[10] Barsky, Mankiw and Zeldes (1986) review the evidence supporting this view.

(Note that $\bar{y}$ is not the mean of y, but the mean of the portion of y that is uncorrelated with z.) Our key conclusions hold even when $\beta = 0$.

The government redistributes labor income through a proportional tax on income above a certain level (y_0) and a proportional rebate on income below that level. Thus, after-tax income in the second period is $y_0 + (1-\tau)(y - y_0)$.[11] Note that any first-period labor income tax is effectively a lump-sum tax since it has neither uncertainty nor labor supply elasticity to interact with. And because there is Ricardian neutrality for changes in the timing of lump-sum taxes (see Chan, 1983), any tax on first-period labor income can be treated as if it were a lump-sum component to second-period labor income taxes.

Solution. Putting together everything above, maximizing (1) is equivalent to maximizing

$$\max_{c,\alpha} \quad u(c) + \mathbf{E}\, v\big(R(w-c) + \alpha\tilde{z} + y_0 + (1-\tau)(\bar{y} + \tilde{\epsilon} + \beta\tilde{z} - y_0)\big). \tag{3}$$

To get to the heart of the mathematical structure of the problem, define $x = Rw + y_0$, $\lambda = (1-\tau)$, $\theta = \alpha + (1-\tau)\beta$, and $\tilde{h} = \bar{y} - y_0 + \tilde{\epsilon}$. Then (3) is equivalent to

$$\max_{c,\theta} \; u(c) + \mathbf{E}\, v(x - Rc + \lambda\tilde{h} + \theta\tilde{z}). \tag{4}$$

Define the pair of functions $C(x,\lambda)$ and $\Theta(x,\lambda)$ as the solution to (4)—

$$\big(C(x,\lambda), \Theta(x,\lambda)\big) = \arg\max_{(c,\theta)} \; u(c) + \mathbf{E}\, v(x - Rc + \lambda\tilde{h} + \theta\tilde{z}). \tag{5}$$

Then the solution to (3) is given by

$$c^* = C(Rw + y_0, 1-\tau) \tag{6}$$

and

$$\alpha^* = \Theta(Rw + y_0, 1-\tau) - (1-\tau)\beta. \tag{7}$$

Our goal is to analyze the effect of changes in the tax rate on c^* and α^*. Differentiating (7) using subscripts for partial derivatives reveals that

$$\frac{d\alpha^*}{d\tau} = -\Theta_\lambda(Rw + y_0, 1-\tau) + \beta. \tag{8}$$

Thus, when $\beta > 0$ (there is a positive correlation between the returns on human capital and financial assets), any positive effect of labor income taxes on risky investment is enhanced. The effect of

[11] Labor income taxes need not be linear as long as the contemplated change in labor income taxes is linear, since one could let $\bar{y}$ represent after-tax labor income under the original tax policy and then let τ represent a linear surtax on what was originally after-tax labor income.

increasing the tax rate τ on the amount of risky investment is always more positive when $\beta > 0$ than it is when $\beta = 0$ (the returns on human capital and financial assets are independent). On the consumption side, (6) implies that

$$\frac{dc^*}{d\tau} = -C_\lambda(Rw + y_0, 1 - \tau), \tag{9}$$

so that β does not alter the effect of the tax rate τ on consumption.

To make progress in evaluating $\frac{d\alpha^*}{d\tau}$ and $\frac{dc^*}{d\tau}$, we must analyze the functions $C(x,\lambda)$ and $\Theta(x,\lambda)$. We begin by imposing some structure on the first and second-period utility functions $u(\cdot)$ and $v(\cdot)$.

First, we assume that $u(\cdot)$ and $v(\cdot)$ are both monotonically increasing, strictly concave functions (i.e., $u'(\cdot) > 0$, $v'(\cdot) > 0$, $u''(\cdot) < 0$ and $v''(\cdot) < 0$). Second, we assume that

$$\frac{d}{dx}\left(\frac{-v''(x)}{v'(x)}\right) < 0, \tag{10}$$

or in words, that $v(\cdot)$ displays decreasing absolute risk aversion. This is a standard assumption that has a sound empirical basis because it is necessary for a positive wealth elasticity of risky investment. In addition to insuring that risky investment does not behave as if it were an inferior good, decreasing absolute risk aversion insures that v''' will be positive—which, in turn, implies a positive precautionary saving motive. Finally, we assume that

$$\frac{d}{dx}\left(\frac{-v'''(x)}{v''(x)}\right) < 0, \tag{11}$$

or in words, that the precautionary saving motive decreases in strength with wealth. As shown in Kimball (1990a,b), $-v'''/v''$—or "absolute prudence"—measures the absolute strength of the precautionary saving motive, just as $-v''/v'$ measures the absolute strength of risk aversion; therefore, the assumption in (11) that this measure is decreasing in the argument is the formal requirement for the absolute strength of the precautionary saving motive to be decreasing in wealth ("decreasing absolute prudence"). This condition is plausible a priori,[12] and is not very restrictive for utility functions that already exhibit decreasing absolute risk aversion, in the sense that almost all commonly used utility functions with decreasing absolute risk aversion also have decreasing absolute prudence.[13] However, if one is looking, it is not difficult to construct a utility function that, over a certain range, satisfies (10) but not (11).

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[12] See the arguments in Kimball (1990b), one of which is the following thought experiment: "Consider a college professor who has $10,000 in the bank, and a Rockefeller who has a net worth of $10,000,000, who have the same preferences except for their differences in initial wealth. If each is forced to face a coin toss at the beginning of the next year, with $5,000 to be gained or lost depending on the outcome, which one will do more extra saving to be ready for the possibility of losing? If one's answer is that the college professor will do more extra saving, it argues for decreasing absolute prudence." More mechanically, Kimball (1990b) shows that absolute prudence is decreasing as long as the wealth elasticity of risk tolerance (which is always equal to 1 for constant relative risk aversion utility) does not increase too rapidly.

[13] For example, all utility functions in the hyperbolic absolute risk aversion class that have (weakly) decreasing

III. The Effect of Labor Income Taxes on Saving and Portfolio Decisions

We are now in a position to describe the effect of changes in the labor income tax rate on both an individual's total saving and saving in a risky financial asset. We do so by proving four propositions characterizing the two functions $C(x, \lambda)$ and $\Theta(x, \lambda)$. Recall that c is consumption in the first period; x is the nonstochastic part of second-period wealth; λ is 1 minus the tax rate τ; and θ equals the amount of explicit risky investment plus the implicit investment in the risky asset through human capital $(1-\tau)\beta$.

Because first-period wealth is held constant, any change in c means there is an opposite change in the level of saving. Inferring changes in risky saving from changes in theta is more complicated, however, because of the term $(1-\tau)\beta$. If the financial and human capital risks are *uncorrelated*, then $\beta = 0$, the extra term disappears, and risky investment is measured by θ. This is the principal case considered below. If the risks are *positively correlated*, our results are strengthened. In this case, financial risky investment equals θ minus the "after-tax beta" of human capital $(1-\tau)\beta$. So a reduction in τ lowers risky investment both by reducing θ (as shown below) and by increasing the after-tax beta of human capital $(1-\tau)\beta$. If the risks are *negatively correlated*, a reduction in τ might increase risky investment (going against our story) because additional risky investment would be desirable to help insure against the increased human capital risk.

