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ACTUAL AND WARRANTED RELATIONS BETWEEN ASSET PRICES

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ABSTRACT

Efficient markets models assert that the price of each asset is equal to the optimal forecast of its ex-post (or fundamental) value, but the models do not imply that the covariances between prices equal the corresponding covariances of ex-post values. We present bounds for covariances and correlations of prices based on the covariance of ex-post values, and show how such bounds can be tightened using information about forecasting variables.

The methods are used to examine the historical covariance between the U.S. and U.K. stock markets 1919-1989. The bounds on the covariance include the actual correlation.

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(1) INTRODUCTION

A variety of efficient markets models can be represented in the form $P_{it} = E_t P^*_{it}$ where P_{it} is the price of asset i at time t and P^*_{it} is its ex-post value, i. e., fundamental value.[1] In this paper, we inquire what such theory implies for the <u>covariance</u> and for the <u>correlation</u> between the prices of assets of the assets in terms of the covariance matrices of the ex-post values. Certainly, there is a common sense presumption that these covariances, correlations, as well as betas and factor loadings, depend on the covariances or correlations of ex-post values, but apparently the actual relations between these covariances or correlations (which are generally interval relations and not functional relations) have never been set forth in a general form before. These are important relations to set forth, since empirical finance is widely concerned with observed correlations among asset prices, and much work is based on the general notion that these have something to do with fundamentals. We will apply our theory to a study of the covariance and correlation of log price-dividend ratios between the United States and the United Kingdom.[2]

Knowing the relations among these covariances and correlations is important for a number of purposes. They would help us to understand

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[1]Typically, ex-post value is a present value of dividends per share at time t. It may have other interpretations as well; for example if p_t is a forward price ex-post value could be the subsequent spot price. Also, prices and ex-post values may be transformed as in the empirical work below.

[2]The transformation of price and of the present value referred to above is its log minus the log dividend. This is a nonlinear transformation, but the transformation can be justified in terms of an approximation to the present value model; see Campbell and Shiller [1988]. The transformation causes the variable P_{it} to be stationary through time.

whether international transmission of asset price movements can be understood in terms of present value models; that is our immediate objective here. Beyond that, they may help us to understand how fundamentals interact with investor information to determine betas or factor loadings of asset prices.

Many empirical studies of relations among financial prices refer either explicitly or implicitly to covariances among fundamentals to motivate the construction of the study or the interpretation of results. For example, Fama and French [1988] examine whether forecasting variables related to business conditions track common variation of expected returns on bonds and stocks for the U. S. 1927-1987. They ask "Are the relations [between returns] consistent with intuition, theory and existing evidence on the exposure of different assets to changes in business conditions?" and they conclude that the results are "comforting."[3] Chen, Roll and Ross [1986], in their study of economic forces in the U. S. stock market 1953-84, in their selection of macroeconomic factors to use for stock market returns appear to base their selection on the likely correlation of fundamentals with these. They write: "Our conclusion is that stock returns are exposed to systematic economic news, that they are priced in accordance with their exposures, and that the news can be measured as innovations in state variables whose identification can be accomplished through simple and intuitive financial theory."[4] Pindyck and Rotemberg [1988] [1990] estimate multiple indicator multiple cause (MIMIC) models of asset prices where the indicators are returns and the causes are economic variables related to fundamentals. They

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[3] Fama and French [1989], pages 3 and 4.

[4] Chen, Roll and Ross [1986], p. 402.

conclude from their study of U. S. stock prices 1969-1987 that "We show that comovements of individual stock prices cannot be justified by economic fundamentals."[5] King, Sentana Wadhwani [1990] estimate a factor model for 16 national stock markets 1970-1988 using 10 macroeconomic variables. They conclude that "The main empirical finding is that only a small proportion of the time-variation in the covariances between national stock markets can be accounted for by observable economic variables."[6]

The key problem in carrying out the objective of reconciling correlations of prices with correlations of fundamentals is that we do not observe the full information set available to market participants to forecast present values, and in the framework of efficient markets theory, we must assume that market participants might have superior information. This means we cannot observe the optimal forecast at time t of P_{it}^*, cannot observe directly its covariance with anything, and thus cannot calculate just what the covariance of prices should be.

