NBER WORKING PAPERS SERIES

LABOR TURNOVER COSTS AND AVERAGE LABOR DEMAND

Giuseppe Bertola

Working Paper No. 3866

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
October 1991

This revision: August, 1991. Larry Ball, Samuel Bentolila, Avinash Dixit, Allan Drazen, and two referees made helpful comments on earlier drafts. Hyeng Keun Koo provided competent research assistance, supported by the John M. Olin Program for the Study of Economic Organization and Public Policy. Work on this paper was financed in part by Consiglio Nazionale delle Ricerche, Progetto Finalizzato *Struttura ed Evoluzione dell'Economia Italiana*, Contract 87.01644.53. This paper is part of NBER's research program in Economic Fluctuations. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

NBER Working Paper #3866
October 1991

LABOR TURNOVER COSTS AND AVERAGE LABOR DEMAND

ABSTRACT

This paper studies simple partial equilibrium models of dynamic labor demand, under certainty. Labor turnover costs may or may not decrease the firm's average labor demand, depending on the form of the revenue function, on the rates of discount and of labor attrition, and on the relative size of hiring and firing costs. With strictly positive discount and labor attrition rates, the firm's optimal policy is partially myopic, and firing costs may well increase average employment even when hiring costs reduce it.

Giuseppe Bertola
Department of Economics
Fisher Hall
Princeton University
Princeton, NJ 08544-1021
CEPR and NBER

1. Introduction

In most countries and sectors, employers are not allowed to shed redundant labor at will in the face of adverse business condition shocks. Rather, employment reductions are regulated by law or by contractual arrangements (see Emerson (1988), Piore(1986), and Lazear (1990)). Partial equilibrium models taking wages and business conditions are given are a useful first step towards understanding the effects of such "employment protection" provisions. Higher turnover costs should clearly reduce the amplitude of employment fluctuations, but their implications for average labor demand are less straightforward to derive and, in general, ambiguous. Obstacles to firing should prevent loss of existing jobs in cyclical downturns, but also discourage hiring in upswings. Marginal employment subsidies, often advocated as a less costly alternative to generalized wage subsidies, have similarly ambiguous effects inasmuch as they should induce rational forward-looking firms to fire more as well as hire more.

The models in Bentolila and Bertola (1990) and Bertola (1990) study these and other issues allowing for simple forms of uncertainty. In this paper, certainty is assumed throughout to avoid unessential complications and to facilitate comparison with earlier work on the subject. In a nonstochastic model of dynamic labor demand similar to those in Nickell (1978) and (1986), the direction of turnover cost effects on average employment depends on three features of the firm's dynamic problem: first, on the relative steepness of labor's marginal revenue product function at firing and hiring times; second, on the size of the discount and labor attrition factors over a typical hiring-firing cycle; and, third, on the relative size of firing and hiring costs. While the first determinant of average labor demand is functional-form dependent, the second and third reflect a general insight: the optimal labor demand path for a discounted objective function is partially myopic, and firing costs (but not hiring costs) tend to increase a myopic firm's average labor demand through a rational ratchet effect.

Section 2 sets up the formal model. Section 3 characterizes average labor demand when the firm's objective is maximization of average (undiscounted) cash flow, and Section 4 shows that positive discount and attrition rates yield asymmetric effects of hiring and firing costs. Section 5 proposes an application, with explicit numerical solutions for specific functional forms. Section 6 briefly discusses the results in light of related research, and concludes.

2. Labor costs and labor demand

Let the revenues of the firm under consideration be a differentiable function $R(L,Z)$ of homogeneous labor L and other variables Z, and let this function be increasing and concave in its first argument. The variables in Z are taken to be exogenous for the purpose of employment determination. For example, the firm might be a perfect competitor producing under decreasing returns to scale, with Z indexing the relative prices of output and inputs other than labor; or it might be endowed with market power, with Z indexing the position of its demand function.

If c denotes labor costs per employee, the static profit-maximizing employment level L^* is determined by the condition

$$M(L^*,Z) = c, \tag{1}$$

where $M(L,Z) \equiv \partial R(L,Z)/\partial L$ is the marginal revenue product of labor (MRPL). In comparative static exercises, a lower L^* corresponds to a higher c for given Z, and different values of Z may increase or decrease L^* depending on how Z affects the position and shape of the MRPL function.

If turnover costs were to be regarded as components of the cost of labor, like wages, one would simply add them to the right-hand side of (1) and conclude that employment must generally be lower if turnover costs are higher. However, the burden of hiring and firing costs is not necessarily reflected in actual payments by the firm, because turnover decisions are discretionary: at times when static maximization of operating profits would call for employment changes, adjustment costs can discourage the firm from hiring or firing. Any such decision *not to* hire or fire affects revenues for given Z and, so to speak, alters the left-hand side of the static first order condition (1) as well as its right-hand side. Hence, the effects of turnover costs on average employment can only be studied in an explicitly dynamic model.

Consider then the problem of a firm faced by certain, periodic fluctuations in revenues. Let cumulated revenues be the integral of the flow revenue function $R(L,Z)$, and let the time path of Z_t be a continuous, differentiable function of time which repeats itself every p time units.[1] It would not be difficult to allow for cyclical variation in wage rates as well, but wages will be assumed constant at $w > 0$ for simplicity, and to model the relative rigidity of compensation in long-term labor contracts.

The firm chooses the dynamic path of employment taking the wage rate as given. Let a unit decrease in employment entail a cost of f when initiated by the firm, modeling the fact that contracts and legislation often make it costly or difficult for the firm to shed redundant workers; voluntary quits, instead, are costless from the firm's point of view. As to hiring costs, let h denote the cost to the firm of a unit increase in employment, reflecting financing of firm-specific human capital net of any marginal employment subsidies. Total turnover costs $f+h$ must of course be positive, to prevent the firm from earning unbounded amounts by very fast turnover; in most realistic situations, both f and h are positive.

Let the firm's planning horizon be infinite, to allow a straightforward definition of long-run average employment and to sidestep the issue of a realistic specification of termination payments to workers at the end of a firm's life. Denoting with $r \geq 0$ the rate of return on the firm's operations and with $\delta \geq 0$ the rate at which voluntary, costless separations occur, the firm should solve the problem

$$\max_{\{X_t\}} V_t \text{ subject to } \dot{L}_\tau = -\delta L_\tau + \dot{X}_\tau \tag{2}$$

where $\dot{X}_t$ is the rate at which the firm hires or fires workers,[2] and where the value of the dynamic employment plan is given by

$$V_t = \int_t^\infty \left(R(L_\tau, Z_\tau) - wL_\tau - C(\dot{X}_\tau)\dot{X}_\tau\right) e^{-r(\tau-t)}\, d\tau \tag{3a}$$

if $r > 0$, and by

$$V_t = \lim_{T\to\infty} \frac{1}{T}\int_t^T \left(R(L_\tau, Z_\tau) - wL_\tau - C(\dot{X}_\tau)\dot{X}_\tau\right) d\tau \tag{3b}$$

if $r = 0$. Formalizing the assumptions above, the piecewise constant unit turnover cost function $C(\dot{X})$ is defined to equal h if $\dot{X} > 0$, $-f$ if $\dot{X} < 0$.