Which of these three cases is most likely? For most people, β is probably close to zero. That is, their labor income risk is primarily idiosyncratic, and their financial risk is primarily aggregate.[14] For entrepreneurs for whom the relevant risky investment is investment in their own company, β will be strongly positive. Employees of brokerage houses or of firms in procyclical industries may also have a substantially positive β. Only for people with skills particularly appropriate for countercyclical industries (for example, bankruptcy lawyers) will β be negative. Since we consider the case of negative β atypical, but wish to be conservative in assuming anything more than nonnegative β, we concentrate on the case of $\beta = 0$ to get a reasonable lower bound for the size of the effect we are interested in.

Given the assumptions of monotonicity, concavity, decreasing absolute risk aversion and decreasing absolute prudence, one can prove the following four propositions about $C(x, \lambda)$ and $\Theta(x, \lambda)$. *Proofs can be found in Appendix A.*

absolute risk aversion (such as constant relative risk aversion or constant absolute risk aversion utility functions) also have decreasing absolute prudence, and any mixture of utility functions that individually have decreasing absolute prudence also has decreasing absolute prudence. Quadratic utility has (weakly) decreasing absolute prudence but not the more basic property of decreasing absolute risk aversion.

[14] Recall that one of the justifications for the distinction between financial and human capital is the much greater difficulty in diversifying the latter.

Proposition 1: *If $u'(\cdot) > 0$, $u''(\cdot) < 0$, $v'(\cdot) > 0$, $v''(\cdot) < 0$, and $v(\cdot)$ exhibits decreasing absolute risk aversion, then*

$$C_x(x, \lambda) > 0 \tag{12}$$

and

$$\Theta_x(x, \lambda) \geq 0. \tag{13}$$

Proposition 1 says that *both consumption and risky investment increase with wealth.* Proving (12) requires only monotonicity and concavity. Proving (13) requires monotonicity, concavity and decreasing absolute risk aversion. Neither (12) nor (13) depends on decreasing absolute prudence.

Proposition 1 implies that the "wealth expansion path" or "Engel curve" for c and θ obtained by holding λ fixed and varying x is an upward-sloping line, as depicted in Figure 1. Proposition 2 implies that an increase in λ shifts the Engel curve downward—toward lower θ for any given level of c. In words, Proposition 2 says that *a decrease in the tax rate shifts the consumer's optimum toward less risky investment* for any given level of consumption.[15]

Proposition 2: *If $u'(\cdot) > 0$, $u''(\cdot) < 0$, $v'(\cdot) > 0$, $v''(\cdot) < 0$, and $v(\cdot)$ exhibits decreasing absolute prudence, then*

$$\Theta_x(x, \lambda)C_\lambda(x, \lambda) - C_x(x, \lambda)\Theta_\lambda(x, \lambda) \geq 0. \tag{14}$$

As shown in Figure 1, (C_x, Θ_x) is the vector along the Engel curve, and $(\Theta_x, -C_x)$ is the downward perpendicular to the Engel curve. Therefore the expression in (14), $\Theta_x C_\lambda - C_x \Theta_\lambda$, is the dot product of $(C_\lambda, \Theta_\lambda)$ with the downward perpendicular to the Engel curve. Proposition 2 says that along with monotonicity and concavity, decreasing absolute prudence is enough to guarantee that the dot product is always positive. This means that an increase in λ moves the point (c, θ) at an acute angle to the downward perpendicular to the Engel curve, and thus shifts the Engel curve down.

Note that the shift of the optimum toward less risky investment for *any given level* of consumption does not mean that an individual will always undertake less risky investment. If an increase in λ results in a large enough increase in consumption, the individual's risky investment may increase as well. Consumption in turn is affected by two opposing forces—the increase in wealth due to the decline in taxes tends to increase consumption, while the increased need for precautionary saving

[15] The simple statement here is for $\beta = 0$. If $\beta > 0$, it makes the level of risky investment that goes along with any given level of consumption fall even more with a reduction in the tax rate τ. If $\beta < 0$, the level of risky investment that goes along with any given level of consumption may rise with a reduction in the tax rate since risky investment would provide insurance for the additional human capital risk.

due to the increase in risk tends to decrease consumption. (Note that we are considering the effects of an uncompensated change in future taxes; below we include the effects of an offsetting change in current taxes.)

The key here is the expected value of the individual's stochastic second-period wealth, $\tilde{h} = \bar{y} - y_0 + \tilde{\epsilon}$, to which the tax is applied. If this value is high enough, then a reduction in the tax rate produces a large enough rise in mean after-tax income to override the precautionary saving effect and raise consumption. With an even larger value for $\mathrm{E}\,\tilde{h}$, a reduction in the tax rate produces a large enough rise in mean after-tax income to override the risk-crowding effect and raise investment in the risky asset. However, if the effect of higher mean after-tax labor income is small enough that additional labor income risk causes a reduction in consumption (*i.e.* the precautionary saving effect dominates the effect of the mean), then additional labor income risk will also cause a reduction in voluntary risk-bearing. Proposition 4 below characterizes the effect of taxes on consumption.

Propositions 2 and 3 are closely related. To explain the connection, it is helpful first to look at Proposition 2 as saying that an increase in labor income risk which leaves precautionary saving unchanged still causes a reduction in the amount of an independent financial risk which is borne. In other words, the negative interaction between independent risks—termed "temperance" by Kimball (forthcoming)—is stronger than the precautionary saving motive. This parallels Drèze and Modigliani's (1972) result that an increase in risk which leaves utility unchanged will still cause an increase in the amount of precautionary saving. In the words of Kimball (forthcoming), "just as decreasing absolute risk aversion implies that prudence is greater than risk aversion, decreasing absolute prudence implies that temperance is greater than prudence."

If decreasing absolute prudence makes temperance stronger than prudence, and decreasing absolute risk aversion makes prudence stronger than risk aversion, then by transitivity, the combination of decreasing absolute prudence and decreasing absolute risk aversion should and does imply that temperance is stronger than risk aversion. Operationally, the combination of decreasing absolute prudence and decreasing absolute risk aversion should guarantee—and by Proposition 3 *does* guarantee—that *even a compensated increase in independent labor income risk to which an individual is indifferent leads to a reduction in other risky investment.*[16] A fortiori, *any increase in independent labor income risk that is not compensated enough to make the individual indifferent*

[16] Pratt and Zeckhauser (1987) note that under their assumptions, if a new insurance policy comes into the market, anyone who voluntarily purchases the policy will do more of other risky investment. Proposition 3 says that the combination of decreasing absolute prudence and decreasing absolute risk aversion is enough to guarantee that a new insurance policy that is voluntarily purchased leads to more of other risky investment even when the consumption/saving decision and the portfolio composition decision have been integrated.

leads to a reduction in other risky investment.

