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A THEORY OF WAR FINANCE

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ABSTRACT

This paper analyzes the financial and war-spending policies of a state that faces a war in which defeat would result in the loss of sovereign power and in which the material consequences, conditional on avoiding defeat, are stochastic. The analysis takes explicit account of the historical experiences of lenders, who face debt repudiation if the state to whom they have lent is defeated and who also face partial default if the material consequences of the war are unfavorable for the debtor state, even if it avoids defeat. In this analysis, the state uses war debt to smooth expected consumption intertemporally in response to temporary war spending, and the state also uses contingent debt servicing to insure realized consumption against the risk associated with the material consequences of the war.

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A Theory of War Finance

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The national debt was the major reason for the British victory [in the Napoleanic Wars]. It had placed huge sums of money at England's disposal at the very moment when she required them. Isaac de Pinto was clearsighted when he wrote in 1771: "The scrupulous and inviolable exactness with which this interest [that on the national debt] has been paid, the idea of parliamentary guarantees, have established England's credit to the point where she has received loans that have surprised and astonished the rest of Europe." He regarded the English victory in the Seven Years' War (1756-1763) as the natural consequence. France's weaknesses, he claimed, lay in her poor credit arrangements. Thomas Mortimer was also right when in 1769 he admired in English public credit "the permanent miracle of her policy, which has inspired both astonishment and fear in the states of Europe." Thirty years earlier, George Berkeley had celebrated it as "the chief advantage which England has over France." – Fernand Braudel, The Perspective of the World, Vol. 3 (Paris, Librairie Armond Colin, 1979; English translation, New York: Harper & Row, 1984), p. 384.

If we can go on giving the army what they want longer than the Germans can do this to theirs, we may appear to win by military prowess. But we shall really have won by financial prowess. – Quoted in Elizabeth Johnson, ed., The Collected Writings of John Maynard Keynes, Activities, 1914-1919, Vol. 16 (London: Macmillan 1971), p. 187.

There can be no time, no state of things, in which credit is not essential to a nation, especially as long as nations in general continue to use it as a resource of war. – Alexander Hamilton, "Second Report on Public Credit," January 16, 1795, reprinted in The Papers of Alexander Hamilton, Vol. 18 (New York: Columbia University Press, 1977).

As indicated by the above quotations (for which we are indebted to Peter Garber), leading historians, economists, and statesmen long have recognized the critical role of financial power in supporting military success. Indeed, in the history of most countries, the incurring of public debt has been associated mainly with the financing of temporarily high levels of public expenditures associated with major wars. After major wars, the ratio of public debt to annual gross national product has declined, reversing most if not all of its wartime increase, with minor wars, peacetime mobilizations, and recessions causing only brief interruptions in this process. This experience accords with the traditional theory of public finance, which focuses on the role of public debt in financing extraordinary public expenditures and, thereby, in smoothing taxation and consumption.

The ability of a sovereign state to issue war debt depends critically on lenders' expectations about the servicing of these debts. Historical experience shows that lenders to a sovereign state at war face two distinct risks that condition their expectations. First, if the stakes in the war include sovereign power itself, then the lenders can expect debt repudiation if the state to whom they have lent is defeated. The victor in a war of sovereign survival typically does not pay the debts of the vanguished. For example, lenders to the American Confederacy, to the Russian Empire, and to the Third Reich all suffered repudiation when defeat in war terminated these sovereign powers.

Second, although the declines in the ratio of public debt to annual gross national product that characterize peacetime usually have resulted from repayment of war debt, sometimes postwar declines in the debt ratio have involved partial defaults. For example, the American experience of servicing war debts in full after the Civil War and World War I is strikingly different from the American experience of partial default by debt restructuring after the Revolutionary War and by inflation after World War II. [See Grossman (1990) for discussion of the relation between war debt and postwar deflations and inflations in the United Kingdom and the United States. See Garber (1991) for discussion of the restructuring of Revolutionary War debt.] Grossman and Van Huyck (1988) interpret such sovereign defaults as unlucky realizations of debt-servicing obligations that are implicitly contingent on the fortunes of the sovereign borrower. In particular, Grossman (1990) attributes the inflation and resulting partial default on war debts in both the United Kingdom and the United States after World War II, a striking exception to the usual postwar experience of deflation, to the unusual competing claims on national resources that were peculiar consequences of World War II.

