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STOCHASTIC EQUILIBRIUM AND EXCHANGE RATE DETERMINATION IN A SMALL OPEN ECONOMY WITH RISK AVERSE OPTIMIZING AGENTS

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ABSTRACT

This paper constructs a stochastic general equilibrium model of a small open economy consisting of risk averse optimizing agents. The stochastic processes describing the rate of monetary growth, government expenditure, private production, and the foreign price level are taken to be exogenous, determining all asset risks and returns, and the equilibrium stochastic processes describing the domestic inflation rate and the exchange rate. The model is used to examine a number of issues. These include: (i) the effects of the means and variances of policy shocks on the equilibrium; (ii) the determinants of the foreign exchange risk premium; (iii) the relationship between net export instability and economic growth.

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I. INTRODUCTION

This paper develops a stochastic general equilibrium model of exchange rate determination, incorporating what we regard as being the most important features of the modern theory of exchange rate determination. Following research of the 1970's and 1980's it is commonly agreed that exchange rates should be viewed as equilibrating variables, more akin to the prices of durable assets, than to prices which equilibrate flows in goods markets. Expectations of present and future conditions relating to the holding returns of international assets should therefore matter to the dynamics of the exchange rate process in a fundamental way.

There are two operational implications of this view. First, in the context of asset market equilibrium, the exchange rate should be treated as part of the mechanism that balances the portfolio choices of asset holders. Simple interest parity conditions in models where assets are interest bearing bonds give way to risk–adjusted parity rules between domestic and foreign assets in more sophisticated models with random returns and a stochastic component to the exchange rate. Second, asset demands should be grounded in rational forward–looking optimization by agents. These demand specifications replace formulations of earlier models of the asset approach to the exchange rate where asset demands were posited as *ad hoc* functions of relative asset returns.[1]

A third element of modern exchange rate theory continues to relate to the role of the exchange rate as the relative price of two currencies. Since the exchange rate is the link between two price levels, the dynamics of the foreign price level relative to the dynamics of the domestic price level are an important determinant of the dynamics of the exchange rate. A fundamental question is the extent to which foreign price disturbances are transmitted to the domestic economy (prices and real activity) in the presence of flexible exchange rates.

The first objective of the paper is to determine the endogenous stochastic process describing the movement in the exchange rate, deriving both its expected or mean component, and its unexpected or stochastic component. Our approach to portfolio selection in a stochastic environment builds on the pathbreaking work of the early 1980's, in which asset demands were determined, with exchange rate dynamics, returns on traded foreign and domestic bonds, and price level randomness all being specified as exogenous Brownian motion processes.[2] In this paper we also describe the relationship between the investor's portfolio choices in the same way, but we endogenize the stochastic processes describing the domestic price level, the exchange rate, and the real rates of return on domestic bonds, equities, and foreign bonds.

At each moment of time, the representative consumer in the domestic economy chooses his consumption of a single produced good, together with the allocation of his stock of wealth between holdings of domestic money, domestic government bonds, equities, and foreign bonds. Foreign bonds and the commodity are internationally traded. Domestically produced output that is not consumed, purchased by the domestic government, or exported, becomes part of the domestic capital stock, the equity claims on which may be traded. The domestic government prints money, floats bonds which are non-traded, and levies taxes to pay for government expenditures and to balance its budget. The domestic firm uses available capital to produce output subject to a random production component. Government expenditure, domestic money and bond creation, and domestic production are all described as continuous time stochastic processes, as is the stochastic process describing the foreign price level. Equilibrium in the goods market and the market for assets then determine the equilibrium stochastic processes for the domestic price level, the exchange rate, the real rates of return on the four assets, consumption, capital formation, and the balance of payments on current account.[3]

The model is therefore a small, but complete, general equilibrium system and extends the framework of Grinols and Turnovsky (1990) to an open economy. The resulting equilibrium is one in which both the means and the variances of the endogenous variables are simultaneously determined.[4] This type of mean–variance optimization framework has formed the basis for much important empirical work pertaining to interest parity relationships, and the determination of exchange rate risk premia; see e.g., Frankel (1986), Giovannini and Jorion (1988), Hodrick and Srivastava (1986), Lewis (1988), among others. The present analysis embeds this financial subsector into a general equilibrium macroeconomic context. It therefore provides a convenient vehicle for examining the properties of the foreign exchange risk premium and how it responds to exogenous changes in the environment. In this paper, we derive an expression for the risk premium (which in general is not zero), analyzing its determinants, and in particular its responsiveness to the exogenous risks impinging on the economy.

Indeed, while we view the model as being capable of addressing a range of other interesting issues, space limitations necessitate restricting ourselves to just a few. After describing the model, we show how the macroeconomic equilibrium can be reduced to a simple pair of equilibrium relationships, which jointly determine the domestic rate of inflation and the expected rate of exchange depreciation. Simple graphical techniques enable us to analyze the effects of monetary and fiscal expansions, as well as the variability in these policies on the equilibrium. From the response of the inflation rate, and the rate of exchange depreciation, the effect on other domestic variables, such as real rates of return can be easily derived.

Secondly, we analyze the effects of changes in the foreign inflation rate and its variance on the domestic economy. The extent to which flexible exchange rates insulate an economy against foreign price shocks was discussed at some length in the 1970's following the worldwide move towards more flexible rates. These analyses were based on the portfolio balance models of the time. It is therefore of interest to use the present model to revisit this important issue. As a final application, we briefly use the model to analyze the relationship between export instability and the rate of growth, a relationship which has undergone substantial empirical investigation.

It is important to emphasize that the implications of this type of mean–variance equilibrium model, in which risks are endogenously determined, can be diametrically opposite to those obtained in more conventional models where the endogeneity of some aspects of the structure is ignored. To anticipate one result, in simple *ad hoc* stochastic models, in which the structural parameters are assumed to be independent of the stochastic structure, an increase in the variance of the foreign price shocks will increase the variance of the nominal exchange rate; see e.g., Turnovsky (1983). In the present model, the reverse turns out to be true. An increase in the variance of foreign price shocks brings about a sufficient readjustment of the equilibrium portfolio, so as to mitigate the effects of foreign price shocks on the exchange rate, thereby reducing the variance of the latter. In effect, the structural changes induced by the change—the key element of the Lucas Critique—are the dominant influence.

The remainder of the paper is structured as follows. Section II presents the components of the open economy, paying particular attention to the description of risk and the structure of assets. Section III describes the solution of the model and the determination of the endogenous stochastic processes taken as given by the representative agents in the model. The next three sections analyze the various issues noted above. The final section summarizes and concludes.

II. THE STOCHASTIC OPTIMIZING OPEN ECONOMY WITH INTERNATIONALLY TRADED CAPITAL

This section of the paper describes the stochastic environment, financial assets, and the choices of consumers, firms, and the government.

A. *Consumers*

The domestic country representative consumer chooses at each moment of time consumption of a single traded good and the allocation of his portfolio wealth between four assets: domestic money, domestic government bonds, internationally traded foreign bonds, and equity claims on internationally traded capital, subject to his personal wealth constraint.

$$W = \frac{M}{P} + \frac{B}{P} + \frac{EB^*}{P} + S \tag{1}$$

where

M = nominal stock of domestic money

P = domestic price level of new consumption goods

B = nominal holdings of domestic bonds

B^* = nominal holdings of foreign traded bonds

E = exchange rate (units of domestic currency per unit foreign currency)

S = real stock of domestic equity measured in units of new consumption good.

Foreign bonds B^* and capital are the only internationally traded assets; M, B are internationally nontraded. With free trade, the exchange rate, E, relates the domestic price of the single traded good to its foreign price Q, according to the purchasing power parity (PPP) relationship

$$P = QE. \tag{2}$$

Over the instant dt the consumer chooses consumption at the rate $C(t)dt$. The consumer's objective is to select his rate of consumption and his portfolio of assets to maximize the expected value of his lifetime utility

$$E_0 \int_0^\infty U\left(C(t), \frac{M(t)}{P(t)}\right) e^{-\rho t} dt \tag{3}$$

subject to the wealth constraint and the stochastic wealth accumulation equation:

$$dW = W[n_1 dR_M + n_2 dR_B + n_3 dR_S + n_4 dR_F] - C(t)dt - dT \tag{4}$$

where

$$n_1 \equiv \frac{M/P}{W} = \text{share of portfolio held in money,}$$

$$n_2 \equiv \frac{B/P}{W} = \text{share of portfolio held in domestic government bonds,}$$

$$n_3 \equiv \frac{S}{W} = \text{share of portfolio held in equity,}$$

$$n_4 = \frac{EB^*/P}{W} = \text{share of portfolio held in foreign bonds,}$$

$$dR_i = \text{rate of return on asset } i,\ i = M, B, F, S,$$

where F denotes the foreign bond, and

$$dT = \text{lump sum taxes paid to the domestic government.}$$

The consumer's objective function reflects utility from the holding of real money balances and from current consumption. For convenience we choose the logarithmic utility function[5]

$$U = \theta \; ln \; C(t) + \delta \; ln \; \frac{M(t)}{P(t)} \qquad \theta + \delta = 1.$$

