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PRICE EQUILIBRIUM, EFFICIENCY, AND DECENTRALIZABILITY  
IN INSURANCE MARKETS

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ABSTRACT

In this paper, we investigate the descriptive and normative properties of competitive equilibrium with moral hazard when firms offer "price contracts" which allow clients to purchase as much insurance as they wish at the quoted prices. We show that a price equilibrium always exists and is one of three types:

- i) zero profit price equilibrium - zero profit, zero effort, full insurance
- ii) positive profit price equilibrium - positive profit, positive effort, partial insurance
- iii) zero insurance price equilibrium - zero insurance, zero profit, positive effort.

We also demonstrate circumstances under which the linear taxation of price insurance allows decentralization of the social optimum (conditional on the unobservability of effort), and when it, does not, whether it is at least utility-improving.

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Price Equilibrium, Efficiency, and Decentralizability in  
Insurance Markets\*

1. Introduction

During the past fifteen years, there has been a growing awareness of moral hazard and related incentive problems. Surprisingly, however, there has been little analysis of the nature of equilibrium -- the conditions under which equilibrium exists, and when it does what its properties are.<sup>1</sup>

In this paper, we investigate the descriptive and normative properties of competitive equilibrium with moral hazard in a particularly simple economic environment, in which there is a single good and a single fixed-damage risk and in which firms offer "price contracts", allowing their clients to purchase as much insurance as they wish at the quoted price.<sup>2</sup>

Pauly [1974] argued that when firms offer actuarially fair price contracts, individuals purchase full insurance - that quantity of insurance which equalizes their marginal utilities of income in the events "accident" and "no accident". He also pointed out that when the accident does not directly affect the utility function, so that equalization of the marginal utilities implies equalization of total utilities, individuals have no incentive to prevent the accident. He concluded that, in these circumstances,

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<sup>1</sup>Exceptions include Pauly [1974], Helpman and Laffont [1975], Stiglitz [1983] (a report on our preliminary work), Hellwig [1983a,b], and Arnott and Stiglitz [1987, 1989].

<sup>2</sup>Elsewhere, we have examined these issues with alternative contract forms (Arnott and Stiglitz [1987]) and have considered the complications which arise with many goods and many risks (Arnott and Stiglitz [1989]).

competitive equilibrium with price insurance is characterized by zero profits, zero "effort" (at accident avoidance), and full insurance. He also suggested that this entails the "overprovision" of insurance.

Pauly's analysis, while insightful, was not conclusive. Is it not possible that with zero effort the price of insurance would be so high that individuals would choose to purchase no insurance? Is it obvious that equilibrium entails firms offering actuarially fair insurance? And does "overprovision" mean simply that, though infeasible, it would be beneficial to restrict insurance purchases so as to stimulate effort, or does it imply that government intervention, in some form, would be beneficial?

In his positive analysis, Pauly erred in neglecting corner solutions and second-order conditions. Helpman and Laffont [1975] demonstrated that moral hazard can give rise to non-convexities and that these non-convexities can cause non-existence of the equilibrium Pauly described, but they did not proceed further. We show that a "price equilibrium" (where all firms offer price contracts) always exists and is unique. It may have the characteristics that Pauly describes; we term such an equilibrium a "zero profit price equilibrium". But there are two other types of price equilibrium as well. In the first, individuals purchase no insurance; we term this a "zero insurance price equilibrium". In the second, insurance is sold, but not at actuarially fair rates; there is positive profit, partial insurance, and positive effort; this type of equilibrium we term a "positive profit price equilibrium".

We also clarify in what sense a price equilibrium entails the overprovision of insurance. In particular, we delineate circumstances under which the linear taxation of price insurance is welfare-improving.

The properties of this competitive equilibrium are very different

from those of the Arrow-Debreu competitive equilibrium with uncertainty, in which insurance markets are characterized by zero profit, full insurance, and typically positive effort, and are efficient. The source of the differences is the asymmetry of information which gives rise to the moral hazard problem, that insurers can observe neither the accident-prevention activities of their clients, nor the Arrow-Debreu states of nature. If an insurer could observe a client's level of effort, he would make the contract contingent on it in such a way that the client chose the efficient level of effort. And if the insurer could observe the states of nature (the probabilities of which are exogenous) he would make the contract contingent on them, and his client would face the appropriate incentives to choose the efficient level of care. But since insurers can observe neither clients' levels of effort nor the states of nature, an incentive problem arises; the more insurance is provided, the smaller is the client's incentive to take care; and there is consequently the tradeoff between risk-bearing and incentives that is the hallmark of the moral hazard problem.

Thus, one would expect that in competitive equilibrium with moral hazard, firms would provide less than complete coverage so as to induce their clients to take some care to prevent the accident. It is therefore surprising that in the zero profit price equilibrium there is full insurance and zero effort. It is also unexpected that moral hazard may give rise to positive profits in equilibrium or to inactivity of the market in the presence of potential gains from trade.

An intuitive explanation of the latter two results is as follows: One may view moral hazard as a form of (negative) informational externality since the provision of insurance drives a wedge between the marginal social

benefit of effort and the marginal private benefit.<sup>3</sup> It is well-known in other contexts that externalities can induce non-convexities (Starrett [1972]), and that is the case here. Moral hazard may cause indifference curves (in the relevant space) to be non-convex, which may in turn cause effort to be discontinuous in the price of insurance. If a firm in a positive profit price equilibrium were to reduce the price of insurance infinitesimally, its clients would reduce effort discontinuously, resulting in the firm making a loss. Relatedly, if a firm in the zero insurance price equilibrium gradually lowers its price, at first its clients purchase no insurance, but suddenly when the price falls to a critical level, rather than purchasing a small amount of insurance, they reduce effort discontinuously and purchase a large amount of insurance, rendering the policy unprofitable.

Whether one finds it surprising that taxing price insurance can be beneficial depends on one's perspective. On one hand, one may view a representative client as contracting with its own representative private insurance company that has no other clients,<sup>4</sup> in which case the informational externality identified earlier is internalized because there are no third parties. Since the firm and the client may contract freely (subject to the form of the contract) one might expect them to reach a contract that is Pareto efficient, conditional on the information available and the contract form (i.e., constrained efficient). Furthermore, with or without this

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<sup>3</sup> If I reduce my effort such that my expected damage costs increase by \$5, and if I have 80% coverage, then I impose a cost of \$4 on the insurance company and ultimately on the company's other clients.

<sup>4</sup> Indeed, this is the way the contracting problem is formulated in our analysis.

perspective on the contracting mechanism, it is not obvious what advantages the government has over the contracting parties, when it has no more information than the firm, that gives it the scope to make a Pareto improvement. From this perspective, our results appear surprising.

On the other hand, the following argument suggests that a small tax on insurance at the zero profit price equilibrium is usually desirable: Such a tax will force the firm to offer insurance at higher than the actuarially fair price. Faced with such a price of insurance, individuals will purchase less than full insurance and expend positive effort. The negative welfare effect of a small movement away from full insurance is second-order. However, the positive welfare effect of an increase in effort is first-order. Thus, if the effect of the increased price of insurance on effort is also first-order, as one might expect, then a small tax on insurance is Pareto-improving. A cruder argument along the same lines is that since the informational externality results in everyone expending too little effort, and since effort can be stimulated by taxing insurance, then taxing insurance will at least partially internalize the informational externality and will therefore be desirable. This argument is flawed since it neglects that the tax also causes the individual to bear more risk. And the previous argument is incomplete and fails to explain what the government can do that insurance firms cannot.

It turns out that the analysis is more subtle than any of the above arguments suggests, being complicated by the nonconvexities to which moral hazard may give rise.

Section 2 presents the basic model and the diagrammatic apparatus we shall employ. Price equilibrium in the absence of government intervention is

analysed in section 3. Section 4 investigates the linear taxation of price insurance. Section 5 discusses issues related to information and government intervention. Section 6 concludes.

## 2. The Basic Model

We employ a very simple model in which individuals are identical (to abstract from adverse selection problems). There is a single, fixed-damage accident, the probability of which,  $p$ , is a function of the level of effort devoted to accident avoidance,  $e$ . Moral hazard arises because the insurer is unable to observe a client's effort, and is hence unable to write insurance contracts contingent on it. In the absence of insurance and in the event of no accident, an individual's consumption is  $w$ , while if there is an accident it is  $w - d$ , where  $d$  is the accident damage. The insured individual receives  $\alpha$  (the "(net) payout" or "benefit") if an accident occurs, and if it does not the insured pays  $\beta$  (the "premium"). Thus, the insurance the individual obtains is characterized by  $(\alpha, \beta)$ . Also, where  $y_0$  is consumption in the event of no accident, and  $y_1$  consumption in the event of accident,

$$y_0 = w - \beta \qquad y_1 = w - d + \alpha \qquad (1)$$

The individual's expected utility is

$$EU = (1-p(e))u_0(y_0, e) + p(e)u_1(y_1, e). \qquad (2)$$

We make the simplifying assumption that the expected utility function has the form<sup>5</sup>

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<sup>5</sup>Some of the complication which arise when the event-contingent utility functions are not separable in consumption and effort are discussed in Arnott and Stiglitz [1988a].



$$EU = (1-p(e))u(y_0) + p(e)u(y_1) - e. \quad (2')$$

This function is separable in consumption and effort, and is event-independent, by which we mean that which event occurs does not affect the utility from consumption--the accident does not cause pain, nor does it alter tastes. There is positive but diminishing marginal utility from consumption. Also, we measure effort so that  $e = 0$  corresponds to minimum effort and assume that  $p' < 0$  and  $p'' > 0$  for  $e > 0$ , and  $\lim_{e \downarrow 0} p(e) = p(0) \equiv \bar{p} < 1$ .

We shall develop our analysis in  $\alpha$ - $\beta$  space (see Figure 1). We define  $q \equiv \frac{\beta}{\alpha}$  to be the price of insurance corresponding to  $(\alpha, \beta)$  (geometrically, it is the slope of a ray from the origin to  $(\alpha, \beta)$ ) and call  $\alpha$  the quantity of insurance. Thus, a price insurance contract at price  $q'$ , under which a client may purchase as much positive insurance as he wishes<sup>6</sup> at price  $q'$ , is drawn as a ray from the origin with slope  $q'$ .

The individual with insurance  $(\alpha, \beta)$  chooses effort to maximize utility; i.e.

$$\max_e EU = (1-p(e))u_0 + p(e)u_1 - e \equiv V(\alpha, \beta), \quad (3)$$

where  $u_0 \equiv u(w-\beta)$  and  $u_1 \equiv u(w-d+\alpha)$ . The first-order condition is

$$-(u_0 - u_1)p'(e) - 1 = 0 \quad \text{if } e > 0; \quad (4)$$

the marginal cost of effort is unity, and the marginal benefit of effort equals the reduction in the probability of accident from an extra unit of effort,  $-p'$ , times the increase in utility from a unit reduction in the

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<sup>6</sup>Under our assumption that the utility-from-consumption function is event-independent, price equilibrium will never entail individuals purchasing a negative quantity of insurance.

probability of accident,  $u_0 - u_1$ . Under the assumption of separable utility,  $e$  is a continuous function of  $\alpha$  and  $\beta$ ,  $e(\alpha, \beta)$ . Furthermore,  $\frac{\partial e}{\partial \alpha} < 0$  and  $\frac{\partial e}{\partial \beta} < 0$  for  $e > 0$ ; as more insurance is provided, the individual reduces effort. Substituting  $e(\alpha, \beta)$  into the expression for expected utility gives  $V(\alpha, \beta)$ , the indifference curves corresponding to which can be plotted in  $\alpha$ - $\beta$  space.