The following conditions are necessary and sufficient for optimality (see e.g. Nickell 1978, 1986):

$$-f \leq \int_t^\infty (M(L_\tau, Z_\tau) - w)\, e^{-(r+\delta)(\tau-t)}\, d\tau \leq h, \qquad \forall t \tag{4}$$

$$-f = \int_t^\infty (M(L_\tau, Z_\tau) - w)\, e^{-(r+\delta)(\tau-t)}\, d\tau, \qquad \text{if } \dot{X}_t < 0 \tag{5}$$

$$\int_t^\infty (M(L_\tau, Z_\tau) - w)\, e^{-(r+\delta)(\tau-t)}\, d\tau = h, \quad \text{if } \dot{X}_t > 0 \tag{6}$$

At all times when the firm chooses to hire or to fire, the cost of optimal marginal turnover decisions should be equal to their effects on future discounted cash flows, computed taking future turnover decisions and labor attrition as given.[3] As the total adjustment cost function is piecewise linear in the hiring and firing rates, unit adjustment costs are (realistically) non negligible even for infinitesimal hiring and firing and the firm does not continuously act on a margin: inaction is optimal when deviations of $R(L_t, Z_t) - wL_t$ from its instantaneous maximum do not justify the cost of adjusting the employment trajectory. Optimal hiring and firing have a bang-bang character, and are interrupted by inaction periods—when (4) holds with strict inequalities and employment is simply allowed to decrease through attrition.

The firm's problem can be solved in two stages: it must decide whether to act rather than passively accept the steady decline of employment at rate δ and, when action is determined to be optimal, it must choose the amount of control to be exercised at every point in time. These decisions are interrelated, of course, since the decision to act cannot be taken optimally unless the amount of control to be applied is already known. When Z_t is a differentiable function of time, however, hiring and firing take place throughout time intervals of strictly positive length, and this makes it straightforward to analyze the latter decision in isolation.[4]

If $\dot{X}_t \neq 0$ in the interior of an interval $(t', t")$, differentiation of (5) or (6) with respect to t yields a local Euler equation:

$$M(L_t, Z_t) - w = (r + \delta)C(\dot{X}_t) \quad \text{if } \dot{X} \neq 0. \tag{7}$$

As hiring or firing is not going to be interrupted immediately, the firm need not look into the future any further than an infinitesimal amount of time to decide *how much* to hire or fire. When the firm is hiring, for example, the marginal unit cost of labor at time t equals the instantaneous wage rate w plus the foregone interest on the hiring cost paid, rh, plus δh, since quitting employees cause the loss to the firm of the amount spent in training and screening them. By concavity of revenues, the sum total of these cost must equal marginal revenues along the optimal employment path.

The firm does have to look far into the future in order to decide when to stop (or resume) hiring (or firing). The next sections solve for the optimal timing of these decisions, considering the $r + \delta = 0$ case first.

3. Inaction and average employment in the undiscounted case

When $r + \delta = 0$, employment is unaffected by the presence of turnover costs whenever the firm is hiring or firing and equation (7) holds. This is the only case considered by Nickell (1978) in his characterization of average employment under firing restrictions. Neither the assumption of no discounting nor that of no labor attrition are realistic, of course, but they usefully isolate some of the general features of the problem under consideration.

In Figure 1, the solid line represent the optimal dynamic employment path in the presence of turnover cost with $r = \delta = 0$, while the dashed line represents L_t^*, the employment path the firm would choose in the absence of turnover costs. By equation (7), the two lines coincide whenever the firm is hiring or firing. If a worker were hired and immediately fired, $h + f$ would be paid with no counteracting benefits: thus, any peak of L_t^* must be located in an inaction spell if $h + f > 0$. The same reasoning applies to troughs of L_t^*. Hence, positive turnover costs imply lower peaks and higher troughs for the employment time path.

The effect of turnover costs on average employment depends on the relationship between the inaction spells around peaks and those around troughs. It is shown in the Appendix that when $r = \delta = 0$ the length and position of the inaction spells are determined only by the sum total of h and f, not by the two parameters separately. To understand this, consider that at the margin an additional hired worker will certainly be fired before a full cycle elapses when employment fluctuates cyclically and there are no quits. Since the future firing cost is not discounted, the firm behaves *as if* it were incurred at the hiring time—and only the sum total of h and f matters for the firm's labor demand.

The appendix also shows that the average MRPL equals the wage if $r = \delta = 0$:

$$\frac{1}{\mathrm{P}} \int_t^{t+\mathrm{P}} M(L_\tau, Z_\tau)\, d\tau = w. \tag{8}$$

This simple relationship makes it possible to explore the role of functional forms in determining the average employment effects of turnover costs. By (8), the excess of the MRPL over the wage around peaks must offset the shortfall of the MRPL below the wage around troughs: but this does not imply that turnover costs should have no effects on average *employment*. If the MRPL is a steeper function of employment during slumps than during booms, the algebraic sum of labor hoarded in the former and restrained hiring in the latter is positive, and average employment is lowered by turnover costs. There is no theoretical reason for this to be the case, however. The contribution of labor to total revenues may well be a flatter function of employment during slumps, especially if hoarded labor ends up being idle or almost idle (see Figure 2a). And if cyclical fluctuations affect the MRPL multiplicatively—as they would in the case of a competitive firm faced by fluctuating prices—then the slope would be steeper during booms. On the other hand, the MRPL for given Z might be so convex in L that workers "not hired" during booms would operate on a relatively flat portion of the MRPL curve (as in Figure 2b).

Sufficient conditions for a positive average employment effect of turnover costs could in principle be found, but would involve high order derivatives of the revenue function (with respect to L and Z) on which theory cannot impose any restrictions. It seems preferable to illustrate by example the three effects outlined above: idle labor, multiplicative fluctuations, and convexity of the MRPL.

Example 1 (additive fluctuations and linear MRPL)
Let $M(L,Z) = \max(Z - bL, 0)$ and $Z_\tau > w$ for some τ. Denoting with $\tilde{L}_t$ the employment path implied by higher turnover costs, and with $\hat{L}_t$ that implied by lower turnover costs, equation (8) yields

$$\int_t^{t+p} \max(Z_\tau - b\tilde{L}_\tau, 0)\, d\tau = \int_t^{t+p} \max\left(Z_\tau - b\hat{L}_\tau, 0\right)\, d\tau$$

Let $\tilde{I}$ and $\hat{I}$ denote the sets of time points where the max operator in the $M(.)$ function, i.e. where labor is idled to prevent its marginal revenue product from becoming negative. More workers are idled during slumps for higher turnover costs, hence $\hat{I} \subseteq \tilde{I}$ and

$$\int_{\tilde{I}} \left(Z_\tau - b\tilde{L}_\tau\right)\, d\tau \leq \int_{\hat{I}} \left(Z_\tau - b\hat{L}_\tau\right)\, d\tau \quad .$$

Removing the max operator from the equation above, we then obtain

$$\int_t^{t+p} \left(Z_\tau - b\tilde{L}_\tau\right)\, d\tau \leq \int_t^{t+p} \left(Z_\tau - b\hat{L}_\tau\right)\, d\tau$$

to imply that $\tilde{L}_t$ is on average larger than $\hat{L}_t$, with equality obtaining if $\tilde{I}$ and $\hat{I}$ are empty: if labor is never idled, the slope of the MRPL function is constant over the cycle and the average level of employment is unaffected by turnover costs.