Proposition 3: *If $u'(\cdot) > 0$, $u''(\cdot) < 0$, $v'(\cdot) > 0$, $v''(\cdot) < 0$, $v(\cdot)$ exhibits decreasing absolute risk aversion and decreasing absolute prudence, and*

$$\frac{\partial}{\partial \lambda}\left(u(c) + \mathrm{E}\, v(x - Rc + \theta \tilde{z} + \lambda \tilde{h})\right)\Big|_{(c,\theta)=(C(x,\lambda),\Theta(x,\lambda))} \leq 0, \tag{15}$$

then

$$\Theta_\lambda(x, \lambda) \leq 0. \tag{16}$$

An increase in λ represents a decrease in the tax rate and thus an increase in labor income risk. With no change in lump-sum taxes, it also represents an increase in wealth, and the combination of increased risk and increased wealth may raise or lower utility. Thus, it is not clear whether a utility-compensating change in lump-sum taxes would be an increase or a decrease. It *is* clear, however, that an increase in λ combined with a large enough increase in lump-sum taxes to leave expected tax payments unchanged would have no wealth effect and would unambiguously lower utility. In other words, the set of inadequately compensated changes in labor income risk necessarily includes those that arise from a tax change that is intertemporally revenue-neutral.[17]

What kinds of tax changes will be revenue-neutral? The answer depends on the interpretation given to the model's risky financial asset.

Consider first the case where the return on the risky financial asset represents the aggregate risk in the economy. Then a tax change is revenue-neutral if and only if $y_0 = \bar{y}$, or equivalently, iff $\mathrm{E}\,\tilde{h} = \mathrm{E}\,\epsilon = 0$. The effect of idiosyncratic labor income risk on government revenue is canceled out by the law of large numbers.[18] Financial risks which the government may appear to bear through the tax system should be considered in fact to have no effect on revenue, because the government can offset any change in its financial risk-bearing through other actions in the financial market.[19] If the government does use the financial market to offset changes in its implicit financial risk-bearing, or if the taxpayers consider government risk-bearing as if it were their own risk-bearing (as they should if the government eventually absorbs the financial risk it bears through stochastic lump-sum

[17] Remember that the model implies Ricardian equivalence for lump-sum taxes so that the timing of lump-sum taxes is of no consequence.

[18] Aggregate labor income risk will still affect both individual incomes and government revenue. Because the government cannot insure individuals against this risk through the tax system, we do not focus on it here. Practically speaking, aggregate human capital risk is small relative to idiosyncratic risk. Formally, aggregate human capital risk can be incorporated into the model by assuming that aggregate human capital risk is captured by the return to the financial asset, at the cost of blurring the distinction between movements in GNP and movements in the stock market.

[19] In its most general form, this principle relies on the envelope theorem.

taxes), then the change in national financial risk-bearing is given directly by the change in θ. In this case, the defining characteristic of a revenue-neutral tax change is that it does not affect the expected present value of the part of labor income left after any implicit investment in the risky financial asset is taken out.

Now consider the case where the return on the risky financial asset represents the return on idiosyncratic projects for which problems of private or semi-private information prevent adequate diversification of returns. Then a small tax change is revenue-neutral if and only if

$$y_0 = \mathbf{E}\,\tilde{y} = \bar{y} + (\beta + \alpha^*)\mathbf{E}\,\tilde{z} = \bar{y} + \theta^*\mathbf{E}\,\tilde{z},$$

or equivalently, iff

$$\mathbf{E}\,\tilde{h} = \mathbf{E}\,[\bar{y} - y_0 + \tilde{\epsilon}] = -\theta^*\mathbf{E}\,\tilde{z} \leq 0.$$

The effects of both idiosyncratic labor income and financial risks on government revenue are canceled out by the law of large numbers. The inequality $-\theta^*\mathbf{E}\,\tilde{z} \leq 0$ follows from the fact that the optimal exposure to a risk is always of the same sign as its expected value. Since in this second case taxes help to diversify financial as well as nonfinancial risks, taxes are more valuable than in the first case where the risky asset represents aggregate financial risk. Thus, a revenue-neutral reduction in τ is even less desirable to individuals here than in the first case, so that Proposition 3 can be applied.

On either interpretation of the financial risky asset, therefore, the following corollary to Proposition 3 effectively guarantees that *an revenue-neutral reduction in τ leads to a reduction in financial risk-bearing.*

Corollary to Proposition 3: *If $u'(\cdot) > 0$, $u''(\cdot) < 0$, $v'(\cdot) > 0$, $v''(\cdot) < 0$, $v(\cdot)$ exhibits decreasing absolute risk aversion and decreasing absolute prudence, and $\mathbf{E}\,\tilde{h} \leq 0$, then $\Theta_\lambda(x, \lambda) \leq 0$.*

Intertemporally revenue-neutral tax changes include changes in the timing of income taxes like those discussed by Barsky, Mankiw and Zeldes (1986) and Kimball and Mankiw (1989). By the Corollary above, a postponement of labor income taxes which shows up here as an increase in τ and a reduction in λ tends to crowd in risky investment.

To review, Proposition 3 says that a decrease in the labor income tax rate in the future causes an individual to do less risky saving as long as the individual is not made better off by the tax change. It might appear that a similarly strong result could be derived about the effect of such a tax change on consumption and total saving, but unfortunately this is not the case.

This lack of a clear result is surprising because the effect on the consumption/saving decision of an increase in labor income risk in the absence of an additional risky investment choice has been settled since Leland (1968), Rothschild and Stiglitz (1971), and Drèze and Modigliani (1972). In the absence of an additional risky investment choice, Leland (1968) shows that a single mean-zero risk leads to reduced consumption as long as $u''' > 0$. Rothschild and Stiglitz (1971) extend Leland's result to mean-preserving spreads. Drèze and Modigliani show that any undesirable risk or undesirable increase in the scale of a risk leads to reduced consumption as long as absolute risk aversion is decreasing. This result indicates that in the absence of an additional risky investment choice, a decrease in the labor income tax rate leads to less consumption and more saving as long as the individual is not made better off by the tax change.

What complicates the analysis of the precautionary saving effect in our model is the interaction of the labor income risk and the financial asset risk. In particular, as shown in Proposition 3, an increase in labor income risk causes the individual to cut back on financial risk. This response could conceivably be large enough to reduce the overall riskiness of the individual's future income, and thus lead to a *reduction* in precautionary saving. For the cutback in financial risk to be so large, however, human capital and the risky financial asset must be very strong substitutes. In fact, as Proposition 4 indicates, they must be such strong substitutes that human capital would have a negative wealth elasticity if its quantity could be freely varied. Proposition 4 addresses the effect of any tax change involving a decrease in the future labor income tax rate which does not make an individual better off, stating that an individual will save more in response to such a tax change whenever human capital would be a normal good if its quantity could be freely varied.

