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PATH DEPENDENCE IN AGGREGATE OUTPUT

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ABSTRACT

This paper studies an economy in which incomplete markets and strong complementarities interact to generate path dependent aggregate output fluctuations. An economy is said to be path dependent when the effect of a shock on the level of aggregate output is permanent in the absence of future offsetting shocks. Extending the model developed in Durlauf [1991(a),(b)], we analyze the evolution of an economy which consists of a countable infinity of industries. The production functions of individual firms in each industry are nonconvex and are linked through localized technological complementarities. The productivity of each firm at t is determined by the production decisions of technologically similar industries at t-1. No markets exist which allow firms and industries to exploit complementarities by coordinating production decisions. This market incompleteness produces several interesting effects on aggregate output behavior. First, multiple stochastic equilibria exist in aggregate activity. These equilibria are distinguished by differences in both the mean and the variance of output. Second, output movements are path dependent as aggregate productivity shocks indefinitely affect real activity by shifting the economy across equilibria. Third, when aggregate shocks are recurrent, the economy cycles between periods of boom and depression. Simulations of example economies illustrate how market incompleteness can produce rich aggregate dynamics.

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Introduction

Recent developments in theoretical macroeconomics have emphasized the potential for multiple, Pareto-rankable equilibria to exist for economies where various Arrow-Debreu assumptions are violated. Authors such as Diamond [1982] and Cooper and John [1988] have emphasized how incomplete markets can lead to coordination failure as economies may become trapped in Pareto-inferior equilibria; Heller [1986, 1990] and Murphy, Shleifer and Vishny [1989] have obtained similar results due to imperfect competition. These different approaches share the idea that strong complementarities in behavior can lead to multiplicity. Intuitively, when technological or demand spillovers make agents sufficiently interdependent, high and low levels of activity can represent internally consistent equilibria in the absence of complete, competitive markets. Most of these models describe multiple steady states in economies rather than multiple nondegenerate time series paths and consequently cannot address issues of aggregate fluctuations.[1] Further, this literature has not shown how economies can shift across equilibria, inducing periods of boom and depression.

A separate empirical literature has concluded that output fluctuations are strongly persistent. Researchers such as Campbell and Mankiw [1987], Durlauf [1989] and Phillips [1990] have concluded from a variety of perspectives that aggregate output in advanced industrialized economies contains a unit root. Perron [1989], on the other band, has argued that real GNP data reflects one or two trend breaks. One interpretation of this result is that the probability density characterizing innovations to a stochastic trend places a large weight on zero. Hamilton [1988] finds evidence of persistence in the sense that the mean of permanent output movements is a function of whether the economy is in a state of boom or recession. Despite differences in both the methodologies and conclusions of work on output persistence, this literature has generally concluded that

[1]Important exceptions to this claim are Diamond and Fudenberg [1990], which describes how self-fulfilling expectations lead to cycles in search models and Heller [1990] which models multiple capital accumulation paths in models with imperfect competition.

long run forecasts of real activity are strongly dependent on some part of contemporary fluctuations.

One interpretation of the many results on output persistence is that real activity is ***path dependent***—the long term behavior of output is affected by the sample path realization of the economy. This notion has been employed to understand such phenomena as the evolution of particular technologies (David [1986,1988], Arthur [1989]), the distribution of trading patterns (Krugman [1990,1991]) and the emergence of multiple equilibria in aggregate activity (Durlauf [1991(a),(b)]). David [1988] argues that path dependent models can provide a general framework for integrating economic theory with economic history. The literature on path dependence has generally argued that the realized pattern of economic activity induces intertemporal complementarities in production, which leads to multiple time series paths for the same economy. As such, this literature contains ideas very similar to the work on multiple equilibria and coordination failure.

One definition of path dependence, which we employ in this paper, is as follows. Suppose that aggregate output in the economy, Y_t, is a measurable function of some set of exogenous variables.[2] Denote the σ-algebra characterizing the history of these exogenous variables as $\Im_t$ which means that $Y_t \in \Im_t$. Innovations to the exogenous variables lie in the changes in the sequence of σ-algebras, i.e. $\Im_{t+1} - \Im_t$. Further, suppose that the stochastic process characterizing the exogenous variables has the property that for all t greater than some fixed date T, $\Im_{t+1} - \Im_t = 0$, which means that no new innovations affect the economy after T. Aggregate output is path dependent if

$$\lim_{s \Rightarrow \infty} Prob(\mathrm{E}(Y_{T+s} \mid \Im_T) - \mathrm{E}(Y_{T+s} \mid \Im_0) = 0) < 1. \tag{1}$$

This definition says that the particular sample path realization of a sequence of

[2]Observe that the mapping from the exogenous variables to Y_t will generally be a function of the stochastic process governing the exogenous variables.

innovations can have indefinite effects on real activity. One may verify that models with unit roots, trend breaks, or state dependent growth rates are all path dependent according to this definition. At the same time, the definition incorporates stationary, nonergodic economies as well as economies which shift between equilibria. Path dependent economies exhibit substantial output persistence as the effects of an economy's realized sample path can have permanent effects in the absence of offsetting future shocks.

The purpose of the current paper is to understand the implications of models of multiple equilibria for path dependence. We do this by modelling coordination problems in an explicitly stochastic framework. As developed in Durlauf [1991(a),(b)], the microeconomic specification of the economy is expressed as a set of conditional probability measures describing how individual agents behave given the economy's history. An aggregate equilibrium exists when one can find a joint probability measure over all agents which is consistent with these conditional measures; multiplicity occurs when more than one such measure exists. This approach, by expressing the equilibrium of the economy as a stochastic process, permits one to describe directly the time series properties of aggregate fluctuations along different equilibrium paths.

Specifically, we examine the capital accumulation problems of a set of infinitely-lived industries. We deviate from standard analyses in two respects. First, firms in each industry face a nonconvex production technology. Second, industries experience technological complementarities as past high production decisions by each industry increase the current productivity of several industries. Learning-by-doing is one example of this phenomenon. Industries do not coordinate production decisions because of incomplete markets. By describing how output levels and productivity evolve as industries interact over time, the model characterizes the impact of complementarities and incomplete markets on the structure of aggregate fluctuations.

Our basic results are threefold. First, we show that with strong complementarities and incomplete markets, multiple stochastic equilibria can exist in aggregate activity. These equilibria are distinguished by differences in both the mean and the variance of output. Second, we illustrate how aggregate output movements will be persistent as aggregate productivity shocks indefinitely affect real activity by shifting the economy across equilibria. Third, we provide conditions on the aggregate productivity shocks which will cause the economy to cycle across equilibria. Although the current model does not exhibit a unit root, one will emerge if deterministic technical change is introduced.

Section I of this paper outlines the evolution of an economy composed of a large set of industries whose production functions are linked by localized intertemporal complementarities. Section II describes how multiple equilibria can arise when the economy experiences industry-specific shocks. Section III explores the implications of aggregate or economy-wide shocks for path dependence. Section IV simulates several examples of the economy to see what sort of time series patterns emerge in aggregate output. Section V contains summary and conclusions. A Technical Appendix follows which contains proofs of all Theorems.

I. A model of interacting industries

Consider a countable infinity of industries indexed by i.[3] Each industry consists of many small, identical firms. All firms produce a homogeneous good; industries are distinguished by distinct production functions rather than distinct outputs. The homogeneous final good may be consumed by the owners of the firms or converted to a capital good which fully depreciates after one period. Industry i's behavior is

[3] Durlauf [1991(a)] derives a general equilibrium version of this economy where consumers are risk neutral, as the expected utility of consumer r takes the form

$$U_{r,t} = \mathrm{E}(\sum_{j=0}^{\infty} \beta^j C_{r,t+j} \mid \mathfrak{F}_t).$$

When agents are risk neutral, the weights β^j correspond to date-zero Arrow-Debreu prices. Our model is therefore a variant of the economy analyzed in Brock and Mirman [1972].

proportional to the behavior of a representative firm which chooses a capital stock sequence $\{K_{i,t}\}$ to maximize the present discounted value of profits $\Pi_{i,t}$

$$\Pi_{i,t} = E(\sum_{j=0}^{\infty} \beta^j (Y_{i,t+j} - K_{i,t+j}) \mid \mathfrak{F}_t). \tag{2}$$

$Y_{i,t}$ equals the output of the i'th industry's representative firm at t; β equals a time invariant one-period discount rate; Initial endowments $Y_{i,0}$ provide starting capital.

Aggregate behavior is determined by the interactions of many heterogeneous industries employing nonconvex technologies. Production occurs with a one period lag; firms at $t-1$ employ both one of two production techniques and a level of capital to determine output at t. Only one technique may be used at a time. Cooper [1987] originally introduced this production function to model coordination problems; Murphy, Shleifer, and Vishny [1989] exploit a similar technology to analyze multiple equilibria in economic development. Milgrom and Roberts [1990] discuss how these sorts of nonconvexities can arise as firms internally coordinate many complementary activities to achieve efficiency. The technique-specific production functions produce $Y_{1,i,t}$ and $Y_{2,i,t}$ through

$$Y_{1,i,t} = f_1(K_{i,t-1} - F, \zeta_{i,t-1}, \xi_{t-1})$$

$$Y_{2,i,t} = f_2(K_{i,t-1}, \eta_{i,t-1}, \xi_{t-1}). \tag{3}$$

$\zeta_{i,t}$ and $\eta_{i,t}$ are industry-specific productivity shocks; ξ_t is an aggregate productivity shock and F is a fixed overhead capital cost. $\zeta_{i,t-1}$, $\eta_{i,t-1}$, and ξ_{t-1} are elements of $\mathfrak{F}_{t-1}$. Recalling that firms within an industry are identical, we define a technique choice variable $\omega_{i,t}$ which equals 1 if technique 1 is used by industry i at t, 0 otherwise and $\underline{\omega}_t = \{\ldots\omega_{i-1,t}, \omega_{i,t}, \omega_{i+1,t}\ldots\}$ which equals the joint set of techniques employed at t.

