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CURRENCY SUBSTITUTION AND THE FLUCTUATIONS OF FOREIGN-EXCHANGE RESERVES WITH CREDIBLY FIXED EXCHANGE RATES

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ABSTRACT

This paper studies the fluctuations of foreign exchange reserves under a regime of credibly fixed exchange rates. The paper considers a variety of assumptions on the determinants of money demand and currency substitution.

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1 Introduction

This paper studies the fluctuation of foreign-exchange reserves in a fixed-exchange-rate regime. A fixed-exchange-rate regime is sustained by two mechanisms: (i) coordination of monetary policies, to insure that central banks do not set the supply of national currencies on a path which is inconsistent with the stated parity, and (ii) a stock of foreign exchange reserves that absorbs temporary deviations of money (domestic credit) supply from the path that is consistent with the stated parity.

The incentives to deviate from a path of currency supply that is consistent with a given parity are typically determined by a government ability to "export the inflation tax," by forcing the neighbor government(s) to accomodate its own expansions of the monetary base. In this paper I assume away any such strategic behavior by monetary authorities, by letting national money supplies (strictly speaking, domestic credit) follow exogenous stochastic processes on which I impose a cointegration restriction. The cointegration restriction insures long-run convergence of money supplies and hence the viability of the fixed exchange rate. This strategy allows me to concentrate on the effects of money demand on the fluctuation of foreign exchange reserves, and the determination of the appropriate level of resources to maintain fixed exchange rates.

Why are foreign exchange reserves needed in a fixed-exchange-rate regime? Even if the monetary authorities' objective is to achieve convergence of money supplies, a stock of foreign exchange reserves is necessary to sustain a fixed-exchange-rate system mainly for two reasons. First, monetary authorities cannot perfectly control the supply of money, and therefore the above-mentioned convergence of money supplies is always subject to errors that need to be absorbed. Second, countries often pursue—at least temporarily—domestic credit policies that are not dictated by foreign concerns: foreign-exchange reserves thus become a buffer stock.[1] This paper, however, refrains from modeling these imperfections explicitly and follows the current literature on monetary models by simply assuming that the supply of domestic credit has a stochastic component that prevents full

[1] Unforeseable fluctuations in money demand are of course not a problem *per se*, since a policy of passively pegging the exchange rate would automatically accommodate them.

synchronization of individual countries' monetary policies.

Determining the equilibrium stocks of reserves requires solving a functional equation: the demand for foreign exchange reserves depends—through the expectations of the public on the store-of-value services supplied by different currencies—on beliefs about a government's ability to withstand future reserve fluctuations, which in turn are determined by the future demand for reserves. To my knowledge, this problem has not yet been solved within the family of stochastic models studied in this paper. Lucas (1982), discussing a model in this family, shows that the gross stock of foreign exchange reserves which allows to withstand *any* shocks in the foreign exchange market has to exceed, at any point in time, the stock of money of any one country in the fixed-rates system. This paper, by contrast, solves the functional equation directly.

With exogenous monetary injections, the dual of reserves fluctuations under fixed exchange rates is of course fluctuations in money demand. Existing models of money in stochastic, general equilibrium models—while subject to the well-known criticism about the role of currency in transactions—are particularly suited to the purposes of this study, since the services provided by money arise endogenously from explicit assumptions. Hence it is possible to determine the sensitivity of the solutions of the problem of reserves-management to various assumptions about the economy, and about the services supplied by different currencies.

This paper surveys three different models of money demand, and considers a number of different combinations of values of the parameters in these models. This strategy permits a rich characterization of the concept of currency substitutability. The most noticeable result of the analysis is that the nature of the substitutability of different currencies is crucially affected by the fixity of the exchange rate. In particular I show that, with a credibly fixed exchange rate, the substitutability of currencies as transactions media can be distinguished from the substitutability of currencies as stores of value.

The paper is organized as follows. Section 2 illustrates two versions of a model of a two-currency economy, where money is used because of a cash-in-advance constraint in the goods markets. This constraint can give rise to a sort of precautionary demand for money, depending on the timing of transactions: the timing conventions adopted here correspond to the models studied by Lucas (1982) and Svensson (1985), respectively. Section 3 discusses an alternative model of transactions services, where cash needs to be used only in the transactions involving national governments, but goods can be purchased with other securities, or with other goods (the purchase of goods with other goods can be implemented in these economy with a system of instantaneous bank credits that are traded in the goods markets). Section 4 reports some illustrative simulations of the models which use realistic stochastic processes for the forcing variables. These simulations yield statistics for the distribution of foreign exchange reserves in the two currencies and permit to determine the minimum size of the total portolio of foreign exchange reserves needed to maintain fixed exchange rates, without ever incurring into negative holdings of any currency. Section 5 contains some concluding remarks.

2 Cash-in-Advance Models

I consider first the well known cash-in-advance models developed by Lucas (1982) and Svensson (1985a). The models have very similar structures, and can be solved applying the same techniques, but have rather distinct implications on the nature of the demand for money. In Lucas's model agents would never hold money balances in excess of their planned purchases as long as nominal interest rates are positive. In Svensson's model, by contrast, money holdings can in equilibrium exceed planned purchases even with positive interest rates, because of the presence of a sort of precautionary motive in money demand.

The general structure of the two models is as follows. There are two goods being produced, which are purchased with two distinct currencies. The goods can be identified with the output of the two countries in the economy, even though by changing the degree of substitution of the two goods one changes also the degree of integration of the two countries, and with very high substitution the output of the two countries is nearly indistinguishable from the viewpoint of consumers. In this setup, the cash-in-advance constraint stands for market imperfections which require the use cash in transactions. As the economies get increasingly integrated (that is, the two goods become increasingly substitutable in consumption), however, the two currencies become themselves closer substitutes in their transactions services.

These models are useful to explore the role of increased substitution in transactions services, but are subject to criticism since in actual economies it appears that, over the relevant time interval, many goods purchases could possibly be paid for with credit rather than currency. Yet, I take the models as a benchmark, a necessary point of departure for more "realistic", and more complex, assumptions on the transactions services of different moneys in an integrated area. The model in section 3 will take this criticism seriously, and will consider an economy where the cash-in-advance constraint is imposed by law.

