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NEW GOODS AND INDEX NUMBERS: U.S. IMPORT PRICES

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ABSTRACT

Researchers constructing index number frequently face the problem of new (or disappearing) goods, for which the price and quantity are not available in some periods. In theory, the correct way to handle a new good is to treat its price before it appears as equal to the reservation price (i.e., where demand is zero); in practice, this method can be difficult to implement. However, if the underlying aggregator function is CES then the reservation price is infinity, and we show that the corresponding price index takes on a very sensible form. We apply this formula to measure the price index for six disaggregate U.S. imports, which have been supplied from many new countries over the past several decades. We find that by incorporating the new supplying countries, the price index for developing countries is significantly lower than would otherwise be measured.

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1. INTRODUCTION

A problem facing researchers constructing index numbers is how to treat new (or disappearing) goods, for which the price and quantity is not available in all periods. One practise is to simply *ignore* the new good in the price index during the first period that it appears, and then include it thereafter. From the consumer viewpoint this practise will often *overstate* the true change in prices, since in the first period that a good appears we should treat its price as lower than previously. Using an example, Diewert (1987, p. 779) argues that: "the above analysis of bias is only illustrative but it does indicate that ignoring new goods could lead to a substantial overestimation of price inflation and a corresponding underestimation of real growth rates, especially in advanced market economies where millions of new goods are introduced each year."

In theory, the correct way to deal with new goods for consumers is to measure their former price as equal to the reservation price, i.e. where demand is zero (see Hicks, 1940, and Fisher and Shell, 1972). In practise, this technique is difficult to implement since reservation prices are not easily estimated. One approach which has been used is to replace the missing price of a new good with its predicted price from a hedonic regression, using the characteristics data from the period when the good is available.[1] This technique has the advantage that it can be implemented whenever characteristics data is available, though it is uncertain whether the predicted price reflects an equilibrium value or its reservation level.[2]

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[1] See Ohta and Griliches (1975, p. 326) for this technique applied to autos, and Cole *et al* (1986) and Triplett (1986) for an application to computer equipment. Kravis (1984) used a regression of prices on product and country dummies to impute missing values. Pollak (1983) provides a general discussion of the theory and practise of dealing with new goods and variety change in index numbers.

[2] That is, if the hedonic regression corresponds to the equilibrium locus described by Rosen (1974), then the predicted price could be an estimate of the

In this paper we shall consider the case where characteristics data are *not* available, and obtain a solution to the "new goods problem" when the underlying aggregator function is CES, with an elasticity of substitution greater than unity. In this case, the reservation price for any good is *infinity*, since quantity approaches zero only for arbitrarily high prices. It turns out that using a reservation price of infinity in one period, and a finite level in another, leads to a very sensible formula for the price (or quantity) index, as derived in section 2. In particular, the bias which would have resulted by ignoring the new good is seen to depend on its share of expenditure in the first period it appears. However, the formula we derive also depends on the *elasticity of substitution*, and is quite sensitive to this value. Thus, in place of having to estimate a reservation price (which is infinity by assumption), we must estimate the elasticity of substitution between any two goods in the index.

In section 3 we extend our results to consider quality and taste change for the goods (while continuing to assume that characteristics data is not available). In sections 4 and 5 we present a fairly general procedure for estimating the elasticity of subsitution, which allows for correlation between the (unobserved) taste parameters, prices and quantities. These methods are applied to data on U.S. import prices in sections 6 and 7.

Since the seminal work of Houthakker and Magee (1969), it has been observed that the income elasticity of demand for imports in the U.S., and in other industrial countries, are significantly greater than unity.[3] Several recent papers (Helkie and Hooper, 1988; Hooper, 1989; Krugman, 1989) have argued that these high income elasticities may be due to the expansion in the range of imports from

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equilibrium price if the missing good were actually available.

[3] The evidence on income elasticities of import demand is surveyed by Goldstein and Khan (1985).

rapidly growing, developing countries. Since these new goods are *not* reflected in existing import price indexes, the estimated import demand equations attribute the rising share of developing country imports to a high income elasticity. It would be preferable to have the new goods incorporated into the price index itself, which we do for six disaggregate U.S. imports for 1964-87.

In examining each of these products, we assume that the goods imported from the various supplying countries are imperfect substitutes: this is the so-called Armington (1969) assumption. Thus, the "new goods" are identified as goods imported from "new supplying countries." Focusing on developing countries, for *three* of the imports we find that a price index which ignores the new suppliers when they first appear overstates the exact index by at least 50% over the entire time period (or about 2% per year), while the upward bias for the other three imports is much less. Our conclusions are given in section 8, and the proofs of Propositions are gathered in the Appendix.

2. CES MODEL

We shall consider a CES aggregator function, which can reflect either the utility function of a consumer or the production function of a firm:

$$y_t = f(x_t, I_t, a) = \left(\sum_{i \in I_t} a_i x_{it}^{\alpha} \right)^{1/\alpha}, \quad a_i > 0 \text{ and } 0 < \alpha < 1, \tag{1}$$

where y_t denotes output or utility, x_{it} is the quantity of good i in period t, $a = (a_1, \ldots, a_N)$ is a vector of parameters, $I_t \subset \{1, \ldots, N\}$ denotes the set of goods available in that period, and x_t denotes the vector of goods x_{it} for $i \in I_t$. The elasticity of substitution is $\sigma = 1/(1-\alpha)$, which exceeds unity by assumption. We shall find it convenient to use a rewritten version of (1):

$$f(x_t, I_t, b) = \left(\sum_{i \in I_t} (b_i x_{it})^{\alpha} \right)^{1/\alpha}, \qquad b_i = a_i^{1/\alpha} > 0. \tag{1'}$$

Let $p_i > 0$ denote the price of good i, with p_t denoting all such prices for $i \in I_t$. Then the unit-cost function corresponding to (1') is:

$$c(p_t, I_t, b) = \left(\sum_{i \in I_t} (p_{it}/b_i)^{-1/\beta} \right)^{-\beta}, \qquad \beta = (1-\alpha)/\alpha > 0, \tag{2}$$

where β is related to the elasticity of substitution by $\beta = 1/(\sigma - 1) > 0$.

To briefly review known results, suppose that the *same* set of goods I are available in periods t-1 and t, and that x_{t-1} and x_t are the cost-minimizing quantities with prices p_{t-1} and p_t, respectively. Diewert (1976) defines an *exact* price index as a function $P(p_{t-1}, p_t, x_{t-1}, x_t, I)$ depending on observed prices and quantities, such that,

$$c(p_t, I, b)/c(p_{t-1}, I, b) = P(p_{t-1}, p_t, x_{t-1}, x_t, I). \tag{3}$$

The remarkable feature of (3) is that the price index itself does not depend on the unknown parameters b_i, $i \in I_t$ From Sato (1976) and Vartia (1976), a formula for the exact price index corresponding to the CES unit-cost function is:

$$P(p_{t-1}, p_t, x_{t-1}, x_t, I) \equiv \prod_{i \in I} (p_{it}/p_{it-1})^{w_{it}(I)}. \tag{4a}$$

This is a geometric mean of the individual price changes, where the weights $w_{it}(I)$ are computed using the cost shares $s_{it}(I)$ in the two periods, as follows:

$$s_{it}(I) \equiv p_{it} x_{it} / \sum_{i \in I} p_{it} x_{it} . \tag{4b}$$

$$w_{it}(I) \equiv \left(\frac{s_{it}(I) - s_{it-1}(I)}{\ln s_{it}(I) - \ln s_{it-1}(I)} \right) \bigg/ \sum_{i \in I} \left(\frac{s_{it}(I) - s_{it-1}(I)}{\ln s_{it}(I) - \ln s_{it-1}(I)} \right). \tag{4c}$$

The numerator on the right of (4c) is the *logarithmic mean* of s_{it} and s_{it-1}, and lies between these cost shares. Then the weights $w_{it}(I)$ are a normalized version of the logarithmic means, and add up to unity. Using L'Hospital's Rule, we see that as $s_{it-1} \rightarrow s_{it}$ for all i, then the weights w_{it} approach s_{it}.

The exact price index in (4) requires that the same goods are available in the two periods. We now show how the formula for the exact index number can be generalized to allow for different - but overlapping - sets of goods in the two periods. To this end, suppose that the goods I_{t-1} and I_t are available in periods t-1 and t, respectively, with $I_t \cap I_{t-1} \neq \emptyset$. We let $I \subset (I_t \cap I_{t-1})$ denote a non-empty set of goods available in both periods, and $P(p_{t-1}, p_t, x_{t-1}, x_t, I)$ denote the price index in (4) which is computed by *only* using data on the common set of goods. The exact price index should equal the ratio $c(p_t, I_t, b)/c(p_{t-1}, I_{t-1}, b)$. Our first result shows how this can be measured with observed prices and quantities:

Proposition 1

For any set of goods $I \subset (I_t \cap I_{t-1})$, $I \neq \emptyset$, we have:

$$c(p_t, I_t, b)/c(p_{t-1}, I_{t-1}, b) = P(p_{t-1}, p_t, x_{t-1}, x_t, I)\ (\lambda_t/\lambda_{t-1})^{\beta}, \tag{5a}$$

$$\text{where } \lambda_r \equiv \sum_{i \in I} p_{ir} x_{ir} \Big/ \sum_{i \in I_r} p_{ir} x_{ir}, \text{ for } r = t-1, t. \tag{5b}$$

The proofs of all Propositions are in the Appendix. To interpret this result, let $I = I_t \cap I_{t-1}$. Then $P(p_{t-1}, p_t, x_{t-1}, x_t, I)$ is the price index computed by *ignoring* all new goods which appear in period t (i.e. goods $i \in I_t$ but $i \notin I_{t-1}$), and *ignoring* all goods which disappear between t-1 and t ($i \in I_{t-1}$ but $i \notin I_t$). The terms λ_r satisfy $0 < \lambda_r \leq 1$, and equal the fraction of expenditure in period r on the goods $i \in I$ relative to the entire set $i \in I_r$. To take a specific example, if there are some new goods in period t - but no disappearing goods - then $\lambda_t < 1$, $\lambda_{t-1} = 1$, and the price index $P(p_{t-1}, p_t, x_{t-1}, x_t, I)$ *overstates* the ratio of unit-costs. This upward bias occurs because the new goods would have zero demand in the first period only if their prices were infinity.[4] In moving to the second period, the prices of these goods therefore fall from infinity to their observed level, and this price decrease is ignored in the index $P(p_{t-1}, p_t, x_{t-1}, x_t, I)$.

