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AN INDICATOR OF FUTURE INFLATION EXTRACTED FROM THE STEEPNESS OF THE INTEREST RATE YIELD CURVE ALONG ITS ENTIRE LENGTH

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ABSTRACT

It is often suggested that the slope of the term structure of interest rates contains information about the expected future path of inflation. Mishkin (1990) has recently shown that the spread between the 12-month and 3-month interest rates helps to predict the difference between the 12-month and 3-month inflation rates. His approach however, lacks a theoretical foundation, other than the (rejected) hypothesis that the real interest rate is constant. This paper applies a simple existing theoretical framework, which allows the real interest rate to vary in the short run but converge to a constant in the long run, to the problem of predicting the inflation spread. It is shown that the appropriate indicator of expected inflation can make use of the entire length of the yield curve, in particular by estimating the steepness of a specific nonlinear transformation of the curve, rather than being restricted to a spread between two points. The resulting indicator, besides having a firmer theoretical foundation does a relatively good job of predicting the inflation rate over the period 1960 to 1988.

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An Indicator of Future Inflation Extracted from the Steepness of the Interest Rate Yield Curve Along its Entire Length

The idea that the slope of the term structure of interest rates can be used as an indicator of whether monetary policy is currently tight or loose is not new. But the idea has generated some new research since 1988 when Manuel Johnson, then Vice-Governor of the Federal Reserve System, announced that the term structure was one of three indicators that he thought might be useful to gauge whether monetary policy was expansionary or not.[1] Frederic Mishkin (1988, 1989, 1990a, 1990b, 1990c, 1990d), in particular, has written an abundant series of papers on this subject, showing that the term structure has predictive power for the change in the inflation rate over the coming period.

The intuitive argument is that when the short-term interest rate is high relative to longer-term interest rates (as in an "inverted yield curve"), this means that monetary policy is currently tight, and that the inflation rate will be falling in the future. The hypothesis tested by Mishkin is a restrictive form of this intuitive argument: it assumes that the real rate of interest, at any given term, is a constant over time. It would follow from this hypothesis that movement in the long-term interest rate reflects one-for-one movement in the expected long-term inflation rate, that movement in the short-term interest rate reflects one-for-one movement in the expected short-term inflation rate, and that movement in the long-short interest differential reflects one-for-one movement in the long-short expected inflation differential.

The hypothesis that the real interest rate is constant is not a very attractive basis for getting at the idea that the term structure of nominal interest rates contains information about the expected future path of inflation. In the first place, the real interest rate is observed to have varied over time, especially in the 1980s. Indeed, Mishkin (1981, 1984, 1991) himself has helped to document this variation thoroughly. In the second place, the intuitive reason for looking at the slope of the yield curve (rather than, for example, at just the one-year interest rate, as an indicator of expected one-year inflation) is that an inverted yield curve means that monetary policy is tight in *real* terms, i.e., that the *real* interest rate is

[1]The other two indicators were the exchange rate and the price of a basket of commodities. Johnson (1988) and Lown (1989).

high and that this will have a negative effect on the real demand for goods. The role of the short-term interest rate is not simply to reflect expected inflation over the short term in the same way that the long-term interest rate reflects expected inflation over the long term. Rather, the long-term interest rate is thought to reflect expected inflation *more fully* than does the short-term interest rate, so that the slope of the yield curve can be thought of as an inverse indicator of the real interest rate. This argument is formalized below.

The paper will show that the Mishkin approach is an overly restrictive way of inferring the expected future path of inflation from the term structure of interest rates. It shows that the appropriate indicator of expected inflation can make use of the entire length of the yield curve, in particular by estimating the steepness of a specific nonlinear transformation of the curve, rather than being restricted to a spread between two points. The resulting indicator, besides having a firmer theoretical foundation, does a better job of predicting the inflation rate over the period 1960 to 1988.

1. A First Illustration of the Restrictiveness of the Mishkin Approach

Let π^{12} be the 12-month inflation rate, π^3 be the 3-month inflation rate, i^{12} be the 12-month interest rate and i^3 be the 3-month interest rate. Mishkin (1988, 1989, 1990a, 1990c) runs the following regression:

$$(\pi^{12} - \pi^3) = a + b(i^{12} - i^3) + u. \tag{1}$$

He finds two principal results. First, the coefficient b is significantly greater than zero, showing that the term structure does contain information useful for predicting the path of the future inflation rate. Second, b is significantly less than 1. This is a rejection of the null hypothesis that real interest rates are constant: if the 12-month real interest rate $(i^{12} - E\pi^{12})$ and the 3-month real interest rate $(i^3 - E\pi^3)$ were both constant (where E is used to identify the *expected* inflation rate), then b should equal 1.

