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CONVERGENCE OF INTERNATIONAL OUTPUT MOVEMENTS

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ABSTRACT

This paper explores the convergence of real per capita output in advanced industrialized economies. We start by observing that in a stochastic environment, convergence in per capita GDP requires that permanent shocks to one economy be associated with permanent shocks to other economies. Convergence is a natural outcome of models where exogenous technical change migrates across countries with similar microeconomic specifications. Conversely, in a world where some component of permanent output movements is due to technical change whereas other components are due to domestic factors, national economies may diverge over time. We formalize a general definition of convergence using the notions of unit roots and cointegration developed in the time series literature. We construct bivariate and multivariate tests of convergence across advanced industrialized economies. Our evidence indicates that one cannot reject the no convergence null. Further, the estimated time series representation of cross-country output deviations exhibits substantial persistence. These results suggest that previous empirical work on convergence has neglected some aspects of the null hypothesis.

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1. Introduction

One of the most striking features of traditional models of economic growth is the implication these models have for cross-country convergence. In standard formulations of the infinite horizon optimal growth problem, various turnpike theorems suggest that steady state per capita output is independent of initial conditions. Further, differences in microeconomic parameters will generate stationary differences in per capita output and will not imply different growth rates, when the forms of utility functions vary.[1] Consequently, when one observes differences in per capita output growth across countries, one must either assume that these countries have dramatically different microeconomic characteristics, such as different production functions or discount rates, or regard these discrepancies as transitory.

Launched primarily by the theoretical work of Romer [1986] and Lucas [1988], much attention has focused on the predictions of dynamic equilibrium models for long term behavior when various Arrow-Debreu assumptions are relaxed. Lucas and Romer have shown that divergence in long term growth can be generated by social increasing returns to scale associated with both physical and human capital. When there exist various types of positive feedbacks from capital formation to production which are not internalized by individual agents, multiple steady states may result. These steady states are indexed by the initial conditions of an economy; consequently, these models predict that convergence will generally not hold. More generally, economies characterized by strong complementarities will possess multiple steady states. For example, Murphy, Shleifer and Vishny [1989] show how increasing returns to scale can induce a multiplicity of steady states because of aggregate demand

[1] For the standard time separable growth model, if rates of time preference vary across countries, consumption and GNP for the more impatient country become asymptotically negligible. Hence cross-country variation across this dimension will not lead to interesting forms of divergence. However, Jones and Manuelli [1990] show that if the marginal product of capital differs sufficiently across countries, then divergence can occur in a competitive equilibrium model. They emphasize how cross-country differences in macroeconomic policies, such as tax rates, can lead to divergence for countries with similar preference and production specifications.

complementarities; Heller [1986] obtains similar results based solely on imperfect competition. This literature has typically concentrated on demonstrating how static economies may exhibit multiple equilibria. However, Heller [1990] has shown that multiplicity can be extended to infinite horizon growth problems. Further, Durlauf [1991a,b] shows how stochastic formulations of complementarities can lead to long run divergence in the sample paths of per capita output for different economies.

An empirical literature exploring convergence has developed in parallel to the new growth theory. Prominent among these contributions is the work of Baumol [1986], DeLong [1988] and Baumol, Blackman, and Wolff [1989]. Most of this research has argued that the historical data is consistent with the convergence hypothesis. In particular, Baumol and his coauthors argue that over long horizons, there is a negative correlation between the initial per capita income of a country and its subsequent growth rate. This correlation means that, on average, relatively poor countries tend to catch up. Barro [1990] has also examined convergence from this perspective using a large cross-section of countries from the Heston-Summers data set. He does not find convergence in the raw correlations but when proxie for human capital development are included the countries do appear to be converging.

The purpose of this paper is to propose a new definition and set of tests of the conve gence hypothesis. Our research differs from much previous empirical work in that we ca the notion of convergence in an explicitly stochastic framework. Technical innovation an capital accumulation are continuing processes which lead to random yet permanent mov ments in per capita output across countries. Convergence in the face of stochastic technic innovations essentially asks whether permanent movements in one country's per capita ou put are associated with permanent movements in other countries' outputs. Recent advanc in time series analysis, notably the theory of cointegration, provide a natural language testing cross-country relationships in permanent output movements. The role of cointeg tion in addressing long run convergence was first explored by Campbell and Mankiw [19 we extend their analysis both through new data analysis and testing methodologies as as by developing additional information on the structure of growth across countries.

Our analysis, which examines annual log real output per capita for 15 OECD economies from 1900 to 1987, leads to two basic conclusions about international output fluctuations. First, we find very little evidence of convergence across the economies. Per capita output deviations do not appear to systematically disappear over time. Second, we find that there is strong evidence of common elements to long run economic fluctuations across countries. As a result, economic growth cannot be reduced exclusively to idiosyncratic, country-specific factors. A relatively small set of common factors interact with individual economic characteristics to determine growth rates.

Our work is most closely related in spirit to a recent paper by Quah [1990] who also finds evidence against convergence. Quah examines whether there is stochastic convergence across (almost all) the capitalist economies listed in the Summers-Heston [1988] international output data set. Our analysis differs from that work in three respects. First, we employ a different econometric framework which leads to alternative tests. Second, we restrict ourselves to analysis of advanced industrialized economies. It seems unreasonable to expect to observe convergence between sub-Saharan and OECD economies on the basis of post-1950 data. Our rejection of convergence therefore is both more surprising and more easily interpreted in terms of different growth models. Third, we examine data sets which extend across the current century. A shorter data set runs the risk of missing long run types of convergence.

Studies similar to ours have been conducted by Campbell and Mankiw [1989] and Cogley [1990] who have explored patterns of persistence in international output. Using quarterly post-1957 data, Campbell and Mankiw demonstrate that 7 OECD economies exhibit both persistence and divergence in output. Cogley, examining 9 OECD economies using a similar data set to the one here, concludes that persistence is substantial for many countries; yet at the same time he argues that common factors generating persistence imply that "long run dynamics prevent output levels from diverging by too much." Results from another paper, (Bernard and Durlauf [1991b]), complement those of both Campbell and Mankiw and Cogley by strongly supporting the persistence hypothesis using a new set of measures and test statistics. On the other hand, this work supports that of Campbell and Mankiw in concluding that there is little evidence of convergence.

This paper consists of a brief theoretical introduction to the testing methodology, a description of the statistics used, and a main empirical section which considers the convergence hypothesis in both bivariate and multivariate settings. Additionally there is a description of the data for the fifteen industrialized countries in our sample preceding the empirical results. The evidence from the cross-country analysis argues against the notion of convergence for the whole sample. Alternatively there do appear to be groups of countries with common stochastic elements as one would expect for some of the proximate, similarly structured economies of Europe.

2. Convergence in stochastic environments

The organizing principles of our empirical work come from employing stochastic definitions for both long term economic fluctuations and convergence. These definitions rely on the notions of unit roots and cointegration in time series. This literature, whose basic ideas are well exposited in Engle and Granger [1987], formalizes the concepts of trends in individual series and of relationships in trends across time series.

By a stochastic trend, or unit root, we mean that part of the time series which is expected to persist into the indefinite future, yet is not predictable from the indefinite past.

Definition 2.1. *Stochastic trend*

$Y_{i,t}$ contains a stochastic trend if it is nonstationary in levels even after removing a linear trend, whereas the process is stationary in first differences. The first difference have the moving average representation

$$\Delta Y_{i,t} = \mu + a(L)\varepsilon_{i,t} = \mu + \sum_{k=0}^{\infty} a_k \varepsilon_{i,t-k} \qquad (2.1)$$

where $\varepsilon_{i,t}$ is white noise distributed $(0, \sigma^2_{\varepsilon_{i,t}})$.

The part of each innovation that persists into the indefinite future is represented as $E(Y_{i,\infty} \mid \varepsilon_{i,t}) = (\sum_{k=0}^{\infty} a_k)\,\varepsilon_{i,t} \neq 0$. The interactions of stochastic trends across countries can be formalized into general definitions of convergence and common trends.

Definition 2.2. *Common stochastic elements in per capita output*

If log per capita outputs in countries i and j satisfy Definition 2.1, then long run growth in $Y_{i,t}$ and $Y_{j,t}$ is determined by a common factor if $Y_{i,t}$ and $Y_{j,t}$ are cointegrated, i.e. there exists a constant γ such that

$$Y_{i,t} = \mu + \gamma Y_{j,t} + \nu_{ij,t} \tag{2.2}$$

where $\nu_{ij,t}$ is distributed $(0, \sigma^2_{\nu_{ij,t}})$ and is stationary in levels.

Definition 2.3. *Stochastic convergence in per capita output*

Log per capita output in country i converges to log per capita output in country j if $Y_{i,t}$ and $Y_{j,t}$ have stochastic trends as in Definition 2.1 and if

$$Y_{i,t} = Y_{j,t} + \vartheta_{ij,t}. \tag{2.3}$$

where $\vartheta_{ij,t}$ is distributed $(0, \sigma^2_{\vartheta_{ij,t}})$ and is stationary in levels.

If a pair of output series satisfies Definition 2.2, but not Definition 2.3, then they will be cointegrated but the expected long run output levels for the pair will not be equal. However, it will remain true that shocks to country i will be related in part to those in country j.

The third definition gives us important testable implications for the convergence question. If it is true that countries with differing 'initial' incomes are converging to similar growth rates *and* levels of output then any pair in such a group will satisfy Definition 2.3.[2] The definition says that the difference between the stochastic components of the two series, $Y_{i,t} - Y_{j,t}$, will be a zero mean stationary process. It is important to exclude a non-zero mean because most tests we employ will look at first differences of the deviations between the series and varying means will help us test the convergence null.