Goods production is from two undepreciable and unreproducible assets, whose shares are traded in the assets markets. The stochastic processes for the output of the two goods are:

$$y_t = \eta_t y_{t-1} \tag{1}$$

$$y_t^* = \psi_{2t} y_t \tag{2}$$

where y and y^* are the domesticand foreign good, respectively, and the stochastic variables $\eta > 0$ and $\psi_2 > 0$ follow first-order Markov processes.

Money is introduced into the economy via lump-sum transfers. The domestic and foreign money supplies follow different process in the two models, reflecting the timing of transactions. In the Lucas model, after the realizations of real shocks and monetary shocks are revealed, transactions in financial markets take place, and agents receive transfers of domestic and foreign money. Hence the process for the domestic money supply is:

$$\bar{M}_t = \omega_t \bar{M}_{t-1}, \tag{3}$$

while the foreign money supply follows the process:

$$\bar{M}_t^* = \psi_{1t} \bar{M}_t \tag{4}$$

with $\omega > 0$ and $\psi_1 > 0$ first-order Markov processes.

In the Svensson model, by contrast, transactions in goods markets occur prior to transactions in asset markets: new money injections at time t can be used to buy goods only at $t+1$. Hence the stochastic processes of domestic and foreign money supplies are:

$$\bar{M}_t = \omega_{t-1}\bar{M}_{t-1} \tag{5}$$
$$\bar{M}_t^* = \psi_{1t-1}\bar{M}_t \tag{6}$$

Where ω and ψ are stochastic variables which also follow first-order Markov processes.

I assume a perfectly pooled equilibrium, which simplifies the analysis considerably, is justified by the focus on money demand of this paper, and corresponds to a situation where markets or institutions—perhaps multinational firms—are sophisticated enough to allow the efficient diversification of idiosyncratic risks. Perfectly pooled equilibria in general require that consumers in different countries be identical in all respects: this assumption does not appear to play a crucial role for the problems I am studying in this paper, and, if anything, helps to clarify the main results. Under this assumption, and to further simplify the notation, I consider a representative resident of the two countries. In both models, the consumer maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{[U(c_t, c_t^*)]^{1-\theta}}{1-\theta} \tag{7}$$

Where c_t and c_t^* is consumption of the domestic and foreign good, respectively, and the function U is a constant-elasticity-of-substitution function, with elasticity parameter $\sigma = 1/\rho$.

As mentioned above, at the beginning of each period the consumer in the Lucas model receives a money transfer in the two currencies, and dividend payments on her stock holdings, and uses them, together with the holdings of domestic and foreign currency, to purchase currency and stocks in the assets market. The currency purchased is then used to buy goods in the goods markets. The proceeds from the sale of the goods is paid as dividends to the firms' stockholders when the assets market opens the following period. The budget constrain in the Lucas model is thus:

$$\pi_t M_t^d + \pi_t^* M_t^{*d} + q_t z_t + q_t^* z_t^*$$

$$
\begin{aligned}
&\leq (q_t + \frac{\pi_t}{\pi_{t-1}} y_{t-1}) z_{t-1} + (q_t^* + \frac{\pi_t^*}{\pi_{t-1}^*} p_{t-1} y_{t-1}^*) z_{t-1}^* \\
&+ \pi_t (M_{t-1}^d - \frac{c_{t-1}}{\pi_{t-1}}) + \pi_t^* (M_{t-1}^{*d} - \frac{p_{t-1} c_{t-1}^*}{\pi_{t-1}^*}) \\
&+ \pi_t (\omega_t - 1) \bar{M}_{t-1} + \pi_t^* (\frac{\psi_{1t}}{\psi_{1t-1}} \omega_t - 1) \bar{M}_{t-1}^* \qquad (8)
\end{aligned}
$$

where p denotes the relative price of the foreign good in terms of the home good, π is the purchasing power of domestic money in terms of the home good, and π^* is the purchasing power of the foreign money, also expressed in terms of the home good. q and q^* are the real prices of stocks, while y and y^* are real dividends. I have set the exchange rate equal to unity to economize in notation. The level of the exchange does not affect change any of the theoretical results in this paper, even though it determines the distribution of foreign exchange reserves, by pinning down the relative value of national money supplies.

In the Svensson model the consumer starts each period with predetermined money holdings, carries out goods purchases, and moves on to the financial markets to receive dividends and acquire money balances for the following period. The budget constraint reflects the different timing of transactions:

$$
\begin{aligned}
&\pi_t M_{t+1}^d + \pi_t^* M_{t+1}^{*d} + q_t z_{t+1} + q_t^* z_{t+1}^* \\
&\leq (q_t + y_t) z_t + (q_t^* + p_t y_t^*) z_t^* \\
&+ \pi_t (M_t^d - \frac{c_t}{\pi_t}) + \pi_t^* (M_t^{*d} - \frac{p_t c_t^*}{\pi_t^*}) \\
&+ \pi_t (\omega_t - 1) \bar{M}_t + \pi_t^* (\frac{\psi_{1t}}{\psi_{1t-1}} \omega_t - 1) \bar{M}_t^* \qquad (9)
\end{aligned}
$$

For both models, the cash-in-advance constraints are:

$$
\begin{aligned}
c_t &\leq \pi_t M_t^d \qquad (10) \\
p_t c_t^* &\leq \pi_t^* M_t^{*d} \qquad (11)
\end{aligned}
$$

The two maximization problems are identical to those studied by Lucas (1982) and Svensson (1985b) except that I adopt the notational shortcut of assuming a single representative individual, rather than working with a representative domestic and foreign residents who end up holding identical shares of all assets, and each consuming half of the endowments of the two goods.