The analysis would require little modification if utility were generalized to the constant elasticity functional form.[6]

B. Prices, Asset Returns and Taxes

The three prices in the model evolve according to the stochastic processes

$$\frac{dP}{P} = pdt + du_p \tag{5a}$$

$$\frac{dQ}{Q} = qdt + du_q \tag{5b}$$

$$\frac{dE}{E} = edt + du_e \tag{5c}$$

where p, q and e, respectively are the instantaneous expected rates of change in the domestic price level, the foreign price level, and the exchange rate, respectively. The terms du_p, du_q and du_e are temporally independent, normally distributed random variables with zero means and instantaneous variances $\sigma_p^2 dt, \sigma_q^2 dt, \sigma_e^2 dt$.[7] Movement in the three prices must be related to one another through equation (2) which implies through stochastic differentiation that[8]

$$\frac{dP}{P} = \frac{dQ}{Q} + \frac{dE}{E} + \left(\frac{dQ}{Q}\right)\left(\frac{dE}{E}\right). \tag{6a}$$

Equating deterministic and stochastic components of equation (6a) implies

$$p = q + e + \sigma_{qe} \tag{6b}$$

and

$$du_p = du_q + du_e \tag{6c}$$

where $\sigma_{qe} dt$ is the instantaneous covariance between du_q and du_e. Equation (6b) describes PPP in terms of mean growth rates, which in a stochastic environment includes the covariance term σ_{qe}; (6c) describes the implication of PPP for the stochastic shocks.

Domestic and foreign bonds are assumed to be short bonds, paying nominal rates of interest i and i^*, respectively. Using the Ito calculus, the real rates of return to domestic consumers on money and the two bonds are respectively

$$dR_M = r_M dt - du_p \tag{7a}$$

where $$r_M = -p + \sigma_p^2$$

$$dR_B = r_B dt - du_p \tag{7b}$$

where $$r_B \equiv i - p + \sigma_p^2$$

$$dR_F = r_F dt - du_q \tag{7c}$$

where $$r_F = i^* - q + \sigma_q^2.$$

The stochastic behavior of the inflation rates introduces random components into these real rates of return. Also, the expected real rate of return increases with the variance of the appropriate inflation rate. This is because the real rates of return are convex functions of the price level; see Fischer (1975).

The real rate of return on equity is specified as

$$dR_S = r_S dt + du_s \tag{7d}$$

where the mean rate of return r_S will be derived below. The stochastic component du_s is temporally independent and normally distributed, with zero mean and variance $\sigma_s^2 dt$ and it too will be determined below.

Taxes are specified by the relationship

$$dT = \tau W dt + W dv \tag{8}$$

where dv is a temporally independent, normally distributed random variable with zero mean and variance $\sigma_v^2 dt$. As we will see below, the parameters τ, dv are set so as to maintain a balanced budget. Given that in equilibrium all real variables, including taxes, will grow at the rate of wealth, τ, dv are essentially the mean and stochastic components of lump sum taxes.

C. Consumer Optimization

The solution of the consumer optimization problem is described in detail in the Appendix.[9] After rewriting, the first order conditions to the consumer's optimization problem become

Choice of Consumption:

$$\frac{C}{W} = \theta\rho \tag{9}$$

Money:

$$n_1 = \frac{\delta\rho}{i} \tag{10a}$$

Domestic Bonds:

$$n_2 = 1 - n_1 - n_3 - n_4 \tag{10b}$$

Equities and Foreign Bonds:

$$\begin{bmatrix} \sigma_p^2 + 2\sigma_{sp} + \sigma_s^2 & \sigma_{pe} + \sigma_{se} \\ \sigma_{pe} + \sigma_{se} & \sigma_e^2 \end{bmatrix} \begin{bmatrix} n_3 \\ n_4 \end{bmatrix} = \begin{bmatrix} r_S - r_B + \sigma_p^2 + \sigma_{sp} + \sigma_{pv} + \sigma_{sv} \\ r_F - r_B + \sigma_{pe} + \sigma_{ve} \end{bmatrix} \tag{10c}$$

where $\sigma_{ij}, i, j = p, s, e, v$ denotes the instantaneous covariance between the terms du_p, du_s, du_e, and dv.

The logarithmic utility function implies a fixed ratio of consumption to wealth. Given the presence of domestic bonds, which share the same risk characteristics, the portfolio share of real money balances depends only upon the nominal interest rate. It is influenced by the stochastic characteristics of the economy only insofar as they affect i, as in equilibrium they do. The portfolio shares of the traded assets n_3, n_4 can be derived from (10c) and can be seen to depend upon the full mean variance–covariance structure of asset returns. Given (10a), (10c), the portfolio share of domestic bonds is then determined residually from the adding up condition (10b).

D. Firms

The flow of new output dY is produced from capital, K, by means of the stochastic linear technology

$$dY = \alpha K dt + \alpha K dy \tag{11}$$

where α is the marginal physical product of capital and dy is a temporally independent, normally distributed, stochastic process with mean zero and variance $\sigma_y^2 dt$. The size of the stochastic disturbance is proportional to the mean rate of output.

With capital being the sole factor of production, the rate of return on equities is

$$dR_S = \frac{dY}{S}.$$

In general, the value of the firm $S = t_q K$, where t_q is the price of capital in terms of new output ("Tobin q"). We assume that capital can be adjusted instantaneously and costlessly, so that $t_q = 1$. Thus it follows that

$$dR_S = \frac{dY}{K} = \alpha dt + \alpha dy \tag{12a}$$

$$r_S = \alpha \tag{12b}$$

$$du_s = \alpha dy. \tag{12c}$$

E. Government Policy

Government policy is characterized by the choice of government expenditures, printing of money and bonds, and tax collection, all of which must be specified subject to the budget constraint

$$\frac{dM}{P+dP} + \frac{dB}{P+dP} = dG + i\frac{B}{P}dt - dT \tag{13}$$

where dG denotes the stochastic rate of real government expenditure.

Government expenditure policy is specified by

$$dG = g\alpha K dt + \alpha K dz \tag{14a}$$

where dz is an intertemporally independent, normally distributed, random variable with zero mean and variance $\sigma_z^2 dt$. This specification means that government spending is a fraction g of the economy's mean level of output, while the stochastic disturbance in government spending varies proportionally to mean expenditures.

Monetary policy is described by the stochastic money growth rule

$$\frac{dM}{M} = \mu dt + dx. \tag{14b}$$

The mean rate of monetary growth μ is subject to a stochastic disturbance dx which is temporally independent and normally distributed having zero mean and variance $\sigma_x^2 dt$.

Government debt policy is formulated in terms of maintaining a fixed ratio of domestic government (nontraded) bonds to money

$$\frac{B}{M} = \lambda \tag{14c}$$

where λ is the policy set by the government.[10] Finally, given the policy specifications (14a) – (14c), both the mean and stochastic component of taxes dT must be set in order to meet the constraint (13).

F. Product Market Equilibrium and Balance of Payments

Net exports of the physical commodity are determined by the excess of production over domestic uses

$$dY - dC - dK - dG.$$

Balance of payments equilibrium, in turn, requires the transfer of new foreign bonds (in excess of interest on earlier issues) to finance net exports of the domestic country. Using the PPP condition (2) and noting that the value of traded bonds $EB^*/P = B^*/Q$, balance of payments equilibrium can be written as

$$\frac{dB^*}{Q + dQ} - i^* \left(\frac{B^*}{Q}\right) dt = dY - dC - dK - dG. \tag{15}$$

Invoking the approximation, $\frac{Q}{Q+dQ} \cong 1 - \frac{dQ}{Q} + \left(\frac{dQ}{Q}\right)^2$, (15) can be rewritten to order $(dQ)^2$

$$d\left(\frac{B^*}{Q}\right) + \frac{B^*}{Q}\left[\frac{dQ}{Q} - \left(\frac{dQ}{Q}\right)^2\right] - i^*\left(\frac{B^*}{Q}\right) dt = dY - dC - dK - dG. \tag{16}$$

Substituting for $dQ/Q, dY$, and dG, from (5b), (11) and (14a) respectively, and applying the rules of stochastic calculus, leads to

$$d\left(\frac{B^*}{Q}\right) + dK = \left[\alpha K(1-g) - C + \frac{B^*}{Q}(i^* - q + \sigma_q^2)\right] dt + \alpha K(dy - dz) - \left(\frac{B^*}{Q}\right) du_q. \tag{17}$$

This equation specifies the rate of accumulation of traded assets in the economy and completes the description of the model.

