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CO-INTEGRATION, AGGREGATE CONSUMPTION, AND THE DEMAND
FOR IMPORTS: A STRUCTURAL ECONOMETRIC INVESTIGATION

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ABSTRACT

This paper uses a two-good version of Hall's (1978) representative agent, permanent income model to derive a structural import demand equation for nondurable consumer goods. Under the identification restriction that taste shocks are stationary, the model is shown to imply that log imports, log domestic goods, and the log relative price of imports are co-integrated.

The data decisively reject the null hypothesis that imports, the relative price of imports, and the consumption of home goods are not co-integrated. We employ the non-linear least squares technique recently proposed by Phillips and Loretan (1990) to estimate the parameters of the import demand equation.

The long-run price elasticity of import demand is estimated to be -0.95. The elasticity of import demand with respect to a permanent increase in real spending is estimated to be 2.20. These estimates fall within the range reported in studies by Helkie and Hooper (1986), Cline (1989), and the many studies surveyed by Goldstein and Kahn (1985).

The message of this paper is that, at least for non-durable consumer goods, it is possible to interpret the traditional import demand equation as a co-integrating regression, and to interpret the price and expenditure elasticities estimated from such a trade equation as a co-integrating vector. Estimates of the co-integrating vector can be used to recover estimates of the utility parameters of the representative household. The similarity between the OLS and Phillips-Loretan estimates of the parameters suggests that the simultaneous equation bias is not large.

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CO-INTEGRATION, AGGREGATE CONSUMPTION, AND THE DEMAND FOR IMPORTS:

A STRUCTURAL ECONOMETRIC INVESTIGATION

Richard H. Clarida

1. Introduction:

This paper uses a two-good version of Hall's (1978) representative agent, permanent income model to derive a structural econometric model of the demand for imported consumer goods. With strongly separable, isoelastic preferences, the log of the demand for foreign goods is shown to be linear in the log of the relative price of imports, the log of the demand for domestic goods, and the log of an unobservable shock to tastes.

The permanent income hypothesis implies that the demand for domestic non-durable goods and the demand for foreign non-durable goods share a common stochastic trend (Stock and Watson (1988)) and that this trend may be identified with the marginal utility of wealth. The data do not reject the null hypothesis that log imports of non-durables and log consumption of domestic non-durables each have a unit root. Since the data also do not reject the hypothesis of a unit root in the relative price of imports, the permanent income hypothesis, along with our specification of preferences and the assumption that shocks to preferences are stationary implies that log imports, log domestic goods, and the log relative price of imports are co-integrated, and that the model's structural parameters - the elasticity of marginal utility with respect to foreign goods consumption, η , and the elasticity of marginal utility with respect to home goods consumption, α , are exactly identified by the co-integrating vector.

Using the approach suggested by Granger and Engle (1987), we find that the data decisively reject the null hypothesis that imports, the relative price of imports, and the consumption of home goods are not-cointegrated. While OLS

might be used to provide an asymptotically consistent estimate of the co-integrating vector, it is subject to the simultaneous equation bias that is likely to be present in our application. To correct for this bias, we employ the non-linear least squares technique recently proposed by Phillips and Loretan (1990) to estimate the parameters of the structural import demand equation.

The results of the empirical work may be summarized as follows. The long-run price elasticity of import demand is estimated to be -0.95 during our sample. Given the precision of the estimate, it is not possible to reject the null hypothesis of a unitary long-run price elasticity, thus putting our estimate in the range of earlier empirical studies (Goldstein and Kahn (1985); Helkie and Hooper (1986); Cline (1989)). The elasticity of import demand with respect to a permanent increase in real spending is estimated to be 2.20 during our sample, roughly the same as reported by Helkie and Hooper (1986), somewhat smaller than reported by Cline (1989), and somewhat larger than the average of the many studies surveyed recently by Goldstein and Kahn (1985). In the context of the optimization problem of the representative household, the Marshallian price elasticity of import demand is not constant but in fact converges to -1 as the share of total spending that falls on imports rises, while the elasticity of import demand with respect to a permanent increase in real spending converges to 1 as the share of spending that falls on imports rises. An advantage of our utility-based, co-integration approach is that, by recovering consistent estimates of the utility parameters via Phillips-Loretan non-linear least squares, we are able to estimate the permanent income elasticity of import demand without having to specify a proxy for permanent income or having to estimate a time series model for actual income.

The paper ends with some concluding remarks.

2. The Model:

We begin by deriving the demand for non-durables foreign goods, F_t , from a standard (Hall (1978)) intertemporal optimization problem. Letting P_t denote the price of imports in terms of domestic goods, H_t the consumption of domestic non-durable goods, A_t assets, y_t labor income, and r_t the real interest rate, the representative household selects (H_t, F_t, A_{t+1}) $t = 0, \dots, T$ so as to:

$$(4) \quad \max E \sum_{t=0}^{t=T} (1 + \delta)^{-t} u(H_t; F_t)$$

s. t.

$$(4') \quad H_t + P_t F_t + A_{t+1} - (1 + r_t) A_t + y_t.$$

The first-order conditions are given by:

$$(5a) \quad u_H = \lambda_t;$$

$$(5b) \quad u_F = \lambda_t P_t;$$

$$(5c) \quad \lambda_t = (1 + \delta)^{-1} E_t(\lambda_{t+1}(1 + r_{t+1}));$$

where λ_t is the Lagrange multiplier on the accumulation constraint (4').