To analyze the effect of these risks of repudiation and of partial default on the ability of a state to finance its war efforts, we set up a model in which war debt is a contingent claim in two respects. First, debt servicing is contingent on the survival of the sovereign borrower. Consequently, the ability of a state to borrow to finance a war depends on the probability of its avoiding defeat in the war. At the same time, however, the probability of avoiding defeat can depend on the ability to finance war spending. Our analysis treats the equilibrium amount of war spending, the sovereign borrower's resulting probability of avoiding defeat, as well as the equilibrium amount of borrowing as a set of endogenous variables to be determined simultaneously.

Second, debt servicing is contingent on the material consequences of the war for the sovereign borrower. In other words, the sovereign borrower uses debt servicing to shift to its lenders the consumption risk associated with the war. This risk reflects both the possibility of war damage as well as the possibilities of being able to extract war reparations (or other forms of booty and tribute) or of having to pay war reparations. In our analysis, the usefulness of contingent debt servicing for insuring realized consumption against the consumption risk associated with war is complementary to the usefulness of borrowing for smoothing expected consumption intertemporally in the face of temporary war spending.

1. Analytical Framework

Consider a simple two-period model. In the first period, the state must fight a war. In the second period, if it avoids defeat in the war, the state enjoys peace.

The state has a resource endowment of one unit in the first period. The state supplements this endowment by borrowing b units in the first period, $b \geq 0$. Out of its endowment and borrowing in the first period, the state finances war spending of g units, $g \geq 0$, and consumption of c_1 units, $c_1 \geq 0$. Thus, the utility of consumption in the first period is given by

$$u(c_1) = u(1 + b - g), \tag{1}$$

where $u' > 0$ and $u'' < 0$ for all $c \geq 0$ and $u(0) = 0$.

[The analysis abstracts from saving. But, if the risk-free interest rate available to the state is small, and if, as we assume, its expected endowment in the second period equals its actual endowment in the first period, then the state has no reason to save in order to supplement its expected endowment in the second period. Also, as we shall see, the assumptions of the model preclude any reason for precautionary saving.]

If the state avoids defeat in the war, then its resource endowment in the second period is $1 - z$ units, where z is an exogenous random variable that represents the material consequences of the war – in particular, the sum of war damage and net reparations paid. Realizations of z have a stationary probability distribution $p(z)$, such that $\sum_z p(z) = 1$, and a mean of zero. The analysis assumes that the realization of z is verifiable, either directly or indirectly. This assumption is critical to the lenders' ability to treat the debts of the state as claims that are contingent on the realization of z.

Out of its endowment of $1 - z$ units in the second period, the state finances debt servicing of s units, $s \geq 0$, and consumption of c_2 units, $c_2 \geq 0$. Debt servicing can be contingent on z and, in general, debt-servicing conforms to a schedule given by $s = S(z;b)$. The analysis assumes that the state is irrevocably committed to this debt-servicing schedule. [This assumption abstracts from the requirement that the debt-servicing schedule must be time consistent. See Grossman and Van Huyck (1988) for a complementary analysis of the time-consistency issue for sovereign debt within a reputational model.]

Because z and s are random variables, the utility of consumption in the second period, conditional on avoiding defeat in the war, is stochastic. Specifically, expected utility in the second period, conditional on avoiding defeat in the war, equals the sum over all possible realizations of z of the products of the probability of each possible realization of z and the value of $u[1 - z - S(z;b)]$ associated with that realization of z.

If the state suffers defeat in the war, then its endowment in the second period is zero. In this case, its expenditures on debt servicing and on consumption are also zero. The possibility of defeat in the war introduces a second stochastic factor into the problem. Specifically, taking account of both the possibility of defeat and the material consequences of the war, expected utility of consumption in the second period is given by

$$E[u(c_2)] = \rho \sum_z p(z)u[1 - z - S(z;b)] + (1-\rho)u(0), \tag{2}$$

where E is the expected value operator and ρ is the probability of avoiding defeat in the war, $0 \leq \rho \leq 1$.