We can only put bounds on the warranted covariances of prices from knowledge of variance matrices of the ex-post values. In section 2 we derive covariance bounds for the case when no forecasting information is available to the econometrician, while in section 3 we show that using more information is helpful both in deriving more efficient covariance bounds and in deriving bounds for the warranted correlation between the two assets. Section 4 contains a description of the pricing theories which we use for stocks, section 5 describes the data. In contrast to most of the aforementioned studies of correlations between assets, we use very long time

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[5]Pindyck and Rotemberg [1990], page i.

[6]King, Sentana and Wadhwani [1990], page i.

series data, extending from 1919 to the present, to enable us to see more of the low frequency variation in fundamentals that might explain correlations in prices, and in contrast to all of these studies we use direct measures of ex-post or fundamental value. Section 6 describes the econometric methodology and section 7 gives the results.

(2) THE CASE OF NO FORECASTING INFORMATION AVAILABLE TO ECONOMETRICIANS

Suppose that we, econometricians, observe only the covariance matrix of the vector $P_t^* = [P_{1t}^*, P_{2t}^*]'$, whose ith element is the present value of the dividends accruing to asset i. The corresponding vector of prices P_t has as its ith element the price of asset i. We will suppose that the present values and corresponding prices have been suitably transformed so that they are stationary, and so that variance matrices $\mathrm{var}(P^*)$ and $\mathrm{var}(P)$ exist.

How large can the covariance between P_{1t} and P_{2t} be, given $\mathrm{var}(P^*)$? To answer this, we must solve a nonlinear programming problem: maximize $\mathrm{cov}(P_{1t}, P_{2t})$ subject to the inequality restrictions implicit in the requirement that $\mathrm{var}(P)$ and $\mathrm{var}(P^*) - \mathrm{var}(P)$ are both positive semidefinite[7]. The positive-semidefinite requirements impose eight inequality restrictions on the elements of the two matrices, two of which are binding at the maximum: $\mathrm{cov}(P_1,P_2) <= \sigma(P_1)\sigma(P_2)$ and $\mathrm{cov}(P_1,P_2) <= \mathrm{cov}(P_1^*,P_2^*) + ((\sigma(P_1^*)^2 - \sigma(P_1)^2)(\sigma(P_2^*)^2 - \sigma(P_2)^2))^{.5}$. The maximized covariance (and the solution to the analogous minimization problem) gives us the following limits on the covariance between P_1 and P_2:

[7] Here, the term positive-semidefinite is taken to allow strictly positive definite matrices as well as singular ones.

$$(\mathrm{cov}(P_1^*,P_2^*)-\sigma(P_1^*)\sigma(P_2^*))/2 \le \mathrm{cov}(P_1,P_2) \le (\mathrm{cov}(P_1^*,P_2^*)+\sigma(P_1^*)\sigma(P_2^*))/2 \quad (1)$$

We shall refer to the range specified in this inequality as the range of warranted covariance between P_1 and P_2. Note that the warranted covariance between prices can exceed the covariance between the present values, even when this covariance is positive. As noted in Shiller [1989], this happens when there is "positive information pooling," when the forecast error $P_{1t}^* - P_{1t}$ is negatively correlated with the forecast error $P_{2t}^* - P_{2t}$. In this case, the variance of the forecast error $P_{1t}^* + P_{2t}^* - (P_{1t} + P_{2t})$ is less than the sum of the variances of the individual forecast errors. In this case, information is more about the aggregate $P_{1t}^* + P_{2t}^*$ than about the individual present values, i. e., the information about the present values is pooled.

Note, for example, from (1), that if P_{1t}^* and P_{2t}^* are highly positively correlated, then P_{1t} and P_{2t} can have both positive or negative covariance, but possible covariances include large positive covariances but only small negative covariances. For another example, note that if P_{1t}^* and P_{2t}^* are uncorrelated and have the same variance, that the covariance between P_{1t} and P_{2t} can range between minus half the variance to plus half the variance.