We place several restrictions on the production technologies. First, each technique fulfills standard curvature conditions. Further, we associate technique 1 with high production. Specifically, net capital $NK_{i,t}$, which equals $K_{i,t}-F$ for technique 1 and $K_{i,t}$ for technique 2, has a strictly higher marginal (and by implication total) product when used with technique 1 than technique 2. A firm chooses technique 1 if it is willing to pay fixed capital costs in exchange for higher output.

Assumption 1. Restrictions on technique-specific production functions

$f_1(NK,\zeta_{i,t},\xi_t)$ and $f_2(NK,\eta_{i,t},\xi_t)$ are measurable functions of $\zeta_{i,t}$, $\eta_{i,t}$, ξ_t, and NK such that

A. $f_1(0,\zeta_{i,t},\xi_t) = f_2(0,\eta_{i,t},\xi_t) = 0.$

B. $\dfrac{\partial f_1(NK,\zeta_{i,t},\xi_t)}{\partial NK} \geq 0,\ \dfrac{\partial f_2(NK,\eta_{i,t},\xi_t)}{\partial NK} \geq 0;\ \dfrac{\partial^2 f_1(NK,\zeta_{i,t},\xi_t)}{\partial NK^2} \leq 0,\ \dfrac{\partial^2 f_2(NK,\eta_{i,t},\xi_t)}{\partial NK^2} \leq 0.$

C. $\dfrac{\partial f_1(0,\zeta_{i,t},\xi_t)}{\partial NK} = \dfrac{\partial f_2(0,\eta_{i,t},\xi_t)}{\partial NK} = \infty;\ \dfrac{\partial f_1(\infty,\zeta_{i,t},\xi_t)}{\partial NK} = \dfrac{\partial f_2(\infty,\eta_{i,t},\xi_t)}{\partial NK} = 0.$

D. $\dfrac{\partial f_1(NK,\zeta_{i,t},\xi_t)}{\partial NK} > \dfrac{\partial f_2(NK,\eta_{i,t},\xi_t)}{\partial NK}.$

Both techniques are assumed to exhibit technological complementarities, as the history of realized activity determines the parameters of the production function at t. Romer's [1986] model of social increasing returns shares this feature. Arrow [1962] postulated that these types of productivity spillovers could occur due to learning-by-doing. Our specification of complementarities differs from Romer's in two respects. First, all complementarities are local as the production function of each firm is affected by the production decisions of a finite number of industries. The index i orders industries by similarity in technology; spillovers occur only between similar technologies. David [1988] and Rosenberg [1982] describe the historical importance of local complementarities in the evolution of technical innovations. Second, our complementarities are explicitly dynamic. Past production decisions affect current productivity, which captures the idea of learning-by-doing.

Specifically, we model the complementarities through the dependence of the productivity shocks $\zeta_{i,t}$ and $\eta_{i,t}$ on the history of technique choices. This form for the complementarities is appropriate when the amount of time spent at an activity is the appropriate metric for the rate of learning-by-doing. These intertemporal complementarities are assumed to be the only source of dependence across shocks. In addition, the aggregate productivity shocks obey a Markov process.[4] $Prob(x \mid y)$ denotes the conditional probability measure of x given information y; $\Delta_{k,l} = \{i-k \ldots i \ldots i+l\}$ indexes the industries which affect industry i's productivity.

Assumption 2. Conditional probability structure of productivity shocks

A. $Prob(\zeta_{i,t} \mid \mathfrak{F}_{t-1}) = Prob(\zeta_{i,t} \mid \omega_{j,t-1}\ \forall\ j\ \in \Delta_{k,l})$.

B. $Prob(\eta_{i,t} \mid \mathfrak{F}_{t-1}) = Prob(\eta_{i,t} \mid \omega_{j,t-1}\ \forall\ j\ \in\ \Delta_{k,l})$.

C. $Prob(\xi_t \mid \mathfrak{F}_{t-1}) = Prob(\xi_t \mid \xi_{t-1})$.

D. The random pairs $\{\zeta_{i,t} - \mathrm{E}(\zeta_{i,t} \mid \mathfrak{F}_{t-1}), \eta_{i,t} - \mathrm{E}(\eta_{i,t} \mid \mathfrak{F}_{t-1})\}$ *are mutually independent of each other and of* $\xi_t - \mathrm{E}(\xi_t \mid \mathfrak{F}_{t-1})\ \forall\ i$.

Markets are assumed to be missing in the sense that individual firms cannot coordinate to exploit complementarities. Consequently, no industry may be compensated for choosing technique 1 in order to expand the production sets of other industries; nor,

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[4]This assumption is made for technical convenience; in particular, all of our results still hold if there is feedback from the level of real activity at $t-1$ to ξ_t.

given our conceptualization of industries as aggregates of many small producers, can firms within an industry strategically choose a technique in order to induce higher future productivity through complementarities. Further, firms are assumed to be unable to combine under joint management in order to internalize the complementarities.

It is straightforward to verify, from standard dynamic programming arguments, that profit maximization by each firm implies that $K_{i,t}$ is chosen to solve

$$\max_{K_{i,t}} \quad sup\Big(\beta f_1(K_{i,t}-F,\zeta_{i,t},\xi_t)-K_{i,t},\beta f_2(K_{i,t},\eta_{i,t},\xi_t)-K_{i,t}\Big) \tag{4}$$

so long as the $K_{i,t}$ is feasible $\forall$ i. These capital choices are feasible whenever aggregate output is sufficiently large at $t-1$. Without loss of generality, we place an additional restriction on the level of output produced each period which ensures that the supply of potential capital is as least as great as the demand implied by eq. (4) in all periods, which renders the economy stationary.

Assumption 3. Lower bounds on available capital

For all realizations of $\zeta_{i,t}$, $\eta_{i,t}$ and ξ_t, $\sum_{i=-\infty}^{\infty} Y_{i,t} > \sum_{i=-\infty}^{\infty} \overline{K}^1_{i,t}(\beta)$, *where* $\overline{K}^1_{i,t}(\beta)$ *fulfills* $1 = \dfrac{\partial \beta f_1(\overline{K}^1_{i,t}(\beta)-F,\zeta_{i,t},\xi_t)}{\partial NK}$.

Under Assumptions 1-3, it is straightforward to verify that the technique choice $\omega_{i,t}$ is a stationary and measurable function of $\zeta_{i,t}$, $\eta_{i,t}$ and ξ_t. Further, Assumption 2 places strong restrictions on the conditional technique choice probabilities.

Theorem 1. Structure of conditional technique choice probabilities

The conditional technique choice probabilities obey stationary measures of the form

$$Prob(\omega_{i,t}\,|\,\Im_{t-1}) = Prob(\omega_{i,t}\,|\,\omega_{j,t-1}\ \forall\ j\ \in \Delta_{k,l},\ \mathrm{E}(\xi_t\,|\,\Im_{t-1})). \tag{5}$$

Once technique choices are determined, one can solve for the optimal levels of capital and output for each firm. In fact, a sufficient condition for the existence of equilibrium capital and output sequences for all firms is the existence of a joint probability measure over all technique choices which is consistent with the conditional measures in Theorem 1. To see this, observe that the optimal choices of output and capital for all industries at all dates obey the same conditional probability structure as the technique choices,

$$Prob(\omega_{i,t}, Y_{i,t}, K_{i,t}\,|\,\Im_{t-1}) = Prob(\omega_{i,t}, Y_{i,t}, K_{i,t}\,|\,\omega_{j,t-1}\ \forall\ j\ \in \Delta_{k,l},\ \mathrm{E}(\xi_t\,|\,\Im_{t-1})), \tag{6}$$

which means that the existence of a joint measure over the technique choices is equivalent to the existence of a joint measure over all output and capital decisions.

The existence of an equilibrium may therefore be verified once it is established that the initial conditions and transition probabilities in this economy always generate a joint probability measure over $\{\underline{\omega}_0, \underline{\omega}_1, \ldots \underline{\omega}_\infty\}$, the set of technique choices over all industries and all dates. The existence of an equilibrium can therefore be reduced to the question of when a set of conditional probabilities may be extended to define a joint probability measure over a set of random variables indexed by $\mathbf{Z}^2$, the two-dimensional lattice of integers. Dobrushin [1968] has given conditions characterizing when such an extension exists.[5] The localized structure of our complementarities ensures that Dobrushin's criteria are satisfied, which leads to Theorem 2.

Theorem 2. Existence of aggregate equilibrium

[5]The Kolmogorov Extension Theorem cannot be directly applied since we are working with conditional probabilities rather than unconditional probabilities over all finite sets of the stochastic process $\{\underline{\omega}_0, \underline{\omega}_1, \ldots \underline{\omega}_\infty\}$. Unlike the Kolmogorov Extension Theorem, Dobrushin's Theorem does not show the joint measure is unique.

For any initial conditions $\underset{\sim}{\omega}_0$ and specification of conditional probabilities over technique choices consistent with Theorem 1, there exists at least one joint probability measure over $\{\underset{\sim}{\omega}_0, \underset{\sim}{\omega}_1, \ldots \underset{\sim}{\omega}_\infty\}$.

II. Long run behavior under industry-specific shocks

In order to see how industry-specific and economy-wide shocks interact to affect the aggregate equilibrium, we first consider the case where $\xi_t = 0$, no economy-wide shocks exist.