The analysis assumes, for simplicity, that potential lenders to the state are risk neutral and require a constant expected rate of return equal to r, $r \geq 0$. [A possible extension would be to introduce risk-averse lenders, in which case the required expected rate of return would not be constant but would depend on the amount lent to the state and on the state's debt-servicing schedule.] Conditional on the state's avoiding defeat in the war, expected debt servicing is the sum over all possible realizations of z of the products of the probability of each possible realization of z and the amount of debt servicing given by the schedule $S(z;b)$ for that realization of z. Alternatively, if the state is defeated in the war, then its debts are repudiated. Thus, lenders have their required expected rate of return if the schedule $S(z;b)$ satisfies

$$(1+r)b = \rho \sum_{z} p(z) S(z;b). \tag{3}$$

To close the model, the analysis assumes that the probability of avoiding defeat in the war is an increasing function of war spending, until the probability reaches unity. Specifically, ρ depends on g according to the simple piece-wise linear function,

$$\rho = \begin{cases} \gamma + \theta g & \text{for} \quad g < \frac{1-\gamma}{\theta} \\ 1 & \text{for} \quad g \geq \frac{1-\gamma}{\theta} \end{cases} \tag{4}$$

where $0 \leq \gamma < 1$ and $0 \leq \theta < 1$. In equation (4), the parameter γ represents the probability that the state can avoid defeat without any war spending, e.g., by passive resistance, and the parameter θ measures the marginal effect of larger g in increasing ρ. The parameters γ and θ incorporate technological factors as well as the strategic responses to the state's war spending by the other parties to the war, including both adversaries and allies. [An interesting extension would be to analyze these strategic interactions, thereby treating γ and θ as endogenous variables derivable from deeper structural parameters.]

The state's problem is to choose in the first period values of war spending g and of borrowing b and a debt-servicing schedule $S(z;b)$. In the second period, the state, if it avoids defeat, simply services its debts according to the schedule $S(z;b)$. The state's objective in making its first-period choices is to maximize total expected utility, denoted by U, where

$$U = u(c_1) + E[u(c_2)], \tag{5}$$

and where $u(c_1)$ is given by equation (1) and $E[u(c_2)]$ is given by equation (2). The relevant constraints are the supply condition for loans, given by equation (3), and the technology for avoiding defeat, given by equation (4).

2. War Spending

As discussed above, we want to analyze both the usefulness of borrowing for smoothing expected consumption intertemporally in the face of temporary war spending and the usefulness of contingent debt servicing for insuring realized consumption against consumption risk associated with the material consequences of war. To focus this analysis, consider first a special case in which the state cannot issue debt. Specifically, replace equation (3) with the constraint $b = 0$. In addition, assume for now that the state's endowment in the second period, conditional on avoiding defeat, is deterministic. Specifically, assume that the realization of z equals zero with probability one.

Under these assumptions, the state has only to choose in the first period an amount of war spending g to maximize

$$U_A = u(1-g) + \rho u(1), \tag{6}$$

where ρ is given by equation (4). Given that $u(c)$ is a concave function of c, U_A is a concave function of g. The Kuhn-Tucker first-order conditions for this problem are

$$\frac{\partial U_A}{\partial g} = 0 \quad \text{with} \quad 0 \leq g \leq \frac{1-\gamma}{\theta}, \text{ or} \tag{7a}$$

$$\frac{\partial U_A}{\partial g} < 0 \quad \text{with} \quad g = 0, \text{ or} \tag{7b}$$

$$\frac{\partial U_A}{\partial g} > 0 \quad \text{with} \quad g = \frac{1-\gamma}{\theta}, \tag{7c}$$

$$\text{where} \quad \frac{\partial U_A}{\partial g} = -u'(1-g) + \theta u(1). \tag{8}$$

Equation (8) shows that the marginal cost of war spending is the marginal utility of the consumption foregone in the first period and that, with no borrowing, the marginal benefit of war spending is simply the increase in the probability of avoiding defeat multiplied by the utility of consumption in the second period conditional on avoiding defeat. Given equation (8), conditions (7a), (7b), and (7c) imply that the state chooses positive war spending if and only if with zero war spending the marginal cost of war spending would be less than the marginal benefit. If this condition is satisfied, then the state increases war spending either to the amount $\frac{1-\gamma}{\theta}$ or to the amount at which the marginal cost equals the marginal benefit, whichever amount is smaller.