It was concluded in Shiller [1989] that, for a transformation of UK and US stock prices indexes 1918 to 1988 (where the transformation consists of dividing price by a long moving average of lagged dividends) $\mathrm{cov}(P_1,P_2)$ exceeded $\mathrm{cov}(P_1^*,P_2^*)$ with a constant discount rate used to compute present values, and there was no evidence of information pooling. It is not surprising, therefore, that the inequality (1) is strongly violated with that data too. However, when the discount rate is allowed to vary with the prime commercial paper rate, the bounds in (1) are no longer violated.[8]

That result, if valid, implies that with constant discount rates there is excess covariance between the U. K. and the U. S. stock prices. But, it does not tell us whether or not there is excess correlation between the two countries' stock prices. Covariance tells us the magnitude of their comovements, but does not tell us whether the two prices closely resemble each other.

In fact, if we have only var(P^*) to work with, lacking any components of the information set that the public uses to forecast, and if this matrix is not singular, then we cannot say anything at all about the warranted correlation between P_1 and P_2. As long as var(P_t^*) is nonsingular we can always write $P_t^* = u_t + v_t$ where u_t and v_t are random vectors uncorrelated with each other, and v_{1t} and v_{2t} are uncorrelated with each other, and all elements have nonzero variances. Suppose that information consists of $u_{1t} + u_{2t}$ + noise, v_{1t} and v_{2t}. As the variance of v_{1t} and v_{2t} are taken to zero, the correlation between prices approaches 1.00. As the variance of noise is increased toward infinity, the correlation approaches zero.

Now suppose that the second asset is the return on the market portfolio, that prices are scaled to 1.00 in the preceding period, that there are no dividends paid this period and that the variance matrix of P^* is conditional on information before this period. Then the conditional

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[8] Variance matrices var(P) and var(P^*) are given in Table 1 of Shiller [1989]. The covariance between P_{1t} and P_{2t} 1919 to 1987 (between the transformed UK and US prices) was reported as 39.73. With a constant discount rate assumption, the upper bound allowed by (1) using the estimated covariance matrices is 8.84. With a discount rate varying with the prime commercial paper rate, the upper bound allowed by (1) using the estimated covariance matrices is 45.18.

beta of the first asset is given by $\beta = \mathrm{cov}(P_1, P_2)/\mathrm{var}(P_2)$. We can always write $P_t^* = u_t + v_t$ where u_t and v_t are random vectors uncorrelated with each other, and as long as $\mathrm{var}(P^*)$ is strictly positive definite, we can take the $\mathrm{var}(u_1)/\mathrm{var}(u_2) = x$ for arbitrary positive x. Suppose the information set consists only of $u_1 + u_2$. Then the beta is x , which can be made anything from 0 to infinity for positive x. It can similarly be shown that beta can also range from 0 to minus infinity by taking the information vector to be $u_1 - u_2$. Thus, the variance matrix of fundamental values places no restrictions at all on beta. It is still possible to put bounds on the correlation between the two prices, or on the beta of an asset,, even without specifying the full information set used by market participants, so long as we know part of the information set used by the market. Using such a subset of public information also allows us to tighten our bounds on the covariance between P_{1t} and P_{2t}.

(3) THE CASE WHEN FORECASTING INFORMATION IS AVAILABLE TO ECONOMETRICIANS

If we know a subset of the information set available by market participants to forecast present values, and thereby observe the variance matrix of the forecast $P_t' = E(P_t^* | I_t)$ where I is the subset of information, then this will allow us to put tighter bounds on the warranted covariance between prices.

We can, following Campbell and Shiller [1988a,b], include the vector of actual prices in the subset of information, since surely the market knows market prices. Under the efficient market hypothesis, then P_t' should equal P_t, and so under the efficient markets hypothesis the covariance between

P_{1t}' and P_{2t}' should equal the covariance between P_{1t} and P_{2t}. A comparison of $\text{cov}(P_{1t},P_{2t})$ with an estimated $\text{cov}(P_{1t}',P_{2t}')$, which should (except for estimation error) be the same, is thus a valid way of testing the efficient markets model. The problem comes in interpreting violations of the efficient markets relation: we cannot take $\text{cov}(P_{1t},P_{2t})$ greater than $\text{cov}(P_{1t}',P_{2t}')$ as evidence of excess covariability. Suppose, for example, that prices are not set by $P_t = E_tP_t^*$ but by $P_t = E_tP_t^* + w_t$, where w_t is a "noise" vector whose variance matrix is diagonal (noise in one asset is independent of noise in the other asset) and which is independent of $E_tP_{it}^*$ (noise is independent of true fundamentals). If P_{it}' (i=1,2) is taken as the projection of P_{it}^* on P_{it}, then we will find that (by usual errors in variables results) the coefficient on price P_{it} is less than one so that the fitted value P_{it}' does not equal P_{it}. Thus, the efficient markets model is (correctly) found to be violated. However, it would be incorrect to infer that it is violated because of excessive covariance between P_{1t} and P_{2t}. The covariance between P_{1t}' and P_{2t}' will be less than the covariance of P_{1t} and P_{2t}, and yet clearly the covariance between P_{1t} and P_{2t} is quite right.

