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ECONOMIC DEVELOPMENT, URBAN UNDEREMPLOYMENT, AND INCOME INEQUALITY

James E. Rauch

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ABSTRACT

The evolution of inequality in permanent income is investigated during the course of a less developed country's transformation from a primarily agricultural to a primarily urban-industrial economy. The source of inequality is market luck in obtaining employment in the protected urban *formal sector* versus employment in the unprotected urban *informal sector." It is shown that with development the log variance measure of inequality in this country tends to follow an "inverted U": it rises when urbanization is low and consequent pressure on the land keeps rural incomes low, making agents willing to incur high risks of "underemployment" in the urban informal sector, and eventually falls after urbanization and consequently rural incomes has increased sufficiently to allow agents to make better than even bets in the industrial sector. These results in combination with new empirical evidence suggest that rather than being an unimportant artifact of the design of inequality indices, inverted-U behavior of inequality may be driven by the important social phenomenon of mass urban underemployment.

James E. Rauch
Department of Economics
University of California,
San Diego
La Jolla, CA 92093
and NBER

1. Introduction

Economic development invariably involves a transfer of labor from the agricultural to the nonagricultural sector, a process that for the purposes of this paper will be identified with urbanization. This transfer appears to take place monotonically over time. A check of the share of economically active population engaged in agriculture as reported in the Food and Agriculture Organization Production Yearbook showed a decline in every year for which data was available for all except one (Ireland) of 121 countries checked. 1 Kuznets (1955) saw this labor transfer as having important consequences for the size distribution of income over time. He hypothesized that income inequality will increase during the early stages of development as population shifts from the agricultural sector where he believed incomes are more equally distributed to the urban sector where he believed incomes are less equally distributed. During the later stages of development this force for inequality is more than offset, he supposed, by growing equality of income distribution within the urban sector due to better adaptation of the children of rural-urban migrants to city economic life and growing political power of urban lower-income groups to effect "protective and supportive legislation" (p. 17). A graph of income inequality against the urbanization rate or per capita income would therefore have the shape of an inverted U.

An important innovation in the formulation of Kuznets's hypothesis was made by Robinson (1976) who showed that, under the assumption of a constant difference between mean incomes in the rural and urban sectors, the inverted-U result required neither that urban income be more unequally distributed than rural income nor that urban income inequality decline as the urban share of the population increased. Specifically he showed that, if the within-sector income distributions remain constant, then overall inequality as

If am referring to data for the years 1960, 1970, and 1975—1984. In 1985 the FAO appears to have revised its definitions since many countries show a dramatic increase in the agricultural share of the economically active population. The 121 countries are the market economies covered in the study of Summers and Heston (1988).

measured by the variance of the logarithms of income is a quadratic function of the share of the population in the urban sector, and achieves its maximum (under reasonable empirical assumptions) for a share between zero and one. The same result was established by Knight (1976) and independently by Fields (1979) for the Gini coefficient for the case where the difference in incomes between the two sectors is the *only* source of inequality.²

Their results led both Knight and Fields to question the importance of the inverted-U path of income inequality. Knight asks (pp. 172-173), "Should we be concerned about an increase in measured inequality [during the upswing of the inverted U] if it simply reflects a relative transfer of people from low- to high-income groups or sectors? No-one is made worse off--total income is allowed to increase--and some of the poor are made better off." Before we accept the process described in the quotation as pertaining to real-world economic development, however, we must look deeper into the situation described by Robinson, Knight, and Fields (hereafter called RKF).

The RKF assumptions are based on the numerous studies of LDCs (see, for example, Tidrick 1975) that have shown that wages for comparably skilled labor tend to be higher in "formal" urban jobs than in rural jobs by a margin that is too large to be attributed to measurement error (such as failure to correct for lower cost of living in rural areas). These studies formed the basis for new models of LDC labor markets, beginning with Todaro (1969) and Harris and Todaro (1970), that are themselves absent from the RKF framework. Like the RKF economy, the Harris-Todaro economy consists of two sectors, which they label agriculture and urban manufacturing. Harris and Todaro argued that the wage in the urban manufacturing sector is set above the competitive equilibrium

²Since reliable time series on income inequality spanning several decades exist for extremely few countries, the Kuznets hypothesis has been empirically tested using cross-country data on the assumption that cross-country and intertemporal "Kuznets curves" are identical. In the most recent study of which I am aware, Papanek and Kim (1986) find a statistically significant quadratic relationship between the Gini coefficient measure of income inequality and the log of per capita GNP using 145 observations for 83 countries. Inclusion of other economic, social, and regional explanatory variables did not weaken this relationship.

wage by institutional forces, which may in practice include trade unions, governments (for example through minimum wage legislation enforced only in urban areas), or both acting together. The key insight of Todaro (1969), used again in a different form in Harris and Todaro (1970), is that unlike in the RKF economy the equilibrium involving the minimum urban wage is not one where there is full employment, with all urban manufacturing jobs filled and the remainder of the labor force employed in agriculture at a wage lower than the original competitive equilibrium wage. Instead, this equilibrium involves an excess supply of workers in the urban sector, with the resulting unemployment or underemployment acting to equate expected utility between the rural and urban sectors. It is this equation that allows labor market equilibrium to exist in the presence of rural-urban wage differentials.

Surprisingly, the way this equation is formulated in Harris and Todaro (1970) and in most of the subsequent theoretical work based on their paper implies that there is no wage inequality within the urban sector or between the urban and rural sectors. It is assumed that in equilibrium the rural wage is equated to the expected urban wage, where the latter equals the urban minimum wage times the ratio of urban employment to the total urban labor force. This assumption can be justified by the following characterization of the urban labor market. Suppose we interpret the static Harris-Todaro equilibrium as a steady state persisting indefinitely in time. Suppose further that the labor market reopens in every period, at the beginning of which workers are drawn at random from the total urban labor pool for the given number of urban manufacturing jobs. Workers not drawn are unemployed during that period. If the draws are independent then by the Law of Large Numbers workers are certain to earn on average the expected wage described above, which should therefore be equal to the certain rural wage in equilibrium. Obviously there is no inequality in lifetime wage income in this model.

More recent research into the nature of LDC labor markets has shown that upon arriving in the city some rural migrants immediately obtain jobs in the protected labor

market, while others wind up in what has become known as the "informal sector", meaning small businesses and self-employment that escape government and union intervention.

The latter group earns less than comparably skilled workers in the former group (Banerjee 1983). While there is some mobility over time from the informal to the formal (protected) sector, most informal sector workers view their situation as permanent (Sethuraman 1981). This more recent research suggests that it is the risk of long-term underemployment in the informal sector that equates expected utility between the urban and rural sectors and thus allows for labor market equilibrium. The risk is substantial, since according to the studies surveyed by Sethuraman (Appendix Table 4, p. 213), the share of the urban labor force in LDC cities engaged in the informal sector ranges from 19 to 69 percent, with a mean of roughly 41 percent. If we modify the Harris-Todaro model in this way, we wind up with three classes of wage earners: rural (agricultural) workers, urban formal sector workers, and urban informal sector workers. We will thus address the evolution of income inequality by examining how the relative sizes and incomes of these three groups change during the course of economic development.