The individual's marginal rate of substitution between the premium and net payout is

$$\left. \frac{d\beta}{d\alpha} \right|_{\bar{V}} = - \frac{V_{\alpha}}{V_{\beta}} = \frac{u_1' p}{u_0' (1-p)} \quad (5)$$

As more insurance is provided, diminishing marginal utility causes  $\frac{u_1'}{u_0'}$  to decrease, but individuals take less care, and as a result  $\frac{p}{1-p}$  increases. For this reason, indifference curves between net payout and premium will not in general be "convex"<sup>7</sup> (see Figure 1).

Even though effort is a continuous function of  $(\alpha, \beta)$ , an individual's purchases of insurance may not be a continuous function of the price of insurance because nonconvexity of the indifference curves may cause the price-consumption lines to be discontinuous.<sup>8</sup> This greatly complicates the analysis to follow.

The set of insurance contracts for which expected returns are

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<sup>7</sup>We say that indifference curves with the normal shape are "convex", even though in  $\alpha$ - $\beta$  space normally-shaped indifference curves are concave according to the definition of concavity.

<sup>8</sup>The price-consumption line is defined in the usual way as the locus of points of maximum utility on price lines (rays through the origin). (Later, we shall have occasion to employ price-consumption lines corresponding to points other than the origin).

non-negative,

$$\pi(\alpha, \beta) \equiv \beta(1-p(e(\alpha, \beta)) - \alpha p(e(\alpha, \beta))) \geq 0, \quad (6)$$

is referred to as the feasibility set and denoted by  $F$ . The boundary of this set is referred to as the resource constraint or zero profit locus (ZPL) (see Figure 1). As one moves up the zero profit locus, effort falls, the probability of accident increases, and to maintain zero profits the ratio  $\frac{\beta}{\alpha}$  must increase. The feasibility set is never convex. It is the lack of convexity of the feasibility set, combined with the lack of quasi-concavity of  $V(\alpha, \beta)$ , which give rise to many of the problems that are the concern of this paper.

It will be useful, before proceeding, to consider some aspects of the geometry of the problem. First, as noted above, because of the possible nonconvexity of indifference curves, price-consumption lines can be discontinuous. Second, because our assumption of separable utility implies that with  $e > 0$  effort falls as  $\alpha$  or  $\beta$  increases, the probability of accident rises monotonically as one moves up any ray from the origin. Third, the zero effort line (ZEL) is the locus of  $(\alpha, \beta)$  such that  $(u_0 - u_1) \lim_{e \downarrow 0} p'(e) + 1 = 0$  and has the slope  $\left. \frac{d\beta}{d\alpha} \right|_{ZEL} = - \frac{u_1'}{u_0'}$ . Effort is zero everywhere beyond the zero effort line. Fourth, the full insurance line (FIL), the locus of  $(\alpha, \beta)$  corresponding to which the marginal utility of consumption is the same in the accident and no-accident events, is  $u_0' = u_1'$ , which with event independence implies  $d = \alpha + \beta$ . If  $\lim_{e \downarrow 0} p'(e) = -\infty$  then  $u_0' = u_1'$  along the zero effort line, which, under our assumption of event-independent utility, implies that the zero effort line coincides with the full insurance line. If, however,

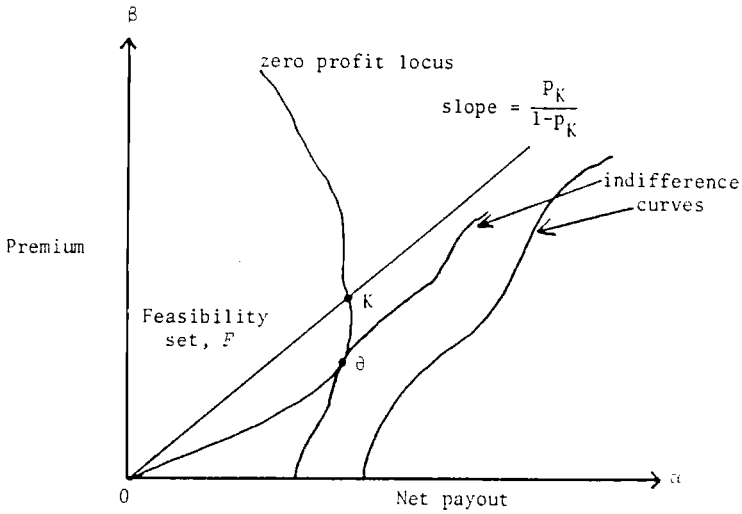


Figure 1: Basic diagram 1, continuum of effort levels  
 i) the zero profit locus is  $\beta(1-p) - \alpha p = 0$   
 ii) the feasibility set is never convex  
 iii) indifference curves are not necessarily convex.

$\lim_{e \rightarrow 0} p'(e)$  is finite, then  $u_0 > u_1$  along the zero effort line, which with event-independent utility implies that less than full insurance is provided along the zero effort line and that the full insurance line lies outside the zero effort line, the case depicted in Figure 2. And fifth, at any point on the ZPL, such as K in Figure 1, the slope of the ray from the origin through the point is  $q_K \equiv \frac{\beta_K}{\alpha_K} = \frac{p_K}{1-p_K}$ .

In some contexts the individual is faced with the choice of how to allocate his time between a discrete number of accident-prevention activities. To illustrate, suppose that there are two activities, a safe activity and a risky activity. One may then interpret  $e$  to be the fraction of time the individual engages in the safe activity, so that  $p(e) = ep^s + (1-e)p^r$ , where  $p^s(p^r)$  is the accident probability associated with the safe (risky) activity per unit time. The indifference curves are never convex; they appear as in Figure 3, where we have also drawn the zero profit locus. This discrete activities model is a limiting case of our general model since  $p' = 0$  or is undefined.

### 3. Price Equilibrium

In a price contract, the firm offering the contract is required to sell to each client as much positive<sup>9</sup> insurance as he wishes to buy at the

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<sup>9</sup> If the price of insurance is high, the individual will purchase a negative amount of insurance if permitted. He will receive a large payout (a negative premium) in the no-accident state, in return for a small payment (a negative net payout) in the accident state.

With the assumed form of the expected utility function, negative purchases of insurance will never occur in equilibrium. To simplify the exposition, we therefore rule out negative purchases of insurance.

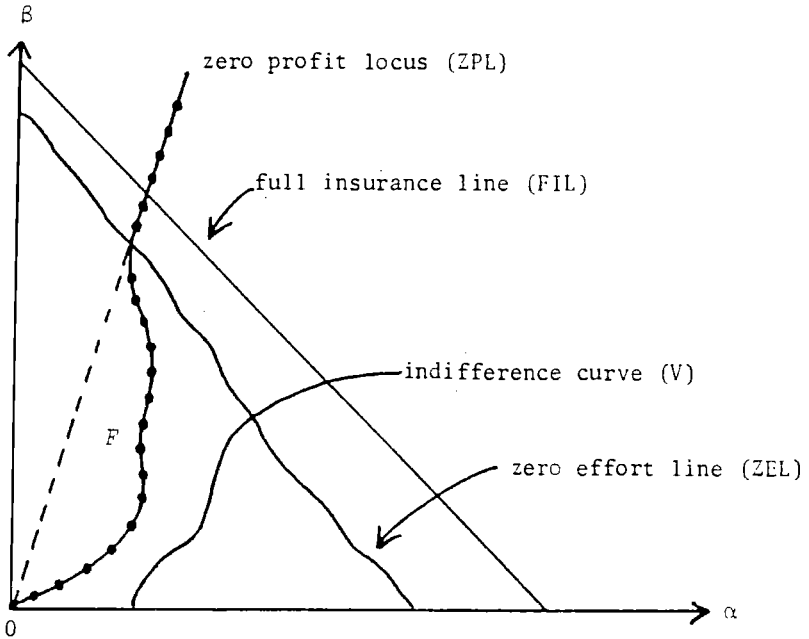


Figure 2: Basic diagram 2,  $\lim_{e \rightarrow 0} p'(e)$  finite

- i) effort is zero beyond the zero effort line
- ii) beyond the zero effort line, the zero profit locus is  $\frac{\beta}{\alpha} = \frac{p(0)}{1-p(0)}$ , a ray going into the origin with slope  $p(0)/(1-p(0))$
- iii) beyond the zero effort line, indifference curves are strictly convex.

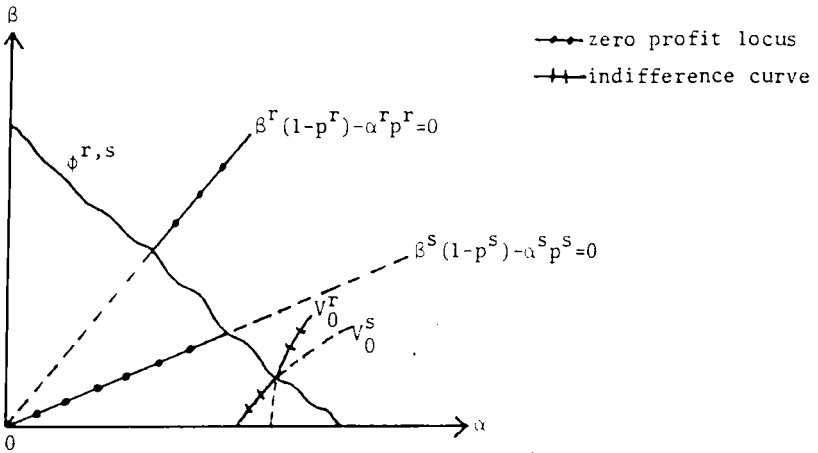


Figure 3: Basic diagram 3, two activities

- i) : The safe activity is undertaken for  $(\alpha, \beta)$  below  $\phi^{r,s}$ , the risky activity above
- ii) :  $V_0^s$  is the indifference curve corresponding to utility level  $V_0$ , contingent on the safe activity, etc.
- iii) :  $\beta^s(1-p^s) - \alpha^s p^s = 0$  is the zero profit locus, contingent on the safe activity, etc.

price<sup>10</sup> quoted in the contract. Thus,  $\beta = q\alpha$ , where  $q$  is the firm's choice variable and  $\alpha$  the client's.

Equilibrium is defined in a similar<sup>11</sup> way to Rothschild-Stiglitz [1976], which examined equilibrium in insurance markets with adverse selection. Each firm is allowed to offer only one price contract, or to do nothing. A Nash equilibrium in price insurance contracts is defined as a set of price contracts such that: i) all contracts offered at least break even; ii) taking as given the contracts offered by incumbent firms (those offering contracts) there is no additional contract which if offered by an entering firm (one not offering a contract) can make a strictly positive profit; and iii) taking as given the set of contracts offered by other incumbent firms, no incumbent firm can increase its profits by altering the contract it offers.

