NBER WORKING PAPERS SERIES

A MICRO ECONOMETRIC MODEL OF CAPITAL UTILIZATION AND RETIREMENT

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Working Paper No. 3568

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
December 1990

The first version of this paper was presented at the Econometric Society Winter Meetings in December 1988 at New York under a slightly different title. I am thankful to the Amdhal Corporation and Mr. Sharad Gavali for running my programs on AMDAHL 1200 VECTOR PROCESSOR. Discussions with and comments from Satya Das, Jeffrey Dubin, Charles Manski, Ariel Pakes and John Rust are appreciated. I am also thankful to the two referees of this journal for many constructive comments. Remaining errors are mine. This paper is part of NBER's research program in Productivity. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

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ABSTRACT

The paper presents a micro econometric model of capital utilization and retirement. Some estimates of a firm's discrete decision problem with regard to an existing piece of capital--whether to operate, hold idle or retire it--are obtained, in the context of the US cement industry, by solving a discrete choice stochastic dynamic programming model. The estimates are then used to simulate effects of product and input price changes, and changes in the size and age of capital on a firm's propensity to operate, hold idle and retire capital.

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A MICRO ECONOMETRIC ANALYSIS OF CAPITAL UTILIZATION AND RETIREMENT

1. Introduction

The study of capital utilization and retirement is important both at the macro and micro level of an economy. At the macro level it is used to explain and predict changes in business cycle activities and in the productivity of factors of production. At the micro level such studies can be used to analyze the impact of demand and cost shocks induced by policy changes and other external factors on the general performance of particular industries in terms of aggregate output, factor employment and productivity and temporary or permanent plant closings. While most studies of capital utilization and retirement are macro oriented this paper attempts to estimate a model of these decisions at the (micro) level of individual production units.

Besides sharing some obvious similarities, capital utilization and retirement at the micro level differs markedly from those at the macro level. One is the lumpiness at zero utilization or complete idling of a production unit that may result from rational choice of an individual firm, the macro counterpart of which is highly unrealistic.[1] For instance, in the data set used in this paper, in the US cement industry there are many

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[1] Existing papers, both theoretical and empirical, that endogeneize capital utilization assume typically that the rate of utilization is continuous and smooth and is necessarily positive at the optimum, e.g., Nadiri and Rosen (1969), Epstein and Denny (1980), Abel (1981), Berndt and Morrison (1981), Bernstein (1983), Merrick (1984) and Morrison (1985). Bentancourt and Clague (1981) regard utilization as a positive but discrete valued variable. The "standard" neoclassical model of Jorgenson (1963, 1974) and its numerous extensions that do not endogeneize capital utilization presume full capital utilization, and moreover, capital retirement choice is ignored in them.

examples of "kilns" (production units) remaining idle throughout the year (see Table 1, column 3).[2] It is worth pointing out here that in another context—namely, labor employment decision by firms—Hamermesh (1989) finds that in the presence of lumpiness (or corner solution) at the appropriate micro level, structural inferences based on more aggregate models that assume smooth and continuous responses are not reliable, and therefore, for estimating behavioral responses he advocates econometric studies at the micro level that allows for corner solutions to occur. The occurrence of such corner solutions at the level of individual production units implies *nonsmooth* changes in the rate of utilization of capital for a plant (averaged over its production units); in such cases the appropriate level of disaggregation for studying utilization should be the level of an individual production unit rather than a plant.

Another important difference with capital utilization and retirement studies at the macro level is that the capital retirement choice in them is typically wrapped with either the replacement decision (e.g. Feldstein and Foot (1971) and Feldstein and Rothschild (1974)) or embodied technological progress (e.g. Salter (1960) and Solow (1969)).[3] But it is quite feasible—and indeed may be optimal—for an individual firm to retire a piece of equipment without replacing it or without any consideration of technological progress.[4] Besides, at the micro level the retirement of a

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[2] A kiln is a huge rotary steel tube lined with fire brick which is the most important piece of capital in cement production and defines the production unit for cement. Details of cement technology are given in Section 3.

[3] Malcomson (1975) and Nickell (1975) are exceptions. Rust (1987), which is a micro study, also treats retirement and replacement as simultaneous decisions.