To measure the right of (5a) we need to know the value of $\beta = 1/(\sigma - 1)$. If the elasticity of substitution σ is very high so that β approaches zero, then $(\lambda_t/\lambda_{t-1})^{\beta}$ will be close to unity, so that the price index obtained by ignoring the new goods is quite adequate. At the other extreme, as σ approaches unity then β approaches infinity and $(\lambda_t/\lambda_{t-1})^{\beta}$ approaches zero (infinity) as $\lambda_t < (>) \lambda_{t-1}$, indicating that the new goods have a very significant effect on unit-costs.[5] Obtaining a suitable estimate of β, together with its standard error, will be the topic of section 4.

3. QUALITY AND TASTE CHANGE

So far we have been assuming that the parameters b of the unit-cost function (2) remain constant over time. As discussed by Fisher and Shell (1972), these parameters can change due to changes in the quality (productivity) of the goods x_{it}, or due to changes in tastes. Note that $b_i = a_i^{1/\alpha}$ are positively related to a_i in the production function (1), so an increase in the productivity of good i will raise b_i, and therefore lower unit-costs. Let b_{ir} denote the productivity or

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[4] Indeed, one proof of Proposition 1 is to show that (5) is obtained as the limiting value of (4) as $p_{ir} \to \infty$ for $i \in I_r$, $i \notin I$, r=t-1,t. While this is not the proof given in the Appendix, a proof of this type using the exact index of Diewert (1976, eq. 4.5) is in an earlier version of this paper, available on request.

[5] For $\sigma \leq 1$ all goods are *essential* to achieve positive production or utility, which is why a new or disappearing good has an infinite effect on unit-costs. For this reason, we exclude $\sigma \leq 1$ from our analysis.

taste parameters in period $r=t-1,t$. In the case of quality change we are interested in measuring the ratio of unit-costs $c(p_t,I_t,b_t)/c(p_{t-1},I_{t-1},b_{t-1})$, using the actual values of b_{ir}. However, in the case of taste change we follow Fisher and Shell (1972) in measuring unit-costs relative to some *constant* taste parameters $\tilde{b}_i$, so the exact index should equal $c(p_t,I_t,\tilde{b})/c(p_{t-1},I_{t-1},\tilde{b})$.[6]

While we shall principally deal with taste change, we note that a corollary of Proposition 1 can handle some forms of quality change. In particular, suppose that there is some non-empty set of goods I in the two periods for which it is known that the productivity parameters are constant, $b_{it-1}=b_{it}$ for $i\varepsilon I$. Then we have:

Corollary 1

If $b_{it-1}=b_{it}$ for $i\varepsilon I \subset (I_t \cap I_{t-1})$, $I \neq \emptyset$, then:

$$c(p_t,I_t,b_t)/c(p_{t-1},I_{t-1},b_{t-1}) = P(p_{t-1},p_t,x_{t-1},x_t,I)\ (\lambda_t/\lambda_{t-1})^{\beta}. \qquad (6)$$

This Corollary follows by treating all goods i which are available in both periods but have $b_{it-1} \neq b_{it}$ as *both* disappearing and new, e.g. manual typewriters in period t-1 having the same index i as electric typewriters in period t, when really they different goods. The changing value of b_{ir} will affect the expenditure on this good in the two periods, and therefore influence the λ_r terms in (6) which exactly reflects the change in unit-costs.

The limitation of Corollary 1 is that the researcher may not know which goods (if any) have constant parameters b_{ir}. In this case, suppose that the price index is constructed as on the right of (5a), where we let $I=I_{t-1} \cap I_t$. The question is whether this price index has any meaning when $b_{t-1} \neq b_t$. The following

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6 Fisher and Shell (1972, p. 9) use $\tilde{b}_i \equiv b_{it}$, whereas we shall allow $\tilde{b}_i$ to lie between a normalized version of b_{it-1} and b_{it} (see Proposition 2).

result answers this in the affirmative, and says that the constructed price index equals the ratio of unit-costs for some *constant* taste parameters $\tilde{b}$. In this result we assume that the quantitities x_{ir} for $i \in I_r$, $r=t-1,t$, are cost-minimizing for the prices $p_{ir} > 0$ and the taste parameters $b_{ir} > 0$, and let $w_{it}(I)$ denote the weights calculated as in (4):

<u>Proposition 2</u>

Letting $I = I_t \cap I_{t-1}$, $I \neq \emptyset$, there exists $\tilde{b}_i > 0$ for $i \in I_{t-1} \cup I_t$ such that:

$$P(p_{t-1}, p_t, x_{t-1}, x_t, I)\ (\lambda_t/\lambda_{t-1})^{\beta} = c(p_t, I_t, \tilde{b})/c(p_{t-1}, I_{t-1}, \tilde{b}), \tag{7}$$

where $\tilde{b}_i$ is between $b_{it-1}/\prod_{i \in I} b_{it-1}^{w_{it}(I)}$ and $b_{it}/\prod_{i \in I} b_{it}^{w_{it}(I)}$ for $i \in I$.

Thus, even when the taste parameters b_{it} are changing over time, a price index calculated as in Proposition 1 can be interpreted as the ratio of unit-costs with constant taste parameters $\tilde{b}_i$. These parameters lie between a normalized version of b_{it-1} and b_{it}. Note that a result similar to Proposition 2 has been obtained by Christensen, Caves and Diewert (1982) for the Tornqvist index and translog function: even when the actual second-order parameters of the translog change over time, the Tornqvist index can be interpreted as the ratio of translog functions with constant parameters. Proposition 2 is quite general in that all of the parameters b_{it} can vary over time, though we require that $\beta = 1/(\sigma-1)$ does not change. In the next two sections we consider how to estimate β, using random variation in the (unobserved) taste parameters to motivate the error term in our estimation. Following this, we shall apply our estimation technique to data on U.S. import prices.

4. STOCHASTIC SPECIFICATION

Suppose that price and quantity data are available over the periods 1,2,...,T. In each period the set $I_t \subset \{1,...,N\}$ of goods is available, where new goods will be appearing and others disappearing over time. We shall let $\varepsilon_{it} \equiv \ln b_{it} - \ln b_{it-1}$ denote the random change in the taste parameter for good i, between two periods where the good is available. To be precise, let $\Omega_i \subset \{2,...,T\}$ denote the set of time periods t for which good i is available in t *and* t-1. We shall let $T_i \leq T$ denote the number of elements in Ω_i, i.e. the number of adjacent time periods for which good i is available.

We continue to assume that x_{t-1} and x_t are cost-minimizing for the prices p_{t-1} and p_t, respectively. The cost shares are computed as $\partial \ln c(p_t,I_t,b_t)/\partial \ln p_{it}$, and so differentiating (2) we obtain:[7]

$$s_{it}^{\beta} = c(p_t,I_t,b_t)\, b_{it} / p_{it}, \quad t \in \Omega_i. \tag{8}$$

Define $\alpha_t \equiv \ln[c(p_t,N_t,b_t)/c(p_{t-1},N_{t-1},b_{t-1})]$ as the ratio of unit costs, where α_t is a random variable since b_{t-1} and b_t are. Letting Δ denote first-differences, we can write the demand equation (8) as:

$$\Delta \ln s_{it} = (\alpha_t/\beta) - (1/\beta)\Delta \ln p_{it} + (1/\beta)\varepsilon_{it}, \quad t \in \Omega_i. \tag{9a}$$

or in "reverse form",

$$\Delta \ln p_{it} = \alpha_t - \beta \Delta \ln s_{it} + \varepsilon_{it}, \quad t \in \Omega_i. \tag{9b}$$

Thus, ε_{it} appears as the error term on the demand equation (9). We will assume that ε_{it} satisfies the following conditions:

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7 To be consistent with our earlier notation, these cost shares are $s_{it}(I_t)$, i.e. are computed relative to the entire set of goods available in each period. For convenience we omit the dependence on I_t.

<u>Assumption 1</u>

(a) $E(\varepsilon_{it})=0$ and ε_{it} is independent of ε_{js} for all $t\varepsilon\Omega_i$, $s\varepsilon\Omega_j$, $i\neq j$;

(b) ε_{it} is stationary with variance $\sigma^2_{\varepsilon i}$ and $\operatorname{plim}_{T_i\to\infty} \frac{1}{T_i} \sum_{t\varepsilon\Omega_i} \varepsilon^2_{it} = \sigma^2_{\varepsilon i}$.

These assumptions are a special case of those in Hansen's (1982) generalized method of moments (GMM), and our estimator will be interpreted as an application of GMM. One difference between our framework and Hansen's, however, it that the data for (9) is an *unbalanced* panel, since each good i is available only in the periods $t\,\varepsilon\,\Omega_i$.

In order to estimate the parameter β, we need to be more specific about the correlation of ε_{it} with the price and market share variables. In preliminary analysis of our data on U.S. import prices, it was found that ε_{it} was correlated with *both* $\Delta \ln s_{it}$ and $\Delta \ln p_{it}$.[8] This can be explained in a conventional supply and demand equilibrium, as follows. Let us write the supply curve for good i in first-differences as:

$$\Delta \ln p_{it} = \gamma \Delta \ln x_{it} + \xi_{it}, \tag{10}$$

where $\gamma \geq 0$ is the inverse supply elasticity and ξ_{it} is a random error. It will be useful to express (10) in terms of market shares s_{it} rather than quantities x_{it}. From (2) we can write the cost-minimizing quantities $x_{it} = y_t\, \partial c(p_t, I_t, b_t)/\partial p_{it}$ as:

$$\Delta \ln x_{it} = \Delta \ln(y_t c_t^{\sigma}) - \sigma \Delta \ln p_{it} + (\sigma-1)\varepsilon_{it}, \tag{11}$$

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[8] In an earlier version of this paper we estimated the "reverse" regression (9b), using both OLS and weighted least squares (WLS) for a particular choice of weights. The terms α_t were treated as fixed-effects. Comparing the OLS and WLS estimates of α_t and β using the test of White (1980), we found that they were significantly different. One explanation for this finding is correlation between ε_{it} and $\Delta \ln s_{it}$ in (9b).

where $c_t \equiv c(p_t, I_t, b_t)$, and $\sigma = (1+\beta)/\beta$ is the elasticity of substitution. Notice that an increase in the taste parameter b_{it} - giving a positive ε_{it} - yields an increase in demand if and only if the elasticity exceeds unity, as we are assuming.