Our definition of convergence is substantially different than that employed by Baumol, DeLong *et al.* (See Bernard and Durlauf [1991a] for a comparison of the alternative definitions.) These authors have tested persistence by performing a cross-section regression which examines whether over a given time period there is a negative correlation between the initial per capita income of a country and its subsequent growth rate. This definition captures the qualitative notion of nations catching up to one another, but does not address the question of whether the economies actually converge. If the world economy experiences a single episode of technical change, it would be surprising if this change did not migrate from richer to poorer countries, which would generate the negative correlation we have described. Our definition of convergence, however, requires that income disparities eventually vanish. Persistence of income disparities is sufficient to reject the turnpike arguments of optima growth models and lead economists to concentrate on idiosyncratic microeconomic factor as a source of growth. It is straightforward to construct a model which generates both negative correlation in per capita income and growth and persistent deviations according t our definition.

It is important to observe that our testing framework, by relying on time series analys of national output movements, presupposes a greater degree of stationarity in the data tha is required for cross-section tests. In particular, we require that the joint autocorrelati function of the first differences of the output series is time invariant. Our tests assume th the initial conditions for the various time series are washed out when the sample momen are computed. In other words, our procedures assume that the sample moments well appr

[2] By 'initial', we mean a date before which all the countries have gained access to simi technologies.

imate the asymptotic population moments of the data. This approximation does not hold when the data consist exclusively of transition dynamics from some initial conditions. Cross-section tests impose no such stationarity. In fact, they are appropriate when one is analyzing how a set of economies evolve from a single event which induces different initial conditions. For example, if one were interested in asking whether the industrial revolution migrated from country to country then a comparison of initial conditions to subsequent growth rates makes sense. Our methodology is appropriate when one regards technical change as an ongoing process where distinct permanent shocks originate at different points in time. A general discussion of the relationships between various tests of convergence is contained in Bernard and Durlauf [1991a]

3. Output relationships across countries

3.1 Econometric methodology

We now turn to the cross-country analysis to look for cointegration and convergence. First we discuss the various statistical techniques employed along with their associated caveats. Then we describe the annual data series. Next we present the empirical results on common stochastic elements to growth for pairs of countries, followed by a pairwise look at convergence, as both were defined in section 2. Finally we use recent techniques developed by Phillips and Ouliaris [1988] for a multi-country look at both cointegration and convergence.

For bivariate output relationships, we can test for the presence of common trends through the use of cointegration techniques. In a companion paper (Bernard and Durlauf [1990b]), we find all 15 countries in our sample exhibit substantial persistence in the univariate output representations. The presence of persistence, or unit roots, in the univariate data naturally suggests the use of cointegration techniques for analyzing the bivariate and multivariate relationships. In particular we employ a methodology described by Engle and Granger [1987] which is based on the estimated residuals of cointegrating regressions. To test for common stochastic elements in a pair of countries we estimate the equation

$$Y_{i,t} = C_{ij} + \gamma_{ij} Y_{j,t} + \xi_{ij,t}. \tag{3.1}$$

The estimated residuals, $\hat{\xi}_{ij,t}$, from this equation are then employed to compute augmented Dickey-Fuller (ADF) statistics for θ_{ij} from a second equation

$$\Delta\xi_{ij,t} = -\theta_{ij}\xi_{ij,t-1} + B(L)\Delta\xi_{ij,t-1} + \varsigma_{ij,t}. \tag{3.2}$$

Since we do not know the actual autoregressive structure, we choose the minimum of the t-statistics over a range of lag lengths.[3] Recommended by Engle and Granger [1987], this statistic allows for non-white noise processes for the stationary series produced by the cointegrating vector. As is well known, the main drawback of this test is its relatively low power against many alternatives. Accordingly, rejections will be taken as a strong signal that the series are cointegrated.

For pairwise convergence, we employ two types of tests to look for unit roots, and thus persistence, in the difference between the output series,

$$DY_{ij,t} = Y_{i,t} - Y_{j,t}. \tag{3.3}$$

First we employ an Augmented-Dickey-Fuller(ADF) statistic to test for the presence of a unit root in $DY_{ij,t}$. Again we choose the minimum of the t-statistics over a range of lag lengths. In addition we use, as a descriptive device, the sum of the coefficients of the second order autoregression of $DY_{ij,t}$. A sum near one indicates a large degree of persistence in the cross-country differences.

[3] Choosing the minimum of the ADF over a range of lag lengths maximizes the possibility of finding common stochastic elements in the output pairs.

Second, we employ a series of spectral-based tests for the first differences of $DY_{ij,t}$. A natural way to test for persistence in output deviations is through the zero frequency of the spectral density of $\Delta DY_{ij,t}$. When a time series is difference stationary, the zero frequency measures the variance of innovations to the stochastic trend; when $f_{\Delta DY_{ij,t}}(0) = 0$, then there is no persistent component to output deviations. However, as documented by Cochrane [1988] and Durlauf [1990b], the standard errors of the zero frequency estimates are typically large. Any rejection of $f_{\Delta DY_{ij,t}}(0) = 1$ can be taken as a relatively strong rejection of persistence in output deviations and thus of divergence. We estimate the zero frequency of the periodogram under Daniell windows of width $\frac{\pi}{16}$ and $\frac{\pi}{8}$. Additionally we employ an alternative spectral based strategy to assess the persistence of output deviations. We first test the spectral properties of output deviations relative to the hypothesis that they are a random walk with drift. Then we look for departures from the pure random walk hypothesis to determine whether these deviations are suggestive of mean reversion.

The simple random walk model,

$$DY_{ij,t} = DY_{ij,t-1} + \eta_{ij,t} \tag{3.4}$$

where $\{\eta_{ij,t}\}$ is a martingale difference sequence, is a useful baseline for measuring the persistence of fluctuations. In this case, shocks are entirely persistent as a contemporaneous output movement is fully incorporated into long term forecasts.

$$\lim_{k \Rightarrow \infty} E(DY_{ij,t+k} \mid \eta_{i,t}) = \eta_{i,t}. \tag{3.5}$$

We may test this null with statistics based upon the properties of the spectral density of first differences of the output deviations series, $f_{\Delta DY_{ij}}(\omega)$. Since the first difference of a random walk with drift is a martingale difference sequence with possibly positive mean, it is possible to capture all second moment implications of the null hypothesis in the requirement that the spectral density be shaped as a rectangle.

$$f_{\Delta DY_{ij}}(\omega) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \sigma_{\Delta DY_{ij}}(k) e^{-ik\omega} = \frac{1}{2\pi} \sigma_{\Delta Y_{ij}}(0) \quad (3.6$$

where $\sigma_{\Delta DY_{ij}}(k)$ is the autocovariance function.

An equivalent way of formulating the null is in terms of the normalized spectral distri- bution function, defined for $\lambda \in [0, \pi]$ as

$$F_{\Delta DY_{ij}}(\lambda) = \frac{2 \int_0^{\lambda} f_{\Delta DY_{ij}}(\omega) d\omega}{\sigma_{\Delta DY_{ij}}(0)}. \quad (3.7$$

Under the null hypothesis,

$$H_0 : F_{\Delta DY_{ij}}(\lambda) = \frac{\lambda}{\pi} \quad (3.8$$

i.e. the spectral distribution function is shaped as a diagonal line. By the Cramér Represen tation Theorem, the spectral density at ω equals the contribution of cycles of period $2\pi/$ to the total variance of $\Delta DY_{ij,t}$. Thus a white noise representation of this process mear that cycles of all lengths between 2 and ∞ contribute equally to the total variance of tl time series. Observe that if $DY_{ij,t}$ exhibits mean reversion, this will be manifested in a lac of spectral power in the lower frequencies relative to white noise.

The normalized spectral distribution function has the advantage that a general asym totic theory exists to formalize tests of the random walk model (Durlauf [1990a]). Tl tests that follow from this asymptotic theory are based on the difference between the samp normalized spectral distribution function and the diagonal shape which holds under the ma tingale difference null. Letting $I_{\Delta DY_{ij},T}(\omega)$ denote the periodogram estimate of the spect density of $\Delta DY_{ij,t}$ over the sample of size T, these deviations can be modelled as a rando function whose domain is $[0, 1]$,

$$\int_0^{\pi t} \left(\frac{I_{\Delta DY_{ij},T}(\omega)}{\hat{\sigma}_{\Delta DY_{ij}}(0)} - \frac{1}{2\pi} \right) d\omega \quad t \in [0, 1].$$

We present two theorems on the behavior of these deviations and their associated test statistics. The proofs of Theorems 3.1 and 3.2 are in Durlauf [1990a].[4]

Theorem 3.1 Distribution of the normalized spectral distribution function.

If $\Delta DY_{ij,t}$ is a martingale difference sequence, then

$$U_T(t) = \sqrt{2T}\int_0^{\pi t}\left(\frac{I_{\Delta DY_{ij},T}(\omega)}{\hat{\sigma}_{\Delta DY_{ij}}(0)} - \frac{1}{2\pi}\right)d\omega \Rightarrow_w U(t)\ t \in [0,1]$$

where $U(t)$ is the Brownian bridge on $t \in [0,1]$.

Theorem 3.2 Spectral distribution function tests.

Under H_0,

(a) $\quad \mathrm{AD}_T = \int_0^1 \frac{U_T(t)^2}{t(1-t)}dt \Rightarrow_w \int_0^1 \frac{U(t)^2}{t(1-t)}dt \triangleq$ the Anderson-Darling statistic.

(b) $\quad \mathrm{CVM}_T = \int_0^1 U_T(t)^2 dt \Rightarrow_w \int_0^1 U(t)^2 dt \triangleq$ the Cramér-von Mises statistic.

(c) $\quad$ *For fixed t, $U_T(t) \Rightarrow_w N(0, t(1-t))$*

AD_T and CVM_T diverge if ΔDY_{ij} is any other MA process.

$U_T(t)$ diverges if $F_{\Delta DY_{ij}}(\lambda) \neq \frac{\lambda}{\pi}$, $t = \frac{\lambda}{\pi}$.

Theorem 3.2 embodies two perspectives in assessing the behavior of output deviations relative to a random walk null. The AD_T and CVM_T statistics represent general tests for spectral shape. These tests are appropriate when a researcher possesses little prior information on the location of the alternative hypothesis to the pure random walk null.

[4] The theorems are valid assuming some technical conditions which we omit. These conditions permit a wide degree of heteroskedasticity in the process.

