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THE INFORMATIVENESS OF PRICES: SEARCH WITH LEARNING AND COST UNCERTAINTY

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ABSTRACT

Aggregate cost uncertainty, arising from real shocks or unanticipated inflation, reduces the informativeness of prices by scrambling relative and aggregate variations. But when agents can acquire additional information, such increased noise may in fact lead them to become better informed, and price competition will intensify. We examine these issues in a model of search with learning, where consumers search optimally from an unknown price distribution while firms price optimally given consumers' search rules. We show that the decisive factor in whether inflation variability increases or reduces the incentive to search, and thereby market efficiency, is the size of informational costs.

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I. Introduction

Consider the problem of a consumer who observes an unexpectedly high price at a gas station. She must estimate how much of it is due to a common factor affecting all other suppliers, such as an oil shock or high inflation working its way through the economy; and how much of it reflects a specific supply or demand shock for this particular seller. If the first explanation is deemed more relevant, it is not worth looking for a better deal elsewhere; in the opposite case it may pay to do so. Similarly, when this consumer observes that an automobile manufacturer is offering large rebates on new cars, she must resolve whether this reflects factors which are specific to this particular brand and make the offer a truly good deal, or whether the whole industry is having a sale. Again, her search behavior will vary with the inferences she draws from observed prices. These inferences are in turn based on her knowledge of the relative variability of idiosyncratic and aggregate shocks, inflation being one of the latter kind.

This problem of search with learning from prices is the main focus of the paper; it is important on several counts. First, from a microeconomic point of view, the inferences which buyers draw from prices underlie their demand functions, and should therefore play an important role in determining how markets react to oil shocks, weather variations, technological innovations, etc. Second, from a macroeconomic point of view, it has implications for what is often thought to be an important source of welfare losses from unanticipated inflation: a deterioration in the information content of prices, operating through increased relative price variability. [1] According to this view, stochastic inflation constitutes a source of aggregate noise in market signals, leading to inefficient allocation decisions. While this is well understood in

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1. Several studies documenting the correlation between the variance of unanticipated inflation and relative price variability (Vining and Elwertowski (1976), Parks (1978), Fischer (1981)) seem to lend support to this idea. Hercowitz (1982), on the other hand, finds that aggregate real shocks, not monetary shocks, explain relative price dispersion in the United States. As the average rate and the variability of inflation are also correlated (e.g. Taylor (1981), Pagan, Hall and Trivedi (1983)), high inflation is often seen as indirectly responsible for any informational costs of unanticipated inflation. The empirical literature on these issues is surveyed extensively in Fischer (1981) and Cukierman (1983).

models where the information structure is exogenous (e.g. Lucas (1973)), what happens when agents can decide to acquire additional information, by searching or otherwise, has not been explored. Finally, another common, but not previously formalized idea, is that sellers can "hide" behind aggregate or inflationary noise to charge higher real prices, taking advantage of consumers' reduced information to increase their markups.[2]

In this paper we attempt a first exploration of these issues by analyzing the effects of aggregate cost uncertainty, arising from real shocks or from unexpected general inflation, on the efficiency of allocations in an oligopolistic search market.[3]

Lucas (1973) and Barro (1976) show that stochastic inflation can cause producers to mistake aggregate price movements for relative ones, leading them to inefficient supply decisions. Cukierman (1979) and Hercowitz (1981) show that if supply elasticities differ across markets, these misperceptions will also manifest themselves through a correlation between the variance of inflation and that of relative prices. Our paper shares these models' central concern with "signal-extraction", but emphasizes the informational problems of consumers as well as producers. More fundamentally, it departs from the traditional literature by recognizing that:

(i) informational costs realistically imply market power. Standard macroeconomic models rely on an extreme asymmetry between informational costs within a market (zero) and across markets (infinite) to sustain the coexistence of perfect competition with imperfect price information. More plausible forms of informational imperfections will generate non-Walrasian prices; the issue then becomes whether inflationary uncertainty will worsen or alleviate preexisting distortions.

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2. For instance, there is a debate over whether oil companies and gasoline retailers took advantage of the confusion generated by the 1990 shutdown of the Alaskan pipeline and 1991 Persian Gulf war to increase their markups.

3. For a related analysis of the effects of anticipated inflation, see Bénabou (1988), (1991).

(ii) information is endogenous: agents' incentives to acquire information and their pricing strategies are determined jointly in equilibrium, and are both affected by the inflation process.[4] For instance, a deterioration in the reliability of price signals due to increased inflationary noise can spur agents to seek more price data, making them actually better informed in equilibrium. Naturally, the costs of acquiring information, such as search costs, will play a central role.

To take account of these points, we build a model with a stochastic environment similar to that of previous models, but with very different information and market structures. Duopolistic firms observe their own production costs, then set prices. Since costs are correlated due to common, inflationary shocks, buyers observing one firm's price learn something about the price of the other. Given these inferences and their knowledge of firms' strategies, they decide whether or not to search. Conversely, when setting prices firms take account of buyers' search rule, as well as of their own inferences about their competitor's cost and price.

Ours is therefore a model of search market equilibrium with Bayesian learning: it combines optimal adaptive search (Rothschild (1973), Rosenfield and Shapiro (1981)) with strategic pricing. To our knowledge this paper, and independent work by Dana (1990) and Fishman (1990) are the first such models. These two interesting papers have a different focus from ours, namely the limited responsiveness of prices to cost shocks, due to partial or complete pooling. They also use much more restrictive (two-state) stochastic cost structures.

Not surprisingly, the problem is quite complex, and this forces us to focus attention on a single market, as opposed to a general equilibrium framework. The question then arises of how to capture the effects of inflation in a microeconomic setting. We identify an increase in

4. This can be viewed as more constructive restatement of the standard criticism of "misperceptions" models, that agents need only obtain macroeconomic price or monetary statistics to become fully informed.

inflation uncertainty with an increase in the variance of industry-wide cost shocks. In effect, we take as given the fact that inflation impacts inter-industry costs, and examine how such shocks affect intra-industry pricing behavior and market performance.

Of course, this is only a partial and crude representation of inflation. First, inflation should affect the demand side as well as the supply side. But this is mainly a timing issue: realistically, when consumers see prices change unexpectedly, their resources have not yet been fully and unambiguously affected by the inflationary shock (if they had, the price change would not be unexpected, and there would be no difficulty in assessing relative prices). Similarly, when firms discover unexpected changes in costs, they have not yet experienced the full increase in nominal demand which consumers will eventually address to them. Second, our model has no money. But if one takes as given that inflation affects nominal costs, there is no substantial problem with calling the numeraire good money. Since all price changes are unexpected by agents given their information (any inflationary trend has already been factored out), the dollar prices which they observe are their best assessments of real prices, and may enter their utility and profit calculations without implying any money illusion.

Thus in spite of the model's obvious limitations, we feel that what we learn from it about the effects of real, aggregate cost shocks remains relevant for genuine, money-driven inflation. The reader who does not share these convictions can maintain a purely "real" reading of the paper; indeed the relationship between the stochastic structure of supply shocks and market efficiency is of interest independently of any possible link to inflation.

The variability of joint cost shocks affects consumers' signal-extraction problem, hence their search rules. This in turn determines the elasticity of demand faced by each firm, hence its pricing strategy and ultimately social welfare. This relationship between aggregate cost or inflation uncertainty, monopoly power and market efficiency is the central focus of the paper.

We identify two major effects of an increase in the variance of inflationary shocks. We refer to the first one as the correlation effect. In a market where search costs are high it reduces search, resulting in higher real prices; when search costs are low, on the contrary, it fosters search, hence lowers prices. The intuition goes as follows. As the variance of joint shocks increases, firms' costs become more correlated, and in equilibrium so do their prices. Bayesian consumers then put more weight on the first observed price and less on their prior, when forming their posterior beliefs about the second firm's price. Thus a high first observation implies a higher conditional mean of the price at the second firm; this lowers the value of search, as it becomes less likely that the observed high price is truly a bad deal. Conversely, if the observed price is low, greater correlation implies a lower conditional mean for the second firm's price. This increases the option value of search, as it becomes more likely that an even better deal can be found. If search costs are high, buyers' reservation price is high, so through the correlation effect inflation variability tends to increase it even more, and with it market power. Conversely if search costs are low, so is buyers' reservation price; inflation variability then tends to decrease it further, making the market more competitive.

An increase in the variance of joint cost shocks affects not only the correlation of costs, but also the variance of their conditional distribution. This in turn increases the conditional variance of prices, and thereby the value of search, as is well known. This is what we term the variance effect; as the variability of inflationary shocks increases, it always promotes search and tends to lower firms' market power.

These insights make clear the main result of the paper: whether inflation uncertainty lowers or raises welfare crucially depends on how costly it is to acquire information. Thus when it is recognized that informational imperfections give rise to market power and endogenous information gathering, the case for information-related welfare losses from variable inflation must be substantially qualified.

The paper proceeds as follows: Section II describes the model and Section III contains the construction of the equilibrium. The effects of changes in inflation variability are discussed in Section IV, both through analytic examples and simulations. Section V concludes.

II. The Model

In this section we present a model of a duopolistic search market equilibrium with Bayesian learning. Buyers' search decisions depend on the inferences they make from observed prices, taking into account firms' strategies. Conversely, firms' pricing decisions incorporate their own inferences about their competitor's prices, and their knowledge of buyers' inference and search rules.

1. Market and information structure. There are two identical firms, with constant marginal costs c_1 and c_2, which are drawn from a symmetric joint distribution with support $[c^-, c^+]\times[c^-, c^+]$, $0 \le c^- < c^+ < +\infty$. For instance, each firm's cost c_i could be the sum or product of a common cost shock θ (e.g. inflation) and a private cost shock γ_i (real cost). Firm i observes its own cost c_i, but not its rival's cost c_j, although c_i provides information about c_j.[5] Buyers do not observe cost realizations but the joint distribution of costs is common knowledge. We denote by $F(c_2 \mid c_1)$ and $f(c_2 \mid c_1)$ the distribution and density of firm 2's

5. For instance if $c_i = \theta + \gamma_i$, $i = 1, 2$, c_1 contains information about c_2 even though firm 1 does not know θ. This assumption fits well with the idea of aggregate uncertainty. Firms are themselves buyers of inputs, and do not have perfect information on whether shocks to materials and labor prices are specific to their own suppliers or economy-wide. In a richer model, of course, firms could also decide to become informed at some cost.

cost, conditional on firm 1's. We assume that $f(c_2 \mid c_1)$ is continuously differentiable in both c_1 and c_2 almost everywhere, and that costs are positively correlated in the following sense:

$$F_2(c_2 \mid c_1) \equiv \frac{\partial F(c_2 \mid c_1)}{\partial c_1} \leq 0, \quad \forall c_1, \tag{1}$$

$$\frac{\partial}{\partial c_1}\left[\frac{f(c_2 \mid c_1)}{1-F(c_2 \mid c_1)}\right]_{c_2=c_1=0} \leq 0, \quad \forall c_1. \tag{2}$$

Condition (1) is just first-order stochastic dominance. (2) is a local hazard rate condition which also means that a higher c_1 signals a higher c_2.[6] It will ensure that a firm's equilibrium expected profit function (conditional on its own cost) is quasi-concave.