Then using (10) and (11) to solve for the price, we obtain:

$$\Delta \ln p_{it} = \pi_t + \rho\varepsilon_{it} + \delta_{it}, \qquad 0 \leq \rho < 1, \quad t\,\varepsilon\,\Omega_i, \tag{12}$$

where $\delta_{it} \equiv \zeta_{it}/(1+\gamma\sigma)$, $\pi_t \equiv [\gamma/(1+\gamma\sigma)]\Delta\ln(y_t c_t^{\sigma})$ and $\rho \equiv \gamma(\sigma-1)/(1+\gamma\sigma)$. The parameter ρ satisfies $0 \leq \rho < (\sigma-1)/\sigma < 1$ with $\sigma > 1$, and $\rho = 0$ if and only if $\gamma = 0$, meaning that the supply curve is horizontal. To interpret (12), a positive value of ε_{it} indicates an increase in the taste parameter b_{it}, and an outward shift in the demand curve for good i when $\sigma > 1$. If this good has an upward sloping supply curve, we obtain a corresponding increase in its price. We shall refer to (12) as a "reduced form" supply curve, and use it instead of (10).

Our stochastic model consists of the two equations (9) and (12). To complete our description we impose the following properties on δ_{it}:

<u>Assumption 2</u>

(a) $E(\delta_{it}) = 0$ and δ_{it} is independent of δ_{js} for all $t\varepsilon\Omega_i$, $s\varepsilon\Omega_j$, $j \neq i$;

(b) δ_{it} is stationary with variance $\sigma^2_{\delta i}$ and $\operatorname{plim}_{T_i\to\infty} \frac{1}{T_i} \sum_{t\varepsilon\Omega_i} \delta^2_{it} = \sigma^2_{\delta i}$.

(c) δ_{it} is independent of ε_{js} for all $t\varepsilon\Omega_i$, $s\varepsilon\Omega_j$, and $\operatorname{plim}_{T\to\infty} \frac{1}{T} \sum_{t\varepsilon\Omega_i\cap\Omega_j} \delta_{it}\varepsilon_{jt} = 0$.

Assumptions 2(a,b) are similar to Assumption 1. Condition (c) states that the error terms in the supply and demand equations are independent for all products i and j. This assumption will be essential to identify and estimate β and ρ, as we consider in the next section.

6. ESTIMATION

To write (9) and (12) in a form more suitable for estimation, suppose that there is some good k which is available in every period 1,...,T. We shall eliminate the (random) terms α_t and π_t in each of (9) and (12) by subtracting the same equation for good k. Denoting $\tilde{\varepsilon}_{it} \equiv \varepsilon_{it} - \varepsilon_{kt}$ and $\tilde{\delta}_{it} \equiv \delta_{it} - \delta_{kt}$ for $i \in I_t$, the resulting equations are,

$$\tilde{\varepsilon}_{it} = (\Delta \ln p_{it} - \Delta \ln p_{kt}) + \beta(\Delta \ln s_{it} - \Delta \ln s_{kt}), \tag{9'}$$

$$\begin{aligned} \tilde{\delta}_{it} &= (\Delta \ln p_{it} - \Delta \ln p_{kt}) - \rho\tilde{\varepsilon}_{it} \\ &= (1-\rho)(\Delta \ln p_{it} - \Delta \ln p_{kt}) - \rho\beta(\Delta \ln s_{it} - \Delta \ln s_{kt}), \end{aligned} \tag{12'}$$

for $t \in \Omega_i$. In order to take advantage of the independence of $\tilde{\varepsilon}_{it}$ and $\tilde{\delta}_{it}$ we can multiply these two equations, and divide by $(1-\rho)>0$, to obtain:

$$Y_{it} = \theta_1 X_{1it} + \theta_2 X_{2it} + u_{it}, \tag{13}$$

where:

$$u_{it} \equiv \tilde{\varepsilon}_{it}\tilde{\delta}_{it}/(1-\rho), \tag{14a}$$

$$Y_{it} \equiv (\Delta \ln p_{it} - \Delta \ln p_{kt})^2, \tag{14b}$$

$$X_{1it} \equiv (\Delta \ln s_{it} - \Delta \ln s_{kt})^2, \tag{14c}$$

$$X_{2it} \equiv (\Delta \ln s_{it} - \Delta \ln s_{kt})(\Delta \ln p_{it} - \Delta \ln p_{kt}), \tag{14d}$$

$$\theta_1 \equiv \rho\beta^2/(1-\rho) \text{ and } \theta_2 \equiv \beta(2\rho-1)/(1-\rho). \tag{14e}$$

The observations in (13) are $t \in \Omega_i$ for $i=1,\ldots,N$, $i \neq k$. Stacking (13) for each of the goods, the total number of observations is $L \equiv \sum_{i \neq k} T_i$. Letting Y denote the Lx1 vector with components Y_{it} which are ordered $t \in \Omega_1$ then $t \in \Omega_2$, etc., X the Lx2 matrix with rows (X_{1it}, X_{2it}) ordered the same way, u the Lx1 vector with components u_{it}, and θ the 2x1 vector (θ_1, θ_2), we can write (13) in the familiar notation,

$$Y = X\theta + u. \tag{13'}$$

Note that the error term u_{it} is correlated with X_{1it} and X_{2it} defined in (14). To control for this, we will introduce instrumental variables (IV). Let ℓ_i denote a T_ix1 vector of 1's, i=1,...,N, i≠k, and define Z as the Lx(N-1) matrix,

$$Z = \begin{pmatrix} \ell_1 & 0 & \dots & 0 \\ 0 & \ell_2 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \cdot & \cdot & \dots & \cdot \\ 0 & 0 & \dots & \ell_N \end{pmatrix}.$$

That is, Z is a matrix of dummy variables for each good i≠k. Then we assert that Z is a valid instrument to estimate (13). To establish the asymptotic properties of using Z, we will let $T \to \infty$ while holding N fixed. We will assume that as $T \to \infty$ then $\text{plim}(T_i/T) \to n_i$, with $0 \le n_i \le 1$ for i=1,...,N. That is, we are assuming each good is almost surely available a fixed portion of the periods (possibly zero) as $T \to \infty$. We then have:

<u>Proposition 3</u>

Under Assumptions 1 and 2:

(a) $\underset{T\to\infty}{\text{plim}} \frac{1}{T} Z'u = 0;$

(b) $\underset{T\to\infty}{\text{plim}} \frac{1}{T} Z'X$ has full column rank if and only if there exist i≠k and j≠k with:

$$n_i n_j \neq 0 \quad \text{and} \quad \left(\frac{\sigma_{\varepsilon i}^2 + \sigma_{\varepsilon k}^2}{\sigma_{\varepsilon j}^2 + \sigma_{\varepsilon k}^2}\right) \neq \left(\frac{\sigma_{\delta i}^2 + \sigma_{\delta k}^2}{\sigma_{\delta j}^2 + \sigma_{\delta k}^2}\right). \tag{15}$$

To prove (a), from (14) we have that plim(Z'u/T) is a (N-1)x1 vector with components $\text{plim}[\sum_{t\in\Omega i} \tilde{\delta}_{it}\tilde{\varepsilon}_{it}/T(1-\rho)]$, which equal zero from Assumption 2(c). That is, the independence of errors across the demand and supply equations implies that the instruments Z are (asymptotically) uncorrelated with u. In part (b), plim(Z'X/T) is a (N-1)x2 matrix with rows $n_i \,\text{plim}[\sum_{t\in\Omega i} X_{1it}/T_i, \sum_{t\in\Omega i} X_{2it}/T_i]$.

That is, for each good we evaluate the mean value of X_{1it} and X_{2it}, and then evaluate the probability limit using (9') and (12'). Condition (15) ensures that the two columns of this matrix are linearly independent, so that the matrix has full column rank: this is the rank condition for identification. To interpret (15), it states that there must be some differences across goods in either the variance of the change in taste parameters ε_{it}, or the variance in the unexplained change in prices δ_{it}.

To see how Proposition 3 is related to the estimation of θ, consider the usual IV estimator:

$$\begin{aligned} \hat{\theta} &= [X'Z(Z'Z)^{-1}Z'X]^{-1}X'Z(Z'Z)Z'Y \\ &= \theta + [X'Z(Z'Z)^{-1}Z'X]^{-1}X'Z(Z'Z)Z'u. \end{aligned} \qquad (16)$$

Under our assumptions, plim(Z'Z/T) is a (N-1)x(N-1) diagonal matrix with $\eta_i \geq 0$ on the diagonal. When (15) is satisfied, at least two of the diagonal elements are strictly positive and correspond to two rows of plim(Z'X/T) that are linearly independent. It follows that plim[(X'Z/T)(Z'Z/T)$^{-1}$(Z'X/T)] has full rank of 2, and so is invertible. Then using Proposition 3(a) we see that plim$\hat{\theta} = \theta$, so that the IV estimator is consistent.

To provide an interpretation of the IV estimator, let $\overline{Y}_i \equiv \sum_{t\varepsilon\Omega i} Y_{it}/T_i$, $\overline{X}_{1i} \equiv \sum_{t\varepsilon\Omega i} X_{1it}/T_i$, $\overline{X}_{2i} \equiv \sum_{t\varepsilon\Omega i} X_{2it}/T_i$ and $\overline{u}_i \equiv \sum_{t\varepsilon\Omega i} u_{it}/T_i$ denote the means of the variables in (13) over good i. Then pre-multiplying (13') by $Z(Z'Z)^{-1}Z'$, we obtain L equations of the form,

$$\overline{Y}_i = \theta_1 \overline{X}_{1it} + \theta_2 \overline{X}_{2it} + \overline{u}_i, \qquad (17)$$

where the equation for good i is repeated T_i times. Thus, the IV estimate $\hat{\theta}$ can be *equivalently* obtained by running weighted least squares (WLS) on the observations $i=1,\ldots,N$, $i \neq k$, in (17), while using T_i as the weights. It is intuitive that the mean values in (17) which are calculated over more time periods should receive greater weight. This interpretation of the IV estimator also shows its direct relationship to Hansen's (1982) GMM:[9] from the independence of $\tilde{\delta}_{it}$ and $\tilde{\varepsilon}_{it}$ we have $Eu_{it}=0$, and we are approximating this moment condition by choosing θ_1 and θ_2 to minimize the (weighted) sum of squared sample moments $\overline{u_i}$.