The statistical rejection of the hypothesis that the real interest rate is constant is consistent with current conventional wisdom, not to mention with the earlier results of Mishkin (1981, 1984, 1991). Nevertheless, we should feel somewhat uncomfortable about the status of the term-structure indicator if the only theoretical basis offered for its use is rejected. We are offered no theoretical framework in which to interpret equation (1) for the case where b is different from 1. Fortunately, there already exists a simple theoretical framework for thinking about the slope of the term structure that allows for variation in the real interest rate.[2]

Only two assumptions are required by this framework: (1) The long-term interest rate reflects expected future short-term interest rates (up to a possible risk premium or liquidity term that does not vary over time).[3] (2) Market participants expect (in the absence of new disturbances such as unexpected changes in monetary policy) that the steady-state inflation rate will become increasingly incorporated into the nominal interest rate with the passage of time.[4] This second assumption will hold, for example, if short-run variation in the real interest rate arises because the price level is sticky and consequently changes in the nominal money supply have real effects in the short run. The two assumptions together imply that the long-term interest rate reflects expected inflation more fully than does the short-term interest rate. This in turn implies that the slope of the term structure can be used to extract indicators of the real interest rate, the current tightness of monetary policy, and the expected future path of the inflation rate.

The purpose of this paper is not solely to present the theoretical framework that is appropriate

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[2] The framework was developed in Frankel (1982). The claims made in this paper were briefly announced in Frankel (1989).

[3] Many researchers have rejected the hypothesis that long-term interest rates offer unbiased forecasts of expected future short-term interest rates. Many have interpreted this finding as evidence against the expectations hypothesis of the term structure of interest rates, and evidence in favor of a time-varying risk premium, in contradiction to the assumption we make here. A second group of researchers, on the other hand, have interpreted the finding as evidence of systematic errors in expectations formation (Shiller, Campbell and Schoenholtz, 1983, and Mankiw and Summers, 1984) or of the pitfalls of econometric tests in small samples (Froot, 1989, and Lewis, 1990). In adopting the expectations hypothesis, we take our inspiration from this second group. (For a survey of work on the term structure, see Shiller, 1987.)

[4] Mishkin (1991, p.25) points out that Irving Fisher originally suggested the "Fisher effect," whereby the expected inflation rate becomes incorporated into the nominal interest rate, as something that takes place in the long run, not in the short run.

for thinking about the term-structure indicator. The appropriate theoretical framework has important practical consequences. It suggests a way of extracting a measure of expected future inflation that is superior to Mishkin's equation (1), regardless whether his coefficient b in that equation is constrained to 1 or not. The suggested technique makes use of points all along the yield curve, rather than restricting the analysis to the two points of maturity that correspond to the maturity of inflation in which we are interested.

Table 1 offers a simple first-pass illustration, for the sample period 1960-1988, of the benefits of making use of the extremities of the yield curve, even when the goal is only, as in Mishkin (1989, 1990a, 1990c), to forecast the path of inflation over the coming year. As in those papers, the dependent variable is the spread in the future inflation rates, $(\pi^{12} - \pi^{3})$. The first column shows the Mishkin approach: the explanatory variable is the interest rate spread $(i^{12} - i^{3})$, which corresponds precisely with the maturities of the inflation rates, as we would want if our theoretical rationale were the hypothesis that real interest rates were constant. The finding is the same as in Mishkin: the coefficient on the term spread is positive and statistically significant. In other words, the term spread does contain information useful for predicting the path of inflation. The second column shows the simplest possible version of the alternative approach: the explanatory variable is the spread between two relatively more extreme points on the yield curve, the 5-year interest rate minus the overnight interest rate. The results show a stronger relationship for the alternative approach than for the Mishkin approach. The t-statistic is almost twice as high, and the R^2 is about three times as high.[5]

It seems evident that the spread between the five-year and short-term interest rates gives us a better measure of the overall steepness of the yield curve, and so does a better job of predicting the path of inflation, than does the spread between the 12-month and 3-month interest rates. A hint of this possibility is revealed by the results in Fama (1990), Mishkin (1990b, 1990d) and Jorion and Mishkin

[5] The overnight federal funds rate used in the second column of Table 1 (and of Tables 3 and 4 below) is quoted on the standard basis of a 360-day year. Most interest rate studies use the federal funds rate as it is. We tried all regressions with the rate recomputed on a more accurate 365-day basis; all statistics were the same as those reported, to the first two decimal places.