We want instead to put bounds on the covariance between P_{1t} and P_{2t} that are violated only when there is in fact excess covariance between the asset prices, and yet we still want to use information about $\text{var}(P')$. We can write $P_t = P_t' + v_t$, where the vector v_t is uncorrelated with P_t' since it represents an error unforecastable from the subset of information used to compute P_t'. To put an upper (lower) bound on $\text{cov}(P_1,P_2)$ we must solve the nonlinear programming problem to maximize (minimize) it in terms of the three elements of $\text{var}(P)$ subject to the inequality restrictions implicit in $\text{var}(P) - \text{var}(P')$ and $\text{var}(P^*) - \text{var}(P)$ both positive semidefinite, in other words, to maximize (minimize) in terms of the three elements of v such that var(v) and var(ϵ^*) - var(v) are positive semidefinite, where var(ϵ^*) = var(P^*) - var(P'). This is really essentially the same maximization problem that we discussed in the preceding section, and the bounds implied by the solution to this problem and by the solution to the corresponding minimization problem are:

$$\mathrm{cov}(P_1',P_2') + (\mathrm{cov}(\epsilon_1^*,\epsilon_2^*)-\sigma(\epsilon_1^*)\sigma(\epsilon_2^*))/2 \le \mathrm{cov}(P_1,P_2)$$
$$\le \mathrm{cov}(P_1',P_2') + (\mathrm{cov}(\epsilon_1^*,\epsilon_2^*)+\sigma(\epsilon_1^*)\sigma(\epsilon_2^*))/2 \quad (2)$$

This inequality can put much tighter bounds on the warranted covariance between P_{1t} and P_{2t}. Suppose, for example, var(P^*) is the identity matrix, so that by (1) cov(P_1,P_2) can range between -.5 and +.5. Suppose, however, that var(P') has all four of its elements equal to 0.5. Then the upper bound to cov(P_1,P_2) is .5, the lower bound is zero: the extra information reduced the range of warranted covariances by a half; moreover, in this case we know that the upper bound to the covariance between P_{1t} and P_{2t} is exactly equal to the covariance between P_{1t}' and P_{2t}'.

Knowing var(P') now enables us to put bounds on the <u>correlation</u> between P_{1t} and P_{2t}. Since v_t is uncorrelated with P_t' and we have:

$$\mathrm{corr}(P_1,P_2) = \frac{\mathrm{cov}(P_1',P_2') + \mathrm{cov}(v_1,v_2)}{((\sigma(P_1')^2+\sigma(v_1)^2)(\sigma(P_2')^2 + \sigma(v_2)^2)^{.5}} \quad (3)$$

We can put maximum and minimum values on this function with respect to var(v) subject to the restriction that var(v) and var(ϵ^*)-var(v) are both positive semidefinite[9]. This will give us bounds on the correlation between P_1 and P_2 that are analogous to the bounds (1) and (2) above. Plainly, so long as $\mathrm{cov}(P_1,P_2)$ is nonzero then this procedure will put some meaningful bounds on the correlation between P_1 and P_2. Since $P_t = P'_t + v_t$ where P'_t and v_t are uncorrelated, and since the variance matrix of v_t is limited by $\mathrm{var}(\epsilon)$, there is no way that perfect positive or perfect negative correlation between P_1 and P_2 can be achieved. By a similar argument, if the second asset is the market portfolio, we can place bounds on the beta between the two assets.

We will discuss below a present value model of stock prices that will allow us to compute $\mathrm{var}(P')$ and $\mathrm{var}(P^*)$ for a certain transformation of stock prices. We will then compute $\mathrm{cov}(P_1,P_2)$ and compare this with $\mathrm{cov}(P_1',P_2')$ as well as the bounds in (2), and compute $\mathrm{corr}(P_1,P_2)$ and compare this with $\mathrm{corr}(P_1',P_2')$ as well as the bounds implied by the maximization of (3).