Having obtained the consistent estimates $\hat{\theta}_1$ and $\hat{\theta}_2$ we can solve for $\hat{\beta}$ and $\hat{\rho}$ from the quadratic equations (14e). So long as $\hat{\theta}_1 > 0$ these equations yield both a positive and negative solution for $\hat{\beta}$, and we shall restrict attention to the positive solution:

<u>Proposition 4</u>

So long as $\hat{\theta}_1 > 0$, then the estimates of β and ρ are:

(a) if $\hat{\theta}_2 > 0$ then $\hat{\rho} = \dfrac{1}{2} + \left(\dfrac{1}{4} - \dfrac{1}{4+(\hat{\theta}_2^2/\hat{\theta}_1)}\right)^{1/2}$,

(b) if $\hat{\theta}_2 < 0$ then $\hat{\rho} = \dfrac{1}{2} - \left(\dfrac{1}{4} - \dfrac{1}{4+(\hat{\theta}_2^2/\hat{\theta}_1)}\right)^{1/2}$,

and in either case $\hat{\beta} = \left(\dfrac{1-\hat{\rho}}{2\hat{\rho}-1}\right)\hat{\theta}_2 > 0$.

In the event that the IV estimate $\hat{\theta}_1 < 0$ then using the formulas in Proposition 4 we fail to obtain estimates for β and ρ in the ranges $\hat{\beta} > 0$ and $0 \le \hat{\rho} < 1$.[10] In particular, if a negative value of $\hat{\beta}$ is obtained then it would be best to conclude conclude that the data do not support the hypothesis of a CES aggregator function with an elasticity of substitution greater than unity; in this

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[9] Except for the unbalanced nature of our panel, our model meets the assumptions of Hansen (1982, Theorem 2.2), so long as the identification condition (15) holds.

[10] Note that if $-4 < \hat{\theta}_2^2/\hat{\theta}_1 < 0$, then we obtain imaginary values for $\hat{\rho}$ and $\hat{\beta}$, while if $\hat{\theta}_2^2/\hat{\theta}_1 < -4$ then $\hat{\rho} \notin [0,1]$ and $\hat{\beta}$ has the sign of $-\hat{\theta}_2$.

case our index number methods cannot be applied. However, whenever $\hat{\theta}_1 > 0$ then we can compute a positive value of $\hat{\beta}$ from Proposition 4, and therefore apply our index number methods.

While the IV estimator in (18) is consistent, it is not the most efficient. To see this, consider that the error term u_{it} in (14a). It has the variance $Eu_{it}^2 = (\sigma_{\varepsilon i}^2 + \sigma_{\varepsilon k}^2)(\sigma_{\delta i}^2 + \sigma_{\delta k}^2)/(1-\rho)^2$, which differs across goods when condition (15) holds. That is, the rank condition for θ_1 and θ_2 to be identified implies heteroscedasticity in the errors. To correct for this we will weight the observations in (13) for good i by the inverse of $\hat{s}_i^2 \equiv \sum_{t\varepsilon\Omega i} \hat{u}_{it}^2/T_i$, where $\hat{u}_{it} \equiv Y_{it} - \hat{\theta}_1 X_{1it} - \hat{\theta}_2 X_{2it}$ are the computed residuals using the initial estimates $\hat{\theta}$. Letting $\hat{S}$ denote the (LxL) diagonal matrix with $\hat{s}_i^2$ repeated T_i times on the diagonal for $i=1,\ldots,N$, $i \neq k$, and with $\hat{X} \equiv Z(Z'Z)^{-1}Z'X$, the weighted IV estimator is:

$$\begin{aligned} \theta^* &= [\hat{X}'\hat{S}^{-1}\hat{X}]^{-1}\hat{X}'\hat{S}^{-1}Y \\ &= \theta + [\hat{X}'\hat{S}^{-1}\hat{X}]^{-1}\hat{X}'\hat{S}^{-1}u, \end{aligned} \tag{18}$$

where the second line follows since it can be shown that $\hat{X}'\hat{S}^{-1}X = \hat{X}'\hat{S}^{-1}\hat{X}$.

White (1982) demonstrates the consistency of θ^* for an unbalanced panel, like we have, when the errors u_{it} are independent over i and t. In this case θ^* is the *efficient* estimator (given the set of instruments), and its covariance matrix is consistently estimated by $[\hat{X}'\hat{S}^{-1}\hat{X}]^{-1}$. When the errors u_{it} were correlated over time, then the efficient estimate θ^* could be calculated as in Newey and West (1987), though there will be little evidence of autocorrelation in our sample. Given the estimate θ^* and its covariance matrix, the estimates β^* and ρ^* are calculated as in Proposition 4, and the variances of β^* and ρ^* can be computed by taking first-order approximations to these formulas around the true parameters (see the proof of Proposition 4 in the Appendix). These estimates will be reported in section 7.

6. U.S. IMPORT PRICES

As discussed in the Introduction, several recent papers have argued that existing indexes of U.S. import prices may be biased since they fail to account for new imported goods. Furthermore, it is possible that the omission of these new goods leads to the high income elasticity of demand for imports which has been found since the work of Houthakker and Magee (1969). Note that even for highly disaggregated products, Grossman (1974) has found income elasticities which exceed unity for U.S. imports from the developing countries, though not from the industrial countries. A possible explanation for this result is that there was substantial growth in the number of developed countries supplying each of these products to the U.S. Following Grossman, we shall distinguish imports from developing and industrial countries.

We shall analyze annual data on U.S. imports of six disaggregated products: stainless steel bars; carbon steel sheets; cotton knit shirts; men's leather athletic shoes; portable typewriters; and color televisions (over 17").[11] These products were chosen as commonly recognized goods which, on initial inspection, showed an increase in the number of countries supplying to the U.S. over time. Our sample period begins between 1964 and 1970, depending on data availability, and continues until 1987.[12] Letting the imports from each country represent a

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[11] The data were collected from U.S. Bureau of the Census (1964-87). The TSUSA numbers corresponding to these products, together with the data, are available on request. Note that in a number cases the products would split during the sample period, such as when portable typewriters begin to distinguish electric and non-electric beginning in 1971. In these cases we simply formed a unit-value of the two (or more) series, and use this as the price variable. This procedure is consistent to the price variable we used before the product was split, which was a unit-value by necessity. Our use of unit-values at the disaggregate level gives some motivation for treating the taste parameters for each product and supplying country as random.

[12] For televisions the data starts in 1970, since before this date receivers of

different variety as in Armington (1969), the "new goods" are identified as goods imported from "new supplying countries." We shall apply our methods to construct an import price index for each of the six products, distinguishing imports from developing and industrialized countries according to the IMF classification.

In this data set, an import from a particular country is "new" if it is supplied from that country in year t, but not in t-1 (and "disappears" if it is supplied from that country in year t-1 but not in t). We note that this definition is partially dependent on the reporting procedures adopted by the Bureau of the Census for the import data. In particular, only countries which exceeded a certain dollar amount in average monthly imports where reported separately, but this dollar amount was raised infrequently over time.[13] This means that at certain dates there was an artificial disappearance of those countries supplying less than the new minimum amount. Rather than make any adjustment for this, we simply accepted the data and list of supplying countries as reported.

In Table 1 we summarize the number of suppliers by product. We see that all of the products have considerable growth in the number of developing countries which supply to the U.S., though the number of industrial countries shows much less fluctuation and no trend in most cases. We also report the value of shipments from each of the developing country (DC) and industrial country (IC) groups. Finally, we report the cumulative value of $\ln(\lambda_t/\lambda_{t-1})$ over the years the

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over 17" and under 17" were not distinguished. For athletic shoes from developing countries the data begins in 1966, while for steel bars and steel sheets from developing countries the data begins in 1967, because before these dates there was no country supplying in consecutive years. All other products begin in 1964.

[13] Countries with less than this dollar amount were lumped together in an "Other Countries" category. We included this as a developing country, but alternatively, could have omitted it from the analysis.

product is available. Taking logs in Proposition 2, we see that the cumulative change in the price index $\sum_t \ln P(p_{t-1}, p_t, x_{t-1}, x_t, I)$, which *ignores* new and disappearing goods, must be reduced by $-\beta\sum_t \ln(\lambda_t/\lambda_{t-1})$ to obtain the exact index. Thus, the *bias* in the index which ignores new suppliers can be defined as:

$$\text{Bias} = \exp[-\beta\sum_t \ln(\lambda_t/\lambda_{t-1})]. \qquad (19)$$

This bias will exceed unity when there are new supplying countries, indicating that ignoring these countries when they first appear overstates the exact price index. We shall report this bias after estimating β, in the next section.

7. ESTIMATION RESULTS

To estimate (13), we first need to select a country k which is used in forming the differences $(\Delta \ln p_{it} - \Delta \ln p_{kt})$ and $(\Delta \ln s_{it} - \Delta \ln s_{kt})$ in (14), ie. the data is differenced with respect to this country. This country should be a supplier of the product in every year. For each of our six products there was a number of countries (primarily industrial countries) which could play this role, but the *only* country which supplied all six of the products to the U.S. in every year was Japan. For this reason we chose Japan as country k, and differenced the data to construct the variables in (14). The parameters of (13) were estimated for each of the six products, while pooling the data over industrial and developing countries. The reason for pooling was that for most products there was a sparcity of DC suppliers in the early years (see Table 1), and in addition, there did not seem to be a strong reason to expect the elasticity of substitution (i.e. β) to differ across the IC and DC groups.