(1991), despite the absence in those papers of any theoretical framework that would predict the finding that the longer-term part of the yield curve reflects expected inflation better than the shorter-term part. Mishkin (1990b) looks at the long-term spread *as a predictor of the future change in the inflation rate over the long term*, and finds a closer relationship than that between $(\pi^{12} - \pi^{3})$ and $(i^{12} - i^{3})$ shown in equation (1):

> "The evidence indicates that there is substantial information in the longer maturity term structure about future inflation... For maturities of six months or less, the term structure contains no information about the future path of inflation, but it does contain a great deal of information about the term structure of real interest rates."

But Mishkin does not consider using the longer term spread to forecast $(\pi^{12} - \pi^{3})$, because of the equation's origins in the theoretical framework in which the real interest rate is constant so that the nominal interest rate of any given term is only useful for predicting inflation of the same term. Similarly, Fama (1990, p.60) finds, "When the forecast horizon is extended, the information in the spread about the real return decays relative to the information about inflation." The model used in this paper provides a simple explanation for such findings.

If the spread between the five-year and overnight interest rates is useful for predicting the path of inflation, it seems logical that one might obtain a still better indicator by using multiple points along the yield curve to get a better estimate of its overall steepness. But a brief inspection of an actual yield curve reveals that the appropriate estimate of the steepness is unlikely to be the slope in a simple linear relationship between the interest rate and its term of maturity: there is more information in the spread between the one-year rate and the two-year rate than there is between the 29-year rate and the 30-year rate. The alternative theoretical framework examined here shows the appropriate nonlinear transformation to apply to the yield curve before estimating its slope, which is another attraction of the model.

The remainder of this paper does three things. It first develops the alternative theoretical framework that allows the real interest rate to vary. It then shows how this framework suggests the appropriate technique to extract a measure of the steepness of the yield curve that uses points from its entire length, rather than from only two maturities.[6] Finally, it shows how this technique offers a predictor of the path of inflation over the year that performs well, relative to the approach illustrated in the first column of Table 1.

2. The Theoretical Framework That Allows the Real Interest Rate to Vary Over Time

Assume that the short-term real interest rate, though not constant, is expected by market participants to converge to a constant in the long run in the absence of future disturbances. Specifically, assume that the short-term nominal interest rate is expected to adjust to the steady-state inflation rate according to a first-order differential equation as follows:

$$\frac{di_t}{dt} = -\delta(i_t - \pi_0^e - r), \tag{2}$$

where i_t is the (instantaneously) short-term interest rate, π_0^e is the exogenous long-run inflation rate expected at time 0, and r is the constant long-run real interest rate, all three of which are not directly observable. δ is the speed of adjustment.

Equation (2) can be taken on its own, as a natural parameterization of how the interest rate adjusts. It is intended to be a way of specifying, as simply as possible, the notion that the real interest rate is not constant in the short run but has a tendency in that direction in the long run. It is also possible to make an economic argument to illustrate a reason why the interest rate might follow the path indicated by equation (2). The argument is that adjustment takes time because goods prices are sticky. When an increase in the expected rate of future monetary growth raises the expected rate of inflation for example, if the short-term interest rate were to rise instantly, real money demand would fall, which is not possible when the price level and thus the real money supply are fixed. But over time, prices rise in response to

[6]The paper thus pursues the extension to Frankel (1982) that was suggested there in footnote 11.

excess goods demand. Thus over time the real money supply is free to fall and the short-term interest rate is free to rise to reflect expected inflation. A textbook model formalizing this argument is specified in Appendix 1. It is demonstrated in this model that rational expectations will take precisely the form of equation (2). But the model in the Appendix is only intended as an example to help motivate the equation.

Equation (2) implies that, at time 0, market participants expect the short-term nominal interest rate at time t to be a weighted average of the long-run nominal interest rate $(\pi_0^e + r)$ and the current short-term nominal interest rate i_0:

$$i_t = [1 - \exp(-\delta t)](\pi_0^e + r) + [\exp(-\delta t)]i_0. \tag{3}$$

The entire analysis that follows hinges on equation (3), representing the tendency of the nominal interest rate to approach $(\pi_0^e + r)$ as time passes, in the absence of new disturbances. It is not directly tested here; the expected inflation rate cannot be directly observed.