[4] For instance, in the data set, 58 kilns were retired while only 3 new ones were bought, and also, plants that retired cement kilns were not necessarily

production unit is a discrete choice.

Hence in tracking capital utilization and retirement at the micro level we develop a model that takes into account the unique features of these decisions as discussed above. In particular, this paper (a) focuses on a firm's decision with regard to its *existing* capital rather than new capital in the form of replacement or addition to capacity,[5] and (b) recognizes that at a given point in time a firm faces three *discrete* choices with regard to its existing capital: operate, hold idle or retire. The simultaneity of these decisions and the methodology used in estimating the parameters of the decision rules are considered novel.

The object of our analysis is the US cement industry. Since cement plants commonly consist of heterogeneous kilns for which corner solutions for utilization are not rare, and further a plant's capital retirement decision is the result of binary choices (retire or not) over each kiln, we study utilization and retirement decisions at the kiln level rather than at the plant level. More specifically the paper obtains estimates of the parameters of a firm's decision rules for operating, holding idle and retiring existing cement kilns. These estimates are then used simulate the impact of variations in output and input prices on the probabilities of operating, holding idle or retiring a cement kiln.

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the ones that bought new kilns. Moreover, there has not been any technological breakthrough in cement production for over two decades or so, and yet retirement of kilns is observed in the mid to late 70s (see Table 1) presumably due to oil price shocks.

[5] The separation between these decisions will be discussed in Section 2.

2. A Model of Capital Utilization and Retirement

2.1. Assumptions

A1. A firm has one or more production units and the cost function associated with one unit is independent of another.

A2. There are fixed costs of maintaining or "holding" a production unit.

Such costs are necessary for retirement to be a viable choice, because otherwise there would be no incentive for a firm to retire or close a production unit that is currently losing as long as there is a positive probability of making positive profits in the future.

A3. Firms are risk neutral and hence maximize expected discounted sum of profits.

A4. Firms are perfectly competitive in the output and input markets.

A5. There is uncertainty about future output and input prices and they follow a joint first order Markov process.[6]

A6. Depreciation of capital or a production unit is in the form of 'input decay' (in the terminology of Feldstein and Rothschild), i.e., it requires increasing amount of one or more variable inputs to produce a unit of output as it gets older.[7]

Assumptions A1 and A4 imply that a firm's profit function is separable

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[6] Even though the theory below holds for any joint first order Markov process, the jointness assumed in the empirical analysis is limited due to insufficient variation in our data on prices. Details are given in Section 4.

[7] It may be remarked that this way of modeling depreciation is no less reasonable than the usual Jorgenson type assumption of 'output decay'. Output decay implies that same amounts of capital of different ages differ in the quantity of services they provide, whereas input decay implies that they differ in quality (even in the absence of technical progress). As Nickell recognizes, the difference in quality implicit in the notion of input decay is critical for capital retirement.

in its production units. As risk neutrality (A3) further implies that the marginal valuation of profits is constant, it follows that the profitability of any production unit is independent of that of another, existing or new. In other words, unlike in the existing models in which the interdependence of marginal revenue products of existing and new capital implies jointness of utilization, retirement and new investment decisions, these decisions are separated in our model.[8] While such separation in the decision making is plausible as they directly follow from A1, A2 and A3 it may not be realistic in many situations. For instance if a firm possessed some market power its marginal revenue would be dependent on the total output from all production units and hence the separation property would not hold. But for our purpose it is attractive as it simplifies an otherwise high dimensional (and computationally highly intractable!) joint decision problem of utilization and retirement of all existing production units and new investment to a trinomial choice—operate, hold idle or retire, for each existing production unit.[9]

2.2. The Discrete-Choice Dynamic Programming Model

The decision problem of a firm is then to choose a sequence of decision rules $I = \left\{ i_t = f_t(x_t, \varepsilon_t, \theta) \right\}_{t=0}^{T}$ to maximize the expected discounted sum of profits from each production unit given by

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[8] More specifically, the jointness in the existing literature is implied from the strictly concave revenue function or the total equilibrium output being strictly concave in capital or both.