Example 2 (multiplicative fluctuations and linear MRPL)
Let $M(L,Z) = \max(Z(a-bL),0)$, with $Z_\tau \geq 0$ for all τ and $Z_\tau > a/w$ for some τ. If no labor is ever idled, we have from (8)

$$\int_t^{t+\mathrm{p}} Z_\tau \left(a - b\tilde{L}_\tau\right) d\tau = \int_t^{t+\mathrm{p}} Z_\tau \left(a - b\hat{L}_\tau\right) d\tau$$

or, defining $\bar{z} \equiv \int_t^{t+\mathrm{p}} Z_\tau \, d\tau$,

$$\int_t^{t+\mathrm{p}} (Z_\tau - \bar{z}) \left(\hat{L}_\tau - \tilde{L}_\tau\right) d\tau = \bar{z} \int_t^{t+\mathrm{p}} \left(\tilde{L}_\tau - \hat{L}_\tau\right) d\tau$$

In slumps, when Z is small, more labor is hoarded if turnover cost are higher. Hence, Z_τ and $\hat{L}_\tau - \tilde{L}_\tau$ are positively correlated, the left-hand side is positive, and employment is on average higher for higher turnover costs. If labor is idled during slumps, the arguments of Example 1 apply here as well.

Example 3 (constant-elasticity MRPL and prohibitive turnover costs)
Let $M(L,Z) = Z\,L^{-\beta}$, $0 < \beta < 1$, and let $Z_\tau > 0$ for all τ. In the absence of turnover costs, employment would equal $L^*_\tau = (Z_\tau/w)^{1/\beta}$ and average employment over the cycle would be given by

$$\frac{1}{\mathrm{p}} \int_t^{t+\mathrm{p}} L^*_\tau \, d\tau = \frac{w^{-1/\beta}}{\mathrm{p}} \int_t^{t+\mathrm{p}} Z_\tau^{1/\beta} \, d\tau$$

Suppose turnover costs are so large as to stabilize employment completely at some level $\bar{L}$. By equation (8), then,

$$\bar{L} = w^{-1/\beta} \left(\int_t^{t+\mathrm{p}} \frac{Z_\tau}{\mathrm{p}} \, d\tau \right)^{1/\beta}$$

By Jensen's inequality,

$$\frac{1}{\mathrm{p}} \int_t^{t+\mathrm{p}} Z_\tau^{1/\beta} \, d\tau \geq \left(\int_t^{t+\mathrm{p}} \frac{Z_\tau}{\mathrm{p}} \, d\tau \right)^{1/\beta}$$

Thus, a constant-elasticity MRPL function is so convex that turnover costs stabilize employment below the average of freely fluctuating employment.

4. Hiring vs. firing costs: discounting and attrition

The effects of hiring costs differ from those of firing costs if the firm, realistically, discounts future cash flows: firing costs have a stronger effect on firing decisions than on hiring ones, and vice versa. Because of discounting, the firm behaves myopically. When firing is being considered, redundancy payments loom large and undiscounted; when hiring, the firm takes into account only the discounted costs of firing.

Labor attrition has similar implications. Since attrition makes some firing unnecessary, only a portion of the (discounted) firing cost is imputed to a new hire, along with the full hiring cost. When firing, the firm should take into account the whole firing cost, but only that portion of the hiring cost that could be saved if the marginal employee were not fired *and* did not quit until the next hiring time.[5]

By equation (7), the MRPL is higher than the wage rate when the firm is hiring if $h > 0$, and lower than the wage rate when the firm is firing if $f > 0$. As to inaction periods, if t and t' are hiring times then (6) implies

$$\int_t^{t'} e^{-(r+\delta)(\tau-t)} \left(M(L_\tau, Z_\tau) - w - (r+\delta)h\right) d\tau = 0. \tag{9}$$

In words, the excess of the MRPL over $w+(r+\delta)h$, discounted at rate $r+\delta$, must integrate to zero over any period that begins and ends with hiring times. This is (trivially) true if hiring occurs continuously throughout (t,t'), and also if one or more inaction spells are included in (t,t'). Symmetrically, if T and T' are both firing times it is easy to obtain from (5)

$$\int_T^{T'} e^{-(r+\delta)(\tau-t)} \left(M(L_\tau, Z_\tau) - w + (r+\delta)f\right) d\tau = 0 \tag{10}$$

Equations (9) and (10), together with (4), uniquely determine the optimal employment path (see e.g. Nickell 1978, 1986).

The Appendix below proves that the average MRPL is lowered by firing costs, increased by hiring costs if r and/or δ are strictly positive. It is not difficult to see that firing costs tend to decrease labor's marginal productivity below its full-flexibility level w. If for example $h = 0$, $f > 0$ then, by equation (7), the MRPL equals w at t and at $t + \mathrm{p}$ if t is a hiring time; and, by (9), the discounted excess of the MRPL over the w averages to zero between t and $t+\mathrm{p}$. Between t and $t+\mathrm{p}$, times when the MRPL exceeds the wage come sooner, and are less heavily discounted, than times when the opposite is true. Thus, the former outweigh the latter when the undiscounted average is considered. Symmetrically, it is not difficult to see that if $h > 0$, $f = 0$ then the MRPL is on average higher than the wage; and the Appendix shows that similar arguments apply when both h and f are different from zero.

When $r + \delta > 0$, then, hiring cost increase the average MRPL and firing costs decrease it. At every point in time, a lower MRPL is associated to higher employment: hence, the effect of hiring and firing costs on average employment depends on their size relative to each other as well as on the relative steepness of the MRPL function in booms and slumps.

5. A numerical example

To highlight interactions between the results of the previous sections, it is useful to examine numerical solutions for a simple special case. Let the reduced-form revenue function be multiplicatively separable in labor input L and exogenous factors Z, and let the elasticity of revenues to employment be constant:

$$R(L, Z) = Z\,\frac{L^{1-\beta}}{1-\beta}, \qquad 0 < \beta < 1 \tag{11}$$

As shown in Example 3 above, in the absence of discounting this commonly assumed functional form would tend to yield a negative effect of turnover costs on average employment. If discounting and attrition over a typical hiring-firing cycle are sufficiently important, however, firing costs (not hiring costs) can increase average employment by the mechanism discussed in Section 4.

As to the time path of the forcing variables, consider the simple trigonometric function

$$Z_\tau = K_1 + K_2 \sin\left(\frac{2\pi}{\mathrm{p}}\tau\right), \qquad K_1 > K_2 > 0. \tag{12}$$

If the period p is one year, such perfectly cyclical behavior of revenues might be interpreted as a stylized model of a firm in a seasonal industry, e.g. a ski resort.