Equilibrium is characterized by a set of first-order necessary conditions—obtained by maximizing (7) over consumption and money demand, subject to the relevant constraints—and a set of market equilibrium conditions. The first order conditions are, for the Lucas model:

$$\beta^t U_t^{-\theta} U_{1t} = E_t\left(\frac{\lambda_{t+1}}{\pi_t}\pi_{t+1}\right) + \mu_t \tag{12}$$

$$\beta^t U_t^{-\theta} U_{2t} = p_t E_t\left(\frac{\lambda_{t+1}}{\pi_t^*}\pi_{t+1}^*\right) + p_t \mu_t^* \tag{13}$$

$$\lambda_t q_t = E_t \lambda_{t+1}\left(q_{t+1} + y_t \frac{\pi_{t+1}}{\pi_t}\right) \tag{14}$$

$$\lambda_t q_t^* = E_t \lambda_{t+1}\left(q_{t+1}^* + p_t y_t^* \frac{\pi_{t+1}^*}{\pi_t^*}\right) \tag{15}$$

$$\lambda_t \pi_t = E_t(\lambda_{t+1}\pi_{t+1}) + \mu_t \pi_t \tag{16}$$

$$\lambda_t \pi_t^* = E_t(\lambda_{t+1}\pi_{t+1}^*) + \mu_t^* \pi_t^* \tag{17}$$

where λ_t are the multipliers associated with the constraint (8) and μ_t and μ_t^* are the multipliers associated with the constraints (10) and (11), respectively. U_t is an abbreviation for $U(c_t, c_t^*)$, and $U_{it}, i = 1,2$ denote the partial derivatives of $U(c_t, c_t^*)$ with respect to its first and second argument, respectively. In the Svensson model the first-order conditions are:

$$\beta^t U_t^{-\theta} U_{1t} = \lambda_t + \mu_t \tag{18}$$

$$\beta^t U_t^{-\theta} U_{2t} = (\lambda_t + \mu_t^*) p_t \tag{19}$$

$$\lambda_t q_t = E_t[\lambda_{t+1}(q_{t+1} + y_{t+1})] \tag{20}$$

$$\lambda_t q_t^* = E_t[\lambda_{t+1}(q_{t+1}^* + p_{t+1} y_{t+1}^*)] \tag{21}$$

$$\lambda_t \pi_t = E_t[(\lambda_{t+1} + \mu_{t+1})\pi_{t+1}] \tag{22}$$

$$\lambda_t \pi_t^* = E_t[(\lambda_{t+1} + \mu_{t+1}^*)\pi_{t+1}^*] \tag{23}$$

The market equilibrium conditions in the goods and stock markets are:

$$c_t = y_t \tag{24}$$

$$c_t^* = y_t^* \tag{25}$$

$$z_t = z_t^* = 1 \quad (26)$$

The third agent in the economy is the Exchange-Rate-Stabilizing Authority (ERSA), which holds a portfolio of domestic and foreign currency, and manages it to maintain, at the given nominal exchange rate, private demand for each currency equal to supply. This institution is of course not the only arrangement which allows to peg the exchange rate. The alternatives—surveyed by Persson (1985)—include:

- an institution that functions as an intermediary between central banks, charging interest on its loans and paying interest on its liabilities. This institution would borrow (sell bonds) from the central bank whose currency is in excess demand, sell the amount borrowed in the foreign exchange market, and lend the proceeds in the other currency.
- the use of a common reserve asset, like gold or a third currency;
- the adoption of a one-sided peg (Helpman (1981)) whereby one of the two countries hold reserves denominated in the currency of the other country, which are used in the foreign exchange market operations.

These alternative arrangements could have different distributional effects whenever injections of domestic credit are distributed selectively to the residents of the two countries.[2] These distributional effects are automatically ruled out here, where a perfectly pooled equilibrium is assumed from the start.

The equilibrium conditions in the money market are thus:

$$\bar{M}_t = M_t^d + R_t \quad (27)$$
$$\bar{M}_t^* = M_t^{*d} + R_t^* \quad (28)$$

The total holdings of the ERSA are:

$$R_t^w = R_t + R_t^* \quad (29)$$

[2] These effects can justify strategic behavior by central banks, if their objectives were to maximize domestic residents' welfare. These complications are left for future extensions of the present study.

Substituting the equilibrium conditions (24)-(28) into the budget constraint, we have:

$$R_t + R_t^* = R_{t-1} + R_{t-1}^* \tag{30}$$

That is the nominal stock of reserves of the ERSA is constant. Since however the world stock of money is allowed to grow over time, I assume that a fraction r of the new injections of $\bar{M}$ is paid to the ERSA, so that, at every time t:

$$R_t = r\bar{M}_t$$

This assumption requires to net out the amount $r(\omega_t - 1)\bar{M}_{t-1}$ from the transfer of domestic currency in equation (8) and $r(\omega_t - 1)\bar{M}_t$ in equation (9).

2.1 The Equilibrium Composition of Foreign Exchange Reserves

This section shows the equilibrium determination of nominal variables, π, π^*, R, and R^*, which can be solved for independently of stock prices q and q^*. The solution algorithm applied is an extension of the one developed by Giovannini and Labadie (1989).

Starting from the Lucas model, notice that the law of one price implies that the prices of either good in terms of either currency have to be the same.[3] Hence:

$$\pi_t = \pi_t^* \tag{31}$$

Equations (31), (12), (13) and (16) imply:

$$\begin{aligned} \beta^t U_t^{-\theta} U_{1t} &= \lambda_t \\ p_t &= \frac{U_{2t}}{U_{1t}} \end{aligned} \tag{32}$$

Furthermore:

$$\mu_t \pi_t + E_t(\lambda_{t+1}\pi_{t+1}) = \mu_t^* \pi_t^* + E_t(\lambda_{t+1}\pi_{t+1}^*) \tag{33}$$

[3] In this model moneys are tradede before goods and therefore any deviation from the law of one price would give rise to arbitrage profit opportunities.

Since in equilibrium the purchasing power of the currencies is the same, so are their store-of-value services, and so must be their transactions services:

$$\mu_t = \mu_t^* \tag{34}$$

The nominal interest rates in the two currencies are equal to the net opportunity cost of a 1-period bond, that is the transactions services of a currency relative to its store-of-value services.[4] Since both the store-of-value and transactions services of the two currencies are the same, nominal interest rates in the two currencies are equal. The equality of nominal interest rates holds because the expected change in the exchange rate is zero, that is $\pi_t = \pi_t^*$, $\forall t$.