[9] It is however recognized that with further development in computing technology and more experience with estimating stochastic dynamic programming models, the formulation and estimation of the simultaneous choices of capital utilization and retirement and new investment in capital could be possible in the near future.

$$\max_{I} \; E_0\left\{\sum_{t=0}^{T} \beta^t u(x_t, \varepsilon_t, i_t, \theta)\right\}, \qquad (1)$$

where

E_0: expectation based on information available at the current period 0

T : end of the physical lifetime of a production unit (about 80 years for a cement kiln)

β : discount rate, $0 \le \beta \le 1$

$u(.)$: (real) instantaneous profit function of the production unit

x : vector of observed exogenous variables--output price, input prices and observed characteristics of a production unit

ε : the vector $(\varepsilon_0, \varepsilon_1, \varepsilon_2)$ where ε_i is the unobserved random component of costs associated with choice i, $i \in C \equiv$ {operate, hold idle or retire}

θ : parameters of the profit function to be estimated.

The instantaneous profit function u(.) equals

$$u(.) = \begin{cases} P_t Q_t - AVC_t Q_t - F_t + \varepsilon_{0t} & \text{if } i = \text{operate} \\ -F_t + \varepsilon_{1t} & \text{if } i = \text{hold idle} \\ SV_t + \varepsilon_{2t} & \text{if } i = \text{retire} \end{cases} \qquad (2)$$

where

P : (real) output price

Q : profit maximizing output when i = operate

AVC : average variable cost function (to be specified later)

F : fixed costs

SV : scrap value of the production unit net of the cost of removing the scrapped unit

The expression (1) is the value function $V_t(x_t, \varepsilon_t, \theta)$, which is the recursive solution to the Bellman equation:

$$V_t(x_t, \varepsilon_t, \theta) = \underset{i \in C}{\text{Max}} \left[u(x_t, i_t, \varepsilon_t, \theta) + \beta EV_t(x_t, i_t, \varepsilon_t, \theta) \right] \tag{3}$$

where

$$EV_t(x_t, i_t, \varepsilon_t, \theta) = \int_{x_{t+1}} \int_{\varepsilon_{t+1}} V_{t+1}(x_{t+1}, \varepsilon_{t+1}, \theta) dp\left(x_{t+1}, \varepsilon_{t+1} \middle| \; x_t, i_t, \varepsilon_t\right). \tag{4}$$

Under certain regularity conditions described in Rust (1988) the optimal choice is given by

$$f(x_t, \varepsilon_t, \theta) = \underset{i \in C}{\text{argmax}} \left\{ u(x_t, i_t, \varepsilon_t, \theta) + \beta EV_t(x_t, i_t, \varepsilon_t, \theta) \right\}. \tag{5}$$

Our objective is to estimate the parameter vector θ and then assess the impact of changes in output and input prices on the utilization and retirement choices implied by (5). In the context of the US cement industry the choice rule (5) refers to the decision making with respect to a cement kiln within a cement plant. It is evident that the estimation of θ will require specifying the current profit function u(.), which in turn will depend on cement technology and available data.

But before we move on to the discussion of technology, data and the estimation procedure, a two-period special case is presented below which illustrates the nature of the solution of the stochastic dynamic programming model.

2.3. Two-Period Special Case: An Illustration

Let the two periods be denoted by t = 0 (present) and 1 (future). Thus T (the terminal period) = 1.