We restrict the conditional probabilities in order to discuss multiplicity and dynamics. Past choices of technique 1 are assumed to improve the current relative productivity of the technique. As a result, technique 1 choices will propagate over time. Further, we assume that $\underset{\sim}{\omega}_t = \underset{\sim}{1}$ is a steady state, which means that when all productivity spillovers are active, the effects are so strong that high production is always optimal.

Assumption 4. Impact of past technique choices on current technique probabilities[6]

Let $\underset{\sim}{\omega}$ and $\underset{\sim}{\omega}'$ denote two realizations of $\underset{\sim}{\omega}_{t-1}$. If $\omega_j \geq \omega'_j \ \forall \ j \in \Delta_{k,l}$, then

A. $Prob(\omega_{i,t} = 1 \mid \omega_{j,t-1} = \omega_j \ \forall \ j \in \Delta_{k,l}) \geq Prob(\omega_{i,t} = 1 \mid \omega_{j,t-1} = \omega'_j \ \forall \ j \in \Delta_{k,l})$.

B. $Prob(\omega_{i,t} = 1 \mid \omega_{j,t-1} = 1 \ \forall \ j \in \Delta_{k,l}) = 1$.

Whenever some industry chooses $\omega_{i,t} = 0$, a positive productivity feedback is lost. Different configurations of choices at $t-1$ determine different production sets and

[6]This assumption can be reformulated in terms of restrictions on the technique-specific production functions.

conditional technique choice probabilities for each industry. We bound the technique choice probabilities from below and above by $\Theta_{k,l}^{min}$ and $\Theta_{k,l}^{max}$ respectively.

$$\Theta_{k,l}^{min} \leq Prob(\omega_{i,t} = 1 \mid \omega_{j,t-1} = 0 \text{ for some } j \in \Delta_{k,l}) \leq \Theta_{k,l}^{max} \tag{7}$$

Since $\underline{\omega}_t = \underline{1}$ is an equilibrium, the aggregate economy exhibits multiple equilibria if for some initial conditions, $\underline{\omega}_t = \underline{1}$ fails to emerge as t grows. Notice that even if $\underline{\omega}_0 = \underline{0}$, favorable productivity shocks will periodically induce industries to produce using technique 1. The choice of technique 1 by one industry, through the complementarities, increases the probability that the technique is subsequently chosen in several industries. With strong spillovers, these effects may build up, allowing $\underline{\omega}_t = \underline{1}$ to emerge from any initial conditions. The model therefore allows us to analyze the stability of a high aggregate output equilibrium from arbitrary initial conditions.

In fact, the limiting behavior of the economy is determined by the bounds $\Theta_{k,l}^{min}$ and $\Theta_{k,l}^{max}$. If the probability of high production by an industry is sufficiently large for all production histories, then the spillover effects induced by spontaneous technique 1 choices cause the economy to iterate towards high production. Alternatively, if technique 1 probabilities are too low in the absence of active spillovers, spontaneous technique 1 choices will not generate sufficient momentum to achieve the $\underline{\omega}_t = \underline{1}$ equilibrium. $\Theta_{k,l}^{min}$ and $\Theta_{k,l}^{max}$ bound the degree of complementarity in the economy. Large values of $\Theta_{k,l}^{min}$ imply complementarities are weak as technique 1 is chosen relatively frequently regardless of the past. Conversely, small values of $\Theta_{k,l}^{max}$ imply strong complementarities; the probability of current high production is very sensitive to past technique choices. Theorem 3 shows how long run industry behavior is jointly determined by initial conditions and conditional technique probabilities.

Theorem 3. Conditions for uniqueness versus multiplicity of long run equilibrium

For every nonnull index set $\Delta_{k,l}$, there exist numbers $0 < \underline{\Theta}_{k,l} < \overline{\Theta}_{k,l} < 1$ such that

A. If $\Theta_{k,l}^{max} \leq \underline{\Theta}_{k,l}$, then $\lim_{t \Rightarrow \infty} Prob(\omega_{i,t} = 1 \mid \underline{\omega}_0 = \underline{0}) < 1$.[7]

If complementarities are sufficiently strong, no industry converges to the high production technique almost surely from economy-wide low production technique initial conditions.

B. If $\Theta_{k,l}^{min} \geq \overline{\Theta}_{k,l}$, then $\lim_{t \Rightarrow \infty} Prob(\omega_{i,t} = 1 \mid \underline{\omega}_0 = \underline{0}) = 1$.

If complementarities are sufficiently weak, each industry converges to the high production technique almost surely from economy-wide low production technique initial conditions.

One can associate $\underline{\omega}_t=\underline{1}$ with the equilibrium which would emerge if all firms chose their production levels cooperatively. If production through technique 1 is sufficiently higher for $\underline{\omega}_t=\underline{1}$ versus any other configuration, then $\underline{\omega}_t=\underline{1}$ emerges as the cooperative equilibrium after one period, as firms and industries will all choose technique 1 at $t = 1$ in order to achieve the productivity spillovers. Consequently, incompleteness of markets lowers the mean and increases the variance of industry and aggregate output along the inefficient equilibrium path, as technique choices fluctuate over time. When industries fail to coordinate, production decisions become dependent on idiosyncratic productivity shocks. Observe that the volatility associated with the inefficient equilibrium is caused by fundamentals. Simulations in Durlauf [1991(a)] show that aggregate output can obey a wide range of AR processes, depending on the values of the transition probabilities.

III. Path dependence and economy-wide shocks

[7] One can also show that $\lim_{t \Rightarrow \infty} Prob(\underline{\omega}_t = 1 \mid \underline{\omega}_0 = \underline{0}) = 0$, the economy almost surely fails to converge to the high production equilibrium.

Now consider the role of the economy-wide shocks ξ_t. By affecting many industries simultaneously, these shocks act in a way analogous to changing the initial conditions of the economy. Path dependence occurs as one realization of ξ_t permanently changes the equilibrium in the absence of future offsetting shocks. In order to illustrate path dependence, it is necessary to restrict both the way in which the aggregate shocks interact with the industry production decisions as well as the structure of the aggregate shocks themselves.

First, we assume that sufficiently unfavorable aggregate productivity draws make the choice of technique 1 unlikely whereas sufficiently favorable draws ensure the use of the technique. This means that particular aggregate productivity realizations can have very powerful aggregate output effects.

Assumption 5. Impact of economy-wide shocks on technique choice

There exist numbers a and b, $a < 0 < b$, with $Prob(\xi_t \leq a)$ and $Prob(\xi_t \geq b)$ both nonzero, such that

A. $Prob(\omega_{i,t} = 1 \mid \xi_t \leq a,\ \omega_{j,t-1} = 1\ \forall\ j \in \Delta_{k,l}) \leq \underline{\Theta}_{k,l}$.

B. $Prob(\omega_{i,t} = 1 \mid \xi_t \geq b,\ \omega_{j,t-1} = 0\ \forall\ j \in \Delta_{k,l}) = 1$.

To understand our final assumption, it is useful to express ξ_t (which by Assumption 2.C is Markov) as

$$\xi_t = g(\xi_{t-1}) + \mu_t, \tag{8}$$

where $\mu_t \in \mathfrak{F}_t - \mathfrak{F}_{t-1}$. We restrict $g(\cdot)$ to ensure that if $\mu_t = 0,\ t > T$, then a realization of $\xi_T \leq a$ ($\geq b$) will not be followed by $\xi_{T+k} \geq b$ ($\leq a$) for some k. A general restriction of this type is necessary if an extreme draw of ξ_T is to have lasting effects; Assumption 6 provides a simple sufficient condition.

Assumption 6. Structure of conditional expectation of aggregate productivity shock

If $\xi_t > 0$ *then* $g(\xi_t) \geq 0$; *if* $\xi_t \leq 0$ *then* $g(\xi_t) \leq 0$.[8]

When Assumptions 5 and 6 hold, economy-wide shocks can have an indefinite effect on real activity.

Theorem 4. Path dependence due to economy-wide shocks

Let $\mu_t = 0 \ \forall \ t > T$ *and* $\Theta_{k,l}^{max} \leq \underline{\Theta}_{k,l}$. *The economy exhibits path dependence as the realization of* ξ_T *affects the limiting technique choice probabilities for all industries.*

A. $\lim_{s \Rightarrow \infty} Prob(\omega_{i,T+s} = 1 \mid \xi_T \leq a) < 1$.

B. $\lim_{s \Rightarrow \infty} Prob(\omega_{i,T+s} = 1 \mid \xi_T \geq b) = 1$.

This result shows how economy-wide shocks and consequently aggregate fluctuations can be persistent. Persistence occurs when economy-wide shocks have the effect of introducing new initial conditions in an economy with multiple equilibria. For example, once many sectors simultaneously decline due to an adverse economy-wide shock, productivity enhancing complementarities are lost until a subsequent favorable economy-wide shock restores them.

Several interpretations beyond productivity can be applied to the economy-wide

[8]For example, this definition is fulfilled if $\xi_t = \rho\xi_{t-1} + \mu_t$, $\rho \geq 0$.

shocks. Interpreting ξ_t as a proxy for the financial sector, the model indicates how the breakdown of financial institutions, such as occurred during the Depression, can cause indefinite output loss. Alternatively, Durlauf [1991(a)] shows how ξ_t can represent the cost of production inputs provided by leading sectors such as transportation or steel. In this case, the growth of leading sectors improves the relative profitability of high production, which can lead to a takeoff in growth as the economy shifts across equilibria.