In the Svensson model, whenever $p_t \neq \frac{U_{2t}}{U_{1t}}$ changes in incipient demand for the two goods would affect the nominal price levels (that is the purchasing power of the two moneys). In equilibrium, then, equation (32) has to hold. Substituting it into the system (18)–(23) it is immediate to obtain equations (31) and (34), and the equality of nominal interest rates.[5]

Both models can be solved using a similar procedure, which is illustrated in detail in the case of the Lucas model. As it will be apparent nominal variables can be solved independently of the stock price, whose discussion is left out, given the focus of the analysis in this paper. Let K^w stand for the ratio of the total stock of money balances in the hands of the public relative to total consumption (the inverse of world velocity). π and π^* are thus:

$$\pi_t = \pi_t^* = \frac{(y_t + p_t y_t^*)K_t^w}{(\bar{M}_t + \bar{M}_t^* - R_t^w)}, \tag{35}$$

where $\bar{M}_t + \bar{M}_t^* - R_t^w$ is the stock of currency in circulation. Similar

[4]In equilibrium:

$$1 + i_t = \frac{\lambda_t \pi_t}{E_t(\lambda_{t+1}\pi_{t+1})},$$

hence

$$i_t = \frac{\mu_t \pi_t}{E_t(\lambda_{t+1}\pi_{t+1})}$$

[5]In the Svensson model, the nominal interest rate is

$$i_t = \frac{E_t(\mu_{t+1}\pi_{t+1})}{E_t(\lambda_{t+1}\pi_{t+1})}.$$

functions are defined for each country:

$$\pi_t = \frac{y_t K_t}{\bar{M}_t - R_t} \tag{36}$$

$$\pi_t^* = \frac{p_t y_t^* K_t^*}{\bar{M}_t^* - R_t^*} \tag{37}$$

Substituting (35) into (16) we have:

$$K_t^w \mu_t = \lambda_t [K_t^w - \kappa_t] \tag{38}$$

where:

$$\kappa_t = E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{y_{t+1} + p_{t+1} y_{t+1}^*}{y_t + p_t y_t^*} \frac{\bar{M}_t + \bar{M}_t^* - R_t^w}{\bar{M}_{t+1} + \bar{M}_{t+1}^* - R_{t+1}^w} K_{t+1}^w \right)$$

or:

$$\kappa_t = E_t[\xi_{t+1} K_{t+1}^w]$$

Note that ξ is only a function of the forcing variables, both directly, and through the definition of λ and p:

$$\xi_{t+1} = \beta \frac{\eta_{t+1}^{1-\theta}}{\omega_{t+1}} \frac{(1 + \psi_{2t+1}^{1-\rho})^{\frac{1-\theta}{1-\rho}}}{(1 + \psi_{2t}^{1-\rho})^{\frac{1-\theta}{1-\rho}}} \frac{(1 + \psi_{1t} - r)}{(1 + \psi_{1t+1} - r)} \tag{39}$$

When the liquidity constrain is binding, $K^w = 1, \mu = \mu^* > 0$, and $K = K^* = 1$. Then:

$$\mu_t = \mu_t^* = \lambda_t [1 - \kappa_t] \tag{40}$$

When the liquidity constraint is not binding $\mu = \mu^* = 0$, and

$$K_t^w = \kappa_t \tag{41}$$

Thus the function K^w is determined by the solution of the following equation:

$$K_t^w = \max[1, \kappa_t] \tag{42}$$

To determine whether the functional equation (42) has a unique solution, define the operator S by:

$$S K_t^w = \kappa_t = E_t[\xi_{t+1} K_{t+1}^w] \tag{43}$$

Then define another operator T as follows:

$$TK_t^w = \max(1, K_t^w) \tag{44}$$

Let H be the composite operator $T \cdot S$:

$$T \cdot S = \max[1, E_t(\xi_{t+1} K_{t+1}^w)]$$

There exist a unique solution to (42) and repeated application of H achieves it if, and only if,

$$E[\xi] < 1 \tag{45}$$

Note that the composite operator takes bounded continuous functions into bounded continuous functions, and is a contraction mapping, under condition (45), since it satisfies Blackwell's sufficient conditions (monotonicity and discounting property). See Giovannini and Labadie (1989) for a detailed proof.

Straightforward (but tedious) algebra can show that, in the case of the Svensson model, the relevant functional equation is identical to (42) with the following expression for ξ:

$$\xi_{t+1} = \beta \frac{\eta_{t+1}^{1-\theta}}{\omega_t} \frac{(1 + \psi_{2t+1}^{1-\rho})^{\frac{1-\theta}{1-\rho}}}{(1 + \psi_{2t}^{1-\rho})^{\frac{1-\theta}{1-\rho}}} \frac{(1 + \psi_{1t-1} - r)}{(1 + \psi_{1t} - r)} \tag{46}$$

To compute the equilibrium stocks of foreign exchange reserves, it is necessary to solve the system of equations (36), (37), (35) and (29). The result is familiar from standard models of the monetary approach to the balance of payments, like those in Dornbusch (1980), Dornbusch and Mussa (1975) and Frenkel and Johnson (1976):

$$R_t = \bar{M}_t - \frac{y_t K_t}{K_t^w (y_t + p_t y_t^*)} (\bar{M}_t + \bar{M}_t^* - R_t^w) \tag{47}$$

$$R_t^* = \bar{M}_t^* - \frac{p_t y_t^* K_t^*}{K_t^w (y_t + p_t y_t^*)} (\bar{M}_t + \bar{M}_t^* - R_t^w) \tag{48}$$

$$K_t^w = K_t \frac{y_t}{(y_t + p_t y_t^*)} + K_t^* \frac{p_t y_t^*}{(y_t + p_t y_t^*)} \tag{49}$$

When the liquidity constraint is binding the distribution of foreign exchange reserves is given by equations (47) and (48), after setting $K^w = K = K^* = 1$.

When the liquidity constraint is not binding, however, there are only three equations to solve for four unknowns: R, R^*, K and K^*. The distribution of reserves is indeterminate. The intuition for this indeterminacy is as follows. When private agents hold money balances in excess of their consumption purchases they are indifferent about the composition of their currency portfolio in excess of their planned goods purchases. The excess of holdings of money over planned purchases are always perfectly substitutable. Hence in these models there are two sources of currency substitution. One arises from the transactions services of the currencies, which can become increasingly similar as the goods become more substitutable. The other originates from the store-of-value services of the two currencies, which are identical as long as exchange-rates are credibly fixed.