It is obvious that individuals will purchase insurance at the lowest available price. Hence, equilibrium, if it exists, will be characterized by a single price,  $\hat{q}$ . Faced with this price, individuals choose  $\hat{\alpha} \geq 0$  such that

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<sup>10</sup> We do not consider random price contracts, where the random price is realized either before (ex ante) or after (ex post) the individual's effort decision. Arnott and Stiglitz [1988b] provides a discussion of randomization with moral hazard when firms offer exclusive quantity contracts.

<sup>11</sup> With either moral hazard or adverse selection, where feasible competitive equilibrium entails rationing the amount of insurance an individual can purchase at the equilibrium price. Rationing is accomplished by insurers offering exclusive (quantity) contracts which specify the amount of insurance

( $\alpha$ ) that a client can purchase at a give price  $\left( q \equiv \frac{\beta}{\alpha} \right)$  and requiring that the client purchase no additional insurance. Such a requirement is feasible if insurers can observe the quantity of insurance their clients purchase. Price equilibrium is of interest when insurers cannot observe the quantity of insurance their clients purchase. Section 5 elaborates on these points.

Rothschild-Stiglitz defined equilibrium for exclusive contracts.



$$\hat{\alpha} = \underset{\alpha}{\operatorname{argmax}} V(\alpha, \hat{q}\alpha). \quad (7)$$

The solution to (7) for all  $q > 0$  characterizes the price-consumption line. For  $q$  such that  $\alpha = 0$  (the price of insurance is so high that the individual would prefer to purchase no insurance), the origin is the relevant point on the price-consumption line.

Firms must make non-negative profits in equilibrium. Thus,  $\hat{q}$  must satisfy  $(\hat{\alpha}\hat{q})(1-\hat{p}) - \hat{\alpha}\hat{p} \geq 0$  which implies either  $\hat{q} \geq \frac{\hat{p}}{1-\hat{p}}$  for  $\hat{\alpha} > 0$  or  $\hat{\alpha} = 0$ , where  $\hat{p} = p(e(\hat{\alpha}, \hat{q}\hat{\alpha}))$ .

It is easy to show that equilibrium exists, is unique, and occurs at the lowest point (corresponding to the lowest price) on the price-consumption line in  $\alpha$ - $\beta$  space consistent with non-negative profits.

At any point on the price-consumption line except the origin, an indifference curve must be tangent to the corresponding price line,<sup>12</sup>

$$q = \frac{u_1'}{u_0'(1-p)}. \quad (8)$$

Furthermore, if zero profits are made

$$\beta(1-p) - \alpha p = 0 \Rightarrow q \equiv \frac{\beta}{\alpha} = \frac{p}{1-p}. \quad (9)$$

Thus, at any point where the price-consumption line intersects the zero profit locus,

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<sup>12</sup>The price-consumption line is the locus of points of maximum utility for the family of price lines  $\beta = q\alpha$  with  $q > 0$  and  $\alpha \geq 0$ . A point of tangency of an indifference curve and a price line  $\beta = q\alpha$  need not be on the price-consumption line. First, because of the possible nonconvexity of indifference curves, there may be multiple points of tangency--i.e., the tangency condition picks out local extrema while only the global maximum is on the price-consumption line. Second, a corner point, the origin, may be the point of maximal utility on a price line.

$$u'_0 = u'_1. \quad (10)$$

But with event-independent utility, equalization of the two event-contingent marginal utilities (full insurance) implies equalization of the total utilities. And from (4), this implies zero effort. Thus, the price-consumption line can intersect the zero profit locus at only one point - the point of intersection of the FIL and ZPL. We label this point E, and term an equilibrium at this point a zero profit price equilibrium, at which there is zero profit, zero effort, and full insurance. Also, we denote the price of insurance at this point  $q^* = p(0)/(1-p(0))$ .

Pauly [1974] considered only the zero profit price equilibrium, and it has generally been assumed that equilibrium is always of this type. In fact, however, there are three (mutually exclusive and collectively exhaustive) types of price equilibrium, of which the zero profit price equilibrium is one.

### 3.1 The zero profit price equilibrium

A necessary condition for the price equilibrium to be of this type is that the price-consumption line intersect the zero profit locus. There are two different situations in which the price-consumption line does not intersect the zero profit locus. The first, shown in Figure 4, occurs where some point F at which insurance is strictly positive, other than E, is the point on the price-consumption line corresponding to  $q^* = p(0)/(1-p(0))$ ; that is, an individual facing the price line  $OQ^*$ , with slope  $q^*$ , would maximize utility at F, not E. The second (illustrated in Figure 5) occurs where the price-consumption line lies everywhere outside the feasibility set, except at the origin; at prices above that corresponding to  $OP_2$ , the individual purchases zero insurance, while at lower prices he purchases a

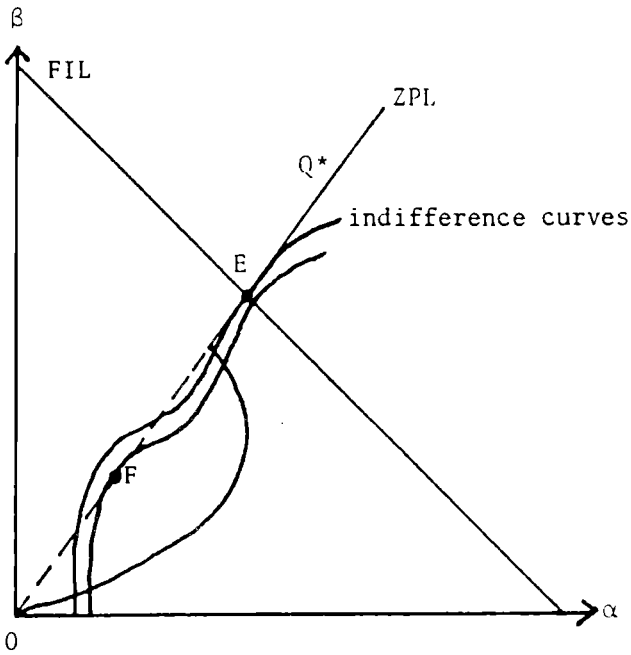


Figure 4:  $E$  need not be on the price-consumption line, i.e. be the point of maximal utility on  $\beta = q^* \alpha$ , where  $q^* = p(0)/(1-p(0))$ .

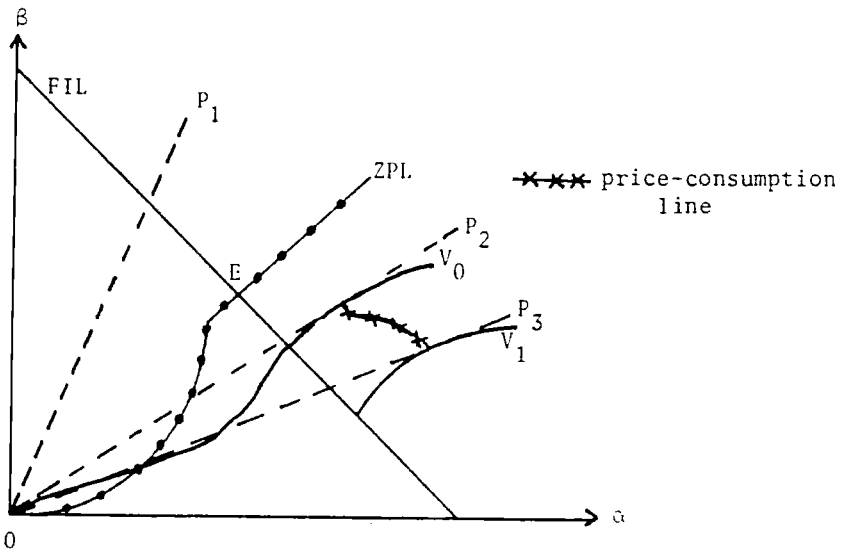


Figure 5: Zero insurance price equilibrium. Observe that the individual's utility is higher at  $0$  than at  $E$ .

large quantity of insurance and expends so little effort that the insurance makes a loss.

Whether either of these two situations occurs depends on the global properties of the utility and probability-of-accident functions. We can, nevertheless, identify a sufficient local condition for non-intersection of the price-consumption line and the zero profit locus when utility is separable and event-independent. It is shown in Figure 6. If the price-consumption line intersects the zero profit locus, it does so at E. A necessary condition for E to be on the price-consumption line is that it be a local utility maximum on the price line  $\beta = q \cdot \alpha$ . This is not possible if, as drawn in Figure 6, the indifference curve through E is concave just below E. In Arnott and Stiglitz [1988a], it is shown that

$$\frac{d^2\beta}{d\alpha^2} \Big|_{\bar{v}} = \begin{cases} -s \left[ (A_1 + sA_0) + \frac{u_1'(p')^3}{(1-p)^2 p p''} \right] & \text{for } e > 0 \\ -s(A_1 + sA_0) & \text{for } e = 0, \end{cases} \quad (11)$$

where  $s \equiv \frac{u_1' p}{u_0(1-p)}$  and  $A_i \equiv -\frac{u_i''}{u_i'}$ ,  $i = 0, 1$ . It follows from (11) that the indifference curve is concave just below E if  $\lim_{e \downarrow 0} \frac{(p')^3}{p''} = -\infty$ . This condition is satisfied if, for example,  $p(e) = \bar{p} - ke^c$  for small  $e$  and  $c < \frac{1}{2}$ .

We have identified a necessary condition for a price equilibrium to be a zero profit price equilibrium; later, we shall indicate a sufficient condition.

### 3.2 The zero insurance price equilibrium

When the price-consumption line lies everywhere outside the

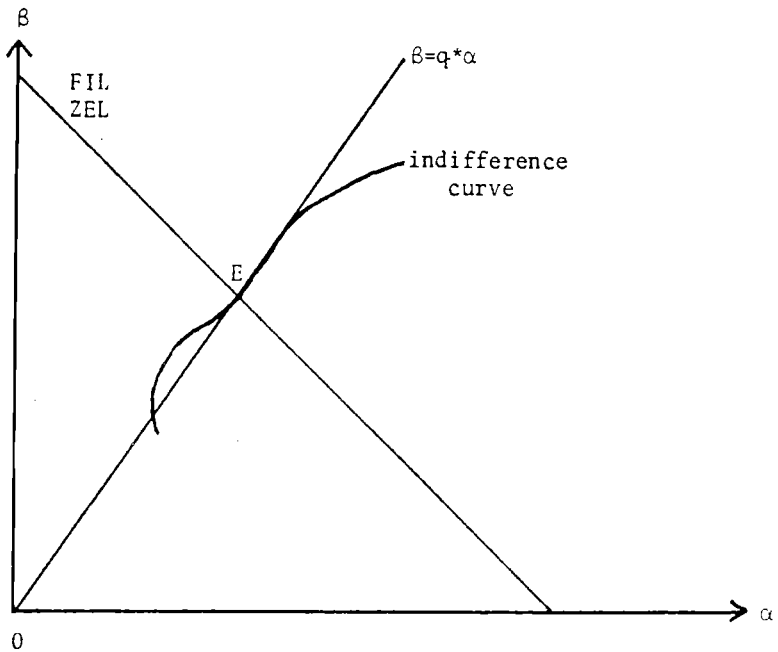


Figure 6: A local condition under which the price-consumption line does not intersect the zero profit locus.

feasibility set except at the origin, there is a zero insurance price equilibrium. This case was illustrated in Figure 5. The zero insurance equilibrium is characterized by zero insurance, zero profits, and positive effort. It should be noted that in this case equilibrium exists, but is inactive, which is different from equilibrium not existing.