If $f = h = 0$, the firm hires half of the time, and fires at all other times. Turnover costs shorten both the hiring and firing periods, inserting inaction spells between them. The shape of the employment path over a cycle is plotted in the top panel of Figure 3. The upper dashed line plots

$$L_\tau^f = \left(\frac{K_1 + K_2 \sin(\frac{2\pi}{\mathrm{p}}\tau)}{w - (r+\delta) f}\right)^{1/\beta},$$

the level of employment when the firm is firing. When the firm is hiring, employment equals

$$L^h_\tau = \left(\frac{K_1 + K_2 \sin(\frac{2\pi}{\mathrm{P}}\tau)}{w + (r+\delta)h} \right)^{1/\beta},$$

plotted in Figure 3 as the lower dashed line.

To solve the dynamic optimization problem one needs to find, in the interval $(0, \mathrm{p})$, the point t (where hiring stops) and the point T (where firing stops). It is not difficult to obtain numerical solutions. The Appendix outlines the solution method, and the tables report selected results.

The results of Table 1 can be interpreted in terms of the "MRPL slope" and "discounting" effects identified in Section 3 and 4. The values of β, K_1, and K_2 matter for the reasons discussed in Section 3: they affect average employment for given average MRPL. The parameters r, p, h, and f all have the dimension of time and, instead, affect average employment through the effect of discounting on average MRPL discussed in Section 4. The quit rate δ affects average MRPL similarly to the discount rate r and has a role in determining average employment for given MRPL as well, because different rates of labor attrition affect the behavior of the MRPL during inaction spells.

Table 2 focuses on the effects of different firing costs. Given other parameter values, a larger f monotonically lowers the average MRPL. For small f, the MRPL slope effect prevails under the assumed functional forms, and average employment is a decreasing function of f; but the discounting effects becomes important as the inaction periods lengthen with larger f, and in panel A of Table 2 average employment is eventually higher than it would be in the absence of turnover costs. Firing costs may become prohibitive *before* the level at which average employment starts increasing, however: this is the case if the attrition and discount rates are small, or if the firm is subject to frequent fluctuations (see panel B in Table 2). Once firing costs are prohibitive their actual size is of course irrelevant, since the firm never pays them.

Similar insights can be gained from Table 3, which illustrates the effects of different cycle lengths p. A longer cycle is equivalent to higher discount and depreciation rates and smaller turnover costs.[6] Thus, firing costs of the same magnitude may increase employment if hiring-firing cycles are long, decrease it if they are short.

6. Discussion of the results and concluding comments

While the model of this paper is much too simple for immediate application to reality and data, the results of the formal analysis above may shed some light on the functioning of complex real-life labor markets. In general, the dynamic nature of employers' labor demand decisions should be taken into consideration both by theoretical work aiming to explain job security legislation and contractual penalties on labor force reductions, and by partial-equilibrium empirical analyses of job security provisions.

If firing costs only improved incumbent workers' tenure prospects and their wage-bargaining position, while decreasing employment on average, it would be difficult to rationalize the favor accorded by workers and by long-lived unions to widely applicable, permanent firing restrictions. By the discounting effect identified above, however, firing costs tend to increase average employment at given wages: thus, job-security provision may improve the long-run employment outlook of all workers, whether currently employed or unemployed. When this is the case, the degree of job security should depend on the political and contractual strength of organized labor relative to that of employers—who obviously favor employment flexibility, since any static or dynamic constraint imposed on a firm's operations reduces its value. In turn, a lower value of investment opportunities may imply a slower growth rate of labor's productivity and wages in general equilibrium; the welfare implications of this are studied in Bertola (1991).

Consideration of the problem's dynamic dimension might also contribute to explain different degrees of job security imposed on different firms in the same institutional environment. It is shown above that long intervals between hiring and firing times may be necessary for firing costs to have positive effects on firms' average labor demand.[7] In reality, firing costs might then reduce average labor demand by firms whose business is highly seasonal or volatile, like ski resorts, restaurants, and marginal firms with little market power; this might explain why employment is typically quite flexible for small firms and firms in the service sector. Table 4 illustrates the point with numerical calculations based on the model of Section 5. The value of the firm is lowered throughout the cycle if firing costs are higher. For large firing costs, however, the present discounted value of the wage bill is increased by firing costs at all times, even at the cyclical peak; and the sum of discounted wages and discounted firing costs (at least part of which might be paid directly to the workers) increases even for small values of f. What is interesting in the table is the sign of the effects: their magnitude is small under the assumed functional forms and parameter values, but much more dramatic results could be obtained either adopting the linear-quadratic forms of Examples 1 and 2 above, or allowing for sharper and longer-lived revenue fluctuations.

As to empirical work, the theoretical model of this paper—like those in related work by Lazear (1990), Bentolila and Bertola (1990), and Bertola (1990)—indicates that turnover costs affect employment dynamics much more than the average level of employment. In particular, theory predicts that job security provisions should have relatively small, functional-form dependent effects on average labor demand. Available data sets probably contain little information about long-run issues, however, and the choice of sample period may crucially affect the results. In fact, Lazear (1990) finds only weak evidence of negative labor-demand effects in static regressions focused on the average effects of job security indicators, while the finding in Bertola (1990) that unemployment is more persistent in high-job-security environments may reflect stabilizing effects of institutional turnover costs on aggregate employment. Applied work should of course take into account many realistic aspects left out of simplified theoretical model like the one studied here. In particular, dynamic restrictions on employment should induce firms to exploit other margins of adjustment, and job security should imply higher volatility of hours worked per employee—as documented by Abraham and Houseman (1989) with U.S. and Japanese evidence—or a more pronounced tendency to contract out parts of the production process. Such flexibility devices would crucially affect the dynamic character of employment, production, and price data in response to similar exogenous shocks and, at a theoretical level, would alter the form of revenues as a function of employment.

The results above are also of some relevance to general equilibrium analysis and, in particular, to joint determination of wages and (contractual or legal) turnover costs. Partial-equilibrium models like that studied above indicate that firms' labor demand may on average be increased by higher firing costs: this implies higher average employment at given wages, and also, on the other side of the same coin, a higher market-clearing wage level if employment is taken as given instead, as done by Bertola (1991) in the context of a full-employment growth model. To endogenize both employment and wages, it would be necessary to model in detail the nature of the markets for labor and for the firm's product. As noted by Lazear (1990), issues of imperfect information and market incompleteness would be crucial to obtain real effects of job security provisions in this context. While optimal contracts should specify employment as well as wages for every state and time, in reality employment is often left to the employer's discretion, at predetermined wages: but employers and workers do include layoff restrictions and work rules in the contracts they negotiate and, in a static context, it has been noted as early as Leontief (1946) that such additional bargaining dimensions might bring real-life contracts closer to the theoretical, fully contingent ones—which may be too complex to draft and difficult to enforce under asymmetric information. The result that job security provisions have small and maybe positive effects on average employment might be relevant to studies of optimal contracts in an explicitly dynamic framework.