In Table 2 we show the estimation results. For each product, the first row shows the the consistent estimates $\hat{\theta}_1$ and $\hat{\theta}_2$ constructed as in (16), with standard errors in parentheses.[14] The consistent estimates $\hat{\beta}$ and $\hat{\rho}$ are then obtained as in Proposition 4. Focusing on these, $\hat{\beta}$ is significantly different from zero for all products. The point estimates correspond to values of the elasticity of substitution $\hat{\sigma}=(1+\hat{\beta})/\hat{\beta}$ ranging from 2.8 (carbon steel sheets) to 6.6 (color televisions). The consistent estimates $\hat{\rho}$ show a considerable variation across products, ranging from zero (carbon steel sheets) to 0.9 (color televisions). From our discussion in section 5, a zero estimate of ρ indicates a horizontal supply curve, and so only for carbon steel sheets would OLS estimation of (9a) have been appropriate; for the other products both prices and market shares should be treated as endogenous variables, as we have done.

The following rows in Table 2 show the *efficient* estimates θ_1^* and θ_2^* calculated as in (18), with β^* and ρ^* again obtained from Proposition 4. The efficient estimates correct for heteroscedasticity, as discussed in section 5. There is a substantial reduction in the standard errors of the efficient estimates, with the t-statistics on β^* all above 5. For most products, the consistent and efficient estimates are reasonably close as compared with their standard errors, and we shall do a formal test for the equality of these estimates below.

In Table 3 we report various test and summary statistics for the products. In the first column we test for autocorrelation of the errors u_{it} in (13). As described in White and Domowitz (1984), this test is performed by taking the computed residuals $\hat{u}_{it} \equiv Y_{it} - \hat{\theta}_1 X_{1it} - \hat{\theta}_2 X_{2it}$, and regressing $\hat{u}_{it}\hat{u}_{it-1}$ on $\hat{X}_{1}^{2}{}_{it}$, $\hat{X}_{2it}^2$ and $\hat{X}_{1it}\hat{X}_{2it}$ (where $\hat{X}_1$ and $\hat{X}_2$ are obtained by regressing X_1 and X_2 on Z). Under the null hypothesis of no correlation between u_{it} and u_{it-1}, the R^2 times the number of observations from this regression is distributed $\chi^2(3)$, with a 90% critical value of 6.25. For all products the null hypothesis is accepted.

[14] The covariance matrix of $\hat{\theta}$ is $(\hat{X}'\hat{X})^{-1}\hat{X}'\hat{S}\hat{X}(\hat{X}'\hat{X})^{-1}$, where $\hat{X} \equiv Z(Z'Z)^{-1}Z'$ and the diagonal matrix $\hat{S}$ is described just above (18).

In the second column of Table 3 we use the approach of Hausman (1978) and White (1982) to test for the difference in the consistent and efficient estimates $\hat{\theta}$ and θ^*. The test statistic $(\theta^*-\hat{\theta})'[V(\hat{\theta})-V(\theta^*)]^{-1}(\theta^*-\hat{\theta})$is distributed $\chi^2(2)$ if our model is correctly specified, with a 90% critical value of 4.61. For all products the test statistic is well below its critical value, which lends support to our overall model specification.

In the third and fourth columns of Table 3, we report the Bias calculated as in (19), which equals the ratio of the cumulative price index which ignores new supplying countries when they first appear and the exact index. Focusing on the developing countries (DC), we see that for three of the imports (carbon steel sheets, stainless steel bars, and portable typewriters) the price index which ignores the new suppliers overstates the exact index by at least 50% over the entire time period (or about 2% per year). The upward bias for the other three DC imports is much less, and for the industrial countries the bias from ignoring the the new suppliers is negligible.

These results can also be seen from the plots of the price indexes for developing countries, in Figures 1-6. In each plot, the line labelled "DCOLD" is import price index calculated as in (4), i.e. ignoring new suppliers when they first appear. The line labelled "DCNEW" is the import price index calculated as in Propositions 1 and 2, i.e. taking account of the new (and disappearing) suppliers. The large upward bias of the former index for carbon steel sheets, stainless steel bars, and portable typewriters is especially evident, with a smaller upward bias for athletic shoes and cotton knit shirts. For color televisions there is a negligible bias from ignoring the new suppliers, because their market share when they initially appear is so small. For the industrial countries the plots of the price indexes look much like Figure 6, i.e. the indexes obtained by either ignoring or including the new (and disappearing) suppliers are very similar.

8. CONCLUSIONS

The contribution of this paper is threefold. First, we have obtained an expression for the exact price index for a CES aggregator function, allowing for different (but overlapping) sets of goods in the periods.[15] The formula we have obtained can be applied even when the taste parameters of the CES function are changing over time. Second, we have proposed a technique for estimating the elasticity of substitution, which permits correlation between the (unobserved) taste parameters, prices, and quantities in a usual demand and supply equilibrium. Third, we have applied this technique to six products imported into the U.S. over 1964-87, distinguishing imports from developing and industrial countries. For the developing countries, the upward bias from ignoring the new supplying countries when they first appear is quite substantial, with a cumulative value exceeding 50% for three of the imports and less for the others.

Our motivation for analysing the import data was the empirical finding that the income elasticity of demand for imports into the U.S. exceeds unity, though this result does not hold for U.S. export demand (see Houthakker and Magee, 1967). These estimates imply that equal growth in the U.S. and abroad can lead to a worsening in the U.S. balance of trade: a troublesome result for policy. Helkie and Hooper (1988), Hooper (1989) and Krugman (1989) argue that new products from developing countries may be responsible for the "artificially" high income elasticity, and the former authors introduce a proxy variable (country capital stocks) into the U.S. import demand equation to try and capture this effect. From the analysis of this paper, it would be preferable to compute the term $\ln(\lambda_t/\lambda_{t-1})$, or its cumulative value, and include it as a variable in the

[15] Analogous results can be obtained for the CES quantity index; see Feenstra and Markusen (1991), who discuss the implications of new intermediate inputs for the measurement of total factor productivity.

import demand equation.

To see why this approach is valid, take the log of the (6) or (7) and multiply by the demand elasticity η to obtain:

$$\eta \ln(c_t/c_{t-1}) = \eta \ln P(p_{t-1}, p_t, x_{t-1}, x_t) + \beta\eta \ln(\lambda_t/\lambda_{t-1}), \tag{20}$$

where (c_t/c_{t-1}) is the ratio of unit-costs (i.e. the exact price index). In our analysis $P(p_{t-1}, p_t, x_{t-1}, x_t)$ has taken on the Sato-Vartia form in (4), though in practise any other import price index could be used. Then rather than including the left side of (20) in a demand equation (since it is not observed), we can include the *two* terms on the right. Notice that the coefficient of the second term can be divided by that on the first to obtain an estimate of β, which is an alternative to the method of estimating β developed in this paper. With the two terms on the right of (20) included in an import demand equation, we would then be interested in whether the income elasticity exceeds unity or not. The author is in the process of assembling the data to calculate $\ln(\lambda_t/\lambda_{t-1})$ over a broad range of U.S. imports, and then estimate the aggregate income elasticity of import demand in this manner.

APPENDIX

Proof of Proposition 1 and Corollary 1

(i) We shall use a more general version of the unit-cost function:

$$c(p_t, I_t, b) = \left(\sum_{i \varepsilon I_t} (p_{it}/b_{it})^{-1/\beta} \right)^{-\beta}, \qquad (2')$$

where we allow the parameters b_{it} to vary over time. Calculating the cost shares as $s_{ir}(I_r) = \partial \ln c(p_r, I_r, b_r)/\partial \ln p_{ir}$, $r=t-1,t$, we obtain:

$$s_{ir}(I_r) = c(p_r, I_r, b_r)^{1/\beta} (b_{ir}/p_{ir})^{1/\beta}. \qquad (A1)$$

Note that $s_{ir}(I_r) = s_{ir}(I)\lambda_r$, $r=t-1,t$, from the definitions in (4b) and (5b). Then take the summation of (A1) over $i \varepsilon I$, and raise to the power β, to obtain:

$$\lambda_r^{\beta} \sum_{i \varepsilon I} s_{ir}(I) = \lambda_r^{\beta} = c(p_r, I_r, b_r) \left(\sum_{i \varepsilon I} (p_{ir}/b_{ir})^{-1/\beta} \right)^{\beta} = \frac{c(p_r, I_r, b_r)}{c(p_r, I, b_r)},$$

where the first equality follows since $s_{ir}(I)$ sum to unity over $i \varepsilon I$, the second from (A1), and the third by definition of $c(p_r, I, b_r)$ as the cost function obtained from (2') when the summation is over $i \varepsilon I \subset I_r$. It follows that,

$$\frac{c(p_t, I_t, b_t)}{c(p_{t-1}, I_{t-1}, b_{t-1})} = \frac{c(p_t, I, b_t)}{c(p_{t-1}, I, b_{t-1})} \left(\frac{\lambda_t}{\lambda_{t-1}} \right)^{\beta}. \qquad (A2)$$

In both Proposition 1 and Corollary 1 we have $b_{it-1} = b_{it}$ for $i \varepsilon I$, and so these results follow by showing that the ratio $c(p_t, I, b_t)/c(p_{t-1}, I, b_{t-1})$ equals the exact price index $P(p_{t-1}, p_t, x_{t-1}, x_t, I)$ in (4a). [Note that any other price index which is exact for the CES, such as Diewert (1976, eq. 4.5), could be used in place of $P(p_{t-1}, p_t, x_{t-1}, x_t, I)$].