When an alternative is well specified, or a researcher possesses a non-diffuse prior over a range of alternatives, then the individual $U_T(t)$ statistics may be more appropriate. For example, if the relevant alternative is long run mean reversion, then the $U_T(t)$ statistic for $t = \frac{\pi}{8}$ would be an appropriate statistic to employ, as it identifies the variance contributions for $[0, \frac{\pi}{8}]$, i.e. cycles of 16 years or longer. Bernard and Durlauf [1991c] conclude that this test has reasonable power against a range of mean reverting alternatives. Further, by examining the spectral distribution function through the $U_T(t)$ statistics, we can completely characterize the second moment properties of the different series.

Finally we turn to multivariate tests for common stochastic components. Unfortunately, there is no natural analogue to the spectral distribution function tests we employ in our bivariate analysis. Hence we rely on analysis of the zero frequency of the spectral density matrix, recalling that the zero frequency in this case measures the variance-covariance matrix of innovations to the various stochastic trends.

For multivariate series, as in the univariate case, common trends and convergence will impose distinct restrictions on the zero frequency of the spectral density matrix. Common trends require that the persistent parts of different time series be proportional; convergence requires that the persistent parts be equal. Let $\vec{Y}_t$ denote the $n \times 1$ vector of output levels, $\Delta\vec{Y}_t$ the first differences of that series, $D\vec{Y}_t$ the $(n-1) \times 1$ vector of output deviations such that $D\vec{Y}_{i,t} = \vec{Y}_{i,t} - \vec{Y}_{n,t}$, and $\Delta D\vec{Y}_t$ the first differences of the deviations. Proportionality of the persistent parts, in a multivariate framework, means that the persistent parts of different series are linearly dependent, which is formalized as

Theorem 3.3 Common factors and spectral density matrix of output differences.

If the number of distinct stochastic trends in $\vec{Y}_t$ is less than n, then $f_{\Delta\vec{Y}}(0)$ is not of full rank.

pf. Engle and Granger [1987].

On the other hand, if several output series have the same persistent parts, the output deviations from a benchmark country must all have zero-valued persistent components.

Theorem 3.4 Complete convergence and spectral density matrix of output deviations.

If all n countries are converging in per capita output, then $f_{\Delta D\tilde{Y}}(0)_{i,i} = 0 \; \forall i$, or equivalently, the rank of $f_{\Delta D\tilde{Y}}(0)$ is 0.

pf. The first implication is immediate from the stationarity of $\Delta DY_{ij,t} \; \forall i,j$. The second implication follows from the application of the Cauchy-Schwarz inequality for the zero frequency of $\Delta DY_{ij,t}$, which implies that $0 = f_{\Delta DY_{ij}}(0) f_{\Delta DY_{kl}}(0) \geq f^2_{\Delta DY_{ij} \Delta DY_{kl}}(0)$.
Q.E.D.

Spectral tests devised by Phillips and Ouliaris [1988] and recently used by Cogley [1990] permit us to determine the number of common trends for the 15 output series and then test for complete convergence. These tests exploit the fact that the spectral density matrix at the zero frequency measures the variance-covariance matrix of the permanent components of output fluctuations in each country. These 15 components can be expressed as linear combinations of orthogonal random variables. The eigenvalues of the zero frequency matrix represent the variances of a particular choice of orthogonal variables. When one or more of these eigenvalues equals zero, the 15 permanent innovations are driven by a smaller number of common factors.

The tests themselves make use of the fact the the spectral density matrix of first differences at the zero frequency will be of rank $q \leq n$ where q is the number of linearly independent stochastic trends in the data and n is the number of series in the sample. This reduction in rank is captured in the eigenvalues of the zero frequency of the spectral density matrix. If the zero frequency matrix is less than full rank, $q < n$, as in Theorem 3.3, then the number of positive eigenvalues will also be $q < n$. If the matrix has zero rank, as in Theorem 3.4, then there will be no positive eigenvalues. The particular Phillips-Ouliaris test we employ is a bounds test that examines the smallest $m = n - q$ eigenvalues to determine if they are close to zero.[5]

4. Data

The data used in both the empirical exercises are annual log real GDP per capita in 1980 international dollars for current boundaries. The series run from 1900-1987 for 15 industrialized countries with the GDP data drawn from Maddison [1989] and the population data from Maddison [1982].[6] Recent years are updated from IFS yearbooks.[7] Figures 1 and 2 present graphs of the fifteen series over the whole sample in levels and logs respectively and Table 1 gives the means and standard deviations of the growth rates. The picture in levels shows dramatic income growth over the period but no absolute narrowing of the overall spread. In logs, however, the narrowing is substantial. From this visual perspective, the convergence hypothesis looks to be an appropriate starting point. In particular, the trends in the various series appear to keep the series within a fixed range. However, Figures 3 and 4 show the fourteen series as deviations from log US output, i.e. $Y_{i,t} - Y_{US,t}$. Output differentials narrow across the whole sample, but there does not appear to be any tendency to converge to the US level.

Other work on convergence, particularly Baumol [1986] and DeLong [1988], has used longer time series from Maddison [1982] for a similar group of countries. Cogley [1990] used this data for a smaller set of countries. However, significant revisions of the data for the pre-World War I period have occurred since their original publication.[8] In light of this we

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[5] Complete descriptions of the Phillips-Ouliaris test statistic and the critical values chosen are in Appendix A.

[6] The countries are: Australia (AL), Austria (AS), Belgium (BE), Canada (CA), Denmark (DE), Finland (FI), France (FR), Germany (GE), Italy (IT), Japan (JA), the Netherlands (NE) Norway (NO), Sweden (SW), the United Kingdom (UK), and the United States (US).

[7] Population data for 1980-1987.

[8] GDP data for Canada, Finland, France, Netherlands, and Sweden have all had major change since Maddison's original book.

choose to use only the revised GDP data available in Maddison's 1989 work. This has the disadvantage of truncating the length of the series by 29 years and possibly missing very long run convergence. However, the data are of substantially higher quality than previous studies as they correspond more closely to current definitions of GDP, given the extra effort in calculating the early years, and allow the inclusion of more countries with uninterrupted series.

Several difficulties remain with our particular data set. First, as DeLong [1988] has argued, the sample includes only countries that have successfully industrialized and therefore is weighted towards accepting the convergence hypothesis. On the other hand, any failure to find convergence will therefore be more persuasive given this bias. Our statistical tests will take no convergence as the null; consequently, DeLong's critique will imply that the size of our tests is larger than the nominal 5% without any implication for the tests' power properties.

More important, however, there are still problems of data quality due to changes in definitions of borders. Germany presents a special problem for tests of convergence. While some may argue that Germany is a prime example of convergence in action due to its rapid growth in the post-WWII period, the numerous boundary changes and population gains and losses for Germany over the entire century make inference difficult. We choose to include Germany for completeness, but we will not stress any conclusions which hinge on Germany's inclusion.

The population data as published in Maddison [1982] are not adjusted to conform to current boundaries, as is the GDP data. Failure to account for boundary changes can lead to large one time income per capita movements as population is gained or lost. For example, GDP per capita in the UK jumps in 1920 without a correction for the loss of the population of Ireland in that year. To avoid these discrete jumps we adjust the population to reflect modern borders.[9] (It should be noted that Cogley [1990] and others do not appear to have

[9] This type of gain or loss affects Belgium, Canada, Denmark, France, Italy, Japan, and the

made these corrections although it is not clear how much, they affect the results.)

The GDP data set also has a few potential difficulties. The year to year movements during the two world wars for Belgium and during WWI for Austria are constructed from GDP estimates of neighboring countries. This means that we will be less likely to reject cointegration for Belgium-France, Belgium-Netherlands or Austria-Germany. Again since our null is no convergence, these linkages increase the possibility of mistaken rejections, which only affects the size of the tests.

5. Empirical results on cointegration and convergence

5.1 Bivariate tests

The first step in determining if the countries in our sample are converging is to see i they have common stochastic elements as defined in Section 2. To do this we use the ADI cointegration tests described above. The results are presented in Table 2. 39 out of 105 pair reject the null hypothesis of no cointegration at the 5% level.

These results give some support to the idea that there are common elements in outpu growth across countries. However, more than one half of the pairs cannot reject the null ‹ no cointegration and thus it is unlikely that the entire sample is driven by a single commc trend. A closer examination of the significant statistics shows that a small subset of tl countries account for almost a third of the rejections for both lag lengths. These countries ar Austria, Belgium, Denmark, France, Italy, and the Netherlands.[10] This is not a surprisi group of countries to share common stochastic elements since every member except Denma has at least one common border with another country in the block. For Belgium and t Netherlands, it would be astounding if we did not see convergence since these two countr

UK at least once. If territory, and thus population, are lost by country X in year T_1, we adj earlier years by extrapolating backward from T_1 using the year-to-year population changes country X.

[10] These six countries plus Germany reject in 18 pairs or almost half the total.

have highly intertwined economies, are members of Benelux and have a common culture. We will make use of this group of countries in the multivariate analysis to help identify the sources of cointegration for larger samples.

Proximity can help explain other cointegrated pairs such as US-Canada, and Finland with Denmark, Norway and Sweden, while former colonial ties may account for Australia-U.K. Geography alone cannot explain all the results. Pairs such as Belgium-Japan and France-Japan remain troublesome. However, cointegration does not necessarily imply convergence to similar output levels.

To better understand the results from the ADF cointegration tests, we look at the distribution of the estimated cointegration coefficients as presented in Figure 5. The coefficients are all near one, although most are statistically significantly different from one. One interpretation of the clustering near unity combined with the lack of cointegration found in the ADF statistics might be that while common shocks may impact economies similarly, there remain country-specific elements in long run growth that induce divergence across many pairs of countries. For example, if $Y_{i,t} = \kappa_{1,t} + \kappa_{2,t}$ and $Y_{j,t} = \kappa_{1,t} + \kappa_{3,t}$ are two income processes where $\kappa_{1,t}, \kappa_{2,t}$, and $\kappa_{3,t}$ are independent random walks, then the OLS estimate of β in $Y_{i,t} = \beta Y_{j,t} + \epsilon_{ij,t}$ will center around 1.