Conditions (7a), (7b), and (7c) and equation (8) also reveal that the chosen amount of war spending depends critically on the value of θ, which measures the marginal effect of war spending in increasing the probability of avoiding defeat. At one extreme, if θ is sufficiently small – specifically, if θ is not larger than $u'(1)/u(1)$ – then the state chooses zero war spending. In this case, ρ equals γ, which measures the effectiveness of passive resistance. At the other extreme, if θ is sufficiently large, then the state chooses sufficient war spending to avoid defeat with probability one – that is, to make ρ equal to unity. Of course, given the constraint of no borrowing, the state can make this choice only if $\frac{1-\gamma}{\theta}$ is smaller than the first-period endowment of one unit. Finally, for intermediate values of θ, the state chooses a positive amount of war spending such that ρ is larger than γ but less than unity. In this range, the chosen value of g and the resulting value of ρ are positively related to θ.

3. Consumption Smoothing

To explore the usefulness of borrowing for smoothing expected consumption intertemporally, assume now that the state can issue war debt according to the supply condition for loans given by equation (3), but continue to assume that the state's endowment in the second period, conditional on avoiding defeat, is deterministic. In this case, the state's problem is to choose in the first period an amount of war spending g and an amount of borrowing b to maximize

$$U_B = u(1 + b - g) + \rho u[1 - S(0; b)], \tag{9}$$

$$\text{where} \quad S(0; b) = b\frac{1 + r}{\rho} \tag{10}$$

and ρ is given by equation (4). Equation (10), which is derived from equation (3), says that, to compensate for the fact that defeat in the war would result in debt repudiation, lenders require the state to pay an interest rate such that one plus the interest rate equals one plus the required expected rate of return divided by the probability of avoiding defeat. Given that $u(c)$ is a concave function of c, it is easy to show that U_B is a strictly concave function of b and g.

The Kuhn-Tucker first-order conditions for this problem are

$$\frac{\partial U_B}{\partial g} = 0 \quad \text{with} \quad 0 \leq g \leq \frac{1 - \gamma}{\theta}, \quad \text{or} \tag{11a}$$

$$\frac{\partial U_B}{\partial g} < 0 \quad \text{with} \quad g = 0, \quad \text{or} \tag{11b}$$

$$\frac{\partial U_B}{\partial g} > 0 \quad \text{with} \quad g = \frac{1-\gamma}{\theta}, \tag{11c}$$

$$\text{where } \frac{\partial U_B}{\partial g} = -u'(1+b-g) + \theta[u(1-b\frac{1+r}{\rho}) + b\frac{1+r}{\rho}u'(1-b\frac{1+r}{\rho})] \tag{12}$$

$$\text{and } \frac{\partial U_B}{\partial b} = 0 \quad \text{with} \quad b \geq 0 \quad \text{or} \tag{13a}$$

$$\frac{\partial U_B}{\partial b} < 0 \quad \text{with} \quad b = 0, \tag{13b}$$

$$\text{where } \frac{\partial U_B}{\partial b} = u'(1+b-g) - (1+r)u'(1-b\frac{1+r}{\rho}). \tag{14}$$

If b were equal to zero, then U_B would be identical to U_A and $\partial U_B/\partial g$ would be identical to $\partial U_A/\partial g$. Moreover, conditions (13a) and (13b) and equation (14) imply that, if g were equal to zero, then the state would choose b equal to zero. In other words, borrowing is useful for smoothing expected consumption intertemporally only if positive war spending is reducing first-period consumption. Accordingly, if θ is sufficiently small that with no borrowing the state would choose zero war spending, then the state continues to choose zero war spending, and also chooses zero borrowing, even if it is allowed to borrow. In other words, the ability to issue war debt cannot cause a policy of passive resistance, if it is optimal, to become suboptimal.