Consider Figure 1. The current product price P_0 and the expected

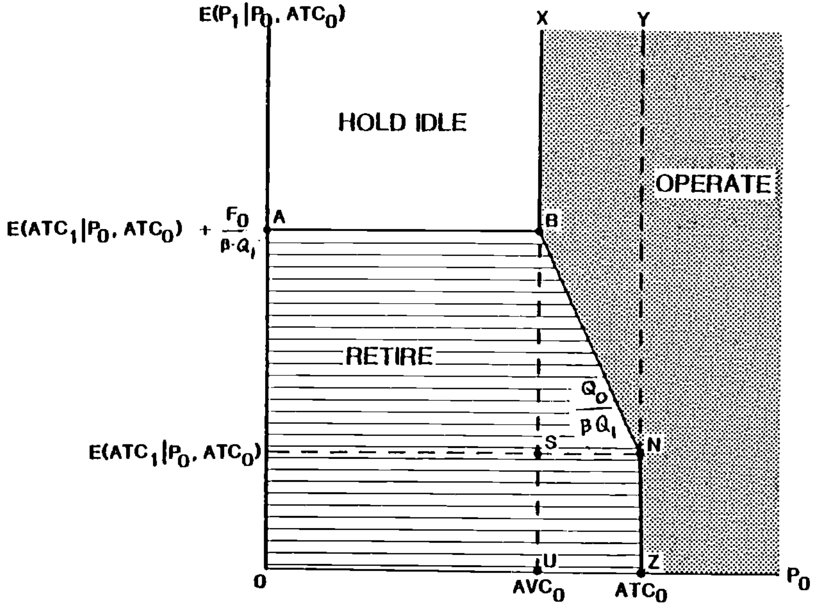
$E(P_1 | P_0, ATC_0)$
X
Y
HOLD IDLE
OPERATE
$E(ATC_1 | P_0, ATC_0) + \frac{F_0}{\beta \cdot Q_1}$
A
B
RETIRE
$\frac{Q_0}{\beta Q_1}$
$E(ATC_1 | P_0, ATC_0)$
S
N
U
Z
0
AVC_0
ATC_0
P_0

FIGURE 1

future product price are measured respectively along the horizontal and vertical axes. If P_0 exceeds the current minimum average total cost, ATC_0, the optimal choice is to operate (at the profit maximizing level of output); this is indicated by the region YNZ.

If $P_0 < ATC_0$, i.e., losses are incurred in the current period, then future profit expectations play a crucial role. Two subcases have to be considered: (a) $AVC_0 \leq P_0 \leq ATC_0$ and (b) $P_0 \leq AVC_0 \leq ATC_0$, where AVC_0 is the current minimum average variable cost. In subcase (a), the optimal current choice is either operate or retire. The reasoning is as follows. If the firm expects to also lose in the future, i.e. $E(P_1 | P_0, ATC_0) < E(ATC_1 | P_0, ATC_0)$, then it is optimal to retire the production unit now. This is indicated by the area USNZ. If the firm expects to make a profit in the future period it will operate or retire depending on whether the discounted future profits exceeds the current loss or not. The dividing line BN whose slope equals $Q_0/(\beta Q_1)$ depicts this choice. In subcase (b), the optimal current choice is either hold idle or retire. If the unit is held idle the current loss equals the fixed costs. Thus it would be worthwhile to incur this loss now by holding the unit idle *rather than retire it* if the discounted future period profit exceeds the current fixed costs, i.e.

$$\beta Q_1 E(P_1 - ATC_1 | P_0, ATC_0) > F_0, \text{or } E(P_1 | P_0, ATC_0) > E(ATC_1 | P_0, ATC_0) + F_0/(\beta Q_1).$$

This is indicated by the region above the line AB. Otherwise, if the reverse of the inequality holds, it is optimal to retire the production unit now, indicated by the area below the line AB.

Figure 1 illustrates the two main features of our analysis: the discreteness of the choices involved as shown by the distinct regions and the role of future expectations.

3. Technology and Data

3.1. Cement Technology[10] *and Its Implication for the Profit Function*

The production of cement begins with the quarrying and crushing of raw material deposits (clay, stone, sand or iron ore). The raw materials are then mixed into a "kiln-feed" either by adding water (the wet process) or without adding water (the dry process). The newer dry process preheats the kiln-feed before it enters the kiln and is the most fuel efficient. (As mentioned earlier, a kiln is a huge rotary steel tube with fire brick lining.) So there are effectively three processes of cement production: wet, dry and dry-with-preheater. The production process of a kiln is irreversible. The kiln-feed enters the kiln at one end and as it moves down the rotating kiln, heated by fuel (coal, oil or gas) to about 2700°F, its chemical composition changes. The product that exits the kiln is called "clinker". The final step is to grind the clinker to get cement.

The kiln is the largest and the most important piece of equipment in cement production. All other auxiliary equipment are built to be consistent with the operation of the kiln. The capacity of each kiln and the number of kilns determine the capacity of a cement plant. Thus, kiln is the "unit of production".