Since the liquidity constraint has to hold with strict inequality for both currencies, it is possible to compute the range of indeterminacy of R and R^*. The two sets of inequalities that have to be satisfied are:

$$M_t^d - \frac{y_t}{\pi_t} < (K^w - 1)\frac{y_t + y_t^* p_t}{\pi_t} \tag{50}$$

$$M_t^d - \frac{y_t}{\pi_t} > 0 \tag{51}$$

$$M_t^{*d} - \frac{p_t y_t^*}{\pi_t^*} < (K^w - 1)\frac{y_t + y_t^* p_t}{\pi_t} \tag{52}$$

$$M_t^{*d} - \frac{p_t y_t^*}{\pi_t^*} > 0 \tag{53}$$

The inequalities above jointly determine the ranges within which R and R^* (or, alternatively, K and K^*) lay whenever the liquidity constraint is not binding.

This completes the description of the procedure to solve for the distribution of foreign exchange reserves. The discussion has also demonstrated that the solution for the nominal variables in the model can be dichotomized from the solution for the real stock prices. Real stock prices, however, are affected by the dynamic behavior of prices and velocity.

3 Money and Taxes

The transactions constraint postulated in the previous section is not justified by specific features of the trading technologies available to individuals, or by any explicit legal restriction. In a multi-currency economy, the additional problem of the cash-in-advance constraint is that goods and asset markets are assumed to be perfectly integrated, and risks perfectly pooled. The nuisance of having to use a specific currency to buy a specific good stands in contrast with the perfect functioning of all financial markets. Yet, the constraint captures one important characteristic of money demand: its geographical distribution. The demand for different currencies is normally unevenly distributed across different countries. Given these difficulties with the standard cash-in-advance model it is desirable to assume that agents can also use financial assets as means of transactions in the goods markets, even though they are obliged by law to use currency on a certain subset of transactions.

In this section I study the case where cash is required by law for all payments by domestic residents to the government, that is cash is required to pay taxes. In turn, cash is used by the government for all its payments to the private sector. The resident of this two-country economy has to deal with two national governments, which, possibly independently, raise taxes. To simplify the analysis, I assume that the governments are not engaged in the production of public goods, but simply rebate the taxes raised in the form of lump-sum transfers. A closed-economy analog of this model is in Giovannini (1989).

The sequence of transactions is as follows. At the beginning of each period the agent receives transfers from the two national governments, dividends from the two firms, monetary transfers from the two central banks, and she carries over any unused money balances from the period period. These resources are used to buy shares, moneys, and goods. Once goods and assets markets, which now operate contemporaneously, close, the agent uses the money balances accumulated to pay taxes, whose amounts in the two currencies are known since the beginning of the period.

Let T_t and T_t^* denote, in terms of the two goods, total taxes paid to the two governments. The growth rates of tax liabilities as a ratio of output in the two goods are also assumed to have a Markov structure similar to that of output growth and money growth:

$$\tau_t = \frac{T_t}{y_t} \quad (54)$$

$$\tau_t^* = \frac{T_t^*}{y_t^*} \quad (55)$$

$$\tau_t = \phi_t \tau_{t-1} \quad (56)$$

$$\tau_t^* = \psi_{3t} \tau_t \quad (57)$$

with $\psi_3, \phi > 0$.

The individual's problem is now:

$$\max_{\{c_t, c_t^*, z_t, z_t^*, M_t^d, M_t^{*d}\}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{[U(c_t, c_t^*)]^{1-\theta}}{1-\theta} \quad (58)$$

subject to:

$$\begin{aligned} & \pi_t M_t^d + \pi_t^* M_t^{*d} + q_t z_t + q_t^* z_t^* + c_t + p_t c_t^* \\ \leq\ & (q_t + y_{t-1}) z_{t-1} + (q_t^* + p_t y_t^*) z_{t-1}^* \\ +\ & \pi_t (M_{t-1}^d - \frac{T_{t-1}}{\pi_{t-1}}) + \pi_t^* (M_{t-1}^{*d} - \frac{p_{t-1} T_{t-1}^*}{\pi_{t-1}^*}) \\ +\ & \pi_t (1-r)(\omega_t - 1)\bar{M}_{t-1} + \pi_t^* (\frac{\psi_{1t}}{\psi_{1t-1}} \omega_t - 1)\bar{M}_{t-1}^* \\ +\ & G_t + G_t^* \end{aligned} \quad (59)$$

$$T_t \leq \pi_t M_t^d \quad (60)$$

$$T_t^* \leq \pi_t^* M_t^{*d} \quad (61)$$

where G and G^* are the transfers from the government. The equilibrium conditions are:

$$c_t = y_t \quad (62)$$

$$c_t^* = y_t^* \quad (63)$$

$$z_t = z_t^* = 1 \quad (64)$$

$$\bar{M}_t = M_t^d + R_t \quad (65)$$

$$\bar{M}_t^* = M_t^{*d} + R_t^* \quad (66)$$

The method of solution of this model is analogous to that of the Lucas model. The only difference is that now the inverse-velocity functions K, K^* and K^w are defined in terms of tax obligations, not in terms of consumption. In equilibrium:

$$\frac{(T_t + p_t T_t^*) K_t^w}{(\bar{M}_t + \bar{M}_t^* - R_t^w)} = \frac{T_t K_t}{\bar{M}_t - R_t} = \frac{p_t T_t^* K_t^*}{\bar{M}_t^* - R_t^*} \tag{67}$$

Repeating the arguments in the previous section, it can be shown that the average velocity function solves the following:

$$K_t^w = \max[1, \kappa_t] \tag{68}$$

where:

$$\kappa_t = E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{T_{t+1} + p_{t+1} T_{t+1}^*}{T_t + p_t T_t^*} \frac{\bar{M}_t + \bar{M}_t^* - R_t^w}{\bar{M}_{t+1} + \bar{M}_{t+1}^* - R_{t+1}^w} K_{t+1}^w \right)$$

And the expression above can be easily simplified in terms of the forcing processes. The portfolio of foreign exchange reserves is thus determined by the following system:

$$R_t = \bar{M}_t - \frac{T_t K_t}{K_t^w (T_t + p_t T_t^*)} (\bar{M}_t + \bar{M}_t^* - R_t^w) \tag{69}$$

$$R_t^* = \bar{M}_t^* - \frac{p_t T_t^* K_t^*}{K_t^w (T_t + p_t T_t^*)} (\bar{M}_t + \bar{M}_t^* - R_t^w) \tag{70}$$

$$K_t^w = K_t \frac{T_t}{(T_t + p_t T_t^*)} + K_t^* \frac{p_t T_t^*}{(T_t + p_t T_t^*)} \tag{71}$$

4 An Illustration

The preceding sections highlighted that the level of reserves consistent with credibly fixed exchange rates depend on the parameters of the forcing variables. In this section I generate quantitative evidence on the importance of the effects discussed above. For the purpose of illustration, I have selected data from two European countries: Germany and France. Let Germany be the "home" country and France the "foreign" country.