We now turn to buyers. There is a continuum of identical consumers, with measure normalized to one. Let $S(p) \equiv \int_p^{\infty} D(r)dr$ be the surplus each of them derives from buying at p, where $D(p) \equiv -S'(p)$ is her demand function in the absence of search. We make the standard assumption that a firm's profit per customer $\Pi(p,c) \equiv (p-c)D(p)$ is strictly quasi-concave in p for all c in $[c^-, c^*]$. Since $\Pi(p,c)$ is the profit function of a monopolist with cost c, let $\Pi_m(c)$ denote its maximum value, achieved at $p_m(c)$.

In addition to conditions (1) and (2), we shall require some more technical assumptions on F, S and Π. Since they offer little insight, we have gathered them in Appendix A at the end of the paper.

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6. If (2) were required at every (c_1, c_2), it would say that the hazard rate for finding c_2, as one moves from c^- to c^*, is decreasing in c_1. Such is the case for instance if $f(c_2 \mid c_1)$ has the monotone likelihood ratio property, i.e., if $f(c_2' \mid c_1)/f(c_2 \mid c_1)$ increases in c_1 for any $c_2' > c_2$. But (2) need only hold as c_2 tends to c_1 from below, so it in fact quite weak. Note also that (1) tends to make (2) hold.

2. Search and learning from prices. Initially, half the buyers observe firm 1's price and half firm 2's price, at no cost. Given the observed price, each buyer must decide whether to purchase or to search and find out the other firm's price. Searching entails a cost σ but allows the consumer to buy at the cheapest of the two prices. The assumption that the first offer can be recalled costlessly means that σ is a pure informational cost, rather than a transportation or communication cost.

Given the first observed price, say p_1, a consumer must first infer the extent to which it reflects a firm-specific shock or a joint shock. She then forms a posterior about the other firm's price, with distribution $G(p_2 \mid p_1)$ and density $g(p_2 \mid p_1)$ on the price support $[\underline{p}, \bar{p}]$. Finally, given these beliefs, the consumer will decide that it is worth finding out p_2 before buying if the expected benefit from search,

$$W(p_1) = \int_{\underline{p}}^{p_1} [S(p_2) - S(p_1)] g(p_2 \mid p_1) dp_2 = \int_{\underline{p}}^{p_1} D(p_2) G(p_2 \mid p_1) dp_2, \quad (3)$$

is larger than the search cost σ; otherwise she will just buy right away at p_1.

Note from (3) that observing a high price has two effects on the expected return from search. For a given distribution, a higher p_1 makes it more likely that a better deal can be found, and this tends to increase $W(p_1)$; but if firms' prices are correlated, a high p_1 is "bad news" about the distribution $G(p_2 \mid p_1)$ of the other firm's price, and this tends to reduce $W(p_1)$. As is well-known from the literature on optimal sampling from an exogenous unknown distribution (e.g. DeGroot (1970)), this learning effect can result in search strategies where a price p_1 is rejected but a higher price $p_1' > p_1$ is accepted. To preserve the reservation price property, Rothschild (1973) makes assumptions on the distribution of prices which ensure that the learning effect is not too strong, so that $W(p_1)$ is monotone. Rosenfield and Shapiro

(1981) impose an alternative condition, namely that the return to an additional search never cross the horizontal line $W(p_1) = \sigma$ from above. In an equilibrium model, however, the price distribution is endogenous; therefore no such assumptions can be made.

We shall in fact follow similar lines of reasoning, but with respect to the exogenous distribution of costs $F(c_2 \mid c_1)$. We mainly focus on equilibria where buyers' search rules have the reservation price property. This is both because it is a rather natural property, required in particular for demand functions to be downward-sloping, and because the model's non-reservation price equilibria (if they exist) are too complicated for us to solve. We show that a pure strategy reservation price equilibrium exists provided that either:

i) Firm's costs are not too correlated (F_2 is not too large),

ii) Buyers' search cost σ is relatively small.

The first assumption will ensure that $W(p_1)$ is monotonic. Alternatively, the second will ensure that $W(p_1)$ never falls below σ once it has risen above.

III. Equilibrium

1. General properties. We look for a symmetric, perfect Bayesian equilibrium of the game between firms and buyers. We use the definition of Fudenberg and Tirole (1991), which imposes the following consistency restrictions on beliefs off the equilibrium path. First, a consumer's observation of firm 1's price p_1 only directly affects her beliefs about firms 1's cost c_1. Thus, if p_1 is off the equilibrium path, there are no restrictions on the consumer's beliefs about c_1; but in determining whether or not to search, she must use these arbitrary beliefs about c_1 to form beliefs about c_2 which are consistent with the joint distribution of costs and Bayes' rule. Secondly, these beliefs about c_2 and the equilibrium strategy must be used to form beliefs

about p_2. This will be important below; for example, if a consumer observes at firm 1 a price less than the lowest price p^- played with positive probability, she will still put zero probability on the other firm's having a price less than p^-.

We identify four different types of equilibria. If search costs are sufficiently high, each firm will be able to charge its monopoly price without triggering any search. If search costs are not quite large enough to support this outcome, there may still be an equilibrium without search, where firms with higher costs bunch at consumers' reservation price to prevent search. These two equilibria are qualitatively similar to those of the Reinganum (1979) model. A new type of equilibrium arises for lower search costs: a pure strategy, reservation price equilibrium with buyers searching at higher prices and firms' markups decreasing to zero as their costs increase toward the maximum c^*. The fourth possible type of equilibrium involves mixed strategies. High cost firms charge prices which make consumers indifferent between buying and searching, and the fraction of consumers which search at any price makes this pricing rule optimal.

We prove the existence of the first, second, or third types of equilibrium under quite intuitive conditions. We do not have any existence result for the mixed strategy equilibrium. Nor can we rule out the possibility that for some parameter values there exists more than one of the four equilibrium types, or even some other, less intuitive type.[7]

The first proposition shows that all equilibria share an intuitive feature: low cost firms charge their monopoly price.

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7. For instance, one can not even exclude equilibria where a firm's price $p(c_1)$ decreases with its cost c_1 over some range. The usual revealed preference argument fails here because c_1 affects firm 1's expected demand function through its correlation with c_2 and p_2. We shall, however, restrict attention throughout the paper to equilibria where $p(c)$ is non-decreasing.

Proposition 1: In any equilibrium of the game, there exists an $\epsilon > 0$ such that a firm with cost $c \in [c^-, c^- + \epsilon]$ sets price equal to $p_m(c)$.

Indeed, since search costs are strictly positive, consumers who observe a price sufficiently close to the lowest price p^- charged in equilibrium will not search. If $p^- < p_m(c^-)$ the firm charging p^- can deviate and raise its price without losing any customers, thereby increasing its profits. This is true a fortiori if $p^- > p_m(c^-)$.

Given Proposition 1, it will be useful to define:

$$V_m(c) = \int_{c^-}^{c} [S(p_m(c_2)) - S(p_m(c))] f(c_2 \mid c) dc_2 = \int_{c^-}^{c} D(p_m(c_2)) p_m'(c_2) F(c_2 \mid c) dc_2 . \quad (4)$$

$V_m(c)$ is the value of search $W(p)$ when observing a price $p = p_m(c)$, if all firms with cost below c charge their monopoly price, and no firm with cost above c charges less than $p_m(c)$. We now move to a characterization of equilibrium, starting with the case of large search costs.

2. Monopolistic equilibrium. When search costs are large enough, the range of monopolistic pricing of Proposition 1 can cover all cost realizations, and consumers will still not search, independent of the price observed at the first store.

Proposition 2: If $V_m(c) < \sigma$ for all c in $[c^-, c^+]$, there exists an equilibrium in which each firm charges its monopoly price $p_m(c)$, and consumers never search.

Given monopoly pricing, if a consumer observes $p_i \in [p^-, p^+]$ at the first store, the value to search is $W(p_i) = V_m(c_i) < \sigma$ so it does not pay to search. As a result, no firm can attract

more than 1/2 of all customers, no matter what price it charges. Therefore $p_m(c)$ is the optimal price, independently of consumers' behavior off the equilibrium path.

3. No-search equilibrium with bunching. When search costs are not large enough to support the equilibrium of Proposition 2, there may still be an equilibrium in which no consumers search. Define c^* as the smallest solution to:

$$V_m(c^*) = \int_{c_*}^{c^*} D(p_m(c_2))p_m'(c_2)F(c_2 \mid c^*)dc_2 = \sigma . \qquad (5)$$

and let $p^* = p_m(c^*)$. A consumer is indifferent between search and purchasing at a store charging p^*, if all firms with cost $c \le c^*$ charge $p_m(c)$ and no firm with cost $c \ge c^*$ charges less than p^*, so that p^* reveals c^*. We shall focus on reservation price equilibria where consumers accept prices p_1 up to p^* but reject higher ones.[8]

Firms with $c \le c^*$ are still able to charge their monopoly price. Consider, however, a firm with cost just above c^*. If it charges its monopoly price, it will induce search; rather than accept the resulting first-order loss in customers (they search and find a lower price with a probability of at least $F(c \mid c)$), it prefers charging p^*, which causes no loss of customers and only a second-order effect on profits per customer. In fact, we show that if $p^* > c^*$, there is an equilibrium in which all firms with cost above c^* charge p^*. In this equilibrium, consumers do not search but prices are constrained by the possibility of search.

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8. It can be shown that any reservation price equilibrium with $p(c)$ non-decreasing must have the same form as those we examine (with p^* simply replaced by $\hat{p} \le p^*$). Moreover those with $\hat{p} < p^*$ necessarily rest on very implausible out-of-equilibrium beliefs. Since the basic features of the equilibrium and the spirit of the results remain unchanged, we do not think it worthwhile to go into the complexities of equilibrium refinement, and simply concentrate on the more natural reservation price equilibrium where $\hat{p} = p^*$.

Proposition 3: If search costs σ are such that $p^* > c^- > c^*$, there exists an equilibrium in which consumers have reservation price p^* and firms' pricing rule is: $p(c) = p_m(c)$ for $c \leq c^-$ and $p(c) = p^*$ for $c^- < c \leq c^-$.

Proof: See Appendix B.

This equilibrium can be sustained by any beliefs which make it profitable to search in response to prices $p_1 > p^*$; for instance, a belief that $c_1 = c^-$. Any firm which deviates to such a price then earns zero profits, while it could earn positive profits by playing its equilibrium strategy. Therefore such deviations will not occur, and this allows the imposed beliefs. Each firm chooses instead the price $p \leq p^*$ which maximizes its profit per customer and prevents search. Finally, by definition of c^-, accepting offers below p^* is optimal for consumers.

The equilibrium types of Propositions 2 and 3 are analogous to those of the Reinganum model, except that consumers' reservation price p^* depends here on the learning which results from the fact that firms' costs are correlated. The other essential difference with Reinganum (1979) is that we analyze a duopoly instead of a continuum of firms; as shown below, this allows search to take place in equilibrium.

4. Reservation price equilibrium with search. Smaller search costs lead to a new but much more difficult case, in which consumers' reservation price p^* is less than c^-. A firm whose cost exceeds p^* can then not avoid search, unless it makes negative profits. If the market contained a continuum of firms, as in Reinganum (1979), such a firm would have to stay out, because its consumers who searched would all find a lower price. However, with only two firms in the market (more generally a finite number), it is possible to charge a price which induces search, but still expect positive profits when one's rival, who follows the same strategy, has an even

higher cost, and hence an even higher price.

We now characterize a pure strategy reservation price equilibrium in which search actually takes place. Firms' pricing strategy is illustrated on Figure 1. For low cost realizations, it is similar to that of Proposition 3: a firm with cost below c^* charges $p_m(c)$ and a firm with cost between c^* and some $c^s > c^*$ charges consumers' reservation price $p^* = p_m(c^*)$. A firm with higher cost realization c, however, charges a price $p_F(c) > p^*$, so all consumers who visit this firm first will search.