The following features of kiln technology and cost function will be relevant for the specification of the current profit function u(.).

First, kilns operate continuously for twenty four hours a day if at all

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[10] A good description of the cement production processes can be found in Witt (1966). The basic cement kiln technology has not changed since 1952.

they are operated. It is because frequent heating and cooling of a kiln can damage its fire brick lining, and the start-up heating cost is very high. In fact, once heated up, plants prefer to operate a kiln almost all year round except for a few days for maintenance, because they typically face capacity constraints during peak demand periods of summer and fall. Thus, typically, *the annual output of a kiln is approximately either zero or its capacity level.* This implies a particular form of the revenue function to be specified later.

Second, there is no jointness of cement production across kilns, which implies that a cement plant's cost function is additively separable in kilns.

Third, there are are essentially five variable inputs in cement production--labor, fuel (coal, oil or gas), electricity, raw material and maintenance. Labor is mainly used in the quarry and for packing. Fuel is largely consumed by the kilns. Electricity is consumed mainly by the auxiliary equipment. Part of maintenance cost is variable and part of it is fixed (e.g. 'winterizing' the kilns each winter). The variable cost function then includes the cost of these inputs.

Fourth, variable inputs are not substitutable. Thus a fixed coefficient technology of cement production is implied. Accordingly, the total variable cost function is linear in the output and input prices or in other words, the average variable cost is independent of the output. However, the marginal cost may increase with the age of the kiln.[11]

Let us turn to the data and its characteristics.

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[11] This has been recognized by McBride (1983, p. 1013).

3.2. *Data*

The data consists of annual observations from 1972 to 1980 on 32 dry process cement plants.[12] A particular plant typically tends to have kilns of one process only. (Among the 32 plants there was only one exception, which had dry process kilns as well as a newer dry-with-preheater process kiln.) The number of kilns in a plant ranged from 1 to 19. There are altogether 987 observations on dry process kilns. Multi-kiln plants tend to have kilns of similar sizes. Data includes each plant's location, clinker production, fuel and electricity consumption and the total number of kilns. For each kiln it includes the vintage, the kind of fuel it uses and its capacity.[13]

Kiln level cement production data are not available although plant level cement production data are. But since our study is at the level of kilns, the kiln level production data are generated from the plant level production data. This is done in the following way.[14]

Because the marginal cost of kilns increases with age of the kiln and multi-kiln plants have kilns of almost identical sizes, a profit maximizing firm will use kilns of a process in the reverse order of their ages. Thus, given the plant level cement production data, the output of the youngest kiln equals the minimum of its capacity and plant output. If plant output is less than the capacity of the youngest kiln, then it is assigned all the output and the rest of kilns in the plant, if any, are assigned zero output. Otherwise, while the youngest kiln is assigned its capacity output, the

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[12] Not dry-with-preheater cement plants.

[13] Thus capacity here is an engineering notion—a name plate rating.

[14] Experts in cement plants whom I have talked with agree with the following procedure.

remaining plant output is assigned toward the capacity of the next to the youngest kiln. Any left over plant output is assigned to the youngest of remaining kilns and so on. In case of two kilns of same age, if output to be assigned is less than the capacity of one kiln then only one kiln is assigned all the output. This is because the high temperature required for kiln operation leads to high start-up costs and hence it is efficient to produce the total output by heating one kiln. But if the output to be assigned is greater than the capacity of one kiln so that both kilns need to be heated up, then they are assumed to be utilized to the same extent.

The annual utilization rate is calculated as the ratio of a kiln output over its annual capacity. The frequency distribution of the kiln utilization rate is given in Table 1. As expected, the utilization rate is concentrated at two extremes: 86% of the observations have either zero or greater than 80% utilization rate.

