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ABSTRACT

At least three types of precautionary motives are directly relevant to an agent's demand for assets. (I.) The precautionary saving motive, or prudence, can cause an agent to respond to a risk by accumulating more wealth. (II.) The desire to moderate total exposure to risk, or temperance, can cause an agent to respond to an unavoidable risk by reducing exposure to other risks even when the other risks are statistically independent of the first. (III.) The precautionary demand for liquidity can cause an agent to respond to a risk by holding more money.

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A *precautionary motive* is any aspect of an agent's preferences which causes a risk to affect decisions *other than* the decision of how strenuously to avoid the risk itself and risks correlated with it (which is governed by *risk aversion*.) A precautionary motive leads an agent to respond to a risk by making adjustments that will help to reduce the expected cost of the risk. *Certainty equivalence*, which is the absence of precautionary motives, arises when an agent has no way to affect the expected cost of a risk.

At least three types of precautionary motives are directly relevant to an agent's demand for assets. (I.) The precautionary saving motive, or *prudence*,[1] can cause an agent to respond to a risk by accumulating more wealth. (II.) The desire to moderate total exposure to risk, which I will call *temperance*, can cause an agent to respond to an unavoidable risk by reducing exposure to other risks even when the other risks are statistically independent of the first. (III.) The precautionary demand for liquidity can cause an agent to respond to a risk by holding more money.

I. Prudence

The simplest model of precautionary saving is a two-period model of the consumption/saving choice in which labor income in the second period is uncertain, but the real interest rate is fixed at zero.[2] In particular, let an expected utility maximizer face the problem

$$\max_x \; u(w - x) + \mathbf{E}\, v(x + \tilde{y}). \tag{1}$$

The decision variable x is saving, $u(\cdot)$ and $v(\cdot)$ are the first-period and second-period utility functions, w is initial wealth (including first-period labor income, which is already known), $\tilde{y}$ is exogenous second-period labor income, $w - x$ is first-period consumption and $x + \tilde{y}$ is second-period consumption. The first order condition for the optimal level of saving x^* is

$$u'(w - x^*) = \mathbf{E}\, v'(x^* + \tilde{y}). \tag{2}$$

The first order condition is shown graphically in Figure 1. The slopes of the curves follow from the assumptions $u''(\cdot) \leq 0$ and $v''(\cdot) \geq 0$.

The condition for additional uncertainty in second-period labor income to lead to increased saving is for second period marginal utility $v'(\cdot)$ to be a convex function.[3] (When $v(\cdot)$ is thrice-differentiable, the condition is $v'''(\cdot) \geq 0$.) If $v'(\cdot)$ is convex, then any mean-preserving spread in its

[1] The first use of the term "prudence" for the precautionary saving motive is in an early version of Kimball (1990a).

[2] If all quantities are measured in present value terms, all of the following equations remain true even if the real interest rate is fixed at some rate other than zero.

[3] Leland (1968) and Rothschild and Stiglitz (1971).

argument will raise its expected value (by replacing the marginal utility at any point by an average of marginal utilities to the left and right of that point);[4] that is, if the mean of ϵ conditional on y is always zero ($\mathrm{E}(\tilde{\epsilon}|y) = 0$) then

$$\mathrm{E}\, v'(x + \tilde{y} + \tilde{\epsilon}) \geq \mathrm{E}\, v'(x + \tilde{y}). \tag{3}$$

As shown in Figure 1, such an increase in expected marginal utility at any given level of saving x results in an increase in the optimal level of saving to x^{**}.