(4) THE DATA

For the U. S., the annual stock price is the Standard and Poor Composite Stock Price Index for January of the year. The dividend is total dividends per share adjusted to index, four quarter total, fourth quarter of the year, backdated before 1926 using the dividend series in Cowles [1939]. The interest rate in the United States is the continuously compounded annual return on 4-6 month prime commercial paper computed from January and July commercial paper rates assuming a 6-month maturity. For the U. K. the

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[9] This will be done by means of numerical methods described in Section VI.

annual stock price is the Barclay de Zoete Wedd (BZW) stock price index for the end of the preceding year, and the dividend is the associated BZW dividend series for the year. The U. K. interest rate is the three-month prime bank bill rate, averaged over the year, as a continuously compounded return. These are the same series as used in Shiller and Beltratti [1990].

For both the U. S. and the U. K. we shall detrend stock prices in each year by using as P_t and P_t^* the log of the price and present value respectively divided by the dividend for the preceding year. This differs from Shiller [1989], where prices were detrended by dividing by a long moving average of dividends, and the resulting ratio was not logged. The dividing of nominal prices by nominal dividends serves to put the data in real terms: the variable P_t may be regarded also as the log of real price divided by real dividend, where the deflator used for both is the same.

(5) THE PRESENT VALUE RELATION

We shall use a log-linearized version of the present-value model, developed by Campbell and Shiller [1988a,b], so that variances and covariances of P^* can be estimated using linear time series methods even though the discount rate in the present value formula is allowed to vary through time. Otherwise, the present value relation would be essentially nonlinear. The model is:

$$P_{st} = E_t P_{st}^* \text{ where } \qquad P_{st}^* = \sum_{n=0}^{\infty} \rho_s^n G_{st+n} + k_s/(1-\rho_s) \qquad (4)$$

and where s = UK (United Kingdom), US (United States).

Here, P_{st} is the log price-dividend ratio for country s, G_{st} is defined as $\Delta d_{st} - i_{st}$, Δd_{st} is the change from the preceding period of log nominal dividends in country s, i_{st} is the nominal one-period interest rate in country s, and k_s and ρ_s are constants of linearization (see Campbell and Shiller [1988a]). For each country, ρ_s was taken to be $\exp(\bar{g}_s - \bar{R}_s)$, where $\bar{g}_s$ is the average rate of growth of dividends and $\bar{R}_s$ is the average return on stocks over the sample, and k_s does not affect our analysis when we calculate a time series for P^*_{st}. Expression (4) says that the log of the price divided by dividend (January log price minus the log total dividends over the preceding year) P_{st} is equal to the expectations at time t of future ex-post value P^*_{st}. Equation (4) is a sort of dynamic Gordon model replacing the original Gordon model, which was a steady-state growth path condition, with a present value relation.[10] The model (4) says simply that stock prices will be high relative to dividends when dividends are expected to grow more than average and/or short-term interest rates are expected to be low in the not-to-distant future, where not-to-distant is defined in terms of the discount parameter ρ_s. By this model the log price-dividend ratio will be stationary if the fundamentals are themselves stationary.

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[10] The Gordon model [1962] says that in a present value model with a steady state growth path for dividends and a constant discount rate the dividend-price ratio is the discount rate minus the growth rate of dividends. The original Gordon model does not apply if the growth rate of dividends or the discount rate is not constant through time.

(6) THE ECONOMETRIC METHODOLOGY

The bounds on the covariances and correlations which we can derive are based on the moments of the vector of ex-post values. We will use two different methods to compute these moments. The first one is the same method usually followed in the literature and proposed by Shiller [1981]; it is based on calculating a time series for P_{st}^* subject to a terminal condition which says that P_{sT}^* in the last year T of the sample is equal to the actual P_{sT} on that date:

$$P_{st}^* = \sum_{k=0}^{T-k-1} \rho_s^k G_{st+k} + \rho_s^{T-t} P_{sT}, \qquad s = US, UK. \tag{5}$$

From these time series one can then estimate the sample covariance matrix for $P_t^* = [P_{USt}^*, P_{UKt}^*]'$ to use in expressions (1)-(3).