It turns out that the log variance measure of inequality in this economy tends to follow an inverted U: it rises when urbanization is low and consequent pressure on the land keeps rural incomes low, making agents willing to incur high risks of underemployment in the urban informal sector, and eventually falls after urbanization and consequently rural incomes has increased sufficiently to allow agents to make better than even bets in the industrial sector. This inverted U in inequality is associated with another inverted U in the share of the informal sector in the total labor force, the upswing of which is driven by the "Todaro paradox" that an increase in the number of urban manufacturing jobs may increase rather than decrease underemployment.

These results are derived in section 2 of the paper, which is divided into three subsections. Section 2.1 introduces a theoretical model. This model is used in section 2.2 to analyze the evolution of underemployment and income inequality during the course of

urbanization. In section 2.3 the model's predictions concerning the informal sector shares of the urban labor force and total labor force are checked against existing data. Section 3 compares the behavior of the Gini coefficient measure of inequality in the model of section 2 with its behavior in the Knight-Fields model. Because the underlying causes for the inverted U in inequality are so different in the two models, they generate very different predictions for the point in the urbanization process at which inequality can be expected to begin to lessen. Section 4 concludes by discussing how this difference in the causes of the inverted U leads to contrasting welfare implications of the evolution of inequality, and how the model of section 2 relates to a popular model of income distribution that does not use the dual economy framework.

2. Urbanization, underemployment and inequality in a Harris-Todaro-type model

2.1 The model

There is no explicit dynamic structure in the RKF papers. Variables are not time dated, and the shift of the population from the low-income rural to the high-income urban sector is an exogenous process. This approach has the virtue of avoiding two distracting issues. First, we are interested in inequality in permanent income. If income varies over an individual's life cycle, an explicit dynamic structure may make computation of inequality in permanent income very difficult. Second, economic growth is required to increase the proportion of high income urban sector jobs in the economy. But an explicit growth model requires that one model savings behavior, preferably by optimizing agents. Analyzing the change in income inequality in a model with more than one sector is already complicated without handling these issues, and in sections 2.2 and 3 I will follow the RKF

³At the outset of his seminal paper on economic growth and income inequality, Kuznets (1955, p. 1) emphasized that to correctly measure income inequality across family units we should "segregate the units whose main income earners are either still in the learning or already in the retired stages of their life cycle—to avoid complicating the picture by including incomes not associated with full—time, full—fledged participation in economic activity."

lead by avoiding them. Nevertheless, I find it useful to sketch out a dynamic structure in this subsection that resolves these issues and is consistent with the RKF framework, because that framework can then be seen as a shorthand for a more comprehensive dynamic general equilibrium model.

I begin with the Diamond (1965) overlapping generations model. In this model agents live for two periods. In their youth they earn wages, consume some of these wages and save the rest. In their old age they retire and consume their savings. If all agents have access to the same competitive asset market then any two agents who earn the same wage when young can be said to have the same permanent income. Intragenerational inequality in permanent income can then be measured simply by intragenerational inequality in wage income. More formally, assume that each individual inelastically supplies one unit of labor services when young and none when old. We can write her budget constraints as follows, suppressing the individual-level subscript to save notation:

$$\begin{aligned} \mathbf{p}_{\mathrm{at}}\mathbf{c}_{\mathrm{at}}^{\mathrm{y}}+\mathbf{c}_{\mathrm{mt}}^{\mathrm{y}}+\mathbf{s}_{\mathrm{t}}&=\mathbf{w}_{\mathrm{t}}\\ \mathbf{p}_{\mathrm{at+1}}\mathbf{c}_{\mathrm{at+1}}^{\mathrm{o}}+\mathbf{c}_{\mathrm{mt+1}}^{\mathrm{o}}&=(1+\mathbf{r}_{\mathrm{t+1}})\mathbf{s}_{\mathrm{t}}, \end{aligned}$$

where the subscript t indicates time period, c_i^y , c_i^o represent the individual's consumption of good i when young and when old, respectively, s represents her savings, w is her wage rate, r_{t+1} is the (perfectly foreseen) interest rate, and p_a is the relative price of agricultural goods in terms of manufactures, the numeraire. Lifetime utility maximization subject to these budget constraints yields an indirect utility function $v(w_t, r_{t+1}, p_{at}, p_{at+1})$, from which a savings function $s(w_t, r_{t+1}, p_{at}, p_{at+1})$ can be derived. It is assumed that individuals are risk averse so that $\frac{\partial^2 v}{\partial w_t} = 0$.

We need our overlapping generations model to generate a monotonic transfer of labor out of the agricultural sector over time. A convenient specification that may accomplish this is the Ricardo-Viner or specific-factors model. In this model a small open economy, whose terms of trade between agriculture and manufacturing are fixed by international markets ($p_{at} = \bar{p}_a$), produces agricultural goods using land and labor and

manufactured goods using capital and labor. If land is fixed in supply while the supply of capital grows over time through saving, the manufacturing sector will expand relative to the agricultural sector and draw labor out of the latter. This specification has the advantage of reproducing over time the typical "life cycle" of a country, which is to start out as a poor exporter of agricultural products and raw materials and end up as a rich exporter of manufactured goods. Another convenient way to generate a monotonic transfer of labor out of the agricultural sector is to rely on deterioration over time in the terms of trade for the agricultural sector due to Engel's law. However, the fact that there was no trend in the world market terms of trade between agriculture and manufactures during the two decades 1960–1979 (World Bank 1986, p. 7) did not eliminate the monotonic transfer of the world's labor from the agricultural sector during that period.

As the number of agricultural workers shrinks, the land-labor ratio increases and the agricultural wage rises. Eventually the agricultural wage may catch up to the institutionally determined urban minimum wage, which then ceases to be binding. I will call the situation where the urban minimum wage is binding the "LDC regime" and the situation where the urban minimum wage is not binding the "mature regime".

More formally, assume that all production takes place under conditions of constant returns to scale, so that all variables can be written in per capita terms without loss of generality. The per capita production functions for formal sector manufactures and agricultural goods are then given respectively by

$$Q_{mt} = F_m(K_t, N_{mt}), Q_{at} = F_a(L, N_{at}),$$

Several of the countries covered by the FAO statistics cited in footnote 1 have experienced close to zero or even negative per capita income growth. Under the Ricardo-Viner model, this is consistent with a decreasing share of the labor force in the agricultural sector only if there is capital accumulation and population growth, where the former does not keep up with the latter so that the capital stock per capita declines but land per capita declines faster. In the model below I assume a constant population for simplicity.