III. MACROECONOMIC EQUILIBRIUM

The elements described in the previous section can now be combined to yield the overall equilibrium of the small open economy. The exogenous factors include the government policy parameters: μ (mean monetary growth), g (mean government expenditure), λ (debt policy), and q (the mean foreign inflation rate). The exogenous stochastic processes include: dx (monetary growth), dz (government expenditure), dy (productivity) and du_q (foreign inflation), all of which are assumed to be mutually uncorrelated. The remaining stochastic terms du_p, du_e, du_s, and dv are all endogenous and can be expressed as functions of the exogenous shocks. The endogenous variances and covariances can then be determined, and the overall mean–variance macroeconomic equilibrium obtained.

At each moment, the economy inherits from the past its stock of physical capital and the nominal quantities of money and bonds. Real money stocks and holdings of real bonds are, of course, determined by endogenous movements in the price level. To this point, no equations determining domestic inflation have been derived. These must be generated by the model and for this purpose we focus on the portfolio balance relationships.

A. *The Price Level*

The consumer optimality conditions (10a) – (10c) imply that if assets have the same stochastic characteristics through time (i.e., the means and variance–covariance matrix of returns is stationary), they will lead to the same allocation of portfolio holdings. Such a recurring equilibrium will be characterized by constant portfolio shares $n_1, \ldots, n_4$ through time. We shall therefore look for an equilibrium in which portfolio shares have this characteristic.

Assuming constant portfolio shares implies $\frac{M/P}{(B^*/Q)+K} = \frac{n_1}{n_3+n_4}$ is constant. The price level can then be written as

$$P = \frac{n_3 + n_4}{n_1} \frac{M}{(B^*/Q) + K}$$

which implies

$$\frac{dP}{P} \equiv pdt + du_p = \frac{dM}{M} - \frac{d[(B^*/Q) + K]}{B^*/Q + K} - \frac{dM}{M} \cdot \frac{d[(B^*/Q) + K]}{B^*/Q + K} + \frac{d[(B^*/Q) + K]^2}{[B^*/Q + K]^2}.$$

Substituting for the terms on the right–hand side by using (14b) and (17), noting that variances are of order dt, and equating the deterministic and stochastic parts of the two sides of this equation, yields

$$p = \mu - \alpha(1-g)\frac{n_3}{n_3+n_4} + \frac{\theta\rho}{n_3+n_4} - \frac{n_4}{n_3+n_4}(i^* - q + \sigma_q^2)$$
$$+ \alpha^2\left(\frac{n_3}{n_3+n_4}\right)^2(\sigma_y^2 + \sigma_z^2) + \left(\frac{n_4}{n_3+n_4}\right)^2\sigma_q^2 \tag{18a}$$

$$du_p = dx - \alpha\left(\frac{n_3}{n_3+n_4}\right)(dy - dz) + \frac{n_4}{n_3+n_4}du_q. \tag{18b}$$

The first of these equations specifies the expected rate of domestic inflation that is consistent with maintaining an unchanging portfolio balance. It varies proportionately with the expected rate of monetary growth and inversely with the expected rate of growth of traded assets. In addition, given portfolio shares n_3 and n_4, it increases with the variance of the growth of traded assets, which in turn depend positively upon the variances of the domestic fiscal and productivity shocks, as well as the foreign price shocks. The second equation determines the endogenous stochastic component of the domestic inflation rate in terms of the stochastic components of money growth, the domestic fiscal and productivity shocks, and the foreign inflation rate. The relative shares of the two traded assets is an important determinant of how these shocks impact on the domestic inflation rate. For example, the impact of foreign price disturbances increases with the share of foreign bonds in the portfolio of traded assets.

B. Determination of Tax Adjustments

Using the approximation $\frac{P}{P+dP} \cong 1 - \frac{dP}{P} + \left(\frac{dP}{P}\right)^2$ to rewrite the left–hand side of the government budget constraint (13), gives

$$\left[\frac{M}{P}\frac{dM}{M} + \frac{B}{P}\frac{dB}{B}\right]\left[1 - \frac{dP}{P} + \left(\frac{dP}{P}\right)^2\right] = dG + i\frac{B}{P}dt - dT.$$

Substituting for the government expenditure policy (14a), the monetary growth rule (14b), debt policy (14c), tax collection (8) and the price evolution (5a), and noting that covariances are of order dt, we obtain to order dt

$$(1+\lambda)\frac{M}{B}[\mu dt + dx] - (1+\lambda)\frac{M}{P}\sigma_{xp}dt$$
$$= \alpha K[gdt + dz] + i\lambda\frac{M}{P}dt - \tau W dt - W dv$$

$\sigma_{xp} \equiv \; cov\ (dx, dp)$. Dividing both sides of this equation by W and equating the deterministic and stochastic parts leads to the two relationships

$$\tau = \alpha n_3 g - (n_1 + n_2)\mu + n_2 i + (n_1 + n_2)\sigma_{xp} \tag{19a}$$

$$dv = \alpha n_3 dz - (n_1 + n_2)dx. \tag{19b}$$

These equations determine the endogenous adjustments in the mean and stochastic component of taxes necessary to finance the government budget for the given policy specifications.

C. Finding the Random Components of Financial Variables' Returns

Since the portfolio shares

$$K/W, \ \frac{M/P}{W}$$

are both constant, stochastically differentiating them gives the following relations between their stochastic movements

$$dk = dw \tag{20a}$$

$$dx - du_p = dw \tag{20b}$$

where dk, dw are the stochastic parts of dK/K and dW/W, respectively. Substituting for (7a), (7b), (7c), (8) and (12a) into (4), we see

$$dw = -(n_1 + n_2)du_p + \alpha n_3 dy - n_4 du_q - dv.$$

Next, substituting for du_p, dv from (18b) and (19b) gives

$$dk = dw = \alpha \frac{n_3}{n_3 + n_4}(dy - dz) - \frac{n_4}{n_3 + n_4} du_q \tag{20c}$$

while combining (18b) with (6c) implies

$$du_e = dx - \alpha \frac{n_3}{n_3 + n_4}(dy - dz) - \frac{n_3}{n_3 + n_4} du_q. \tag{20d}$$

Equations (12c), (18b), (19b), (20c), (20d) therefore collect the information about the random components of equity returns, the price level, taxes, capital formation and real wealth formation, and the exchange rate, respectively. These equations are semi-reduced forms since they involve portfolio shares n_1, n_2, n_3, n_4 but once the latter are known, they will become full reduced forms.[11]

D. Core Equations for Full Reduced Form Solution

Using the semi-reduced forms, the following variances and covariances can be computed

$$\sigma_p^2 = \sigma_x^2 + \left(\frac{\alpha n_3}{n_3 + n_4}\right)^2 (\sigma_y^2 + \sigma_z^2) + \left(\frac{n_4}{n_3 + n_4}\right)^2 \sigma_q^2 \tag{21a}$$

$$\sigma_s^2 = \alpha^2 \sigma_y^2 \tag{21b}$$

$$\sigma_e^2 = \sigma_x^2 + \left(\frac{\alpha n_3}{n_3 + n_4}\right)^2 (\sigma_y^2 + \sigma_z^2) + \left(\frac{n_3}{n_3 + n_4}\right)^2 \sigma_q^2 \tag{21c}$$

$$\sigma_{ps} = -\frac{\alpha^2 n_3}{n_3 + n_4}\sigma_y^2 \tag{21d}$$

$$\sigma_{pe} = \sigma_x^2 + \left(\frac{\alpha n_3}{n_3 + n_4}\right)^2 (\sigma_y^2 + \sigma_z^2) - \frac{n_3 n_4}{(n_3 + n_4)^2}\sigma_q^2 \tag{21e}$$

$$\sigma_{pv} = -(n_1 + n_2)\sigma_x^2 + \frac{\alpha^2 n_3^2}{n_3 + n_4}\sigma_z^2 \tag{21f}$$

$$\sigma_{se} = -\frac{\alpha^2 n_3}{n_3 + n_4}\sigma_y^2 \tag{21g}$$

$$\sigma_{sv} = 0 \tag{21h}$$

$$\sigma_{ev} = -(n_1 + n_2)\sigma_x^2 + \frac{\alpha^2 n_3^2}{n_3 + n_4}\sigma_z^2 \tag{21i}$$

$$\sigma_{eq} = -\frac{n_3}{n_3 + n_4}\sigma_q^2. \tag{21j}$$

Substituting these expressions into the consumer optimality conditions enables the core equilibrium relationships to be derived. For this purpose it is notationally convenient to define

$$\omega \equiv \frac{n_3}{n_3 + n_4}$$

the share of capital in the traded portion of the consumer's portfolio. With this notation, and upon substitution, the first row of (10c) can be written as

$$r_S - r_B = \alpha^2\omega(1-\omega)\sigma_y^2 - \alpha^2\omega^2\sigma_z^2 - (1-\omega)^2\sigma_q^2. \tag{22}$$

This equation expresses the differential expected real rate of return between domestic bonds and equities, both of which are risky assets to the domestic consumer. Assuming $0 < \omega < 1$, this may quite plausibly be negative.