4.1 Methodology

While it might be plausible to take estimated processes for η, ψ_2, ϕ and ψ_3 from the data, the changes in monetary regimes in the second postwar period do not allow to fit ω and ψ_1 processes that are necessarily consistent with the model. For this reason I fit the process for ω directly from the German data, but assume a process for ψ_1 that satisfies certain properties—even though it does not resemble the experience of the second postwar period.

The procedure I follow is:

- estimate joint stochastic processes for (η, ψ_2, ω) and for $(\eta, \psi_2, \omega, \phi, \psi_3)$ using data from France and West Germany;[6]
- add to these estimated VAR's alternative assumed processes for ψ_1, which is taken to be uncorrelated with the other variables, and to follow first-order autoregressive processes that are consistent with the (nonstochastic) steady state: $\psi_1 = 1$.
- fit a Markov model to the augmented joint processes, by discretizing the state-space with Tauchen's (1986) quadrature method;
- compute numerical solutions of the model (the functions that solve the equations described above are, with a discrete state-space, vectors of size equal to the number of elements in the state space);
- simulate the models using the solutions.

The data are annual and are obtained from the International Monetary Fund's *International Financial Statistics*. Endowment in the two countries is measured by Gross Domestic Product while the supply of domestic money is Germany's base money.[7] Because the fixed exchange rate is normalized to 1, ψ_2 was normalized to have an average of 1. I fit two first-order VAR (one corresponding to each model) on η, ψ_2 and ω over the period from 1953 to 1988, after having tested for, and rejected, an alternative second order specification.[8]

[6] The variable ω_t in the Svensson model is equal to the ratio of the future stock of money to the current stock of money. Hence a separate vector autoregression has to be estimated for that model.

[7] While the supply of domestic credit would have been more appropriate, IFS data do not permit to compute a reliable estimate of total domestic credit, since it is not clear whether "government deposits" and "other items-net" should be included or not.

[8] The likelihood ratio test statistic for the null hypothesis that the increase in the likelihood of a second order system versus a first order system is is negligible had a significant level of 17 percent in the case of hte Lucas model, of 3 percent in the case of the Svensson model.

Following Tauchen's procedure, these VAR's are used to produce a number of state-space vectors and transition matrices, each one corresponding to a specific assumption on the process followed by ψ_1. When the Markov models are simulated and the same VAR is estimated on the simulated variables, the result is extremely close to the original estimates.[9] In the case of the Lucas model, the state space is represented by the current realizations of the exogenous variables. η and ψ_2 are allowed to take three values each, ω four values, while ψ_1 can take two values. The size of the state space in the Lucas model is thus 72 (all functions are thus vectors of 72 elements). In the case of the Svensson model, equation (45) reveals that ψ_{1t-1} also enters the state space. Hence in that case all functions are vectors of size 144.

To solve the "money-and-taxes" model, I use data on government revenue from *International Financial Statistics* (line 81) for both countries. These data include social security and extra-budgetary operations after 1970 (in Germany) and 1972 (in France). τ and τ^* are constructed assuming that the ratio of social-security and extra-budgetary operations to GDP over the years preceding 1970 and 1972 are equal to the ones observed in those years. A first-order VAR fit over the 1955–87 period on the forcing variables arranged in the vector $(\eta_t, \psi_{2t}, \omega_t, \phi_t, \psi_{3t})'$, was also found superior to the alternative, second order specification.[10] The variables η, ψ_2, ω and ψ_1 are allowed to take the same number of values as in the two models above (3, 3, 4 and 2, respectively), while ϕ and ψ_3 can take two values each. As a result, the size of the state-space in the money-and-taxes model is 288: all functions are vectors of that size. As above, the simulated VAR under these assumptions was close to the original one.

While the VAR's do not restrict any of the exogenous variables to take on positive values only, the discretization procedure I adopted ended up choosing only positive realizations. The alternative state-space vectors and probability transition matrices are then used in the numerical simulations of the models. Recall from the discussion in section 2.1 that whenever the cash-in-advance constraint is not

[9] This information is not reported here, but is of course available.

[10] The likelihood-ratio test statistic for the alternative, second order system had a marginal significance level of about 3 percent.

binding, the distribution of foreign exchange reserves is indeterminate. In the simulations these cases are treated by sampling foreign exchange reserves from a uniform distribution whose boundaries are determined by equations (50) to (53).

4.2 Results

For each model I compute the equilibrium distribution of reserves varying three parameters:

1. The (reciprocal of the) elasticity of substitution between the two goods, which I set equal to 0 (infinite substitutability), 0.025 and 0.5, respectively. This is the same experiment carried out by Woodford (1990) and Weil (1990). The motivation for this experiment is the association of the substitutability between the two goods with the integration of the payments system in the two countries. An increase in the substitutability between the two goods is an increase in the substitutability of the transactions services of the two currencies. In the case of the money-and-taxes model, I assume that the elasticity of substitution between the two goods is equal to 1 (and consumption shares of the two goods are equal). The reason is that the model where money is used to pay taxes money is meant to describe a situation where there are only legal restrictions in the use of money: money is no more used in the payment system, except for legal reasons.

2. The (reciprocal of the) elasticity of intertemporal substitution in consumption, which I vary from 0.5 to 2.

3. The ratio of the value of the portfolio of the ERSA to domestic money, which I set equal to 5, 10 and 15 percent.

4. The first-order autocorrelation coefficient of the ratio of the two money stocks, which I set equal to .2 and .8. These values are chosen by adjusting the constant term in the autoregression, so that the steady state value for ψ_1 is equal to 1. In the first case, and in the absence of additional shocks, an exogenous innovation is reduced to less 10 percent of its original value in about 3 years. In the second case the lag exceeds 9 years.

5. The standard error of the innovation in the ψ_1 process (ϵ), which I set equal to 5 and 10 percent per year.

The last parameters in the model, the utility discount rate, is set equal to 1 percent per annum.[11] After solving the models, I generate a sample of 5,000 realizations of the exogenous varables, and compute the distribution of foreign exchange reserves.