⁵An agricultural wage that catches up with the urban minimum wage is consistent with the behavior of the informal sector share of the urban labor force found in the data, as described in section 2.3 below.

where Q_i is per capita output of good i, K is the per capita stock of capital, L is the per capita stock of land, and N_i is the share of the labor force in sector i. We can then define "intensive" production functions

$$q_{mt} = Q_{mt}/N_{mt} = f_m(K_t/N_{mt}), \ q_{at} = Q_{at}/N_{at} = f_a(L/N_{at}),$$
 where $f_i'>0$, $f_i''<0$, and the Inada conditions are satisfied so that f_i' approaches zero (infinity) as its argument approaches infinity (zero). Competition in factor markets insures that in every period factors are paid their marginal revenue products. Under the LDC regime a unique capital-labor ratio in formal sector manufacturing \bar{k} is determined by $f_m - \bar{k} f_m' = \bar{w}_m$, where \bar{w}_m is the urban minimum wage. It follows that given the capital stock K_t the formal sector share of the labor force N_{mt} is determined, and that the marginal product of capital in terms of manufactures is fixed at $f_m'(\bar{k}) = \bar{r}$. Under the mature regime the informal sector ceases to exist so that $N_{at} = 1 - N_{mt}$. It follows that given K_t the allocation of the labor force N_{mt} is determined by equating the marginal

How should we model production in the informal sector? All studies agree that one of the most important distinctions between formal and informal sector production is the wide difference in capital intensity of production. The studies of LDC cities cited by Sethuraman (1981, p. 36) show the average informal sector capital-labor ratio to range

value products of labor in agriculture and manufacturing. Thus under either regime we

need only know that K_t increases monotonically to know that N_{mt} increases

monotonically.6

In the absence of international capital mobility, the addition to the capital stock in any period is determined entirely by the savings behavior of the country's young residents in the previous period. In general, the young have the option of investing in either capital or land. This means that the time path of the price of land plays a crucial role in the allocation of savings between capital formation and land holding. For example, if the price of land is sufficiently high land holding could absorb all savings, leaving capital formation at zero. The dynamics in the neighborhood of the steady state of a model corresponding to our mature regime have been worked out by Eaton (1987). Unfortunately, we need to be able to analyze the global dynamics of our model, and in particular to derive the conditions under which K will increase monotonically starting from any initial value. Sufficient conditions are worked out in Rauch (1990).

between 5 and 50 percent of the formal sector capital-labor ratio, with the modal value being in the neighborhood of 15 percent. Part of this differential is undoubtedly due to the lower wage rate paid in the informal sector, but part is also due to the much higher cost of capital there. This difference in cost of credit results from the fact that commercial banks are themselves part of the formal sector, so that informal sector firms do not have access to them. For simplicity, then, we can assume that the informal sector employs no capital, in which case if the informal sector produces manufactures the wage earned by an informal sector worker under competition equals the reciprocal of the labor input coefficient that completely describes the informal sector technology. The image the reader should bear in mind is of the lucky formal sector worker who gets a job in a modern textile factory regulated by the government and a union contract, and the unlucky informal sector worker who makes the same product using traditional handicraft methods in an unregulated "sweatshop".

Denote by \underline{w} the wage determined by competition and the informal sector technology, and by $w_{at}(N_{at})$ the wage determined by competitive factor pricing in agriculture, where $w'_{at} < 0$ since increased pressure on the land lowers the marginal product of labor. By assumption we have $\underline{w} < \overline{w}_m$, and in migration equilibrium it must also be the case that $w_{at} \geq \underline{w}$, since anyone can come to the city and set up an informal sector shop without needing any capital. To know more about the relationship between w_{at} , \overline{w}_m , and \underline{w} than just these two inequalities we need to specify the expected utility equation between workers in the rural and urban sectors that is at the core of any Harris-Todaro-type model.8 This is derived as follows. I assume that when the young are

where everyone strictly prefers to work in the agricultural sector. In order to avoid the

If have constructed a much more detailed model of informal sector wage determination in Rauch (forthcoming 1991). This paper also contains extensive references to other theoretical work on the informal sector that are omitted here for the sake of brevity.
Suppose that at time zero a country is completely "undeveloped": it has no modern sector, so that $N_{m0} = 0$. In this case equation of expected utility between urban and rural workers at time zero will not be possible if $w_a(1) > \underline{w}$: the economy is at a corner solution

born in period t they can choose to become either rural or urban workers. If they make the latter choice they have a chance of landing a modern manufacturing job equal to $N_{\rm mt}/(1-N_{\rm at})$, the ratio of such jobs available to the entire urban labor force. As we saw above, $N_{\rm mt}$ is determined by savings decisions made in the preceding period, while $N_{\rm at}$ is known by perfect foresight. The probability that an urban worker will wind up underemployed in the informal sector is just $1-[N_{\rm mt}/(1-N_{\rm at})]=(1-N_{\rm at}-N_{\rm mt})/(1-N_{\rm at})$. The young are therefore indifferent between becoming rural or urban workers when

$$\mathbf{v}(\mathbf{w}_{at},\mathbf{r}_{t+1}) = \begin{bmatrix} \mathbf{N}_{mt} \\ 1 - \mathbf{N}_{at} \end{bmatrix} \mathbf{v}(\overline{\mathbf{w}}_{m},\mathbf{r}_{t+1}) + \begin{bmatrix} 1 - \mathbf{N}_{at} - \mathbf{N}_{mt} \\ 1 - \mathbf{N}_{at} \end{bmatrix} \mathbf{v}(\underline{\mathbf{w}},\mathbf{r}_{t+1}),$$

where we have suppressed the constant \bar{p}_a in the indirect utility function v. Note that this equation is relevant only in the LDC regime, since in the mature regime we have full employment $(N_{at}+N_{mt}=1)$ with $w_{at}=w_t \geq \overline{w}_m$. Moreover, in the LDC regime r_{t+1} is fixed at \bar{r} , so that all variables in this equation are contemporaneous. We can therefore drop all time subscripts without loss of information, and replace this equation with

$$v_a = [N_m/(1 - N_a)]\overline{v} + [(1 - N_a - N_m)/(1 - N_a)]\underline{v},$$
 (1)

where $v_a = v(w_a, \overline{r})$, $\overline{v} = v(\overline{w_m}, \overline{r})$, and $\underline{v} = v(\underline{w}, \overline{r})$. Taking the monotonic increase in N_m as given, we have regained the simplicity of the RKF framework, behind which lies the implicit dynamic structure presented in this subsection.

2.2 The evolution of underemployment and inequality

We begin by showing that as formal sector employment expands with economic growth in the model of section 2.1, the share of the labor force in agriculture must decline monotonically. Rewrite equation (1) as

$$(1 - N_a)(v_a - \underline{v}) = N_m(\overline{v} - \underline{v}). \tag{1'}$$

We know from our assumption that $w_a(1) \leq \underline{w}$ (see footnote 8) that for $N_m = 0$, N_a is

mathematical inconvenience of this case we assume throughout that $w_a(1) \leq \underline{w}$, which when the inequality is strict simply amounts to the historically accurate assumption that there is some manufacturing activity even in an economy without any modern industry.

determined by $w_a(N_a) = \underline{w}$. As N_m increases, N_a must decrease since the left-hand side of equation (1') is monotonically decreasing in N_a given that v_a is increasing in w_a . This proof implies, given the Inada conditions on f_a' , that for some $N_m < 1$ N_a will have declined sufficiently to yield $w_a = \overline{w}_m$. Calling this critical formal sector labor share \overline{N}_m , it follows from equation (1') that $N_m = \overline{N}_m$ implies $N_a = 1 - \overline{N}_m$.