This result may appear to contradict studies which suggest that the real return on equities substantially exceeds that on bonds; see e.g., Mehra and Prescott (1985). The apparent inconsistency can be reconciled by noting that the definition of r_B in the present analysis includes the equilibrium variance of the inflation rate σ_p^2. If instead, one (incorrectly) ignores this term and defines the expected real rate of return on bonds by the more familiar expression $r'_B \equiv i - p$, then (22) implies the equilibrium relationship

$$r_S - r'_B = \sigma_x^2 + \alpha^2\omega\sigma_y^2. \tag{22'}$$

In general, r_S will *exceed* r'_B, and may do so by substantial margins if the variances σ_x^2, σ_y^2 are sufficiently large.

In any event, substituting further for r_S and r_B into (22), this relationship may be expressed in the equivalent form

$$\alpha = i - p + \sigma_x^2 + \alpha^2 \omega \sigma_y^2. \tag{23}$$

Turning to the second equation in (10c), upon substitution this can be written as

$$r_F - r_B = -\alpha^2 \omega^2 (\sigma_y^2 + \sigma_z^2) + \omega(1-\omega)\sigma_q^2 \tag{24}$$

expressing the differential expected real rate of return between domestic and traded bonds. This will be considered in detail in our discussion of the foreign exchange risk premium, in Section V below. Substituting for r_B and r_F from (7b), (7c), noting (6b), and substituting for σ_{qe} and σ_p^2,

$$i = i^* + e - \sigma_x^2. \tag{25}$$

Observe that in the absence of domestic monetary risk ($\sigma_x^2 = 0$), (25) is simply the condition of uncovered interest parity. The fact that uncovered interest parity can obtain even when agents are risk averse (as they are here), has been emphasized recently by Engel (1990) and our analysis provides a simple example.

Finally, substituting from (21j) into (6b) gives

$$p = q + e - \omega \sigma_q^2 \tag{26}$$

and using (14c) to rewrite (10b) implies

$$(1+\lambda)n_1 + n_3 + n_4 = 1. \tag{27}$$

Equations (23), (25), relating to the choice of portfolio shares, and equations (18a), (26), relating to the determination of the mean inflation rate and exchange rate, therefore form a core system of equations in the endogenous variables i, e, p, and $\omega \equiv n_3/(n_3+n_4)$. These can be solved in terms of the exogenous variables relating to monetary, expenditure, and debt policies $(\mu, \sigma_x^2, g, \sigma_z^2, \lambda)$; foreign interest and inflation variables (i^*, q, σ_q^2); and the variance of domestic productivity shocks σ_y^2. Having determined i, n_1 immediately follows from (10a), and given n_1 and ω, n_3 and n_4 can be obtained from (27). Moreover, other variables such as r_B etc. can also be determined, by referring to their definitions.

For analytical purposes it is convenient to reduce the equilibrium further to a pair of equations involving p and e. This can be done as follows. First, solving (23), (25) and (26) for ω, yields

$$\omega \equiv \frac{n_3}{n_3+n_4} = \frac{\alpha-(i^*-q)}{\alpha^2\sigma_y^2+\sigma_q^2} \equiv \omega(\alpha, i^*-q, \sigma_y^2, \sigma_q^2). \tag{28}$$

Equation (28) is a simple, intuitively appealing, expression for the share of capital in the traded portion of the investor's portfolio. An increase in either σ_q^2 or σ_y^2 will induce a shift towards traded bonds, away from traded capital. A rise in the former will raise r_F, doing so by an amount which more than compensates for the increased risk, thereby inducing a substitution in favor of this asset. A rise in the latter will increase the risk associated with equities, while leaving their expected return unchanged, and this will have a similar effect.

Using (25) to eliminate i in (23) and replacing ω from (28), gives the following relationship between e and p, which will maintain equilibrium between the real rates of return

(RR) $$p = i^* + e - \alpha + \alpha^2\sigma_y^2\omega(\alpha, i^*-q, \sigma_y^2, \sigma_q^2).$$

Next, using (10a), (25) to rewrite (27) yields

$$n_3+n_4 = 1 - \frac{(1+\lambda)\rho\delta}{i^*+e-\sigma_x^2} \equiv \psi(i^*+e-\sigma_x^2, \lambda). \tag{29}$$

Also, using (28) and (29) to replace the portfolio shares n_3 and n_4 in (18a), leads to the following relationship between e and p which will maintain portfolio balance

$$\text{(PP)} \quad \begin{aligned} p = \mu - (i^* - q) + [i^* - q - \sigma_q^2 - \alpha(1-g)]\omega(\alpha, i^* - q, \sigma_y^2, \sigma_q^2) \\ + [\alpha^2(\sigma_y^2 + \sigma_z^2) + \sigma_q^2]\omega(\cdot)^2 + \frac{\theta\rho}{\psi(i^* + e - \sigma_x^2, \lambda)}. \end{aligned}$$

The core equations (RR) and (PP) form a two–equation system in e and p which can be graphed as in Figure 1. Once e and p are determined, the other variables can be obtained derivatively. In particular, the stochastic process generating the equilibrium exchange rate is given by

$$\frac{dE}{E} = \tilde{e}\ dt + dx - \tilde{\omega}[\alpha(dy - dz) + du_q]$$

where $\tilde{\omega}$ is determined from (28) and $\tilde{e}$ is the equilibrium obtained from the point of intersection of the RR and PP curves.

IV. POLICY SHOCKS, FOREIGN INFLATION, AND DOMESTIC EQUILIBRIUM

A. *Domestic Policy and Policy Risk*

Before turning to the issue of the exchange risk premium, we discuss the basic behavior of the model with respect to policy and outside influences. The equilibrium expected rates of exchange depreciation and inflation are obtained at the intersection of the RR and PP curves in Fig. 1. The RR locus is a positively sloped 45° line with intercept $-(\alpha - i^*) + \alpha^2\sigma_y^2\omega$. The PP locus is a negatively sloped rectangular hyperbola, having the asymptotes as indicated. As these curves are drawn, they illustrate the case where the equilibrium domestic inflation rate and rate of exchange depreciation are both positive.

Domestic monetary policy as described by the parameters (μ, σ_x^2) and fiscal policy as described by (g, σ_z^2) impact on the domestic economy through shifts in the PP curve.

An increase in the expected monetary growth rate μ raises the PP curve, shifting the equilibrium from A to B. Both the domestic inflation rate and the expected rate of exchange depreciation increase by equal amounts, though in each case by a smaller percentage than the increase in μ. An increase in σ_x^2 has the reverse effect, shifting the PP curve down and reducing both e and p.

The effects of an increase in the expected share of government expenditure g also raises the PP curve, leading to higher equilibrium values for e and p. The same is true for an increase in its variance, σ_z^2, as long as the domestic economy holds positive stocks of both foreign bonds and domestic capital, so that $\omega > 0$, as we shall assume.

It is important to mention that any of these policy changes (or other policy shocks) are associated with a concurrent one–time discrete jump in the level of the exchange rate E. This is necessary to maintain portfolio balance in stock terms. To see this, we write the monetary equilibrium as

$$n_1 \equiv \frac{M/QE}{M/QE + B/QE + B^*/Q + K}. \tag{30}$$

We see that the rise in e from say an increase in the monetary growth rate μ, implies a higher domestic nominal interest rate [see equation (25)], and therefore a reduction in the equilibrium share of money balances n_1 [see equation (10a)]. Given that M, K, B, B^*, Q are predetermined, this requires a one time depreciation of the domestic exchange rate E, for (30) to be maintained.

Finally, turning to the variances of the stochastic components, du_e, du_p, substituting for ω into (21a), (21c)

$$\sigma_e^2 = \sigma_x^2 + \omega^2[\alpha^2(\sigma_y^2 + \sigma_z^2) + \sigma_q^2] \tag{21a'}$$

$$\sigma_p^2 = \sigma_x^2 + \omega^2\alpha^2(\sigma_y^2 + \sigma_z^2) + (1-\omega)^2\sigma_q^2. \tag{21c'}$$

Since ω is independent of domestic government policy parameters, it follows that the variances σ_e^2, σ_p^2, are independent of the means of domestic government policy instruments, and simple increasing functions of their variances, σ_x^2, σ_z^2.