Tables 1 to 6 report the highlights of the numerical simulations. The tables contain, in the last two columns on the right, the range (the difference between the maximum and minimum values) of the holdings of domestic currency by the ERSA, as a fraction of the total stock of domestic money, as well as the minimum value of the holdings of the domestic currency by the ERSA, also expressed as a fraction of the total stock of domestic money. The range represents a measure of the volatility of the holdings of the ERSA. The columns on the left are the parameters whose values have been changed in the simulations.

In the case of the Lucas model (Tables 1 and 2), as well as in the money-and-taxes model (Table 6), I find that the liquidity constraint is always binding. This result implies that the interest rates produced by the simulations of the two models are always positive, and hence constitutes a weak empirical support of the models. When the liquidity constraint is always binding, equations (47)–(48) and (69)–(70) show that only current realizations of relative money supplies and relative money demands affect the composition of foreign exchange reserves, while expectations about the persistence of shocks have no effects. Indeed, both changes in the elasticity of intertemporal substitution and changes in the persistence of the relative money-supply disturbances do not affect the results in the simulations of these two models. By contrast, I find that in the case of the Svensson model the liquidity constraint is not binding in several states.

As argued above, in the Lucas and Svensson models, the elasticity of substitution of consumption of the two goods is a proxy for the susbtitutability of the two currencies' transactions services. In the simulations I study the effects of variations in this parame-

[11] Some experiments indicated that different values for this parameter do not affect the results considerably.

ter: an increase in ρ is a decrease of the elasticity of substitution of the two goods. The results of these experiments show that there is a noticeable difference between the stochastic, fixed-exchange-rates model studied here and the perfect-foresight models of Woodford (1990) and Weil (1990). These authors find that, in their models, increasing the substitutability between the two goods "worsens" the problems of multiple steady-state equilibria, and the possibility of implosive or explosive equilibrium paths. In the models studied here, the condition for the existence and uniqueness of a rational-expectations monetary equilibrium is the condition for the existence and uniqueness of the solution to the functional equation (42). It is easily checked, by inspection of equations (39) and (46), that these conditions are altered by changes in the parameter ρ, but not pathologically. Indeed, they are always satisfied in the numerical simulations that I have performed. This observation, however, applies only to the class of equilibria studied in this paper, which are fixed functions of the relevant state variables.[12]

While the "good" properties of monetary equilibria in my models are not affected by increasing the substitutability between the two goods, the volatility of foreign exchange reserves is. In the case of the Lucas model under all parameter combinations the volatility of reserves increases with higher substitutability between the two goods, as shown by the increase in the range of the ratio $R/\bar{M}$. The intuition for this result is straightforward. When the two goods are perfectly substitutable in consumption their relative price is fixed. Recall that in the case of the Lucas model the data implies that the cash-in-advance constraint is always binding. Under these conditions exogenous fluctuations in goods' supplies are not offset by fluctuations in the relative price and equations (10) and (11) (expressed with equalities) imply that the volatility of money demand necessarily increases. In the case of the Svensson model this intuition does not apply exactly since the liquidity constraint is not always binding. Yet, tables 3, 4 and 5 suggest that in the majority of cases an increase of the substitutability of the two currencies' transactions services increases the volatility of foreign exchange reserves

[12] The paper, for example, does not consider "sunspot" equilibria, where extrinsic variables are allowed to affect the demand for reserves. An analysis of sunspot equilibria is beyond the scope of this paper.

also in the Svensson model.

Intertemporal factors play a role in the Svensson model, where under some states of the world agents hold more cash than their planned purchases. Tables 3, 4 and 5 reveal that as θ increases (lower intertemporal substitution and higher risk aversion) the volatility of foreign exchange reserves goes down. A comparison of tables 3 and 5 allows to determine the effects of changes in the persistence of relative money demand shocks. In 14 out of 18 cases the volatility of reserves increases with an increase of the persistence of relative money supply shocks.

Finally, for all models, doubling the volatility of the innovation of the relative-money-supply process increases the volatility of reserves noticeably. An increase in the size of the ERSA is in the largest majority of cases associated with a decrease in the range of domestic currency holdings. When the total stock of foreign exchange reserves is equal to 15 percent of the domestic country's money stock, the holdings of domestic currency seldom reach negative values.

5 Concluding Observations

This paper has discussed the determination of foreign exchange reserves with credibly fixed exchange rates. The paper highlighted the role of substitutability of currencies in a fixed exchange-rate system. I have shown that higher substitutability of transactions services leads to higher variability of foreign exchange reserves, and that whenever money is held purely for store-of-value purposes currencies are perfect substitutes, and the composition of foreign exchange reserves is indeterminate.

The analysis in this paper can be applied to determine the level of foreign exchange reserves that makes a fixed-exchange-rate system credible. Under the assumption that agents' expectations are based on "fundamentals" exclusively (that is, the state space is specified as in the analysis above, and does not include any extraneous variables), one can find the size of the portfolio of the ERSA such that the holdings of any one of the currencies never falls below zero. If exchange-rates are never changed as long as foreign exchange reserves are positive, the chosen size of the ERSA sustains credibly fixed exchange rates. The numerical illustrations presented seem to indicate that in the data I used—and given the assumptions about relative money supply shocks—a size of the ERSA of about 20 percent of the total stock of domestic money would sustain a credibly[13] fixed exchange rate.

The analysis could also be extended to study a regime of managed floating, which requires to introduce some feedbacks to the exchange rate, and perhaps a richer specification of money supply processes. This extension could allow to determine whether in a managed floating regime the fluctuation of foreign exchange reserves exceeds that occurring under credibly fixed rates.

[13]Notice, to avoid confusion, that the exchange rate is *credible* in the sense described in this section. In the rest of this paper, where nonnegativity constraints on the holdings of the ERSA are not imposed, the fixed exchange rate is always credible.