We now derive $p_F(c)$. If firm i charges a price p_i which induces search, it will sell to all consumers if its rival has a higher price, and to none if its rival has a lower price. Therefore firm i's probability of making sales is the probability that firm j has a higher price, conditional on firm i's true cost c_i, and its expected profits are:

$$\Psi(p_i, c_i) = \Pi(p_i, c_i) \cdot [1 - Prob[p(c_j) \leq p_i \mid c_i)]], \tag{6}$$

where $p(\cdot)$ is the equilibrium pricing strategy. Assume for now that $p(\cdot)$ is increasing and differentiable on $[c^s, c^*]$, with $p(c^s) > p^*$; this will be verified below. We can then rewrite:

$$Prob[p(c_j) \leq p_i \mid c_i] = F[p_F^{-1}(p_i) \mid c_i]$$

and differentiate (6) with respect to p_i to obtain the first-order condition which $p(c)$ must solve:[9]

$$p'(c) = \frac{f(c \mid c)}{1 - F(c \mid c)} \cdot \frac{\Pi(p(c), c)}{\Pi_p(p(c), c)}. \tag{7}$$

This differential equation must be satisfied along the price path in the region of costs $[c^s, c^*]$ which lead to search. The boundary condition is found by considering a firm with the highest

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9. Since charging $p > p^*$ leads consumers to become fully informed, it is not surprising that this pricing rule is quite similar to the optimal bidding rule in an auction with correlated values (Milgrom and Weber (1982)). The difference, and source of difficulty, is that $\Pi(p, c)$ cannot be expressed as a function $U(p - c)$.

EQUILIBRIUM PRICING STRATEGY WITH SEARCH

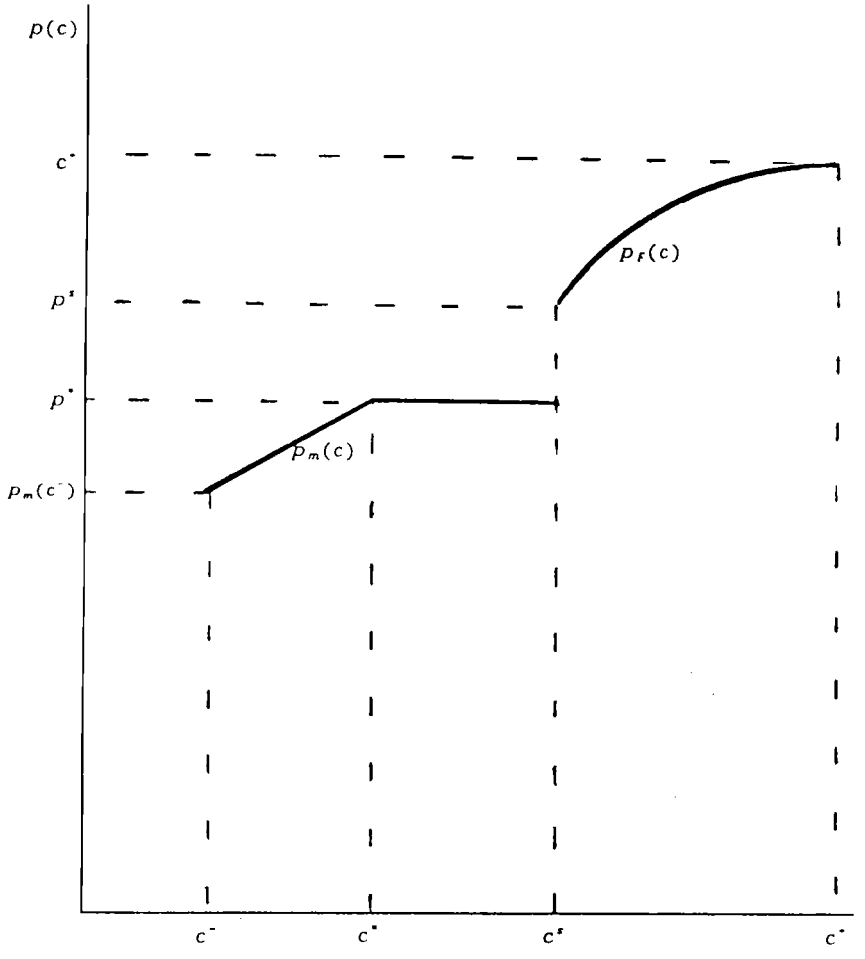

Figure 1

possible cost, c^*. Such a firm makes no sales since consumers search and are sure to find a lower price. Therefore, it must be the case that $p(c^*) = c^*$; otherwise the firm would lower its price by a little bit and make positive expected profits, since it would make sales at a price above cost whenever its rival's price was higher. Because $p(c^*) = c^*$, (7) does not satisfy Lipschitz conditions at (c^*, c^*), standard theorems are not applicable.[10] Instead, we construct the solution as the fixed-point of a contraction mapping; this is where the technical conditions of Appendix A are needed.

Lemma 1: The differential equation (7) on (c^-, c^*), with terminal condition $p(c^*) = c^*$ has a unique solution $p_F(c)$, satisfying $c \le p_F(c) < p_m(c)$, with equality only at c^*. Moreover, $p_F'(c) > 0$, for all c in $[c^-, c^*]$.

Proof: See Appendix B.

While $p_F(c)$ solves the first-order condition (7), it remains to prove that it really characterizes optimal prices for $c \ge c^s$. First, using condition (2) on the distribution of costs, we show that equilibrium profits are strictly quasiconcave.

Lemma 2: Assume that buyers have reservation price $p^* = p_m(c^*)$. If firms with cost above some level $c^s > c^*$ charge $p_F(c)$, while firms with cost $c \le c^s$ charge $\min(p_m(c), p^*)$, then a firm's profits from charging any price $p \ge p^s$ are:

$$\Psi(p, c) = \Pi(p, c).[1 - F(p_F^{-1}(p) \mid c)]. \qquad (8)$$

They are strictly quasiconcave and maximized over $p \in [p^s, c^*]$ at $p = p_F(c)$.

Proof: See Appendix B.

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10. The Lipschitz condition is also not satisfied at points $(p_m(c), c)$.

The final step in characterizing firms' strategies is to find the threshold cost c^s separating those which prefer to charge p^* from those which prefer to charge $p_F(c) > p^*$. The first strategy prevents search but the second yields greater profit per consumer, if they come back. At c^s a firm is indifferent between the two, so c^s is defined by:

$$\left[1 - \frac{1}{2}F(c^s \mid c^s)\right] \cdot \Pi(p^*, c^s) = [1 - F(c^s \mid c^s)] \cdot \Pi(p_F(c^s), c^s). \qquad (9)$$

The left-hand side represents profits from charging p^*. A firm which does this sells to all 1/2 consumers who visit it first and to all consumers who visit the other firm first and observe a price above p^*. Given the symmetry of the pricing strategy, this is just 1/2 times the probability that the other firm has cost above c^s, i.e. $\frac{1}{2}[1 - F(c^s \mid c^s)]$. The right-hand represents profits from charging $p_F(c^s)$. A firm which does this sells to all customers who visit it first, search, find a higher price and return; given symmetry, this is $\frac{1}{2}[1 - F(c^s \mid c^s)]$ customers. It also sells to customers who visit the other firm first, search, and find its lower price, i.e. to another $\frac{1}{2}[1 - F(c^s \mid c^s)]$ customers.

Lemma 3: Assume that buyers have reservation price $p^* = p_m(c^*)$. There exists a unique $c^s \in (c^s, c^*]$, such that the following strategies are mutual best responses for firms: they charge $p_m(c)$ if $c \in [c^-, c^*]$, p^* if $c \in [c^*, c^s)$, and $p_F(c)$ if $c \in [c^s, c^-]$. Moreover, c^s and $p^s = p_F(c^s)$ are continuous and non-decreasing in c^*.

Proof: See Appendix B.

This result concludes the characterization of firms' strategies: if there exists a pure strategy equilibrium where consumers have reservation price p^*, it is uniquely defined by c^*, $p_F(\cdot)$, and c^s, which are the unique solutions to (5), (7), and (9), respectively. It only remains to verify that, given firms' strategies, optimal buyer search is indeed characterized by the reservation

price p^*, i.e. that $W(p_1) > \sigma$ if and only if $p_1 > p^*$. This is where issues associated with equilibrium learning will be most important.

Consider first $p_1 < p^*$; then $W(p_1) = V_m(c_1) < V_m(c^*) = \sigma$, by definition of c^* in (5); so consumers buy at p^*. Next, if $p_1 = p^*$, consumers only infer that $c_1 \in [c^*, c^s)$, so

$$W(p_1) = \int_{c^-}^{c^*} D(p_m(c_2)) p_m'(c_2) F(c_2 \mid c^* \le c_1 < c^s) dc_2$$

$$\le \int_{c^-}^{c^*} D(p_m(c_2)) p_m'(c_2) F(c_2 \mid c_1 = c^*) dc_2 = V_m(c^*) = \sigma,$$

and they still do not want to search. For out-of-equilibrium prices $p_1 \in (p^*, p^s)$, assume that they lead to the belief that $c_1 = c^*$, so that:

$$W(p_1) = \int_{p^-}^{p_1} D(p_2) G(p_2 \mid p_1) dp_2 = \int_{p^-}^{p^*} D(p_2) G(p_2 \mid p_1) dp_2 + [S(p^*) - S(p_1)] G(p^* \mid p_1)$$

$$= V_m(c^*) + [S(p^*) - S(p_1)] F(c^* \mid c^*) > \sigma,$$

due to (4) and $c^* > c^-$. More generally, any out-of-equilibrium beliefs which put sufficient weight on c_1 being closer to c^* than to c^s will lead to search at prices $p_1 \in (p^*, p^s)$.[11]

Finally, buyers will search at any $p_1 \ge p^s$ if and only if $W(p_1) > \sigma$; given firms' strategies, this is equivalent to $V(c_1) \ge \sigma$ for all $c_1 \ge c^s$, where:

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11. The fact that consumers' beliefs do not remain monotonic in the observed price as it moves off the equilibrium path is admittedly unappealing. But one has to choose between such monotonicity and the reservation price property. Indeed, due to bunching, consumers who observe their reservation price p^* strictly do not want to search; if observing $p^* + \epsilon$ did not lead them to infer a lower c_1, hence a lower c_2 and p_2, they would also not want to search, a contradiction.

$$V(c_1) = \int_{c^-}^{c^*} D(p_m(c_2))p_m'(c_2)F(c_2|c_1)dc_2 + [S(p^*) - S(p^s)]F(c^s|c_1)$$

$$+ \int_{c^s}^{c_1} D(p_F(c_2))p_F'(c_2)F(c_2|c_1)dc_2 . \qquad (10)$$

Since $\sigma = V(c^*)$ by definition of c^*, the condition for search becomes:

$$[S(p^*) - S(p^s)]F(c^s|c_1) + \int_{c^s}^{c_1} D(p_F(c_2))p_F'(c_2)F(c_2|c_1)dc_2 >$$

$$\int_{c^-}^{c^*} D(p_m(c_2))p_m'(c_2)[F(c_2|c^*) - F(c_2|c_1)]dc_2 , \qquad \forall c_1 \geq c^s . \qquad (11)$$

The left hand side, which we shall denote as $\xi(c_1, \sigma)$, is the direct effect of finding the price p^s instead of p^*: it is the additional incentive to search which results from the prospect of finding a price $p^* \leq p_2 < p_1$ (conditional on c_1). The right hand side is the "bad news" from inferring c_1 rather than c^*, about the likelihood of finding a price $p_2 < p^*$; For buyers to have a reservation price rule, this disincentive to search at high prices must not be too large; we shall denote it as $\chi(c_1, \sigma)$. From (11) we see that $\chi(c_1, \sigma)$ will be small in two intuitive cases, which respectively extend the insights of Rosenfield and Shapiro (1981) and of Rothschild (1973) to an equilibrium context.