The bivariate cointegration tests are therefore suggestive of both common and idiosyncratic components to output fluctuations. Recall from Definitions 2.2 and 2.3 that common trends are necessary but not sufficient for convergence. Consequently, the evidence of common bivariate trends in the data may be consistent with divergence in per capita output. For example, different economies may process similar production sets differently. We now examine the behavior of output deviations across the 15 countries.

The first of our convergence tests parallels the cointegration tests in that it explores the presence of unit roots in the output deviation series $DY_{ij,t}$. Table 3 presents the minima of the Augmented-Dickey-Fuller statistics for $DY_{ij,t}$ for lag lengths, $k = 1, \ldots, 5$. This test rejects if the data are inconsistent with a unit root in the cross-country output differences.

Thus a rejection means that a pair of countries exhibits convergence as defined in Equation 2.3. The difference in the results from the cointegration tests is immediately apparent. Only 6 pairs of countries reject the no convergence null hypothesis at the five per cent level, o which five are in the group of six European countries.[11]

Since the ADF tests may possess low power against some mean reverting alternative in such small samples, we now turn to point estimates of the time series structure of th differences between pairs of output. These estimates provide some indication as to wheth our rejections of convergence are due to estimated autoregressive roots near unity or to larg standard errors leading to large confidence intervals on those roots. Table 4 contains th sums of the second order autoregression coefficients of $DY_{ij,t}$. Numbers near 1.0 indicate large amount of persistence in the output differentials. Only 7 of the 105 pairs produce sum less than 0.90 and 59 of the numbers are greater than or equal to 0.97. Among the ne combinations that appear to be stationary using 0.9 as a cutoff are Australia-UK, Austri Italy, Denmark-Netherlands and Finland-Germany. Except for the last pair all are eith geographically contiguous or tied by former colonial relations. While the point estimates a slightly more suggestive of stationarity, the large majority of the country pairs give evider of substantial persistence in output disparities.

Lastly in testing for bivariate convergence, we turn to the estimators based on spect density and distribution functions. First we estimate the zero frequency of output dev tions, $Y_i - Y_j$. As mentioned earlier windowed zero frequency estimates have large varian and thus are unlikely to provide definitive answers on the degree of persistence in the c put deviations series. Low estimates of the zero frequency will be taken as an indica of pairwise convergence. We then test for pure random walks in $Y_i - Y_j$ with the AD CVM statistics and then examine the point estimates of the spectral distribution funct Remembering the null hypothesis of the $U_T(t)$ statistics is that the differences of deviat in output, $DY_{ij,t}$, is a random walk, we will look for rejections that indicate an alternati long run mean reversion, which is equivalent to convergence. Since the spectral distribu

[11] Allowing for a constant difference in logs did not change the results markedly.

function summarizes all second moment information in the each data series, the mean reversion alternative will be preferred if there is a lack of power in the low frequencies. Again, rejections of the pure random walk null may not indicate mean reversion and hence convergence when there exists excess power in the low frequencies of the spectrum of $\Delta DY_{ij,t}$. Given our alternative of long run mean reversion, we will concentrate on the frequency bands $[0, \frac{\pi}{8}]$ and $[0, \frac{2\pi}{8}]$.

The windowed zero frequency estimates are shown in Table 5. Ten country pairs reject of the hypothesis that the zero frequency of output deviations equals one for the $\frac{\pi}{16}$ window. Seven of the ten pairs are in the group of European countries listed above. If we look for low point estimates, those below 0.25, we find that 14 of 17 pairs come from the group of six European countries. On the other end, there are 20 estimates greater than one. This suggests that some pairs may be converging while some are diverging. The standard deviation of the $\frac{\pi}{8}$ window estimates is so large that even a point estimate of zero does not reject the hypothesis that the zero frequency equals one. The general magnitude of the estimates remains unchanged.

Table 6 shows a number of rejections for the random walk null hypothesis by the AD and CVM statistics. 39 of the 105 pairs reject at the 5% level for both statistics and an additional 11 reject for one test. On the other hand, 52% of the combinations fail to reject the random walk null for either statistic, which argues against convergence for the whole sample. We must again look more closely at the spectral distribution estimates to determine the nature of these rejections.

The spectral distribution point estimates and the $U_T(t)$ statistics for $[0, \frac{\pi}{8}]$ in Table 7, above the diagonal, gives some evidence for the alternative of mean reversion. As Table 7 indicates, only two of the 105 pairs reject the random walk null at the 5% level because of too little power in these frequencies; however, a total 66 pairs have deficient power to some extent, i.e the point estimate is below 0.125. Leaving aside Germany and the six European countries which have already shown a high degree of cointegration, the number of combinations that display too little power drops to 37 of a possible 76. The point estimates of $f(\frac{\pi}{8})$ for these 37 indicate some evidence of substantial deficiencies in spectral power. In particular, 15 pairs possess less than $\frac{2}{3}$ of the power of the white noise null in the $[0, \frac{\pi}{8}]$ interval and 5 pairs have less than half of the power under that null.

The evidence here is certainly ambiguous. Over this frequency range, many pairs have less persistence in the output differential than is consistent with a random walk. In particular, the six European countries which are pairwise cointegrated also appear to be converging. Additionally, other pairs including Germany, whose univariate output series is also deficient from $[0, \frac{\pi}{8}]$, show signs of mean reversion. However, there are 17 pairs of countries whose output differential displays more power in the $[0, \frac{\pi}{8}]$ range than do either of the output series by themselves. This is powerful evidence against convergence for these combinations.

Turning to the results for $[0, \frac{2\pi}{8}]$, also in Table 7, we find many more rejections of the random walk null at the 5% level, most of them due to excess power. Three more pairs do show up negative and significant, Germany-Japan, Germany-Italy and Denmark-Norway; but now 28 combinations reject because of too much power. In particular the U.S. rejects in 8 instances. This is interesting because the U.S. is considered to be the 'leading' country in the post-war years and thus the other countries should be converging or catching up to the U.S per capita output levels. Only the U.S.-Canadian statistic even shows up with the negative sign required for convergence, but this is to be expected, given the high degree of interaction between the economies. Canada also rejects with 7 countries as would make sense if U.S. and Canadian output levels are converging.

There still is some evidence of too little power for 30 of 105 pairs. However, excluding Germany and the European six, only 10 other combinations are deficient. These results confirm that while convergence again appears unlikely for all 15 countries, there are subsets of countries for which the convergence hypothesis cannot be rejected.[12]

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[12] The overall pairwise results on cointegration and convergence are robust to changes in the testing methodology, see Bernard [1991]

5.2. Multivariate tests

Finally we consider tests for cointegration and convergence with more than two time series. This section gives us a sense of the extent of common stochastic components for blocks of countries. If idiosyncratic elements dominate for every country then we would expect to find n distinct roots for n countries. If countries converge then we would expect to find 1 distinct root. If the number of significant roots lies between these extremes, this indicates the presence of common elements in international output. As an alternative measure of the number of common trends, we look at the cumulative percentage of the sum of the roots. If the first $p < n$ largest roots contribute 95% or more of the sum, then we conclude that there are p important common stochastic trends for the block.[13]

To use these various measures, we must arrange the countries into groups. We first consider all 15 countries together. We initially test for cointegration by performing the Phillips-Ouliaris bounds tests on the first difference of output, $\Delta\vec{Y}_t$. Next, we repeat the exercise for those six European countries which displayed a high degree of cointegration in the pairwise results. Finally, to determine if the removal of those six countries affects the results from the original 15, we run the tests on the remaining nine.

Second we examine the behavior of output deviations. In order to identify convergence for the 15 country group, we test using $\Delta D\vec{Y}_t$, having subtracted off the US output, as in Theorem 3.4. We separately test for convergence in the 6 European economies exhibiting substantial cointegration by examining output deviations from France and in the 9 remaining countries by examining output deviations from the US.

In Table 8, we present the Phillips-Ouliaris bounds tests for cointegration and the cumulative sums of the eigenvalues for the groups mentioned above.[14] There are two distinct tests for each group. First if the upper bound is less than the critical value for a given p, we can reject the null hypothesis that there are p or more distinct roots. If the lower bound is

[13] Cogley [1990] uses a similar measure.

[14] K, the size of the Daniell window was chosen to be $T^{0.6}$, or 27 for our sample.

greater than the same critical value then we accept the hypothesis that there are at least j distinct roots.

Table 9 contains the results of the convergence tests and presents the upper and lowe bounds on the largest eigenvalue of output deviations as well as cumulated sums of eigen values for the different groups of countries. Here, if the lower bound on the largest root i greater than the critical level we can accept the no convergence null.

For the fifteen country sample and critical value C_1 $(= \frac{m}{n})$[15], we reject the null hy pothesis that there are 7 or more distinct roots and we accept the null that there at least distinct roots. With the alternative critical value of 5% of the sum of the eigenvalues, C_2 we again reject for 7 or more distinct roots but now accept for at least 5. This leads us t posit that there is a large common stochastic component over the sample. The six larges roots account for 96.7% of the total, coinciding with the results from the test statistics. O the other hand, the largest root accounts for barely 50% and the largest two roots for abou 75% of total variance, which argues against the existence of just a single common factor, a is required for convergence. The direct convergence test in Table 9 accepts the no conve gence null for both critical levels as the largest eigenvalue is statistically different from zer Observe that over 95% of the output deviation variance is attributable to the first 4 factor Overall the output deviations exhibit somewhat greater concentration of variance in few roots than do the output levels. For example, the largest factor contributes 52% of the tot variance to levels whereas the largest factor in the deviations contributes 74%. This sugges that there are some common elements which are cancelled out in the deviations.

Turning to the results for the six European countries which were largely cointegrat in the pairwise exercise, we reject the null that there are are 4 or more distinct roots wi both the C_1 and C_2 critical values and accept the null that there are at least 3, again wi both values. 97.6% of the sum comes from the three largest eigenvalues. Even in this samp we do not find evidence for complete convergence as the largest eigenvalue is statistica

[15] See Appendix A.

significant. Interestingly, a comparison of the cumulated eigenvalue contributions in Tables 8 and 9 reveals that the variance contributions of the first few factors are comparable for the levels and deviations, suggesting that France does not contain a common factor for the remaining economies in the subsample.