Appendix

Fact 1 If the firm maximizes undiscounted cash flows and there are no voluntary quits, then the dynamic path of employment depends only on the total turnover cost $h+f$, not on h and f separately.

proof: by equation (7), employment is unaffected by turnover costs whenever the firm is hiring or firing. If the firm hires at t and fires at T', the interval (t, T') necessarily includes an inaction spell if $h+f>0$; equations (5) and (6) then yield

$$\int_t^{T'} (M(L_\tau, Z_\tau) - w)\ d\tau = h + f$$

and the total excess of marginal revenues over inaction (the shaded area in Figure 1) must equal total turnover costs. Similarly, if the firm fires at T and hires at t' then

$$\int_T^{t'} (M(L_\tau, Z_\tau) - w)\ d\tau = -h - f$$

These equations determine the length and location of inaction spells, which thus depend only on $h+f$.

Fact 2 If the firm maximizes undiscounted cash flows and there are no voluntary quits, then the average marginal revenue product of labor over a cycle always equals the wage rate.

proof: If the firm hires at t, then it hires at $t+\mathbf{p}$ as well. Hence

$$\int_t^{t+\mathbf{p}} (M(L_\tau, Z_\tau) - w)\ d\tau = h - h = 0$$

Similar considerations apply to all other points along the cycle, to prove the fact.

Fact 3 The average marginal revenue product of labor is lower than the wage rate if $f>0$, $h=0$, and the difference between average MRPL and the wage is large in absolute value the larger is f.

proof: suppose hiring is interrupted only once between t and t' and let $t_1 \geq t$ be the time when hiring stops, t_2 be the first time after t_1 such that when $M(L_{t_2}, Z_{t_2}) = w$, and let $t_3 \leq t'$ be the time when hiring resumes.

Since $M(L_\tau, Z_\tau) - w > 0$ for $\tau \in [t_1, t_2]$, and given $r+\delta>0$, we have

$$\int_{t_1}^{t_2} e^{-(r+\delta)(\tau-t)} (M(L_\tau, Z_\tau) - w)\ d\tau > e^{-(r+\delta)(t_2-t)} \int_{t_1}^{t_2} (M(L_\tau, Z_\tau) - w)\ d\tau$$

Noting that $M(L_\tau, Z_\tau) - w < 0$ for $\tau \in [t_2, t_3]$, we also have

$$\int_{t_2}^{t_3} e^{-(r+\delta)(\tau-t)} \left(M(L_\tau, Z_\tau) - w\right) d\tau > e^{-(r+\delta)(t_2-t)} \int_{t_2}^{t_3} \left(M(L_\tau, Z_\tau) - w\right) d\tau$$

As a larger f is associated with larger absolute values of $M(L_\tau, Z_\tau) - w$, these inequalities are strengthened by a larger f.

The discounted integral of $M(L_\tau, Z_\tau) - w$ is zero between hiring times, and $M(L_\tau, Z_\tau) - w = 0$ for $\tau \in [t, t_1] \cup [t_3, t']$, when the firm is hiring; thus, summing the above, we obtain

$$0 = \int_t^{t'} e^{-(r+\delta)(\tau-t)} \left(M(L_\tau, Z_\tau) - w\right) d\tau > e^{-(r+\delta)(t_2-t)} \int_t^{t'} \left(M(L_\tau, Z_\tau) - w\right) d\tau$$

This proves the fact if hiring is interrupted only once during the cycle. Similar arguments are valid for each interruption of hiring f more than one occurs, and the proof of the general case can be completed by discounted sum.

Fact 4 The MRPL is higher than the wage rate if $f = 0$, $h > 0$, and the more so the larger is h.

proof: Similar to the proof of Fact 3.

Fact 5 The dynamic path of employment is the same if the structure of labor costs is given by w, h, f or by $\tilde{w}, \tilde{h}, \tilde{f}$ such that

$$\tilde{w} + (r+\delta)\tilde{h} = w + (r+\delta)h \qquad \text{and} \qquad \tilde{w} - (r+\delta)\tilde{f} = w - (r+\delta)f$$

proof: it is easy to show that the same L_τ path satisfies equations (4-7) and (8) for either set of parameters.

Fact 6 If both f and h are different from zero, then the average MRPL is lowered by larger f for given h, and vice versa.

proof: Fact 5 allows a given set of labor cost parameters to be transformed into an equivalent one with $\tilde{f}$ or $\tilde{h}$ equal to zero. Fact 3 or Fact 4 then yield the result.

On numerical solution of the model in Section 5

Inserting the functional forms in (11) and (12) into equations (4) and (5), some rearranging yields

$$\int_t^{T'} e^{-(r+\delta)(\tau-t)} \left(\left(K_1 + K_2 \sin(\frac{2\pi}{\text{P}}\tau) \right) \left(L_t e^{-\delta(\tau-t)} \right)^{-\beta} - w \right) d\tau = h + f\, e^{-(r+\delta)(T'-t)}$$

where

$$L_t = \left(\frac{K_1 + K_2 \sin(\frac{2\pi}{\text{P}} t)}{w + (r+\delta)h} \right)^{1/\beta}$$

and T' is the first time after t such that

$$\left(\frac{K_1 + K_2 \sin(\frac{2\pi}{\text{P}} T')}{w - (r+\delta)f} \right)^{1/\beta} = L_t\, e^{-\delta(T'-t)}$$

(If δ is large, T' may fail to exist: then the firm never fires, and t and t' should be chosen to satisfy the appropriate optimality conditions.)

Similarly,

$$\int_T^{t'} e^{-(r+\delta)(\tau-T)} \left(\left(K_1 + K_2 \sin(\frac{2\pi}{\text{P}}\tau) \right) \left(L_T e^{-\delta(\tau-t)} \right)^{-\beta} - w \right) d\tau = -h\, e^{-(r+\delta)(t'-T)} - f$$

where

$$L_T = \left(\frac{K_1 + K_2 \sin(\frac{2\pi}{\text{P}} T)}{w - (r+\delta)f} \right)^{1/\beta}$$

and t' is the first time after T such that

$$\left(\frac{K_1 + K_2 \sin(\frac{2\pi}{\text{P}} t')}{w + (r+\delta)h} \right)^{1/\beta} = L_T\, e^{-\delta(t'-T)}$$

These integrals are obtained in closed form using

$$\int e^{\lambda x} \sin(\gamma x)\, dx = \frac{\lambda e^{\lambda x}}{\gamma^2 + \lambda^2} \left(\sin(\gamma x) - \frac{\gamma}{\lambda} \cos(\gamma x) \right)$$

and it is not difficult to devise an iterative procedure to determine t, t', T, and T' from these equations. Average employment and average MRPL can then be computed by numerical integration.

Footnotes

[1] This is the simple dynamic framework of Nickell (1978). Bentolila and Bertola (1990) allow for a stochastic Z_t process with continuous, almost nowhere differentiable sample paths; Bertola (1990, 1991) models Z_t as a piecewise constant function of time, with infrequent jumps.