We need only one other assumption: i_0^τ, defined as the interest rate on τ-maturity bonds (issued at time 0), is the average of the expected instantaneously short-term interest rates between time 0 and time τ, plus a possible liquidity premium term k_τ:[7]

$$i_0^\tau = \frac{1}{\tau} \int_0^\tau i_t \, dt + k_\tau \tag{4}$$

By integrating (3), we find that the τ-maturity interest rate can be represented as another weighted average:

$$i_0^\tau = (1 - w_\tau)(\pi_0^e + r) + (w_\tau) i_0 + k_\tau. \tag{5}$$

where the weights are given by

$$(w_\tau) = \frac{1 - \exp(-\delta\tau)}{\delta\tau}.$$

The equation can also be written as:

$$i_0^\tau = k_\tau + i_0 + (\pi_0^e + r - i_0)(1 - w_\tau). \tag{5'}$$

For any given point in time, we can observe the Treasury security rate for two or more maturities, τ_1

[7]In order for equation (4) to hold as an arithmetic average (rather than as a geometric average), and in order to enable us to work generally with linear equations, all rates are defined logarithmically. For small rates, the log of one plus the rate is numerically close to the rate itself, which justifies the reference in the text to i as "the interest rate," etc. It would be desirable to try our technique on other interest rate series, for example that calculated in the Appendix to Shiller (1987).

and τ_2 (say, 3 months and 5 years). Then (given an estimate of δ) we can solve for the unknown $(\pi_0^e+r-i_0)$, which is the appropriately-calculated slope of the yield curve at that point in time. The relationship is illustrated schematically in Figure 1. The lower half of the graph illustrates the non-linear relationship between the term τ and the weight $(1-w_\tau)$; the upper half illustrates the linear relationship between $(1-w_\tau)$ and the interest rate i_0^τ of the corresponding term. At any point in time, e.g., $t=0$, the τ-term nominal interest rate is a weighted average of $(\pi_0^\tau+r)$, the hypothetical infinitely long-term interest rate, and i_0, the hypothetical infinitely short-term interest rate. The longer the term of maturity, the greater the weight given to the former and the less to the latter. Although any two points would be sufficient to construct an appropriate estimate of the difference between the extremities (π_0^e+r) and i_0 that reflects the steepness of the yield curve, a better technique would be to apply OLS regression to however many points along the curve are available. One can see from equation $(5')$ that the coefficient in a regression of i_0^τ against $\{1-[1-\exp(-\delta\tau)]/\delta\tau\}$ is the appropriate measure of $(\pi_0^e+r-i_0)$, the overall steepness of the yield curve. (If we want to allow for a liquidity premium or risk premium that -- although constant over time -- varies with the term to maturity, then we have to subtract from the interest rate i_0^τ the time-series average of i_t^τ for all t, as an estimate of the corresponding premium term k_τ.)

Appendix 2 develops the point that the slope of the yield curve, as a measure of the current looseness of monetary policy, reflects the likely future path of inflation. It shows that the difference between the expected 12-month inflation rate and the expected 3-month inflation rate, which is the variable examined by Mishkin, is proportional to $(\pi_0^e+r-i_0)$.

3. Empirical Results

We now turn to the estimation. We begin by taking an estimate for δ, the parameter governing the speed with which the nominal interest rate incorporates the expected inflation rate, from Frankel (1982): $\delta=.4$. For each monthly observation over the period January 1960 to December 1988, we estimate the steepness of the yield curve at that point in time by running the regression

$$i_t^\tau = B0_t + B1_t[1 - \frac{1-\exp(-\delta\tau)}{\delta\tau}]; \tag{6}$$

the coefficient $B1_t$ is the appropriate estimate of the measure of steepness $(\pi_t^e + r - i_t)$.[8] Then we see whether the time series $B1_t$ can help forecast the difference between the future 12-month inflation rate and the future 3-month inflation rate. We leave for future work such obvious refinements as iterating over δ to obtain an optimal estimate of that parameter.

The results of the second stage of this procedure are reported for the period 1960 to 1988 in Table 2. Table 2a shows the most proper case, where the k_τ premiums estimated as the average term structure over time, have been subtracted from the interest rates (in log form) before running the regressions at each point in time to estimate $B1_t$. [Table 2b assumes that the liquidity or term premiums are zero and so dispenses with subtracting off the means. Results there are reported in two forms: with interest rates expressed in level form, because that is the way that Mishkin and most other authors apparently do it, and then with interest rates expressed in the form of the log of 1 plus the interest rate, because that is the form correctly suited to the expectations hypothesis for the term structure of interest rates -- equation (4) -- and for the rest of the model that follows. We find very similar results regardless which form is used.[9]]

The t-statistic is significant at the 99 per cent level, indicating that the steepness of the yield curve does contain useful information regarding the future path of inflation. Both the t-statistic and the R^2, when compared to those in the first column of Table 1, indicate that our measure extracted from the interest rate yield curve along its entire length, is a considerably better indicator than the simple term spread used by Mishkin.

It is often suggested that the change in Federal Reserve operating procedures of October 1979

[8] If the model held perfectly, then it might seem that the OLS regressions that are estimated at each point in time would fit the yield curve perfectly. But of course they do not. (The R^2s, averaged across all time series observations, is .37.) How should one think about the error term in such a regression? The easiest interpretation of the error term at a given maturity may be that it is a small liquidity premium, which is assumed to be independent.