We shall assume that u is strongly separable and satisfies:

$$(6) \quad u(H_t, F_t) = D_t H_t^{1-\alpha} (1 - \alpha)^{-1} + B_t F_t^{1-\eta} (1 - \eta)^{-1};$$

where B_t and D_t are random, stationary shocks to preferences with means B and D respectively. Hodrick (1989) employs this specification of preferences in a

recent study in which he derives a structural exchange rate equation from the intertemporal optimization problem of a representative household in a two-country world. Using (6), (5a) and (5b) are easily solved for the optimal consumption of domestic and foreign goods as a function of λ_t and P_t :

$$(7a) \quad H_t = \lambda_t^{-1/\alpha} D_t^{1/\alpha};$$

$$(7b) \quad F_t = \lambda_t^{-1/\eta} P_t^{-1/\eta} B_t^{1/\eta}.$$

Using the fact that:

$$(8) \quad H_t^{\alpha/\eta} = \lambda_t^{-1/\eta} D_t^{1/\eta};$$

we may express the demand for foreign goods as:

$$(9) \quad F_t = H_t^{\alpha/\eta} P_t^{-1/\eta} (B_t/D_t)^{1/\eta}.$$

Letting lower case letters denote logs, we obtain:

$$(10) \quad f_t = \gamma_t - (1/\eta)p_t + (\alpha/\eta)h_t + e_t;$$

where $\gamma_t = (b_0 + b_1t - d_0 - d_1t)/\eta$ is the difference between the linearly deterministic components of the log shocks to preferences divided by η and

$$(11) \quad e_t = (b_t - b_0 - b_1t)/\eta - (d_t - d_0 - d_1t)/\eta.$$

A well known property of the standard permanent income model is that the marginal utility of consumption follows a martingale (Hall (1978)). In the context of our two-good specification, the marginal utility of wealth follows a martingale, as does the marginal utility of consuming an extra home good, while the marginal utility of consuming an extra foreign good divided by the relative price of foreign goods follows a martingale. If the variance in forecasting the marginal utility of wealth λ_t is small, $\log \lambda_t$ itself is well approximated by the following random walk:

$$(12) \quad \log \lambda_t \approx (\delta - r) + \log \lambda_{t-1} + (\lambda_t - E_{t-1}(\lambda_t))/\lambda_t.$$

Taking logs of both sides of (7a) and (7b) we obtain:

$$(13) \quad f_t = b/\eta - (1/\eta)p_t - (1/\eta)\log \lambda_t;$$

$$(14) \quad h_t = d/\alpha - (1/\alpha)\log \lambda_t.$$

Thus, the permanent income hypothesis implies that the log consumption of foreign goods and the log consumption of home goods share a common stochastic trend, and that this trend can be identified with the marginal utility of wealth, $\log \lambda_t$.

While the theory implies that the log consumption of home goods, h_t and foreign goods, f_t share a common stochastic trend, these two variables are not necessarily co-integrated (Granger and Engle (1987)). In fact, as is revealed by equation (10),

$$(10) \quad f_t = \gamma_t - (1/\eta)p_t + (\alpha/\eta)h_t + e_t;$$

the permanent income hypothesis implies that f_t and h_t are co-integrated if and only if the log relative price of foreign goods, p_t , is stationary. By contrast, if the log relative price of foreign goods is non-stationary in levels, the permanent income hypothesis - along with the identifying restriction that the shocks to preferences b_t and d_t are stationary - implies that f_t , h_t , and p_t must be co-integrated. Furthermore, by the results of Stock and Watson (1988), the existence of two stochastic trends among three non-stationary variables implies that there exists a unique (at least up to a scale factor) co-integrating vector. In the context of our model, if two stochastic trends are found to be present in the data, these trends can be identified with the log marginal utility of wealth $\log \lambda_t$ and the log relative price of foreign goods p_t . The unique co-integrating vector is $[1, 1/\eta, -\alpha/\eta]'$, as is defined by equation (10).

It follows that, in a co-integrating regression of f_t on p_t and h_t , the utility parameters η and α - the elasticities of marginal utility with respect to foreign and home goods - are just identified. Holding constant the marginal utility of wealth, the elasticity of the demand for foreign goods with respect to the relative price of foreign goods is given by $\epsilon_{f,p;\lambda} = -1/\eta$, the coefficient on p_t in the co-integrating regression of f_t on p_t and h_t . The elasticity of the demand for foreign goods with respect to the consumption of domestic goods - the correct "activity variable" on the right-hand-side of the import demand equation (10) derived from the theory - is given by $\epsilon_{f,h;p} = \alpha/\eta$, the coefficient on h_t in the co-integrating regression of f_t on p_t and h_t . In Section 4, after presenting estimates of α and η , we shall use (7) and these estimates to obtain estimates of the Marshallian price elasticity of import demand holding constant real expenditure $C = H + PF$, $\epsilon_{f,p;C}$, as well as of the elasticity of import demand with respect to a permanent increase in real spending, $\epsilon_{f,C;p}$.