The second method which we use does not involve computation of a time series for P_t^*. Defining $G_t = [G_{USt}, G_{UKt}]'$, and defining ρ as a 2x2 matrix with ρ_{US} and ρ_{UK} on the diagonal, then from (4) P_t^* $\Sigma(k{=}o,\infty)\rho^k G_{t+k}$, (plus a constant which we will disregard) and so $var(P_t^*)$ is given by:

$$var(P_t^*) = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \rho^j cov(G_{t+j}, G'_{t+k})\rho^k \tag{6}$$

Using (6) to estimate $var(P_t^*)$ of course involves estimating the complete autocovariance function for G_t at all leads and lags. Unfortunately, for a given sample of data, we cannot estimate all the infinite series of covariance matrices, and we have to truncate the estimated autocovariance function after a finite number of lags. According to Box and Jenkins [1974] one should not go beyond the covariance at lag n/4, where n is the sample size. We report results for lag n/4 as well as for lag 30. Note that in (6), future covariances are multiplied by the terms ρ_{US} and ρ_{UK}, which are less than one; this means that autocovariances at long lags are already given less weight because of the very definition of the vector P_t^*, so that truncation is not likely to affect the results much.

As to the bound on the covariance and the correlation which we have derived for the case when some forecasting information is available to econometricians, that is (2) and (3), one can see that the information contained in the perfect foresight price must be supplemented with information contained in the econometrician's estimate of the fundamental price of the assets. An econometric model is therefore necessary to this purpose.

Following previous work by Campbell and Shiller [1988a,b] we use vector autoregressions to test the models and to calculate the expectations of future fundamentals given an a priori specified information set. In the case of a VAR of order 1[11] we consider the following vector:

$$x_t = [P_{USt}, G_{USt-1}, P_{UKt}, G_{UKt-1}]' \tag{7}$$

where variables are demeaned. Note that the G_{st}, s = US, UK, are lagged in this vector, so that it contains only information known by the agents at the beginning of period t.

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[11]We consider in the text only the first-order VAR case, since higher-order VARs can be easily treated with the same methodology after putting them into a first order "companion form" VAR, as described in Campbell and Shiller [1988a,b].

We assume an autoregressive form for the vector x:

$$x_{t+1} = A\ x_t + a_t \tag{8}$$

where a_t is a white noise term with a covariance matrix which can have non-zero contemporaneous correlations. The model (4) implies:

$$P_{st} = P'_{st} \quad s = US,\ UK,\ \text{where},$$
$$P'_{USt} = e2'\ A\ (I - \rho_{US}\ A)^{-1}\ x_t \tag{9a}$$
$$P'_{UKt} = e4'\ A\ (I - \rho_{UK}\ A)^{-1}\ x_t \tag{9b}$$

where ei is a vector of zeros apart from the i-th element which is equal to 1. The expressions (9) in turn imply the following cross-equation restrictions on the estimated matrix A:

$$e1'(I - \rho_{US}A) = e2'\ A \tag{10a}$$
$$e3'(I - \rho_{UK}A) = e4'\ A \tag{10a}$$

We test these linear restrictions by means of Wald tests. Beyond testing the models, if we are willing to identify expectations with linear projections, we can use (9) to derive the expectations of future fundamentals under the hypothesis that the model is true (see Campbell and Shiller [1988a,b]), and then use these estimated values to compute what in the previous sections was defined with the variable $\bar{P}'$. Then we can use P' to compute the theoretical covariances and correlations between the two assets, the ones that should hold under the null hypothesis that there is no noise in market prices.

In order to consider the possibility of small sample bias we calculate empirical distributions for all the statistics which we report in the tables[12] by a Monte Carlo experiment which generates 2,000 series of the variables contained in the vector x subject to the restrictions that the models for the two assets are true. We report both numerical standard errors for the statistics, and the p-value corresponding to the empirical distribution.

We can also use our P′ to calculate the bounds in expressions (2) and (3). Again we can use two methods to compute the covariance bounds. One possibility is to compute P^* with a terminal value, compute P′ from the VAR, calculate $\varepsilon^* = P^* - P'$, as with expression (5). From these time series one can compute the variance matrices $var(P^*)$, $var(P')$, $var(\varepsilon^*)$. This guarantees that all matrices are positive semidefinite. The second possibility is to calculate $var(P^*)$ from the covariance matrix of G_t using (6), and then calculate $var(\varepsilon^*)$ as the difference between $var(P^*)$ and $var(P')$, where the last is computed from the time series of P′.