Since the size of the informal sector is given by $1 - N_a - N_m$, we see that expansion of the formal or modern sector has conflicting effects on the underemployment rate. When an increase in the number of formal sector jobs causes a more than matching decrease in the agricultural labor force $(dN_a/dN_m < -1)$, the underemployment rate increases, while otherwise the underemployment rate decreases or remains constant. In our model, then, an increase in underemployment implies satisfaction of the "Todaro paradox" that an increase in employment in the urban formal sector increases urban underemployment.

PROPOSITION 1: The economy exhibits the Todaro paradox when the formal sector share of the labor force is sufficiently small, provided agricultural productivity is sufficiently high relative to informal sector productivity, and does not exhibit the Todaro paradox when the formal sector labor share is sufficiently large.

PROOF: From equation (1') we can derive

$$dN_a/dN_m = (\overline{v} - \underline{v})/[(1 - N_a)v_a'w_a' - (v_a - \underline{v})]. \tag{2}$$

Given that workers are risk averse, equation (1) implies that the average nonagricultural wage exceeds the agricultural wage for $0 < N_{\rm m} < \bar{N}_{\rm m}$. It can easily be shown that this difference between the mean urban wage and the rural wage first increases and then decreases as urbanization proceeds. Perhaps surprisingly, this is consistent with the finding of Chenery and Syrquin (1975, pp. 52-53) that the shortfall in primary sector labor productivity relative to the rest of the economy first increases and then decreases as per capita GNP grows.

¹⁰Strictly speaking we can only think of the share of the labor force in the informal sector $1-N_a-N_m$ as identically equal to "the underemployment rate" if $w_a(1) \geq w$, since only in this case will no one ever work in the informal sector except as a result of the inappropriately high (above competitive equilibrium) formal sector wage rate.

Consider the extreme formal sector labor shares $N_m = 0$ and $N_m = \bar{N}_m$. In the latter case $w_a = \overline{w}_m$ and $N_a = 1 - \bar{N}_m$, so $dN_a/dN_m > -1$ follows immediately since $v_a' > 0$ and $w_a' < 0$. In the former case $w_a = \underline{w}$, so the denominator on the right hand side of equation (2) reduces to $(1 - N_a)v_a'w_a'$. As $w_a(1)$ approaches \underline{w} from below, this negative expression tends to zero since N_a tends to one, so we need only choose $w_a(1)$ close enough to \underline{w} to insure that $dN_a/dN_m < -1$ for $N_m = 0$.

It turns out that under stronger assumptions we can extend Proposition 1 to show that dN_a/dN_m increases monotonically from a value below -1 to a value above -1, in which case the underemployment rate follows an inverted U that peaks when $dN_a/dN_m = -1$. PROPOSITION 2: Under sufficient conditions that are satisfied when (for example) the agricultural production function is Cobb-Douglas and the intertemporal utility function is log-linear, the underemployment rate follows an inverted U as the economy urbanizes. PROOF: See Appendix.

Equivalently, Proposition 2 states that the Todaro paradox that formal sector job creation increases rather than decreases underemployment holds most strongly at low levels of urbanization and gradually weakens and eventually ceases to hold as urbanization increases. The intuition is that as urbanization proceeds, pressure on the land decreases and agricultural incomes rise, making agents less willing to reject a certain rural income and risk urban underemployment in response to creation of new jobs in the urban formal sector.

We now turn to consideration of the evolution of inequality in permanent income. We measure inequality using the log variance, the measure of inequality used by Robinson (1976). Unlike the Gini coefficient (used in section 3 below), the log variance can be decomposed into the sum of within sector inequalities and between sector inequalities. It therefore allows us to analyze separately the evolution of the contributions to total inequality of inequality within the urban sector and inequality between the urban and rural sectors. Following Robinson, we denote the log mean and log variance of permanent

income in the rural and urban sectors by Y_a , Y_u and σ_a^2 , σ_u^2 , respectively. The overall log mean is given by $Y = N_a Y_a + (1 - N_a) Y_u$ and the overall log variance is given by $\sigma^2 = N_a \sigma_a^2 + (1 - N_a) \sigma_u^2 + N_a (Y_a - Y)^2 + (1 - N_a) (Y_u - Y)^2.$

We also know that

$$Y_a = \log w_a \text{ and } Y_u = [N_m/(1 - N_a)]\log \overline{w}_m + [(1 - N_a - N_m)/(1 - N_a)]\log \underline{w}.$$

Now suppose we make the assumption of intertemporally log-linear utility. It is easily shown that $v(w_t, r_{t+1})$ is now separable into $\log w_t$ plus a function of r_{t+1} (and \bar{p}_a). But we can then see from equation (1) that $Y_a = Y_u = Y!$ Since $\sigma_a^2 = 0$, it follows immediately that

$$\sigma^2 = (1 - N_a)\sigma_u^2, \tag{3}$$

so that the evolution of inequality in permanent income will result from the interaction of monotonically increasing urbanization with the non-monotonic path of inequality within the urban sector. By computing σ_{11}^{2} and substituting the result into equation (3) we obtain

$$\sigma^2 = N_m (\log \overline{w}_m - \log w_a)^2 + (1 - N_a - N_m)(\log w_a - \log \underline{w})^2. \tag{4}$$

It is shown in the Appendix that substitution for N_m in equation (4) using equation (1') yields, after some simplification,

$$\sigma^2 = (1 - N_a)(\log \overline{w}_m - \log w_a)(\log w_a - \log \underline{w}). \tag{5}$$

We can now prove

PROPOSITION 3 (Kuznets's inverted U): Under sufficient conditions that are satisfied when (for example) the agricultural production function is Cobb-Douglas, the log variance of permanent income given by equation (5) follows an inverted U as the economy urbanizes, attaining its peak for a level of urbanization $I - N_a$ greater than that for which the rural wage equals the geometric mean urban wage.

PROOF: It is easily shown that $(\log \overline{w}_m - \log w_a)(\log w_a - \log \underline{w})$ is maximized when $\log w_a = (\log \overline{w}_m + \log \underline{w})/2$ or $w_a = (\overline{w}_m \underline{w})^{1/2}$. It is then clear from equation (5) that any maximum for σ^2 must occur at a level of urbanization higher than that determined by $w_a = (\overline{w}_m \underline{w})^{1/2}$. In the Appendix sufficient conditions for only one such maximum to exist

are derived, and it is shown that these conditions are satisfied when the agricultural production function is Cobb-Douglas.