Proposition 4: *If $u'(\cdot) > 0$, $u''(\cdot) < 0$, $v'(\cdot) > 0$, $v''(\cdot) < 0$ and*

$$\frac{\partial}{\partial \lambda}\left(u(c) + \mathrm{E}\, v(x - Rc + \theta\tilde{z} + \lambda\tilde{h})\right)\Big|_{(c,\theta)=(C(x,\lambda),\Theta(x,\lambda))} = 0, \tag{17}$$

then $C_\lambda(x,\lambda) \leq 0$ if and only if the optimal choice of λ would have a positive wealth elasticity at (x,λ).

Equation (17) says that if the quantity of human capital risk λ could be freely varied, the first-order condition for λ would be satisfied at (x,λ). Using notation defined in Appendix A, Proposition 4 says that if the agent is indifferent to a marginal change in λ, then

$$\mathrm{sign}(C_\lambda) = -\mathrm{sign}\left(\frac{d\lambda^*}{dx}\right). \tag{18}$$

The economic logic behind Proposition 4 is based on the link between two aspects of individual behavior. The first is the effect on saving of an expected-utility-preserving increase in labor income risk *in our model;* the second is the effect of wealth on the optimal amount of human capital *in a portfolio problem with two risky assets.* Both effects reflect the same complementarity (in the normal case) or substitutability (in the inferior case) between saving and human capital risk-bearing. The relevant notion of complementarity or substitutability is one that takes full account of optimal adjustments in holdings of the financial asset. Thus the complementarity between saving and risky investment guaranteed by decreasing absolute risk aversion when there is only one risky asset[20] may be consistent with substitutability between saving and human capital risk-bearing once optimal adjustment in the other risky asset is taken into account.

Unfortunately, it is difficult to analyze the condition that human capital be a normal good. The existing literature on the portfolio problem with two risky assets provides little guidance as to what is needed to guarantee that two assets *with independent returns* both have positive wealth elasticities. (β has no effect on the wealth elasticity of human capital, since optimal adjustment in financial asset holdings cancels out any undesired changes in financial risk-bearing implicit in human capital holdings. Therefore, one can assume without loss of generality that the human capital and the financial asset have independent returns.) Hart (1975) shows that without the assumption of independence, conditions stringent enough to guarantee that the mix of risky securities does not depend on wealth at all can guarantee a positive wealth elasticity for every security. The additional assumption of independent returns gives some hope that weaker conditions might guarantee positive wealth elasticities, but despite considerable efforts we have *not* been able to rule out inferiority even for one of a pair of assets with independent *two-point distributions* for any utility function with decreasing absolute prudence[21] and decreasing absolute risk aversion.[22] Our approach establishes clear results about the effect of labor income risk on investment in other risky assets, but casts some doubt on the generality of previous results about the effect of labor income risk on total saving.[23]

[20] See Kimball (forthcoming).

[21] *Increasing* absolute prudence, by implying complementarity between two independent assets, could guarantee that the optimal quantities of the two assets would go up and down together and could guarantee positive wealth elasticities in conjunction with decreasing absolute risk aversion, but we view increasing absolute prudence as implausible. Moreover, the proof in Kimball (1991a) that (assuming monotonicity and concavity) globally decreasing absolute prudence implies globally decreasing absolute risk aversion can easily be converted to show that given the Inada condition $u'(\infty) = 0$, globally *increasing* absolute prudence implies globally *increasing* absolute risk aversion. Assuming *increasing* absolute prudence and strictly *decreasing* absolute risk aversion globally is so restrictive that it forces $u'(\infty)$ to be strictly positive.

[22] Unfortunately, we have not been able to find an example of inferiority, either. One situation favorable for the existence of an inferior asset even when the two assets are independent is when one asset is skewed strongly to the right, while the other is skewed strongly to the left. Decreasing absolute prudence implies that increases in wealth make v''' decline in relation to $-v''$, or in other words, that skewness preference declines in relation to risk aversion. As a result, an agent might shift strongly toward the negatively skewed risk and away from the positively skewed risk as his or her wealth increases.

[23] It may seem surprising that we can establish such clear results about labor income taxes and financial risk-taking while the literature on capital income taxes and financial risk-taking is replete with ambiguities. The

We cannot guarantee that saving will always rise in response to a compensated increase in labor income risk, or even that it will always rise in response to an uncompensated increase in that kind of risk.

IV. The Social and Private Value of Investing in Risky Assets

The propositions in section III show that, given plausible assumptions about individuals' preferences, the reduction in idiosyncratic labor income risk provided by taxes increases individuals' financial risk-taking. We turn now to the question of whether an increase in financial risk-taking is beneficial to the economy.

Domar and Musgrave write that "there is no question that increased risk-taking ... is highly desirable" (p. 391). They never explain their claim. Nor have most of the researchers who have followed them in analyzing risk-taking. It seems clear that encouraging risky investment through public policy is appropriate only if private markets generate less of such investment than is socially optimal.[24] (Even then, the benefits must be weighed against the other distortions caused by the policy, such as the effect of income taxes on total saving and work effort.) But why would private markets generate too little risky investment?

First, imperfections in the financial capital market may inhibit risky investment. Consider an entrepreneur who has private information about the value of a project, whose success also depends on his or her own effort. Then adverse selection and moral hazard problems may inhibit the participation of outside investors, so that it is impossible for the entrepreneur to diversify away the idiosyncratic risk of the project (a possibility mentioned in the discussion of Proposition 3).[25] In that case the idiosyncratic risk, which society as a whole can easily diversify away, drives a wedge between the private and social value of the project. More generally, any transactions costs in trading financial assets will prevent complete diversification and create a wedge between private and social value.

Second, the lack of a full-scale market for human capital may inhibit risky investment. Be-

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main explanation for the difference is that individuals cannot trade away their risky human capital in the way that they can trade away risky financial assets.

[24] Note that the correct measure of risk is social risk, comprising the private (net-of-tax) uncertainty and the variability of government revenues. In our model, taxes only reduce idiosyncratic risk, not aggregate risk, so the only change in social risk is due to whatever effect changes in the demand for financial assets has on the quantity of risky physical investment. We follow the literature in assuming that if individuals demand more risky assets, more risky projects will be undertaken. Feldstein (1983), for example, asserts that "the net rates of return on capital in different uses are not generally equal but reflect the risk-return preferences of investors and their equilibrium portfolio compositions" (p. 17).