Our econometric analysis requires information on whether during a given year a kiln in the data is operated, held idle or retired. The information on kiln retirement is directly given in the data set (58 out of 987 kilns were retired over the sample period). But kilns that are either operated or held idle have to be classified according to the utilization rates. Although, ideally, kilns that have zero utilization rate but are not retired should be classified as held idle and the rest operated, the cut-off point for "held idle" and "operated" is taken at 20%, i.e., kilns whose utilization rate ranged from 0 to 0.2 and which are not retired are identified as held idle and the rest operated. This is because in the data set there are two out of 58 retired kilns, which have utilization rates, 0.17 and 0.2. (This happened because an old kiln may be used part of a year and then retired in the same year.) This cut-off point at 0.2 rather than at 0 utilization rate is hardly significant because, as Table 1 shows, only 1% of kilns have utilization rates which are positive but less than 0.2. In other words the cement technology implies that capacity utilization is either nearly zero or nearly full; however, the choice between the two is economic.[15]

Given the above classification, the proportions of kilns operated, held idle and retired each year in the sample period are shown in Table 2. Table 2 also contains summary statistics of the age of kilns, cement price and input prices such as electricity price, fuel price and wage rate (all relative to CPI).[16] However, data on the prices of the remaining variable inputs—raw material and maintenance—are not available and hence the sum of these two costs are denoted as unobserved costs.

In Table 2 it is seen that the oil price shock of 1974 resulted in a significant increase in the real fuel price in 1974-75. It stayed high until 1979 and then decreased in 1980. Over the sample period the real wage did not change much, while the real electricity price had an increasing trend

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[15] This notion of optimal capacity or capital utilization is different from that in the standard literature in which the optimal utilization choice arises when there are other quasi-fixed inputs like labor besides capital and it is costly to change output and utilization by changing such inputs (see, for example, Abel (1981) and Shapiro (1986)).

[16] The data on cement price and input prices were obtained from the following sources. The cement prices in each state were obtained from the *Minerals Yearbook*. The hourly wage rates were derived from the earnings and hours data given for the cement industry in *Employment and Earnings* published by the Bureau of Labor Statistics. State level fuel and electricity prices were obtained from the *Energy Price and Expenditure Data Report (1970-1980)* which is published every ten years by the US Department of Energy. In the case where a kiln could use more than one source of fuel, the fuel price relevant for the kiln in any year is taken to be the one that is the cheapest in that year in the state in which the kiln was located. In order to abstract from the differences in price due to differences in the general price level, the cement price and all the input prices are deflated by the CPI in each state.

and was particularly high in 1978 and in 1980. Following the oil shock of 1974 there was a marked increase in kiln idling and retirement from 1974 to 1975. Kiln idleness however steadily declined beginning 1975. Kiln retirement stayed high relative to the pre-oil-shock era, and in two particular years, 1978 and 1980, there were "blips" in retirement. We will examine later the performance of our model in explaining these "blips" and predicting the overall pattern of utilization and retirement during the sample period.

4. Econometric Analysis

In this section we specify the current profit function u(.), identify the parameter vector θ that enters the decision rule (5) with regard to each kiln, and discuss its estimation. Using the estimates the model is then simulated to provide predictions; these are presented in Section 5.

4.1. Specification of the Profit Function

If the kiln is held idle, the current revenues from it are obviously zero. If it is operated, the current revenues equal P.K, where P ≡ (real) cement price and K ≡ capacity output since, as explained in Section 3, it is optimal to produce the capacity output if the kiln is operated at all. If the kiln is retired the current revenues are equal to SV, the net scrap value of the kiln.

Turn next to the cost side. Among the five variable inputs, electricity, fuel, labor, raw materials and maintenance, data are available only on the electricity and fuel prices and wage rate. Accordingly, the (real) average variable cost function, AVC--which is independent of the output--is specified as:

$$AVC = ACU + AVLI.W + AVFI.PF + AVEI.PE \qquad (6)$$

where

ACU: the average cost of unobserved variable inputs (raw material and maintenance)

AVLI, AVFI, AVEI: average variable labor, fuel and electricity input

W, PF, PE: hourly wage rate, fuel price, electricity price.

With respect to ACU, we assume that it is proportional to the price of cement, the rationale being that price of cement reflects the strength of demand for cement, which will be positively related to the demand for raw materials and maintenance inputs. Hence,

$$ACU = \alpha P,^{17} \tag{7}$$

where α is a positive constant. The constancy of α is justified by the fixed coefficient nature of the cement technology.