3. The Data

The NIPA accounts provide quarterly, seasonally adjusted nominal and 1982 dollar data on non-durable consumer goods imports beginning with 1967:1. The theory presented in Section 2 models the consumption of foreign goods, not their importation. Because importers maintain inventories, imports of foreign goods are a noisy signal of consumption of foreign goods. The NIPA accounts do not provide data on the spending on or consumption of domestically produced consumer goods, but of course do provide quarterly, seasonally adjusted nominal and 1982 dollar data on non-durables consumption. I will first show that, for plausible target inventory behavior, log imports of foreign goods are equal to log consumption of foreign goods plus a stationary, serially correlated disturbance. I will then use this relationship to derive a measure of log home goods consumption that is equal to the true value of log home goods consumption plus noise, noise that is stationary if f_t , p_t , and h_t are co-integrated. I conclude this section with a discussion of the impact of measurement error on the estimation of the model.

Consider the following target inventory model. Letting M_t denote the 1982 dollar value of imports received during quarter t that were ordered at the beginning of quarter $t-1$, letting I_{t-1} denote the 1982 dollar value of inventories of foreign goods on hand at the end of quarter $t-1$, and letting I_t^D denote the 1982 dollar value of desired inventories of foreign goods on hand at the end of quarter t , we suppose that imports are determined by:

$$(15) \quad M_t = E_{t-1}F_t + E_{t-1}I_t^D - I_{t-1};$$

where

$$(16) \quad I_t = I_{t-1} + M_t - F_t.$$

It follows immediately that:

$$(17) \quad E_{t-1}I_t^D - I_t = F_t - E_{t-1}F_t;$$

which can be used, along with (15) to show that:

$$(18) \quad M_t = F_t - (F_t - E_{t-1}F_t) + (F_{t-1} - E_{t-2}F_{t-1}) + E_{t-1}I_t^D - E_{t-2}I_{t-1}^D.$$

If desired inventories are proportional to sales

$$(19) \quad I_t^D = kF_t;$$

we see that imports are equal to consumption of foreign goods plus noise:

$$(20) \quad M_t = F_t + k(F_t - F_{t-1}) - (1+k)(F_t - E_{t-1}F_t) + (1+k)(F_{t-1} - E_{t-2}F_{t-1}).$$

Dividing both sides of (20) by F_t and taking logs we see that

$$(21) \quad m_t \approx f_t + z_t;$$

where:

$$(22) \quad z_t = k(F_t - F_{t-1})/F_t - (1+k)(F_t - E_{t-1}F_t)/F_t + (1+k)(F_{t-1} - E_{t-2}F_{t-1})/F_t.$$

Thus, if both the growth rate in foreign goods consumption the variance in forecasting foreign goods consumption are small, the log of imports, m_t is approximately equal to log foreign goods consumption f_t plus stationary noise z_t .

Consider next the consumption of domestically produced non-durable goods. Our measurement of H_t is defined as follows:

$$(23) \quad H'_t = (E_t - P_{Ft}M_t)/P_{Ht}$$

where E_t is the NIPA definition of quarter t consumption of non-durable goods valued in current dollars, P_{Ft} is the NIPA deflator for non-durable consumer goods imports, and P_{Ht} is the producer price index for non-durable consumer goods. A constant, or even random but stationary mark-up of the unobservable deflator for home goods over the ppi for home goods could be incorporated without changing the thrust of the argument. It follows that:

$$(24) \quad H'_t = H_t + P_t(F_t - M_t);$$

where $P_t = P_{Ft}/P_{Ht}$, H_t is the 1982 dollar value of quarter t consumption of domestic non-durable goods, H'_t is the 1982 dollar value of measured quarter t consumption of domestic goods, and F_t is the 1982 dollar value of quarter t consumption of imported non-durable goods. Dividing both sides (24) by H_t and using (20) we obtain:

$$(25) \quad h'_t = h_t + u_t;$$

where $u_t = -k(F_t - F_{t-1})/H_t + (1+k)(F_t - E_{t-1}F_t)/H_t - (1+k)(F_{t-1} - E_{t-2}F_{t-1})/H_t$.

We now investigate the impact of measurement error on the co-integrating equation. Using (21) and (25) to substitute for f_t and h_t in (10) we obtain:

$$(26) \quad m_t = \gamma_t - (1/\eta)p_t + (\alpha/\eta)h'_t + v_t;$$

where:

$$(27) \quad v_t = e_t + z_t - (\alpha/\eta)u_t.$$

The stationarity of e_t is assumed, and the stationarity of z_t is implied by target inventory behavior. Since $u_t = -z_t P_t F_t / H_t$, u_t will be stationary if f_t , p_t , and h_t are co-integrated with co-integrating vector $[1, 1, -1]$.

Notwithstanding these theoretical predictions, the co-integration of m_t , p_t and h'_t is an empirical issue. A rejection of the null hypothesis that m_t , p_t and h'_t are not co-integrated and that v_t has a unit root will be consistent with co-integration among f_t , p_t , and h_t and the stationarity of e_t . Moreover, if m_t , p_t , and h'_t are co-integrated and the null of a unit root in v_t is rejected, the parameters of interest, α and η , can be recovered from the co-integrating vector defined by equation (26), $[1, 1/\eta, -\alpha/\eta]'$.