[25] An entrepreneur's labor income risk will be highly correlated with his or her financial capital risk, implying that β is strongly positive. As noted in the discussion of Proposition 3, increasing the tax rate in this case is especially likely to increase the entrepreneur's risky capital investment.

cause of this missing market, the Arrow-Debreu welfare theorems do not hold; specifically, since human capital risk is undiversifiable to the individual, but largely diversifiable to the economy as a whole, there is no presumption that the optimal amount of risky investment (in human *or* financial capital) is undertaken. Indeed, there is some presumption that the optimal conditions can be approached more closely by diversifying the idiosyncratic human capital risk through the tax system. By reducing the socially unproductive risk that individuals bear, the government can encourage individuals to bear more socially productive risk.[26]

Third, the social return to risky investment will exceed the private return if there are important technological spillovers or other positive externalities from such investment. Adam Jaffe (1986) presents evidence that the extent of technological spillovers is substantial. Andrei Shleifer and Robert Vishny (1987) argue that aggregate demand externalities in an imperfectly competitive economy make the optimal amount of risky investment greater than the amount chosen (in the absence of taxes) by profit-maximizing firms.

Fourth, other aspects of public policy may reduce the amount of risky investment. In particular, capital income taxes at both the corporate and personal levels may have powerful effects on risk-taking in investment.[27]

V. Financial Crowding In of Investment

The results in Section III imply that an increase in government debt will crowd in risky investment whenever any of the following conditions is satisfied:

(1) the increase in debt represents a change only in the timing of taxes, with no change in the present value of government expenditures and thus no change in the expected present value of taxes faced by an individual (the Corollary to Proposition 3);

(2) the increased future taxes have a strong enough insurance effect that the taxes raise expected utility (Proposition 3); or

(3) the tax changes lead to higher current consumption (Proposition 2).

Thus we concur with Friedman (1978) that crowding in will occur when the government reduces taxes now and pays off the debt with higher taxes in the future.[28] But our results are based on

[26] Through its coercive power of taxation, the government is able to overcome adverse selection problems that may hinder private insurance.

[27] Unfortunately, as is clear from Sandmo's (1985) survey, there is little theoretical or empirical consensus on the direction or size of these effects.

[28] Frankel (1985) estimates that portfolio effects on rates of return are very small, but that crowding in of equity investment is more likely than crowding out. We can foresee two ways in which an empirical analysis based on our approach would differ from that of Frankel. First, Frankel does not allow for the effects of future

a neoclassical foundation of expected utility maximization in the absence of complete insurance markets. Further, the individual in our model is fully aware of the future tax liability, which allows us to show that financial crowding in can coexist with Ricardian equivalence of lump-sum tax reschedulings, which is not obvious from Friedman's treatment.

VI. Conclusion

Individual's ability to earn labor income in the future is often their most valuable asset, but it carries with it a large and mostly undiversifiable and unmarketable risk. A decrease in current taxes combined with an offsetting future increase in proportional or progressive labor income taxes provides insurance against this risk. Because most labor income risk is idiosyncratic risk, individual uncertainty can be reduced with no increase in the uncertainty of government revenue.

Barsky, Mankiw and Zeldes (1986) show that the reduction in idiosyncratic labor income risk acts through the precautionary saving motive to reduce saving relative to the Ricardian benchmark. We show that the reduction in idiosyncratic labor income risk affects portfolio decisions as well.

By an analysis of the effect of uninsured idiosyncratic labor income risk on the joint saving/portfolio composition decision in a two-period model, we show that—given plausible restrictions on preferences—any change in taxes that lowers labor income risk and does not make an individual worse off will lead the individual to invest more in the risky security, even if its return is statistically independent of the labor income risk. A deferral of labor income taxes with no change in their expected value is one such tax change.

One obvious direction for further research is to study models with more than two periods. Kimball (1990b) gives one idea of how our results might be extended to multiperiod models. In a multiperiod model, the absolute risk aversion of the value function is equal to the product of the absolute risk aversion of the underlying period utility function and the marginal propensity to consume out of wealth. Under conditions similar to those we assume, idiosyncratic labor income risk raises the marginal propensity to consume out of wealth *and* raises the absolute risk aversion of the underlying period utility function by lowering consumption.

To sketch the multiperiod extension of our results in a slightly different way, Breeden (1986) finds that for continuous-time diffusion processes, the expected rate of return differential between

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tax liabilities, which play an important role in our analysis. Second, Frankel constrains government debt to affect asset demands only through changing the market portfolio and therefore the covariances of various asset returns with the market portfolio. In our approach, the risk aversion of the indirect utility function depends on expected future tax rates; therefore, changing those rates changes the market risk premia. Until empirical tests are implemented with these features, the case cannot be closed on the direction and magnitude of the effect of government debt on risky investment.

risky and riskless securities should be equal to the product of an agent's underlying risk aversion and the covariance of the rate of return differential with consumption growth. Grossman and Shiller (1982) show that this relationship can be aggregated: the market risk premium should be equal to the product of a weighted average of agents' underlying risk aversions and the covariance between the rate of return differential and aggregate consumption growth. Idiosyncratic labor income risk raises the premium for holding risky assets (1) by lowering consumption—and therefore underlying risk aversion—and (2) by raising the marginal propensity to consume out of wealth—and therefore the covariance of consumption with the returns on risky securities in which agents have substantial positions.

Another direction for further research is to consider more carefully the desirability of encouraging investment in risky assets. Much of the literature on capital taxes and financial risk-taking has assumed that risk-taking should be encouraged, but this view has not been well justified. We presented several explanations of why private markets may generate too little risky investment, but clearly more work remains to be done in this area. If the need for more risky investment in the economy is even more urgent than the need for more investment overall, the positive effect of the insurance provided by future labor income taxes on *risky* investment may outweigh the negative effect of such insurance on overall saving and investment, *or* may tip the balance between this negative effect and the *direct* positive effect of such insurance on welfare.

Appendix A

Proofs of Propositions 1–4

Five lemmas prepare the ground for proofs of Propositions 1–4. Define

$$J(c,\theta;x,\lambda) = u(c) + \mathrm{E}\,v(x - Rc + \lambda\tilde{h} + \theta\tilde{z}). \qquad (A.1)$$

Then

$$\big(C(x,\lambda), \Theta(x,\lambda)\big) = \arg\max_{c,\theta} \; J(c,\theta;x,\lambda). \qquad (A.2)$$

Lemmas 1–5 characterize the function J. All of the results here assume that the risks $\tilde{z}$ and $\tilde{h}$ are nondegenerate and statistically independent of each other and that $u'(\cdot) > 0$, $u''(\cdot) < 0$, $v'(\cdot) > 0$, $v''(\cdot) < 0$. Lemmas 3 and 4 rely in addition on v having decreasing absolute risk aversion. Lemma 5 relies on v having decreasing absolute prudence, but does not rely on decreasing absolute risk aversion. Subscripts denote partial derivatives.