### 3.3 The positive profit price equilibrium

This can be an equilibrium only if any firm that offers an insurance policy with a lower price loses money; thus, the price-consumption line must have a discontinuity across the zero profit locus. This is illustrated in Figure 7. H is the (lower jump) point on the price-consumption line corresponding to the lowest price at which non-negative profits are made,  $q^H$ . Firms offering insurance at price  $q^H$  make positive profits. Any firm offering a contract at a lower price would lose money, since at prices lower than  $q^H$ , the price-consumption line lies outside the zero profit locus. When the price of insurance is lowered just slightly below  $q^H$ , individuals purchase much more insurance and take much less care, with the result that the insurance contract becomes unprofitable. Such an equilibrium is characterized by positive profits, positive effort, and partial insurance.<sup>13</sup>

Equilibrium is always of this type when the individual purchases a positive amount of insurance at price  $q^*$ , but not that corresponding to E.

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<sup>13</sup>Since effort decreases in the amount of insurance offered, if effort is zero at H, it is zero beyond H. This implies that the indifference curve passing through H is convex beyond H, which is inconsistent with a jump discontinuity from H to H' (see Figure 7). Since H is on the price-consumption line

$$\left. \frac{dB}{d\alpha} \right|_{\hat{v}} = \frac{u_1' p^H}{u_0' (1-p^H)} = q^H > \frac{p^H}{1-p^H} \text{ (positive profits)} \Rightarrow u_1' > u_0' \Rightarrow \text{partial insurance.}$$

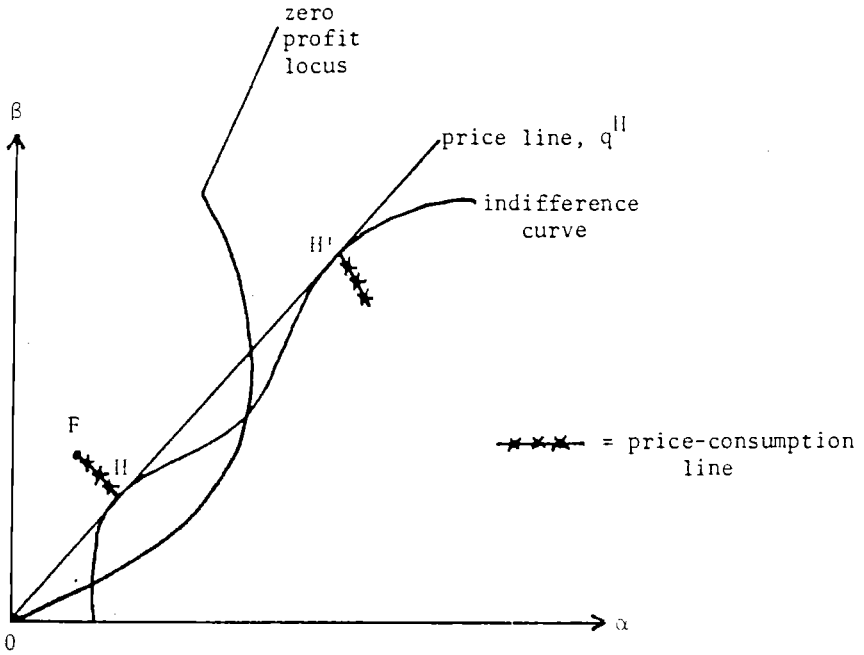


Figure 7: The price-consumption line can be discontinuous and not intersect the zero profit locus. Note that the point  $F$  corresponds to the point  $F$  in Figure 4.



In this situation, which is illustrated in Figure 4, the price-consumption line does not intersect the zero profit locus. Equilibrium may also be of this type even when the price-consumption line intersects the zero profit locus. Relatedly, a necessary and sufficient condition for the price equilibrium to be of the zero profit type is that the price-consumption line intersect the zero profit locus and that no point on the price-consumption line corresponding to a price less than  $q^*$  lie inside the feasibility set.<sup>14</sup>

Drawing the above results together, we have:

Proposition 1: When only price contracts are admitted, equilibrium always exists and is unique. It occurs at the lowest point on the price-consumption line consistent with non-negative profits (which may be the origin) and is of one of the following three types.

1. Zero insurance price equilibrium. This occurs when the price-consumption line lies everywhere outside the feasibility set except at the origin, and as a result the market is inactive.
2. Positive profit price equilibrium. Positive profits, positive effort, partial insurance. This occurs when the price-consumption line is discontinuous across the zero profit locus for some  $q < q^*$ .
3. Zero profit price equilibrium. Zero profits, zero effort, full insurance. This occurs when the price-consumption line intersects the zero profit locus and no point on the price-consumption line

---

<sup>14</sup> Proposition 1 examined which of the three types of price equilibrium obtains in terms of the properties of the price-consumption line. Our discussion implies a simpler, but less complete, characterization in terms of the point of maximal utility on the price line  $\beta = q^*\alpha$ : 1) If the point of maximal utility is not at the origin, the price equilibrium cannot be of the zero insurance type. 11) If the point of maximal utility is neither at the origin nor at E, the price equilibrium is of the positive profit type.

with  $q < q^*$  lies in the feasibility set.

It is obvious that the positive profit price equilibrium and the zero insurance price equilibrium<sup>15</sup> can occur only when indifference curves are nonconvex. Furthermore, the price equilibrium is always a zero profit price equilibrium when indifference curves are convex.

#### 4. On the Desirability of the Linear Taxation of Price Insurance

Can the price equilibrium be improved on by government intervention? This depends of course on what informational and contractual advantages the government has vis-à-vis the private sector. The issues involved are quite subtle and require a lengthy discussion. To improve the readability of the paper, we simply assume in this section that the government can observe neither an individual's effort nor his aggregate insurance purchases, but it is able to monitor the total sales of insurance by each company.

We first determine conditions under which the optimum conditional on the unobservability of effort can be decentralized using linear<sup>16</sup> taxation of insurance. Then, for situations where the optimum cannot be so decentralized, we investigate the best that can be done.

##### 4.1 The optimum conditional on the unobservability of effort

Return to Figure 1. The point  $\theta$  is the point of maximum utility on the zero profit locus. Imagine that only the government provides insurance.

---

<sup>15</sup> Turn to Figure 5. If indifference curves are convex, E is always preferred to the origin. Hence, if insurance firms were to offer insurance at the price  $q^*$ , E would upset the origin as an equilibrium.

<sup>16</sup> There is nothing to be gained from the non-linear taxation of an insurance company's total sales of insurance. All this would do is determine a profit-maximizing sales volume for each firm.

It is easy to check that Figure 1 is unaltered, except that the zero profit locus becomes the government budget constraint or the economy's resource constraint. Thus,  $\theta$  is the point of maximum utility conditional on the unobservability of effort.<sup>17</sup> We shall refer to  $\theta$  simply as "the optimum".

Under our assumptions, the indifference curve passing through the origin is strictly steeper than the zero profit locus at the origin. This implies that the optimum is never at the origin. Furthermore, since  $\theta$  is on the resource constraint, which corresponds to the zero profit locus, the optimum can never coincide with the positive profit price equilibrium. And finally, except where the optimum entails zero effort, an empirically uninteresting case, the optimum does not coincide with the zero profit price equilibrium. Hence, conditional on the unobservability of effort, there are almost always potential welfare gains relative to the price equilibrium. Can any or all of these gains be realized from the linear taxation of insurance?

#### 4.2 Decentralization of the optimum

With the linear taxation of insurance firms' total sales, the consumer price of insurance is defined naturally as the ratio of the premium payable to the insurance company in the no-accident event to the net payout from the insurance company in the event of accident. The tax revenue is redistributed to individuals in lump-sum fashion (though we admit the possibility that part of it may be destroyed if doing so improves expected utility). Thus,

$$y_0 = w - q\alpha + l \qquad y_1 = w - d + \alpha + l, \qquad (12)$$

---

<sup>17</sup>A utility improvement over  $\theta$  may be possible through randomization of insurance premia and payouts. See Arnott and Stiglitz [1988b]. We ignore this possibility.

where  $l$  is the lump-sum transfer. With taxation, the natural space to work in is  $(\alpha+l, q\alpha-l)$ . Note that zero insurance purchased corresponds to the point  $(l, -l)$  in this space.

The major result of this section is

Proposition 2a: A set of sufficient conditions under which the optimum can be attained when firms provide price insurance and there is linear taxation of aggregate insurance sales is:

- i) Indifference curves be convex; and
- ii)  $(1-\hat{p})\hat{q}-\hat{p} > (1-\tilde{p})\tilde{q}-\tilde{p}$  on the  $\hat{l}$ -price-consumption line  $\forall \tilde{q} < \hat{q}$ , where  $\hat{p}$  is the probability of accident at the optimum,  $\hat{q}$  the consumer price of insurance at the optimum,  $\hat{l}$  the lump-sum subsidy from the government to each individual at the optimum, and the  $\hat{l}$ -price-consumption line the price consumption line with  $(\hat{l}, -\hat{l})$  as origin.

We shall first assume convexity of indifference curves and demonstrate that with convexity, condition ii) is both necessary and sufficient for decentralizability of the optimum with linear taxation. We shall then show why nonconvexity of indifference curves may prevent decentralization of the optimum via linear taxation.

#### 4.2.1 decentralization with convex indifference curves

To start, we set up an artificial planning problem and then investigate decentralization. We imagine that the planner is able to choose the price of insurance, but not the quantity, and redistributes any profits through a lump-sum subsidy to consumers,  $l$ . In this case, the government's problem is to

$$\max_{q, e, \alpha, \ell} EU = u(w - q\alpha + \ell)(1 - p(e)) + u(w - d + \alpha + \ell)p(e) - e$$

$$\text{s. t.} \quad \text{i) } q\alpha(1 - p(e)) - \alpha p(e) \geq \ell \quad (13)$$

$$\text{ii) } (e, \alpha) = \underset{e, \alpha}{\text{argmax}} (EU; q, \ell)$$

i) is the resource constraint; and ii) the constraints imposed by the individual's choosing how much effort to expend and how much insurance to purchase, taking both the price of insurance and the lump-sum payment as given. We have put an inequality constraint on i) since we permit the government to destroy resources.