A. Measuring Prudence

Not only does convex marginal utility guarantee a positive precautionary saving motive, *more* convex marginal utility induces a *stronger* precautionary saving motive. The marginal utility function $v_2'(\cdot)$ is "more convex" than $v_1'(\cdot)$ if $v_2'(x) = \phi(v_1'(x))$ where $\phi(\cdot)$ is a monotonically increasing, convex function. If $v_2'(\cdot)$ is more convex than $v_1'(\cdot)$, it guarantees that, at least when starting from a situation of certain second-period income y,[5] the set of (not necessarily zero-mean) random variables that will raise $\mathrm{E}\, v_2'(\cdot)$ is larger than the set of random variables that will raise $\mathrm{E}\, v_1'(\cdot)$. To show that every random variable that raises $\mathrm{E}\, v_1'(\cdot)$ also raises $\mathrm{E}\, v_2'(\cdot)$ (but not the converse) it is enough to show that any random variable that leaves $\mathrm{E}\, v_1'(\cdot)$ unchanged raises $\mathrm{E}\, v_2'(\cdot)$. Given any mean-zero random variable $\tilde{\epsilon}$ one can typically choose a constant[6] psi_1^* so that

$$\mathrm{E}\, v_1'(x + y + \tilde{\epsilon} + \psi_1^*) = \mathrm{E}\, v_1'(x + y); \tag{4}$$

that is, so that the composite random variable $\tilde{\epsilon} + \psi_1^*$ has no effect on $\mathrm{E}\, v_1'(\cdot)$. Then

$$\begin{aligned} \mathrm{E}\, v_2'(x + y + \tilde{\epsilon} + \psi_1^*) &= \mathrm{E}\, \phi(v_1'(x + y + \tilde{\epsilon} + \psi_1^*)) \\ &\geq \phi(\mathrm{E}\, v_1'(x + y + \tilde{\epsilon} + \psi_1^*)) \\ &= \phi(v_1'(x + y)) \\ &= v_2'(x + y). \end{aligned} \tag{5}$$

The middle inequality follows from Jensen's inequality—the fact that uncertainty tends to raise the expected value of convex functions such as $\phi(\cdot)$. Equation (4) says that the quantity ψ_1^* is a *compensating precautionary premium* which cancels out the effect of the mean-zero risk $\tilde{\epsilon}$ on

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[4] Rothschild and Stiglitz (1970) and (1971).

[5] Kimball (1990a) discusses conditions under which the stipulation of initial certainty can be relaxed.

[6] One might note that if $v_1'(\cdot)$ is strictly convex, ψ_1^* must be positive, but this has no role in the following proof.

$\mathbf{E}\,v_1'(\cdot)$. But (5) says that ψ_1^* is inadequate to cancel out the effect of $\tilde{\epsilon}$ on $\mathbf{E}\,v_2'(\cdot)$. Graphically, the rightward shift of the $\mathbf{E}\,v'(\cdot)$ curve in Figure 1 due to the risk $\tilde{\epsilon}$ is equal to ψ_1^* for the second-period utility function $v_1(\cdot)$ but is greater than ψ_1^* for the more concave second-period utility function $v_2(\cdot)$. The magnitude of the shift of the $\mathbf{E}\,v'(\cdot)$ curve determines, in turn, the amount of additional precautionary saving caused by $\tilde{\epsilon}$.

The foregoing proof that more convex marginal utility induces a stronger precautionary saving motive is identical in form to Pratt's (1964) proof that more concave utility induces stronger risk aversion. The principle that the concavity or convexity of different functions can be measured by the Pratt-Arrow measure of absolute risk aversion[7] $\frac{-v''(\cdot)}{v'(\cdot)}$ can also be applied to precautionary saving. The corresponding measure of the convexity of *marginal* utility is *absolute prudence*[8]: $\frac{-v'''(\cdot)}{v''(\cdot)}$. In particular, $v_2'(\cdot)$ is more convex than $v_1'(\cdot)$ if and only if $-\frac{v_2'''(\cdot)}{v_2''(\cdot)} \geq -\frac{v_1'''(\cdot)}{v_1''(\cdot)}$,[9] just as one utility function is more concave than another if its absolute risk aversion is greater.