Fluctuations between the high and low equilibria will be triggered by movements in the economy-wide shocks. The properties of ξ_t will determine whether the long run behavior of aggregate output exhibits multiple equilibria in the following sense. When the events $(\xi_t \leq a)$ and $(\xi_t \geq b)$ are recurrent, i.e. ξ_t enters each of the sets $(-\infty, a)$ and (b, ∞) infinitely often, then long run forecasts of the economy are unaffected by history in the sense that any sample path history of the economy will, with probability 1, be reversed by some future realization of the economy-wide shock.

Conversely, if the event $(\xi_t \neq 0)$ is nonrecurrent, then the economy-wide shocks will have permanent effects since the events $(\xi_t \leq a)$ and $(\xi_t \geq b)$ will, with probability 1, occur only a finite number of times. By Theorem 2, in the absence of economy-wide shocks in all periods, the long run behavior of the economy can depend on initial conditions. Further, so long as $Prob(\xi_1 \leq a)$, $Prob(\xi_1 \geq b)$, $Prob(\xi_t = 0 \mid \xi_{t-1} \leq a)$ and $Prob(\xi_t = 0 \mid \xi_{t-1} \geq b)$ are all nonzero, then two different sample path realizations of the same economy can converge to different average levels of output, as either high or low production initial conditions may precede the period when each ξ_t becomes 0. This specific example illustrates the more general proposition that models where initial conditions matter may be thought of as path dependent models with special assumptions on the distributions of certain variables. Theorem 5 summarizes these ideas.

Theorem 5. Economy-wide shocks and the long run properties of aggregate output

Let $\Theta_{k,l}^{max} \leq \underline{\Theta}_{k,l}$.

A. If the events $Prob(\xi_t \leq a)$ and $Prob(\xi_t \geq b)$ are recurrent, then the economy will cycle infinitely often between periods of high and low activity. Long run forecasts of the economy are not history dependent; for fixed T

$$\lim_{s \Rightarrow \infty} \mathrm{E}(\underline{\omega}_{T+s} \mid \mathfrak{F}_T) = \lim_{s \Rightarrow \infty} \mathrm{E}(\underline{\omega}_{T+s} \mid \mathfrak{F}_0). \quad (9)$$

B. If the event $(\xi_t \neq 0)$ is nonrecurrent, and if $Prob(\xi_1 \leq a)$, $Prob(\xi_1 \geq b)$, $Prob(\xi_t = 0 \mid \xi_{t-1} \leq a)$ and $Prob(\xi_t = 0 \mid \xi_{t-1} \geq b)$ are all nonzero, then long run forecasts of the level of aggregate activity are history dependent; for large enough T

$$\lim_{s \Rightarrow \infty} \mathrm{E}(\underline{\omega}_{T+s} \mid \mathfrak{F}_T) \neq \lim_{s \Rightarrow \infty} \mathrm{E}(\underline{\omega}_{T+s} \mid \mathfrak{F}_0). \quad (10)$$

Either equilibrium described in Theorem 3 can emerge.

IV. Time series properties of aggregate output

In this section, we simulate the aggregate economy to see what sort of patterns emerge in aggregate output fluctuations. We simulate economies based on the interaction range $\Delta_{1,1}=\{i-1,i,i+1\}$. In each simulation, we construct a finite approximation to the infinite economy consisting of 500 industries. Output per period by each industry is normalized to equal 0 or 1. A $\{0,1\}$ support for output may be justified by generalizing Assumption 2 to model the two techniques as

$$Y_{1,i,t} = \overline{Y}_1 \text{ if } K_{i,t-1} \geq \overline{K}_1(\zeta_{i,t-1},\xi_{t-1})$$
$$= 0 \text{ } otherwise.$$

$$Y_{2,i,t} = \overline{Y}_2 \text{ if } K_{i,t-1} \geq \overline{K}_2(\eta_{i,t-1},\xi_{t-1})$$

$$= 0 \ otherwise. \tag{11}$$

and then normalizing output.[9] In this specification, each firm produces a fixed output level for each technique, $\overline{Y}_1$ or $\overline{Y}_2$, given a fixed capital input of $\overline{K}_1(\zeta_{i,t-1},\xi_{t-1})$ or $\overline{K}_2(\eta_{i,t-1},\xi_{t-1})$ respectively. Under the specification, the productivity shocks act to affect capital input requirements rather than output.

In constructing the simulations, it is necessary to place some restrictions on the conditional production probabilities. First, we assume that the economy-wide shock ξ_t is a Markov process with state space $\{-1,0,1\}$ and transition matrix $\mathbf{P}$.

We equate the values -1 with the event $(\xi_t \leq a)$ and 1 with the event $(\xi_t \geq b)$ as described in Section III. We correspondingly define the two conditional probabilities

$$Prob(\omega_{i,t} = 1 \mid \omega_{j,t-1} \forall\, j \in \Delta_{k,l},\, \xi_t = -1) = \Theta_3 \tag{12}$$

and

$$Prob(\omega_{i,t} = 1 \mid \omega_{j,t-1} \forall\, j \in \Delta_{k,l},\, \xi_t = 1) = 1. \tag{13}$$

Finally, we reduce the number of relevant transition probability parameters to 3 when the economy-wide shock equals 0 by assuming that the associated conditional probabilities obey

$$Prob(\omega_{i,t} = 1 \mid \sum_{j=-1}^{1} \omega_{i-j,t-1} = 3,\, \xi_t = 0) = 1$$

$$Prob(\omega_{i,t} = 1 \mid \sum_{j=-1}^{1} \omega_{i-j,t-1} = 2,\, \xi_t = 0) = \Theta_1$$

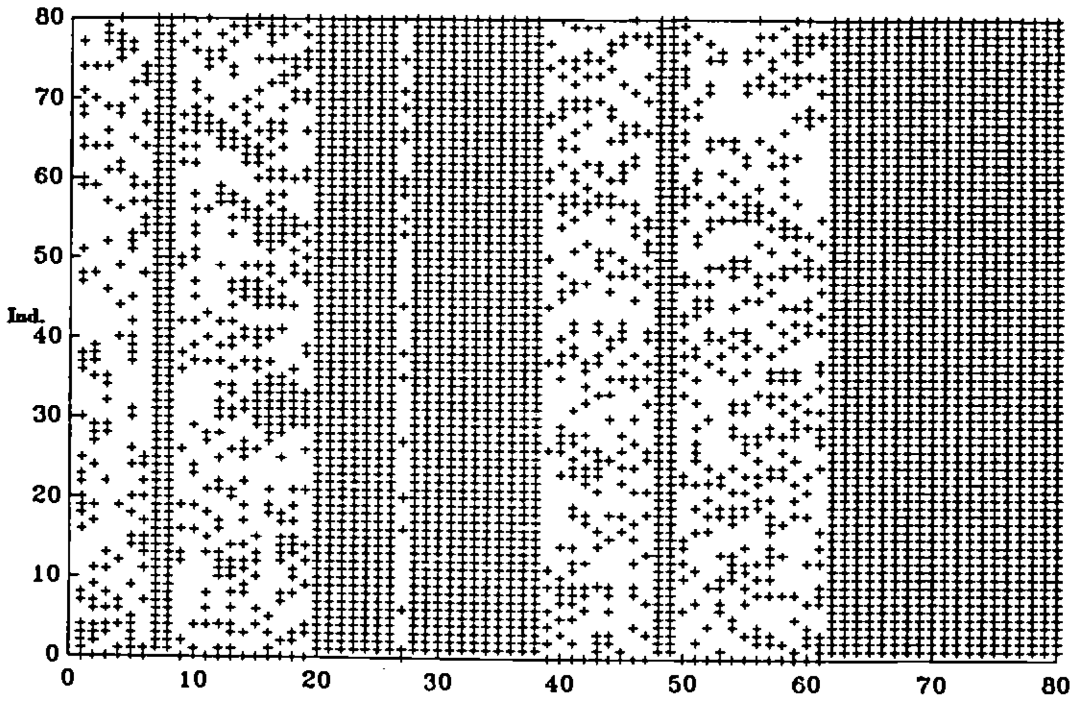
[9]We assume that for all realizations of $\zeta_{i,t-1}$, $\eta_{i,t-1}$, and ξ_{t-1}, 1) $\overline{Y}_1 > \overline{Y}_2$, 2) $\overline{K}_1(\zeta_{i,t-1},\xi_{t-1}) > \overline{K}_2(\eta_{i,t-1},\xi_{t-1})$ 3) $\beta\overline{Y}_2 > \overline{K}_2(\eta_{i,t-1},\xi_{t-1})$, 4) $\overline{Y}_2 > \overline{K}_1(\zeta_{i,t-1},\xi_{t-1})$ in order to preserve the structure of the conditional probability measures of the two techniques as described in Sections II and III.

Figure 1

80-Period Sample Path Realisation for 80-Industry Cross-Section

$\Delta_{1,1}$ Complementarity Range

P_1 transition matrix
$\Theta_1 = .40$
$\Theta_2 = .35$
$\Theta_3 = .30$



+ denotes high production by industry *i* at *t*

Aggregate Output Equation: $Y_t = .15 + .77\,Y_{t-1} + \epsilon_t$

$$Prob(\omega_{i,t} = 1 \mid \sum_{j=-1}^{1} \omega_{i-j,t-1} = 1,\ \xi_t = 0) = \Theta_2$$

$$Prob(\omega_{i,t} = 1 \mid \sum_{j=-1}^{1} \omega_{i-j,t-1} = 0,\ \xi_t = 0) = \Theta_3. \qquad (14)$$

This structure can be interpreted as saying each local complementarity has the same effect on the production function. Simulations of this structure have shown that the model is nonergodic when all transition probabilities are below .45.