If $\partial U_B/\partial g$ and $\partial U_A/\partial g$, evaluated at g and b equal to zero, are positive, then the state again increases war spending either to the amount $\frac{1-\gamma}{\theta}$ or to the amount at which $\partial U_B/\partial g$ equals zero – that is, the amount at which the marginal cost of war spending equals the marginal benefit – whichever amount is smaller. With a positive and nonnegligible amount of g, as long as the required expected rate of return is small, $\partial U_B/\partial b$, evaluated at b equal to zero, would be positive. Accordingly, if positive war spending is optimal, then the state chooses a positive amount of borrowing and the ability to issue war debt increases total expected utility. In fact, conditions (13a) and (13b) and equation (14) imply that, if the required expected rate of return is small, then the state chooses an amount of borrowing such that consumption in the first period and consumption in second period, conditional on avoiding defeat, are approximately equal. This amount of borrowing would be positive, but less than g, if g is positive and nonnegligible.

The choice of positive amount of borrowing affects both the marginal cost and the marginal benefit of war spending. Positive borrowing decreases the marginal cost of war spending by increasing consumption in the first period. Positive borrowing decreases the marginal benefit of war spending by decreasing consumption in the second period, but it also increases the marginal benefit by introducing the effect of war spending in increasing the probability of avoiding defeat and, thereby, reducing the interest rate that lenders charge. It is easy to show that, for a given amount of g, the net effect of an increase in b is to increase $\partial U_B/\partial g$. Specifically, if with no borrowing the state would choose an amount of war spending such that ρ is between γ and unity, then the ability to borrow causes the state to increase war spending and its probability of avoiding defeat in the war. Moreover, with borrowing allowed, the state can choose the amount of war spending that makes ρ equal to unity even if this amount of war spending exceeds the first-period resource endowment – that is, even if θ is smaller than $1-\gamma$.

Consider the specific utility function

$$u(c) = c^{1-\beta}, \quad 0 < \beta < 1. \tag{15}$$

In this utility function, the parameter β is the absolute value of the elasticity of marginal utility, and is sometimes called the coefficient of relative risk aversion. Also, assume for convenience that the required expected rate of return r equals zero. Given these assumptions, the expressions for $\partial U_B/\partial g$ and $\partial U_B/\partial b$ in equations (12) and (14) become

$$\frac{\partial U_B}{\partial g} = -\frac{1-\beta}{(1+b-g)^{\beta}} + \frac{\theta(1-\beta\frac{b}{\rho})}{(1-\frac{b}{\rho})^{\beta}}$$

$$\text{and} \quad \frac{\partial U_B}{\partial b} = \frac{1-\beta}{(1+b-g)^{\beta}} - \frac{1-\beta}{(1-\frac{b}{\rho})^{\beta}}.$$

Under these specifications, conditions (11a), (11b), (11c), (13a), and (13b) imply that the state chooses an amount of war spending, denoted by g^*, such that

$$g^* = \begin{cases} \frac{1+\gamma}{1-\theta}(1-\frac{1-\beta}{\theta}) & \text{if } 1-\beta < \theta < 1-\beta\frac{1+\gamma}{2} \\ 0 & \text{if } \theta \leq 1-\beta \\ \frac{1-\gamma}{\theta} & \text{if } \theta \geq 1-\beta\frac{1+\gamma}{2}, \end{cases} \tag{16}$$

and an amount of borrowing, denoted by b^*, such that

$$b^* = \frac{\rho}{1+\rho}g^*. \tag{17}$$

Equations (16) and (17) show exactly how the chosen amount of war spending and borrowing depend on the parameters θ and γ and on the nature of the utility function, here summarized by the single parameter β. Specifically, equation (16) says that the state chooses zero war spending if and only if θ is not larger than $1-\beta$. The comparison of

θ and $1-\beta$ is relevant because θ measures the marginal benefit of war spending in increasing ρ and $1-\beta$ measures the marginal utility of the consumption foregone by choosing positive war spending. Equation (16) also says that the state chooses sufficient war spending to make ρ equal to unity if and only if θ is larger than a function that is decreasing in β and γ. Finally, between these extreme cases, war spending is an increasing function of γ, β, and θ. A higher γ induces more war spending because higher γ implies higher ρ and, hence, a lower interest rate and more borrowing.