Turn to Figure 8a. Draw in the line tangent to the optimum point  $\theta$  and extend it back until it intersects the 45°-line in the lower quadrant. The point of intersection,  $\theta'$ , is  $(\hat{\ell}, -\hat{\ell})$ , where  $\hat{\ell}$  is the optimal lump-sum transfer, and the slope of the line,  $\hat{q}$ , gives the optimal price of insurance. If the individual is given a lump-sum transfer  $\hat{\ell}$  and allowed to purchase as much insurance as he wants at the price  $\hat{q}$ , he will choose the optimum point  $\theta$ . Furthermore, since  $\theta$  is on the resource constraint, the government breaks even on the provision of insurance, i.e. the revenue from insurance premia just covers insurance payouts plus lump-sum transfers.

This result by itself is not of great practical interest since it is hard to envision a situation where the government is the sole provider of insurance and is restricted to price insurance. But it does suggest a decentralization mechanism for which the optimum may be achievable.

The mechanism is as follows: Let  $\hat{\alpha}$  denote insurance purchases at the optimum. The government taxes total insurance purchases and distributes the revenue so obtained as lump-sum transfers to consumers. If the optimum is decentralizable by this mechanism, then the tax rate must satisfy  $\hat{t}\hat{\alpha} = \hat{\ell}$

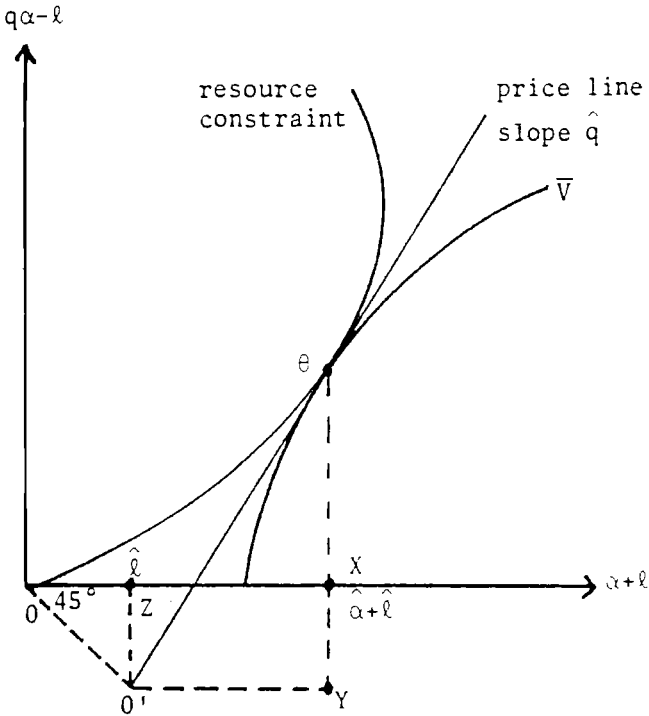


Figure 8a: Convex indifference curves.

$$\text{Note: } \hat{q} = \frac{\theta Y}{O'Y}, \hat{t} = \frac{\hat{\ell}}{\hat{\alpha}} = \frac{OZ}{O'Y} = \frac{O'Z}{O'Y} = \frac{XY}{O'Y}$$

$$\text{producer price} = \hat{q} - \hat{t} = \frac{\theta Y - XY}{O'Y} = \frac{\theta X}{O'Y} = \frac{\theta X}{ZX}$$

or  $\hat{t} = \frac{\hat{l}}{\hat{\alpha}}$ . How will firms respond? Competition between firms will result in their offering insurance at the lowest price consistent with making non-negative profits. With consumer price  $\hat{q}$ , tax rate  $\hat{t}$ , and lump-sum payment  $\hat{l}$ , the government is making zero profits at  $\theta$  by construction, and since the economy is on the resource constraint, firms too are making zero profits. But this is not enough to ensure decentralizability. It also needs to be demonstrated that firms will make negative profits at all lower prices of insurance. Now, a firm's profits per unit of insurance are

$$\pi = (1-p)q - p - \hat{t}. \quad (14)$$

If it lowers its consumer price from  $\hat{q}$  to say  $\tilde{q}$ , the individual will choose the point of maximum utility on the price line having slope  $\tilde{q}$  and origin  $(\hat{l}, -\hat{l})$ . We term the locus of points so generated, the  $\hat{l}$ -price-consumption line. Thus, condition ii) of Proposition 2a is the requirement that the firm make negative profits at all lower prices than  $\hat{q}$ .

Now,

$$\left. \left( \frac{d\pi}{dq} \right) \right|_{\hat{l}\text{-PC}} = \left. \left( (1-p) - (q+1)p' \frac{de}{dq} \right) \right|_{\hat{l}\text{-PC}}. \quad (15)$$

where  $\hat{l}$ -PC denotes "evaluated along the  $\hat{l}$ -price-consumption line". Ordinarily, one expects  $\frac{de}{dq} > 0$  - that a decrease in the price of insurance will discourage effort and increase the probability of accident - in which case  $\frac{d\pi}{dq} > 0$ . But a decrease in the price of insurance can, with decreasing absolute risk aversion, have such a strong income effect that individuals purchase less insurance; and this effect can be strong enough that the increase in the individual's effort induced by the purchase of less insurance causes the probability of accident to fall by so much that profits increase. In this case, a reduction in the price of insurance below  $\hat{q}$  puts the economy

outside the resource constraint, but the government's revenue losses resulting from the individual's reduced purchases of insurance are so large that insurance firms make a profit. This case strikes us as decidedly abnormal; hence, we describe the situation where condition ii) of Proposition 2 is satisfied as the normal case.

The decentralization mechanism described above requires government intervention, i.e. the market cannot mimic what the government does. Suppose otherwise and that equilibrium entails different firms offering different contracts, some with higher-priced insurance and a larger lump-sum subsidy to each client, others with lower-priced insurance and a smaller subsidy. Individuals would have an incentive to purchase an infinitesimal amount of insurance from each firm, to obtain the maximum possible subsidy, and to purchase the rest of their insurance from a firm offering insurance at the lower price. This is not an equilibrium, since the high-price, high-subsidy firms make a loss. Suppose instead that equilibrium entails all  $n$  firms (possibly one, constrained by potential entry) offering the policy  $\left(\hat{q}, \frac{\hat{z}}{n}\right)$ . An entering firm can upset this equilibrium by offering lower-priced insurance with no subsidy. Thus, when firms are constrained to selling clients as much insurance as they want at the quoted price, the unique equilibrium without government intervention is the price equilibrium. It should be evident that the one advantage the government has over the collectivity of firms is its monopoly on taxation, which in turn derives from its monopoly on legal coercion. In section 5, we shall examine more closely the rationale for government intervention in this context.

#### 4.2.2 nonconvexity of indifference curves

Nonconvexity of indifference curves provides another reason why the



optimum may not be decentralizable using the linear taxation of insurance. This is shown in Figure 8b; at the price of insurance corresponding to the optimum, the individual would choose to purchase insurance characterized by  $G$  rather than  $\theta$ . The optimum may, however, be decentralizable even when the indifference curve through  $\theta$  is nonconvex. What matters is whether  $\theta$  is on the  $\hat{l}$ -price-consumption line. Thus, Proposition 2a may be strengthened and simplified:

Proposition 2b: Necessary and sufficient conditions under which the optimum can be attained when firms provide price insurance and there is linear taxation of aggregate insurance sales are:

- i) The optimum lies on the  $\hat{l}$ -price-consumption line
- ii)  $(1-\hat{p})\hat{q}-\hat{p} > (1-\bar{p})\bar{q}-\bar{p}$  on the  $\hat{l}$ -price-consumption line  $\forall \bar{q} < \hat{q}$ .

#### 4.3 Ameliorative taxation when the optimum cannot be decentralized

The above discussion raises two related questions: If the optimum is not decentralizable through linear taxation when individual insurance purchases are unmonitorable, what is the best that can be done? And is taxation at some rate always better than no taxation at all?

##### 4.3.1 determination of the constrained optimum

We treat the former question first. To answer it, we construct a decentralizability locus; the best that can be done is then the point of maximum utility on the decentralizability locus. A representative point on the decentralizability locus is constructed as follows, as shown in Figure 8: Imagine the government paying a lump-sum subsidy  $\bar{l}$ . This moves the origin of the individual's budget line in  $(\alpha+l, \alpha-l)$  space to  $(\bar{l}, -\bar{l})$ . Draw in the  $\bar{l}$ -price-consumption line, and determine the lowest point (i.e., the point for which the price of insurance is lowest) on this price-consumption line

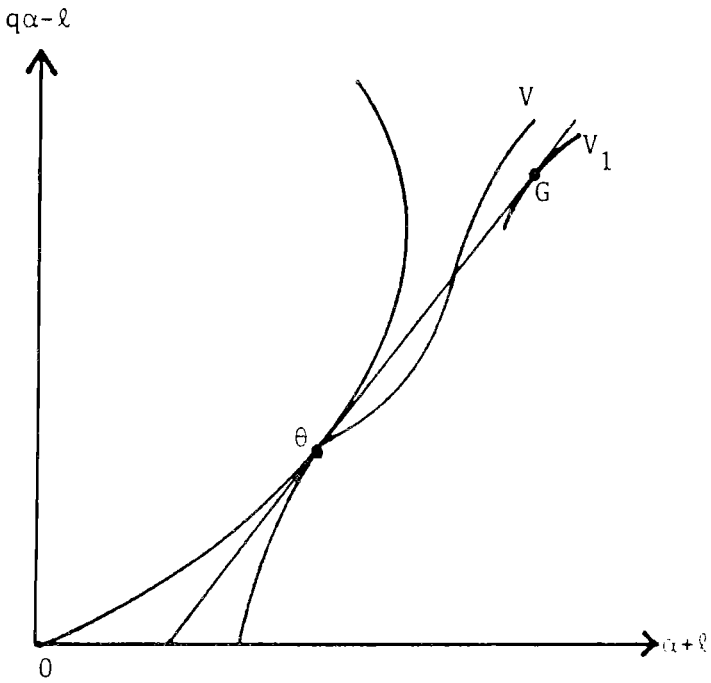


Figure 8b: Non-convex indifference curves.

$(\alpha^*(\bar{l}) + \bar{l}, q^*(\bar{l})\alpha^*(\bar{l}) - \bar{l})$  such that there exists a tax rate on insurance,  $\bar{t}$ , for which:

- i) the government at least balances its budget,  $\bar{t}\bar{\alpha} \geq \bar{l}$ ;
- ii) insurance firms at least break even,  $(1-\bar{p})\bar{q} - \bar{p} - \bar{t} \geq 0$ ; and
- iii) insurance firms cannot increase profits by lowering the price of insurance.

In Figure 9, M is the point on the decentralizability locus associated with  $\bar{l}$ , as long as condition iii), which does not have a neat geometric characterization, is satisfied. Repeating this procedure for every  $\bar{l}$  generates the decentralizability locus. The "constrained optimum" - the best that can be done with linear taxation of insurance and firms selling price insurance - is then the point of maximum utility on the decentralizability locus. Associated with this is the constrained optimal government policy  $(t^*, l^*)$ . Note that the constrained optimum may entail insurance firms making positive profits and/or the government destroying some tax revenue; in the former case, the constrained optimum is defined attaching zero welfare weight to such profits.