As to the correlations between the two assets, we generate upper and lower bounds by means of numerical methods. We use a Monte Carlo program that generates random positive definite matrices var(v), and which tests then if $var(\varepsilon^*)-var(v)$ is also positive semidefinite. If it passes the test, the program calculates the correlation coefficient using expression (3). After repeating the exercise 4,000 times we pick the highest and the lowest correlation. In particular, we make the diagonal elements of var(v)

[12] These statistics are the Wald tests, and the ratio between theoretical and actual covariances and correlations.

uniform from zero to corresponding diagonal elements of var(ϵ^*). In each iteration we compute from these diagonal elements $\sigma(v_1)\sigma(v_2)$, and make off diagonal elements of var(v) uniform from $-\sigma(v_1)\sigma(v_2)$ to $+\sigma(v_1)\sigma(v_2)$. So var(v) is positive semidefinite, and the diagonal elements of var(ϵ^*)-var(v) are nonnegative. We then only need to check that the determinant of var(ϵ^*)-var(v) is nonnegative in each iteration.

Both for the covariance bounds and the correlation bound we calculate standard errors by means of Monte Carlo simulations. In this case we generate 4,000 series of variables from the estimated VAR and we use them to calculate the standard errors of the bounds across iterations.

(7) RESULTS

Table 1, panel, A shows that the Wald tests usually reject the restrictions (10), and this is similar to previous results (Campbell and Shiller [1988], Beltratti [1989] and Shiller and Beltratti [1990]. We report both asymptotic p-values, and p-values from the empirical distribution function obtained from the restricted model. Note that the asymptotic standard errors sometimes overreject the model, though the differences are minimal even for large order VARs.

Table 1 Panel B shows that the correlation between the estimated warranted prices P' tends to be higher than the correlation between prices, but that the covariance between the estimated warranted prices tends to be lower than the covariance between the actual prices. This sort of difference between the results with covariances and with correlations has been noted before (see for example, Campbell and Shiller [1988b]); the difference reflects the estimated "excess volatility" of both markets, which drives up covariances but not correlations of actual prices relative to warranted values. Of course, these results take no account of the possibility that the market may have superior information from that used to make estimated P', and hence we turn to the covariance and correlation bounds.

Table 2 reports results for the covariance bound that can be derived when no information is available to the econometrician, that is the bounds given in expression (1). The actual covariance is within the bounds in all cases, but close to the upper bound. There is not much difference between the results obtained by estimating the covariance matrix of the time series of P^* or by estimating the autocovariance function of fundamentals when only (n/4) terms are included in the last. However, when 30 terms are considered the upper bound gets much closer to the actual value.

The same structure of results appears in Table 3, when the information set contained in the estimated VAR is used for the covariance bounds given by expression (2). Again, the actual covariance is usually within the bounds. Again a long estimated autocovariance function tends to lower the upper bound. Results from VARs of order 1, 2 and 3 are not very different from each other.

Finally, also the actual correlations shown in Table 4 are in general inside the bounds computed using expression (3). When only one lag is used in the vector autoregression, the estimated correlation bounds are extremely wide, allowing almost anything from no correlation to perfect positive correlation. The bounds are substantially tighter when more lags are introduced, reflecting the information available in the further lagged values.

Conclusion

We are unable to reject the hypothesis that the covariance and correlation between the U. S. and U. K. log price-dividend ratios is in accordance with the present value model. The bounds on covariances and correlations are quite wide and usually embrace the actual covariance and correlations. This does not rule that if a larger information set were used we might have been able to get narrower bounds on covariances and correlations, and might then have been able to reject the model. Note also that in this paper, in contrast with some results in Shiller [1989], time varying interest rates are used to discount in the present value formulae.