Table 1:

Lucas Model:

$\sigma(\epsilon) = 5\%$ *per annum*

ρ	r	$range(R/\bar{M})$	$\min(R/\bar{M})$
0.0000	0.0500	0.1092	-0.0401
0.0000	0.1000	0.1077	-0.0141
0.0000	0.1500	0.1062	0.0119
0.0250	0.0500	0.1078	-0.0391
0.0250	0.1000	0.1063	-0.0131
0.0250	0.1500	0.1048	0.0129
0.5000	0.0500	0.0797	-0.0201
0.5000	0.1000	0.0789	0.0054
0.5000	0.1500	0.0781	0.0310

Notes: ϵ is the innovation in the process followed by ψ_1, the ratio of foreign to domestic money stock. ρ is the reciprocal of the elasticity of substitution between the goods, r is the ratio of total reserves to the stock of money of the home country. R is the stock of domestic-currency reserves in the hands of the stabilization authority, $\bar{M}$ is the *total* stock of domestic money.

Table 2:

Lucas Model:

$\sigma(\epsilon) = 10\%$ *per annum*

ρ	r	$range(R/\bar{M})$	$\min(R/\bar{M})$
0.0000	0.0500	0.1472	-0.0661
0.0000	0.1000	0.1460	-0.0401
0.0000	0.1500	0.1449	-0.0141
0.0250	0.0500	0.1460	-0.0651
0.0250	0.1000	0.1449	-0.0391
0.0250	0.1500	0.1437	-0.0131
0.5000	0.0500	0.1236	-0.0456
0.5000	0.1000	0.1231	-0.0201
0.5000	0.1500	0.1225	0.0054

Notes: See Table 1.

Table 3:

Svensson Model:

$\psi_{1t} = 0.8 + 0.2\psi_{1t-1} + \epsilon_t$

$\sigma(\epsilon) = 5\%$ *per annum*

ρ	θ	r	$range(R/\bar{M})$	$\min(R/\bar{M})$
0.0000	0.5000	0.0500	0.2036	-0.0861
0.0000	0.5000	0.1000	0.2031	-0.0585
0.0000	0.5000	0.1500	0.1983	-0.0245
0.0000	2.0000	0.0500	0.1384	-0.0439
0.0000	2.0000	0.1000	0.1342	-0.0178
0.0000	2.0000	0.1500	0.1286	0.0083
0.0250	0.5000	0.0500	0.1866	-0.0631
0.0250	0.5000	0.1000	0.1758	-0.0325
0.0250	0.5000	0.1500	0.1643	-0.0013
0.0250	2.0000	0.0500	0.1299	-0.0428
0.0250	2.0000	0.1000	0.1360	-0.0167
0.0250	2.0000	0.1500	0.1221	0.0093
0.5000	0.5000	0.0500	0.1702	-0.0537
0.5000	0.5000	0.1000	0.1784	-0.0384
0.5000	0.5000	0.1500	0.1803	-0.0107
0.5000	2.0000	0.0500	0.0974	-0.0220
0.5000	2.0000	0.1000	0.0997	0.0036
0.5000	2.0000	0.1500	0.0887	0.0291

Notes: See Table 1. θ is the reciprocal of the elasticity of intertemporal substitution in consumption.

Table 4:

Svensson Model:

$$\psi_{1t} = 0.8 + 0.2\psi_{1t-1} + \epsilon_t$$

$$\sigma(\epsilon) = 10\% \textit{ per annum}$$

ρ	θ	r	$range(R/\bar{M})$	$\min(R/\bar{M})$
0.0000	0.5000	0.0500	0.3301	-0.1297
0.0000	0.5000	0.1000	0.2737	-0.0683
0.0000	0.5000	0.1500	0.3241	-0.0749
0.0000	2.0000	0.0500	0.2264	-0.0700
0.0000	2.0000	0.1000	0.2212	-0.0439
0.0000	2.0000	0.1500	0.2310	-0.0178
0.0250	0.5000	0.0500	0.3124	-0.1201
0.0250	0.5000	0.1000	0.2986	-0.0679
0.0250	0.5000	0.1500	0.3013	-0.0512
0.0250	2.0000	0.0500	0.2028	-0.0689
0.0250	2.0000	0.1000	0.2297	-0.0428
0.0250	2.0000	0.1500	0.2099	-0.0167
0.5000	0.5000	0.0500	0.2900	-0.0906
0.5000	0.5000	0.1000	0.2799	-0.0769
0.5000	0.5000	0.1500	0.2934	-0.0493
0.5000	2.0000	0.0500	0.1916	-0.0475
0.5000	2.0000	0.1000	0.1886	-0.0220
0.5000	2.0000	0.1500	0.2008	0.0036

Notes: See Table 1. θ is the reciprocal of the elasticity of intertemporal substitution in consumption.

Table 5:

Svensson Model:

$\psi_{1t} = 0.2 + 0.8\psi_{1t-1} + \epsilon_t$

$\sigma(\epsilon) = 5\%$ *per annum*

ρ	θ	r	$range(R/\tilde{M})$	$\min(R/\tilde{M})$
0.0000	0.5000	0.0500	0.2116	-0.0872
0.0000	0.5000	0.1000	0.1836	-0.0372
0.0000	0.5000	0.1500	0.1771	-0.0055
0.0000	2.0000	0.0500	0.1390	-0.0439
0.0000	2.0000	0.1000	0.1352	-0.0178
0.0000	2.0000	0.1500	0.1530	0.0083
0.0250	0.5000	0.0500	0.1948	-0.0774
0.0250	0.5000	0.1000	0.1992	-0.0509
0.0250	0.5000	0.1500	0.2050	-0.0264
0.0250	2.0000	0.0500	0.1373	-0.0428
0.0250	2.0000	0.1000	0.1358	-0.0167
0.0250	2.0000	0.1500	0.1353	0.0093
0.5000	0.5000	0.0500	0.1900	-0.0650
0.5000	0.5000	0.1000	0.1769	-0.0341
0.5000	0.5000	0.1500	0.1878	-0.0112
0.5000	2.0000	0.0500	0.1023	-0.0220
0.5000	2.0000	0.1000	0.1051	0.0036
0.5000	2.0000	0.1500	0.1058	0.0291

Notes: See Table 1. θ is the reciprocal of the elasticity of intertemporal substitution in consumption.

Table 6:

Money-and-Taxes Model

$\sigma(\epsilon)$	r	$range(R/\bar{M})$	$\min(R/\bar{M})$
5 %	0.050	0.0872	-0.0194
5 %	0.100	0.0862	0.0061
5 %	0.150	0.0852	0.0316
10 %	0.050	0.1372	-0.0449
10 %	0.100	0.1362	-0.0194
10 %	0.150	0.1353	0.0061

Notes: See Table 1.

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