(ii) To establish that (4a) is exact for (2), write the cost shares as $s_{ir}(I)^{\beta} = c(p_r,I,b)(b_i/p_{ir})$ from (A1), r=t-1,t. Then,

$$\frac{c(p_t,I,b)}{c(p_{t-1},I,b)} = \frac{s_{it}(I)^{\beta}\, p_{it}}{s_{it-1}(I)^{\beta}\, p_{it-1}} \quad \text{for } i \varepsilon I . \tag{A3}$$

Take a geometric mean of (A3) using the weights $w_{it}(I)$ to obtain:

$$\frac{c(p_t,I,b)}{c(p_{t-1},I,b)} = P(p_{t-1},p_t,x_{t-1},x_t,I) \prod_{i \varepsilon I} [s_{it}(I)^{\beta}/s_{it-1}(I)^{\beta}]^{\,w_{it}(I)} . \tag{A4}$$

where $P(p_{t-1},p_t,x_{t-1},x_t,I)$ is defined by (4a). To show that the product on the right of (A4) equals unity, take its natural log to obtain:

$$\sum_{i \varepsilon I} w_{it}(I)\,\beta[\ln s_{it}(I) - \ln s_{it-1}] = \frac{\beta \sum_{i \varepsilon I} [s_{it}(I) - s_{it-1}(I)]}{\sum_{i \varepsilon I} \left(\frac{s_{it}(I) - s_{it-1}(I)}{\ln s_{it}(I) - \ln s_{it-1}(I)} \right)} = 0 ,$$

where the first equality follows from the definition of $w_{it}(I)$ in (4c), and the second equality since the cost shares $s_{ir}(I)$ sum to unity over $i \varepsilon I$, r=t-1,t. It follows that (4a) is exact for (2). QED

<u>Proof of Proposition 2</u>

(i) First, we shall show that for $i \varepsilon I$ we can choose $\tilde{b}_i$ such that:

$$\frac{c(p_t,I,\tilde{b})}{c(p_{t-1},I,\tilde{b})} = P(p_{t-1},p_t,x_{t-1},x_t,I) = \prod_{i \varepsilon I} (p_{it}/p_{it-1})^{w_{it}(I)} . \tag{A5}$$

From (A4), we know that the ratio of unit-costs on the left of (A5) equals:

$$\frac{c(p_t, I, \tilde{b})}{c(p_{t-1}, I, \tilde{b})} = \prod_{i \in I} (p_{it}/p_{it-1})^{\tilde{w}_{it}(I)}, \tag{A6}$$

where $\tilde{w}_{it}(I)$ are the weights calculated as in (4c) but using the cost shares $\tilde{s}_{ir}(I) = \partial \ln c(p_r, I, \tilde{b})/\partial \ln p_{ir}$, $r=t-1,t$. Thus, a *sufficient* condition for (A5) to hold is that there exists $\tilde{b}_i$ for $i \in I$ such that,

$$w_{it}(I) = \tilde{w}_{it}(I), \quad i \in I. \tag{A7}$$

From (4c), condition (A7) will hold iff there exists $k_1 > 0$ such that,

$$\frac{s_{it}(I) - s_{it-1}(I)}{\ln s_{it}(I) - \ln s_{it-1}(I)} = k_1 \left(\frac{\tilde{s}_{it}(I) - \tilde{s}_{it-1}(I)}{\ln \tilde{s}_{it}(I) - \ln \tilde{s}_{it-1}(I)} \right), \quad i \in I. \tag{A8}$$

We can calculate the shares $\tilde{s}_{ir}(I)$ as in (A1) but using $\tilde{b}_i$ in place of b_{ir}. Let $\pi \equiv c(p_t, I, \tilde{b})/c(p_{t-1}, I, \tilde{b})$ denote the ratio of unit-costs. Then the denominator on the right side of (A8) equals $[\ln \pi - \ln(p_{it-1}/p_{it})]/\beta$. If this expression is zero then we can replace the bracketed term on the right side of (A8) by its limiting value of $\tilde{s}_{it-1}(I) = \tilde{s}_{it}(I)$, and adapt what follows to solve for $\tilde{b}_i$. So without loss of generality suppose that $[\ln \pi - \ln(p_{it-1}/p_{it})]/\beta \neq 0$. Then (A8) holds iff,

$$\left(\frac{s_{it}(I) - s_{it-1}(I)}{\ln s_{it}(I) - \ln s_{it-1}(I)} \right) \left(\ln \pi - \ln \left(\frac{p_{it-1}}{p_{it}} \right) \right) = \beta k_1 \left(\frac{(\tilde{b}_i/p_{it})^{1/\beta}}{\sum_{i \in I} (\tilde{b}_i/p_{it})^{1/\beta}} - \frac{(\tilde{b}_i/p_{it-1})^{1/\beta}}{\sum_{i \in I} (\tilde{b}_i/p_{it-1})^{1/\beta}} \right) \tag{A9}$$

for $i \in I$. We will only be able to solve for the vector $\tilde{b}$ up to a scalar multiple, so let $k_2 \equiv \sum_{i \in I} (\tilde{b}_i/p_{it})^{1/\beta}$, where we are free to choose $k_2 > 0$. Then multiplying (A9) by k_2 and rearranging terms, we can uniquely solve for $\tilde{b}_i$ as,

$$\tilde{b}_i^{1/\beta} = \left(\frac{k_2}{\beta k_1} \right) \left(\frac{s_{it}(I) - s_{it-1}(I)}{\ln s_{it}(I) - \ln s_{it-1}(I)} \right) \left(\frac{\ln \pi - \ln(p_{it}/p_{it-1})}{p_{it}^{-1/\beta} - p_{it-1}^{-1/\beta} \pi^{-1/\beta}} \right) > 0. \tag{A10}$$

In addition, we can solve for k_1 by multiplying (A10) by $p_{it-1}^{1/\beta}$, summing over $i \varepsilon I$, and rearranging terms to obtain:

$$k_1 = \frac{\pi^{-1/\beta}}{\beta} \sum_{i \varepsilon I} \left(\frac{s_{it}(I) - s_{it-1}(I)}{\ln s_{it}(I) - \ln s_{it-1}(I)} \right) \left(\frac{\ln \pi - \ln(p_{it}/p_{it-1})}{p_{it}^{-1/\beta} - p_{it-1}^{-1/\beta} \pi^{-1/\beta}} \right) > 0. \tag{A11}$$

In summary, π can be evaluated as the right side of (A5). Then k_1 can be evaluated from (A11), and for any choice of $k_2 > 0$, $\tilde{b}_i$ is obtained from (A10). It follows that (A7) holds by construction, and so (A5) also holds.

(ii) Next, we must show that $\tilde{b}_i$ evaluated as in (A10) lies between the bounds described in Proposition 2. The cost shares $s_{ir}(I)$ are evaluated as in (A1). Without loss of generality, we can normalize the price vectors p_r by a scalar multiple in each period so that $c(p_r, I, b_r) = 1$, $r = t-1, t$. Denoting $B_i \equiv b_{it}/b_{it-1}$, (A10) can be written as,

$$\tilde{b}_i^{1/\beta} = b_{it}^{1/\beta} \left(\frac{k_2}{k_1} \right) \left(\frac{p_{it}^{-1/\beta} - p_{it-1}^{-1/\beta} B_i^{-1/\beta}}{\ln B_i - \ln(p_{it}/p_{it-1})} \right) \left(\frac{\ln \pi - \ln(p_{it}/p_{it-1})}{p_{it}^{-1/\beta} - p_{it-1}^{-1/\beta} \pi^{-1/\beta}} \right) \tag{A12a}$$

$$= \pi^{1/\beta} b_{it-1}^{1/\beta} \left(\frac{k_2}{k_1} \right) \left(\frac{p_{it}^{-1/\beta} B_i^{1/\beta} - p_{it-1}^{-1/\beta}}{\ln B_i - \ln(p_{it}/p_{it-1})} \right) \left(\frac{\ln \pi - \ln(p_{it}/p_{it-1})}{p_{it}^{-1/\beta} \pi^{1/\beta} - p_{it-1}^{-1/\beta}} \right). \tag{A12b}$$

From concavity of the natural log function we have $1 - (1/z) \le \ln z \le z - 1$, and letting $z = (\pi p_{it-1}/p_{it})^{1/\beta}$ it follows that,

$$\left(1 - \pi^{-1/\beta} \left(\frac{p_{it}}{p_{it-1}} \right)^{1/\beta} \right) \le \frac{1}{\beta} \left(\ln \pi - \ln \left(\frac{p_{it}}{p_{it-1}} \right) \right) \le \left(\pi^{1/\beta} \left(\frac{p_{it1}}{p_{it}} \right)^{1/\beta} - 1 \right). \tag{A13}$$

Notice that the last bracketed terms in (A12a,b) are the reciprocal of the previous bracketed terms, but with $B_i = (b_{it}/b_{it-1})$ appearing instead of π. Suppose that $B_i \ge \pi$. Using (A13), we can show that:

$$\frac{d}{dB_i}\left(\frac{p_{it}^{-1/\beta} - p_{it-1}^{-1/\beta} B_i^{-1/\beta}}{\ln B_i - \ln(p_{it}/p_{it-1})}\right) \leq 0 \text{ and } \frac{d}{dB_i}\left(\frac{p_{it}^{-1/\beta} B_i^{1/\beta} - p_{it-1}^{-1/\beta}}{\ln B_i - \ln(p_{it}/p_{it-1})}\right) \geq 0 .$$

It follows by comparing the bracketed terms in (A12) that:

$$\pi^{1/\beta} b_{it-1}^{1/\beta}(k_2/k_1) \leq \tilde{b}_i^{1/\beta} \leq b_{it}^{1/\beta}(k_2/k_1), \tag{A14}$$

while if $B_i \leq \pi$ then these inequalities would be reversed. Express π from (A5) in the following manner:

$$\pi = \prod_{i\varepsilon I}\left(\frac{p_{it}}{p_{it-1}}\right)^{w_{it}(I)} = \prod_{i\varepsilon I}\left(\frac{b_{it}}{b_{it-1}}\right)^{w_{it}(I)} \prod_{i\varepsilon I}\left(\frac{p_{it}/b_{it}}{p_{it-1}/b_{it-1}}\right)^{w_{it}(I)}. \tag{A15}$$

A straightforward extension of (A3) and (A4) allowing for $b_{it} \neq b_{it-1}$ shows that the final product in (A15) equals $c(p_t,I,b_t)/c(p_{t-1},I,b_{t-1})$. But this is unity by our normalization of prices, so that π equals $\prod_{i\varepsilon I}(b_{it}/b_{it-1})^{w_{it}(I)}$. Then choose k_2 such that $(k_2/k_1)^{\beta} = \prod_{i\varepsilon I}(1/b_{it})^{w_{it}(I)}$. Raising (A14) to the power β, the bounds on $\tilde{b}_i$ in Proposition 2 are obtained.