For this case, then, Kuznets's original inverted-U hypothesis is confirmed: permanent income inequality increases with development as urbanization shifts population from the low inequality rural sector ($\sigma_a^2 = 0$) to the high inequality urban sector ($\sigma_u^2 = (\log \overline{w}_m - \log w_a)(\log w_a - \log w_b)$), and eventually declines with development due to lessened inequality in the urban sector (occurring for some urbanization higher than that at which $w_a = (\overline{w}_m \underline{w})^{1/2}$). These results suggest that rather than being an unimportant artifact of the design of inequality indices, inverted-U behavior of inequality may be driven by the important social phenomenon of mass urban underemployment. The last proposition tightens the relationship between these two variables.

PROPOSITION 4: When the agricultural production function is Cobb-Douglas and the intertemporal utility function is log-linear, as the economy urbanizes the log variance in permanent income cannot decline until after the underemployment rate declines.

PROOF: It follows from Proposition 2 and equation (2) that the peak underemployment rate occurs when $(\overline{v} - \underline{v})/[(1 - N_a)v_a'w_a' - (v_a - \underline{v})] = -1$, or

 $\log \overline{w}_{m} - \log \underline{w} = \log w_{a} - \log \underline{w} - (1 - N_{a})w'_{a}/w_{a}$ since utility is intertemporally log-linear. It is easily shown that in the Cobb-Douglas case $w_{a}(N_{a}) = w_{a}(1)N_{a}^{-\phi}$, where ϕ is the share of land in the value of agricultural production. Substituting this fact into the last expression and simplifying yields

$$\log \overline{w}_{m} - \log w_{a}(1) = \phi[(1 - N_{a})/N_{a} - \log N_{a}]. \tag{6}$$

Note that the right hand side of equation (6) is monotonically decreasing in N_a , and that if (contrary to assumption) $\overline{w}_m = w_a(1)$ we have $N_a = 1$. It follows that the level of urbanization $1 - N_a$ at which the underemployment rate peaks increases monotonically with $\overline{w}_m/w_a(1)$, a fact that is not surprising in light of the intuition behind Proposition 2. Our proof is complete if the value of N_a determined by $\log w_a = (\log \overline{w}_m + \log \underline{w})/2$ or

$$(\log \overline{w}_{m} + \log \underline{w})/2 - \log w_{a}(1) = -\phi \log N_{a}$$
 (7)

is not greater than the value of N_a determined by equation (6), since the peak log variance occurs for a still lower value of N_a. In the Appendix the proof is completed.

2.3 Empirical investigation of the model: A beginning

Is there evidence for our view of the causes of inverted-U behavior of income inequality? We need incomes data reported by agricultural, urban formal, and urban informal sectors for a number of countries and years. Unfortunately, incomes data disaggregated by formal versus informal sector have typically been collected as part of one-time surveys of particular urban areas rather than as part of ongoing nationwide censes. Moreover, the definitions of the informal sector tend to vary from survey to survey. While it therefore appears that the accuracy of our model's description of inequality behavior cannot be assessed empirically at the present time, the model makes other predictions that are necessary though not sufficient for this description to be accurate. If these are supported by the stylized facts, we can at least claim that further investigation of our inequality results as better data becomes available is a promising direction for future research.¹¹

Underlying our inequality results in the previous subsection are results on labor market behavior. In particular, two predictions concerning the behavior of the informal sector that can be checked against available data emerge from the analysis. First, the informal sector share of the urban labor force $(1 - N_a - N_m)/(1 - N_a)$ should decrease with the level of urbanization. This follows from equation (1) and the monotonic decline of the share of the labor force in agriculture and consequent monotonic increase in w_a relative to \overline{w}_m and \underline{w} as urbanization proceeds. Second, Proposition 2 predicts that the informal sector share of the total labor force or underemployment rate $1 - N_a - N_m$ should follow an inverted U with urbanization. To facilitate the following empirical discussion of these

¹¹Greenwood and Jovanovic (1990) make an important theoretical contribution by showing that inverted-U behavior of inequality can be based on differential participation in financial markets, but they do not empirically evaluate any *new* predictions made by their theory.

predictions, we denote by URB and UNDER the variables used to measure $1 - N_a$ and $1 - N_a - N_m$, respectively. The ratio $(1 - N_a - N_m)/(1 - N_a)$ is measured by UNDER/URB, which is given the mnemonic SHARE.

Unlike the incomes data we would like, there does exist data on UNDER and URB (and therefore SHARE) that were collected for many countries on a nationwide basis using standardized definitions. This apparently unique data set was collected by PREALC (1982) under the direction of Victor E. Tokman, a leading scholar of the informal sector. It is based on the decennial censes taken by almost every Latin American country. The variables of interest to us are defined as follows. URB is the non-primary share of the economically active population. It is further disaggregated into the classifications formal, informal, and wage-earning domestic service. Workers are classified as informal if they are either self-employed or unpaid family help, excluding professionals and technicians. This definition of UNDER is more restrictive than we would like since it excludes the many wage earners in firms small enough to be bypassed by unions and government regulation. ¹²
We simply have to assume that the ratio of those omitted from UNDER to those included does not vary in any systematic way with URB.

The PREALC data covers 17 Latin American countries 13 for the years 1950, 1960, 1970, and 1980. Perhaps more troubling than the obvious limitation resulting from inclusion in our sample of only one region of the globe is the fact that the countries covered are all classified by the World Bank as either "lower middle income" or "upper middle income": no countries classified as low income or high income are included. Presumably this truncation of the sample at the low and high ends decreases the likelihood of finding an

¹²Two studies of Latin American cities cited in Portes et al. (1989, pp. 17 and 97) expand the definition of the urban informal sector to include wage workers without social security protection. This causes a shift from the formal to the informal sector of 3.1 percent of the urban labor force in Montevideo in 1983 and 11.8 percent in Bogotá in 1984.

¹³The countries included are Argentina, Bolivia, Brazil, Chile, Colombia, Costa Rica, Dominican Republic, Ecuador, El Salvador, Guatemala, Honduras, Mexico, Nicaragua, Panama, Peru, Uruguay, and Venezuela.

inverted U in UNDER. Nevertheless, Table 1 shows that there is substantial variation

Table 1

Variable	Mean	Std. Dev.	Minimum	Maximum
URB	50.9	16.6	18.9	84.4
UNDER	11.3	3.4	4.5	20.4
SHARE	23.8	8.4	10.9	44.0

Note: All variables are expressed in percent.

in the data. The reader may also note from Table 1 that the mean of SHARE is substantially below the figure of roughly 41 percent (cited in the introduction to this paper) computed from the surveys of LDC cities listed by Sethuraman (1981). This can be partly explained by the undercounting of the informal sector mentioned above since the survey definitions, though not standardized, were typically more inclusive than the PREALC definition. Moreover, if our hypothesis concerning the decline of SHARE with URB is correct, the difference may also be explained by the inclusion of observations for several low income countries (accounting for 7 out of 32 observations, with a mean SHARE of 48 percent) in the Sethuraman sample.

As in all studies that have checked for the existence of an inverted U in income distribution data (see the extensive references in Papanek and Kyn (1986)), we will seek to establish the stylized facts by applying ordinary least squares regressions to pooled data, a procedure that may be less troubling when working with a relatively homogeneous sample of countries such as Latin America. Table 2 shows that both predictions of our model concerning labor market behavior are strongly supported by the PREALC data.