We summarize the above results in:

Proposition 3: The constrained optimum is the point of maximum utility on the decentralizability locus. If the optimum point,  $\theta$ , is on the decentralizability locus, then the optimum can be decentralized through linear taxation when firms offer price insurance; otherwise, it cannot be.

The information required to solve for the constrained optimum is substantial. Thus, it is of interest to investigate under what circumstances a welfare-improving tax can be found with less information. We first

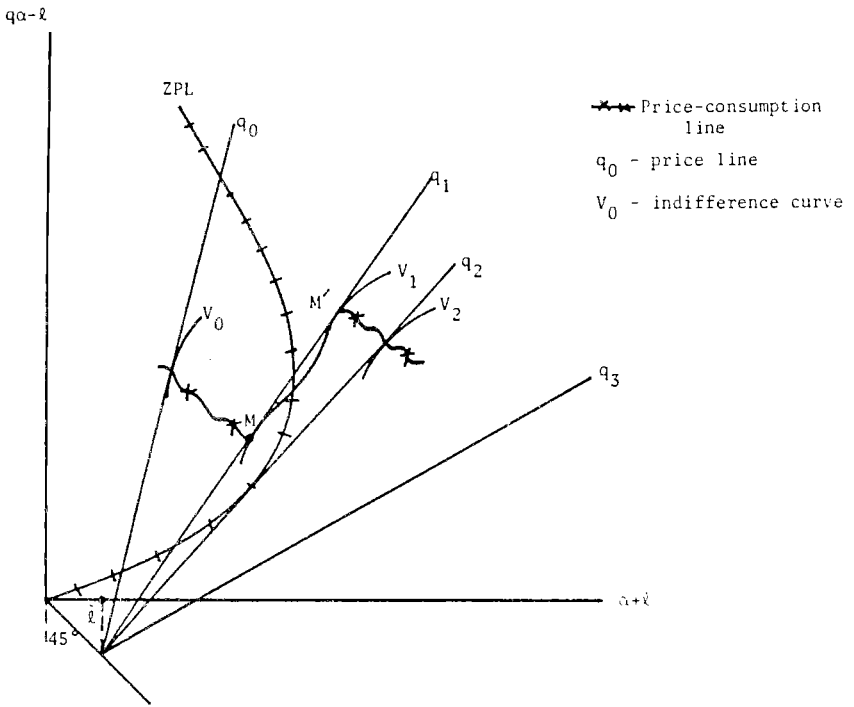


Figure 9: Construction of the decentralizability locus.

investigate when a small tax is desirable.

#### 4.3.2 when a small tax is welfare-improving

In the remainder of the section, we shall focus on the case where the price equilibrium is a zero profit price equilibrium, and assume that insurance firms cannot increase profits by lowering the price of insurance.

It turns out that the desirability of a small tax on insurance in this case depends on whether the zero effort line coincides with the full insurance line, and on the properties of the zero profit locus and indifference curves in the neighborhood of the zero profit price equilibrium.

The relevant properties of the zero profit locus and indifference curves can be inferred from their slopes. The slope of an indifference curve is given by (5), while the slope of the zero profit locus, from Arnott and Stiglitz [1988a], is

$$\frac{d\beta}{d\alpha}\Big|_{ZPL} = \begin{cases} \left[ p - \frac{(\alpha+\beta)u_1'(p')^3}{p''} \right] & \left[ (1-p) + \frac{(\alpha+\beta)u_0'(p')^3}{p''} \right] & \text{with } e > 0 \\ p(0)/(1-p(0)) & & \text{with } e = 0 \end{cases} \quad (16)$$

We noted earlier that the price equilibrium is not a zero profit price equilibrium if  $\lim_{e \downarrow 0} \frac{(p')^3}{p''} = -\infty$ . This leaves us with four relevant cases, all of which are possible,<sup>18</sup> depending on the properties of the probability-of-accident function:

<sup>18</sup>Case I:  $p(e) = \bar{p} - ke^{\frac{1}{2}}$ ; case II:  $p(e) = \bar{p} - ke^{\epsilon}$ ,  $1 < \epsilon < \frac{1}{2}$ ;

case III:  $p(e) = \bar{p} \exp(-ze)$ ; case IV:  $p(e) = \bar{p} - k_0e + k_1e^{\beta}$ ,  $\beta \in (1,2)$ .

$$-\lim_{e \rightarrow 0} \frac{(p')^3}{p''}$$

finite

&gt; 0                      0

 $-\lim_{e \rightarrow 0} p'$                       infinite

finite

I	II
III	IV

Let us start with case I, which is drawn in Figure 10a. From (4) when  $\lim_{e \rightarrow 0} p' = -\infty$ , the ZEL and the FIL coincide. From (16), it follows that there is a slope discontinuity in the zero profit locus at E, with a discontinuous increase in the slope from below to above E. And from (5), the indifference curves have a continuous slope as they cross the zero effort line. Since the indifference curve through E,  $V_0$ , is tangent to the ZPL "just above" E, it follows from the geometry of the problem that utility is higher at a point slightly below E on the ZPL, such as E', than at E. Furthermore, since E is the point of maximum utility on  $\beta = q \cdot \alpha$ , E' is the point of maximum utility on the line tangent to  $V_1$  at E' and is therefore decentralizable. Thus, a small tax unambiguously increases welfare. This case corresponds to the intuition given in the introduction. A small tax has a negative second-order effect on welfare through exposing the individual to more risk via less-than-full insurance, but a positive first-order effect on welfare through its stimulation of effort.

In case III, shown in Figure 10b, and in case IV, the full insurance line lies strictly outside the zero effort line. In these cases, a small tax on insurance generates a negative second-order welfare effect by disqualifying the event-contingent marginal utilities and a zero effect on effort, and is therefore unambiguously harmful.

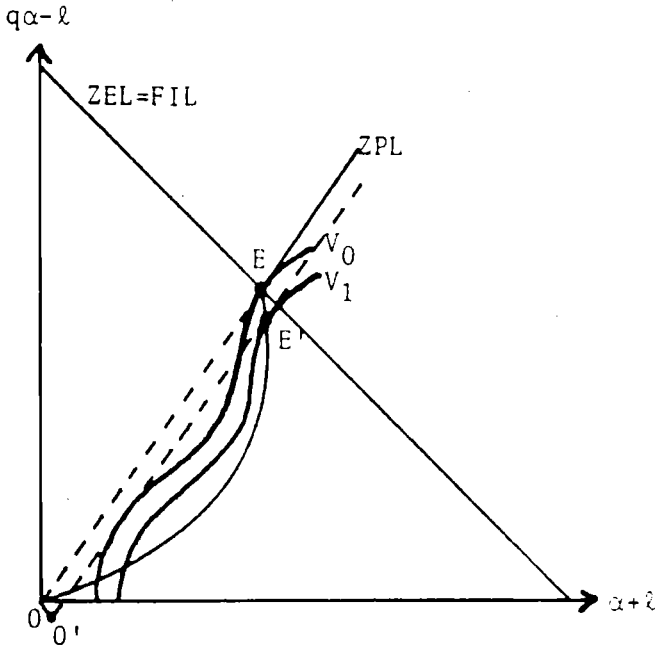


Figure 10a: In case I, a small tax unambiguously increases welfare.

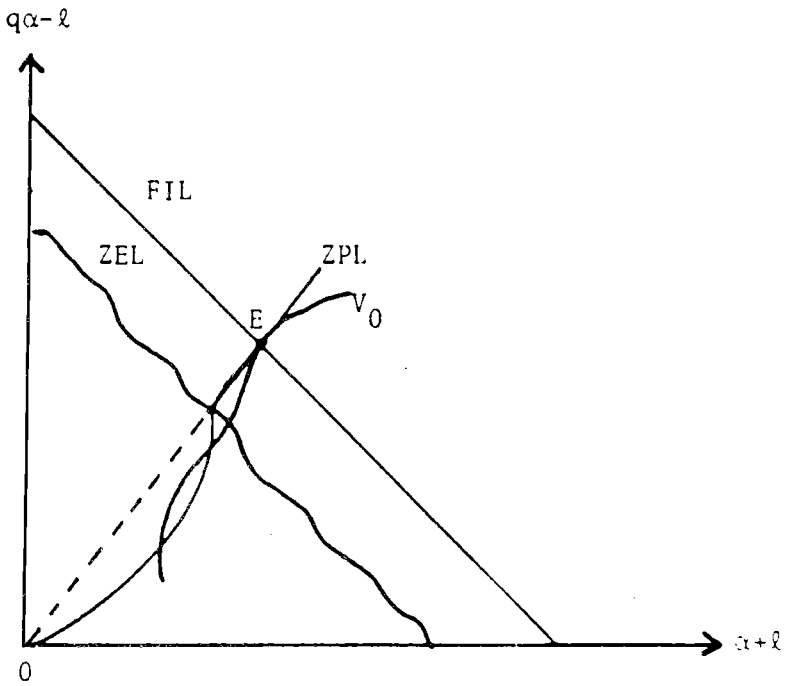


Figure 10b: In cases III and IV, a small tax unambiguously decreases welfare.



The reader can picture case II by considering Figure 10a, but without the slope discontinuity in the zero profit locus at E. A small tax on insurance is desirable if the indifference curve  $V_0$  has less curvature just below E than the ZPL,<sup>19</sup> and is undesirable otherwise. This is the case where both the risk and effort-stimulating welfare effects of a small tax are second-order.

We summarize the above results in:

Proposition 4: A small tax on insurance imposed at the zero profit price equilibrium has two opposing effects on welfare. The tax results in the individual receiving less-than-full insurance, which has a negative second-order welfare effect. The tax may also stimulate effort, which has a positive welfare effect. Which effect dominates depends on the properties of the probability-of-accident function as effort approaches zero, since this determines the order of magnitude of the effort-stimulating effect on welfare.

Employing simpler arguments, it may also be shown that a small tax on insurance at a positive profits price equilibrium is always welfare-improving, and almost never causes an inactive market (a zero insurance price equilibrium) to become active.

#### 4.3.3 when there is a large welfare-improving tax

A large welfare-improving tax exists if any part of the decentralizability locus lies on a lower indifference curve (corresponding to higher utility) than the price equilibrium. In this subsection, we briefly

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<sup>19</sup>From (16), the curvature of the ZPL in the positive effort region contains terms in  $p'''$ . Since there are no natural restrictions on the sign or magnitude of  $p'''$ , either the ZPL or  $V_0$  can have greater curvature.

investigate the circumstances under which this will be the case.

If the price equilibrium is a positive profit price equilibrium, there is always a large, welfare-improving tax.<sup>20</sup> This is shown in Figure 11.  $H$  is the positive profit price equilibrium point. The price line through  $H$  and  $H'$  provides an "upper cover" of the indifference curve  $V_0$ , which is supported by the points  $H$  and  $H'$ . By the continuity of indifference curves, it must almost always be the case that for some distance below  $V_0$ , the indifference curves have the characteristic that the lower point supporting the upper cover lies inside the feasibility set while the upper point lies outside it. This is illustrated by the points  $I$  and  $I'$  which lie on  $V_1$ . Furthermore, the point  $I$  is decentralizable with a positive tax on insurance. The tax rate must be positive since, otherwise, contrary to a property of the positive profit price equilibrium,  $H$  would not be the lowest point on the 0-price-consumption line inside the feasibility set.<sup>21</sup> Also, at  $I$ , an insurance firm has no incentive to lower its price since if it did, the equilibrium would switch from  $I$  to  $I'$ , the government would run a budget surplus (since  $\alpha_{I'} > \alpha_I$ ), while the economy would operate at a loss, implying that the firm would operate at a loss.