4. Testing for Unit Roots and Stochastic Trends

We begin by reporting the results obtained from Fuller(1976) and Augmented Dickey-Fuller(1981) tests of the null hypothesis that the series m_t , p_t , and h'_t possess a unit root. The alternative hypothesis is that these series are stationary about a deterministic trend. The Fuller test is just a t-test that the coefficient β is equal to zero in the following regression:

$$(28) \quad \Delta x_t = \mu_0 + \mu_1 t + \beta x_{t-1} + \epsilon_{xt}.$$

The ADF test allows Δx_t to be serially correlated, and is a joint F test that $\mu_1 - \beta = 0$ in the following regression:

$$(29) \quad \Delta x_t = \mu_0 + \mu_1 t + \beta x_{t-1} + \rho \Delta x_{t-1} + \epsilon_{xt}.$$

The results of these tests are reported in Table 1 and are easily summarized. Neither test can reject at even the 10% level the null hypothesis of a unit root in any of the three variables m_t , p_t , and h'_t . With no strong evidence against the null hypothesis of a unit root in m_t , p_t , or h'_t , we turn next to an investigation of the number of stochastic trends that are present among the three variables in our system.

Stock and Watson (1988) demonstrate that any system of m $I(1)$ variables has a common trends representation, and that in a system comprised of m $I(1)$ variables being driven by $n \leq m$ common trends, the number of linearly independent co-integrating vectors must equal $m - n$. It follows immediately from Stock and Watson's result that if there exists one common trend among m variables, then all $m(m-1)/2$ possible pairs of these variables must be co-integrated. Of course,

if there exists $n = m - 1$ common trends among m variables, the co-integrating vector is unique up to scale.

The theory, along with the unit-root results reported in Table 1, predicts that two common trends, one identified with the log marginal utility of wealth $\log \lambda_t$ and the other identified with the relative price of imports p_t , should be driving the non-stationary components of the system's three variables, m_t , p_t , and h'_t . It follows that the parameters of interest, α and η , can be recovered from the unique co-integrating vector defined by equation (26), $[1, 1/\eta, -\alpha/\eta]'$. Alternatively, if shocks that drive $\log \lambda_t$ also drive the non-stationary component of p_t (or vis-versa), there is only one common trend in the system and all three possible pairs of variables should be co-integrated.

We now test the null hypothesis that none of the three possible pairs of the system's three variables are co-integrated. If we fail to reject this hypothesis, then there exists either two or three common trends among the system's three variables. We test for co-integration by using the approach suggested by Granger and Engle (1987). To test the null hypothesis that m_t and h'_t are not co-integrated, we first run the regression:

$$(30) \quad m_t = \mu_0 + \beta h'_t + \epsilon_{mh't}$$

We then regress changes in the estimated residuals, $\Delta \epsilon_{mh't}$ on one lagged level of the residual and the lagged change:

$$(31) \quad \Delta \epsilon_{mh't} = \delta_0 \epsilon_{mh't-1} + \rho \Delta \epsilon_{mh't-1} + e_{mh't}$$

The test is just a t-test on the coefficient δ_0 ; the appropriate critical values are those reported in Engle and Yoo (1987) since the co-integrating regression has a constant term. We also allow for the alternative that m_t and h'_t are stationary about a deterministic trend by first running the regression:

$$(30a) \quad m_t = \mu_0 + \mu_1 t + \beta h'_t + \epsilon_{mh't};$$

and then estimating:

$$(31a) \quad \Delta \epsilon_{mh't} = \delta_1 \epsilon_{mh't-1} + \rho \Delta \epsilon_{mh't-1} + e_{mh't}.$$

The appropriate critical values for a t-test of $\delta_1 = 0$ are those reported in Phillips and Ouliaris (1989) since the co-integrating regression has a trend term.

The results of these tests are reported in Table 2 and are easily summarized. Neither test can reject at even the 10% level the null hypothesis that any of the three pairs of variables (m_t, h'_t) , (m_t, p_t) , and (h'_t, p_t) are not co-integrated. With no strong evidence against the null hypothesis that there does not exist one common trend among m_t , p_t , or h'_t , we turn next to a test of the hypothesis that there exists two common trends, as predicted by the theory.

Theory, along with the unit root results reported in Table 1, predicts that m_t , p_t , and h'_t are co-integrated with co-integrating vector $[1, 1/\eta, -\alpha/\eta]'$:

$$(26) \quad m_t = \gamma_t - (1/\eta)p_t + (\alpha/\eta)h'_t + v_t.$$

In light of the results reported in Table 2, a rejection of the null of no co-integration among m_t , p_t , and h'_t is evidence in favor of the model. Moreover, a rejection of the null of no co-integration implies that the parameters of interest, α and η , can be identified from the data. Granger and Engle (1987) suggest estimating $[1, 1/\eta, -\alpha/\eta]'$ directly from the first-stage OLS regression:

$$(32) \quad m_t = \mu_0 + \mu_1 t + \beta_1 p_t + \beta_2 h'_t + \epsilon_{\text{mph}'t}.$$

If it is found that, in the Dickey-Fuller regression:

$$(33) \quad \Delta \epsilon_{\text{mph}'t} = \delta_1 \epsilon_{\text{mph}'t-1} + \zeta_t;$$

δ_1 is significantly negative, the OLS estimates of $[1, 1/\eta, -\alpha/\eta]'$ given by $[1, -\beta_1, \beta_2]'$ are consistent, despite the fact that v_t is correlated with p_t and h'_t and is also likely to be serially correlated.