Particularly impressive is the performance of the quadratic specification of the dependence of the underemployment rate on urbanization compared to the naive linear specification that assumes that the urban informal sector expands part passu with the rest of the urban sector. The quadratic results imply that the peak of the inverted U in the

Table 2

	Dependent Variable: SHARE	Dependent Variable:UNDER	
Variable	Coefficient (t-value)	Coefficient (t-value)	Coefficient (t-value)
CONSTANT	36.9 (14.5)	8.2 (6.3)	-3.0 (0.8)
URB	-0.30 (5.93)	0.061 (2 .52)	0.52 (3.51)
(URB) ²			-0.0043 (3.13)
R ²	0.35	0.09	0.21
σ̂	6.8	3.3	3.1

Note: Number of observations = 68.

underemployment rate occurs when 61.0 percent of the labor force has left the primary sector. If these results are representative of labor market behavior in the world as a whole, they hold out considerable promise for future empirical support of our model's predictions concerning the source of inverted-U behavior of income inequality.

3. The Gini coefficient and "turning points" in inequality

The Gini coefficient is probably the most widely used measure of income inequality. In this section I will show that it is a property of the model of Knight (1976) and Fields (1979) that the Gini coefficient must decline monotonically after more than half of the labor force has left the rural sector. This is also true if their model is modified to allow reduced pressure on the land to increase the agricultural wage relative to the urban wage. In other words, in a two sector model with no within-sector inequality, a country's Gini coefficient measure of income inequality must reach its maximum at a relatively early stage in the country's economic development, unless the income differential between the two sectors widens with urbanization. In contrast, the share of agricultural employment for

which the Gini coefficient reaches its maximum is not restricted for the model of section 2.

The issue is important since in the majority of LDCs more than 50 percent of the labor force remained in agriculture as of 1987, according to United Nations figures. 14

The Gini coefficient measure of income inequality is defined by

$$G = \sum_{i} \sum_{j} |\mathbf{w}_{i} - \mathbf{w}_{j}| / 2N^{2} \mu$$

where N is the number of workers and μ is the mean of their wages (permanent incomes). Moore (1990) points out that the Gini coefficient can be interpreted as one-half of the expected difference between the incomes of two workers chosen randomly (with replacement) from the economy, with the difference expressed as a percentage of the mean income of the society. In a two sector model with no inequality within sectors the mean income μ equals $(1 - N_m)w_a + N_m\overline{w}_m$ (since $N_a = 1 - N_m$), while the numerator of G can be shown to equal $2N_m(1 - N_m)(\overline{w}_m - w_a)N^2$. We thus define

$$G1(N_m) = N_m (1 - N_m)(\overline{w}_m - w_a) / [(1 - N_m)w_a + N_m \overline{w}_m],$$
 (8)

which is just a rearrangement of the expression obtained by Knight (1976, p. 175) and Fields (1979, p. 343, n. 10). By inspection we can see that G1 = 0 when either $N_m = 0$ or $w_a = \overline{w}_m$, and G1 > 0 when $0 < N_m < 1$ and $w_a < \overline{w}_m$. We can also see that the term $N_m(1 - N_m)$ in the numerator reaches a maximum for $N_m = 1/2$ while the other term $(\overline{w}_m - w_a)$ in the numerator is constant (if w_a is fixed) or declines monotonically with N_m (if w_a rises as the agricultural labor force declines). Since the denominator increases monotonically with N_m , we can conclude that G1 must decline monotonically for $N_m \ge 1/2$. 15

¹⁴The aforementioned paper of Papanek and Kyn (1986) found that the Gini coefficient tended to peak at a per capita GDP of roughly \$400 (in 1964 U. S. prices). At that income level, the "predicted" primary share of the labor force (for a country of medium size) is 43.8 percent according to Chenery and Syrquin (1975, p. 21).

¹⁵We know from Knight and Fields that G1 has only one local maximum for $0 < N_{\rm m} < 1$ when $\overline{w}_{\rm m} - w_{\rm a}$ is constant, so that it follows an inverted U as the economy urbanizes. To prove the same when their model is modified to allow reduced pressure on the land to

Intuitively, when all inequality is between-sector it tends to be maximized when the two sectors are of equal size. Normalization by mean income causes the maximum to occur for an urban labor force share less than one-half since mean income increases with urbanization. Allowing the rural-urban wage differential to shrink as urbanization proceeds causes the maximum to occur at an even lower level of urbanization, as can be seen from equation (8) by letting w_a increase with N_m rather than remaining constant. In the model of section 2, however, any peak in inequality need bear no particular relation to the point where the urban and rural labor forces are equal since between-sector inequality need no longer make the dominant contribution to total inequality. The Gini coefficient for the model of section 2 is given by

$$\frac{N_{\underline{a}}N_{\underline{m}}(\overline{w}_{\underline{m}} - w_{\underline{a}}) + N_{\underline{a}}(1 - N_{\underline{a}} - N_{\underline{m}})(w_{\underline{a}} - \underline{w}) + N_{\underline{m}}(1 - N_{\underline{a}} - N_{\underline{m}})(\overline{w}_{\underline{m}} - \underline{w})}{N_{\underline{a}}w_{\underline{a}} + N_{\underline{m}}\overline{w}_{\underline{m}} + (1 - N_{\underline{a}} - N_{\underline{m}})\underline{w}}$$
(9)

I have not been able to prove that G2 follows an inverted U for the special case for which the log variance was proved to follow an inverted U, i.e., where utility is intertemporally log-linear and the agricultural production function is Cobb-Douglas. Nevertheless, G2 does follow an inverted U for all numerical simulations of this special case that I have computed.

Let us refer to the urbanization level (equals 1 - N_a) at which G2 attains its maximum as the "turning point." Can we expect the turning point for an economy described by the special case of the model of section 2 to exceed 50 percent? We can begin to answer this question by recalling the proof of Proposition 4 above, where it was noted

increase the agricultural wage, it is sufficient to show that G1 $^{\prime\prime}<0$ for $0< N_{\rm m}<1/2$. The conditions under which this holds, and a proof that these conditions are met when the agricultural production function is Cobb—Douglas, are available from the author on request. This result confirms, at least for the Cobb—Douglas case, Braulke's (1983) finding using the RKF framework with estimated parameters that the Gini coefficient plotted against $N_{\rm m}$ follows an inverted U even when reduction of the rural-urban income differential takes place.

that the level of urbanization at which the underemployment rate peaks increases with the ratio $\overline{\mathbf{w}}_{m}/\mathbf{w}_{n}(1)$. We can also see from equation (6) that this level of urbanization decreases with ϕ , the land share of agricultural output, which also gives the elasticity of w₂ with respect to a decrease in the agricultural labor force. We can guess from Proposition 4 that the turning point would be affected by these parameters in the same direction as the level of urbanization at which the underemployment rate peaks, and this guess is supported by numerical simulations, though again analytical results are not forthcoming. We can therefore minimize the likelihood of finding a high turning point by setting w₂(1) equal to w, its maximum permissible value. The parameters affecting the turning point are then $\overline{w}_m/\underline{w}$ and ϕ . (Since the Gini coefficient is scale invariant, the absolute wage differential does not matter.) For the sake of brevity, we focus on the case where the formal sector wage exceeds the informal sector wage by 50 percent, near the lower end of the range in the studies of Latin American cities surveyed by Tokman (1990, p. 102) for a worker in a large formal sector firm versus a worker in an informal sector shop. In this case it can be shown that ϕ must be increased to roughly one-third (the labor share of agricultural output must be reduced to roughly two-thirds) in order to reduce the turning point to 50 percent. It follows that, in contrast to the models of the previous literature, for a broad range of realistic parameter values an economy described by the special case of the model of section 2 that has passed the early stages of development cannot be assured of falling income inequality (as measured by the Gini coefficient) in the absence of any policy initiatives.