[9] The coefficient when interest rates are expressed in log form is always roughly 100 times the coefficient when interest rates are expressed in the form of percentage points. The t-statistics are quite similar.

marked a structural break in relationships among the money supply, inflation and interest rates. For this reason we tried testing the term structure equations separately on the two sub-periods before and after that date. The top half of Table 3a shows all three methods of forecasting the inflation spread for the sub-period January 1960 to September 1979: the Mishkin approach which uses the spread between the 12-month and 3-month interest rates, the simplest version of the alternative approach, which uses the spread between the 5-year and overnight interest rates, and the more sophisticated version of the alternative approach, which uses the nonlinear estimate of the steepness of the entire length of the yield curve. [Table 3b does the same for the percentage form rather than the log form; in the case of the alternative approach in the third column it also dispenses with subtracting off means.] The R^2 is more than three times as great for the sophisticated form, and more than seven times as great for the simpler form.

The bottom half of the table shows the same exercise for the second sub-period, October 1979 to December 1988. There is a change in the results between the two sample periods, though only the downward shift in the constant term is statistically significant. (The upward shift in the coefficient on B1 is not significant, once the proper correction for the moving average error process is applied.[10]) In the first sub-period, the sophisticated form of the alternative approach does not do as good a job of forecasting the inflation spread as well as the simple form, even though both do better than the Mishkin approach. In the second sub-period, both versions of the alternative approach again dominate the Mishkin approach. Now, however, the more sophisticated version dominates the simpler version as well as dominating the Mishkin approach.

In short, our technique appears to have the advantage, in addition to being based in a theoretical framework, of doing a relatively good job of forecasting inflation.

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[10] The corrected t-statistic on the coefficient shift is 1.230 [not reported in the table]. The regressions on the simple spreads in the first two columns show the same pattern: the decrease in the constant term between the two sub-periods is statistically significant, but the increase in the coefficient is not.

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APPENDIX 1

A Simple Macroeconomic Model of Adjustment in the Price Level and Interest Rate

Equation (2) embodies the proposition that the real interest rate has a tendency to adjust over time toward a constant. This proposition would follow from a wide variety of macroeconomic models that included a money-demand equation and price-adjustment equation. This appendix specifies one such model, of textbook familiarity, demonstrating that the rationally expected path of the nominal interest rate is of the same form as equation (2).

$$y-\bar{y}=-\gamma\,(i-\pi^{e}-r), \tag{A.1}$$

$$m-p=\phi y-\lambda i, \tag{A.2}$$

$$\frac{dp}{dt}=\rho\,(y-\bar{y})+\pi^{e}, \tag{A.3}$$

where

y is the log of output,
$\bar{y}$ is the log of normal or potential output,
i is the short-term nominal interest rate,
π^{e} is the expected long-run inflation rate,
m is the log of the money supply, and
p is the log of the price level.

(A.1) is an IS relationship; it says that the output gap is related to the current real interest rate through investment demand.[11] We note immediately that, in the long run, when $y=\bar{y}$, we have $i=\pi^{e}+r$.

(A.2) is an LM relationship; it says that real money demand depends positively on income, with an elasticity of ϕ, and negatively on the interest rate, with a semielasticity of λ. In the long run, $dm/dt=dp/dt=\pi$.

(A.3) is a supply relationship; it says that the rate of price change is given by the sum of an excess-demand term and the expected steady-state inflation rate.

Differentiating (A.2), we find

$$\frac{dm}{dt}-\frac{dp}{dt}=\frac{\phi dy}{dt}-\frac{\lambda di}{dt}. \tag{A.2$'$}$$

[11] The description of $i-\pi^{e}$ as "the real interest rate" is not quite proper, since i is short term and π^{e} is long term. But the model would be unchanged if the interest and inflation rates were specified to be of the same term--for example, the short term:

$$y-\bar{y}=-\psi\left(i-\frac{dp}{dt}-r\right).$$

We simply substitute (A.3) and solve for $(y-\bar{y})$:

$$y-\bar{y}=-\frac{\psi}{1-\psi\rho}\,(i-\pi^{e}-r).$$

This equation is the same as (A.1), with γ defined as $\psi/(1-\psi\rho)$.

Differentiating (A.1), we have

$$\frac{dy}{dt} = -\gamma \frac{di}{dt}. \tag{A.1$'$}$$

We substitute (A.1′) and (A.3) into (A.2′):

$$\pi - \left[\rho\,(y - \bar{y}) + \pi^{e}\right] = -(\phi\gamma + \lambda)\,\frac{di}{dt}.$$

Finally, we substitute in (A.1), assume perfect foresight ($\pi = \pi^e$), and solve for the expected rate of change of the interest rate:

$$\frac{di}{dt} = -\delta\,(i - \pi^{e} - r), \tag{2}$$

where $\delta \equiv \rho\gamma/(\phi\gamma + \lambda)$. This is equation (2) in the text.