The first one is when search is inexpensive, i.e. σ is small, making c^* close to c^-. The following proposition formalizes this intuition.

Proposition 4: If search costs are relatively low, there exists a pure-strategy, reservation-price equilibrium characterized by critical cost levels c^* and c^s, with $c^- < c^* < c^s < c^+$. Buyers have reservation price $p^* = p_m(c^*)$. Firms charge $p_m(c)$ if $c \in [c^-, c^*]$, p^* if $c \in [c^*, c^s)$, and $p_F(c)$ if $c \in [c^s, c^+]$, with $c \leq p_F(c) \leq p_m(c)$.

Proof: See Appendix B.

The second case in which a reservation price rule is optimal is when a firm's cost does not reveal too much information about that of its competitor, i.e., when $F_2(c_2 \mid c_1)$ is not too large. The two distribution functions in χ are then close to one another.

Proposition 5: If firms' costs are not too correlated, i.e., if,

$$\bar{F}_2 = \max\{F_2(c_2 \mid c_1) \mid c^- \le c_2 \le c_1 \le c^*\}$$

is not too large, there exists a pure-strategy reservation price equilibrium. For σ above some level σ^*, it involves no search and corresponds to that of Proposition 2 or 3, depending on whether $V_m(c^*)$ is larger or smaller than σ. For $\sigma < \sigma^*$, it involves search and corresponds to the equilibrium in Proposition 4.

Proof: See Appendix B.

5. Mixed strategy equilibrium. If the assumptions of low search costs or low correlation do not hold, there may be no reservation price equilibrium, as one should expect. Indeed, simulations suggest that there exists an intermediate range of search costs in which none of the three types of equilibria discussed above exists (see Section IV). We must therefore turn briefly to a fourth type of equilibrium, which involves mixed strategies by consumers and generally does not have the reservation price property.

In such an equilibrium, pricing at low levels of costs remains unchanged, i.e., $p(c) = p_m(c)$, for $c \le c^*$. From c^* to c^- the pricing rule $p_R(c)$ makes consumers indifferent between searching and not, and they will randomize this decision. Thus for all $c_1 \in [c^*, c^-]$:

$$\int_{c^*}^{c^*} [S(p_m(c_2)) - S(p_R(c_1))] f(c_2 \mid c_1) dc_2 + \int_{c^*}^{c_1} [S(p_R(c_2)) - S(p_R(c_1))] f(c_2 \mid c_1) dc_2 = \sigma. \quad (12)$$

Differentiating this expression with respect to c_1 yields $p_R'(c_1)$ as a function of all $p_R(c_2)$, for $c_2 < c_1$; this allows the function $p_R(\cdot)$ to be constructed, moving up from the initial condition $p_R(c^*) = p^*$. The fraction $\omega_R(p)$ of consumers who search at any price $p > p^*$ must then make the pricing rule $p_R(c)$ optimal; for all $c_1 \in (c^*, c^-]$:

$$p_R(c_1) \in argmax\left\{ \Pi(p, c_1)\left[1 - \omega_R(p) + \int_{p_R^{-1}(p)}^{c^-} \omega_R(p_R(c_2)) f(c_2 \mid c_1) dc_2 \right], p \geq p^* \right\} \quad (13)$$

Note that in this type of equilibrium $p_R(c^*) > c^*$. The highest-cost firm makes positive profits because not all its consumers search; yet if it raised its price they all would search, and it would then make zero profits.

Such an equilibrium is somewhat less appealing intuitively than the previous reservation price equilibria. It is also much more difficult to construct for a general specification: one must show the existence of solutions $p_R(c)$ to (12) and $\omega_R(p) \in [0, 1]$ to (13), both of which are extremely complicated. We have not established general conditions under which this equilibrium exists, but report below on simulations using simple functional forms which indicate that it does exists in the intermediate range where none of the other three types do.

IV. The Effects of Inflation Uncertainty

We are now ready to analyze how equilibrium pricing and search rules are affected by changes in the underlying distribution of costs. Before doing so, however, it worth pointing out the impact of individual cost realizations on firms' markups. Consider the equilibrium depicted in Figure 1. If an aggregate cost shock pushes both c_1 and c_2 from (c^*, c^s) to somewhat above c^s, both firms will significantly increase their markups, raising prices toward their respective

monopoly levels. They know that this will trigger search, but each is betting on its competitors' cost and price being at least as high as its own. This is an interesting feature of the model: in response to an oil shock, retail markups on gasoline may jump up.

Let us now turn to our main subject of interest, and examine how an increase in the uncertainty of the inflationary process impacts firms' market power, the amount of search, profits and consumer surplus.

First we demonstrate the two most important intuitions, regarding what we call the correlation effect and the variance effect. This is done by analyzing the comparative statics of c^*, using a particularly convenient specification where costs are log-linear and demand is iso-elastic. As explained below, c^* provides an intuitive but only partial indicator of monopoly power in the model. So in the second part of this section, we present a variety of simulations. These are carried out using an alternative specification, both for technical reasons and as a check on the robustness of the results derived in the first part. They confirm that the correlation and variance effects provide insights into the full equilibrium effects of inflationary uncertainty in this model.

Recall that c^* characterizes consumer's reservation price $p^* = p_m(c^*)$, and is therefore the cost above which firms are prevented from charging their monopoly price. In the no-search equilibrium of Proposition 3, it is clear that an increase in c^* to $c^{*\prime}$ results in prices which are equal up to c^* and greater above. The effects accompanying an increase in c^* in the search equilibrium of Proposition 4 are more complicated, since the differential equation (7) giving $p_F(c)$ is affected in a complicated manner by the underlying changes in the inflation process. This is where we must resort to simulations.

Case 1: Log-Normal Costs, Iso-Elastic Demand

Let us now denote firms' costs as C_i, i = 1,2, and assume that $c_i = \log C_i = \bar{c} + \theta + \gamma_i$, where $\theta \sim \mathcal{N}(0, v_\theta)$, $\gamma_i \sim \mathcal{N}(0, v_\gamma)$ and $\theta, \gamma_1, \gamma_2$ are independent.

The distribution of c_2 conditional on c_1 is therefore normal, with mean $\rho c_1 + (1-\rho)\bar{c}$ and variance $(1-\rho^2)(v_\theta + v_\gamma)$, where $\rho = v_\theta/(v_\theta + v_\gamma)$ is the correlation coefficient of c_1 and c_2. We shall examine how v_θ, which measures aggregate cost or inflation uncertainty, affects the conditional distribution $F(c_1 \mid c_2)$ and consumers' return to search.

Assume that demand is iso-elastic, $D(P) = P^{-\eta}$, $\eta > 1$, so that the log of the monopoly price is: $p_m(c) = c + \log\left(\frac{\eta}{\eta-1}\right)$. The unconditional distribution of $p_m(c)$ is normal, with mean $\bar{p} = \bar{c} + \log\left(\frac{\eta}{\eta-1}\right)$ and variance v_θ. The conditional distribution of $p_m(c_2)$, given $c_1 = c$, is normal with mean $\mu(p_m(c))$ and variance s^2, where we define:

$$\mu(p) = \rho p + (1-\rho)\bar{p}; \qquad s^2 = (1-\rho^2)(v_\theta + v_\gamma) \qquad (14).$$

To better demonstrate the two effects of inflation uncertainty, let us for the moment consider ρ and s^2, rather than v_θ and v_γ, as the parameters of interest. Consumer surplus at the monopoly price is:

$$S(p_m(c)) = \frac{1}{\eta-1}\exp[(1-\eta)p_m(c)].$$

Therefore the return to search in the region of monopoly prices is:

$$V_m(c) = \frac{1}{\eta-1}\int_{-\infty}^{p_m(c)} \frac{1}{\sqrt{2\pi}s}\{\exp[(1-\eta)p_2] - \exp[(1-\eta)p_m(c)]\}\exp\left[-\frac{(p_2-\mu)^2}{2s^2}\right]dp_2.$$

with $\mu = \mu(p_m(c))$. Rewriting in terms of the distribution Φ and density ϕ of a standard normal:

$$V_m(c) = \frac{1}{\eta-1}\exp\left[-\frac{1}{2}(\eta-1)[2\bar{p}+2\rho(p_m(c)-\bar{p})-s^2(\eta-1)]\right]\cdot\Phi\left[\frac{(1-\rho)(p_m(c)-\bar{p})+s^2(\eta-1)}{s}\right]$$

$$-\frac{1}{\eta-1}\exp[-(\eta-1)p_m(c)]\cdot\Phi\left[\frac{(1-\rho)(p_m(c)-\bar{p})}{s}\right]. \qquad (15)$$

Hence:

$$\frac{\partial V_m(c)}{\partial\rho} = -(p_m(c)-\bar{p})\cdot\exp-\frac{1}{2}(\eta-1)[2\bar{p}+2\rho(p_m(c)-\bar{p})-s^2(\eta-1)]$$

$$\cdot\Phi\left[\frac{(1-\rho)(p_m(c)-\bar{p})+s^2(\eta-1)}{s}\right].$$

$$\frac{\partial V_m(c)}{\partial s} = \phi\left[\frac{(1-\rho)(p_m(c)-\bar{p})+s^2(\eta-1)}{s}\right]+s(\eta-1)\Phi\left[\frac{(1-\rho)(p_m(c)-\bar{p})+s^2(\eta-1)}{s}\right]$$

$$\cdot\exp-\frac{1}{2}(\eta-1)[2\bar{p}+2\rho(p_m(c)-\bar{p})-s^2(\eta-1)].$$

So finally:

$$sgn\frac{\partial V_m(c)}{\partial\rho} = sgn(\bar{p}-p_m(c)) = sgn(\bar{c}-c) \qquad (16)$$

$$\frac{\partial V_m(c)}{\partial s} > 0. \qquad (17)$$

The first result (16) shows what we call the correlation effect. Recall that the mean of the distribution of c_2 conditional on $c_1 = c$ is a weighted average of the observation c and the unconditional mean $\bar{c}$, with weights ρ and $1-\rho$, respectively. This conditional mean is thus increasing in ρ if $c > \bar{c}$, and decreasing if $c < \bar{c}$. By increasing ρ, the first effect of an increase in v_θ is therefore to raise the value of search at $c < \bar{c}$, and to lower it at $c > \bar{c}$.[12] This

12. Of course, what really determines search is not the conditional distribution of cost c_2, but the conditional distribution of surplus $S(p_2)$; see (3). The difference between the two involves the equilibrium pricing rule $p(c)$ as well as the convexity of $S(p)$. The discussion above is only meant to give the main qualitative intuitions.

correlation effect captures the idea that inflation or aggregate cost uncertainty makes people search less when they see a high price, because they think it more likely that things are just as bad elsewhere. However, it also makes them search more when they see a low price, because they think it more likely that even better bargains can be found.