B. Prudence and Risk Aversion

The strength of the precautionary saving motive can be compared to the strength of risk aversion by comparing absolute prudence to the Pratt-Arrow measure of absolute risk aversion. Straightforward calculation shows that

$$\left(\frac{-v'''(x)}{v''(x)}\right) - \left(\frac{-v''(x)}{v'(x)}\right) = -\frac{d}{dx}\ln\left(\frac{-v''(x)}{v'(x)}\right). \tag{6}$$

In words, the difference between absolute prudence and absolute risk aversion is equal to the rate at which the logarithm of absolute risk aversion decreases with second-period wealth. Thus, the assumption of decreasing absolute risk aversion—which is necessary and for a positive wealth elasticity of risky investment—implies that absolute prudence is greater than absolute risk aversion, and therefore that expected marginal utility reacts more strongly to risk than expected utility does. In particular, a risk that is compensated so as to leave expected utility unaffected will always raise expected second-period marginal utility and lead to increased saving, as long as second-period utility $v(\cdot)$ has decreasing absolute risk aversion.

The effect on saving of a risk to which an agent is indifferent is the *Drèze-Modigliani (1972) substitution effect.* Intuitively, the reason for a connection between a positive Drèze-Modigliani substitution effect and a positive wealth elasticity of risky investment is that both reflect the same

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[7] Pratt (1964) and Arrow (1965).

[8] Kimball (1990a).

[9] This statement assumes that $v_1''(\cdot) < 0$ and $v_2''(\cdot) < 0$.

complementarity—that is, the same positive interaction—between saving and risk.[10] This can be shown most clearly when the quantity of risk is continuously variable (though the same logic applies to the case of discrete changes in the quantity of risk to which an agent is initially indifferent). Let

$$J(x, \alpha) = u(w - x) + \mathrm{E}\, v(x + \alpha \tilde{z}), \tag{7}$$

where α is a continuously variable amount of the risk $\tilde{z}$, and x is saving, as before. Beginning at the global optimum for both x and α, consider the dual thought experiments of (i) adjusting α optimally when x is forcibly altered by a small amount and (ii) adjusting x optimally when α is forcibly altered by a small amount. Since saving, x, is the portion of wealth relevant for portfolio choices, experiment (i) indicates the sign of the wealth elasticity of risky investment. Since the first order condition for α guarantees that a small change in α away from the optimum has no effect on expected utility on the margin, experiment (ii) isolates the Drèze-Modigliani substitution effect of risk on saving.

(i) When α is adjusted optimally in response to an enforced change in x according to the function $\mathbf{a}(x)$, the first-order condition $J_\alpha(x, \mathbf{a}(x)) = 0$ must hold for any value of x. Differentiating with respect to x and solving for $\mathbf{a}'(x)$ reveals that

$$\mathbf{a}'(x) = J_{\alpha x}(\cdot)/\big(-J_{\alpha\alpha}(\cdot)\big). \tag{8}$$

Concavity of the underlying utility functions $u(\cdot)$ and $v(\cdot)$ guarantees that $-J_{\alpha\alpha}(\cdot) \geq 0$; therefore, the sign of $\mathbf{a}'(x)$ is the same as the sign of $J_{\alpha x}(\cdot)$. Decreasing absolute risk aversion guarantees that $\mathbf{a}'(x) \geq 0$ and therefore that $J_{\alpha x}(\cdot) \geq 0$.