In the case of the zero profit price equilibrium which does not coincide with the optimum, there may not be a large, welfare-improving tax. An example where there is not is shown in Figure 12. The relevant characteristic of the indifference curves between  $V_0$  and  $V_1$  is that they are

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<sup>20</sup>Essentially the same argument establishes that a small tax is welfare-improving at a positive profit price equilibrium.

<sup>21</sup>Redraw Fig. 11 with a negative tax rate and lump-sum transfer. Then there is a point on  $V_1$  on the 0-price-consumption line inside the feasibility set.

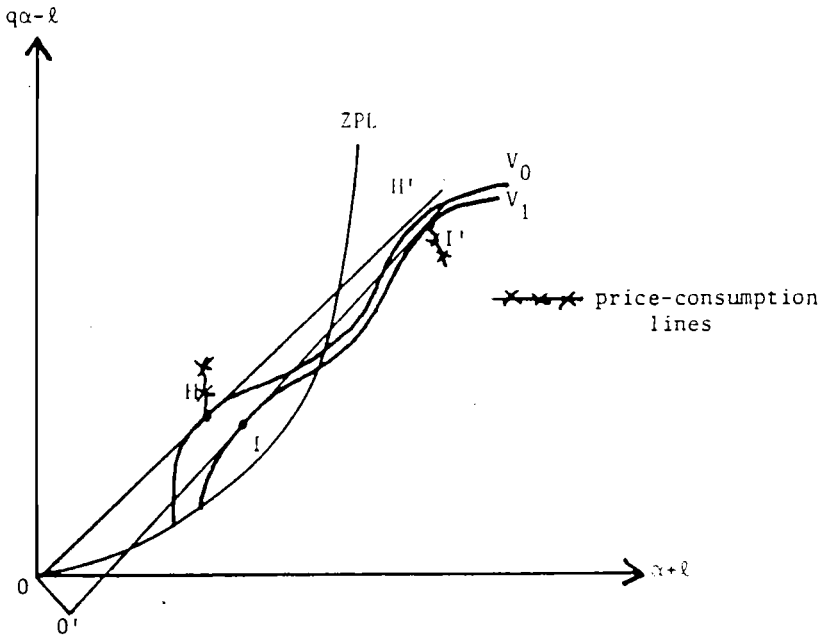


Figure 11: If the price equilibrium is a positive profit price equilibrium, there is always a large, welfare-improving tax.

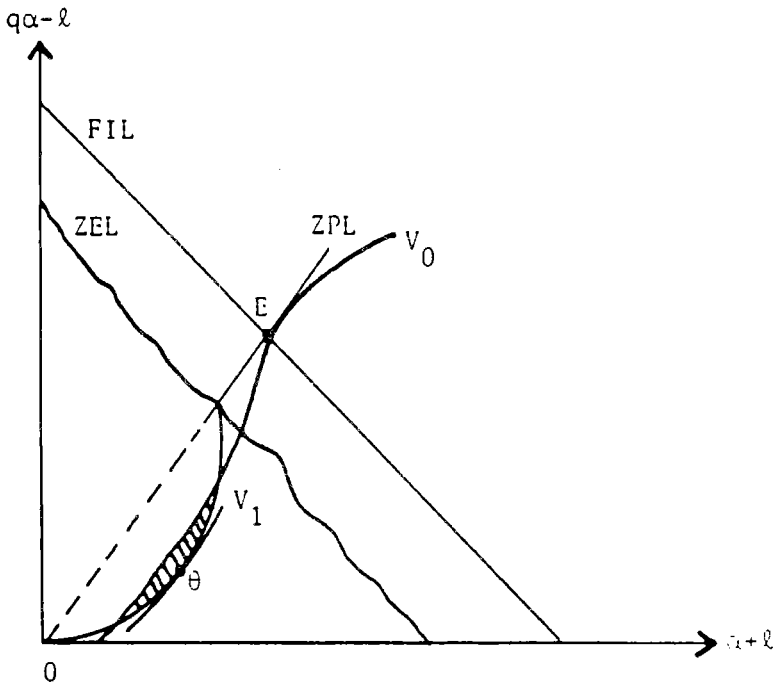


Figure 12: With a zero profit price equilibrium, there may not be a large, welfare-improving tax.

nonconvex inside the feasibility set. As a result, no feasible, utility-improving insurance allocations (the set of such allocations is shown by the hatched area in the Figure) can be decentralized with a linear price system. There may also be no large, welfare-improving tax with a zero insurance price equilibrium.

We conclude this subsection with

Proposition 5: Whether there is a large, welfare-improving tax depends on global, rather than local, properties of the utility and probability-of-accident functions. A sufficient condition for the non-existence of a welfare-improving tax is that indifference curves be non-convex in the region of feasible, utility-improving insurance allocations.

## 5. Information and Government Intervention

Whenever one encounters a model in which there is scope for welfare-improving government intervention, one should ask: What is it that the government can do that private agents cannot? Relatedly, has the model unduly restricted the actions of private agents or given the government unreasonably broad powers? And, finally, has the model been consistent in its treatment of information or has it artificially provided the government with informational advantages over the market?

### 5.1 Price insurance

We restricted firms to offering price insurance. Is this reasonable?

Elsewhere (Arnott and Stiglitz [1987]) we have shown that if an insurance firm can restrict or ration the total amount of insurance its clients purchase, it will generally do so. A crude intuition is that, since

moral hazard causes individuals to expend too little effort, it is beneficial to stimulate effort by restricting the amount of insurance they can purchase at any price. This intuition is incomplete, however, since it neglects that restricting the amount of insurance also exposes the individual to more risk. A more precise intuition is as follows: One may view insurance as a commodity, where  $\alpha$  is the quantity of insurance sold to a particular individual and  $p\alpha$  is the cost to the producer of providing the  $\alpha$  units. Unlike other commodities, the cost of production is a function of the price charged. So let us define marginal and average cost with price fixed. Then

$$MC(\alpha; q) = \frac{\partial(p(e(\alpha, \alpha q))\alpha)}{\partial\alpha} = p + p' \left( \frac{\partial e}{\partial\alpha} + q \frac{\partial e}{\partial\beta} \right)$$

$$AC(\alpha; q) = p.$$

Since  $\frac{\partial e}{\partial\alpha} < 0$ ,  $\frac{\partial e}{\partial\beta} < 0$ , and  $p' < 0$ , then  $MC(\alpha; q) > AC(\alpha; q)$ ; the marginal cost of providing insurance to a particular individual exceeds the average cost. This stands in contrast to regular commodities where the marginal and average costs of providing units of a commodity to a particular individual coincide. If the firm sets price equal to marginal cost, it makes a profit on the sale of insurance, which is typically inconsistent with competitive equilibrium. If the firm sets price equal to average cost, it breaks even but individuals purchase an excessive amount of insurance since price is less than marginal cost, which entails an efficiency loss. Efficiency and competitiveness can be simultaneously achieved if insurers employ non-linear pricing or if they offer a contract which provides the efficient amount of insurance and charges total cost for that amount; with zero profits, this entails rationing insurance purchases priced at average cost. Since the costs of providing insurance depend on the client's total purchases of insurance, these contract

forms are fully effective only if the insurance company can either enforce the requirement that its client purchase no additional insurance or make the insurance contract contingent on the client's supplementary insurance purchases.

Thus, the simplest potentially efficient contract form entails a "quantity" contract,  $(\alpha, \beta)$ , and a requirement that the client purchase no supplementary insurance. Elsewhere (Arnott and Stiglitz [1987]) we have termed this form of contract an "exclusive (quantity) contract" since the firm insists on being the client's exclusive seller, and have referred to the corresponding competitive equilibrium as an exclusive contract equilibrium. It is easy to prove, as the above argument suggests, that the exclusive contract equilibrium is at  $\theta$ , and is therefore constrained efficient conditional on the unobservability of effort.<sup>22</sup> It is also easy to prove that exclusive contracts, when they are enforceable, dominate price contracts.

Thus, price insurance is of interest only when insurers are unable

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<sup>22</sup>The literature contains apparently conflicting results on the efficiency of the exclusive contract equilibrium with moral hazard when there is more than one consumer good. While Prescott and Townsend [1984] claim that the equilibrium is constrained efficient, and Shavell [1979] seems to have demonstrated a similar result, Arnott and Stiglitz [1986, 1989] and Greenwald and Stiglitz [1986] have claimed otherwise. The differing results are not a consequence of errors in analysis, nor of differences in either the equilibrium or welfare concepts. Rather, they arise from differences in assumptions concerning what is observable. Prescott and Townsend assume that everything except effort and the underlying states of the world is observable; their insurance contracts are written contingent not only on clients' insurance purchases from other firms, but also on the level of consumption of all goods. Since this is formally equivalent to the firm offering a contract which specifies a client's total insurance purchases and consumption of all goods, Prescott and Townsend essentially assume complete exclusivity. Arnott and Stiglitz meanwhile, assume that insurance contracts cannot be made contingent on the individual's consumption of all goods, while Greenwald and Stiglitz consider both the Prescott-Townsend and Arnott-Stiglitz cases.

to enforce exclusivity. Two questions arise. First, if exclusivity is unenforceable, will insurance contracts be price contracts? Second, under what circumstances are insurers unable to enforce exclusivity?

We have considered the first question at length in a companion paper, Arnott and Stiglitz [1987]. Price contracts are at our extreme, where insurers have no control over the quantity of insurance their clients purchase; exclusive contracts are at the other extreme, where insurers have complete control. Persuasive modelling of the realistic middle ground, where exclusivity is not perfectly enforceable, but where firms attempt nonetheless to restrict their clients' aggregate insurance purchases has proved difficult: For example, Hellwig [1983b], Stiglitz [1983], and Arnott and Stiglitz [1987] analyse the situation where each firm sells a non-exclusive quantity contract. An equilibrium may exist where all incumbent firms offer the same large contract. The individual prefers to purchase one rather than an integer multiple of this contract. But if any supplementary utility-improving insurance contract is offered, he purchases that insurance plus an integer multiple of the "equilibrium" contracts, and as a result decreases effort discontinuously. If this renders all supplementary contracts unprofitable, the equilibrium contracts are protected against entry. In such an equilibrium, firms have some control over their clients' purchases even though exclusivity is unenforceable. There are other equilibria of this general type, but with different contract forms. All these equilibria are, however, delicate, complex, and informationally expensive. Thus, one can defend analysis of price equilibria on two grounds. First, examination of the price equilibrium is instructive because it is so much simpler than analyses of alternative equilibria when exclusivity is unenforceable.



Second, it may be that these alternative equilibria do not in fact occur because of transactions costs, broadly interpreted.