We also consider the behavior of the sample after removing these six European countries from the larger set. The nine remaining countries display a large number of distinct roots. We reject the null that there are 7 or more distinct roots, but we can accept the null that there are at least 5 for both critical values. The largest five sum to 95.7% of the total. It appears that removing the six countries has dropped the number of distinct roots from the 15 country case by at most one. We still do not have completely idiosyncratic components dominating the sample. The convergence test for this subsample once again accepts the no convergence hypothesis.

These results do not support either of the extreme hypotheses that countries converge or that they are entirely dominated by idiosyncratic elements. There is substantial evidence for common stochastic components, particularly in the European sub-sample.[16]

6. Conclusions

This paper attempts to answer empirically the question of whether there is convergence in output per capita across countries. This question is important to a large body of recent macroeconomic theory as new models of economic growth, in contrast to traditional formulations generating turnpike results, have shown how long run divergence can occur across economies.

We first construct a stochastic definition of convergence based on the theory of integrated time series. Time series for per capita output of different countries can fail to converge only if the persistent parts of the time series are distinct. Consequently, we are able to

[16] Using a different multivariate methodology, Bernard [1991] finds similar results for these samples.

identify the common elements of long term growth across economies by examining whether deviations in aggregate output series contain unit roots; in addition, we can determine whether aggregate output in different economies is determined by common factors through cointegration analysis.

Our analysis of the relationship among long term output movements across countries reveals little evidence of convergence. Both time and frequency domain approaches indicate that there is substantial persistence in per capita output deviations. Virtually all of our hypothesis tests accept the null hypothesis of no convergence for both bivariate and multivariate data samples. Our rejection of convergence holds despite the argument by DeLong [1988] that concentration on OECD economies will bias empirical work towards accepting convergence. On the other hand, we find evidence that there is substantial cointegration across OECD economies. Further, we find that the number of integrated processes driving the 15 countries' output series appears to be on the order of 4-6. Our results therefore imply that there is clearly some set of common factors which jointly determines international output growth.

Overall, our conclusions on the absence of convergence are consistent with either the class of macroeconomic growth models which emphasizes the potential for multiple steady state equilibria due to complementarities or those Arrow-Debreu models where microeconomic differences can cause divergence. This ambiguity is natural given the atheoretical nature of the data analysis. An important next step in empirical work on convergence is the estimation of different stochastic growth models to determine the mapping of our reduced form results into structural inferences. In turn, structural estimation will permit the evaluation of whether divergence in aggregate economies is indicative of essential deviations from the competitive equilibrium paradigm.

Appendix A. The Phillips-Ouliaris Bounds Test

In order to test for cointegration and the number of common stochastic trends, we estimate the zero frequency of

$$\widetilde{\Delta \vec{Y}} = \Delta \vec{Y} - \overline{\Delta Y} \tag{A1}$$

where $\overline{\Delta Y}$, a scalar, is the average growth rate of output across countries. The smoothed estimate of the spectral density matrix at the zero frequency, $f_{\widetilde{\Delta \vec{Y}}\widetilde{\Delta \vec{Y}}}(0)$, using the real parts of the periodogram estimates, is given by

$$\hat{f}_{\widetilde{\Delta \vec{Y}}\widetilde{\Delta \vec{Y}}}(0) = \frac{2\pi}{2k+1}\left[I_{\widetilde{\Delta \vec{Y}}\widetilde{\Delta \vec{Y}}}(0) + \sum_{s=1}^{k} Re\left(I_{\widetilde{\Delta \vec{Y}}\widetilde{\Delta \vec{Y}}}\left(\frac{2\pi s}{T}\right)\right)\right] \tag{A2}$$

with k the number of ordinates used in the rectangular (Daniell) filter. The resulting estimated spectral density matrix is then decomposed into its ordered eigenvalues, $\lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_n$. The ratio of the sum of the smallest m eigenvalues to the sum of all the eigenvalues is calculated and its upper and lower bounds determined.

$$\Lambda_p = \frac{\sum_{j=1}^{m} \lambda_j}{\sum_{j=1}^{n} \lambda_j} \quad where\ p = n - m + 1 \tag{A3}$$

$$\Lambda_{p,U} = \Lambda_p + \frac{z_\alpha B}{k^{1/2}} \tag{A4}$$

$$\Lambda_{p,L} = \Lambda_p - \frac{z_\alpha B}{k^{1/2}} \tag{A5}$$

where z_α is the α-percent critical value of the standard normal distribution and

$$B = \frac{\left[\left(\sum_{j=1}^{m} \lambda_j\right)^2 \left(\sum_{j=m+1}^{n} \lambda_j^2\right) + \left(\sum_{j=m+1}^{n} \lambda_j\right)^2 \left(\sum_{j=1}^{m} \lambda_j^2\right)\right]^{\frac{1}{2}}}{\left(\sum_{j=1}^{n} \lambda_j\right)^2}. \tag{A6}$$

If the upper bound, $\Lambda_{p,U}$, is less than the critical value we reject the null hypothesis that there are p or more distinct roots. If the lower bound, $\Lambda_{p,L}$, is greater than the critical value then we accept the null hypothesis that there are at least p distinct roots. As emphasized by Phillips and Ouliaris, there is no preassigned critical value selected for these statistics. However, the tests are designed so that if there are zero eigenvalues, the power of the test will go to one asymptotically. If the matrix is of full rank but some of the roots are small, there is a relatively high probability of mistakenly rejecting the null.

We define some critical values for the two null hypotheses. For the purpose of identifying whether the series are cointegrated, we examine the bounds of λ_p relative to two critical values, $C_1 = 0.10\frac{m}{n}$ and $C_2 = 0.05$. These critical values assess the average of the m smallest eigenvalues in comparison to the average of all the eigenvalues. Consequently we compare the average variance of the smallest m factors to the average variance of all n factors. Using the upper bound statistic and the critical value, C_1, we reject the null of $n - m + 1(q+1)$ or more distinct roots if the sum of the smallest m eigenvalues is less than $10m\%$ of the sum of all the eigenvalues. Interpreted differently, we reject if the upper bound is less than $m \times 10\%$ of the average root. Employing the same critical value and the lower bound statistic, we accept the null of at least $q+1$ distinct roots if the sum of the smallest m eigenvalues is greater than $m \times 10\%$ of the average root. One employs C_2 in an analogous fashion.

The null hypothesis of no convergence requires that all eigenvalues for the matrix $f_{\Delta D\tilde{Y}}(0)$ equal zero. This means that we need to test for $p = 1$. In order to do this, we consider the largest eigenvalue for $f_{\Delta D\tilde{Y}}(0)$, denoted as $\vec{\lambda}_n$.[17] Asymptotic upper and lower $100(1-\alpha)\%$ confidence intervals for this eigenvalue are

[17] The results for this non-normalized eigenvalue are reported in Table 9. Additionally we confirmed the results using the spectral density matrix at the zero frequency normalized by the variance-covariance matrix for the first differences, a normalization suggested by Phillips and Ouliaris for series denominated in different units.

$$\vec{\lambda}_{n,U} = \vec{\lambda}_n + \frac{\bar{\lambda}_{n,} z_\alpha}{K^{1/2}} \tag{A7}$$

$$\vec{\lambda}_{n,L} = \vec{\lambda}_n - \frac{\bar{\lambda}_{n,} z_\alpha}{K^{1/2}} \tag{A8}$$

respectively. We then follow the same rule as before: reject the null of no convergence if the upper bound is less than C_1 (or C_2), accept the null of no convergence if the lower bound is greater than C_1 (or C_2).

Bibliography

Barro, R. [1990]. "Economic Growth in a Cross Section of Countries." *Quarterly Journal of Economics*, forthcoming.

Baumol, W. [1986]. "Productivity Growth, Convergence, and Welfare: What the Long-Run Data Show." *American Economic Review*, 76, 1072-1085.

Baumol, W., Blackman, S. and Wolff, E. [1989]. *Productivity Growth: The Long View.* Cambridge, MIT Press.

Bernard, A. B. [1990]. "Testing for Random Walks in Exchange Rates." Working Paper, Stanford University.

_______ [1991]. "Empirical Implications of The Convergence Hypothesis." Working Paper, Stanford University.

Bernard, A. B. and S. N. Durlauf. [1991a]. "Interpreting Tests of the Convergence Hypothesis." Working Paper in progress, Stanford University.

_______ [1991b]. "Persistence in International Output Movements." Working Paper in progress, Stanford University.

_______ [1991c]. "Power Properties of Martingale Tests." Working Paper in progress, Stanford University.

Campbell, J. Y. and N. G. Mankiw. [1987]. "Are Output Fluctuations Transitory." *Quarterly Journal of Economics*, CII, 857-880.

_______ [1989]. "International Evidence on the Persistence of Economic Fluctuations." *Journal of Monetary Economics*, 23, 319-333.

Cochrane, J. [1988]. "How Big is the Random Walk in GNP?" *Journal of Political Economy*, 96, 893-920.

Cogley, T. [1990]. "International Evidence on The Size of the Random Walk in Output." *Journal of Political Economy*, 98, 501-518.

DeLong, J. B. [1988]. "Productivity Growth, Convergence, and Welfare: Comment." *American Economic Review*, 78, 1138-1154.

Durlauf, S. N. [1989]. "Output Persistence, Economic Structure and the Choice of Stabilization Policy." *Brookings Papers on Economic Activity*, 2, 69-116.

_______ [1990a]. "Spectral Based Tests of the Martingale Hypothesis." *Journal of Econometrics*, forthcoming.

_______ [1990b]. "Time Series Properties of Aggregate Output Fluctuations." Working Paper, Stanford University.

_______ [1991a]. "Nonergodic Economic Growth." Working Paper, Stanford University.

_______ [1991b]. "Path Dependence in Aggregate Output." Working Paper, Stanford University.

Engle, R. F. and C. W. J. Granger. [1987]. "Cointegration and Error-Correction: Representation, Estimation, and Testing." *Econometrica*, 55, 251-276.