(ii) When x is adjusted optimally in response to an enforced change in α according to the function $\mathbf{x}(\alpha)$, the first-order condition $J_x(\mathbf{x}(\alpha), \alpha) = 0$ must hold for any value of α. Differentiating with respect to α and solving for $\mathbf{x}'(\alpha)$ reveals that

$$\mathbf{x}'(\alpha) = J_{x\alpha}(\cdot)/\big(-J_{xx}(\cdot)\big). \tag{9}$$

Concavity of the underlying utility functions $u(\cdot)$ and $v(\cdot)$ guarantees that $-J_{xx}(\cdot) \geq 0$; therefore, the sign of $\mathbf{x}'(\alpha)$ is the same as the sign of $J_{x\alpha}(\cdot)$. At the global optimum (x^*, α^*),

$$J_{x\alpha}(\mathbf{x}(\alpha^*), \alpha^*) = J_{x\alpha}(x^*, \alpha^*) = J_{\alpha x}(x^*, \alpha^*) = J_{\alpha x}(x^*, \mathbf{a}(x^*)). \tag{10}$$

[10] Complementarity between saving and risk can also be characterized as substitutability between saving and insurance.

Therefore, at the global optimum $\mathbf{x}'(\alpha^*)$ has the same sign as $\mathbf{a}'(x)$. Decreasing absolute risk aversion guarantees that $\mathbf{x}'(\alpha^*) \geq 0$—the Drèze-Modigliani substitution effect is positive.

The key to the relationship between the sign of $\mathbf{a}'(x^*)$, the effect of an enforced increase in saving on the optimal level of risky investment, and $\mathbf{x}'(\alpha^*)$, the effect of an enforced increase in risky investment on saving, is the symmetry of the cross derivatives: $J_{x\alpha}(x^*, \alpha^*) = J_{\alpha x}(x^*, \alpha^*)$. Intuitively, additional risk makes saving more attractive if and only if additional saving makes risk more attractive because both effects reflect the same (Pareto-Edgeworth-Samuelson[11]) complementarity between saving and risk in the case of *decreasing* absolute risk aversion or the same substitutability between saving and risk in the case of *increasing* absolute risk aversion. The complementarity between saving and risk engendered by decreasing absolute risk aversion makes the precautionary saving motive stronger than risk aversion in the sense that all undesirable risks cause increased saving and even some risks that have a high enough mean to be desirable cause increased saving.

II. Temperance

Labor income risk or other types of risks that are difficult to avoid can affect not only the total amount of saving, but also the fraction of saving devoted to risky investment. It is called *hedging* when an unavoidable risk affects the freely chosen quantity of investment in another risk due to a correlation between the two risks. But it is reasonable to think that an unavoidable risk might lead an agent to reduce exposure to another risk even if the two risks are statistically independent.[12] This tendency can be called *temperance*, in the sense of moderation in accepting risks.

Temperance cannot be measured as neatly as risk aversion or prudence, but it can be shown that just as decreasing absolute risk aversion implies that prudence is greater than risk aversion, decreasing absolute *prudence* implies that *temperance* is greater than *prudence*. More precisely, decreasing absolute prudence implies that any two statistically independent risks (or certain kinds of increases in risk) which individually raise expected marginal utility are substitutes in the sense of interacting negatively in their effect on expected utility.[13] This means that any risk that leads to increased precautionary saving reduces an agent's demand for risky assets both in absolute terms and as a fraction of total saving.[14]

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[11] Samuelson (1974) discusses different notions of complementarity. Pareto-Edgeworth complementarity is defined as a positive cross derivative of a utility function. Samuelson (1974) focusses on the cross derivative of a von Neumann-Morgenstern utility function in his preferred sixth definition of complementarity, which can be stretched to apply to an indirect utility function derived from a von Neumann-Morgenstern utility function, as here.

[12] Pratt and Zeckhauser (1987) make the original argument to this effect.

[13] Kimball (1990b.)

[14] Elmendorf and Kimball (1991).

Intuitively, the reason for a connection between temperance and decreasing absolute prudence is that, when both risks are present, the first risk, interacting with decreasing absolute prudence, tends to make expected marginal utility conditional on the realization of the second risk fall faster.[15] Since risk aversion results from a negative correlation between marginal utility and the realization of a risk, the faster expected marginal utility conditional on the realization of the second risk falls, the more risk averse the agent will act toward the second risk.