Recent research, as summarized in the excellent recent survey of Campbell and Perron (1991), has documented that, with the samples sizes available for macroeconomic time series research, the OLS estimate of the co-integrating vector can be severely biased. Furthermore, it is not possible to test hypotheses about the parameters of the co-integrating vector when these are estimated by OLS (Campbell and Perron (1991), p. 56). Fortunately, both Stock and Watson (1989) and Phillips and Loretan (1990) have discovered tractable methods for obtaining asymptotically FIML estimates of the co-integrating vector. For this reason, we will rely on the co-integrating regression primarily for its estimates of $\epsilon_{\text{mph}'t}$ and $\Delta \epsilon_{\text{mph}'t}$ that are needed to test the null of no co-integration among m_t , p_t , h'_t .

5. Co-integration, Consumption, and the Demand for Imports: Empirical Results

The results of the Granger-Engle test of the null hypothesis that m_t , p_t , h'_t are not co-integrated are presented in the top panel Table 3. The critical values are those reported in Phillips and Ouliaris (1989) since both a constant and a linear time trend are included in (32), the co-integrating regression. It is seen that the estimated value of δ_1 is -0.4119 with a standard error of 0.0863 and a t ratio of -4.774. Under the null hypothesis that $\Delta\epsilon_{\text{mph},t}$ is a random walk, the estimated δ_1 is significant at the 1% level using the Phillips-Ouliaris critical values. If instead we use the critical values for a Dickey-Fuller test with 100 observations computed from a Monte Carlo and reported in Figure 1, a no co-integration null can be rejected at the 2.5 percent level.

In light of the results reported in Table 2, we conclude that the data are consistent with the prediction of the model that exactly two stochastic trends and thus one co-integrating vector describe the data. The OLS estimate of the co-integrating vector is [1, 0.96, -2.33]. This implies an OLS estimate of η , minus the elasticity of marginal utility with respect to the consumption of foreign goods, of $\eta^{\text{ols}} = 1.04$ and an OLS estimate of α , minus the elasticity of marginal utility with respect to the consumption of home goods, of $\alpha^{\text{ols}} = 2.37$.

As discussed above, if v_t is correlated with the regressors p_t and h'_t , OLS estimates of the co-integrating vector can be severely biased in small samples. We would expect the structural preference shock, b_t to be positively correlated with p_t . That is, a transitory rise in consumption of foreign goods brought about by a jump in b_t would be positively correlated with p_t and thus negatively correlated with $-p_t$. We would also expect the structural preference shock, d_t to be positively correlated with h_t . It follows that $e_t = (b_t - d_t)/\eta - \gamma_t/\eta$ is likely to be negatively correlated with the regressors in equation (26).

Phillips and Loretan (1990) propose a parametric procedure for estimating the co-integrating vector in an equation in which the variables are in fact known to be co-integrated. The Phillips and Loretan approach tackles the simultaneity problem by including lagged and lead values of the change in the regressors. The approach deals with the autocorrelation in the residuals by including lagged values of the stationary deviation from the co-integrating relationship. Phillips and Loretan prove that the estimates of the co-integrating vector obtained from this approach are asymptotically FIML. They also show that the likelihood ratio test can be used to test hypotheses about the parameters of the co-integrating vector.

Let y_t denote the vector $[1, t, p_t, h'_t]'$ and let θ denote the vector $[\mu_0, \mu_1, \beta_1, \beta_2]'$. The Phillips-Loretan equation is given by:

$$(34) \quad m_t = \theta' y_t + \rho(m_{t-1} - \theta' y_{t-1}) + \sum_{j=-\tau}^{j=\tau} \varphi_j \Delta p_{t-j} + \sum_{j=-\tau}^{j=\tau} v_j \Delta h'_{t-j} + \epsilon_{mt}.$$

The θ vector is estimated by non-linear least squares. The implied estimates of θ along with standard errors are reported in Table 4.

As shown in Table 4, the NLS estimate is quite similar to the OLS estimate of the co-integrating vector. The NLS estimate of the co-integrating vector is $[1, 0.94, -2.21]$. This implies a NLS estimate of η , minus the elasticity of marginal utility with respect to the consumption of foreign goods, of $\eta^{nls} = 1.05$ and a NLS estimate of α , minus the elasticity of marginal utility with respect to the consumption of home goods, of $\alpha^{nls} = 2.27$.