Heller, W. P. [1986]. "Coordination Failure Under Complete Markets with Applications to Effective Demand." in *Essays in Honor of Kenneth J. Arrow, Volume II*, W. P. Heller, R. M. Starr and D. A. Starrett, eds.

_______ [1990]. "Perfect Foresight Coordination Failure with Savings and Investment." Working Paper, UC San Diego.

Jones, L. and R. Manuelli. [1990]. "A Convex Model of Equilibrium Growth." *Journal of Political Economy*, 98, 5, 1008-38.

King, R., C. Plosser, J. Stock and M. Watson. [1987]. "Stochastic Trends and Economic Fluctuations." Working Paper, University of Rochester.

Lucas, R. E. [1988]. "On the Mechanics of Economic Development." *Journal of Monetary Economics*, 22, 3-42.

Maddison, A. [1982]. *Phases of Capitalist Development.* Oxford, Oxford University Press.

_______. [1989]. *The World Economy in the 20th Century.* Paris, Development Centre of the Organisation for Economic Co-operation and Development.

Murphy, K., A. Shleifer and R. Vishny. [1989]. "Industrialization and the Big Push." *Journal of Political Economy*, 97, 1003-1026.

Phillips, P.C.B. and S. Ouliaris. [1988]. "Testing for Cointegration Using Principal Components Methods." *Journal of Economic Dynamics and Control*, 12, 205-30.

Quah, D. [1990]. "International Patterns of Growth: Persistence in Cross-Country Disparities." Working Paper, MIT.

Romer, P. [1986]. "Increasing Returns and Long Run Growth." *Journal of Political Economy*, 94, 1002-1037.

Summers, R., and A. Heston. [1988]. "A New Set of International Comparisons of Real Product and Price Levels Estimates for 130 Countries, 1950-1985." *Review of Income and Wealth*, 34, 1-25.

Figure 1: GDP Per Capita - 15 Countries 1900-1987

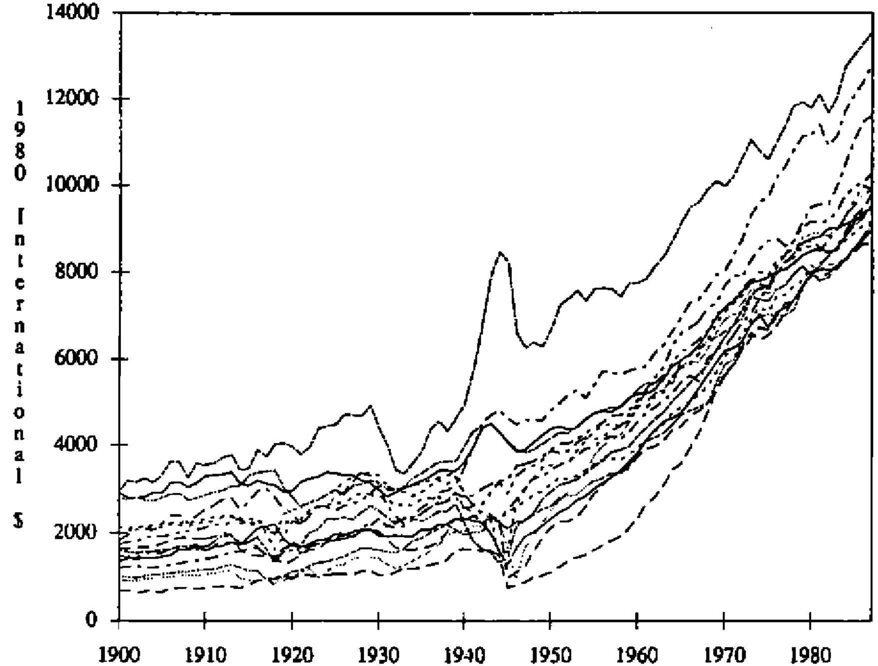

Figure 2: Log GDP per Capita - 15 Countries

1900-1987

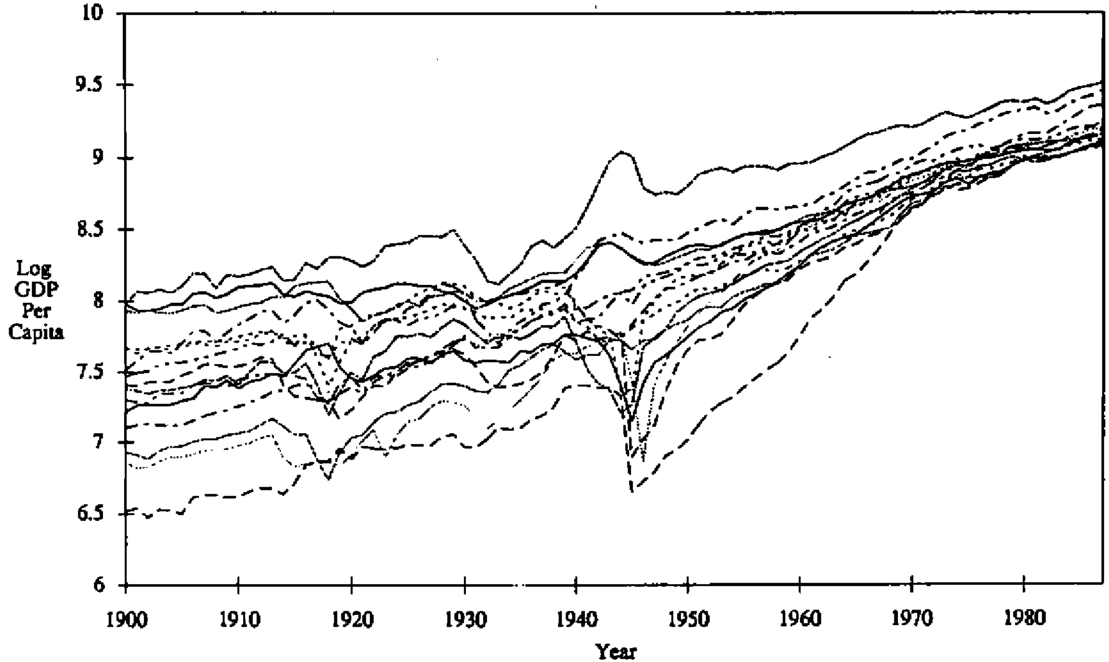

Figure 3: Difference from US Log GDP Per Capita
7 European Countries 1900-1987

(Austria, Belgium, Denmark, France, Germany, Italy, Netherlands)

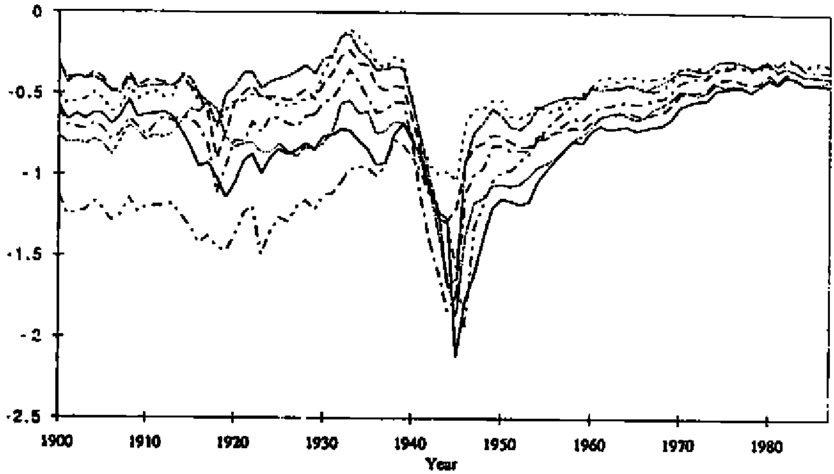

Figure 4: Difference from US Log GDP Per Capita
Non-European, Scandinavia and the UK 1900-1987

(Australia, Canada, Finland, Japan, Norway, Sweden, United Kingdom)

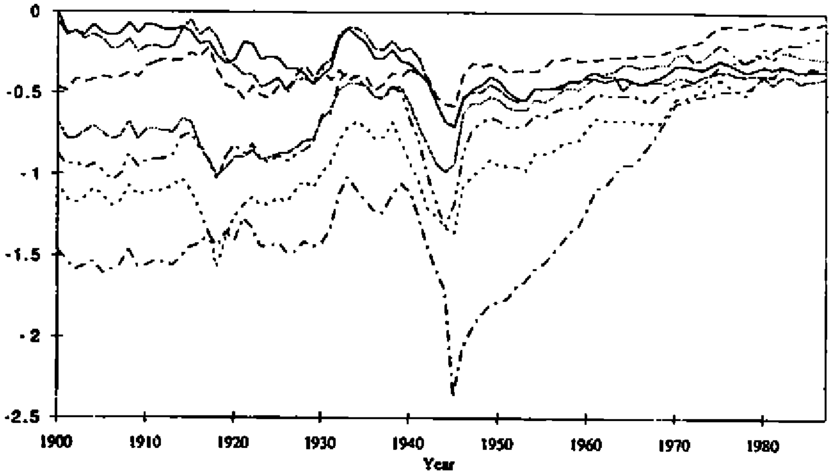

Figure 5: Histogram of Estimated Cointegrating Coefficients

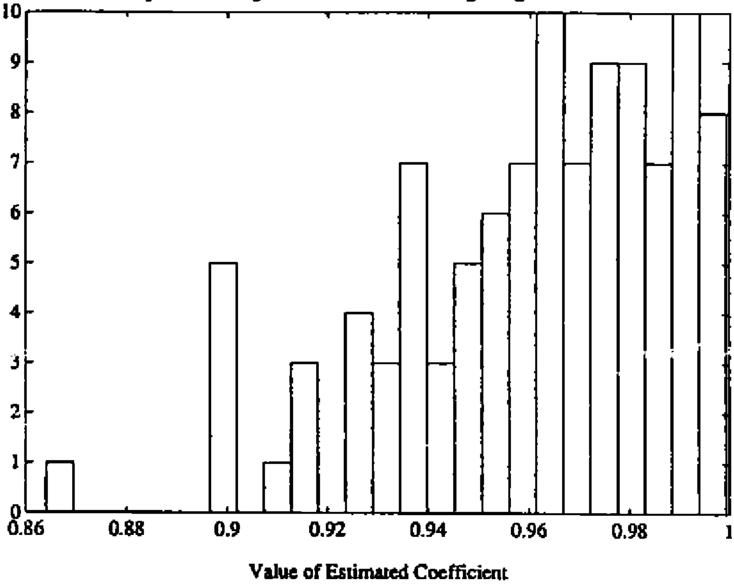

Table 1: Means and Standard Deviations of Output Growth Rates
15 Countries, 1900-1987