The second result (17) shows what we call the variance effect. Given that buyers can return to the first store costlessly, an increase in the variance s^2 of the conditional distribution increases the option value of search. But note that an increase in v_θ, the unconditional variance of the joint cost shock, does lead to such an increase in the conditional variance:

$$\frac{\partial s^2}{\partial v_\theta} = \frac{\partial}{\partial v_\theta}\left[1-\left(\frac{v_\theta}{v_\theta+v_\gamma}\right)^2\right](v_\theta+v_\gamma) = \frac{v_\theta^2}{(v_\theta+v_\gamma)^2} > 0;$$

This raises the value of search and tends to reduce the market power of firms.

We now examine how these two effects impact consumers' reservation price and firms' pricing. We focus on the case where search matters, i.e. where $c^* = \inf\{c \mid V_m(c) = \sigma\} < \infty$. Then $V_m'(c^*) > 0$, so (16) and (17) imply:

$$sgn\frac{\partial c^*}{\partial p} = sgn[p_m(c^*) - \bar{p}] = sgn(c^* - \bar{c}) \tag{18}$$

$$\frac{\partial c^*}{\partial s} < 0. \tag{19}$$

If c^* exceeds $\bar{c}$, the correlation effect tends to increase it further, resulting in monopoly markups over a wider range of costs; the variance effect, however, works in the opposite direction. By definition, such a configuration with $c^* > \bar{c}$ occurs when search is relatively costly. With a relatively low cost of search, on the other hand, c^* is less than $\bar{c}$; in this case both the variance and the correlation effects reduce it even more, and the market becomes more competitive.

While this log-normal case is very specific, the intuitions behind the correlation and variance effects seem quite robust. One can think in general of the variability of inflation, or of any common shock to firms' costs, as having two effects on the conditional distribution of costs (and through equilibrium pricing, of surplus). First, by making costs more correlated, it shifts $F(c_2 \mid c_1)$, upward for high c_1, downward for low c_1; second, since it is a source of additional uncertainty, it causes a mean-preserving spread in the shifted distribution. The generality of these intuitions is also supported by the simulations, using a very different specification, reported below.

In addition to these two central, information-related effects, inflationary uncertainty has other consequences in our model. First, greater price variability in itself tends to increase consumer surplus, which is convex in price; this is a feature of partial equilibrium, where the marginal utility of income is constant. Similarly, equilibrium profits may be positively affected because they depend in part on monopoly profits, which are convex in cost.[13] Neither of these effects is really interesting; in particular, they have nothing to do with information, since they occur even when search is impossible and consumers always buy at the monopoly price. When performing comparative statics, we shall therefore normalize surplus, profits and welfare by their respective values in a monopolistic market. This will allow us to isolate the effects which are really due to the interaction of the inflationary process with the informational role of prices.

The last, and much more interesting effect of inflation uncertainty is the following. Even though an increase in the value of search raises each firm's elasticity of demand, an increase in search activity shifts consumers and purchases toward the firm with lower cost. Since these firms are more profitable, this tends to increase total expected profits and efficiency at the same time.

The log-normal specification used above yields very clear, closed-form results for the

13. Note however that firms face both ex-ante and ex-post uncertainty, because they set their price after learning their cost, but before learning that of their competitor.

effects of inflation uncertainty on c^*. On the other hand it does not allow us to construct (even numerically) a full equilibrium with search. This is because it does not satisfy the assumption $c^* < +\infty$, so that one can not use $p_F(c^*) = c^*$ as a terminal condition to solve (7). We therefore turn below to an alternative specification; it also serves as a robustness check on the insights just derived.

Case 2: Uniform Costs, Linear Demand

Let us now assume that costs are the sum of a joint cost shock and a private cost shock, both uniformly distributed: $c_i = \bar{c} + \theta + \gamma_i$, with $\theta \sim U[-a, a]$, $\gamma_i \sim U[-b, b]$ and γ_1, γ_2, θ independent. Demand is taken to be linear: $D(p) = A - p$.

We assume that $a < b$, which reduces the number of cases to analyze but is not essential; a represents the volatility of inflation. The unconditional density of costs is the familiar, trapeze-shaped, sum of two uniform distributions. But of greater interest is $F(c_2 \mid c_1)$, or equivalently the conditional distribution of θ, given c_1. Inferring c_1 from a price observation causes a consumer to update his beliefs about the joint shock θ as follows:

if $c_1 \in [\bar{c} - a - b, \bar{c} + a - b]$ the posterior of θ is $\theta \sim U[-a, c_1 - \bar{c} + b]$;

if $c_1 \in [\bar{c} + a - b, \bar{c} - a + b]$ the posterior of θ is $\theta \sim U[-a, a]$;

if $c_1 \in [\bar{c} - a + b, \bar{c} + a + b]$ the posterior of θ is $\theta \sim U[c_1 - \bar{c} - b, a]$.

Note that if c_1 falls in the intermediate region there is no learning. However, if c_1 falls in the lowest region, the conditional expectation of $c_2 = \bar{c} + \theta + \gamma_2$ is less than $\bar{c}$, and decreasing in the variability a. On the contrary, if c_1 falls in the highest region the conditional expectation of c_2 is above $\bar{c}$, and increasing in a. Thus the correlation effect works here in a way similar to the log-normal case. The same is true of the variance effect, since the supports of θ and c_2, given c_1, always widen as a increases.

We now look at a number of simulations of this example, in order to get some feeling for the relative size of the various effects of an increase in aggregate uncertainty. In all simulations, $D(p) = 15 - p$, $\bar{c} = 6$, and $b = 3$. We allow search costs σ and the dispersion of the joint cost shock α to vary. The results for low, intermediate, and high search costs are given in Tables 1, 2, and 3, respectively. We define these terms so that low search costs lead to a c^* well below the unconditional mean of $\bar{c} = 6$, intermediate search costs lead to a c^* near $\bar{c}$, and high search costs lead to a c^* well above $\bar{c}$.

Looking first at the effects of σ (say, for $\alpha = 1.50$ or 2.0), we see that as it increases, the equilibrium first involves reservation price strategies and search ("type 3"), then mixed strategies ("type 4"), then monopolistic pricing plus bunching at p^* ("type 2"), and finally unconstrained monopolistic pricing ("type 1"). These results support the intuitive way in which we associated, in Section III, each type of equilibrium to a different range of search costs.

Next we turn to our main subject of interest: the effects of inflation uncertainty on monopoly power and on the components of welfare in equilibrium.

Table 1 reports the case of low search costs; c^* is decreasing in α, because both the variance and correlation effects make search more valuable. This reduction in firms' market power with increases in the variability of joint cost shocks leads to gains in consumer surplus, both in absolute terms and relative to the monopoly level. Conversely, profits decrease, once the convexity effect discused earlier is eliminated by normalizing Π by the monopoly level Π^m

Finally, the reason why Π / Π^m does not fall more as α rise is that increased search raises the likelihood that consumers will purchase at a low-cost firm, which has higher profits per customer. The better matching of consumers to more efficient firms thus mitigates the decrease in markups. This can be seen from the columns of Table 1 which report the price $E(p)$ paid by the average consumer and the cost $E(c)$ incurred for the average customer. At higher levels

LOW SEARCH COSTS: $\sigma = 0.5$

a	$Type$	c^*	c^s	p^s	$E(c)$	$E(p)$	Π	CS	W	Π/Π^m	CS/CS^m	W/W^m
0.10	2	4.4747	--	--	6.0000	9.6466	19.791	14.248	34.039	0.942	1.357	1.081
0.50	2	4.4435	--	--	6.0000	9.6315	19.691	14.430	34.121	0.937	1.373	1.082
1.00	3	4.3420	9.6651	9.7756	5.9860	9.5816	19.687	14.697	34.384	0.934	1.394	1.087
1.50	3	3.9877	9.4448	9.7844	5.9004	9.4066	19.745	15.550	35.295	0.932	1.468	1.111
2.00	3	3.4317	9.0605	9.6659	5.7316	9.0955	19.793	16.923	36.716	0.928	1.587	1.147
2.50	3	2.8796	8.6373	9.5031	5.5029	8.7325	19.880	18.281	38.161	0.924	1.699	1.182

Table 1

NOTES:
In all simulations $D(p) = 15-p$, $\bar{c} = 6$, $b = 3$.

Type refers to the equilibrium pricing strategy in the following way:

1 = $p(c) = p_m(c) \quad \forall c$. No search.

2 = $p(c) = p_m(c) \quad \forall c \le c^*$, $p(c) = p^* \quad \forall c > c^*$. No search.

3 = $p(c) = p_m(c) \quad \forall c < c^*$, $p(c) = p^* \quad \forall c \in [c^*, c^s)$, $p(c) = p_s(c) \quad \forall c \ge c^s$. Search above p^* .

4 = $p(c) = p_m(c) \quad \forall c < c^*$, $p(c) = p_s(c) \quad \forall c \ge c^*$. Consumers search with a mixed strategy above $Max\{c^*, \bar{c} - a + b\}$.

E(p) and E(c) are, respectively, the expected price that a customer pays and the expected cost at the store she purchases from.

Π , CS , W are, respectively, expected industry profits per consumer, expected surplus per consumer (net of search costs) and their sum.

Π/Π^m , CS/CS^m , W/W^m are, respectively, expected industry profits per consumer divided by a monopolist's expected profits per consumer, expected surplus per consumer (net of search costs) divided by a consumer's expected surplus if there is a monopolist, and the expected welfare (consumer plus producer surplus) per consumer divided by the expected welfare per consumer if there is a monopolist.

INTERMEDIATE SEARCH COSTS: $\sigma = 2.5$

α	Type	c^*	c^s	p^s	$E(c)$	$E(p)$	Π	CS	W	Π/Π^m	CS/CS^m	W/W^m
0.10	2	6.5259	--	--	6.0000	10.2448	20.790	11.476	32.266	0.990	1.093	1.024
0.50	2	6.5098	--	--	6.0000	10.2381	20.798	11.510	32.308	0.989	1.095	1.025
1.00	2	6.4592	--	--	6.0000	10.2171	20.820	11.782	32.602	0.988	1.118	1.031
1.50	2	6.3740	--	--	6.0000	10.1814	20.854	11.801	32.655	0.984	1.114	1.027
2.00	4	6.2524	--	--	5.9482	10.2333	21.358	11.581	32.939	1.003	1.086	1.029
2.50	4	6.0917	--	--	5.9431	10.2504	21.584	11.573	33.157	1.003	1.076	1.027

Table 2

NOTES: See Table 1.

HIGH SEARCH COSTS: $\sigma = 5.0$

α	Type	c^*	c^s	p^s	$E(c)$	$E(p)$	Π	CS	W	Π/Π^m	CS/CS^m	W/W^m
0.10	2	8.3300	--	--	6.0000	10.481	20.986	10.561	31.547	0.999	1.006	1.001
0.50	2	8.3165	--	--	6.0000	10.477	21.014	10.584	31.598	1.000	1.007	1.002
1.00	2	--	--	--	6.0000	10.496	21.082	10.554	31.636	1.000	1.001	1.000
1.50	1	--	--	--	6.0000	10.500	21.188	10.594	31.782	1.000	1.000	1.000
2.00	1	--	--	--	6.0000	10.500	21.333	10.667	32.000	1.000	1.000	1.000
2.50	1	--	--	--	6.0000	10.500	21.521	32.280	32.280	1.000	1.000	1.000

Table 3

NOTES: See Table 1.

of α, the decrease in price is offset by the decrease in cost.