B. Foreign Inflation and Inflation Risk

Early discussions of flexible exchange rates focused on the question of the extent to which such a regime insulates the domestic economy against foreign price disturbances.[12] Using a portfolio balance model, Turnovsky (1977) showed that a necessary and sufficient condition for a flexible exchange rate regime to provide perfect insulation against changes in the foreign inflation rate is that the rest of the world be 'Fisherian,' in the sense that the foreign nominal interest rate fully adjust to changes in the foreign inflation rate;

$$\text{i.e.} \qquad \frac{di^*}{dq} = 1. \tag{31}$$

If this condition holds, then $de/dq = -1$ and the higher foreign inflation rate leads to an equivalent decrease in the rate of depreciation of the domestic currency, thereby fully insulating the domestic economy from the foreign inflationary shock. This same condition was also later shown to provide perfect insulation in a deterministic intertemporal utility maximizing model; see Turnovsky (1985).

Recalling the definitions of ω and ψ, it is seen that the (PP) and (RR) schedules are functions of $i^* - q$, and $i^* + e$. It therefore follows that (31) is also a necessary and sufficient condition to ensure perfect insulation against changes in the foreign mean inflation rate, in the present stochastic setting as well. Geometrically, the increase in q subject to (31) will lead to an upward shift in the RR curve, accompanied by a downward shift in the PP curve, such that the equilibrium value of p is unchanged, while e falls by the exact amount of the increase in q. With ω remaining unchanged, it follows that all real rates of return in the domestic economy remain unaffected, so that full insulation is obtained.

The effect of an increase in the variance of the foreign inflation rate is more complicated. In general, an increase in σ_q^2 will cause both PP and RR curves to shift down. This implies an unambiguous reduction in the mean domestic rate of inflation, while e may either rise or fall, depending upon the relative shifts. In the case where $\sigma_y^2 = 0$, the RR curve remains unchanged, and in this case e falls unambiguously, as well.

An increase in σ_q^2 has two effects on the variances σ_e^2, σ_p^2. Focusing on σ_e^2, first the direct effect of an increase in σ_q^2, will, given the portfolio balance structure (i.e., given ω), cause σ_e^2 to rise. At the same time, the increase in σ_q^2 causes a portfolio adjustment away from capital (i.e., ω declines), mitigating the effects of a given level of variance, σ_q^2. On balance the portfolio adjustment effect dominates, so that somewhat paradoxically, an increase in the variance of the foreign inflation rate, *reduces* the variance of the nominal exchange rate.

V. FOREIGN EXCHANGE RISK PREMIUM

The present stochastic general equilibrium model offers a convenient vehicle for examining the determinants of the foreign exchange risk premium. Adapting a measure used by Engel (1990) and others to the present continuous time framework, we define the risk premium over the period $(t, t+dt)$ by[13]

$$\Omega(t, t+dt) \equiv 1 - \frac{F(t, t+dt)}{\Phi(t, t+dt)} \tag{32}$$

where

$$\Phi(t, t+dt) \equiv \frac{E_t[E(t+dt)/P(t+dt)]}{E_t[1/P(t+dt)]}.$$

E_t is conditional expectation formed at time t and $F(t, t+dt)$ is the forward exchange rate at time t, for the future time $t+dt$. This measure expresses the risk premium in units comparable to interest rates. If agents are risk neutral and markets efficient,

$\Phi(t,t+dt) = F(t,t+dt)$ and the risk premium $\Omega(t,t+dt) = 0$. As defined, a positive risk premium on the foreign asset implies $\Omega > 0$.

If we assume covered interest parity (CIP), then

$$1 + idt = (1 + i^{*}dt)\frac{F(t,t+dt)}{E(t)}. \tag{33}$$

We also assume that as $dt \rightarrow 0$, the spot and forward markets converge, so that $F(t,t) = E(t)$. Thus for small dt,

$$F(t,t+dt) \cong E(t)[1 + f(t)dt] \tag{34}$$

where $f(t) \equiv [\partial F(t,t)/\partial t]/E(t)$ is the instantaneous rate of forward premium (or discount) on foreign exchange. Substituting (34) into (33), we get the usual continuous time approximation to the CIP condition

$$i = i^{*} + f. \tag{35}$$

To calculate the risk premium, we use (5a) and (5c) to write

$$\frac{E(t+dt)}{P(t+dt)} = \frac{E(t)[1 + edt + du_e]}{P(t)[1 + pdt + du_p]}$$

which may be approximated to the order dt by

$$\frac{E(t+dt)}{P(t+dt)} \cong \frac{E(t)}{P(t)}[1 + (e - p - \sigma_{ep} + \sigma_p^2)dt + du_e - du_p]$$

while, similarly

$$\frac{1}{P(t+dt)} \cong \frac{1}{P(t)}[1 + (\sigma_p^2 - p)dt - du_p].$$

Taking expected values of these expressions, and substituting into (32), yields

$$\Omega = 1 - \frac{(1 + f dt)(1 + (\sigma_p^2 - p)dt)}{1 + (e - p - \sigma_{ep} + \sigma_p^2)dt} \tag{36}$$

which to the order dt is approximately

$$\Omega \cong -(f - e + \sigma_{ep} + \sigma_p^2)dt.$$

Using (25), (35), this expression is

$$\Omega \cong (\sigma_x^2 - \sigma_{ep})dt$$

and substituting for σ_{ep} from(21e), implies

$$\Omega \cong [-\alpha^2\omega^2(\sigma_y^2 + \sigma_z^2) + \omega(1-\omega)\sigma_q^2]dt = (r_F - r_B)dt. \tag{37}$$

Thus under the conditions of CIP, the risk premium as measured by Ω, is proportional to the differential expected real rate of return between foreign and domestic bonds, (24).[14]

Equation (37) expresses the risk premium in terms of exogenous determinants and it can be seen to be either positive or negative, depending upon the relative magnitudes of the domestic and foreign sources of risk. It is immediate that an increase in the domestic sources of real risk, σ_y^2, σ_z^2, both lower the risk premium on foreign bonds, as one would expect. The risk premium on foreign bonds declines with the expected rate of foreign inflation, if $1 > \omega > \frac{1}{2}$, i.e., if the domestic economy holds more traded equities than traded bonds. But it is independent of any change in q which is accompanied by an equal change in the foreign interest rate. An increase in foreign price risk, σ_q^2, directly raises the risk premium on foreign bonds. It also induces a shift in the traded portion of investors' portfolios in their favor, thereby serving to reduce the equilibrium risk premium. The net effect depends upon ω, and under the reasonable condition $1 > \omega > \frac{1}{2}$, the risk premium will rise.

Under what conditions is the risk premium zero? One simple condition where this will be so is if $\alpha = i^* - q$, i.e., the expected real rate of return on equities equals the foreign real interest rate, unadjusted for the foreign inflation variance. From (28) this implies $\omega = 0$, so that domestic investors hold no equities. In this case, the equilibrium stochastic processes describing the real returns on the interest earning assets can be readily shown to be

$$dR_B = (i^* - q + \sigma_q^2)dt - (dx + du_q) \tag{38a}$$

$$dR_F = (i^* - q + \sigma_q^2)dt - du_q \tag{38b}$$

$$dR_S = (i^* - q)dt + \alpha dy. \tag{38c}$$

Note that while domestic and foreign bonds have the same expected real rates of return, their stochastic components are not identical. However, the two assets are perfect substitutes, since their β's (i.e., their systematic risk as measured by their covariance with the market risk), and denoted by β_B, β_F, respectively, are both the same, namely $\beta_B = \beta_F = 1$. Equities have a lower expected return and $\beta_S = 0$.[15] But the lower risk is insufficient to compensate for the lower return and the asset is not held in the equilibrium portfolio of domestic investors.

An alternative condition which implies a zero foreign exchange risk premium is

$$\omega^* \equiv \frac{\alpha - i^* + q}{\alpha^2\sigma_y^2 + \sigma_q^2} = \frac{\sigma_q^2}{\alpha^2(\sigma_y^2 + \sigma_z^2) + \sigma_q^2}. \tag{39}$$

In this case, the equilibrium stochastic processes characterizing the rates of return are

$$dR_B = (i^* - q + \sigma_q^2)dt - [dx - \alpha\omega^*(dy - dz) + (1 - \omega^*)du_q] \tag{38a'}$$

$$dR_F = (i^* - q + \sigma_q^2)dt - du_q \tag{38b}$$

$$dR_S = \alpha dt + \alpha dy. \tag{38c'}$$

Again, the two bonds are perfect substitutes, having equal rates of return and equal β coefficients, $\beta_B = \beta_F = 1$. The expected rate of return on equities is lower, but the β coefficient, $\beta_S = \alpha^2\omega^*\sigma_y^2/\sigma_w^2$, is now sufficiently reduced to induce domestic investors to hold this asset. The difference between the returns on equities and bonds is accounted for by the presence of fiscal risk. If $\sigma_z^2 = 0$, (39) becomes $\alpha = i^* - q + \sigma_q^2$, and all three assets have identical expected returns and β coefficients, equal to unity.