[2] As the adjustment cost function is not strictly convex, $\dot{X}_\tau$ need not exist in general. Under the differentiability and continuity assumption made above, however, X_τ is always differentiable except possibly at the beginning of time.

[3] Hiring (firing) one unit of labor at time t increases (decreases) labor input by $e^{-\delta(\tau-t)}$ at time τ when future hiring and firing decisions are, by the envelope theorem, taken as given. A constant, certain rate of labor attrition approximates real-life situations in which any employee quits with probability $e^{-\delta}$ in a unit of time.

[4] In Bentolila and Bertola (1990) and Bertola (1990, 1991) the exogenous variables are not differentiable as a function of time, and hiring and firing occur at isolated time points.

[5] Other characteristics of the labor market are also relevant on this margin. As a referee points out, not all (if any) of the specific training financed by the firm is lost upon firing if the employer can hope to recall laid-off employees at a later date.

[6] It is easy to verify by a change of variables that the same employment path satisfies the optimality conditions, under the functional forms assumed in this section, if r and δ are multiplied by some constant and h, f, and p are divided by the same.

[7] Bentolila and Bertola (1990) represent firms' business conditions by a geometric Brownian motion process, the continuous time equivalent of a random walk with drift, and find that firing costs increase employment even for very small discount and attrition rates. The spectral density of a nonstationary process diverges at zero, and this makes it roughly equivalent to infinitely long cycles in the framework considered here.

REFERENCES

Abraham, Katharine G. and Susan N. Houseman (1989), "Job Security and Work Force Adjustment: How Different are U.S. and Japanese Practices?," *Journal of the Japanese and International Economies* **3**, 500-521

Bentolila, Samuel, and Giuseppe Bertola (1990), "Firing Costs and Labour Demand: How Bad Is Eurosclerosis?," *Review of Economic Studies* **57**, 381-402

Bertola, Giuseppe (1990), "Job Security, Employment and Wages," *European Economic Review* **34**, 851-886

Bertola, Giuseppe (1991), "Flexibility, Investment, and Growth," working paper (CEPR D.P.#422; revised)

Emerson, Michael (1988), "Regulation or Deregulation of the Labor Market," *European Economic Review* **32**, 775-817

Lazear, Edward P. (1990), "Job Security Provisions and Employment," *Quarterly Journal of Economics* **CV**, 699-726

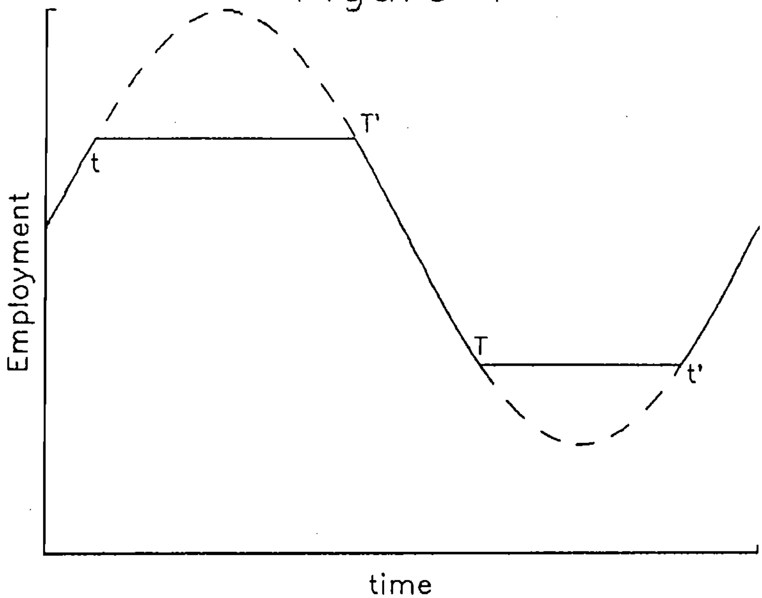
Leontief, Wassily (1990), "The Pure Theory of the Guaranteed Annual Wage Contract," *Journal of Political Economy* **LIV**, 72-76

Nickell, Stephen J. (1978), "Fixed Costs, Employment, and Labour Demand Over the Cycle," *Economica* **43**, 329-345

Nickell, Stephen J. (1986), "Dynamic Models of Labor Demand," in O. Ashenfelter and D. Layard (eds.), *Handbook of Labor Economics*, Volume 2, North Holland

Piore, Michael (1986), "Perspectives on Labor Market Flexibility," *Industrial Relations* **2**, 146-166