The foregoing perfect-foresight formulation can be transformed to a stochastic one by introducing future disturbances to the level (m) and trend (π) of the money supply. As long as these disturbances have expectation zero, (2) will describe the rationally expected path of i. We could even allow for purely transitory disturbances in m.

APPENDIX 2

The Relationship Between the Inflation Spread and the Steepness of the Yield Curve

From Appendix 1, we combine (A3) and the first equation in footnote 11:

$$\frac{dp}{dt} = \pi_0^e - \gamma\rho\,(i_t - r). \tag{A.4}$$

Expected inflation over the term τ is then given by the expectation of

$$\begin{aligned}
\pi_0^\tau &= \frac{p_\tau - p_0}{\tau} \\
&= \frac{1}{\tau}\int_0^\tau \frac{dp}{dt}\,dt \\
&= \frac{\tau}{\tau}\pi_0 - \frac{\gamma\rho}{\tau}\int_0^\tau (i_t - r)\,dt \\
&= \pi_0 - \gamma\rho\left[\frac{1}{\tau}\int_0^\tau i_t\,dt - r\right] \\
&= \pi_0 - \gamma\rho\left[i_0^\tau - k_\tau - r\right] \quad \text{equation (4) in the text} \\
&= \pi_0 - \gamma\rho\left[i_0 + (1-w_\tau)(\pi_0 + r - i_0) - r\right] \quad \text{from } (5') \\
&= \pi_0 + \gamma\rho\left[r - i_0\right] - \gamma\rho\,(1-w_\tau)(\pi_0 + r - i_0).
\end{aligned} \tag{A.5}$$

This is the expression for the τ-period inflation rate. Following Mishkin, however, we wish to predict the *difference* between the τ_2-period and τ_1-period inflation rates:

$$\pi_0^{\tau_2} - \pi_0^{\tau_1} = \gamma\rho\left(w_{\tau_2} - w_{\tau_1}\right)\left(\pi_0^e + r - i_0\right). \tag{A.6}$$

This shows that the expected change in the inflation rate is simply proportionate to $(\pi_0 + r - i_0)$, which is the steepness of the yield curve. This is the proposition claimed at the end of section 2 and implemented in section 3.

DATA APPENDIX

The interest rates used in this paper are monthly averages of daily figures and are from the Citibase data bank. The data also appear in the Federal Reserve System Release G.13, and in the Federal Reserve Bulletin, Table 1.35. The following rates are used, based on availability:

1960:01 - 1969:06: federal funds rate (adjusted to be comparable to bond rates), the 3- and 6-month Treasury bill rates from the Secondary Market (adjusted to be comparable to bond rates), the 1-year, 3-year, 5-year, 10-year, and 20-year Treasury bond rates.

1969:07 - 1976:05: all of the above plus the 7-year Treasury bond rate.

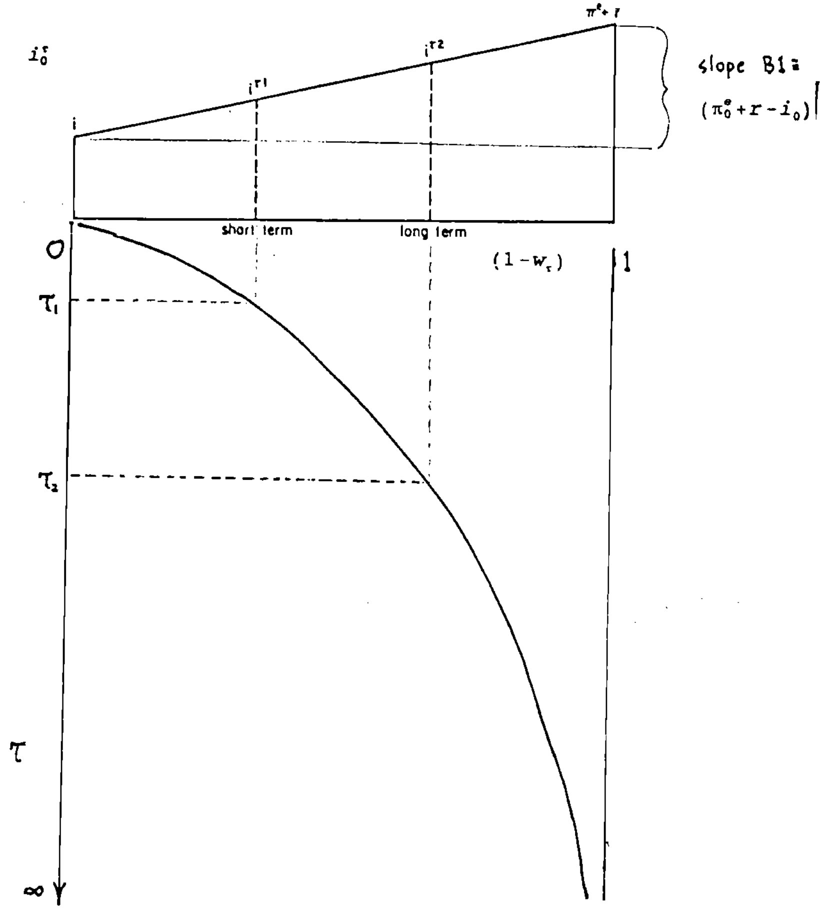
1976:06 - 1977:02: all of the above plus the 2-year Treasury bond rate.