These restrictions specify the transition probabilities for all possible technique choice histories. By varying **P** and Θ_i, one can affect the time series properties of the economy. For each simulation, we start with $\underline{\omega}_0 = \underline{0}$ and allow the economy to run for 2000 periods. In each case, a time series is computed for aggregate output. Each regression was was constructed by using the last 1000 observations for all 500 industries.

Our first set of simulations examines the behavior of the economy when the transition probabilities obey

$$\mathbf{P}_1 = \begin{bmatrix} .8 & .1 & .1 \\ .1 & .8 & .1 \\ .1 & .1 & .8 \end{bmatrix} \qquad (15)$$

and $\Theta_1 = .4$, $\Theta_2 = .35$, $\Theta_3 = .3$. This specification has two important features. First, economy-wide shocks are highly correlated. Second, the Θ_i values are such that the economy possesses multiple equilibria when the economy-wide shocks equal 0. A sample path realization of 80 industries over 80 periods is shown in Figure 1. As the Figure indicates, the economy exhibits two separate regimes with substantially different levels of mean activity and output volatility.[10] Notice that even in periods of low aggregate

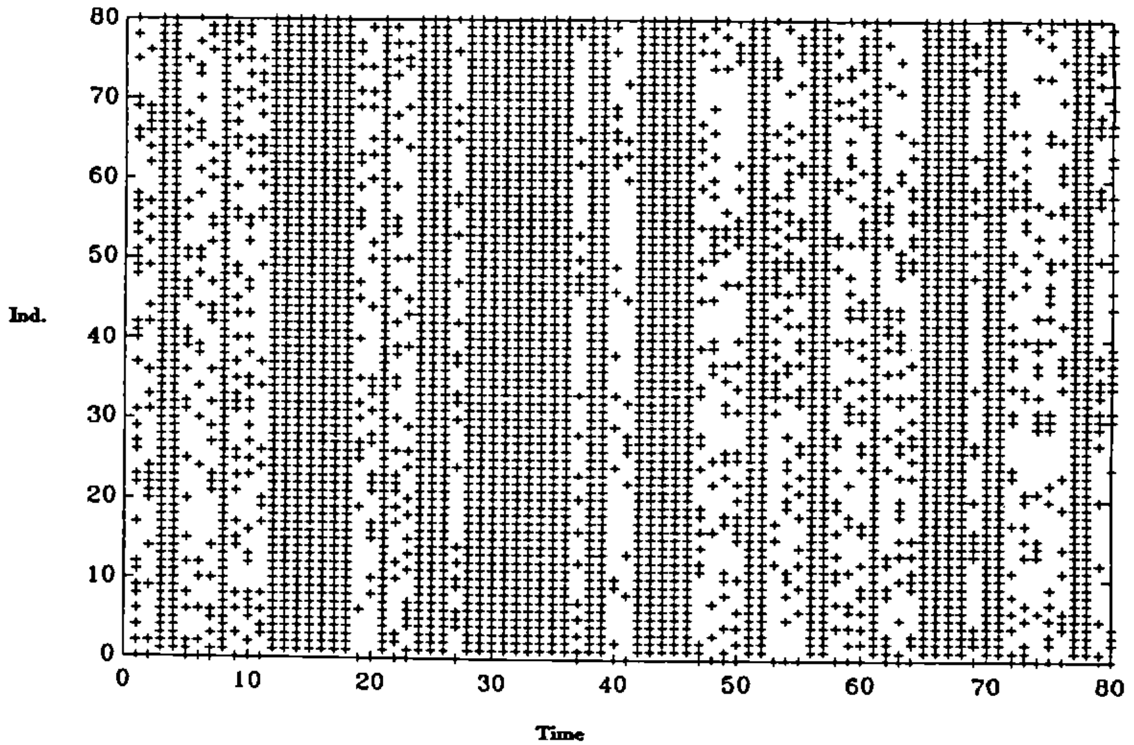
[10]Durlauf [1991(a)] shows that aggregate output under the inefficient equilibrium obeys $Y_t = .18 + .49\,Y_{t-1} + .07\,Y_{t-2} + \epsilon_t$ when the economy-wide shocks are always 0.

Figure 2

80-Period Sample Path Realization for
80-Industry Cross-Section

$\Delta_{1,1}$ Complementarity Range

P_2 transition matrix
$\Theta_1 = .40$
$\Theta_2 = .35$
$\Theta_3 = .30$



+ denotes high production by industry i at t

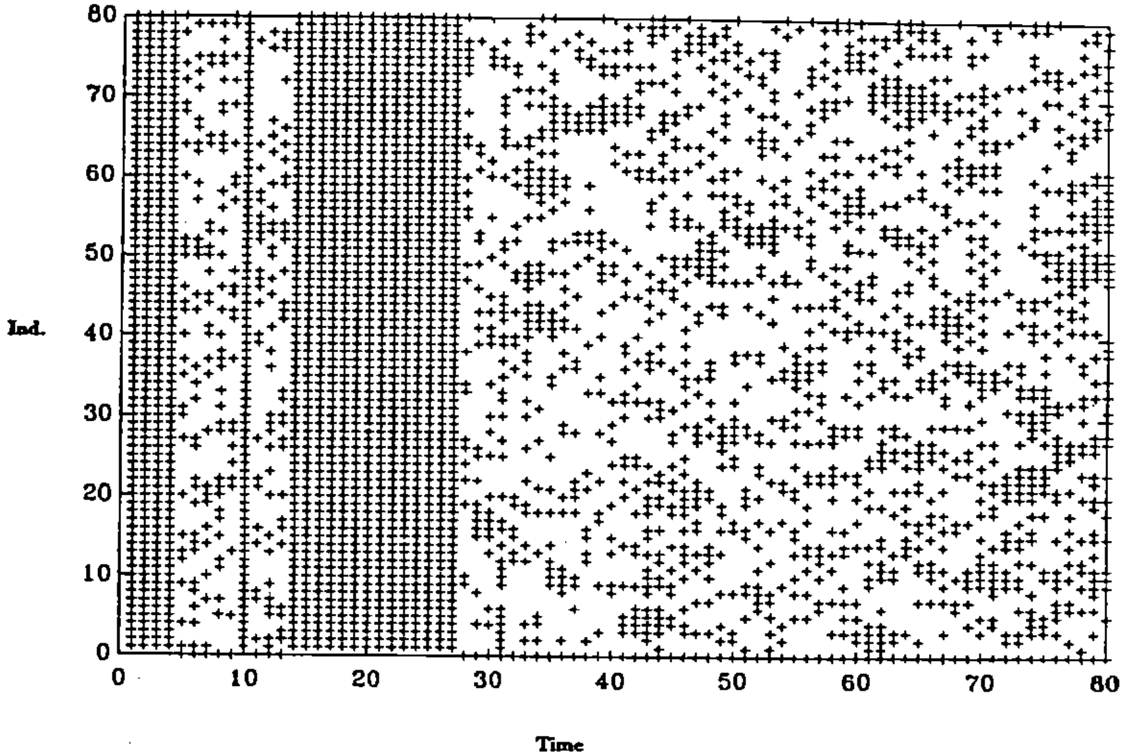
Aggregate Output Equation: $Y_t = .42 + .38\,Y_{t-1} + \epsilon_t$

Figure 3

80-Period Sample Path Realization for
80-Industry Cross-Section

$\Delta_{1,1}$ Complementarity Range

$\mathbf{P}_3$ transition matrix
$\Theta_1 = .40$
$\Theta_2 = .35$
$\Theta_3 = .30$



+ denotes high production by industry i at t

Aggregate Output Equation: $Y_t = .14 + .79\,Y_{t-1} + \epsilon_t$

output, some groups of industries experience periods of boom. This occurs due to the interactions of the spontaneous production probability Θ_3 with the productivity spillovers. This feature illustrates how the model captures heterogeneity in industry behavior during periods of economic decline. Aggregate output in this economy obeys

$$Y_t = .15 + .77 Y_{t-1} + \epsilon_t. \tag{16}$$

Figure 2 illustrates the same economy when the aggregate shocks are governed by

$$\mathbf{P}_2 = \begin{bmatrix} .4 & .3 & .3 \\ .3 & .4 & .3 \\ .3 & .3 & .4 \end{bmatrix} \tag{17}$$

which means that the economy-wide shocks exhibit little persistence. In this case, aggregate output follows

$$Y_t = .42 + .38 Y_{t-1} + \epsilon_t. \tag{18}$$

The main difference between the two economies is that the degree of output persistence is greatly reduced when the economy-wide shocks approach white noise. The AR coefficient for aggregate output is reduced from .77 to .38. As Figure 2 shows, the economy shifts between the two regimes quite frequently.