Lemma 1: *For all c, θ, x and λ,*

$$J_{cc}(c,\theta;x,\lambda) = R^2 J_{xx}(c,\theta;x,\lambda) + u''(c), \qquad (A.3)$$

$$J_{c\theta}(c,\theta;x,\lambda) = -RJ_{x\theta}(c,\theta;x,\lambda), \qquad (A.4)$$

$$J_{cx}(c,\theta;x,\lambda) = -RJ_{xx}(c,\theta;x,\lambda), \qquad (A.5)$$

$$J_{c\lambda}(c,\theta;x,\lambda) = -RJ_{x\lambda}(c,\theta;x,\lambda), \qquad (A.6)$$

$$J_{\theta\theta} < 0 \qquad (A.7)$$

and

$$J_{\lambda\lambda} < 0. \qquad (A.8)$$

Proof. Differentiate (A.1).■

Lemma 2: *For all c, θ, x and λ,*

$$J_{cc}J_{\theta\theta} - J_{c\theta}^2 > R^2[J_{xx}J_{\theta\theta} - J_{\theta x}^2] > 0, \qquad (A.9)$$

$$J_{cc}J_{\lambda\lambda} - J_{c\lambda}^2 > R^2[J_{xx}J_{\lambda\lambda} - J_{x\lambda}^2] > 0, \qquad (A.10)$$

$$J_{cc}J_{xx} - J_{cx}^2 > 0 \qquad (A.11)$$

and

$$J_{\theta\theta}J_{\lambda\lambda} - J_{\theta\lambda}^2 > 0. \qquad (A.12)$$

Proof. Any sum of concave functions and any expectation over concave functions is concave. Since both u and v are jointly concave in all c, θ, x, λ for particular realizations of $\tilde{z}$ and $\tilde{h}$, the function J is jointly concave in all four arguments. The strict inequalities follow from $u''(\cdot) < 0$, $v''(\cdot) < 0$, (A.7), (A.8) and from the nondegeneracy and independence of $\tilde{z}$ and $\tilde{h}$.[29]∎

Lemma 3: *For any random variable $\tilde{q}$, the derived utility function*

$$\hat{v}(x) = \mathrm{E}\, v(x + \tilde{q}) \qquad (A.13)$$

inherits decreasing absolute risk aversion from v.

Proof. Both Nachman (1982) and Kihlstrom, Romer and Williams (1981) give proofs of Lemma 3. Since decreasing absolute risk aversion is equivalent to convexity of $\ln(v'(x))$, Lemma 3 is a consequence of Artin's 1931 Theorem that log-convexity is preserved under expectations. Marshall and Olkin (1979) give a particularly simple proof of Artin's theorem based on the fact that any sum of or expectation over positive semidefinite matrices is positive semidefinite. Decreasing absolute risk aversion of v implies convexity of $\ln(v'(x + \tilde{q}))$ for each realization of $\tilde{q}$, which is equivalent to positive semidefiniteness of the matrix

$$\begin{bmatrix} v'(x+\tilde{q}) & v'(x+\tilde{q}+\delta) \\ v'(x+\tilde{q}+\delta) & v'(x+q+2\delta) \end{bmatrix}. \qquad (A.14)$$

This implies positive definiteness of

$$\begin{bmatrix} \mathrm{E}\, v'(x+\tilde{q}) & \mathrm{E}\, v'(x+\tilde{q}+\delta) \\ \mathrm{E}\, v'(x+\tilde{q}+\delta) & \mathrm{E}\, v'(x+\tilde{q}+2\delta) \end{bmatrix} \qquad (A.15)$$

and decreasing absolute risk aversion for $\hat{v}(\cdot)$.∎

Lemma 4: *If $\theta \geq 0$, then $J_{\theta x}(c,\theta;x,\lambda) \geq 0$ wherever $J_\theta(c,\theta;x,\lambda) \leq 0$. Similarly, if $\lambda \geq 0$, then $J_{\lambda x}(c,\theta;x,\lambda) \geq 0$ wherever $J_\lambda(c,\theta;x,\lambda) \leq 0$.*

Proof. By the symmetry between θ and λ in the definition of J (A.1), proving one half of Lemma 4 is enough to prove both halves. Letting $\tilde{q} = \lambda\tilde{h} - Rc$ in (A.13),

$$J_\theta(c,\theta;x,\lambda) = \mathrm{E}\,\tilde{z}v'(x - Rc + \theta\tilde{z} + \lambda\tilde{h}) = \mathrm{E}\,\tilde{z}\hat{v}'(x + \theta\tilde{z}) \qquad (A.16)$$

[29] The Cauchy-Schwartz inequalities which apply to the expanded versions of (A.9–A.12) hold with equality only when the random variables are perfectly correlated or when one of the random variables is degenerate.

and

$$J_{\theta x}(c,\theta;x,\lambda) = \mathrm{E}\,\tilde{z}v''(x - Rc + \theta\tilde{z} + \lambda\tilde{h}) = \mathrm{E}\,\tilde{z}\hat{v}''(x+\theta\tilde{z}) \tag{A.17}$$

because of the independence of $\tilde{z}$ and $\tilde{h}$. Decreasing absolute risk aversion of $\hat{v}$ makes $\frac{\hat{v}''(x+\theta\tilde{z})}{\hat{v}'(x+\theta\tilde{z})}$ an increasing function of $\tilde{z}$, so that

$$\frac{\hat{v}''(x+\theta\tilde{z})}{\hat{v}'(x+\theta\tilde{z})} - \frac{\hat{v}''(x)}{\hat{v}'(x)}$$

has the same sign as $\tilde{z}$. Thus,

$$J_{\theta}(c,\theta;x,\lambda) = \mathrm{E}\,\tilde{z}\hat{v}'(x+\theta\tilde{z}) \leq 0 \tag{A.18}$$

implies

$$\begin{aligned} J_{\theta}(c,\theta;x,\lambda) &= \mathrm{E}\,\tilde{z}\hat{v}''(x+\theta\tilde{z}) \\ &\geq \mathrm{E}\,\tilde{z}\hat{v}''(x+\theta\tilde{z}) - \frac{\hat{v}''(x)}{\hat{v}'(x)}\mathrm{E}\,\tilde{z}\hat{v}'(x+\theta\tilde{z}) \\ &= \mathrm{E}\,\tilde{z}\left[\frac{\hat{v}''(x+\theta\tilde{z})}{\hat{v}'(x+\theta\tilde{z})} - \frac{\hat{v}''(x)}{\hat{v}'(x)}\right]\hat{v}'(x+\theta\tilde{z}) \\ &\geq 0. \blacksquare \end{aligned} \tag{A.19}$$

Lemma 5: *For any c, x, $\theta \leq 0$ and $\lambda \geq 0$,*

$$J_{\theta\lambda}(c,\theta;x,\lambda)J_{xx}(c,\theta;x,\lambda) - J_{\theta x}(c,\theta;x,\lambda)J_{\lambda x}(c,\theta;x,\lambda) \geq 0 \tag{A.20}$$