For comparison, Domowitz and Hakkio (1985), Hodrick and Srivastava (1986) consider models in which the risk premium is zero, where there is no government spending, expenditure shares are constant, and monetary and real shocks are independent. Engel (1990) reports a zero risk premium for a model in which spending shares are constant, the share of output devoted to government is constant, and money shocks are independent of endowment shocks (there is no production). The differences we have obtained can be accounted for by noting the differences in model specification. Specifically, in our model, since dz is not proportional to dy, government expenditure is not a constant share of output; production shocks are proportional to output levels; and foreign inflationary shocks are introduced.

VI. CAPITAL ACCUMULATION AND THE BALANCE OF PAYMENTS

The constancy of the equilibrium portfolio shares implies

$$\frac{B^*}{Q} = \frac{n_4}{n_3}K; \quad d\left(\frac{B^*}{Q}\right) = \frac{n_4}{n_3}dK.$$

Combining these two equations with (17), we see that in equilibrium, the rate of growth of capital and the rate of accumulation of traded bonds both follow the stochastic process

$$\frac{dK}{K} = \frac{d(B^*/Q)}{B^*/Q} = \{\omega[\alpha(1-g) - \frac{\theta\rho}{n_3}] + (1-\omega)[i^* - q + \sigma_q^2]\}dt \\ + \alpha\omega(dy - dz) - (1-\omega)du_q. \tag{40}$$

The mean rate of growth comprises two components. The first, $\omega[\alpha(1-g) - \theta\rho/n_3]$, is associated with the growth of domestic output; the second, $(1-\omega)[i^* - q + \sigma_q^2]$, is the growth attributable to the interest earnings from abroad. In general, the equilibrium we are considering is one in which capital is steadily accumulating or decumulating, though at a sufficiently slow rate, as to be sustainable in the sense of being consistent with the intertemporal budget constraint (transversality condition) facing the economy.[16] This characteristic is a consequence of: (i) the logarithmic utility function, and (ii) the linear production function.

By studying (40), the effects of means and variances of government policy on the means and variances of growth and the current account balance, can be analyzed. Rather than pursue this, as a final application of our framework, we shall show how it offers insight into another issue, which has been extensively discussed. This is the question of the effects of export instability on investment and growth.[17] Several authors have studied correlations between measures of export instability and growth with a variety of conflicting results. For example, Kenen and Voivodes (1972) find the investment–GNP ratio to be negatively correlated with export instability; by contrast Yotopoulos and Nugent (1976) find a positive correlation. Other studies, focusing on the growth of GNP and export instability, also get conflicting results. The present framework is able to offer a conciliation of these findings.[18]

To eliminate problems of dimensionality, we normalize net exports $dX = dY - dC - dG - dK$, by the growing stock of capital, writing

$$\frac{dX}{K} = (1-\omega)\left[\alpha(1-g) - \frac{\theta\rho}{n_3} - (i^* - q + \sigma_q^2)\right] dt + (1-\omega)[\alpha(dy - dz) + du_q]. \qquad (41)$$

The variance of this measure of net exports is therefore

$$E\left(\frac{dX}{K}\right)^2 = (1-\omega)^2[\alpha^2(\sigma_y^2 + \sigma_z^2) + \sigma_q^2]dt \qquad (42)$$

and the issue of the effects of export instability on growth, thus centers around the relationship between (42) and the deterministic component of the growth rate (40).

In the present framework, export instability is endogenous and will reflect the exogenous sources of risk, σ_z^2, σ_y^2, and σ_q^2. We therefore turn to each in turn.

An increase in fiscal variability, σ_z^2, has no effect on ω, and will therefore raise export instability by the amount $(1-\omega)^2\alpha^2$. With ω unchanged, its only effect on the mean growth rate is through n_3. An increase in σ_z^2 has been shown to increase e, which from (25) implies a higher domestic interest rate i, which in turn from (29), will raise the total proportion of traded assets $(n_3 + n_4)$ in the domestic agent's portfolio. Given an unchanging ω, this implies an increase in n_3 and therefore a reduction in consumption, so that expected growth will therefore rise. Export instability arising from domestic fiscal variability will therefore be *positively* correlated with growth.

An increase in domestic production instability is more complicated. It will raise export instability on two counts. In addition to having a direct effect, analogous to the fiscal variability just discussed, it will also cause a portfolio shift among the traded portion of the portfolio towards bonds ($1-\omega$ increases) and this will induce additional instability into exports.

To consider what happens to mean growth, we focus on the simplest case where $\sigma_z^2 = \sigma_q^2 = g = 0$, when the deterministic component of (40) reduces to

$$\frac{[\alpha - (i^* - q)]^2}{\alpha\sigma_y^2} - \frac{\theta\rho}{n_3 + n_4}.$$

It is immediate that an increase in σ_y^2, by inducing a shift away from capital, will reduce the first component. Under these same assumptions, it will also lower e, and hence i, thereby lowering $n_3 + n_4$, raising consumption, and lowering growth further. In this case, higher export instability resulting from higher variability in domestic production will therefore be *negatively* correlated with growth; see also Brock (1991). If $g > 0$, and the other sources of risk are present, the results become more ambiguous.

Finally, an increase in σ_q^2, while generally similar to σ_y^2, is less clear cut. It will raise export instability, both directly and by lowering ω. The latter will tend to reduce growth, but this is offset by a positive effect, resulting from the higher real rate of return on foreign securities. Thus, export instability resulting from variability in foreign inflation may be either *positively* or *negatively* correlated with growth.

In summary, our model is consistent with all patterns of correlation between export instability and domestic growth. The critical element is the origin of the export instability.

VII. SUMMARY AND CONCLUSIONS

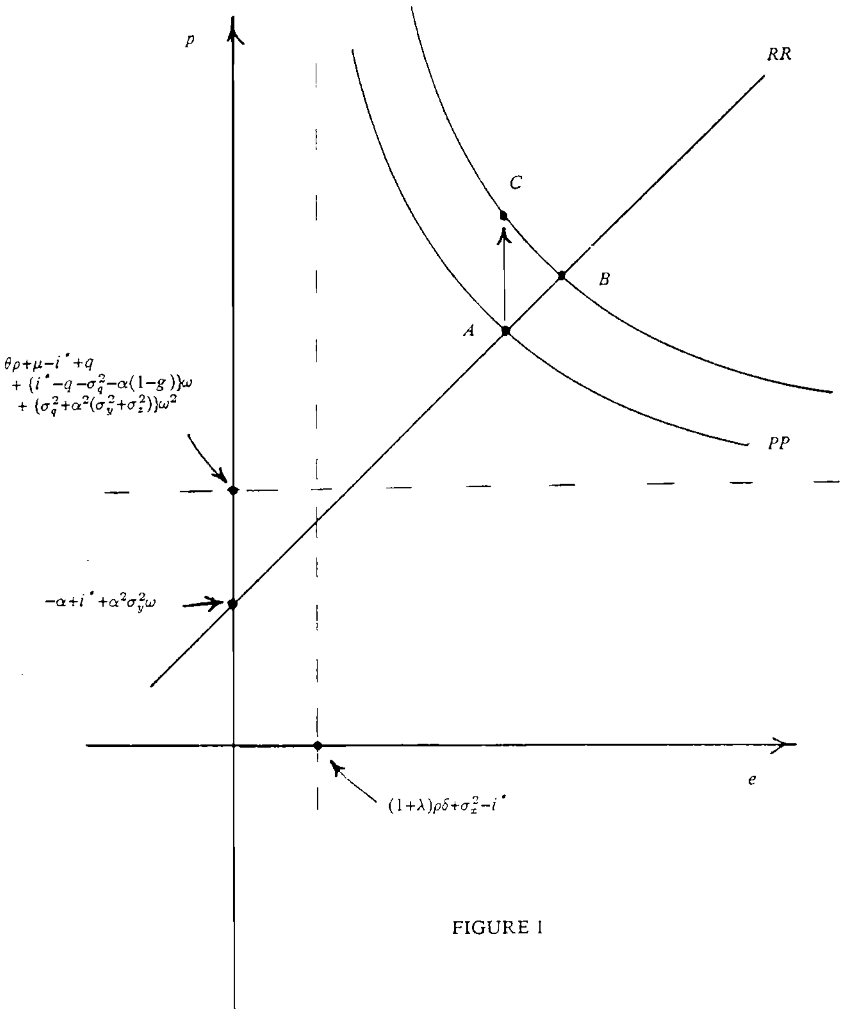
This paper has derived the stochastic equilibrium of a small open economy from the intertemporal optimization of risk averse representative agents. The key characteristic of the equilibrium is that it involves the joint determination of the means and variances of the relevant economic variables, in terms of the first two moments of the exogenous stochastic processes impinging on the economy. This mean-variance equilibrium is then used to explore a number of interesting issues.