We now use these NLS estimates of η and α to construct estimates of the familiar Marshallian price elasticity and the permanent expenditure elasticity of the demand for imports. If total real expenditure $C = H + PF$ is to remain

constant in the face of an increase in the relative price of foreign goods, (7) can be used to show that:

$$(35) \quad (\eta - 1)(1 - s)d\log P/\eta - [s/\alpha + (1 - s)/\eta]d\log \lambda;$$

where s is the share of spending that falls on domestic goods. Substituting for $\log \lambda$ in (13), we obtain the expression for the Marshallian price elasticity:

$$(36) \quad \epsilon_{I,P;C} = -(1/\eta)[1 - (1 - \eta)(1 - s)/((\eta s/\alpha) + (1 - s))].$$

Since our estimate of η , $\eta^{nl_s} = 1.06$ exceeds 1, the estimated Marshallian elasticity must, in absolute value, exceed $1/\eta^{nl_s} = 0.94$. In our sample $(1 - s)$, the share of total non-durables spending that falls on imports, rises from 0.01 in 1967 to 0.04 in 1990. Using our estimate of $\alpha^{nl_s} = 2.27$, we determine that, in our sample, the Marshallian price elasticity of the demand for imports falls in the following range:

$$(37) \quad 0.94 \leq \epsilon_{I,P;C} \leq 0.95.$$

We now derive an expression for the elasticity of import demand with respect to a permanent increase in real expenditure, holding constant the relative price of imports. From (13) and (14), we see that the source of such a permanent rise in real spending must be a permanent decline in the marginal utility of wealth. Using (7) it is straightforward to show that:

$$(38) \quad d\log C = -(s/\alpha + (1 - s)/\eta)d\log \lambda.$$

Substituting for $\log \lambda$ in (13) and differentiating with respect to $\log C$, we obtain:

$$(39) \quad \epsilon_{f,C;p} = (\alpha/\eta)[1/(s + (\alpha/\eta)(1 - s))].$$

Thus, since α^{nls} exceeds η^{nls} , the elasticity of import demand with respect to a permanent rise in real expenditure is bounded above by 2.21, the NLS estimate of β_2 . Using the fact that $(1 - s)$ is less than 0.04 in our sample, we obtain:

$$(40) \quad 2.12 \leq \epsilon_{f,C;p} \leq 2.21.$$

These elasticity estimates are firmly in the range of those reported in the many studies surveyed by Goldstein and Kahn (1985), and those reported by Helkie and Hooper (1986) and Cline (1989). However, it should be pointed out that the Marshallian price elasticity and the permanent expenditure elasticity are not constant if, as is the case in our sample, the share of spending that falls on imports is changing over time. Indeed, it is easily verified from (36) and (39) that, as the share of spending on imports, $(1 - s)$, rises over time, the permanent expenditure elasticity must decline over time from 2.21 to 1.00, while the Marshallian price elasticity must rise - in absolute value - over time from -0.94 to -1.00. An excellent recent paper by Marquez (1991) emphasizes the importance of allowing for time varying elasticities in empirical trade models.

One message of this paper is that, at least for non-durable consumer goods, it is possible to interpret the traditional import demand equation as a co-integrating regression, and to interpret the price and expenditure elasticities estimated from such a trade equation as a co-integrating vector. The striking

similarity between the OLS and Phillips-Loretan estimates suggests that the simultaneous equation bias is not large.

A second message of this paper is that the permanent income theory, along with the empirically testable restriction that the log relative price of imports and the log marginal utility of wealth are not co-integrated, predicts that the co-integrating vector for (f_t, p_t, h_t) is unique, and that estimates of this co-vector can be used to identify exactly the parameters of the household utility function. An expenditure elasticity in excess of unity is consistent with the theory when the concavity of the sub-utility function for home goods exceeds the concavity of the sub-utility function for foreign goods. Our estimate is that the elasticity of the marginal utility of home goods consumption, α , is a bit more than twice the elasticity of the marginal utility of foreign goods consumption.

6. Concluding Remarks

This paper has employed a two-good version of Hall's (1978) representative agent, permanent income model to derive a structural econometric specification of the demand for imported consumer goods. With separable, isoelastic preferences, the log of the demand for foreign goods is shown to be linear in the log of the relative price of imports, the log of the demand for domestic goods, and the log of an unobservable shock to tastes.

A recent paper by Ceglowski (1991) also employs the permanent income hypothesis and the utility function (6) to derive two alternative equations that can be used to estimate the parameters α and η . Ceglowski shows that, in our notation:

$$(41) \quad m_{t+1} - m_t - c_1 + (1/\eta)(i_t - \log(P_{Ft+1}/P_{Ft})) + z_{t+1};$$

where i_t is the nominal interest rate and z_{t+1} is referred to as a rational error in forecasting the intertemporal marginal rate of substitution, but, as is acknowledged on page 127 of Ceglowski (1991), it also includes the error in using imports to proxy for consumption of foreign goods. To estimate $(1/\eta)$ in (41), her equation (5), Ceglowski employs an instrumental variables procedure to correct for the correlation between the forecast error and the ex post real interest rate. Ceglowski also demonstrates that

$$(42) \quad m_{t+1} - c_2 + (1/\eta)(i_t - \log(P_{Ft+1}/P_{Ft})) + (\alpha/\eta)h'_t + z_{t+1}.$$

To estimate $(1/\eta)$ and (α/η) in (42), her (7), Ceglowski employs instrumental variables to correct for the correlation between $(1_t - \log(P_{Ft+1}/P_{Rt}))$ and z_{t+1} . She finds that IV estimation of these two equations give dramatically different estimates of $(1/\eta)$. In fact, the IV estimate of $(1/\eta)$ from equation (41) is 0.39 - less than half the estimate of $(1/\eta)$ obtained from (42) - with a standard error of 0.68. Ceglowski reports estimates of the Marshallian price elasticity $\epsilon_{f,r;c}$; she does not investigate the theoretical predictions and empirical implications of the co-integration relationship that is the focus of this paper, nor does she derive, discuss, relate to the permanent income hypothesis, nor estimate the permanent expenditure elasticity $\epsilon_{f,c;p}$.