Proof. Decreasing absolute prudence is equivalent to convexity of $\ln(-v'')$. Therefore, for any four quantities z_1, z_2, h_1 and h_2,

$$\ln(-v''(x+\theta z_1+\lambda h_1)) + \ln(-v''(x+\theta z_2+\lambda h_2)) \geq \ln(-v''(x+\theta z_1+\lambda h_2)) + \ln(-v''(x+\theta z_1+\lambda h_2) \tag{A.21}$$

when $(z_2 - z_1)(h_2 - h_1) \geq 0$, with the direction of the inequality in (A.21) reversed when $(z_2 - z_1)(h_2 - h_1) \leq 0$. Exponentiating both sides of (A.21) and subtracting, the quantity

$$v''(x+\theta z_1+\lambda h_1)v''(x+\theta z_2\lambda h_2) - v''(x+\theta z_1+\lambda h_2)v''(x+\theta z_1\lambda h_2)$$

always has the same sign as $(z_2 - z_1)(h_2 - h_1)$. Thus, if $\tilde{z}_1$, $\tilde{z}_2$, $\tilde{h}_1$ and $\tilde{h}_2$ are mutually independent random variables with $\tilde{z}_1$ and $\tilde{z}_2$ having the same distribution as $\tilde{z}$, and $\tilde{h}_1$ and $\tilde{h}_2$ having the same distribution as $\tilde{h}$, then

(A.22)

$$
\begin{aligned}
J_{\theta\lambda}(c,\theta;x,\lambda)J_{xx}(c,\theta;x,\lambda) - J_{\theta x}(c,\theta;x,\lambda)J_{\lambda x}(c,\theta;x,\lambda) &= \{\mathbf{E}\,\tilde{z}\tilde{h}v''(x+\theta z_1+\lambda h_1)\}\{\mathbf{E}\,v''(x+\theta z_1+\lambda h_1)\} \\
&\quad - \{\mathbf{E}\,\tilde{z}v''(x+\theta z_1+\lambda h_1)\}\{\mathbf{E}\,\tilde{h}v''(x+\theta z_1+\lambda h_1)\} \\
&= \frac{1}{4}\mathbf{E}\,(\tilde{z}_2-\tilde{z}_1)(\tilde{h}_2-\tilde{h}_1)[v''(x+\theta z_1+\lambda h_1)v''(x+\theta z_2\lambda h_2) \\
&\quad - v''(x+\theta z_1+\lambda h_2)v''(x+\theta z_1\lambda h_2)] \\
&\geq 0.\blacksquare
\end{aligned}
$$

Remark: If v exhibits *increasing* absolute prudence over the interval of interest, essentially the same proof can be used to show that $J_{\theta\lambda}J_{xx} - J_{\theta x}J_{\lambda x} \leq 0$. This converse to Lemma 5 allows one to establish the converse to Proposition 2.■

Proof of Proposition 1

The agent's problem can be rewritten as

$$
max_{c,\theta}\; J(c,\theta;x,\lambda). \tag{A.23}
$$

The first-order conditions are

$$
J_c\big(C(x,\lambda),\Theta(x,\lambda);x,\lambda\big) = 0 \tag{A.24}
$$

and

$$
J_\theta\big(C(x,\lambda),\Theta(x,\lambda);x,\lambda\big) = 0 \tag{A.25}
$$

Differentiating (A.24) and (A.25) with respect to x and λ and arranging the results in matrix form yields

$$
\begin{bmatrix} J_{cc} & J_{c\theta} \\ J_{c\theta} & J_{\theta\theta} \end{bmatrix}\begin{bmatrix} C_x & C_\lambda \\ \Theta_x & \Theta_\lambda \end{bmatrix} = -\begin{bmatrix} J_{cx} & J_{c\lambda} \\ J_{\theta x} & J_{\theta\lambda} \end{bmatrix}. \tag{A.26}
$$

Define

$$
A = \begin{bmatrix} J_{cc} & J_{c\theta} \\ J_{c\theta} & J_{\theta\theta} \end{bmatrix}. \tag{A.27}
$$

Then by Lemma 1,

$$
\begin{aligned}
A^{-1} &= \frac{1}{J_{cc}J_{\theta\theta} - J_{c\theta}^2}\begin{bmatrix} J_{\theta\theta} & -J_{c\theta} \\ -J_{c\theta} & J_{cc} \end{bmatrix} \\
&= \Delta^{-1}\begin{bmatrix} J_{\theta\theta} & RJ_{\theta x} \\ RJ_{\theta x} & R^2J_{xx}+u'' \end{bmatrix},
\end{aligned} \tag{A.28}
$$

where

$$
\Delta = J_{cc}J_{\theta\theta} - J_{c\theta}^2 > 0 \tag{A.29}
$$

by Lemma 2. Multiplying both sides of (A.26) on the left by A^{-1} and restricting our attention for now to the left column of the result,

$$\begin{aligned}\begin{bmatrix} C_x \\ \Theta_x \end{bmatrix} &= -A^{-1}\begin{bmatrix} J_{cx} \\ J_{\theta x}\end{bmatrix} \\ &= -\Delta^{-1}\begin{bmatrix} J_{\theta\theta} & RJ_{\theta x} \\ RJ_{\theta x} & R^2 J_{xx} + u'' \end{bmatrix}\begin{bmatrix} -RJ_{xx} \\ J_{\theta x}\end{bmatrix} \\ &= \Delta^{-1}\begin{bmatrix} R(J_{\theta\theta}J_{xx} - J_{\theta x}^2) \\ -u''J_{\theta x}\end{bmatrix}.\end{aligned} \tag{A.30}$$

The effect of wealth on c given by C_x is positive by Lemma 2. The effect of wealth on θ given by Θ_x is positive by Lemma 4, in conjunction with the first-order condition (A.25).∎

Proof of Proposition 2

Since the determinant of the product of two matrices is equal to the product of the determinants, (A.26) implies

$$[J_{cc}J_{\theta\theta} - J_{c\theta}^2][C_x\Theta_\lambda - C_\lambda\Theta_x] = -[J_{cx}J_{\theta\lambda} - J_{c\lambda}J_{\theta\lambda}] \tag{A.31}$$

and therefore

$$\begin{aligned}[C_\lambda\Theta_x - C_x\Theta_\lambda] &= \frac{[J_{cx}J_{\theta\lambda} - J_{c\lambda}J_{\theta\lambda}]}{[J_{cc}J_{\theta\theta} - J_{c\theta}^2]} \\ &= \frac{R}{\Delta}[J_{\theta\lambda}J_{xx} - J_{\theta x}J_{\lambda x}] \\ &\geq 0,\end{aligned} \tag{A.32}$$

by Lemma 1, Lemma 5 and (A.29).∎

Remark: As noted in the remark to Lemma 5, increasing absolute prudence on any interval implies that $J_{\theta\lambda}J_{xx} - J_{\theta x}J_{\lambda x} \leq 0$, and therefore that $C_\lambda\Theta_x - C_x\Theta_\lambda \leq 0$. Since it is always possible to choose x, $\tilde{z}$ and $\lambda\tilde{h}$ in such a way that only the values of v on a small interval matter, decreasing absolute prudence is a necessary condition for (A.32) to hold for any x, $\tilde{z}$ and $\lambda\tilde{h}$.∎