Equation (17) says that borrowing is positive if and only if war spending is positive. Moreover, if war spending is positive, then borrowing equals a fraction of war spending. This fraction increases with the amount of war spending. It approaches $\frac{\gamma}{1+\gamma}$ as g^* approaches zero and it approaches $1/2$ as g^* approaches $\frac{1-\gamma}{\theta}$. A larger amount of war spending implies more borrowing both because more war spending reduces first-period consumption and because, by raising the probability of avoiding defeat and thereby lowering the required interest rate, more war spending increases second-period consumption.

4. Risk Shifting

Now, let us consider the state's problem in the original model in which its resource endowment in the second period is stochastic and in which it can issue debt the servicing of which is contingent on the realization of z, the material consequences of the war. In this problem, the state has to choose in the first period an amount of war spending g, an amount of borrowing b, and a debt-servicing schedule $S(z;b)$ that relates the amount of debt servicing in the second period to the realization of z. The state's objective now is to maximize

$$U_C = u(1+b-g) + \rho \sum_z p(z) u[1 - z - S(z;b)], \tag{18}$$

where the debt-servicing schedule $S(z;b)$ satisfies equation (3), and ρ is given by equation (4). We assume, for simplicity, that the constraint $S(z;b) \geq 0$ is not binding for any realization of z. This assumption implies that the amount of war debt issued to smooth consumption intertemporally is sufficiently large that the state would be able to shift the risk associated with the realization of z optimally to its lenders while still providing nonnegative debt servicing for all realizations of z. [See Grossman and Van Huyck (1988) for a complementary analysis that allows for the analogous constraint to be binding.]

To analyze the state's choice problem, form the Lagrangian expression

$$L = u(1+b-g) + \rho \sum_z p(z)u[1-z-S(z;b)] - \lambda[(1+r)b - \rho \sum_z p(z)S(z;b)],$$

where λ is a Lagrangian multiplier. The Kuhn-Tucker first-order conditions for the maximization of L are

$$\frac{\partial L}{\partial g} = 0 \quad \text{with} \quad 0 \leq g \leq \frac{1-\gamma}{\theta}, \quad \text{or} \tag{19a}$$

$$\frac{\partial L}{\partial g} < 0 \quad \text{with} \quad g = 0, \quad \text{or} \tag{19b}$$

$$\frac{\partial L}{\partial g} > 0 \quad \text{with} \quad g = \frac{1-\gamma}{\theta}, \tag{19c}$$

$$\text{where} \quad \frac{\partial L}{\partial g} = -u'(1+b-g) + \theta \sum_z p(z)u[1-z-S(z;b)] + \lambda\theta \sum_z p(z)S(z;b), \tag{20}$$

$$\text{and} \quad \frac{\partial L}{\partial b} = 0 \quad \text{with} \quad b \geq 0, \quad \text{or} \tag{21a}$$

$$\frac{\partial L}{\partial b} < 0 \quad \text{with} \quad b = 0, \tag{21b}$$

where $$\frac{\partial L}{\partial b} = u'(1+b-g) - \rho \sum_z p(z) u'[1-z-S(z;b)] \frac{\partial S(z;b)}{\partial b} - \lambda[1+r-\rho \sum_z p(z) \frac{\partial S(z;b)}{\partial b}], \tag{22}$$

$$\frac{\partial L}{\partial [S(z;b)]} = -\rho p(z) u'[1-z-S(z;b)] + \lambda \rho p(z) = 0 \text{ for each possible realization of } z, \tag{23}$$

$$\text{and } (1+r)b = \rho \sum_z p(z) S(z;b). \tag{24}$$

The set of equations given by (23) implies

$$u'[1-z-S(z;b)] = \lambda \text{ for each possible realization of } z. \tag{25}$$

Accordingly, the state chooses a debt-servicing schedule that yields the same value of second-period consumption, $1 - z - S(z;b)$, for each possible realization of z. This choice implies that the state shifts to its lenders all risk associated with the material consequences of the war. [This result, of course, depends on the assumption that the lenders require a constant expected rate of return, which is independent of the chosen debt-servicing schedule. The fact that all risk is shifted to lenders also means that the state has no reason to save for precautionary reasons.]