Country	Mean*
AL	0.0136 (0.033)
AS	0.0192 (0.114)
BE	0.0162 (0.051)
CA	0.0224 (0.054)
DE	0.0199 (0.044)
FI	0.0257 (0.050)
FR	0.0204 (0.080)
GE	0.0270 (0.105)
IT	0.0217 (0.065)
JA	0.0307 (0.091)
NE	0.0167 (0.084)
NO	0.0259 (0.043)
SW	0.0222 (0.034)
UK	0.0137 (0.038)
US	0.0176 (0.062)

* Standard deviations in parentheses

Table 2

PAIRWISE COINTEGRATION TESTS ON LOG PER CAPITA OUTPUT+
15 Countries - 1900-1987

	AL	AS	BE	CA	DE	FI	FR	GE	IT	JA	NE	NO	SW	UK
AL	----													
AS	-3.16	---												
BE	-2.71	-7.30*	---											
CA	-3.03	-2.57	-1.67	---										
DE	-2.76	-3.09	-2.41	-2.33	---									
FI	-2.17	-3.20*	-1.52	-2.37	-4.40*	---								
FR	-2.69	-6.13*	-5.53*	-1.76	-2.85	-2.00	---							
GE	-3.42*	-3.23*	-3.87*	-3.55*	-6.39*	-7.29*	-5.01*	---						
IT	-2.55	-4.86*	-5.38*	-1.46	-2.92	-2.39	-4.79*	-4.36*	---					
JA	-1.87	-2.45	-4.23*	-1.29	-2.24	-1.82	-3.52*	-2.73	-3.81*	---				
NE	-3.05	-6.02*	-6.70*	-2.48	-4.52*	-2.87	-11.09*	-5.25*	-5.33*	-4.32*	---			
NO	-1.86	-3.13	-1.28	-1.85	-5.31*	-4.18*	-1.86	-5.31*	-2.59	-2.34	-2.84	---		
SW	-2.17	-2.90	-1.48	-2.31	-2.49	-4.30*	-1.51	-7.02*	-1.89	-1.61	-2.53	-2.14	---	
UK	-3.32*	-3.23*	-2.79	-2.54	-3.70*	-3.60*	-2.68	-4.89*	-2.33	-2.58	-3.16	-3.01	-3.42*	---
US	-2.52	-2.53	-1.58	-4.37*	-2.56	-2.60	-1.98	-2.89	-1.97	-1.88	-2.67	-2.60	-3.19*	-2.57

* Denotes significant at 5%

+ Statistics above the diagonal are Augmented- Dickey-Fuller w/1 lag; below the diagonal , ADF w/5 lags. Critical values are taken from Engle-Granger [1987].

The equations estimated were :

$$Y_{i,t} = C_{ij} + \gamma_{ij} Y_{j,t} + \xi_{ij,t} \qquad (3.1)$$

$$\Delta\xi_{ij,t} = -\theta_{ij}\xi_{ij,t-1} + B(L)\Delta\xi_{ij,t-1} + \zeta_{ij,t} \qquad (3.2)$$

Table 3

PAIRWISE CONVERGENCE TESTS ON LOG PER CAPITA OUTPUT+
15 Countries - 1900-1987

	AL	AS	BE	CA	DE	FI	FR	GE	IT	JA	NE	NO	SW	UK
AL	---													
AS	-0.72	---												
BE	-0.72	-2.38	---											
CA	-1.17	-1.16	-1.04	---										
DE	-0.47	-1.49	-1.75	-1.57	---									
FI	1.05	-1.89	1.26	-0.02	0.39	---								
FR	-0.35	-4.08*	-1.02	-0.82	-1.19	-0.46	---							
GE	0.26	-0.81	-0.49	-0.41	-0.78	-4.62*	-2.01	---						
IT	0.13	-4.31*	-1.22	-0.49	-0.95	-1.28	-3.35*	-2.00	---					
JA	0.71	0.48	0.25	0.09	0.23	-0.61	-0.16	-1.40	0.07	---				
NE	-1.58	-2.07	-3.80*	-1.99	-4.49*	0.06	-0.74	-0.96	-0.92	0.10	---			
NO	0.58	-2.52	0.19	-0.08	-0.39	-1.14	-1.71	-2.12	-2.29	-0.48	-0.57	---		
SW	0.71	-1.82	-0.31	-0.57	-0.60	0.04	-1.10	-1.39	-1.15	-0.09	-1.17	-1.08	---	
UK	-2.85	-0.85	-1.00	-1.05	-0.90	0.70	-0.48	0.11	0.15	0.61	-1.88	0.58	0.35	---
US	-1.18	-0.60	-0.55	-0.07	-0.38	0.48	-0.31	0.14	-0.12	0.43	-0.94	0.34	0.17	-0.98

* Denotes significant at 5%

\+ Statistics are $\min_k$(ADF) where k is the number of lags in B(L) and ranges from 1 to 5. Critical values are taken from Engle-Granger[1987].

The equation estimated was : $\Delta DY_{ij,t} = -\theta_{ij} DY_{ij,t-1} + B(L)\Delta DY_{ij,t-1} + \zeta_{ij,t}$

Table 4

SUMS OF AR(2) COEFFICIENTS OF CROSS-COUNTRY DIFFERENCES IN LOG OUTPUT PER CAPITA
15 Countries - 1900-1987

	AL	AS	BE	CA	DE	FI	FR	GE	IT	JA	NE	NO	SW	UK	US
AL	—														
AS	0.98	—													
BE	0.97	0.91	—												
CA	0.96	0.98	0.97	—											
DE	0.95	0.95	0.94	0.93	—										
FI	0.98	0.90	0.98	0.99	0.99	—									
FR	0.97	0.81	0.94	0.96	0.95	0.96	—								
GE	0.98	0.95	0.96	0.98	0.97	0.81	0.92	—							
IT	0.98	0.78	0.92	0.98	0.96	0.93	0.82	0.93	—						
JA	0.99	0.98	0.98	0.99	0.99	0.97	0.97	0.97	0.98	—					
NE	0.92	0.94	0.81	0.92	0.76	0.97	0.96	0.95	0.96	0.98	—				
NO	0.98	0.91	0.97	0.97	0.97	0.98	0.93	0.93	0.95	0.98	0.96	—			
SW	0.98	0.94	0.98	0.97	0.97	0.99	0.94	0.95	0.36	0.98	0.94	0.96	—		
UK	0.84	0.97	0.96	0.97	0.93	0.98	0.96	0.97	0.98	0.98	0.90	0.98	0.98	—	
US	0.99	0.99	0.99	0.98	0.98	0.99	0.98	0.99	0.99	0.99	0.97	0.98	0.99	0.99	—

The equation estimated was : $DY_{ij,t} = \beta_{1,ij} DY_{ij,t-1} + \beta_{2,ij} DY_{ij,t-2} + \varepsilon_{ij,t}$

Table 5

ZERO FREQUENCY SPECTRAL DENSITY ESTIMATES+
CHANGES IN CROSS-COUNTRY DIFFERENCES IN LOG OUTPUT PER CAPITA
15 Countries - 1900-1987

	AL	AS	BE	CA	DE	FI	FR	GE	IT	JA	NE	NO	SW	UK	US
AL	–	0.34	0.51	0.81	0.46	1.24	0.58	0.54	0.94	2.08	0.49	0.95	1.18	0.10*	0.40
AS	0.63	–	0.08*	0.50	0.22	0.30	0.06*	0.27	0.15*	1.18	0.11*	0.28	0.34	0.38	0.49
BE	1.08	0.30	–	0.92	0.37	1.76	0.38	0.34	0.24	0.71	0.18	1.46	1.77	0.44	0.69
CA	0.72	0.80	1.17	–	0.56	0.71	0.72	0.43	0.95	1.63	0.69	0.53	0.76	1.14	0.38
DE	0.94	0.48	0.47	1.01	–	0.79	0.39	0.23	0.33	1.07	0.23	0.54	1.17	0.41	0.42
FI	1.27	0.36	1.24	0.93	0.59	–	0.85	0.15*	0.65	1.00	0.75	0.29	0.52	0.79	0.69
FR	1.22	0.38	0.59	1.30	0.65	1.02	–	0.15*	0.08*	0.38	0.15*	0.93	1.01	0.61	0.57
GE	0.65	0.17	0.42	0.76	0.37	0.26	0.33	–	0.27	0.70	0.20	0.22	0.14*	0.49	0.51
IT	1.26	0.49	0.62	1.73	0.58	0.69	0.72	0.25	–	1.01	0.20	0.67	1.01	1.29	0.73
JA	1.67	1.84	1.23	1.70	1.42	1.27	1.19	0.58	0.99	–	0.5[illegible]	0.86	1.43	1.88	1.75
NE	0.95	0.35	0.26	1.08	0.39	0.75	0.21	0.28	0.70	1.11	–	0.87	0.84	0.50	0.47
NO	1.36	0.43	1.41	1.37	0.54	0.65	0.96	0.33	0.52	1.0[illegible]	0.75	–	1.11	0.93	0.45
SW	1.23	0.53	1.53	0.99	0.92	0.54	1.37	0.37	1.14	1.47	1.02	1.68	–	0.83	0.50
UK	0.71	0.58	0.97	2.11	0.77	0.93	1.09	0.51	0.97	1.22	0.84	1.02	1.17	–	0.30
US	0.78	0.85	1.32	0.61	1.32	1.30	1.31	0.97	1.46	1.72	1.01	1.51	1.27	1.58	–