TABLE 1: Wald Tests and Comovement Measures

Panel A: results from VAR estimation
Tests of Restrictions Expressions (10)

Lags	1	2	3
Country: US			
Wald test:			
asymptotic p-value	0.009	0.014	0.015
p-value from e.d.f.	0.014	0.026	0.029
Country: UK			
Wald test:			
asymptotic p-value	0.000	0.000	0.000
p-value from e.d.f.	0.000	0.000	0.000

Panel B: Comovements Between Stock Markets

$\mathrm{Corr}(P_{US},P_{UK})$: 0.470 $\qquad$ $\mathrm{Cov}(P_{US},P_{UK})$: 0.033877

Warranted Comovements Estimated Using Expressions (9):

$\mathrm{Corr}(P'_{US},P'_{UK}) - \mathrm{Corr}(P_{US},P_{UK})$	0.113	0.393	0.417
numerical std. error	0.339	0.205	0.179
std. error from e.d.f.	0.184	0.224	0.241
$\mathrm{Cov}(P'_{US},P'_{UK}) - \mathrm{Cov}(P_{US},P_{UK})$	-0.029	-0.018	-0.011
numerical std. error	0.004	0.013	0.016
std. error from e.d.f.	0.023	0.027	0.028

Note: Sample period is 1919-1989.

TABLE 2: Covariance bounds from eqn (1) in the text

Actual $Cov(P_{US}, P_{UK}) = 0.033877$

	Lower Bound	Upper Bound
a. Var(P*) computed from time series of P*, using expression (5).		
	-0.007864	0.045279
	(0.012107)	(0.026580)
b. $var(P^*)$ is computed from the estimated autocovariance function of fundamentals up to 30 lags, using expression (6).		
	-0.009551	0.035559
	(0.012309)	(0.018608)
c. $Var(P^*)$ is computed from the estimated autocovariance function of fundamentals up to (n/4) lags, where n is the number of observations, using expression (6).		
	-0.005157	0.048412
	(0.012107)	(0.026580)

Note: The numbers in parentheses are standard errors obtained from a Monte Carlo simulation. Sample period is 1919-1989.

TABLE 3: Covariance bounds from eqn (2) in the text

Actual $Cov(P_{US}, P_{UK}) = 0.033877$

Order of the VAR	Lower bound	Upper bound
a. Var(P*) computed from time series of P*, using expression (5).		
1	-0.004374	0.048823
	(0.012458)	(0.025063)
2	0.003075	0.045601
	(0.015201)	(0.029568)
3	0.004996	0.050163
	(0.026828)	(0.042309)
b. $Var(P^*)$ is computed from the estimated autocovariance function of fundamentals up to 30 lags, using expression (6).		
1	-0.003321	0.033885
	(0.013186)	(0.018712)
2	0.003131	0.034206
	(0.014322)	(0.018150)
3	0.008383	0.033310
	(0.017621)	(0.019741)
c. $Var(P^*)$ is computed from the estimated autocovariance function of fundamentals up to (n/4) lags, where n is the number of observations, using expression (6).		
1	0.001062	0.046749
	(0.012458)	(0.025063)
2	0.007541	0.047042
	(0.014445)	(0.018146)
3	0.012774	0.046167
	(0.017732)	(0.019281)

Note: Each sub-panel reports results from VAR of order 1, 2 and 3. The number in parentheses are standard errors obtained from a Monte Carlo simulation. Sample period is 1919-1989.

TABLE 4: Correlation bounds from eqn (3) in the text

Actual corr(P_{US}, P_{UK}) = 0.470

a. Var(P*) computed from time series of P*, using expression (5).

Order of the VAR	Lower Bound	Upper Bound
1	-0.212	0.929
	(0.268)	(0.106)
2	0.167	0.933
	(0.298)	(0.094)
3	0.186	0.932
	(0.274)	(0.058)

b. Var(P^*) is computed from the estimated autocovariance function of fundamentals up to 30 lags, using expression (6).

1	0.135	0.929
	(0.185)	(0.098)
2	0.437	0.940
	(0.213)	(0.082)
3	0.415	0.919
	(0.256)	(0.061)

c. Var(P^*) is computed from the estimated autocovariance function of fundamentals up to (n/4) lags, where n is the number of observations, using expression (6).

1	-0.148	0.899
	(0.268)	(0.106)
2	0.183	0.932
	(0.279)	(0.093)
3	0.324	0.896
	(0.248)	(0.059)

Note: Each sub-panel reports results from VAR of order 1, 2 and 3. The number in parentheses are standard errors obtained from a Monte Carlo simulation. Sample period is 1919-1989.

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