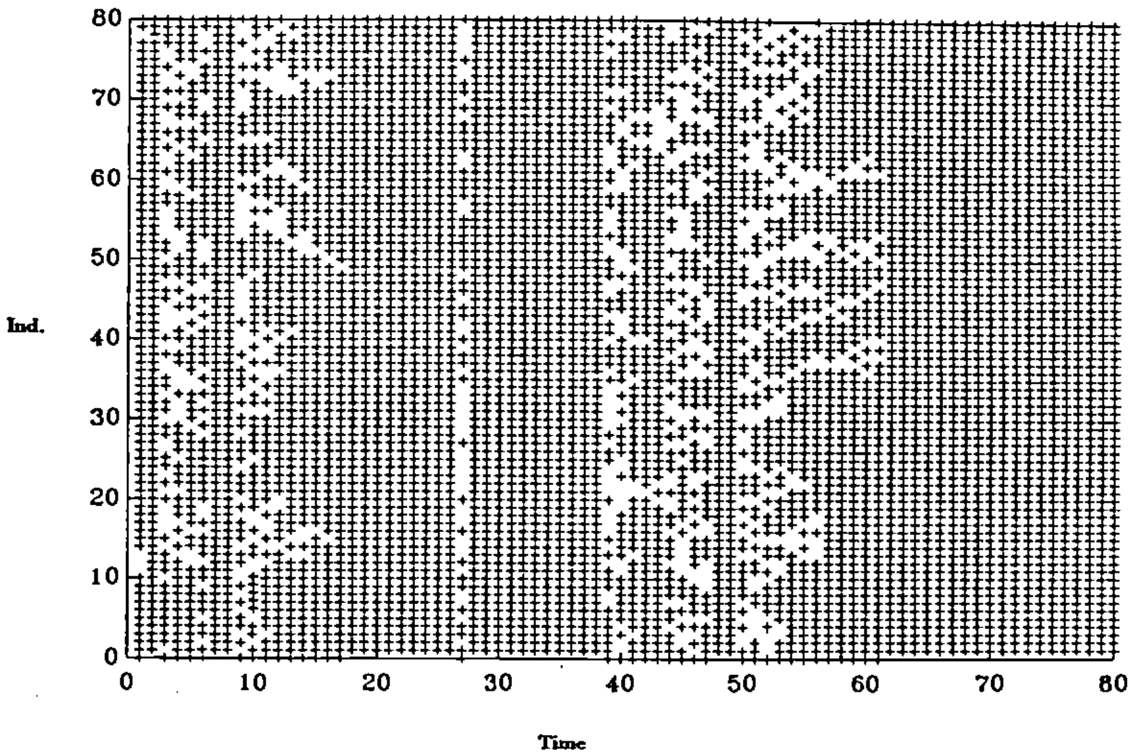
Finally, Figure 3 illustrates the economy when the aggregate shocks are uncorrelated yet tend to be concentrated around zero, i.e.

Figure 4

80-Period Sample Path Realization for 80-Industry Cross-Section

$\Delta_{1,1}$ Complementarity Range

P_1 transition matrix
$\Theta_1 = .70$
$\Theta_2 = .65$
$\Theta_3 = .60$



+ denotes high production by industry i at t

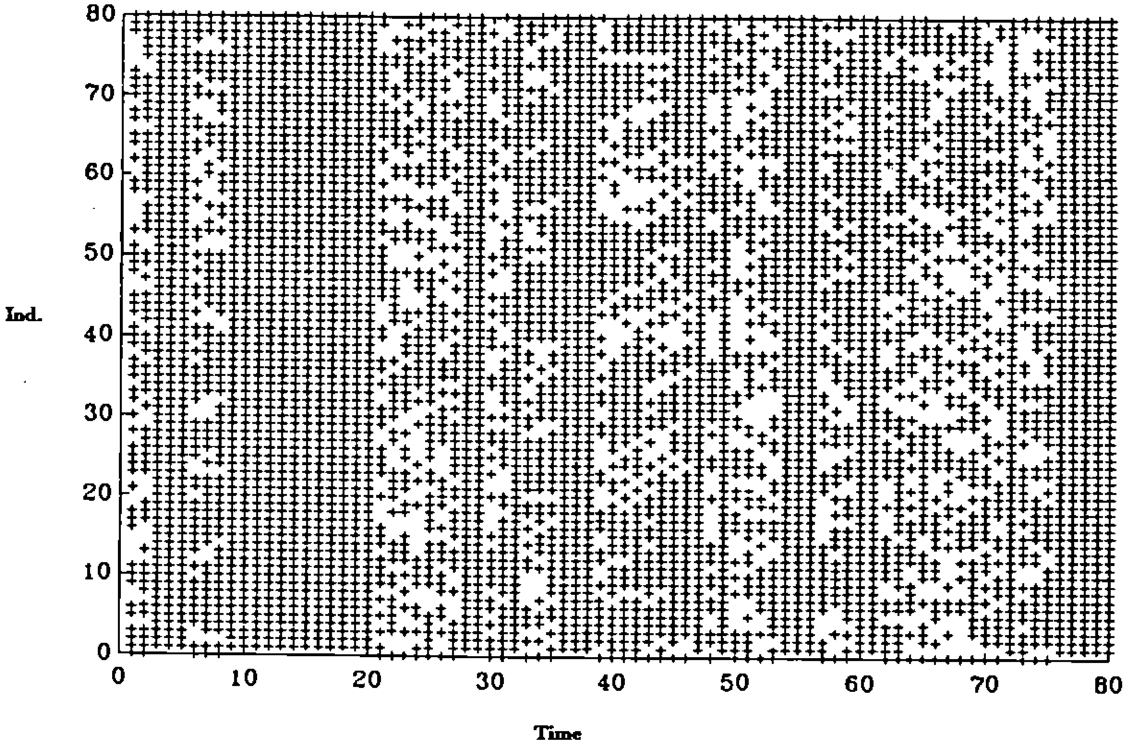
Aggregate Output Equation: $Y_t = .21 + .75\,Y_{t-1} + \epsilon_t$

Figure 5

80-Period Sample Path Realization for 80-Industry Cross-Section

$\Delta_{1,1}$ Complementarity Range

P_2 transition matrix
$\Theta_1 = .70$
$\Theta_2 = .65$
$\Theta_3 = .60$



+ denotes high production by industry i at t

Aggregate Output Equation: $Y_t = .64 + .24Y_{t-1} + \epsilon_t$

$$\mathbf{P}_3 = \begin{bmatrix} .1 & .8 & .1 \\ .1 & .8 & .1 \\ .1 & .8 & .1 \end{bmatrix} \quad (19)$$

In this case, aggregate output is described by

$$Y_t = .14 + .79\,Y_{t-1} + \epsilon_t. \quad (20)$$

a process which exhibits substantial persistence. This equation best illustrates how the incomplete markets structure acts as a propagation mechanism as white noise productivity shocks lead to substantial autoregression in tbe aggregate output process.[11]

Our next three simulations consider the behavior of the economy when only one limiting equilibrium exists. This is done by setting $\Theta_1 = .7$, $\Theta_2 = .65$, $\Theta_3 = .6$. Figure 4 illustrates the behavior of a time series cross-section for the transition matrix $\mathbf{P}_1$. In this case, the aggregate output equation is

$$Y_t = .21 + .75\,Y_{t-1} + \epsilon_t. \quad (21)$$

For the transition matrix $\mathbf{P}_2$, aggregate output follows

$$Y_t = .64 + .24\,Y_{t-1} + \epsilon_t. \quad (22)$$

A realization of this economy may be seen in Figure 5.

Finally, the $\mathbf{P}_3$ transition matrix generates the aggregate output equation

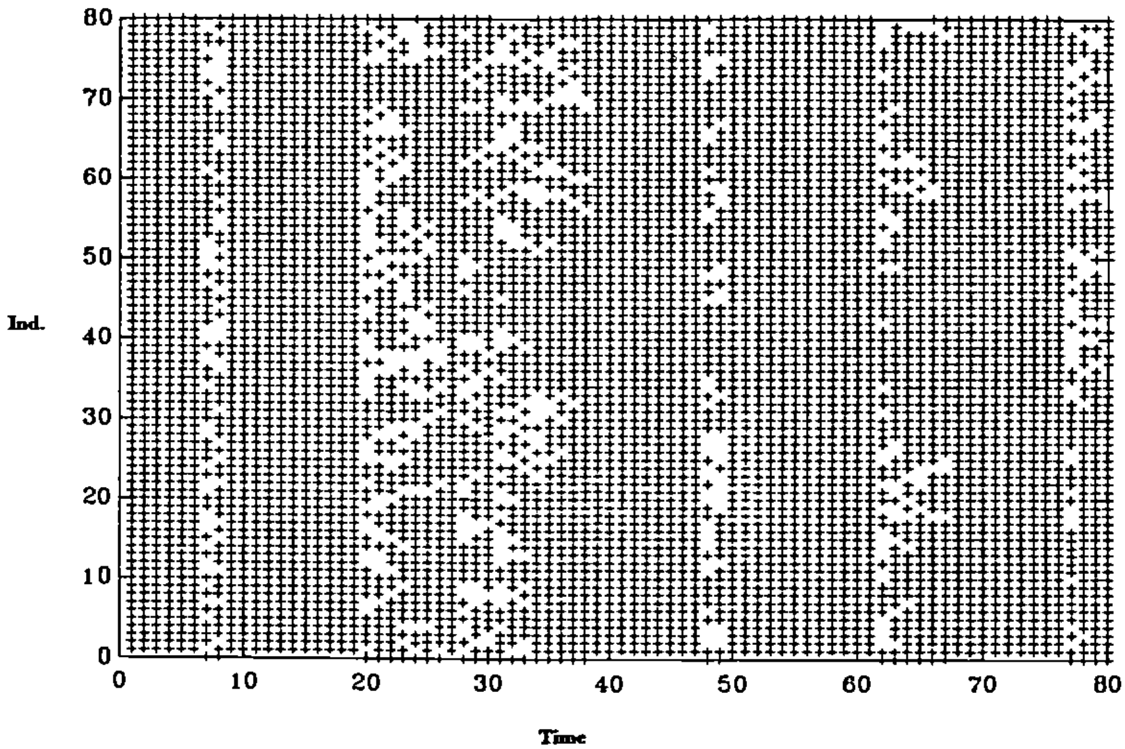
[11] Recalling our earlier discussion, if the expected payoff from cooperation is high enough, then the complete markets equilibrium is $\underset{\sim}{\omega}_t = \underset{\sim}{1}\ \forall\ t$, even in the presence of aggregate shocks, which means that the complete markets equilibrium will exhibit no volatility.