1977:03 - 1986:12: all of the above plus the 30-year Treasury bond rate.

1987:01 - 1988:12: all of the above minus the 20-year Treasury bond rate.

The inflation rates used in this paper are constructed from the seasonally adjusted consumer price index, which can be obtained from the Citibase data bank or from the U.S. Department of Labor, Bureau of Labor Statistics.

FIGURE 1.—EXTRAPOLATION FROM INTEREST RATES TO THE EXPECTED INFLATION RATE



The upper graph shows the relationship between interest rates along the yield curve and the weight $[1-w_\tau]$, as in equation (5'), $i_0^\tau = k_\tau + i_0 + (\pi_0^e + r - i_0)[1-w_\tau]$.

The lower graph shows the nonlinear relationship between the term to maturity τ, and the weight, given by $[1-w_\tau] = [1 - \frac{1-\exp(-\delta\tau)}{\delta\tau}]$.

Table 1

Forecasting the Inflation Spread (12 month - 3 month)

from Interest Rate Term Spreads

Monthly Observations, January 1960 - December 1988

	Mishkin Approach	Alternative Approach
Interest rate term spread	12-month minus 3-month	5-year minus overnight fed. funds
Constant term	- 0.46	- 0.25
t Statistic corrected*	- 2.06	- 1.58
uncorrected	- 3.61	- 2.83
Coefficient of term spread	0.74	0.34
t Statistic corrected*	2.37	3.83
uncorrected	3.92	6.35
Adjusted R^2	.040	.102

Note: Regression is $(\pi^{12}-\pi^{3})_t = a+b(i^{\tau 2}-i^{\tau 1})_t+u_t$, where $(\pi^{12}-\pi^{3})_t$ is the difference between the 12-month and 3-month (realized) inflation rates from time "t" forward, and $(i^{\tau 2}-i^{\tau 1})_t$ is the interest rate spread at time "t" (12-month minus 3-month, or else 5-year minus overnight federal funds rate, as the case may be).

* t-statistics corrected for the moving average error process (introduced by overlapping monthly observations of year-ahead forecasts). The uncorrected t-statistics are biased upwards.

Table 2a:

Forecasting the Inflation Spread from the Steepness of the Yield Curve Along its Entire Length

Monthly observations: Jan. 1960 - Dec. 1988

$(\pi^{12} - \pi^{3})_t = a + b(B1_t) + u$, where $(B1_t)$ is the steepness of the yield curve

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Interest rates expressed in log form. In estimating B1, each interest rate is first expressed as a deviation from the sample mean for its term, an estimate of k_τ.

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constant term	0.054
standard error (uncorrected)	0.088
t statistic (uncorrected)	0.613
t statistic (corrected*)	0.358
coefficient	27.966
standard error (uncorrected)	4.200
t statistic (uncorrected)	6.659
t statistic (corrected*)	3.985

R^2: .114. Adjusted R^2: .111

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* t-statistics corrected for the moving average error process (introduced by overlapping monthly observations of year-ahead forecasts). The uncorrected t-statistics are biased upwards.

Table 2b:

Forecasting the Inflation Spread from the Steepness of the Yield Curve Along its Entire Length

Monthly observations:
Jan. 1960 - Dec. 1988

$$(\pi^{12} - \pi^{3})_t = a + b(B1_t) + u,$$

where

$(B1_t)$ is the steepness of the yield curve

Interest rates expressed in percent form

constant term	-0.346
standard error (uncorrected)	0.094
t statistic (uncorrected)	-3.684
t statistic (corrected*)	-2.113
coefficient	0.246
standard error (uncorrected)	0.039
t statistic (uncorrected)	6.241
t statistic (corrected*)	3.684

R^2: .101. Adjusted R^2: .099

Interest rates expressed in log form

constant term	-0.335
standard error (uncorrected)	0.093
t statistic (uncorrected)	-3.596
t statistic (corrected*)	-2.065
coefficient	26.807
standard error (uncorrected)	4.288
t statistic (uncorrected)	6.252
t statistic (corrected*)	3.690

R^2: .102. Adjusted R^2: .099

* t-statistics corrected for the moving average error process (introduced by overlapping monthly observations of year-ahead forecasts). The uncorrected t-statistics are biased upwards.