(iii) Finally, use (A2) to express the right side of (7) as:

$$\frac{c(p_t,I_t,\tilde{b})}{c(p_{t-1},I_{t-1},\tilde{b})} = \frac{c(p_t,I,\tilde{b})}{c(p_{t-1},I,\tilde{b})}\left(\frac{\tilde{\lambda}_t}{\tilde{\lambda}_{t-1}}\right)^{\beta}, \tag{A16}$$

where $\tilde{\lambda}_r$ is defined as in (5b) but using the quantities $\tilde{x}_{ir} = y_t \partial c(p_r,I_r,\tilde{b})/\partial p_{ir}$ evaluated with the taste parameters $\tilde{b}$. Letting $\tilde{s}_{ir}(I_r) = \partial \ln c(p_r,I_r,\tilde{b})/\partial p_{ir}$ denote the cost shares, we can write $\tilde{\lambda}_r$ as:

$$\tilde{\lambda}_r = 1 - \sum_{i\varepsilon I_r/I} \tilde{s}_{ir}(I_r) = 1 - \sum_{i\varepsilon I_r/I} c(p_r,I,\tilde{b})^{1/\beta}(\tilde{b}_i/p_{ir})^{1/\beta}, \tag{A17}$$

where $i\varepsilon I_r/I$ means $i\varepsilon I_r$ but $i\notin I$, and the second equality follows from (A1). We assert that $\tilde{b}_i$ can be chosen for $i \in I_r/I$, $r=t-1,t$, such that $(\tilde{\lambda}_t/\tilde{\lambda}_{t-1}) = (\lambda_t/\lambda_{t-1})$. This follows since as $\tilde{b}_{ir}$ ranges between zero and infinity in (A17) then $\tilde{\lambda}_r$ ranges between unity and zero, so by appropriate choice of $\tilde{b}_{ir}$ for $I \in I_r/I$ we can obtain $\tilde{\lambda}_r = \lambda_r$, $r=t-1,t$. We have shown in part (i) that the first term on the right of (A16) equals $P(p_{t-1},p_t,x_{t-1},x_t,I)$, and with $\tilde{\lambda}_r = \lambda_r$ then (7) is established. QED

Proof of Proposition 3

(a) This is proved in the text.

(b) By definition of the instruments Z, plim(Z'X/T) is a (N-1)x2 matrix with rows $\text{plim}[\sum_{t\in\Omega i} X_{1it}/T, \sum_{t\in\Omega i} X_{2it}/T]$. From (9'), (12') and (14) we have:

$$X_{1it} = (\Delta \ln p_{it} - \Delta \ln p_{kt})^2/\beta^2 - 2(\Delta \ln p_{it} - \Delta \ln p_{kt})\tilde{\varepsilon}_{it}/\beta^2 + \tilde{\varepsilon}_{it}^2/\beta^2$$
$$= (1-\rho)^2\tilde{\varepsilon}_{it}^2/\beta^2 - 2(1-\rho)\tilde{\varepsilon}_{it}\tilde{\delta}_{it}/\beta^2 + \tilde{\delta}_{it}^2/\beta^2$$

and,

$$X_{2it} = -(\Delta \ln p_{it} - \Delta \ln p_{kt})^2/\beta + (\Delta \ln p_{it} - \Delta \ln p_{kt})\tilde{\varepsilon}_{it}/\beta$$
$$= \rho(1-\rho)\tilde{\varepsilon}_{it}^2/\beta + (1-2\rho)\tilde{\varepsilon}_{it}\tilde{\delta}_{it}/\beta - \tilde{\delta}_{it}^2/\beta.$$

Using Assumptions 1 and 2, we see that:

$$\underset{T\to\infty}{\text{plim}} \sum_{t\in\Omega_i} X_{1it}/T = n_i[(1-\rho)^2(\sigma_{\varepsilon i}^2 + \sigma_{\varepsilon k}^2) + (\sigma_{\delta i}^2 + \sigma_{\delta k}^2)]/\beta^2, \qquad \text{(A18a)}$$

$$\underset{T\to\infty}{\text{plim}} \sum_{t\in\Omega_i} X_{2it}/T = n_i[\rho(1-\rho)^2(\sigma_{\varepsilon i}^2 + \sigma_{\varepsilon k}^2) - (\sigma_{\delta i}^2 + \sigma_{\delta k}^2)]/\beta. \qquad \text{(A18b)}$$

Thus, (A18) for a given good $i \neq k$ will form one row of plim(Z'X/T), while (A18) for another good $j \neq k$ will form another row. By taking the determinant of the 2x2 matrix formed from these rows, it is straightforward to check that the two rows are linearly independent iff (15) holds. QED

Proof of Proposition 4

(i) Written in terms of the parameter estimates, (14e) becomes,

$$\hat{\theta}_1 = \hat{\rho}\hat{\beta}^2/(1-\hat{\rho}) \text{ and } \hat{\theta}_2 = \hat{\beta}(2\hat{\rho}-1)/(1-\hat{\rho}). \quad \text{(A19)}$$

Eliminate $\hat{\beta}$ in (A19) by writing $\hat{\theta}_2^2/\hat{\theta}_1 = (2\hat{\rho}-1)^2/\hat{\rho}(1-\hat{\rho})$. It follows that $\hat{\rho}^2[4+(\hat{\theta}_2^2/\hat{\theta}_1)] - \rho[4+(\hat{\theta}_2^2/\hat{\theta}_1)] + 1 = 0$. The solution to this quadratic equation is,

$$\hat{\rho} = \frac{1}{2} \pm \left(\frac{1}{4} - \frac{1}{4+(\hat{\theta}_2^2/\hat{\theta}_1)}\right)^{1/2}. \quad \text{(A20)}$$

Then $\hat{\beta} = [(1-\hat{\rho})/(2\hat{\rho}-1)]\hat{\theta}_2$, so we choose the value for $\hat{\rho}$ in (A20) which will give $\hat{\beta} > 0$, i.e. choose $\hat{\rho} > 1/2$ when $\hat{\theta}_2 > 0$, and $\hat{\rho} < 1/2$ when $\hat{\theta}_2 < 0$.

(ii) To compute the variances of $\hat{\beta}$ and $\hat{\rho}$, take the first-order approximation to (A20) around the true parameters θ_1 and θ_2 to obtain:

$$\operatorname{var}\hat{\rho} = \frac{1}{4}\left(\frac{1}{4} - \frac{1}{4+(\hat{\theta}_2^2/\hat{\theta}_1)}\right)^{-1} \left(\frac{1}{4+(\hat{\theta}_2^2/\hat{\theta}_1)}\right)^4 \times \left(\left(\frac{\hat{\theta}_2}{\hat{\theta}_1}\right)^4 \operatorname{var}\hat{\theta}_1 + 4\left(\frac{\hat{\theta}_2}{\hat{\theta}_1}\right)^2 \operatorname{var}\hat{\theta}_2 - 4\left(\frac{\hat{\theta}_2}{\hat{\theta}_1}\right)^3 \operatorname{cov}(\hat{\theta}_1,\hat{\theta}_2)\right).$$

Also from (A20) we can calculate that,

$$\operatorname{cov}(\hat{\rho},\hat{\theta}_2) = \pm\frac{1}{2}\left(\frac{1}{4} - \frac{1}{4+(\hat{\theta}_2^2/\hat{\theta}_1)}\right)^{-1/2} \left(\frac{1}{4+(\hat{\theta}_2^2/\hat{\theta}_1)}\right)^2 \left(2\left(\frac{\hat{\theta}_2}{\hat{\theta}_1}\right)\operatorname{var}\hat{\theta}_2 - \left(\frac{\hat{\theta}_2}{\hat{\theta}_1}\right)^2 \operatorname{cov}(\hat{\theta}_1,\hat{\theta}_2)\right),$$

with the ± sign being chosen as in (A20). Since $\hat{\beta} = [(1-\hat{\rho})/(2\hat{\rho}-1)]\hat{\theta}_2$, taking a first-order approximation we have:

$$\operatorname{var}\hat{\beta} = \left(\frac{1-\hat{\rho}}{2\hat{\rho}-1}\right)^2 \operatorname{var}\hat{\theta}_2 + \frac{\hat{\theta}_2^2}{(2\hat{\rho}-1)^4}\operatorname{var}\hat{\rho} - \frac{2\hat{\theta}_2(1-\hat{\rho})}{(2\hat{\rho}-1)^3}\operatorname{cov}(\hat{\rho},\hat{\theta}_2).$$

Combining the above results provides the formula for $\operatorname{var}\hat{\beta}$. QED

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Table 1. Data on U.S. Imports

	Developing Countries			Industrial Countries		
	1967	1977	1987	1967	1977	1987
ATHLETIC SHOES						
No. of Suppliers	5	11	17	11	9	10
Value ($million)	0.07	191	966	3.2	49	79
$\sum_t \ln(\lambda_t/\lambda_{t-1})$	-0.20	-0.40	-0.40	-0.008	0.003	-0.023
COTTON KNIT SHIRTS						
No. of Suppliers	11	21	40	13	9	11
Value ($million)	2.8	59	462	3.8	4.3	25
$\sum_t \ln(\lambda_t/\lambda_{t-1})$	-0.36	-0.50	-0.53	-0.006	0.036	0.023
CARBON STEEL SHEETS						
No. of Suppliers	2	6	12	9	12	14
Value ($million)	0.13	101	247	139	816	640
$\sum_t \ln(\lambda_t/\lambda_{t-1})$	0	-0.88	-1.15	0.006	-0.019	-0.049
STAINLESS STEEL BARS						
No. of Suppliers	1	4	6	9	8	9
Value ($million)	.0014	1.3	5.3	3.5	24	35
$\sum_t \ln(\lambda_t/\lambda_{t-1})$	0	-1.02	-1.02	-0.031	0.050	0.037
PORTABLE TYPEWRITERS						
No. of Suppliers	5	9	10	12	10	4
Value ($million)	0.75	17	64	41	94	33
$\sum_t \ln(\lambda_t/\lambda_{t-1})$	-0.23	-0.78	-0.83	-0.001	0.002	0.018
COLOR TELEVISIONS [a]						
No. of Suppliers	2	4	9	4	3	5
Value ($million)	0.46	46	513	63	258	148
$\sum_t \ln(\lambda_t/\lambda_{t-1})$	0	0.032	-0.021	0	-0.0005	0.024

Notes:

a For color televisions, the 1967 data is for 1970, the first year in which data is available for over 17" screens.