In this paper, we have shown that the permanent income hypothesis implies that the demand for domestic goods and the demand for foreign goods each have a unit root, a prediction that is not rejected by the data. Since the data also do not reject the hypothesis of a unit root in the relative price of imports, the assumption that shocks to preferences are stationary implies that log imports, log domestic goods, and the log relative price are co-integrated. Using the approach of Granger and Engle (1987) we were able to decisively reject the null hypothesis that imports, the relative price of imports, and the consumption of home goods are not co-integrated.

The estimation technique proposed by Phillips and Loretan (1990) was employed to estimate the parameters of the structural import demand equation. The long-run price elasticity of import demand was estimated to be -0.95. The elasticity of import demand with respect to a permanent increase in real spending was estimated to be 2.20, roughly the same as reported by Helkie and Hooper (1986), somewhat smaller than reported by Cline (1989), and somewhat larger than the average of the many studies surveyed recently by Goldstein and Kahn (1985).

In the context of the optimization problem of the representative household, the Marshallian price elasticity of import demand is not constant but in fact converges to -1 as the share of total spending that falls on imports rises, while the elasticity of import demand with respect to a permanent increase in real spending converges to 1 as the share of spending that falls on imports rises. An advantage of our utility-based, co-integration approach is that, by recovering consistent estimates of the utility parameters via Phillips-Loretan non-linear least squares, we are able to estimate the permanent income elasticity of import demand without having to specify a proxy for permanent income or having to estimate a time series model for actual income.

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TABLE 1
Testing for Unit Roots

The Fuller Regression: $\Delta x_t = \mu_0 + \mu_1 t + \beta x_{t-1} + \epsilon_{xt}$.

Variable	Estimated β	t-ratio
-----	-----	-----
m_t	-0.0958	-2.080
p_t	-0.0670	-1.755
h_t	-0.0149	-0.646

The Fuller (1976) critical values from Table 8.5.2 for a sample size of 100 are:

- 3.15 at the 10 percent level;
- 3.45 at the 5 percent level;
- 4.04 at the 1 percent level.

The sample is 1968:2 through 1990:2. Variables are as defined in the text.

The Dickey-Fuller Regression: $\Delta x_t = \mu_0 + \mu_1 t + \beta x_{t-1} + \rho \Delta x_{t-1} + \epsilon_{xt}$.

Variable	Estimated β	Estimated μ_1	F-Statistics (2,87)
-----	-----	-----	-----
m_t	-0.0882 (0.0473)	0.0014 (0.0008)	1.7372
p_t	-0.0555 (0.0380)	0.0001 (0.0002)	1.1956
h_t	-0.0240 (0.0222)	0.0001 (0.0001)	0.9166

The Dickey-Fuller(1981) critical values for F from Table VI for sample 100 are:

- 5.47 at the 10 percent level;
- 6.49 at the 5 percent level;
- 8.73 at the 1 percent level.

The sample is 1968:2 through 1990:2. All three equations were re-estimated with four, three, and two lags of Δx_t , and the lag length for calculating the F-test was chosen as recommended by Campbell and Perron (1991). Using this approach, the null hypothesis of a unit root in h_t was never rejected at even the 10 percent level.

TABLE 2

Testing for A Single Stochastic Trend

The Co-integrating Regression: $x_t = \mu_0 + \mu_1 t + \beta y_t + \epsilon_{xyt}$.

The Dickey-Fuller Regression: $\Delta \epsilon_{xyt} = \delta_1 \epsilon_{xyt-1} + \rho \Delta \epsilon_{xyt-1} + e_{xyt}$.
(Augmented)

Variables	Estimated δ_1	t-ratio
$[m_t, h_t]$	-0.1508	-2.2500
$[m_t, p_t]$	-0.0958	-1.9827
$[p_t, h_t]$	-0.0543	-1.4414

The Phillips-Ouliaris(1989) asymptotic critical values from Table IIc are:

- 3.51 at the 10 percent level;
- 3.80 at the 5 percent level;
- 4.36 at the 1 percent level.

The sample is 1968:2 through 1990:2. The data are defined in the text.

The Co-integrating Regression: $x_t = \mu_0 + \beta y_t + \epsilon_{xyt}$.

The Dickey-Fuller Regression: $\Delta \epsilon_{xyt} = \delta_0 \epsilon_{xyt-1} + \rho \Delta \epsilon_{xyt-1} + e_{xyt}$.

Variables	Estimated δ_0	t-ratio
$[m_t, h_t]$	-0.0382	-1.2857
$[m_t, p_t]$	-0.0360	-1.0900
$[p_t, h_t]$	-0.0444	-1.5008

The Engle-Yoo (1987) critical values from Table 2 for a sample of 100 are:

- 3.03 at the 10 percent level;
- 3.37 at the 5 percent level;
- 4.07 at the 1 percent level.