We now turn to the second question: When is exclusivity enforceable? It is if insurance companies can monitor their clients' insurance purchases from other firms. Actually, less information is required, since exclusivity can be enforced if insurance companies can monitor only their clients' payouts in the event of accident. Monitoring can be achieved by direct observation or by communication between insurance firms concerning their clients' purchases. Direct observation is costly, but, given the current information technology, the sharing of information by insurance firms can be achieved at quite modest cost. Will firms share such information? Hellwig [1983a] has argued that there will never be complete information sharing between firms concerning their sale of insurance, in the sense that if all firms but one exchange information on their clients' purchases, the remaining firm has an incentive not to communicate. The intuition behind this result is simple: Since individuals are rationed at the exclusive contract equilibrium, the renegade firm can sell a small, supplementary quantity contract at greater than the market price. With more insurance, individuals will decrease effort, which will render the "exclusive" contracts unprofitable. But if the renegade does not sell too much supplementary insurance to each individual, he can make a profit because he sells insurance at above the market price. There are no doubt other reasons why insurance firms are reluctant to share information.

In fact, insurance contracts seem to be characterized by imperfect exclusivity. Most insurance contracts attempt to restrict the aggregate quantity of insurance its clients purchase through exclusivity provisions;

insurance companies do attempt to monitor their clients' payouts from other insurers in the event of an accident;<sup>23</sup> and insurance firms do exchange information on their clients' purchases in some situations. But there are other factors at work which undermine insurers' attempts at exclusivity. In insurance markets, we have already noted that exclusivity provisions are costly to enforce, and that both clients and rival insurance firms have an incentive to circumvent such provisions. Furthermore, for most types of risk, the individual supplements market insurance with various kinds of non-market insurance, which are typically very imperfectly monitored by the insurance company. For example, if an individual gets sick, he receives benefits not only from his insurance company, but also from his employer (compensated sick leave), his family and friends, and the government (deductibility of uncovered medical expenses).<sup>24</sup> Also, where insurance is provided, but not in explicit insurance contracts, exclusivity provisions typically seem even more difficult to enforce. For instance, where an employer provides output-related insurance, he would like to restrict his employees' outside employment so that they work harder on the job, but in most situations cannot achieve this.

Taking all these considerations into account suggests that insurers'

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<sup>23</sup>In the paper we have focussed on insurance markets. Insurance is also provided in principal-agent relationships. Credit markets are a good example. A bank bases its willingness to lend on the borrower's current debt situation and may attempt to restrict future indebtedness during the loan period. Provisos ensuring seniority to existing debt holders do not suffice, as the recent experience with firms' recapitalization demonstrates. Exclusivity-like provisions are even more difficult to enforce in the context of LDC debt (Eaton, Gersovitz, and Stiglitz [1986], Kletzer [1984]).

<sup>24</sup>This point is developed in Arnott and Stiglitz [1990].

success in enforcing exclusivity should vary considerably from market to market. Furthermore, the modelling of imperfect exclusivity will no doubt prove both subtle and complex.

Thus, our defence of analysing price equilibrium is that it is a tractable and insightful extreme case, and may provide a good approximation to reality for some types of insurance.

## 5.2 Potential advantages of government intervention

We first consider whether the government is informationally-advantaged vis-à-vis the market, and then consider whether it is contractually-advantaged.

### 5.2.1 Informational advantages

The government, unlike the collectivity of firms, can require that all insurance policies be registered. This results in full information sharing which we have argued would not occur in the absence of government intervention. The advantage that the government has over the collectivity of firms in this context derives from its monopoly on legislation and the legal enforcement of contracts. It does not need to employ compulsion, since it can declare illegal or refuse to enforce unregistered policies.

The government may utilize the information acquired from registration in a variety of ways: it can nationalize the provision of insurance; it can regulate the market, by for example imposing a ceiling on each individual's aggregate insurance purchases; it can impose non-linear taxation, essentially forcing individuals to choose insurance packages from along the zero profit locus; or it can make information on each individual's insurance purchases available to insurance companies.

The registration of sales is particularly effective in the context

of insurance. Since insurance is intrinsically non-transferable, the problem of secondary markets does not arise, as it would for example if the government were to require the registration of cigarette sales in an attempt to ration each individual's cigarette consumption.

But there are presumably forms of insurance for which the registration of individual policies is excessively costly, or is deemed to constitute an invasion of privacy. Then the government has a more limited informational advantage over the market. It can employ its powers of legal compulsion to require that insurance firms report their aggregate sales of insurance. The government can then employ the linear taxation of insurance. This is the informational situation we have considered in the paper.

#### 5.2.2 contractual advantages

The government has one obvious contractual advantage over the market. Through the nationalization of insurance or the regulation of a private monopoly insurer, it can effect contractual forms that may not be sustainable in an unregulated market; in the context of the paper, either institution results in exclusive contracts which we have seen may not be sustainable in competitive equilibrium without government intervention. Here the advantage of the government over the market stems from its powers of proscription.

There are well-known problems with both nationalized industries and regulated monopolies. Thus, it is of interest to enquire whether the government can employ its unique powers to support contractual forms within the market that would be unsustainable without government intervention. This is precisely what the linear taxation of insurance achieves. The market, with the help of the government but not without it, is able to offer

insurance in the form of a two-part tariff, a subsidy  $\hat{t}$  for participating in the market, plus a unit price of insurance  $\hat{q}$ . This is not as efficient as having each individual face a non-linear price schedule, but is the best that can be done, given that each individual's aggregate insurance purchases are unobservable. Here the contractual advantages of the government over the market stem from its monopoly on the coercive power of taxation.<sup>25</sup>

### 5.2.3 alternative intuitions

Let us briefly review the intuitions given in the introduction concerning the desirability of taxing price insurance.

Two arguments were presented against the taxation of price insurance. The first was that a representative client contracting freely with his own representative private insurance company should negotiate an efficient contract, conditional on the information available and the contract form. As far as it goes, this argument is correct. It neglects, however, that government intervention, here in the form of taxation, can support forms of contract that are unsustainable without government intervention - the market along with the government can offer a two-part tariff, which the market alone cannot. The second was that it is not obvious what advantages the government has over the market; the above discussion indicates what these advantages are.

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<sup>25</sup>Taxation has another role to play when moral hazard is present. In Arnott and Stiglitz [1986] we argued that shadow prices deviate from market prices when moral hazard is present. One may regard moral hazard as a distortion, associated with which is a deadweight loss (relative to the corresponding economy with symmetric information). The consumption of any good alters the magnitude of this deadweight loss; for example, the consumption of a fire extinguisher likely reduces the deadweight loss associated with fire insurance. The second best is achieved by setting commodity taxes equal to the corresponding marginal deadweight loss.

Two arguments were presented in favor of the taxation of price insurance. The first argument was essentially correct, failing to recognize only that whether, in the neighborhood of the zero profit equilibrium, the effect of a small increase in the price of insurance on effort is first-order, second-order, or nil, depends on the properties of indifference curves and the zero profit locus near the zero profit equilibrium. The cruder argument that taxation is desirable because moral hazard results in deficient effort and taxation stimulates effort is seriously flawed. If correct, it would imply that the taxation of insurance at the exclusive contract equilibrium is beneficial, which we have shown to be false. The argument fails to recognize that while taxation stimulates effort, it also causes the individual to bear more risk. Rather, it is necessary to determine whether with moral hazard the market achieves the right balance, relative to the first best (with effort observable), between deficient effort and incomplete insurance.

#### 6. Concluding Comments

This paper provided an analysis of the positive and normative properties of price equilibrium in insurance markets with moral hazard. In a price equilibrium, insurance firms offer price contracts which allow clients to purchase as much insurance as they wish at the quoted price. We demonstrated that a price equilibrium always exists and is of one of three types:

- i) zero profit price equilibrium - zero profit, zero effort, full insurance
- ii) positive profit price equilibrium - positive profit, positive effort, partial insurance

- iii) zero insurance price equilibrium - zero insurance, zero profit, positive effort.

And we identified the circumstances under which each of the three obtains. The possibility of the zero insurance and positive profit price equilibria stems from the non-convexities (of indifference curves in the relevant space) to which moral hazard can give rise.

We showed that a price equilibrium is generally not Pareto efficient, conditional on the unobservability of effort, and enquired whether a utility improvement can be made by taxing insurance.

Elsewhere (Arnott and Stiglitz [1987]) we have argued that when an individual's aggregate insurance purchases are observable, the market provides "exclusive" insurance contracts which ration the amount of insurance that can be purchased at any price. Thus, price equilibrium is of interest only when an individual's aggregate insurance purchases are unobservable by market insurers. We assumed that in this case the government cannot observe an individual's aggregate insurance purchases either, and is therefore restricted to the linear taxation of insurance.

We found that there is scope for utility-improving linear taxation<sup>26</sup> of price insurance. In some cases, the optimum (conditional on the unobservability of effort) can be attained through the linear taxation of insurance; in others, a utility improvement can be achieved through taxation, even though the optimum cannot, and in yet other cases, taxation is ineffectual. We derived the conditions under which each of these results

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<sup>26</sup>Since adverse selection and moral hazard almost invariably occur together, before it is possible to estimate optimal tax rates in practical situations, it will be necessary to develop the welfare economics of moral hazard and adverse selection together.

obtains. These conditions are complex and relate to global rather than local properties of the probability-of-accident and utility functions.

We pointed out that the unobservability of an individual's aggregate insurance purchases does not imply that competitive equilibrium is a price equilibrium. Other contract forms are possible. However, these alternative contract forms are informationally expensive, and the equilibria corresponding to them are delicate. Furthermore, on most types of insurance contracts, insurers attempt to ration and monitor their clients' aggregate insurance purchases, which would appear to cast doubt on the empirical relevance of price contracts. However, for reasons we discussed, these attempts may not be very successful. Analysis of the intermediate cases between full and zero observability of individuals' aggregate insurance purchases is both subtle and complex. Thus, we defended our focus on price equilibrium on two grounds. First, equilibrium in some insurance markets may approximate a price equilibrium. Second, examination of extreme and simple cases is often instructive. For example, on the basis of our analysis, we strongly conjecture that the taxation of insurance is potentially desirable in most Real World insurance markets because of moral hazard.

We were careful in choosing our assumptions that the government have no exogenous informational advantage over the private sector. On what basis, then, is government intervention potentially utility improving in this context? The general answer is that the government has a monopoly on legal compulsion and proscription. More specifically, the government can require that firms reveal their total insurance sales, which they would not necessarily do voluntarily. This gives the government an endogenous informational advantage. Through its coercive powers of taxation, it can use



the information collected to apply a linear tax on insurance. In turn, the linear tax on insurance allows the government to support a contractual form (a two-part tariff) that cannot be sustained as a competitive equilibrium in the absence of government intervention.

Admittedly, our analysis has not addressed the question of whether the taxes imposed by the government would actually be welfare-improving. What we have established is that an ideal government may, through intervention, be able to improve the economy's performance. But actual governments are not ideal. To establish that the benefits of government intervention exceed the costs, it will be necessary, on the benefit side, to estimate the deadweight loss associated with the inefficiency we have identified, and on the cost side, to develop models of the public sector that capture the inefficiencies to which it is prone.

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