Table 2. Parameter Estimates

	L^a	θ_1	θ_2	β	ρ
ATHLETIC SHOES	420	0.071	-0.034	0.283	0.468
		(0.014)	(0.195)	(0.119)	(0.180)
	Efficient[b]	0.065	0.064	0.225	0.562
		(0.0058)	(0.056)	(0.030)	(0.054)
COTTON KNIT SHIRTS	651	0.124	0.029	0.337	0.521
		(0.030)	(0.142)	(0.101)	(0.103)
	Efficient	0.111	0.068	0.300	0.551
		(0.011)	(0.047)	(0.034)	(0.036)
CARBON STEEL SHEETS	353	-0.00018	-0.556	0.555	-0.00057
		(0.013)	(0.268)	(0.257)	(0.041)
	Efficient	0.0050	-0.368	0.381	0.033
		(0.0027)	(0.069)	(0.066)	(0.021)
STAINLESS STEEL BARS	220	0.074	-0.356	0.503	0.227
		(0.018)	(0.225)	(0.176)	(0.123)
	Efficient	0.066	-0.399	0.525	0.193
		(0.010)	(0.103)	(0.095)	(0.041)
PORTABLE TYPEWRITERS	312	0.220	-0.061	0.500	0.468
		(0.067)	(0.137)	(0.075)	(0.063)
	Efficient	0.211	-0.124	0.526	0.433
		(0.039)	(0.082)	(0.046)	(0.032)
COLOR TELEVISIONS	133	0.207	0.994	0.178	0.868
		(0.057)	(0.376)	(0.052)	(0.059)
	Efficient	0.183	0.940	0.165	0.870
		(0.031)	(0.167)	(0.027)	(0.028)

Notes

Standard errors are in parentheses.

a L is the number of observations, over years and supplying countries.

b The efficient estimates correct for heteroscedasticity.

Table 3. Test and Summary Statistics

	Auto-Correlation[a]	Specification Test[b]	Index Bias[c] DC	IC
ATHLETIC SHOES	3.79	0.301	1.09	1.01
COTTON KNIT SHIRTS	1.80	0.216	1.17	0.99
CARBON STEEL SHEETS	2.47	0.525	1.55	1.02
STAINLESS STEEL BARS	5.71	0.315	1.71	0.98
PORTABLE TYPEWRITERS	1.79	0.318	1.55	0.99
COLOR TELEVISIONS	2.78	0.310	1.00	1.00

Notes

a Distributed $\chi^2(3)$ under the null hypothesis of no autocorrelation, with a 90% critical value of 6.25.

b Computed as $(\theta^* - \hat{\theta})'[V(\hat{\theta}) - V(\theta^*)]^{-1}(\theta^* - \hat{\theta})$, where θ^* is the efficient estimate and $\hat{\theta}$ is consistent. This statistic is distributed $\chi^2(2)$ under the null hypothesis of no specification error, with a 90% critical value of 4.61.

c Equals $\exp[-\beta^* \sum_t \ln(\lambda_t/\lambda_{t-1})]$, and is the ratio of the cumulative price index which ignores new supplying countries when they first appear and the exact index. DC refers to developing countries, and IC to industrial countries.

FIGURE 1

ATHLETIC SHOES -- DEVELOPING COUNTRIES

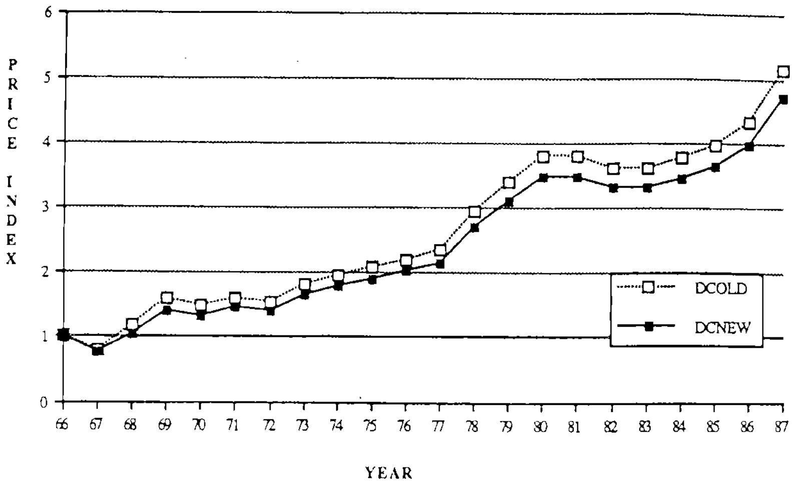

FIGURE 2

COTTON KNIT SHIRTS -- DEVELOPING COUNTRIES

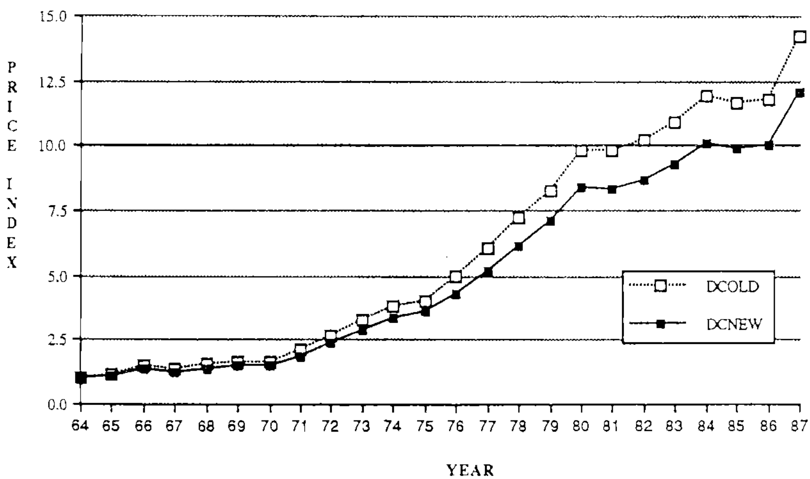

FIGURE 3

CARBON STEEL SHEETS -- DEVELOPING COUNTRIES

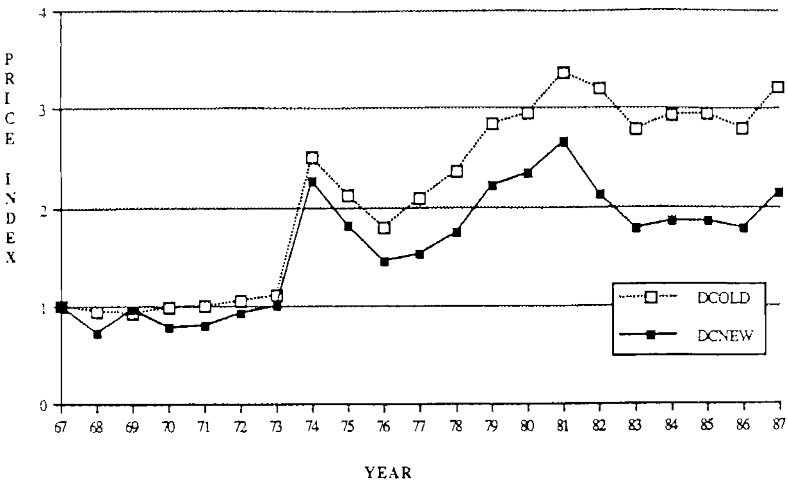

FIGURE 4

STAINLESS STEEL BARS -- DEVELOPING COUNTRIES

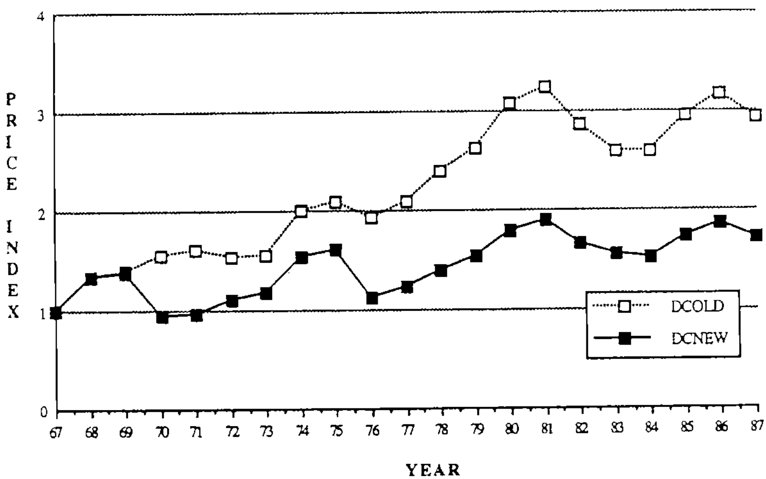

FIGURE 5

PORTABLE TYPEWRITERS -- DEVELOPING COUNTRIES

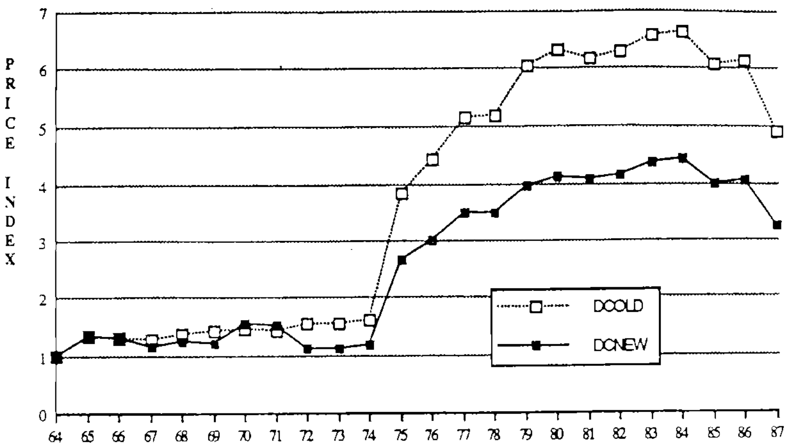

FIGURE 6

COLOR TV -- DEVELOPING COUNTRIES