First, we have considered how the means and variances of domestic government policy impact on the economy. Our model yields the expected proposition that an increase in the (expected) monetary growth rate or share of government expenditure will both raise the (expected) inflation rate and rate of exchange depreciation. More interesting are the findings that an increase in the variance of the monetary growth rate operates in the reverse direction from the mean, reducing expected inflation and the rate of exchange depreciation, while by contrast, a larger variance of fiscal policy operates in the same direction as does an increase in its mean.

Secondly, we have analyzed the effects of changes in the mean foreign inflation rate and its variance on the equilibrium. We have confirmed that flexible rates will provide perfect insulation against changes in the foreign inflation rate, which leave the foreign real rate of return unchanged. An increase in the variance of the foreign inflation rate will impact on the domestic economy, in particular lowering the expected rate of inflation. Also, somewhat paradoxically, we have shown that it will reduce the variance of the nominal exchange rate.

Thirdly, we have used the model to study the determinants of the foreign exchange risk premium and to consider conditions under which it is zero. Finally, we have considered the relationship between export instability and growth implied by the model and shown how it is capable of reconciling the seemingly contradictory evidence on this issue.

We view the equilibrium framework we have developed as being a rich and fruitful one, capable of addressing many other interesting issues. Some of these will require appropriate extensions of the model. For example, to address issues pertaining to the variability of the real exchange rate, will require the introduction of a second commodity, which is a straightforward thing to do. Being grounded in utility maximization, the model provides a natural approach to analyzing the welfare implications of macroeconomic policy shocks, such as those we have been discussing. It also may be used to compare the relative merits of alternative exchange rate regimes and to consider the question of the optimal choice of regime. This is a topic we plan to pursue in subsequent research.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$+ (i^*-q-\sigma_q^2-\alpha(1-g))\omega$
$+ (\sigma_q^2+\alpha^2(\sigma_y^2+\sigma_z^2))\omega^2$
$-\alpha+i^*+\alpha^2\sigma_y^2\omega$
e
$(1+\lambda)\rho\delta+\sigma_z^2-i^*$

FIGURE 1

FOOTNOTES

[1]For a discussion of the portfolio balance approach to exchange rate determination and the balance of payments, see Branson and Henderson (1985).

[2]Branson and Henderson (1985) review the literature and discuss asset demands in a three asset model, in which the exchange rate and price level are stochastic. However, their model does not endogenize the stochastic returns on assets, the exchange rate, or the price level. It is therefore only a partial equilibrium approach.

[3]The demand for money in the presence of domestic interest paying bonds is generated by positing a utility value for domestic money.

[4]In many respects, the model can be viewed as being a mean–variance version of the Lucas (1982) model. While it is more special in this respect, it does allow for capital accumulation and growth, it includes a separate government sector with monetary and fiscal instruments and has the advantage of being able to offer explicit insights into the specific questions we are addressing.

[5]For simplicity, we assume that consumers do not ascribe direct utility to government expenditure. This assumption is inessential and can be easily relaxed.

[6]See e.g., Merton (1971).

[7]The assumption of Brownian motion, as specified by these stochastic processes, is convenient in that it leads to a mean–variance equilibrium in a natural way. As will become evident below, we will allow E and therefore P to undergo discrete jumps at instances where once–and–for–all unanticipated discrete changes, such as policy changes, occur. Elsewhere, they will follow the continuous (but non–differentiable) processes specified in (5).

[8]For a lucid discussion of the methods of stochastic calculus, see e.g., Malliaris and Brock (1982).

[9]This is a standard problem in stochastic calculus; see e.g., Merton (1971), Malliaris and Brock (1982).

[10]This specification can be thought of as being a stochastic version of a balanced growth equilibrium assumption. Our representation of debt policy in this form is dictated by the analytical constraint that the stochastic model be represented by a single state variable, as we have done. More general specifications of debt policy will introduce a second state variable into the dynamics and become highly intractable.

[11]The stochastic relationships can be used to determine the covariances between various key quantities such as capital flows, investment, savings, etc., as they respond to the exogenous shocks; see e.g., Stockman and Svensson (1987), Stulz (1988). We do not pursue this aspect here, except to calculate those covariances which are necessary to solve for the reduced fom equilibrium; see equations (21).

[12]This issue was discussed at some length in the 1970's using the portfolio balance framework; see e.g., Floyd (1978), Laidler (1977), Turnovsky (1977). In this discussion the distinction was drawn between once-and-for-all increases in the foreign price level, on the one hand, and increases in the steady foreign rate of inflation, on the other. Our discussion here does not consider the former.

[13]See also Stockman (1978), Frankel (1979) and Sibert (1989) for discussion of the risk premium.

[14]Notice that we have used the CIP condition (35) to eliminate f from the expression for the risk premium. Other authors eliminate $F(t,t+dt)$ using the consumer optimality conditions, thereby expressing the risk premium in terms involving the marginal utility of consumption; see e.g., Stockman (1978), Sibert (1989) and Engel (1990). As noted by Stockman (1978, p. 165) these expressions are equivalent.

[15]In general, the β coefficient of asset i, having a stochastic component in its rate of return du_i, is $\beta_i = cov\ (dw, du_i)/var\ (dw)$, where it should be recalled that dw is given by (20c). Assuming $\omega = 0$, we see that $\sigma_w^2 = \sigma_q^2$ and noting (38), $\beta_B = \beta_F = 1, \beta_S = 0$ immediately follow.

[16]Writing the stochastic differential equation (40) as

$$dK = \delta K dt + K dk$$

the solution can be written as

$$K = K_0 e^{(\delta - \frac{1}{2}\sigma_k^2)t} + k(t) - k_0$$

with the transversality condition being

$$\lim_{t \to \infty} EKe^{-\int dR_S} = 0.$$

The point about the slow rate of growth can be seen most clearly by considering the nonstochastic case, when $dk \equiv 0, \sigma_q^2 = 0$, and in equilibrium $\alpha = i^* - q$. The solution for K in this case is

$$K = K_0 e^{[\alpha - \omega[g + \theta\rho/n_3]]t}$$

which grows at a slower rate than the real rate of return, so that the transversality condition

$$\lim_{t \to \infty} Ke^{-\alpha t} = \lim_{t \to \infty} K_0 e^{-\omega[g+\theta\rho/n_3]t} = 0$$

is clearly met.

[17]This issue is examined analytically by Brock (1991). However, his model is more restrictive than the present one, and in particular, the only source of stochastic shocks are domestic productivity disturbances.

[18]Our model does not distinguish between investment and growth of GNP; they are the same. Voivodas (1974) and Özler and Harrigan (1988) find a negative correlation between instability and growth rates of GNP; Yotopoulos and Nugent (1976) obtain a positive correlation; and Kenen and Voivodas (1972) find no correlation.

APPENDIX

Solution to the Consumer's Optimization Problem

The consumer's stochastic optimization problem is to choose consumption and portfolio shares to

$$Max \quad E\int_0^{\infty}[\theta \; ln \; C + \delta \; ln[n_1 W]]e^{-\rho t}dt \qquad \theta + \delta = 1 \tag{A.1a}$$

subject to the stochastic wealth accumulation equation

$$dW = \psi W dt + W d\phi \tag{A.1b}$$

where for notational convenience,

$$\psi \equiv n_1 r_M + n_2 r_B + n_3 r_S + n_4 r_F - \frac{C}{W} - \tau$$

$$d\phi \equiv -(n_1 + n_2)du_p + n_3 du_s - n_4 du_q - dv$$

$$\chi \equiv \lim_{dt \to 0} \frac{E(d\phi)^2}{dt} = (n_1 + n_2)^2\sigma_p^2 + n_3^2\sigma_s^2 + n_4^2\sigma_q^2 + \sigma_v^2$$

$$- 2(n_1 + n_2)n_3\sigma_{sp} + 2(n_1 + n_2)\sigma_{pv} - 2n_3\sigma_{sv} + 2(n_1 + n_2)n_4\sigma_{pq}$$

$$+ 2n_4\sigma_{qv} - 2n_3 n_4\sigma_{sq}.$$

We define the differential generator of the value function $V(W,t)$ by

$$L_W[V(W,t)] \equiv \frac{\partial V}{\partial t} + \psi W \frac{\partial V}{\partial W} + \frac{1}{2}\chi W^2 \frac{\partial^2 V}{\partial W^2}. \tag{A.2}$$

In particular, given the exponential time discounting, V can be taken to be of the time separable form

$$V(W,t) \equiv e^{-\rho t}X(W).$$

The formal optimization problem is now to choose C, n_1, n_2, n_3, n_4 to maximize the Lagrangean expression

$$e^{-\rho t}[\theta \ ln \ C + \delta \ ln(n_1 W)] + L_W[e^{-\rho t}X(W)] + \frac{\eta e^{-\rho t}}{\rho}[1 - n_1 - n_2 - n_3 - n_4]. \tag{A.3}$$