III. Precautionary Demand for Liquidity

In the presence of unavoidable risk, prudence leads to increased saving. Temperance channels most of the extra saving into safe assets and can even lead to an absolute shift out of risky assets. The *precautionary demand for liquidity* leads to increased holding of *liquid* assets in response to risk.[16]

Unlike prudence and temperance, the precautionary demand for liquidity does not depend critically on the shape of the underlying utility function. It arises instead from the convexity induced in the marginal value of money by liquidity constraints. Consider a simple cash-in-advance model in which the level of cash balances must be chosen before the size of a random cash infusion θ is known. Pushing the consumption/saving decision emphasized above into the background, let the direct utility function be[17]

$$U(c) + final\ wealth = U(c) + w + \theta - c - im, \tag{11}$$

where c is immediate consumption, $U(c)$ is the concave utility from immediate consumption, w is initial wealth, i is the cost of money holding, and m is the initial quantity of money the agent chooses to hold. The indirect utility of money is

$$V(m, \theta) = \max_{c}\ U(c) + w + \theta - c - im \qquad s.t.\ c \leq m + \theta. \tag{12}$$

[15] Decreasing absolute prudence means that the first risk tends to raise expected marginal utility more at low levels of wealth than at high levels of wealth. Thus, expected marginal utility tends to fall faster with wealth in the presence of the first risk than in its absence. When looking at expected marginal utility conditional on the realization of the second risk, the realization of the second risk has an effect similar to the level of initial wealth.

[16] I cannot review here all of the literature on the precautionary demand for liquidity. One recent article with many references is Svensson (1985).

[17] If the direct utility function were in turn derived from a multiperiod optimization problem, the utility of final wealth would, in general, be convex rather than linear, but if the length of a period is short compared to the length of an agent's horizon, the marginal utility of final wealth will be little affected by the one-period disturbance in the cash infusion θ, so that assuming linear utility of final wealth is a sensible approximation here. Of course, this means that the precautionary saving and risk crowding effects described above are important only when there is substantial uncertainty about *permanent* labor income. By contrast, the precautionary demand for liquidity can arise even in response to uncertainty about temporary fluctuations in cash infusions and cash requirements.

The Lagrangian for this maximization problem is

$$\mathcal{L} = U(c) + w + \theta - c - im + \lambda(m + \theta - c). \tag{13}$$

The first-order condition for the optimal choice of c is

$$U'(c) - 1 = \lambda. \tag{14}$$

The Kuhn-Tucker condition for the Lagrange multiplier λ is

$$\lambda = 0 \quad \text{or} \quad \lambda \geq 0 \textit{ and } c = m + \theta. \tag{15}$$

The envelope theorem, together with (14) and (15), indicates the marginal value of money:

$$V_m(m, \theta) = \lambda - i = \max(U'(m + \theta) - 1 - i, -i). \tag{16}$$

Figure 2 illustrates the marginal value of money graphically. Even if $U(c)$ is quadratic, so that the underlying marginal utility of consumption is linear, the marginal value of money is a convex function of $m + \theta$ because of the kink induced where the cash-in-advance constraint stops binding.[18] Since the problem of choosing the optimal initial amount of money,

$$\max_m \mathrm{E}\, V(m, \tilde{\theta}), \tag{17}$$

has the first-order condition

$$\mathrm{E}\, V_m(m, \tilde{\theta}) = 0, \tag{18}$$

convexity of the marginal value of money as a function of $m + \theta$ means that an agent will react to uncertainty about the size of the cash infusion θ by increasing initial money holding m.

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[18] If $U'(c)$ is itself convex, the cash-in-advance constraint interacting with the convexity of $U'(c)$ leads to a precautionary demand for liquidity even if the constraint is certain to bind, making the kink irrelevant.