In order for $1 - z - S(z;b)$ to the independent of z, the debt-servicing schedule $S(z;b)$ must have the form $K - z$. Substituting $K - z$ for $S(z;b)$ in equation (24) reveals that K must equal $b\frac{1+r}{\rho}$. Thus, equations (23) and (24) together imply

$$S(z;b) = b\frac{1+r}{\rho} - z \tag{26}$$

$$\text{and } c_2 = 1 - b\frac{1+r}{\rho} \text{ for all possible realizations of } z. \tag{27}$$

Equation (26) implies

$$\frac{\partial S(z;b)}{\partial b} = \frac{1+r}{\rho} \text{ for all possible realizations of } z. \tag{28}$$

Substituting equations (25) and (26) into equation (20), we see that the expression for $\partial L/\partial g$ is the same as the expression for $\partial U_B/\partial g$ in equation (13). Moreover, substituting equations (26) and (28) into equation (22), we see that the expression for $\partial L/\partial b$ is the same as the expression for $\partial U_B/\partial b$ in equation (15). Thus, given an efficient contingent servicing schedule that shifts all risk associated with the material consequences of war from the state to its lenders, the introduction of this risk does not alter the amounts of war spending and borrowing that the state chooses. [The assumption that the constraint $S(z;b) \geq 0$ is not binding means that this amount of borrowing is large enough that equation (26) yields a nonnegative value of $S(z;b)$ for all possible realizations of z. Otherwise, as discussed in Grossman and Van Huyck (1988), efficient risk shifting would require additional borrowing for the purpose of accumulating liquid reserves.]

5. Summary

This paper has analyzed the financial and war-spending policies of a state that faces a war in which defeat would result in the loss of sovereign power and in which the material consequences, conditional on avoiding defeat, are stochastic. The analysis takes explicit account of the historical experiences of lenders, who face debt repudiation if the state to whom they have lent is defeated and who also face partial default if the material consequences of the war are unfavorable for the debtor state, even if it avoids defeat. The experience of partial default, in particular, suggests that, whereas as in traditional public finance the state uses the issuance of war debt to smooth expected consumption intertemporally in response to temporary war spending, the state also uses contingent servicing of war debt to insure realized consumption against the risk associated with the material consequences of the war.

Within this framework, the analysis derives the state's optimal choices of the amount of war spending and resulting probability of avoiding defeat, of the amount of borrowing, and of the contingent servicing schedule to attach to its debts. This derivation allows for the lenders' reaction to the risks of repudiation and default and, specifically, for the interaction between the effect of the ability to borrow on the amount of war spending and the effect of war spending in increasing the probability of avoiding defeat and, consequently, in decreasing the cost of borrowing.

The state's choice of the amount of war spending involves weighing the cost of war spending in reducing wartime consumption against the benefit of war spending in increasing the probability of avoiding defeat. The effect of war spending on the probability of avoiding defeat is critical both because defeat precludes the enjoyment of future consumption and because the cost of borrowing is negatively related to the probability of avoiding defeat. The analysis shows that whether positive war spending or passive resistance is the optimal policy is independent of the state's ability to borrow. But, the analysis also shows how, if positive war spending is optimal, then the ability to borrow causes the state to choose a larger amount of war spending and a higher probability of avoiding defeat.

Finally, the analysis shows how the two functions of war debt – intertemporal consumption smoothing and risk shifting – are complementary. In particular, the state issues war debt in order to smooth consumption intertemporally and attaches a contingent servicing schedule to this debt in order to shift to its lenders the risks associated with the material consequences of the war. In the simplest case, the amount of war debt issued to smooth consumption intertemporally is sufficient to facilitate efficient risk shifting without violating the nonnegativity constraint on debt servicing, the efficient contingent servicing schedule shifts all risk associated with the material consequences of war from the state to its lenders, and both the optimal amount of war spending and the optimal amount of war debt are invariant with respect to the existence of this risk.

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