Taking partial derivatives of this expression and cancelling $e^{-\rho t}$ yields

$$\frac{\theta}{C} - \frac{dX}{dW} = 0 \tag{A.4a}$$

$$\frac{\delta}{n_1} + r_M W \frac{dX}{dW} + [(n_1 + n_2)\sigma_p^2 - n_3\sigma_{sp} + \sigma_{pv} + n_4\sigma_{pq}]W^2 \frac{d^2X}{dW^2} = \frac{\eta}{\rho} \tag{A.4b}$$

$$r_B W \frac{dX}{dW} + [(n_1 + n_2)\sigma_p^2 - n_3\sigma_{sp} + \sigma_{pv} + n_4\sigma_{pq}]W^2 \frac{d^2X}{dW^2} = \frac{\eta}{\rho} \tag{A.4c}$$

$$r_S W \frac{dX}{dW} + [n_3\sigma_s^2 - (n_1 + n_2)\sigma_{sp} - \sigma_{sv} - n_4\sigma_{sq}] \frac{d^2X}{dW^2} = \frac{\eta}{\rho} \tag{A.4d}$$

$$r_F W \frac{dX}{dW} + [n_4\sigma_q^2 + (n_1 + n_2)\sigma_{pq} + \sigma_{qv} - n_3\sigma_{sq}] \frac{d^2X}{dW^2} = \frac{\eta}{\rho} \tag{A.4e}$$

$$n_1 + n_2 + n_3 + n_4 = 1. \tag{A.4f}$$

These equations determine the optimal values for C, n_1, n_2, n_3, n_4, as functions of the derivatives dX/dW and d^2X/dW^2 of the value function $X(W)$. In addition, this function must satisfy the stochastic Bellman equation

$$\max_{C, n_1, n_2, n_3, n_4} [[\theta \ ln \ C + \delta \ ln[n_1 W]]e^{-\rho t} + L_W[e^{-\rho t}X(W)] = 0. \tag{A.5}$$

This involves substituting for the optimized values obtained from (A.4) and solving the resulting differential equation for $X(W)$, namely

$$\theta \ ln \ \hat{C} + \delta[ln \ \hat{n}_1 + ln \ W] - \rho X(W) + \hat{\psi} W \left(\frac{dX}{dW}\right) + \frac{1}{2}\hat{\chi} W^2 \frac{d^2X}{dW^2} = 0 \tag{A.6}$$

where $\hat{}$ denotes optimized values.

The solution strategy is essentially by trial and error, finding a function $X(W)$ that satisfies both the optimality conditions and the Bellman equation. In searching for such a solution we shall assume that the consumption ratio C/W and portfolio shares n_i, are all constant, as in fact they turn out in equilibrium to be, so that ψ and χ are also constants. We postulate a solution of the form

$$X(W) = b_0 + b_1 \; ln \; W \tag{A.7}$$

where b_0, b_1 are to be determined. This equation immediately implies

$$\frac{dX(W)}{dW} = \frac{b_1}{W}; \quad \frac{d^2X}{dW^2} = -\frac{b_1}{W^2} \tag{A.8}$$

with the optimality condition for consumption being

$$\hat{C} = \left(\frac{\theta}{b_1}\right) W$$

so that

$$ln \; \hat{C} = ln \; \theta - ln \; b_1 + ln \; W.$$

Substituting into the Bellman equation (A.6) leads to

$$\theta[ln \; \theta - ln \; b_1 + ln \; W] + \delta[ln \; \hat{n}_1 + ln \; W] - \rho[b_0 + b_1 \; ln \; W] + \hat{\psi} b_1 - \frac{1}{2}\hat{\chi} b_1 = 0.$$

This consists of constants and terms involving $ln \; W$. The function (A.7) will be a viable solution provided b_1 and b_0 are chosen to satisfy

$$b_1 = \frac{\theta + \delta}{\rho} = \frac{1}{\rho} \tag{A.9a}$$

$$\rho b_0 = \theta \; ln \; \theta\rho + \delta \; ln \hat{n}_1 + \frac{1}{\rho}(\hat{\psi} - \frac{1}{2}\hat{\chi}). \tag{A.9b}$$

The form of the value function is therefore

$$X = b_0 + \frac{ln \; W}{\rho}$$

and substituting for $dX/dW = 1/\rho W, d^2X/dW^2 = -1/\rho W^2$ into the differential generator and simplifying equations (A.4) yields the expressions (9) and (10) of the text. One can further establish that the transversality condition

$$\lim_{t \to \infty} E[e^{-\rho t} X(W)] = 0$$

is satisfied.

REFERENCES

Branson, W.H. and D.W. Henderson, "The Specification and Influence of Asset Markets," in R.W. Jones and P.B. Kenen (eds.) *Handbook in International Economics*, North-Holland, Amsterdam, 1985.

Brock, P.L., "Export Instability and the Economic Performance of Developing Countries," *Journal of Economic Dynamics and Control* 15, 1991, 129–147.

Domowitz, I. and C. Hakkio, "Conditional Variance and the Risk Premium in the Foreign Exchange Market," *Journal of International Economics* 19, 1985, 47–66.

Engel, C., "On the Foreign Exchange Risk Premium in a General Equilibrium Model," University of Virginia, 1990.

Fischer, S., "The Demand for Indexed Bonds," *Journal of Political Economy* 83, 1975, 509–534.

Floyd, J.E., "The Asset Market Theory of the Exchange Rate: A Comment," *Scandinavian Journal of Economics* 80, 1978, 100–103.

Frankel, J.A., "The Diversifiability of Exchange Risk," *Journal of International Economics* 9, 1979, 379–393.

Frankel, J.A., "The Implications of Mean–Variance Optimization for Four Questions in International Macroeconomics," *Journal of International Money and Finance* 5, 1986, S53–S75.

Giovannini, A. and P. Jorion, "Interest Rates and Risk Premia in the Stock Market and in the Foreign Exchange Market," *Journal of International Money and Finance* 6, 1987, 107–123.

Grinols, E.L. and S.J. Turnovsky, "Extrinsic Risk, Financial Market Volatility and Macroeconomic Performance," Universities of Illinois and Washington, 1990.

Hodrick, R.J. and S. Srivastava, "The Covariation of Risk Premiums and Expected Future Spot Exchange Rates," *Journal of International Money and Finance* 5, 1986, 5–21.

Kenen, P.B. and C.S. Voidodas, "Export Instability and Economic Growth," *Kyklos* 25, 1972, 701–803.

Laidler, D.E.W., "Expectations and the Behaviour of Prices and Output Under Flexible Exchange Rates," *Economica* 44, 1977, 327–336.

Lewis, K.K., "Inflation Risk and Asset Market Disturbances: The Mean–Variance Model Revisited," *Journal of International Money and Finance* 7, 1988, 273–288.

Lucas, R.E., "Interest Rates and Currency Prices in a Two Country World," *Journal of Monetary Economics*, 335–359, 1982.

Malliaris, A.G. and W.A. Brock, *Stochastic Methods in Economics and Finance*, North–Holland, Amsterdam, 1982.

Mehra, R. and E.C. Prescott, "The Equity Premium: A Puzzle," *Journal of Monetary Economics* 15, 1985, 145–161.

Merton, R.C., "Optimum Consumption Rules in a Continuous–Time Model," *Journal of Economic Theory* 3, 1971, 373–413.

Özler, S. and J. Harrigan, "Export Instability and Growth," Department of Economics Working Paper No. 486, UCLA, 1988.

Sibert, A., "The Risk Premium in the Foreign Exchange Market," *Journal of Money, Credit, and Banking* 21, 1989, 49–65.

Stockman, A., "Risk, Information, and Forward Exchange Rates," in J.A. Frenkel and H.G. Johnson (eds.) *The Economics of Exchange Rates*, Addison–Wesley, Reading, MA, 1978.

Stockman, A. and L. Svensson, "Capital Flows, Investment, and Exchange Rates," *Journal of Monetary Economics* 19, 1987, 171–201.

Stulz, R., "Capital Mobility and the Current Account," *Journal of International Money and Finance* 7, 1988, 167–180.

Turnovsky, S.J., *Macroeconomic Analysis and Stabilization Policy*, Cambridge, U.K., 1977.

Turnovsky, S.J., "Exchange Market Intervention Policies in a Small Open Economy," in J.S. Bhandari and B.H. Putnam (eds.), *Economic Interdependence and Flexible Exchange Rates*, MIT Press, Cambridge, MA, 1983.

Turnovsky, S.J., "Domestic and Foreign Disturbances in an Optimizing Model of Exchange Rate Determination," *Journal of International Money and Finance* 4, 1985, 151–171.

Voivodas, C.S., "The Effect of Foreign Exchange Instability on Growth," *Review of Economics and Statistics* 56, 1974, 410–412.

Yotopoulos, P.A. and J.B. Nugent, *Economics of Development: Empirical Investigations*, Harper and Row, New York, 1976.