Figure 6

80-Period Sample Path Realisation for 80-Industry Cross-Section

$\Delta_{1,1}$ Complementarity Range

P_3 transition matrix
$\Theta_1 = .70$
$\Theta_2 = .65$
$\Theta_3 = .60$



+ denotes high production by industry *i* at *t*

Aggregate Output Equation: $Y_t = .46 + .50Y_{t-1} + \epsilon_t$

$$Y_t = .46 + .50Y_{t-1} + \epsilon_t, \quad (23)$$

As before, substantial persistence can be generated by white noise shocks interacting with the dynamic complementarities. A sample realization appears in Figure 6.

One interesting feature of Figures 4 to 6 is the evolution towards the high equilibrium after negative productivity shocks. As the Figures indicate (and the model analytically implies for these parameter values), the percentage of industries choosing low production gradually declines after $\xi_t = -1$ realizations. This behavior suggests a reason why business cycles may exhibit sharp rapid declines in output during recessions and gradual periods of recovery. When the high production equilibrium is stable, aggregate output will gradually adjust towards the equilibrium after negative economy-wide shocks. On the other hand, when there are two equilibria, there is no tendency for the economy to correct itself after large output declines. Consequently, our model implies a relationship between the number of equilibria and the degree of asymmetry of the business cycle.

V. Summary and conclusions

This paper has explored how economies can exhibit multiple equilibria and output persistence as a consequence of dynamic coordination failure. These features arise when strong technological complementarities interact with incomplete markets. Low production initial conditions prevent an economy from realizing local technological spillovers. Further, economy-wide shocks can generate indefinite movements in total output as local productivity feedbacks induced by complementarities emerge or disappear. The model exhibits both path dependence of shocks as well as a mechanism for reversals of booms and downturns.

One application of these ideas is to explore whether output behavior during the Depression and World War II can be described as movements across equilibria. Most analyses of output behavior during the 1930's and 1940's have interpreted the Depression and recovery as the consequence of two large offsetting shocks rather than a result of one shock which interacted with some sort of self-correcting mechanism in the aggregate economy. The idea that the Depression was not self-correcting, yet was overcome by a large aggregate demand shock is compatible with the model in this paper, when multiple equilibria are present. An important empirical extension of the current paper is the identification of complementarities in the time series patterns of industrial production which are sufficiently strong to be consistent with our model. This approach is pursued in Cooper and Durlauf [1991].

Technical Appendix

Proof of Theorem 1

If a firm were constrained to use technique 1 each period, standard Euler equation arguments (see Brock and Mirman [1972]) imply that an optimal capital sequence $\{K_{1,i,t}\}$ is implicitly defined by

$$\pi_1(\zeta_{i,t},\xi_t) = \max_{K_{1,i,t}} \beta f_1(K_{1,i,t}-F,\zeta_{i,t},\xi_t)-K_{1,i,t} \tag{A.1}$$

whereas if the firm were constrained to use technique 2 each period, an optimal capital sequence would obey

$$\pi_2(\eta_{i,t},\xi_t) = \max_{K_{2,i,t}} \beta f_2(K_{2,i,t},\eta_{i,t},\xi_t)-K_{2,i,t} \tag{A.2}$$

By our assumptions, $\pi_1(\zeta_{i,t},\xi_t)$ and $\pi_2(\eta_{i,t},\xi_t)$ are measurable functions of the productivity shocks.

Let $\omega_{i,t}=1$ with probability 1 if $\pi_1(\zeta_{i,t}\xi_t) > \pi_2(\eta_{i,t},\xi_t)$, $\omega_{i,t}=1$ with probability $\lambda(\zeta_{i,t},\eta_{i,t},\xi_t)$ if $\pi_1(\zeta_{i,t}\xi_t) = \pi_2(\eta_{i,t},\xi_t)$, and $\omega_{i,t}=0$ with probability 1 if $\pi_1(\zeta_{i,t}\xi_t) < \pi_2(\eta_{i,t},\xi_t)$. This says that if one technique generates higher one period profits than the other, it is chosen with certainty, whereas if the techniques generate identical profits, technique 1 is chosen according to a time invariant function of the productivity shocks. Any such rule generates a sequence of technique choices which are consistent with the solution to a representative firm's maximization problem. Since all firms within an industry are identical, we can conclude that $\omega_{i,t}$ is a measurable function of $\zeta_{i,t},\eta_{i,t}$ and ξ_t. We can rewrite $\omega_{i,t}$ as

$$\omega_{i,t}(\mathrm{E}(\zeta_{i,t} \mid \mathfrak{F}_{t-1}), \mathrm{E}(\eta_{i,t} \mid \mathfrak{F}_{t-1}), \mathrm{E}(\xi_t \mid \mathfrak{F}_{t-1}),$$
$$\zeta_{i,t} - \mathrm{E}(\zeta_{i,t} \mid \mathfrak{F}_{t-1}), \eta_{i,t} - \mathrm{E}(\eta_{i,t} \mid \mathfrak{F}_{t-1}), \xi_t - \mathrm{E}(\xi_t \mid \mathfrak{F}_{t-1})) \qquad \text{(A.3)}$$

Conditioning $\omega_{i,t}$ on $\mathfrak{F}_{t-1}$ means that the terms $\zeta_{i,t} - \mathrm{E}(\zeta_{i,t} \mid \mathfrak{F}_{t-1})$, $\eta_{i,t} - \mathrm{E}(\eta_{i,t} \mid \mathfrak{F}_{t-1})$, and $\xi_t - \mathrm{E}(\xi_t \mid \mathfrak{F}_{t-1})$ can be integrated out in (A.3), which immediately yields Theorem 1, given the restrictions on the conditional probability measures of $\zeta_{i,t}$, $\eta_{i,t}$ and ξ_t in Assumption 2.

Proof of Theorem 2

Dobrushin [1968] provides conditions for proving the existence of a joint probability measure which is consistent with a given set of conditional measures. We verify Theorem 1 by proving the existence of a joint measure over the random vectors $\psi_{i,t} = \{\zeta_{i,t}, \eta_{i,t}, \xi_t, \omega_{i,t}\}$. Observe that the random vectors $\psi_{i,t}$ can be indexed by $\mathbb{Z}^2$; we let $\underline{\psi}_t = \{\ldots\{\zeta_{i-1,t}, \eta_{i-1,t}, \xi_t, \omega_{i-1,t}\}, \{\zeta_{i,t}, \eta_{i,t}, \xi_t, \omega_{i,t}\}\ldots\}$ denote the joint realizations of the random vectors at each t and $\Psi_t = \{\underline{\psi}_0, \underline{\psi}_1, \ldots, \underline{\psi}_t\}$ denote the history of the random vectors up to t.

The first condition for showing the existence of a joint measure is to show that conditional probabilities can be consistently defined over all finite combinations of $\psi_{i,t}$. To see this, specify any initial conditions $\underline{\psi}_0$. Given the specification of a stochastic process for ξ_1, and the conditional probability structure specified in Assumption 2 for $\zeta_{i,1}$ and $\eta_{i,1}$, one can compute the conditional probabilities for any $\psi_{i,1}$ as well as for any finite set in Ψ_1. Repeating this procedure, it is possible to assign probabilities for any finite set in Ψ_t. Letting $t \Rightarrow \infty$, this means that all conditional probabilities over finite sets can be consistently defined.

Second, we need to verify that for any finite set S and any $\delta > 0$, there exists a finite set of elements, $\Gamma(S, \delta)$, $S \not\subseteq \Gamma(S, \delta)$, such that

$$|\,Prob(S\,|\,\Gamma) - Prob(S\,|\,\Psi_\infty - S)\,| \;\leq\; \delta. \tag{A.4}$$

This condition immediately holds for the probability structure we have examined. Consider the case $S = \psi_{i,t}$ where the range of interactions is $\Delta_{k,l}$. Choose the surrounding set Γ as

$$\Gamma = \{\psi_{p,q} \; such \; that \; 0 < |p-i| \leq k+l, \; 0 < |q-t| \leq k+l\}.$$

Let Γ' be any set of elements such that $\Gamma' \cap \Gamma = \Gamma' \cap \psi_{i,t} = \phi$. It is clear, given the localized $\Delta_{k,l}$ conditional probability structure for $\zeta_{i,t}$ and $\eta_{i,t}$, and the fact that ξ_t is a common element of all elements of $\underline{\psi}_t$, that the conditional probability of any Γ', given Γ, is equal to the conditional probability given Γ and $\psi_{i,t}$.

$$Prob(\Gamma'\,|\,\psi_{i,t},\Gamma) = Prob(\Gamma'\,|\,\Gamma) \tag{A.5}$$

Since this is true for all sets Γ', it is also true for $\Psi_\infty - \Gamma - \psi_{i,t}$, i.e.

$$Prob(\Psi_\infty - \Gamma - \psi_{i,t}\,|\,\psi_{i,t},\Gamma) = Prob(\Psi_\infty - \Gamma - \psi_{i,t}\,|\,\Gamma) \tag{A.6}$$

or

$$\frac{Prob(\psi_{i,t},\, \Gamma,\, \Psi_\infty - \Gamma - \psi_{i,t})}{Prob(\psi_{i,t},\, \Gamma)} = \frac{Prob(\Gamma,\, \Psi_\infty - \Gamma - \psi_{i,t})}{Prob(\Gamma)}, \tag{A.7}$$

which implies

$$\frac{Prob(\psi_{i,t},\, \Gamma,\, \Psi_\infty - \Gamma - \psi_{i,t})}{Prob(\Gamma,\, \Psi_\infty - \Gamma - \psi_{i,t})} = \frac{Prob(\psi_{i,t},\, \Gamma)}{Prob(\Gamma)} \tag{A.8}$$

or

$$Prob(\psi_{i,t} \mid \Psi_\infty - \psi_{i,t}) - Prob(\psi_{i,t} \mid \Gamma) = 0, \tag{A.9}$$

which shows that (A.4) holds for $S = \psi_{i,t}$. This argument generalizes to any finite set S, which proves that a joint measure exists over Ψ_∞ and hence over $\{\underset{\sim}{\omega}_0, \underset{\sim}{\omega}_1, \ldots \underset{\sim}{\omega}_\infty\}$.