Furthermore, as kiln operation is very fuel intensive, it is likely that AVFI increases with the age of a kiln--a form of "input decay". In keeping with the existing literature (e.g. Feldstein and Rothschild), we assume a constant rate, δ, of input decay, i.e., $AVFI_{t+1} = (1+\delta)AVFI_t$. This difference equation implies

$$AVFI_t = (1+\delta)^{A_t} AVFI_0, \tag{8}$$

where A_t is the age of the kiln at time t and $AVFI_0$ is the average variable fuel input of a new kiln.[18]

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[17] The inclusion of a constant term in (7) led to identification problems in estimating the AVC function. The estimation procedure is discussed in the next subsection.

[18] Since labor is used mainly for quarrying and packing, labor requirement is unlikely to change as a kiln ages. Also, electricity is used mainly by the auxiliary equipment and for lighting buildings, and its usage is unlikely to

Substituting (7) and (8) in (6), the average variable cost function is written as

$$AVC_t = \alpha P_t + AVLI.W_t + (1+\delta)^{A_t} AVFI_0.PF_t + AVEI.PE_t \tag{9}$$

Now consider the fixed costs, which consists largely of mechanical and electrical maintenance of kilns. These costs are assumed to increase proportionately with the capacity of the kiln and the age of the kiln. Hence

$$F = h.K + d.A, \quad h, \; d > 0 \tag{10}$$

where F is the fixed costs, h can be interpreted as the holding cost per unit of kiln capacity and A is the age of a kiln.

Collecting the expressions for total revenue, variable costs given in (9) and fixed costs given in (10), the instantaneous profit function of a kiln can now be written as

$$u(.) = \begin{cases} P.K - AVC.K - F + \varepsilon_0 & \text{if } i = \text{operate} \\ -F + \varepsilon_1 & \text{if } i = \text{hold idle} \\ SV + \varepsilon_2 & \text{if } i = \text{retire} \end{cases} \tag{11}$$

ε_0, ε_1 and ε_2 are respectively the random disturbances associated with the choices, "operate", "hold idle" and "retire" respectively.

The exogenous variables are: K, A, P, W, PE and PF, and the estimable parameters of the profit function are: $\theta = (\alpha, \delta, AVLI, AVFI_0, AVEI, h, d, SV)$.

4.2. *Outline of the Estimation Procedure*

The parameter vector θ is estimated in the context of the dynamic

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vary with the age of a kiln.

programming model given in (1), where u(.) is specified in (8), (9), (10) and (11). The choices being discrete there are no closed form solutions or explicit estimating equations. Instead, the estimation involves solving the dynamic programming model numerically.

Compared to the existing literature on the estimation of discrete choice dynamic programming models, e.g. Miller (1984), Wolpin (1984, 1987), Pakes (1986) and Rust (1987), there are three distinguishing features of our estimation exercise here. First, our model involves more choice variables and more exogenous variables, which are implied by the nature of our decision problem. However, the 'computational burden' and the 'curse of dimensionality' of dynamic programming models are well known. Hence, all the parameters are not estimated simultaneously. A two-step procedure is used instead (discussed below). Second, in discrete choice models the parameters are typically identifiable only up to a positive scale constant. But the two-step procedure enables us to obtain consistent estimates of *individual* parameters. Third, while discrete choice models are typically estimated by assuming specific distributions for the disturbances, we obtain estimates of as many parameters as feasible, given the data and the existing econometric methodologies, without assuming specific distributions.

Step 1 involves estimating the parameters of the AVC function—α, δ, AVLI, $AVFI_0$ and AVEI—which does not require solving the dynamic programming model for the following reasons. Since plant level data are available for fuel and electricity consumption (but not for other inputs), δ, $AVFI_0$ and AVEI—the parameters that are associated with fuel and electricity inputs—are estimated by regression. Conditional on the regression estimates the remaining parameters, α and AVLI, of the AVC function are then estimated by identifying a static decision rule nested in the firm's dynamic programming model: "do not operate a kiln if its current AVC exceeds the current price P"—which is illustrated by the region to the left of UX in Figure 1. This decision rule is discrete, but because it is static semiparametric estimates could be obtained; Manski's (1985) maximum score procedure was used for this purpose. For details, see Das (1990). The estimates of the parameters of the AVC function are reported in Table 3.