Proof of Proposition 3

Calculating C_λ and Θ_λ just as we calculated C_x and Θ_x (that is, by multiplying both sides of A.26 on the left by A^{-1}),

$$\begin{aligned}\begin{bmatrix} C_\lambda \\ \Theta_\lambda \end{bmatrix} &= -A^{-1}\begin{bmatrix} J_{c\lambda} \\ J_{\theta\lambda}\end{bmatrix} \\ &= -\Delta^{-1}\begin{bmatrix} J_{\theta\theta} & RJ_{\theta x} \\ RJ_{\theta x} & R^2 J_{xx} + u'' \end{bmatrix}\begin{bmatrix} -RJ_{x\lambda} \\ J_{\theta\lambda}\end{bmatrix} \\ &= \Delta^{-1}\begin{bmatrix} R(J_{\theta\theta}J_{x\lambda} - J_{\theta x}J_{\theta\lambda}) \\ R^2(J_{\theta x}J_{x\lambda} - J_{xx}J_{\theta\lambda}) - u''J_{\theta\lambda}\end{bmatrix}.\end{aligned} \tag{A.33}$$

Lemma 5 implies that $R^2(J_{\theta x}J_{x\lambda} - J_{xx}J_{\theta\lambda}) \leq 0$. Moreover, since $J_{xx} < 0$, Lemma 5 implies that

$$J_{\theta\lambda} \leq \frac{J_{\theta x}J_{\lambda x}}{J_{xx}}. \tag{A.34}$$

The first-order condition $J_\theta = 0$, together with Lemma 4, implies that $J_{\theta x} \geq 0$, and the assumption of Proposition 3 that $J_\lambda \leq 0$, together with Lemma 4, implies that $J_{\lambda x} \geq 0$. Therefore, (A.34) guarantees that

$$-u''J_{\theta\lambda} \leq -\frac{u''J_{\theta x}J_{\lambda x}}{J_{xx}} \leq 0. \tag{A.35}$$

Thus, $\Theta_\lambda \leq 0$ whenever $J_\lambda \leq 0$.■

Proof of Proposition 4

From (A.33)

$$\mathrm{sign}\big(C_\lambda(x,\lambda)\big) = \mathrm{sign}\big(J_{\theta\theta}J_{x\lambda} - J_{\theta x}J_{\theta\lambda}\big). \tag{A.36}$$

If λ were chosen optimally, the first-order condition

$$J_\lambda(c,\theta;x,\lambda) = 0 \tag{A.37}$$

would always hold. All three of the variables c, θ and λ would be functions of x alone. Differentiating (A.37) and the other two first-order conditions with respect to x would yield the matrix equation

$$\begin{bmatrix} J_{cc} & J_{c\theta} & J_{c\lambda} \\ J_{c\theta} & J_{\theta\theta} & J_{\theta\lambda} \\ J_{c\lambda} & J_{\theta\lambda} & J_{\lambda\lambda} \end{bmatrix} \begin{bmatrix} \frac{dc^*}{dx} \\ \frac{d\theta^*}{dx} \\ \frac{d\lambda^*}{dx} \end{bmatrix} = -\begin{bmatrix} J_{cx} \\ J_{\theta x} \\ J_{\lambda x} \end{bmatrix}. \tag{A.38}$$

Define

$$B = \begin{bmatrix} J_{cc} & J_{c\theta} & J_{c\lambda} \\ J_{c\theta} & J_{\theta\theta} & J_{\theta\lambda} \\ J_{c\lambda} & J_{\theta\lambda} & J_{\lambda\lambda} \end{bmatrix}. \tag{A.39}$$

By Cramer's rule,

$$\begin{aligned} \frac{d\lambda^*}{dx} &= \frac{-1}{\det(B)} \begin{vmatrix} J_{cc} & J_{c\theta} & J_{cx} \\ J_{c\theta} & J_{\theta\theta} & J_{\theta x} \\ J_{c\lambda} & J_{\theta\lambda} & J_{\lambda x} \end{vmatrix} \\ &= \frac{-1}{\det(B)} \begin{vmatrix} u'' & J_{c\theta} & J_{cx} \\ 0 & J_{\theta\theta} & J_{\theta x} \\ 0 & J_{\theta\lambda} & J_{\lambda x} \end{vmatrix} \\ &= \frac{-u''}{\det(B)}[J_{\theta\theta}J_{\lambda x} - J_{\theta\lambda}J_{\theta x}], \end{aligned} \tag{A.40}$$

where the second line results from subtracting R times the third column from the first column of the determinant in the numerator on the first line. Since B is a negative definite 3×3 matrix, $\det(B) < 0$ and

$$sign\left(\frac{d\lambda^*}{dx}\right) = -sign(J_{\theta\theta}J_{\lambda x} - J_{\theta\lambda}J_{\theta x}) = -sign(C_\lambda).■ \tag{A.41}$$

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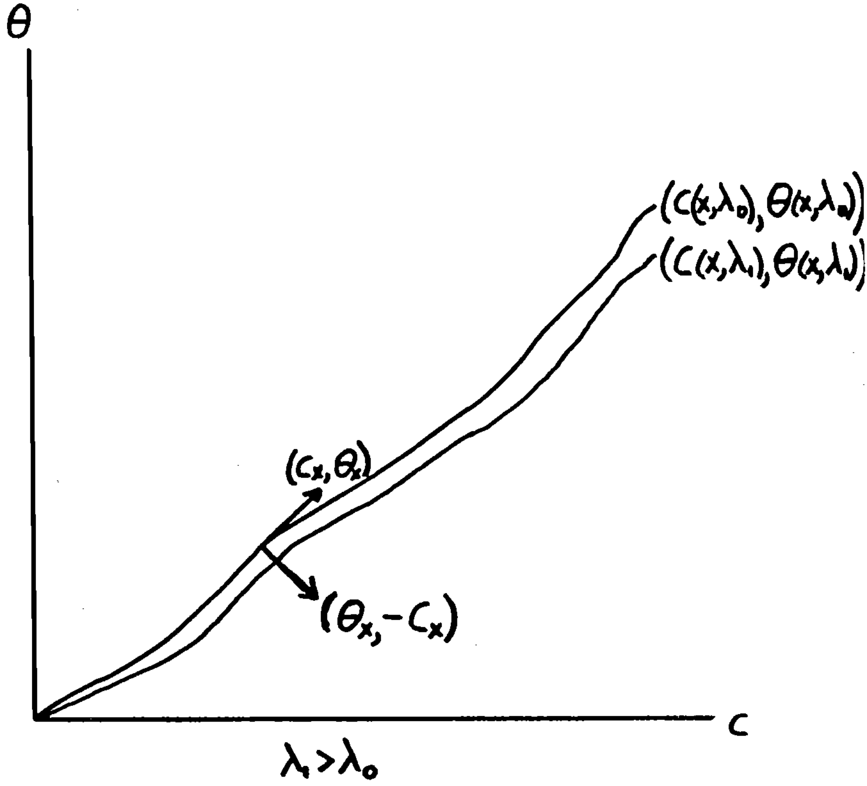

Figure 1