Table 2 reports simulation results for intermediate values of σ. When c^* is near $\bar{c}$, the correlation effect affects it very little. The variance effect alone acts to reduce c^* as α increases, but this reduction is very limited compared to Table 1. Similarly, the total effect of inflation variability on consumer surplus and profits is small relative to the previous case. These simulations also indicate that the determination of c^* is not the entire story, because for $\alpha \geq 1$ c^* is decreasing, yet (normalized) profits increase and consumer surplus decreases. Interestingly, total expected profits can exceed the monopoly level: as explained earlier, search transfers sales from the high cost to the low cost firm, so the relevant average cost is below the average cost for a monopolist.

Table 3 reports the results when c^* is significantly greater than $\bar{c}$. In all equilibria, there is no search. Note that c^* first decreases with α, then increases at higher levels: the correlation effect eventually becomes dominant. In this higher range, (normalized) consumer surplus falls, profits rise, and welfare declines.

The lesson from these simulations is that one needs to know a great deal more about market structure before one can say that increases in inflation uncertainty reduce the informativeness of prices and therefore decrease welfare. When observing additional prices is cheap, it is possible for the benefits from an increase in the variance of joint cost shocks to outweigh the losses. At higher levels of search cost, on the other hand, we show that the conventional argument is correct. Interestingly, Bénabou's (1991) analysis of the effects of anticipated inflation leads to rather similar conclusions about the importance of market structure, and in particular the size of informational costs.

V. Conclusion

Aggregate cost uncertainty, whether due to common input prices or to stochastic inflation, reduces the information content of prices by making it difficult to separate relative and aggregate price variations. In this paper we explore how this mechanism operates in an environment where agents can decide to enhance their information via search. We study how the stochastic structure of shocks, consumer search and oligopolistic pricing interact in a single product market.

The results indicate that the a priori case for welfare losses from inflation associated with reduced informativeness of prices has to be substantially qualified when one allows for endogenous information acquisition and price-setting. Indeed, inflationary noise can lead agents to seek more information, so that in equilibrium they are in fact better informed, and prices reflect increased competition. We show that the decisive factor in whether inflation uncertainty improves or deteriorates market efficiency is the size of informational costs.

Another contribution of this paper is that it develops an equilibrium model in which Bayesian consumers search optimally from an unknown price distribution, and firms price optimally given the learning and search rules of consumers. We hope that this analysis will be useful for attaining a better understanding of the relation between pricing and search behavior in general.

Appendix A: Technical Conditions

In addition to (1) and (2), we shall assume that the distribution of costs satisfies:

$$f(c \mid c) > 0, \quad \forall c \in [c^-, c^+]. \tag{A.1}$$

where $f(c^- \mid c^-) = \lim_{c \to c^-} f(c \mid c)$ and $f(c^+ \mid c^+) = \lim_{c \to c^+} f(c \mid c)$. These limits can be positive even with $f(c^- \mid c_1) = 0$ or $f(c^+ \mid c_1) = 0$ for all $c_1 \in (s^-, c^+)$. Note that (A.1) implies $F(c \mid c) > 0$, for all $c > c^-$.

Turning now to the demand side, we assume that $-D'(p) = S'(p)$ is bounded on $[c^-, c^+]$, i.e. $\Delta = \sup\{-D'(p) \mid c^- \le p \le c^+\} < \infty$. This may require $c^- > 0$. Finally, strict quasi-concavity of $\Pi(\cdot, c)$ implies:

$$\rho(p, c) = \frac{\Pi_p(p, c)}{p_m(c) - p} > 0, \quad \forall p, \ c \le p < p_m(c).$$

This also holds in the limit at $p_m(c)$, since $\Pi_{pp}(p_m(c), c) < 0$. Therefore, by uniform continuity on the compact set $K = \{(c, p) \mid c^- \le c \le c^+, c \le p \le p_m(c)\}$:

$$0 < m = \min\{\rho(p, c), (c, p) \in K\} \le \max\{\rho(p, c), (c, p) \in K\} = M < \infty. \tag{A.2}$$

We shall assume:

$$\Pi_m(c^-) \ge S(c^-) - S(c^+) + M\left(\frac{\Delta}{m}\right)^2 (c^+ - c^-)^2. \tag{A.3}$$

This requires that monopoly profit functions $\Pi(p, c)$ be neither too flat nor too spiked, that monopoly profits for the most efficient firm be sufficiently large, and that the range of possible costs (c^-, c^+) not be too wide. Condition (A.3) will ensure the existence of a solution to the differential equation defining the optimal price strategy of firms with high costs.

Appendix B: Proofs

Proof of Proposition 3: In this equilibrium, consumers' search rule is to search if and only if the first observed price exceeds p^*. By the definition of c^*, no consumer wishes to search at prices below p^*. At p^*, all a consumer knows is that $c_1 \in [c^*, c^+]$. By the definition of c^*, she is indifferent between searching and not if she observes p^* and knows that $c_1 = c^*$. However, in this equilibrium, observing p^* only reveals that $c_1 \ge c^*$. Given the positive correlation in costs this implies that the consumer's beliefs of c_2 are at least as great as if she knew $c_1 = c^*$. This combined with the (weak) monotonicity of the pricing rule implies that she does not wish to search if she observes

p^* in the proposed equilibrium. This shows that consumer search decisions are optimal on the equilibrium path, where all prices are below $p_m(c^*)$. If a consumer observes a price above p^*, her beliefs must be such that it pays for him to search. Believing that $c_1 = c^*$, or more generally that c_1 is close to c^* is sufficient to ensure that she does wish to search.

Pricing rules are clearly optimal. If any firm deviates to a price greater than p^* it gets zero consumers, since they search and find a lower price. Thus, so long as a firm charges a price no greater than p^*, it gets half the consumers. Thus, the optimal price is $p_m(c)$ unless it exceeds p^* in which case (given the quasi-concavity of profits) the optimal price is p^*. Q.E.D.

Proof of Lemma 1: Let C_0 be the space of continuous functions on $[0, c^*]$. We shall work within the subset:

$$C = \{p(\cdot) \in C_0 \mid p(c) \in [c, p_m(c)] \ \forall c\}. \tag{B.1}$$

Denote by $\nu(c) = \frac{f(c|c)}{1-F(c|c)}$ the hazard rate entering (7); then $p(\cdot)$ solves (7) if and only if the function

$$I(c) = \Pi(p(c), c) \tag{B.2}$$

obeys the differential equation:

$$I'(c) = \Pi_p(p(c), c)p'(c) - D(p(c)) = \nu(c)I(c) - D(p(c)) \tag{B.3}$$

with terminal condition $I(c^*) = \Pi(c^*, c^*) = 0$. Integrating (B.3) backwards:

$$I(c) = \int_c^{c^*} D(p(x))e^{-\int_c^x \nu(y)dy} dx = J(p(\cdot), c). \tag{B.4}$$

This integral is convergent at c^*, since $p(x) \geq x$ implies:

$$I(c) \leq \int_c^{c^*} D(x)dx = S(c) - S(c^*).$$

We have transformed the differential equation (7) into an equivalent integral equation:

$$J(p(\cdot), c) = \Pi(p(c), c), \quad \forall c. \tag{B.5}$$

By assumption $\Pi(\cdot, c)$ is strictly quasi-concave, hence can be inverted from $[0, \Pi_m(c)]$ into $[c, p_m(c)]$. Hence, with obvious notation, the fixed point formulation (clearly $I(c) \in [0, \Pi_m(c)]$) is:

$$p(c) = \Pi(\cdot, c)^{-1}(J(p(\cdot), c)). \tag{B.6}$$

We now show that the mapping $T: p(\cdot) \to Tp(\cdot)$, where $Tp(c)$ is the r.h.s. of (B.6), is a contraction on C endowed with the sup norm: $|p| = \sup_{c \in [0, c^+]} |p(c)|$. Let $p(\cdot) \in C$; by construction, $Tp(c) \in [c, p_m(c)]$. $\forall c$.

Moreover, it is easily verified that $\Pi(\cdot, c)^{-1}(\bar{\Pi})$ is jointly continuous in $(c, \bar{\Pi})$, for all $c \in [c^-, c^+]$ and for all $\bar{\Pi} \in [0, \Pi_m(c)]$. Since $J(p(\cdot), c) = I(c)$ is clearly continuous in c, Tp is then continuous, hence $Tp(\cdot) \in C$. Consider now $(p, q) \in C \times C$ and any $c \in [c^-, c^+]$. We have:

$$|Tp(c) - Tq(c)| = |\Pi(\cdot, c)^{-1}(J(p(\cdot), c) - \Pi(\cdot, c)^{-1}(J(q(\cdot), c))|. \tag{B.7}$$

Note that $\Pi(\cdot, c)^{-1}$ has derivative $1 / \Pi_p(\Pi(\cdot, c)^{-1}, c)$, which is unbounded. For all $X, Y \in [0, \Pi_m(c)]$, with $X \geq Y$ denote $x = \Pi(\cdot, c)^{-1}(X)$, $y = \Pi(\cdot, c)^{-1}(Y)$. We claim:

$$\frac{1}{M} \cdot \frac{X - Y}{p_m(c) - y} \leq x - y \leq \frac{1}{m} \cdot \frac{X - Y}{p_m(c) - x}. \tag{B.8}$$

Indeed there exists Z, $X \geq Z \geq Y$, or $z = \Pi(\cdot, c)^{-1}(Z)$, $x \geq z \geq y$ such that $\frac{X - Y}{x - y} = \Pi_p(z, c)$. But, by (A.1), the definition of m and M:

$$m \leq \frac{\Pi_p(z, c)}{p_m(c) - z} \leq M.$$

Inequality (B.8) then follows from $y \leq z \leq x \leq p_m(c)$. Next, apply the first inequality in (B.8) with $X' = \Pi_m(c)$, $x' = p_m(c)$, $Y' = X$, $y' = x$:

$$[p_m(c) - x]^2 \geq \frac{\Pi_m(c) - X}{M} \quad \text{or} \quad p_m(c) - x \geq \sqrt{\frac{\Pi_m(c) - X}{M}} \tag{B.9}$$

Finally, replace (B.9) in the second part of (B.8), to obtain:

$$\Pi(\cdot, c)^{-1}(X) - \Pi(\cdot, c)^{-1}(Y) = x - y \leq \frac{\sqrt{M}}{m} \frac{X - Y}{\sqrt{\Pi_m(c) - X}}. \tag{B.10}$$

Therefore,

$$|Tp(c) - Tq(c)| \leq \frac{\sqrt{M}}{m} \cdot \frac{\left| \int_c^{c^+} e^{-\int_c^x \nu(y) dy} [D(p(x)) - D(q(x))] dx \right|}{\sqrt{\Pi_m(c) - J(p(\cdot), c)}}$$

$$\leq \frac{\sqrt{M}}{m} \cdot \Delta \frac{\int_c^{c^+} e^{-\int_c^x \nu(y) dy} |p(x) - q(x)| dx}{\sqrt{\Pi_m(c) - J(p(\cdot), c)}}$$

$$\leq |p-q| \cdot \frac{\Delta\sqrt{M}}{m} \cdot \frac{c^* - c}{\sqrt{\Pi_m(c) - J(p(\cdot), c)}}. \qquad (B.11)$$

Thus T will be a contraction if, for all $c \in [0, c^*]$,

$$\Pi_m(c) - J(p(\cdot), c) > M\left(\frac{\Delta}{m}\right)^2 (c^* - c^-)^2. \qquad (B.12)$$

Indeed, uniform continuity will then imply that (B.12) holds with M replaced by $M/\sqrt{\beta}$, for some $\beta \in (0,1)$, so that $|Tp - Tq| < \beta |p - q|$. But note that we have,

$$\Pi_m(c) - J(p(\cdot), c) = \Pi_m(c) - \int_c^{c^*} e^{-\int_c^x \nu(y)dy} D(p(x))dx > \Pi_m(c) - \int_c^{c^*} D(x)dx = \Pi_m(c) - S(c) + S(c^*),$$

since $p(c) > c$. Because, $\Pi_m(c) - S(c)$ is increasing, (B.12) then holds by assumption (A.3). This concludes the proof of the existence and uniqueness of $p_f(c)$.