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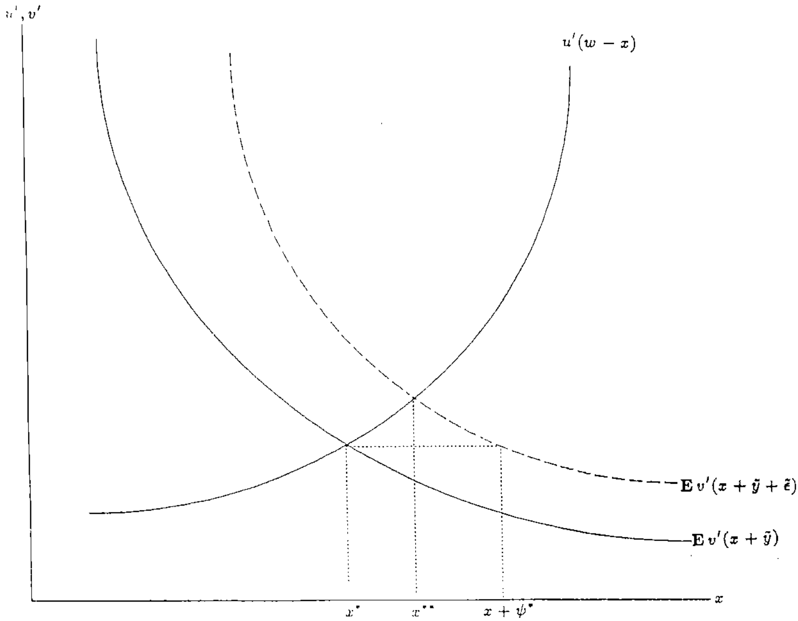

u', v'
$u'(w - x)$
$\mathbf{E}\, v'(x + \tilde{y} + \tilde{\epsilon})$
$\mathbf{E}\, v'(x + \tilde{y})$
x
x^*
x^{**}
$x + \psi^*$

Figure 1

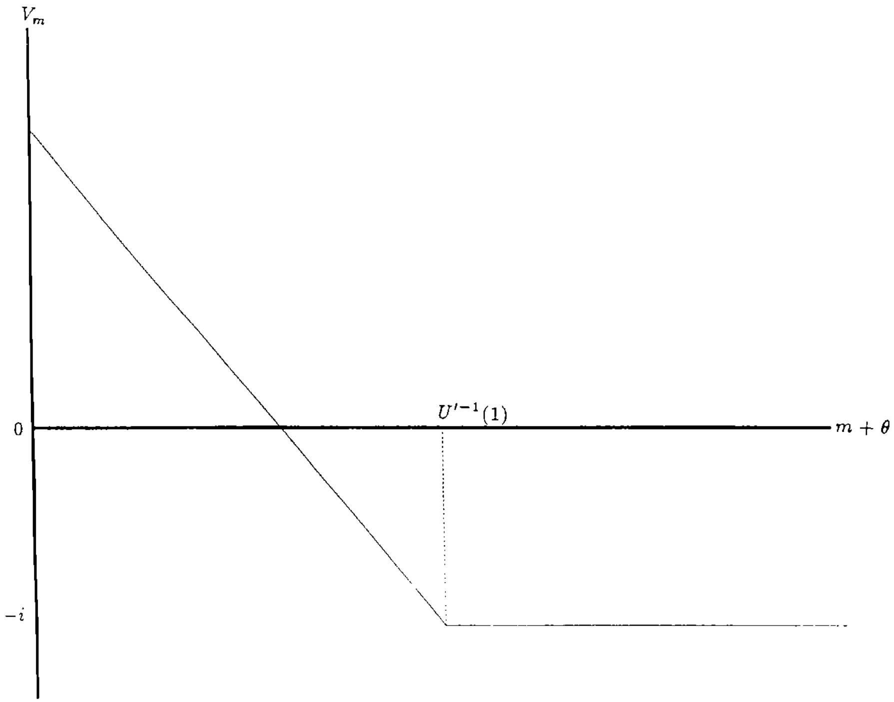

Figure 2