4. Conclusions: Interpreting inverted-U behavior of income inequality

Fields (1987) has criticized the usefulness of traditional inequality indices for measuring changes in income inequality with economic development because of their performance in dual economy models. His argument is strengthened when those models are modified to allow for the agricultural wage to rise as pressure on the land is reduced, since income inequality as measured by the Gini coefficient grows for some time even though the

income of the poorer individuals (rural workers) is catching up to that of the richer individuals (urban workers) and the numbers of the former are decreasing. Since this runs counter to our intuitive notions of the conditions under which income inequality can be said to be increasing, Fields (1987) proposes that for the purpose of measuring change in income inequality during the course of economic development we discard the traditional indices in favor of ones that do not have the inverted-U property under the conditions of pure "modern sector enlargement" (the fixed rural-urban income differential case). Moore (1990) argues against Fields's proposal. The present paper suggests that both sides of this debate may be misguided because they are using an incomplete economic model where there is no equilibrium condition that determines the intersectoral allocation of the labor force. In the complete model of section 2, the rise and fall of income inequality as measured by the log variance are closely related to the rise and fall of the proportion of the labor force that falls into the poorest class, the underemployed. This is in accord with rather than counter to our intuitive notions of the conditions under which income inequality should increase and decrease. Far from being an unimportant artifact of traditional inequality indices, then, the inverted U reflects the rise and fall of urban slums and of the share of the population with disappointed expectations, which are phenomena of political as well as economic significance.

Perhaps surprisingly, the model of section 2 can be seen as a complement to the human-capital-based explanation of income inequality associated with the University of Chicago. According to this school of thought, exemplified by the work of Becker and Tomes (1979), inequality among identically endowed individuals is generated over time by differences in "market luck", which parents can pass on to their children by investing in the children's human capital. Market luck is exactly the source of inequality in the model of section 2: the lucky wind up in the formal sector while the unlucky wind up in the informal sector. Over time, market luck first becomes more important, as the movement from an agrarian to an industrial society opens up more opportunities for both success and

failure, and later becomes less important, simply because rising incomes in the agricultural sector provide a "safety net" that allows everyone to make much safer bets in the industrial sector. From this point of view, then, market luck is the driving force behind the inverted U, and the advantage of the model of section 2 over the human capital model of income inequality is its ability to endogenize market luck. Its disadvantage is its inability to incorporate "inheritance" of market luck through parental investment in children.

Overcoming this disadvantage should be a subject for future research.

Appendix: Proofs of Propositions concerning underemployment and inequality

Proof of Proposition 2: We can see from equation (2) that dN_a/dN_m is monotonically increasing as N_m increases from 0 to \bar{N}_m if and only if the denominator on the right hand side is monotonically decreasing. We have

$$\begin{split} (\mathrm{d}/\mathrm{dN_m}) & [(1-\mathrm{N_a}) \mathrm{v_a'w_a'} - (\mathrm{v_a} - \underline{\mathrm{v}})] = \\ & [-\mathrm{v_a'w_a'} + (1-\mathrm{N_a}) \mathrm{v_a'w_a'}' + (1-\mathrm{N_a}) \mathrm{v_a''}(\mathrm{w_a'})^2 - \mathrm{v_a'w_a'}] (\mathrm{dN_a}/\mathrm{dN_m}). \end{split}$$

Multiplying the last expression through by wa/va and rearranging gives us

$$\{-2w_aw'_a + (1-N_a)[w_aw'_a - C(w'_a)^2]\}(dN_a/dN_m),$$

where C is the coefficient of relative risk aversion. This expression can be shown to be negative in the case where the agricultural production function is Cobb-Douglas and the intertemporal utility function is log-linear (C = 1). Since $dN_a/dN_m < 0$ and $w_a' < 0$, it is sufficient to show that $w_aw_a'' > C(w_a')^2 = (w_a')^2$. The Cobb-Douglas assumption gives us $f_a(L/N_a) = B(L/N_a)^\phi, \ 0 < \phi < 1$. Competitive factor pricing yields $w_a(N_a) = \bar{p}_a(1-\phi)B(L/N_a)^\phi = w_a(1)N_a^{-\phi}$. Using this formula we derive $w_a' = -\phi w_a(1)N_a^{-\phi-1} \text{ and } w_a'' = (\phi+1)\phi w_a(1)N_a^{-\phi-2}. \text{ We then have } w_aw_a'' = [w_a(1)]^2(\phi+1)\phi(N_a)^{-2\phi-2} > (w_a')^2 = [w_a(1)]^2 \phi^2(N_a)^{-2\phi-2}.$

Proof of Proposition 3: From equation (1'), in the intertemporally log-linear utility case we have $N_m = (1 - N_a)(\log w_a - \log w)/(\log w_m - \log w)$. Substituting this into equation (4) gives us

$$\sigma^{2} = \frac{(1 - N_{a})[(\log w_{a} - \log \underline{w})(\log \overline{w}_{m} - \log w_{a})^{2} + (\log \overline{w}_{m} - \log w_{a})(\log w_{a} - \log \underline{w})^{2}]}{\log \overline{w}_{m} - \log \underline{w}}$$

$$= \frac{(1 - N_{a})(\log \overline{w}_{m} - \log w_{a})(\log w_{a} - \log \underline{w})(\log \overline{w}_{m} - \log \underline{w})}{\log \overline{w}_{m} - \log \underline{w}}$$

$$= (1 - N_{a})(\log \overline{w}_{m} - \log w_{a})(\log w_{a} - \log \underline{w})$$

which is equation (5) in the text.