* Denotes significantly different from one at 5%

+ Statistics above the diagonal are zero frequency estimates of $Y_i - Y_j$ with a Daniell window of $\pi/16$
Statistics below the diagonal are zero frequency estimates of $Y_i - Y_j$ with a Daniell window of $\pi/8$

Table 6

RANDOM WALK TESTS+
CHANGES IN CROSS-COUNTRY DIFFERENCES IN LOG OUTPUT PER CAPITA
15 Countries - 1900-1987

	AL	AS	BE	CA	DE	FI	FR	GE	IT	JA	NE	NO	SW	UK	US
AL	— —														
AS	0.05 (0.49)	— —													
BE	0.60* (3.36*)	0.08 (0.95)	— —												
CA	0.48* (3.40*)	0.02 (0.24)	1.29* (8.19*)	— —											
DE	0.33 (1.86)	0.33 (2.35)	0.43 (2.79*)	0.84* (5.37*)	— —										
FI	0.32 (1.72)	0.05 (0.70)	0.08 (0.60)	0.87* (5.37*)	0.24 (1.46)	— —									
FR	0.69* (4.10*)	0.19 (1.44)	0.26 (1.55)	1.19* (7.72*)	0.51* (2.61*)	0.71* (4.12*)	— —								
GE	0.16 (1.35)	3.89* (21.23*)	0.13 (1.24)	0.08 (0.59)	0.10 (1.07)	0.25 (2.24)	0.23 (1.94)	— —							
IT	0.60* (3.69*)	0.14 (1.13)	0.56* (3.19*)	0.95* (6.24*)	1.02* (5.80*)	0.76* (4.40*)	0.45 (2.65*)	0.29 (2.74*)	— —						
JA	0.04 (0.41)	0.26 (2.20)	0.30 (1.90)	0.15 (1.35)	0.87* (5.56*)	0.41 (2.70*)	0.56* (3.57*)	1.74* (9.46*)	0.05 (0.37)	— —					
NE	0.56* (3.20*)	0.09 (0.98)	0.42 (2.87*)	0.62* (3.85*)	0.39 (2.46)	0.67* (3.63*)	0.26 (2.68*)	0.33 (2.88*)	0.06 (0.69)	0.29 (1.78)	— —				
NO	0.46* (2.71*)	0.03 (0.53)	0.10 (0.62)	1.54* (9.82*)	0.38 (3.14*)	0.16 (1.22)	0.07 (0.45)	0.19 (1.56)	0.07 (0.65)	0.02 (0.23)	0.14 (0.96)	— —			
SW	0.64* (3.37*)	0.08 (0.67)	0.20 (1.13)	0.93* (5.56*)	0.14 (0.67)	0.07 (0.70)	0.59* (3.69*)	0.16 (1.56)	0.95* (5.29*)	0.33 (2.13)	0.34 (1.89)	0.08 (0.82)	— —		
UK	0.32 (2.41)	0.02 (0.34)	0.58* (3.64*)	0.26 (2.06)	0.15 (1.16)	0.56* (3.45*)	0.71* (4.41*)	0.11 (1.11)	0.20 (1.08)	0.05 (0.32)	0.45 (2.66*)	0.42 (2.72*)	0.45 (2.86*)	— —	
US	0.46* (2.78*)	0.03 (0.27)	1.41* (8.76*)	0.15 (1.07)	0.85* (5.56*)	0.71* (4.41*)	1.34* (8.28*)	0.05 (0.35)	1.11* (6.62*)	0.11 (0.97)	0.91* (5.31*)	1.32* (8.45*)	0.93* (5.65*)	0.23 (1.76)	— —

+ Cramer-von-Mises statistics w/o parentheses. Anderson-Darling statistics w/parentheses.

* Denotes significant at 5% level.

Table 7

SPECTRAL DISTRIBUTION FUNCTION ESTIMATES+
CHANGES IN CROSS-COUNTRY DIFFERENCES IN LOG OUTPUT PER CAPITA
15 Countries - 1900-1987

	AL	AS	BE	CA	DE	FI	FR	GE	IT	JA	NE	NO	SW	UK	US
AL	—	0.08	0.13	0.07	0.11	0.10	0.15	0.06	0.15	0.17	0.12	0.13	0.11	0.09	0.09
AS	0.26	—	0.04	0.10	0.07	0.04	0.05	0.02*	0.07	0.21	0.05	0.06	0.07	0.08	0.11
BE	0.36	0.24	—	0.14	0.06	0.09	0.07	0.05	0.07	0.13	0.03	0.13	0.17	0.12	0.16
CA	0.41*	0.29	0.46*	—	0.13	0.11	0.17	0.10	0.22*	0.21	0.14	0.18	0.13	0.23*	0.07
DE	0.28	0.34	0.13	0.40*	—	0.07	0.08	0.05	0.07	0.17	0.05	0.05	0.12	0.09	0.17
FI	0.28	0.27	0.20	0.40*	0.19	—	0.12	0.03	0.08	0.16	0.08	0.09	0.06	0.08	0.15
FR	0.42*	0.25	0.25	0.49*	0.32	0.37	—	0.04	0.09	0.14	0.02*	0.11	0.17	0.13	0.16
GE	0.17	0.05*	0.18	0.21	0.18	0.16	0.16	—	0.03	0.07	0.03	0.05	0.05	0.05	0.11
IT	0.37	0.22	0.28	0.43*	0.33	0.32	0.35	0.09*	—	0.11	0.08	0.06	0.14	0.11	0.18
JA	0.31	0.37	0.37	0.37	0.45*	0.41*	0.42*	0.11*	0.26	—	0.12	0.13	0.18	0.12	0.20
NE	0.38*	0.22	0.15	0.39	0.21	0.30	0.08	0.12	0.32	0.37	—	0.08	0.12	0.10	0.13
NO	0.34	0.23	0.22	0.46*	0.09*	0.27	0.31	0.16	0.24	0.31	0.21	—	0.21	0.06	0.18
SW	0.32	0.27	0.28	0.40*	0.22	0.20	0.42*	0.16	0.36	0.39*	0.31	0.31	—	0.11	0.15
UK	0.36	0.26	0.42*	0.36	0.34	0.37	0.45*	0.17	0.30	0.26	0.38*	0.38*	0.39*	—	0.19
US	0.32	0.31	0.47*	0.23	0.43*	0.41*	0.49*	0.25	0.40*	0.34	0.40*	0.45*	0.42*	0.34	—

+ Figures above the diagonal are spectral distributions for $[0, \pi/8]$
Figures below the diagonal are spectral distributions for $[0, 2\pi/8]$

* Denotes $U_T(t)$ statistic significant at 5%

Table 8: Phillips-Ouliaris Bounds Tests for Cointegration

Bounds Tests

All Countries

P	Lower	Upper
15	0.0001	0.0002
14	0.0003	0.0005
13	0.0006	0.0011
12	0.0015	0.0026
11	0.0030	0.0052
10	0.0053	0.0088
9	0.0083	0.0136
8	0.0140	0.0229
7	0.0250	0.0411*+
6	0.0418	0.0681
5	0.0651+	0.1042
4	0.1068*	0.1694
3	0.1881	0.2932
2	0.3868	0.5672

6 European Countries

P	Lower	Upper
6	0.0019	0.0043
5	0.0048	0.0093
4	0.0149	0.0289*+
3	0.0582*+	0.1124
2	0.2271	0.3970

Remaining 9 Countries

P	Lower	Upper
9	0.0019	0.0041
8	0.0055	0.0101
7	0.0121	0.0209*+
6	0.0312	0.0541
5	0.0647*+	0.1079
4	0.1039	0.1661
3	0.2018	0.3142
2	0.4084	0.5893

If the upper bound is below the critical value, reject null of P or more distinct roots.
If the lower bound is above the critical value, accept null of at least P distinct roots.

* Significant at 0.10m/n where n is the number of countries and m is the number of roots = 0.

+ Significant at 5% of the sum of the roots.

Cumulative Percentage from p Largest Eigenvalues

	All Countries		6 European Countries		Remaining 9 Countries	
	P	Cumulated %	P	Cumulated %	P	Cumulated %
Largest	1	0.5230	1	0.6880	1	0.5012
	2	0.7594	2	0.9147	2	0.7420
	3	0.8619	3	0.9781	3	0.8650
	4	0.9153	4	0.9929	4	0.9137
	5	0.9450	5	0.9969	5	0.9574
	6	0.9669	6	1.0000	6	0.9835
	7	0.9816			7	0.9922
	8	0.9890			8	0.9970
	9	0.9929			9	1.0000
	10	0.9959				
	11	0.9980				
	12	0.9991				
	13	0.9996				
	14	0.9999				
Smallest	15	1.0000				

Table 9: Phillips-Ouliaris Bounds Tests for Convergence

Bounds Tests**

All Countries			6 European Countries			Remaining 9 Countries		
p	Lower	Upper	p	Lower	Upper	p	Lower	Upper
1	1.6028*+	3.0877	1	0.6837*+	1.3171	1	0.4621*+	0.8901

If the upper bound is below the critical value for the largest root, reject null of no convergence.
If the lower bound is above the critical value for the largest root, accept null of no convergence.

* Significant for critical value of 0.10.
\+ Significant for critical value of 0.05.

** These statistics are calculated on the vector of first differences of GDPi - GDPk.

For all countries, the US is subtracted off.
For the 6 European countries, France is subtracted off.
For the remaining 9 countries, the US is subtracted off.

Cumulative Percentage from p Largest Eigenvalues

	All Countries		6 European Countries		Remaining 9 Countries	
	p	Cumulated %	p	Cumulated %	p	Cumulated %
Largest	1	0.7403	1	0.6785	1	0.6876
	2	0.8826	2	0.8846	2	0.8626
	3	0.9247	3	0.9780	3	0.9162
	4	0.9552	4	0.9951	4	0.9591
	5	0.9734	5	1.0000	5	0.9781
	6	0.9841			6	0.9913
	7	0.9911			7	0.9969
	8	0.9941			8	1.0000
	9	0.9965				
	10	0.9980				
	11	0.9992				
	12	0.9997				
	13	0.9999				
Smallest	14	1.0000				