It remains to show that $p_f'(c) > 0$, $\forall c \in [c^-, c^*]$. For $c \in [c^-, c^*)$, $\Pi(p_f(c), c) > 0$ by (B.4) while $f(c \mid c) > 0$ by assumption (A.1), hence the result, by (7). The case of $p_f'(c^*)$ is more complicated because both the numerator and denominator in (7) go to zero as c goes to c^*. For all $\epsilon > 0$,

$$f(c^* - \epsilon \mid c^* - \epsilon)\Pi(p_f(c^* - \epsilon), c^* - \epsilon) - [1 - F(c^* - \epsilon \mid c^* - \epsilon)]\Pi_p(p_f(c^* - \epsilon), c^* - \epsilon)p_f'(c^* - \epsilon) = 0. \qquad (B.13)$$

But,

$$F(c^* - \epsilon \mid c^* - \epsilon) - F(c^* \mid c^*) = [f(c^* \mid c^*) + F_2(c^* \mid c^*)](-\epsilon) + o(\epsilon);$$

$$\Pi(p_f(c^*), c^*) - \Pi(p_f(c^* - \epsilon), c^* - \epsilon) = [\Pi_p(p_f(c^* - \epsilon), c^* - \epsilon)p_f'(c^* - \epsilon) + \Pi_c(p_f(c^* - \epsilon), c^* - \epsilon)]\epsilon + o(\epsilon)$$

$$= [\Pi_p(c^* \mid c^*)p_f'(c^* - \epsilon) + \Pi_c(c^*, c^*)]\epsilon + o(\epsilon).$$

Since $F(c^* \mid c^*) = 1$ and $\Pi(p_f(c^*), c^*) = 0$, (B.13) becomes:

$$\{f(c^* \mid c^*)[-\Pi_p(c^* \mid c^*)p_f'(c^* - \epsilon) + D(c^*)] - [f(c^* \mid c^*) + F_2(c^* \mid c^*)]\Pi_p(c^*, c^*)p_f'(c^* - \epsilon)\}\epsilon = o(\epsilon),$$

or

$$p_f'(c^* - \epsilon)[2f(c^* \mid c^*) + F_2(c^* \mid c^*)]\Pi_p(c^*, c^*) = f(c^* \mid c^*)D(c^*) + o(1)$$

which means that the limit $p_f'(c^*) = \lim_{\epsilon \to 0} p_f'(c^* - \epsilon)$ exists and, since $\Pi_p(c^* \mid c^*) = D(c^*)$,

$$p_f'(c^*) = \frac{f(c^* \mid c^*)}{2f(c^* \mid c^*) + F_2(c^* \mid c^*)} > \frac{1}{2}.$$

This proves the result. Q.E.D.

Proof of Lemma 2: Consumers observe $p_1 \geq p^*$; given that they search, they will come back only if the other firm has $p_2 > p_1$. Given that $p_f(c_2) < p^*$ if $c_2 < c^*$ and $p_f(c_2)$ is increasing in c_2 for $c_2 \geq c^*$, this occurs if and only if $p_f(c_2) > p_1$ and has probability $F(p_f^{-1}(p_1) \mid c)$; hence the form of $\Psi(p, c)$. The first-order condition for maximization $\Psi_p(p, c) = 0$ is precisely the differential equation (7), to which $p_f(c)$ is the only solution. Thus it only remains to show that $\Psi_{pp}(p_f(c), c) < 0$, implying that equilibrium profits (for $c \geq c^*$) are strictly quasiconcave in p. But by the implicit function theorem,

$$p_f'(c) = \frac{-\Psi_{pc}(p_f(c), c)}{\Psi_{pp}(p_f(c), c)}.$$

Given that $p_F' > 0$, the second-order condition becomes $\Psi_{pc}(p_f(c), c) > 0$. But,

$$\Psi_c(p, c) = -D(p)[1 - F(p_f^{-1}(p) \mid c)] - \Pi(p, c) F_2(p_f^{-1}(p) \mid c),$$

$$\Psi_{pc}(p_f(c), c) = -D'(p_f(c))[1 - F(c \mid c)] + D(p_f(c)) \cdot \frac{f(c \mid c)}{p_f'(c)} - \Pi(p, c) \cdot \frac{f_2(c \mid c)}{p_f'(c)} - \Pi_p(p(c), c) F_2(c \mid c).$$

The first two terms are positive. Replacing $p_F'(c)$ from (7), the sum of the last two terms has the sign of:

$$-\Pi_p(p_f(c), c)[f_2(c \mid c)(1 - F(c \mid c)) + F_2(c \mid c) f(c \mid c)] > 0,$$

from assumption (A.1) and the fact that $\Pi_p(p_f(c), c) > 0$. Q.E.D.

Proof of Lemma 3: We first find c^*, the point of indifference between p^* and $p_f(c)$ (when $c^* < c^*$), by examining:

$$\delta(c) = \Pi(p^*, c)\left[1 - \frac{1}{2}F(c \mid c)\right] - \Pi(p_f(c), c)[1 - F(c \mid c)]. \qquad (B.14)$$

We first show that if $\delta(c) = 0$, then $\delta'(c) < 0$. Indeed, $\delta(c) = 0$ if and only if,

$$\Pi(p^*, c) = \frac{1 - F(c \mid c)}{1 - \frac{1}{2}F(c \mid c)} \Pi(p_f(c), c) < \Pi(p_f(c), c). \qquad (B.15)$$

Then:

$$\delta'(c) = -D(p^*)\left[1-\frac{1}{2}F(c\mid c)\right] - D(p_f(c))[1-F(c\mid c)] - \Pi_p(p_f(c),c)[1-F(c\mid c)]p_f'(c)$$

$$-\left[\frac{1}{2}\Pi(p^*,c)-\Pi(p_f(c),c)\right][f(c\mid c)+F_2(c\mid c)]$$

$$= -D(p^*)\left[1-\frac{1}{2}F(c\mid c)\right]\left[1-\frac{p^*-c}{p_f(c)-c}\right] - \frac{1}{2}\Pi(p^*,c)f(c\mid c)$$

$$+\left[\Pi(p_f(c),c)-\frac{1}{2}\Pi(p^*,c)\right]F_2(c\mid c)$$

where we used both (7) and (B.15). Now (B.15) and (2) imply that the last term is negative, and also that $p^* < p_f(c)$ since $\Pi(\cdot,c)$ is increasing on $[c, p_m(c)]$, which contains p^* and $p_f(c)$. This in turn implies that the first term above is also negative. Since the second term is always negative, we have showed that $\delta'(c) < 0$ whenever $\delta(c) = 0$.

Therefore, $\delta(\cdot)$ can have at most one zero. Moreover,

$$\delta(c^*) = \Pi(p^*,c^*)\left[1-\frac{1}{2}F(c^*\mid c^*)\right] - \Pi(p_f(c^*),c^*)[1-F(c^*\mid c^*)] > 0$$

because $p^* = p_m(c^*) > p_f(c^*)$, so $\Pi(p^*,c^*) > \Pi(p_f(c^*),c^*)$, while $\delta(c^*) = \frac{1}{2}\Pi(p^*,c^*) = \frac{1}{2}(p^*-c^*)D(p^*)$ has the sign of p^*-c^*. Thus two cases can arise:

(i) If $p^* \geq c^*$, $\delta(c^*) \geq 0$, so all firms with cost in $[c^*, c^*]$ prefer charging p^* to $p_f(c)$ and, therefore, also (by Lemma 2) to any price above p^*. Thus p^* maximizes profits.

(ii) If $p^* < c^*$, $\delta(\cdot)$ has a unique zero $c' \in (c^*, c^*)$, and a firm with $c \geq c'$ prefers charging $p_f(c)$ to p^*: since $c > c^*$, p^* is its preferred price among those which do not induce search. By Lemma 2, $p_f(c)$ is its preferred price among those which induce search. Thus $p_f(c)$ is the globally optimal price for $c \geq c'$. For $c \in [c^*, c')$, $\delta(c) < 0$ so the firm would rather charge p^* than $p_f(c)$, and also than any $p \leq p^*$, since there is no search below $p^* \leq p_m(c)$ and $\Pi(\cdot,c)$ increases on $[c, p_m(c)]$. Finally, a firm with $c < c^*$ clearly will prefer charging $p_m(c)$. Note that the uniqueness of the solution c' to $\delta(c) = 0$ ensures that it is continuous in c^*. Moreover, for $c > c^*$, $p^* < p_m(c)$ so $\Pi(p^*,c)$ and $\delta(c)$ increase in p^* or c^*. Therefore, c' increases in c^*, and so does $p' = p_f(c')$, since $p_f(\cdot)$ is increasing and independent of c^*. Q.E.D.

Proof of Proposition 4: We shall make the dependence of c^*, $c^\#$, etc., on σ explicit, by denoting them as c_σ^*, $c_\sigma^\#$, etc. To show that (14) holds when σ is small enough, we examine more closely the determination of c^* and $c^\#$ for small σ. Recall that,

$$V_m(c^*) = \int_{c^-}^{c^*} D(p_m(c_2))p_m'(c_2)F(c_2 \mid c^*)dc_2 = \sigma .$$

Even if $V_m(\cdot)$ is not monotonic on all of $[c^-, c^*]$, it is monotonic up to some (maximal) $c^1 \in (c^-, c^*]$, because $V_m'(c^-) > 0$. Moreover, $V_m(c) > 0$ in $[c^1, c^*]$, so $V_m(\cdot)$ is bounded away from zero on this interval, i.e., $V_m(c) \geq \underline{V}_m > 0$. Since $V_m(c^-) = 0$, there exists a unique $c^{**} \in [c^-, c^1]$ such that: $\forall c \in [c^-, c^*]$ $V_m(c) \geq \underline{V}_m$ if and only if $c \geq c^{**}$. Let $\sigma^{**} = V_m(c^{**}) = \underline{V}_m$. Then for all $\sigma \leq \sigma^{**}$, $\exists! c_\sigma^\# \in [c^-, c^{**}] \forall c \in [c^-c^*]$, $V_m(c) > \sigma$ if and only if $c > c_\sigma^\#$, namely $c_\sigma^\# = \max\{c \in [c^-, c^{**}] \mid V_m(c) \leq \sigma\}$. Thus for $\sigma \leq \sigma^{**}$, monopoly pricing can be sustained up to the cost c^*, and not above. Clearly, as σ decreases from σ^{**} to zero, $c_\sigma^\#$ decreases to c^-. In particular, we shall assume that $c_\sigma^\# < c_m(c^*)$, i.e, $p_\sigma^\# = p_m(c_\sigma^\#) < c^*$.