Make the change of variable 1 - $N_a = U$, so that w_a is now a function of (1 - U). Clearly σ^2 attains any maxima for U less than \bar{N}_m but greater than is determined by $\log w_a = (\log \overline{w}_m + \log w)/2$. To prove that there is only one such maximum, it is sufficient to show that σ^2 is concave in U for U in this range. We have

$$\begin{split} \frac{d\sigma^{2}}{dU} &= -U(\log \overline{w}_{m} - \log w_{a})(w_{a}'/w_{a}) + U(w_{a}'/w_{a})(\log w_{a} - \log \underline{w}) \\ &+ (\log \overline{w}_{m} - \log w_{a})(\log w_{a} - \log \underline{w}). \\ \frac{d^{2}\sigma^{2}}{(dU)^{2}} &= U(\log \overline{w}_{m} - \log w_{a})(\frac{w_{a}w_{a}'' - (w_{a}')^{2}}{w_{a}^{2}}) - U(w_{a}'/w_{a})(w_{a}'/w_{a}) \\ &- (\log \overline{w}_{m} - \log w_{a})(w_{a}'/w_{a}) + U(w_{a}'/w_{a})(-w_{a}'/w_{a}) \\ &- U(\frac{w_{a}w_{a}'' - (w_{a}')^{2}}{w_{a}^{2}})(\log w_{a} - \log \underline{w}) + (w_{a}'/w_{a})(\log w_{a} - \log \underline{w}) \\ &- (\log \overline{w}_{m} - \log w_{a})(w_{a}'/w_{a}) + (w_{a}'/w_{a})(\log w_{a} - \log \underline{w}) \\ &- (\log \overline{w}_{m} - \log w_{a})(w_{a}'/w_{a}) + (w_{a}'/w_{a})(\log w_{a} - \log \underline{w}) \\ &= -2U(w_{a}'/w_{a})^{2} + [U(\frac{w_{a}w_{a}'' - (w_{a}')^{2}}{w_{a}^{2}}) - 2(w_{a}'/w_{a})](\log \overline{w}_{m} + \log \underline{w} - 2\log w_{a}) < 0 \end{split}$$

for U greater than that determined by $\log w_a = (\log \overline{w}_m + \log \underline{w})/2$ since $w_a w_a' - (w_a')^2 > 0$ in the Cobb-Douglas case as shown in the proof of Proposition 2.

Proof of Proposition 4: Denote by \hat{N}_a and \hat{N}_a the values of N_a determined by equations (6) and (7), respectively. We need to show that $\hat{N}_a \geq \hat{N}_a$, implying $1 - \hat{N}_a \leq 1 - \hat{N}_a$: the level of urbanization at which the underemployment rate peaks is lower than that at which the log variance in permanent income peaks. We begin by noting that since $\log \overline{w}_m > (\log \overline{w}_m + \log \underline{w})/2$, the larger is $w_a(1)$ the larger is the left hand side of equation (6) relative to equation (7). Since the right hand sides of both equations are monotonically declining in N_a , we make it hardest to obtain our result by setting $w_a(1)$ equal to its maximum permissible value, \underline{w} . We can now write equations (6) and (7) as $\log \overline{w}_m - \log \underline{w} = \phi(\hat{N}_a^{-1} - 1 - \log \hat{N}_a)$ and $\log \overline{w}_m - \log \underline{w} = -2\phi \log \bar{N}_a$, respectively. Since for $0 < N_a < 1$

we have N_a^{-1} - 1 > -log N_a , it follows that the right hand side of equation (6) exceeds that of equation (7) for given N_a , hence $\hat{N}_a > \bar{N}_a$.

REFERENCES

- Banerjee, Biswajit. "The Role of the Informal Sector in the Migration Process: A Test of Probabilistic Migration Models and Labour Market Segmentation for India."

 Oxford Economic Papers 35 (1983): 399-422.
- Becker, Gary S. and Nigel Tomes. "An Equilibrium Theory of the Distribution of Income and Intergenerational Mobility." *Journal of Political Economy* 87 (December 1979): 1153-1189.
- Braulke, Michael. "A Note on Kuznets' U." Review of Economics and Statistics 65 (February 1983): 135-139.
- Chenery, Hollis, and Moises Syrquin, Patterns of Development, 1950-1970. Oxford: Oxford University Press, 1975.
- Diamond, Peter A. "National Debt in a Neoclassical Growth Model." American Economic Review 55 (December 1965): 1126-1150.
- Eaton, Jonathan. "A Dynamic Specific Factors Model of International Trade." Review of Economic Studies 54 (April 1987): 325-338.
- Fields, Gary S. "A Welfare Economic Approach to Growth and Distribution in the Dual Economy." Quarterly Journal of Economics 93 (August 1979): 325-353.
- . "Measuring Inequality Change in an Economy with Income Growth."

 Journal of Development Economics 26 (August 1987): 357-374.
- Greenwood, Jeremy, and Boyan Jovanovic. "Financial Development, Growth, and the Distribution of Income." Journal of Political Economy 98 (October 1990): 1076-1107.
- Harris, John R. and Michael P. Todaro. "Migration, Unemployment, and Development: A Two-Sector Analysis." American Economic Review 60 (March 1970): 126-142.
- Knight, John B. "Explaining Income Distribution in Less Developed Countries: A Framework and an Agenda." Oxford Bulletin of Economics and Statistics 38 (August 1976): 161-177.
- Kuznets, Simon. "Economic Growth and Income Inequality." American Economic Review 45 (March 1955): 1-28.
- Moore, Robert E. "Measuring Inequality Change in an Economy with Income Growth: Reassessment." Journal of Development Economics 32 (January 1990): 205-210.
- Papanek, Gustav F. and Oldrich Kyn. "The Effect on Income Distribution of Development, the Growth Rate and Economic Strategy." Journal of Development Economics 23 (September 1986): 55-65.

- Portes, Alejandro, Manuel Castells, and Lauren A. Benton, eds. The Informal Economy: Studies in Advanced and Less Developed Countries. Baltimore: Johns Hopkins University Press, 1989.
- PREALC (Programa Regional del Empleo para América Latina y el Caribe). Mercado de Trabajo en Cifras, 1950-1980. Santiago: Oficina International del Trabajo, 1982.
- Rauch, James E. "Urbanization, Underemployment, and the Size Distribution of Income: Harris-Todaro and Kuznets's Inverted U." University of California, San Diego Discussion Paper 89-39 (August 1989), revised 1990.
- Rauch, James E. "Modelling the Informal Sector Formally." Journal of Development Economics, forthcoming 1991.
- Robinson, Sherman. "A Note on the U Hypothesis Relating Income Inequality and Economic Development." American Economic Review 66 (June 1976): 437-440.
- Sethuraman, S. V., ed. The Urban Informal Sector in Developing Countries. Geneva: International Labour Office, 1981.
- Summers, Robert and Alan Heston. "A New Set of International Comparisons of Real Product and Price Levels Estimates for 130 Countries, 1950-1985." Review of Income and Wealth 34 (March 1988).
- Tidrick, Gene M. "Wage Spillover and Unemployment in a Wage-Gap Economy: The Jamaican Case." Economic Development and Cultural Change 23 (January 1975): 306-324.
- Todaro, Michael P. "A Model of Labor Migration and Urban Unemployment in Less Developed Countries." American Economic Review 59 (March 1969): 138-148.
- Tokman, Victor E. "The Informal Sector in Latin America: Fifteen Years Later." In David Turnham, Bernard Salomé, and Antoine Schwarz, eds., The Informal Sector Revisited. Paris: OECD, 1990, 94-110.
- World Bank. World Development Report. New York: Oxford University Press, 1986.