Figure 1



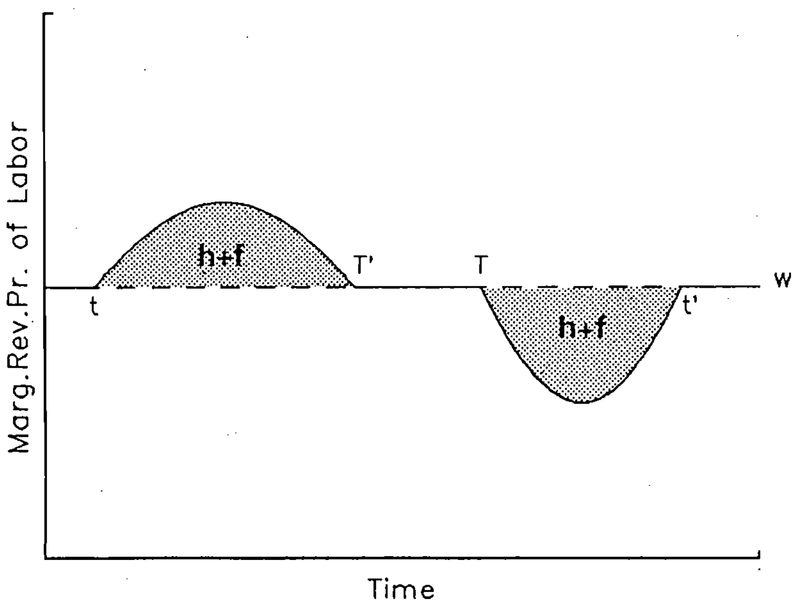


Figure 2

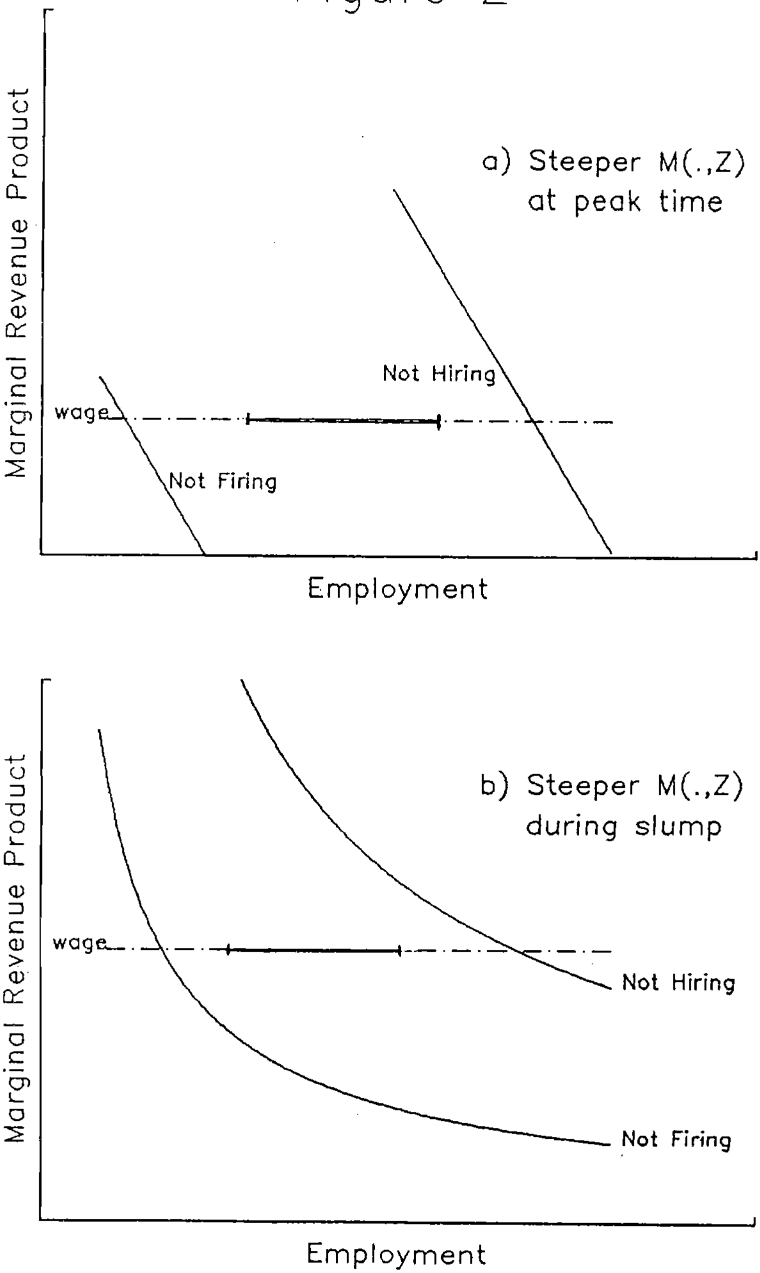
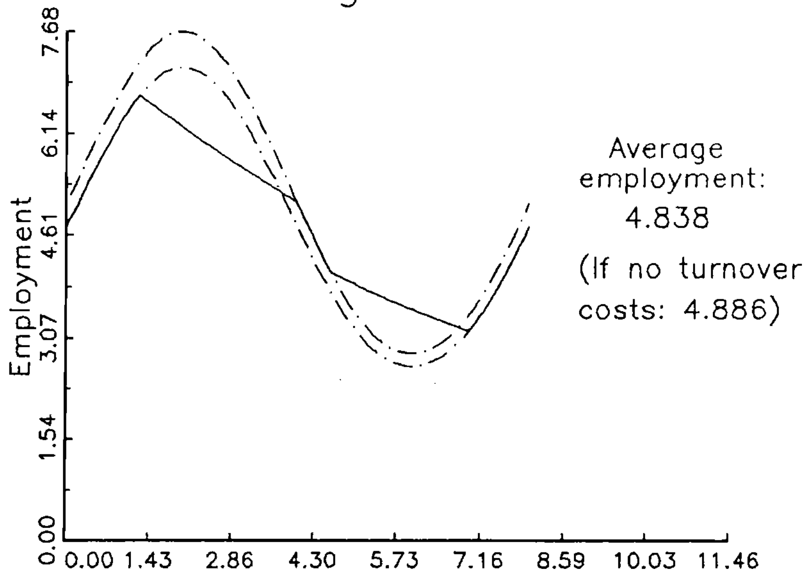

Marginal Revenue Product
a) Steeper M(.,Z)
at peak time
Not Hiring
wage
Not Firing
Employment
Marginal Revenue Product
b) Steeper M(.,Z)
during slump
wage
Not Hiring
Not Firing
Employment