Let us turn next to the determination of $c_\sigma^\sharp$. The differential equation (7) and its solution $p_f(\cdot)$ do not depend on σ, while $c_\sigma^\sharp$ is defined by (see Lemma 3):

$$\Pi(p_\sigma^\#, c_\sigma^\sharp)\left[1 - \frac{1}{2}F(c_\sigma^\sharp \mid c_\sigma^\sharp)\right] = \Pi(p_f(c_\sigma^\sharp), c_\sigma^\sharp)[1 - F(c_\sigma^\sharp \mid c_\sigma^\sharp)] .$$

(Recall that $p_\sigma^\# < c^*$ ensures that a unique solution $c_\sigma^\sharp \in (c_\sigma^\#, c^*)$ exists). As σ decreases to zero, $p_\sigma^\#$ decreases to $p_0^\# = p_m(c^-)$ so by Lemma 3, $c_\sigma^\sharp$ decreases to a limit $c_0^\sharp \geq c^-$. In fact $c_\sigma^\sharp$ remains bounded away from c^-, otherwise in the limit: $\Pi(p_m(c^-), c^-) = \Pi(p_f(c^-), c^-)$, which is impossible since $c^- < p_f(c^-) < p_m(c^-)$ and $\Pi(\cdot, c^-)$ is strictly quasiconcave.

We are now ready to examine (14) for low values of σ. First, for all $c_1 \geq c_\sigma^\sharp$:

$$\chi(c_1, \sigma) \leq \int_{c^-}^{c^*} D(p_m(c_2))p_m'(c_2)F(c_2 \mid c^*)dc_2 = \sigma \qquad (B.16)$$

so $\chi(c_1, \sigma)$ goes to zero (uniformly in c_1) with σ. On the other hand we show that:

$$\underline{\xi}(\sigma) = \min\{\xi(\sigma, c_1), c_1 \in [c_\sigma^\#, c^*]\} > 0 \qquad (B.17)$$

for σ low enough. First note, that for all $\sigma \leq \sigma^{**}$ and $c_1 \geq c_\sigma^\sharp \geq c_0^\sharp$:

$$|\xi(\sigma, c_1) - \xi(0, c_1)| \leq |S(p_\sigma^\#) - S(p_0^\#)| + |S(p_\sigma^\sharp) - S(p_0^\sharp)| + \int_{c_0^\sharp}^{c_\sigma^\sharp} D(p_f(c_2))p_f'(c_2)dc_2$$

$$= |S(p_\sigma^\#) - S(p_0^\#)| + 2|S(p_\sigma^\sharp) - S(p_0^\sharp)|. \qquad (B.18)$$

so $\xi(\sigma, c_1)$ converges to $\xi(0, c_1)$ uniformly in c_1 as σ goes to zero. Therefore, by continuity (B.17) will hold if $\xi(0) > 0$, i.e., for all $c_1 \in [c_0^*, c^*]$:

$$\xi(0, c_1) = [S(p_0^*) - S(p_f(c_0^*))]F(c_0^* \mid c_1) + \int_{c_0^*}^{c_1} D(p_f(c_2))p_f'(c_2)F(c_2 \mid c_1)dc_2 > 0. \quad (B.19)$$

Since $p_0^* = p_f(c_0^*) > p_0^* = p_m(c^*)$, the first term can only be zero if $F(c_0^* \mid c_1) = 0$, which by assumption (A.1) requires $c_1 > c_0^*$. The second term in (B.19) can then only be zero if the integrand $D(p_f(c_2))p_f'(c_2)F(c_2 \mid c_1)$, which is continuous and non-negative, is identically zero on $[c_0^*, c_1]$. But $D(p_f(c_1)) \geq D(c^*) > 0$; by Lemma 1, $p_f'(c_1) > 0$, and by assumption (A.1), $F(c_1 \mid c_1) > 0$, so this cannot be for c_2 near c_1, and (B.19) must therefore hold. Thus (B.17) holds, which, with (B.16), implies that for σ low enough (below some $\sigma' \in (0, \sigma'']$), $V(c_1) - \sigma = \xi(\sigma, c_1) - \chi(\sigma, c_1) > 0$, $\forall c_1 \in [c_1^*, c^*]$. Q.E.D.

Proof of Proposition 5: We consider distributions $F(c_2 \mid c_1)$ satisfying (1),(2), and (A.1), and for which $\overline{F}_2 = \sup\{F_2(c_2 \mid c_1),\ c^* \leq c_2 \leq c_1 \leq c^*\}$ is small. For instance, if $c_i = \bar{c} + \theta + \gamma_i$, $i = 1, 2$, where $\theta \sim U[-\alpha, \alpha]$, $\gamma_i \sim U[-b, b]$, θ, γ_i are independent and $0 < \alpha < b$, on can show that $\overline{F}_2 \leq \frac{1}{4b}$; so it suffices that b be large enough. We first show that if $\overline{F}_2$ is low enough:

(i) $V_m(c_1)$, the returns to search under monopoly pricing at $p = p_m(c)$, is increasing.

This defines c^*, uniquely; if $p_m(c^*) > c^*$ the equilibrium corresponds to either Proposition 2 or Proposition 3; if $p_m(c^*) < c^*$, we can construct $p_f(\cdot)$ and define $c^* \in (c^*, c^*]$. Then for $\overline{F}_2$ low enough, we show:

(ii) $V(c_1)$, the equilibrium return to search at $p = p_f(c)$, $c \geq c^*$, is increasing;

(iii) $V(c^*) > \sigma$, so that searching above p^* is optimal.

This will prove the theorem.

As before with σ, we shall make the dependence of c^*, c^*, etc., on F explicit by denoting them as c_F^*, c_F^*, etc. Since,

$$V_m'(c_1) = D(p_m(c_1))p_m'(c_1)F(c_1 \mid c_1) + \int_{c^*}^{c_1} D(p_m(c_2))p_m'(c_2)F_2(c_2 \mid c_1)dc_2,$$

(i) will hold if for all $c_1 \in (c^*, c^*]$:

$$\overline{F}_2 \leq \frac{D(p_m(c_1))p_m'(c_1)F(c_1 \mid c_1)}{S(p_m(c^*)) - S(p_m(c_1))}. \quad (B.20)$$

We can impose (B.20) directly because the r.h.s. is a continuous, positive function of c_1, and therefore bounded away from zero. Indeed, the r.h.s. of (B.20) is strictly positive for $c_1 > c^-$, and applying L'Hopital's rule, it has limit $f(c^- \mid c^-) > 0$ at $c_1 = c^-$. Alternatively, using the convexity of $S(p)$, (B.20) is implied by,

$$\bar{F}_2 \leq \frac{D(p_m(c^-))}{D(p_m(c^-))} \min_{c_1 \in [c^-, c^+]} \left[\frac{p_m'(c_1) F(c_1 \mid c_1)}{p_m(c_1) - p_m(c^-)} \right]. \tag{B.21}$$

where similarly the term in brackets is bounded away from zero on $[c^-, c^+]$, due to (3) and L'Hopital's rule.

For $c_1 \geq c_F^*$, we have:

$$V'(c_1) = D(p_f(c_1)) p_f'(c_1) F(c_1 \mid c_1) + \int_{c^-}^{c_F^*} D(p_m(c_2)) p_m'(c_2) F_2(c_2 \mid c_1) dc_2$$

$$+ [S(p_F^*) - S(p_F^*)] F_2(c_F^* \mid c_1) + \int_{c_F^*}^{c_1} D(p_f(c_2)) p_f'(c_2) F_2(c_2 \mid c_1) dc_2$$

$$\geq D(p_f(c_1)) p_f'(c_1) F(c_1 \mid c_1) - \bar{F}_2 [S(p_m(c^-)) - S(p_m(c_F^*)) + S(p_F^*) - S(p_F^*) + S(p_F^*) - S(p_f(c_1))]$$

$$\geq D(p_f(c_1)) p_f'(c_1) F(c_1 \mid c_1) - \bar{F}_2 [S(p_m(c^-)) - S(c_1)].$$

But,

$$p_f'(c_1) = \frac{f(c_1 \mid c_1)}{1 - F(c_1 \mid c_1)} \cdot \frac{\Pi(p_f(c_1), c_1)}{\Pi_p(p_f(c_1), c_1)} \geq \frac{f(c_1 \mid c_1)}{1 - F(c_1 \mid c_1)} \cdot \frac{\Pi(p_m(c_1), c_1)}{D(p_f(c_1))} = \frac{f(c_1 \mid c_1)}{1 - F(c_1 \mid c_1)} \cdot \frac{\Pi_m(c_1)}{D(p_f(c_1))},$$

so,

$$V'(c_1) \geq \frac{f(c_1 \mid c_1) F(c_1 \mid c_1)}{1 - F(c_1 \mid c_1)} \Pi_m(c_1) - \bar{F}_2 [S(p^-) - S(c_1)].$$

Thus, (ii) will hold if:

$$\bar{F}_2 \leq \min_{c \in [c_F^*, c^+]} \left[\frac{f(c \mid c) F(c \mid c)}{1 - F(c \mid c)} \right] \cdot \frac{\Pi_m(c^-)}{S(p_m(c^-)) - S(c^-)}. \tag{B.22}$$

By assumption (A.1), the r.h.s. is strictly positive, but both sides of (B.22) involve the distribution F. Nonetheless, if we consider a one parameter family of conditional distributions $F^\lambda(c_2 \mid c_1)$, $\lambda \in [0, \bar{\lambda}]$ which satisfy assumptions (1), (2) and (A.1), and such that F^λ, f^λ depend continuously on λ, with $F^0 = 0$; then as λ goes to zero, so does the l.h.s. of (B.22) whereas the minimum in the r.h.s. converges to (by L'Hopital's rule):

$$\min_{c \in [c_F^{0*}, c^+]} \frac{f^0(c \mid c) F^0(c \mid c)}{1 - F^0(c \mid c)} > 0,$$

since $c_f^{\circ} > c^*$. Therefore, (B.22) holds for F^{λ} with λ small enough.

Finally, by (14), (iii) will hold if:

$$V(c_f^*) = [S(p_f^*) - S(p_f^*)]F(c_f^* \mid c_f^*) \geq \int_{c^*}^{c_f^*} D(p_m(c_2))p_m'(c_2)[F(c_2 \mid c_f^*) - F(c_2 \mid c_f^*)]dc_2 ,$$

for which it suffices that:

$$\bar{F}_2 \leq \frac{S(p_f^*) - S(p_f^*)}{c_f^* - c_f^*} \cdot \frac{F(c_f^* \mid c_f^*)}{S(p^*) - S(p_f^*)} . \qquad (B.23)$$

Again, this condition involves F on both sides; moreover, it requires that the function $p_f(\cdot)$ be computed, so as to find c_f^*. Nonetheless, given a family of distribution $F^{\lambda}(c_2 \mid c_1)$ with the properties described above, for small λ the l.h.s. of (B.23) will be small, while the r.h.s. will be close to the finite value corresponding to F°. This is because the equality (12) defining c_f^* always requires $p_f^* > p_f^*$, unless $\Pi(p^*, c_f^*) = 0$, i.e., $c_f^* = p^* = p_f^*$; but $c = p_f(c)$ is only possible at $c = c^*$. Thus $p_{f^{\circ}}^* = p_{f^{\circ}}^*$ would require $p_{f^{\circ}}^* = c^*$ which can be excluded by focusing (as we have) on the case where $\int_{c^*}^{c^*} D(p_m(c_2))p_m'(c_2)F^{\circ}(c_2 \mid c^*)dc_2 > \sigma$. Q.E.D.

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