Table 3a

Forecasting the Inflation Spread (12 month - 3 month)
on Two Subperiods
Monthly Observations

Interest rates expressed in log form.

In estimating B1, each interest rate is first expressed as a deviation from the sample mean for its term, as an estimate of k_t.

Measure of Slope of Yield Curve	Mishkin Approach: 12-month minus 3-month spread	Alternative Approach: 5-year minus overnight spread	Alternative Approach: Nonlinear measure of steepness B1
January 1960 to Sept. 1979			
Constant term	-0.083	-0.257	0.128
t Statistic uncorrected	-.572	-.272	1.391
corrected*	-.326	-.149	0.746
Coefficient of slope	43.517	27.826	18.055
t Statistic uncorrected	1.533	4.030	3.389
corrected*	.906	2.280	1.890
R^2	.010	.065	.047
adjusted R^2	.006	.061	.043
Oct. 1979 to Dec. 1988			
Constant term	-1.263	-0.733	-0.466
t Statistic uncorrected	-4.956	-3.940	-2.647
corrected*	-3.158	-2.823	-1.955
Coefficient of slope	136.077	48.334	36.318
t Statistic uncorrected	4.232	4.657	5.222
corrected*	2.752	3.244	3.641
R^2	.141	.166	.200
adjusted R^2	.133	.158	.193

* t-statistics corrected for the moving average error process (introduced by overlapping monthly observations of year-ahead forecasts). The uncorrected t-statistics are biased upwards.

Table 3b

Forecasting the Inflation Spread (12 month - 3 month) on Two Subperiods
Monthly Observations

Interest rates expressed in percent form

Measure of Slope of Yield Curve	Mishkin Approach	Alternative Approach	
	12-month minus 3-month spread	5-year minus overnight spread	Nonlinear measure of steepness B1
January 1960 to Sept. 1979			
Constant term	-0.08	-0.02	-0.08
t Statistic uncorrected	-.54	-.23	-.75
corrected*	-.31	-.13	-.41
Coefficient of slope	0.40	0.26	1.00
t Statistic uncorrected	1.50	4.10	2.84
corrected*	.88	2.32	1.58
R^2	.009	.067	.033
adjusted R^2	.005	.063	.029
Oct. 1979 to Dec. 1988			
Constant term	-1.23	-0.71	-1.16
t Statistic uncorrected	-4.88	-3.86	-5.42
corrected*	-3.10	-2.77	-3.97
Coefficient of slope	1.20	0.43	4.17
t Statistic uncorrected	4.15	4.66	5.31
corrected*	2.69	3.25	3.74
R^2	.136	.166	.205
adjusted R^2	.128	.158	.198

* t-statistics corrected for the moving average error process (introduced by overlapping monthly observations of year-ahead forecasts). The uncorrected t-statistics are biased upwards.

[now omitted] Table 4

Forecasting the Inflation Spread (12 month - 3 month)
on Two Subperiods
Monthly Observations

Interest rates expressed in log form

	Mishkin Approach	Alternative Approach	
Measure of Slope of Yield Curve	12-month minus 3-month spread	5-year minus overnight spread	Nonlinear measure of steepness B1
January 1960 to Sept. 1979			
Constant term	-0.083	-0.257	-0.059
t Statistic uncorrected	-.572	-.272	-.599
corrected*	-.326	-.149	-.326
Coefficient of slope	43.517	27.826	19.887
t Statistic uncorrected	1.533	4.030	3.694
corrected*	.906	2.280	2.069
R^2	.010	.065	.055
adjusted R^2	.006	.061	.051
Oct. 1979 to Dec. 1988			
Constant term	-1.263	-0.733	-0.926
t Statistic uncorrected	-4.956	-3.940	-4.787
corrected*	-3.158	-2.823	-3.468
Coefficient of slope	136.077	48.334	36.500
t Statistic uncorrected	4.232	4.657	-5.205
corrected*	2.752	3.244	3.619
R^2	.141	.166	.199
adjusted R^2	.133	.158	.192

* t-statistics corrected for the moving average error process (introduced by overlapping monthly observations of year-ahead forecasts). The uncorrected t-statistics are biased upwards.