Figure 3



δ=0.10 r=0.10 P=8.00 β=0.70

K_1=3.00 K_2=1.00 w=1.00 h=0.05 f=0.20

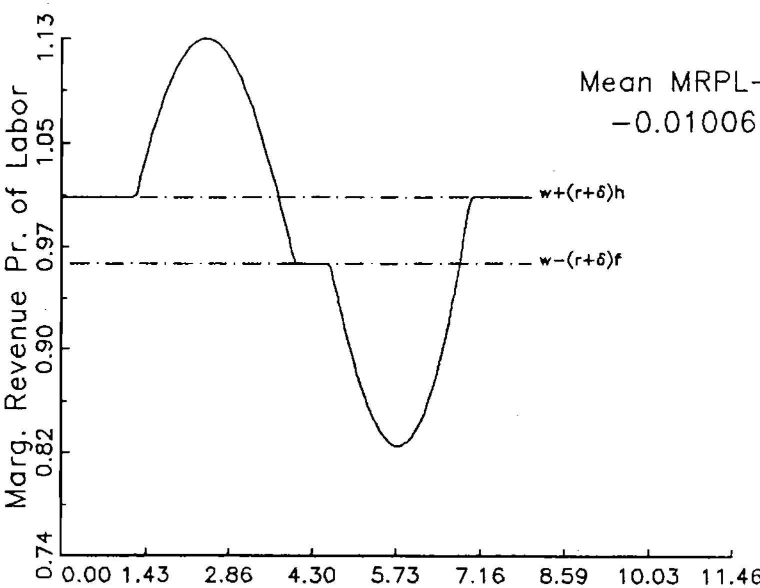


TABLE 1

Long-run effects of various differences in parameters

Parameter values (" = unchanged from line above)									Boundaries of inaction spells:				average M.R.P.L. - wage	average employment	avg.emp. if h=f=0	
P	δ	r	w	β	h	f	K1	K2	t1	t2	t3	t4		(A)	(B)	(A)/(B)
10	0.15	0.15	1	0.7	0.0	0.5	1.5	1.0	1.77	5.10	6.23	8.61	-0.06008	1.90283	1.90953	0.99649
"	0.10	"	"	"	"	"	"	"	1.52	4.83	6.29	8.65	-0.05386	1.88520	1.90953	0.98726
"	0.20	"	"	"	"	"	"	"	2.03	5.37	6.16	8.58	-0.06483	1.91819	1.90953	1.00454
"	0.15	"	"	"	"	0.3	"	"	1.95	4.75	6.46	8.35	-0.03649	1.89302	1.90953	0.99135
"	"	"	"	"	"	0.7	"	"	1.65	5.39	6.03	8.86	-0.08341	1.93140	1.90953	1.01145
"	"	0.10	"	"	"	0.5	"	"	1.73	5.04	6.21	8.58	-0.05043	1.87878	1.90953	0.98390
"	"	0.20	"	"	"	"	"	"	1.81	5.16	6.25	8.65	-0.06962	1.92711	1.90953	1.00921
"	"	0.15	"	0.5	"	"	"	"	1.59	4.92	6.30	8.67	-0.06364	2.64530	2.75000	0.96193
"	"	"	"	0.9	"	"	"	"	1.96	5.28	6.16	8.55	-0.05654	1.60719	1.59183	1.00965
20	"	"	"	0.7	"	"	"	"	5.62	10.95	12.76	16.39	-0.04738	1.93543	1.90953	1.01356
5	"	"	"	"	"	"	"	1.0	0.50	2.56	2.91	4.58	-0.06703	1.85313	1.90953	0.97046
10	"	"	"	"	"	"	3.0	"	2.33	N.A.	N.A.	8.32	-0.02112	4.94849	4.88621	1.01274
"	"	"	"	"	"	"	1.5	0.5	2.33	N.A.	N.A.	8.32	-0.02112	1.83836	1.81522	1.01274
"	"	"	"	"	0.1	"	"	1.0	1.72	5.22	6.14	8.72	-0.04182	1.83302	1.90953	0.95993
"	"	"	"	"	0.2	"	"	"	1.67	5.33	6.07	8.81	-0.02351	1.77028	1.90953	0.92708
"	"	"	"	"	0.2	0.0	"	"	2.10	4.50	6.61	8.17	0.03546	1.74323	1.90953	0.91291
"	"	"	"	"	-0.1	0.5	"	"	1.84	4.96	6.32	8.50	-0.07830	1.98083	1.90953	1.03734

TABLE 2

Effects of different firing costs

A) A long-cycle firm

Parameter values:

P	δ	r	w	β	h	f	K1	K2
10	0.05	0.15	1	0.7	0	see	1.5	1.0

f	Boundaries of inaction spells: t1	t2	t3	t4	average M.R.P.L. - wage	average employment (A)	avg.emp. if h=f=0 (B)	(A)/(B)
0.005	2.37	3.09	7.24	7.67	-0.00047	1.90844	1.90953	0.99943
0.01	2.28	3.19	7.19	7.73	-0.00095	1.90742	1.90953	0.99889
0.05	1.98	3.54	6.99	7.94	-0.00469	1.89983	1.90953	0.99492
0.20	1.60	4.04	6.69	8.28	-0.01864	1.87943	1.90953	0.98424
0.40	1.35	4.42	6.45	8.57	-0.03702	1.86624	1.90953	0.97733
0.60	1.18	4.69	6.25	8.81	-0.05525	1.86772	1.90953	0.97810
0.70	1.11	4.81	6.16	8.93	-0.06437	1.87449	1.90953	0.98165
0.80	1.06	4.92	6.08	9.05	-0.07346	1.88562	1.90953	0.98748
0.90	1.00	5.02	5.99	9.17	-0.08260	1.90166	1.90953	0.99588
1.10	0.91	5.21	5.82	9.41	-0.10099	1.95082	1.90953	1.02162
1.20	0.87	5.30	5.74	9.54	-0.11025	1.98539	1.90953	1.03973
1.30	0.83	5.39	5.65	9.67	-0.11961	2.02796	1.90953	1.06202
1.40	0.79	5.47	5.57	9.81	-0.12908	2.07968	1.90953	1.08911
1.50	0.77	N.A.	N.A.	9.89	-0.13460	2.11407	1.90953	1.10711

B) A short-cycle firm

Parameter values:

P	δ	r	w	β	h	f	K1	K2
1	0.15	0.15	1	0.7	0	see	1.5	1.0

f	Boundaries of inaction spells: t1	t2	t3	t4	average M.R.P.L. - wage	average employment (A)	avg.emp. if h=f=0 (B)	(A)/(B)
0.005	0.18	0.33	0.70	0.80	-0.00070	1.88943	1.90953	0.98947
0.01	0.16	0.35	0.69	0.81	-0.00145	1.87120	1.90953	0.97993
0.05	0.10	0.42	0.63	0.86	-0.00734	1.76849	1.90953	0.92614
0.20	0.02	N.A.	N.A.	0.99	-0.02864	1.83363	1.90953	0.96025

TABLE 3

Effects of different period length

Parameter values:

P	δ	r	w	β	h	f	K1	K2
see	0.15	0.15	1	0.7	0	0.5	1.5	1.0

P	Boundaries of inaction spells: t1	t2	t3	t4	average M.R.P.L. - wage	average employment (A)	avg.emp. if h=f=0 (B)	(A)/(B)
1	0.02	N.A.	N.A.	0.99	-0.02864	1.83363	1.90953	0.96025
2	0.07	N.A.	N.A.	1.97	-0.05188	1.87603	1.90953	0.98245
6	0.71	3.05	3.58	5.39	-0.06558	1.86227	1.90953	0.97525
10	1.77	5.10	6.23	8.61	-0.06008	1.90283	1.90953	0.99649
15	3.50	7.89	9.52	12.54	-0.05362	1.92736	1.90953	1.00934

TABLE 4

Present discounted value effects of different firing costs

Parameter values:

P	δ	r	w	B	h	f	K1	K2
10	0.20	0.10	1	0.7	0.1	see	1.5	1.0

Firing cost f =	0.00000	0.10000	0.20000	0.50000	1.00000
(t1) Stop hiring	2.49969	2.30896	2.17926	1.93359	1.79443
(t2) Start firing	4.42413	4.71985	4.94019	5.43060	N.A.
(t3) Stop firing	6.71752	6.53618	6.39602	6.05354	N.A.
(t4) Start hiring	7.93518	8.12531	8.27393	8.64227	8.94257
average MRPL - wage	0.01854	0.00705	-0.00409	-0.03682	-0.06499
(A) average employment	1.81929	1.81371	1.81115	1.82577	1.87293
(B) " " if f=h=0	1.90953	1.90953	1.90953	1.90953	1.90953
(A)/(B)	0.95274	0.94982	0.94848	0.95613	0.98083

<u>The following evaluated at the cyclical peak:</u>

(C) PDV of revenues	63.97969	63.80046	63.65964	63.42859	63.43474
(D) Discounted Wage Bill	18.47797	18.33917	18.25186	18.26791	18.65257
(E) PDV of hiring costs	0.35430	0.33937	0.32883	0.29823	0.26847
(F) PDV of firing costs	0.00000	0.13130	0.21965	0.26631	0.00000
(C)-(D)-(E)-(F) = Firm value	45.14742	44.98930	44.85953	44.59748	44.51370

<u>The following evaluated at the cyclical trough:</u>

(C) PDV of revenues	61.37753	61.28814	61.22026	61.16100	61.30506
(D) Discounted Wage Bill	17.79507	17.73300	17.70383	17.84185	18.31418
(E) PDV of hiring costs	0.58492	0.56094	0.54333	0.48843	0.44419
(F) PDV of firing costs	0.00000	0.07898	0.13447	0.16462	0.00000
(C)-(D)-(E)-(F) = Firm value	42.99755	42.91492	42.83910	42.66670	42.54670