Proof of Theorem 3

This Theorem is proven in Durlauf [1991(a)].

Proof of Theorem 4

Theorem 4 is proved if we can show that for any vector $\underset{\sim}{\omega}$,

$$\begin{gathered} Prob(\underset{\sim}{\omega}_{T+1} \geq \underset{\sim}{\omega} \mid \xi_T \leq a,\ \mu_t = 0\ \forall\ t > T,\ \Theta_{k,l}^{max} \leq \underline{\Theta}_{k,l}) \leq \\ Prob(\underset{\sim}{\omega}_1 \geq \underset{\sim}{\omega} \mid \underset{\sim}{\omega}_0 = \underline{0},\ \xi_t = 0\ \forall\ t,\ \Theta_{k,l}^{max} = \underline{\Theta}_{k,l},\ \Theta_{k,l}^{max} = \Theta_{k,l}^{min}) \end{gathered} \tag{A.10}$$

and

$$\begin{gathered} Prob(\underset{\sim}{\omega}_{T+1} \geq \underset{\sim}{\omega} \mid \xi_T \geq b,\ \mu_t = 0\ \forall\ t > T,\ \Theta_{k,l}^{max} \leq \underline{\Theta}_{k,l}) \geq \\ Prob(\underset{\sim}{\omega}_1 \geq \underset{\sim}{\omega} \mid \underset{\sim}{\omega}_0 = \underline{1},\ \xi_t = 0\ \forall\ t,\ \Theta_{k,l}^{max} = \underline{\Theta}_{k,l},\ \Theta_{k,l}^{max} = \Theta_{k,l}^{min}). \end{gathered} \tag{A.11}$$

To see that eqs. (A.10) and (A.11) are sufficient to verify the Theorem, observe that those inequalities imply, given 1) Assumption 4.A, which shows that the conditional probability

$$Prob(\underset{\sim}{\omega}_{i,T+s} = 1 \mid \underset{\sim}{\omega}_0 = \underset{\sim}{\omega},\ \xi_t = 0\ \forall\ t) \tag{A.12}$$

is weakly increasing in $\underset{\sim}{\omega}$, 2) Assumption 5 which bounds the conditional probability of high production at t if $\xi_t \leq a$ or $\xi_t \geq b$, and 3) Assumption 6, which restricts the evolution of the aggregate shock after T, that

$$Prob(\omega_{i,T+s}=1 \mid \xi_T \leq a,\ \mu_t = 0\ \forall\ t > T,\ \Theta_{k,l}^{max} \leq \underline{\Theta}_{k,l}) \leq$$
$$Prob(\underset{\sim}{\omega}_{i,T+s}=1 \mid \underset{\sim}{\omega}_T = \underset{\sim}{0},\ \xi_t = 0\ \forall\ t > T,\ \Theta_{k,l}^{max} = \underline{\Theta}_{k,l},\ \Theta_{k,l}^{max} = \Theta_{k,l}^{min}) \quad (A.13)$$

and

$$Prob(\underset{\sim}{\omega}_{i,T+s}=1 \mid \xi_T \geq b,\ \mu_t = 0\ \forall\ t > T,\ \Theta_{k,l}^{max} \leq \underline{\Theta}_{k,l}) \geq$$
$$Prob(\underset{\sim}{\omega}_{i,T+s}=1 \mid \underset{\sim}{\omega}_T = \underset{\sim}{1},\ \xi_t = 0\ \forall\ t > T,\ \Theta_{k,l}^{max} = \underline{\Theta}_{k,l},\ \Theta_{k,l}^{max} = \Theta_{k,l}^{min}), \quad (A.14)$$

$\forall\ s > 0$. Second, note that Theorem 3 shows that if $\Theta_{k,l}^{max} \leq \underline{\Theta}_{k,l}$,

$$\lim_{s \Rightarrow \infty} Prob(\underset{\sim}{\omega}_{i,T+s}=1 \mid \underset{\sim}{\omega}_T = \underset{\sim}{1},\ \xi_t = 0\ \forall\ t > T,\ \Theta_{k,l}^{max} = \underline{\Theta}_{k,l},\ \Theta_{k,l}^{max} = \Theta_{k,l}^{min}) >$$
$$\lim_{s \Rightarrow \infty} Prob(\underset{\sim}{\omega}_{i,T+s}=1 \mid \underset{\sim}{\omega}_T = \underset{\sim}{0},\ \xi_t = 0\ \forall\ t > T,\ \Theta_{k,l}^{max} = \underline{\Theta}_{k,l},\ \Theta_{k,l}^{max} = \Theta_{k,l}^{min}). \quad (A.15)$$

which combined with (A.12) and (A.13) would verify Theorem 4.

(A.11) is immediate since both probabilities in the weak inequality equal 1 under our assumptions. To see that (A.10) holds, observe that

$$Prob(\underset{\sim}{\omega}_{T+1} \geq \underset{\sim}{\omega} \mid \xi_T \leq a,\ \mu_t = 0\ \forall\ t > T,\ \Theta_{k,l}^{max} \leq \underline{\Theta}_{k,l}) \leq$$
$$Prob(\underset{\sim}{\omega}_{T+1} \geq \underset{\sim}{\omega} \mid \underset{\sim}{\omega}_T = \underset{\sim}{1},\ \xi_T \leq a,\ \mu_t = 0\ \forall\ t > T,\ \Theta_{k,l}^{max} \leq \underline{\Theta}_{k,l}) \quad (A.16)$$

Observe further that 1) by Assumption 5.A, $Prob(\omega_{i,T+1} = 1 \mid \underset{\sim}{\omega}_T = \underset{\sim}{1},\ \xi_T \leq a,\ \mu_t = 0\ \forall\ t > T,\ \Theta_{k,l}^{max} \leq \underline{\Theta}_{k,l}) \leq \underline{\Theta}_{k,l}\ \ \forall\ \ i$, and 2) by Assumption 2.D, the variables $\omega_{i,T+1} - E(\omega_{i,T+1} \mid \underset{\sim}{\omega}_T = \underset{\sim}{1},\ \xi_T \leq a,\ \mu_t = 0\ \forall\ t > T,\ \Theta_{k,l}^{max} \leq \underline{\Theta}_{k,l})$ are independent across all i. On the other hand, 1) by eq. (7), $Prob(\omega_{i,1} = 1 \mid \underset{\sim}{\omega}_0 = \underset{\sim}{0},\ \xi_t = 0\ \forall\ t,\ \Theta_{k,l}^{max} = \underline{\Theta}_{k,l},$

$\Theta_{k,l}^{max} = \Theta_{k,l}^{min}) = \underline{\Theta}_{k,l}$, and 2) by Assumption 2.D, the variables $\omega_{i,1} - E(\omega_{i,1} \mid \underline{\omega}_0 = \underline{0}$, $\xi_t = 0 \ \forall \ t$, $\Theta_{k,l}^{max} = \underline{\Theta}_{k,l}$, $\Theta_{k,l}^{max} = \Theta_{k,l}^{min})$ are independent across all i. Given Assumption 6, it therefore follows that

$$Prob(\underline{\omega}_{T+1} \geq \underline{\omega} \mid \underline{\omega}_T = \underline{1}, \xi_T \leq a, \mu_t = 0 \ \forall \ t > T, \Theta_{k,l}^{max} \leq \underline{\Theta}_{k,l}) \leq$$
$$Prob(\underline{\omega}_1 \geq \underline{\omega} \mid \underline{\omega}_0 = \underline{0}, \xi_t = 0 \ \forall \ t, \Theta_{k,l}^{max} = \underline{\Theta}_{k,l}, \Theta_{k,l}^{max} = \Theta_{k,l}^{min}). \qquad (A.17)$$

Eqs. (A.15) and (A.16) imply eq. (A.10), which completes the proof.

Proof of Theorem 5

Part A of the Theorem follows immediately from the definition of a recurrent Markov process. If the events $(\xi \leq a)$ and $(\xi \geq b)$ each occur infinitely often, then the impact of any sample path realization of the economy up to t on forecasts of the level of output at $t+s$ will become zero with probability 1 if the forecast horizon s is sufficiently large. To verify Part B, observe that if the event $(\xi_t \neq 0)$ is nonrecurrent, then 0 must be an absorbing state for ξ_t. Consequently, either of the equilibria characterized in Theorem 3 can emerge if both $(\xi \leq a)$ and $(\xi \geq b)$ are possible candidates to be the last event before ξ_t becomes zero. If $Prob(\xi_1 \leq a)$, $Prob(\xi_1 \geq b)$, $Prob(\xi_t = 0 \mid \xi_{t-1} \leq a)$ and $Prob(\xi_t = 0 \mid \xi_{t-1} \geq b)$ are all nonzero, then either equilibrium can emerge with positive probability. Further, for large enough T, $\xi_{T+s} = 0$ with probability 1 $\forall \ s > 0$, which means that the last nonzero ξ_t will permanently affect conditional long run forecasts.

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