The sample is 1968:2 through 1990:2. All three equations were re-estimated with four, three, and two lags of $\Delta \epsilon_{xyt}$, and the lag length for calculating the t-test was chosen as recommended by Campbell and Perron (1991). Using this approach, the null hypothesis of no co-integration among any pair of $[m_t, h_t, p_t]$ was never rejected at even the 10 percent level.

TABLE 3

Testing for Co-Integration

The Co-integrating Regression: $m_t = \mu_0 + \mu_1 t + \beta_1 p_t + \beta_2 h'_t + \epsilon_{\text{mph}'t}$

The Dickey-Fuller Regression: $\Delta \epsilon_{\text{mph}'t} = \delta_1 \epsilon_{\text{mph}'t-1} + \zeta_t$

Estimated δ_1	t-ratio
-0.4119	-4.7740*

The Phillips-Ouliaris(1989) critical values from Table IIC are:

- 3.84 at the 10 percent level;
- 4.16 at the 5 percent level;
- 4.65 at the 1 percent level*.

The augmented Dickey-Fuller regression:

$$\Delta \epsilon_{\text{mph}'t} = \delta_1 \epsilon_{\text{mph}'t-1} + \rho_1 \Delta \epsilon_{\text{mph}'t-1} + \dots + \rho_4 \Delta \epsilon_{\text{mph}'t-4} + \zeta_t$$

was also estimated and the lag length used to calculate the t-statistic for δ_1 was chosen as recommended by Campbell and Perron (1991). As none of the ρ_j was significant, the t-test for the significance of δ_1 is based on the simple Dickey-Fuller regression. See Figure 1 for critical values of Dickey-Fuller t-ratio with 100 observations obtained from Monte Carlo simulation.3

The OLS estimates of the parameters are:

Coefficient	Estimated Value
μ_0	-6.4105 (0.1661)
μ_1	0.0170 (0.0004)
β_1	-0.9577 (0.0684)
β_2	2.3258 (0.1386)

The R^2 is 0.979892. The Durbin-Watson statistic is 0.8107.
The sample is 1967:2 through 1990:2. Variables defined in text.

TABLE 4

Estimation of the Parameters
Phillips and Loretan(1990) Non-Linear Least Squares

Phillips-Loretan equation with $\theta = [\mu_0, \mu_1, \beta_1, \beta_2]'$:

$$m_t = \theta' y_t + \rho(m_{t-1} - \theta' y_{t-1}) + \sum_{j=1}^{j-1} \varphi_j \Delta p_{t-j} + \sum_{j=1}^{j-1} v_j \Delta h'_{t-j} + \epsilon_{mt}$$

The non-linear least squares estimates of θ are:

Coefficient	Estimated Value
μ_0	-6.2096 (0.3289)
μ_1	0.0164 (0.0008)
β_1	-0.9404 (0.1366)
β_2	2.2062 (0.2721)

The implied elasticities are:

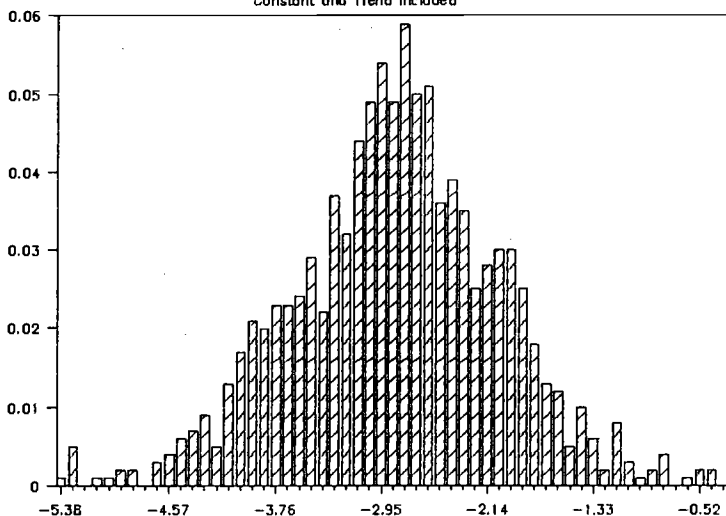
Elasticity	Estimated Value
$\epsilon_{z,p;c}$	-0.95
$\epsilon_{z,c;p}$	2.20

The elasticities are derived in the text, equations (36) and (39). The Phillips-Loretan equation was estimated with up to $\tau = 3$ leads and lags and with up to 2 lags of the equilibrium error with no significant difference in the results.

FIGURE 1

Dickey-Fuller t Distribution, $n = 3$

Constant and Trend Included



Size	Critical Value
0.010	-5.010
0.025	-4.494
0.050	-4.183
0.100	-3.918

Critical values obtained from Monte Carlo experiment. 100 observations on 3 independent random walks with drift 0.02 and normal (0,0.02) innovations were drawn, a first-stage regression, with a constant and trend included, was run, generating 99 observations on $\Delta\epsilon_{xyzt}$. The Dickey-Fuller regression:

$$\Delta\epsilon_{xyzt} = \delta_1\epsilon_{xyzt-1} + \zeta_t;$$

was then run, and the t-statistic on δ_1 was computed and stored. This process was repeated 1000 times. The empirical distribution is plotted in Figure 1.