Given these estimates, in this paper the remaining parameters--h, d and SV--are estimated by using Rust's (1988) method that solves the dynamic programming model at each iteration of a maximum likelihood routine. This is step 2. A drawback of this two-step procedure is that the standard errors of h, d and SV are biased downward and cannot be corrected because a feasible closed form of the standard errors of the maximum score estimates are not yet known. Hence the significance of the estimates should be cautiously evaluated.[19]

We now present the procedure for estimating h, d and SV—a subset of the vector θ in (1). These estimates are obtained, conditional on the estimated AVC function, by maximizing the likelihood function for the controlled stochastic process $\{i_t, x_t\}$ for a sample of N kilns with the nth kiln having T_n observations:

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[19] Although our estimates of h, d and SV are consistent there may be some doubt about whether they converge at rate $\sqrt{N}$ because the rate of convergence of our semiparametric estimates is less than $\sqrt{N}$. We still expect our estimates to converge at rate $\sqrt{N}$ based on the work of Ahn and Manski (1990). They have proved that, in binary choice dynamic optimization models, when nonparametrically obtained estimates of some parameters or functions (which have a rate of convergence less than $\sqrt{N}$) are used to estimate other parameters, the second stage estimates still converge at the rate $\sqrt{N}$ and are asymptotically normal. We expect that this holds in our case although its formal proof is a topic of future research.

$$L = \prod_{n=1}^{N} \prod_{t=1}^{T_n} p(i_{nt} | x_{nt}, \theta), \tag{12}$$

where $p(i_{nt} | x_{nt}, \theta)$ are the optimal choice probabilities, given x_{nt} and θ. These choice probabilities are obtained from the optimal decision rule $i_{nt} = f(x_{nt}, \varepsilon_{nt}, \theta)$, as given in (5), and by specifying a distribution for ε.

However, as (5) shows, the choice probabilities depend on the value at t+1 as expected at t--$EV_t(x_t, i_t, \varepsilon_t)$--which does not have a closed form solution. Hence at each iteration of a maximum likelihood routine, $EV_t(.)$ and its derivatives are to be numerically computed by backward induction and then used to evaluate the likelihood function L and its derivatives.

In our model the $EV_t(.)$ function involves a nine-dimensional integral as $\varepsilon = (\varepsilon_0, \varepsilon_1, \varepsilon_2)$ and there are six exogenous variables, x = (K, A, P, PE, PF, W).

As in Rust, we make the following assumption on the joint distribution of x_{t+1} and ε_{t+1}.

A7: $$p\left(x_{t+1}, \varepsilon_{t+1} | x_t, i_t, \varepsilon_t\right) = q\left(\varepsilon_{t+1} | x_{t+1}\right) p\left(x_{t+1} | i_t, x_t\right),$$

where q(.) is the density of ε and $p\left(x_{t+1} | i_t, x_t\right)$ is the Markovian law of motion for the observed state variables. Rust calls this the 'conditional independence' assumption. Assumption A7 involves two restrictions. First, x_{t+1} is a sufficient statistic for ε_{t+1}, which implies that any serial correlation in ε_{t+1} is transmitted entirely through the vector x_{t+1}. Second, the distribution of x_{t+1} depends only on x_t, not on ε_t.[20]

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[20] In our model this is reasonable because among the components of x for a given kiln, K (capacity) does not change over time, A (age) changes in a deterministic manner and (real) observed prices are determined either

The assumption A7 implies that $EV_t(x_t, i_t, \varepsilon_t, \theta)$ is no longer a function of ε_t, i.e., $EV_t(x_t, i_t, \varepsilon_t, \theta) = EV_t(x_t, i_t, \theta)$.[21] This greatly simplifies the computation; $EV_t(.)$ can be computed only on the state space for x by using finite grid approximation. Hence we have

$$EV_t(x_t, i_t, \varepsilon_t, \theta) = \begin{cases} EV_t(x_t, i_t, \theta) \equiv EV_t(x_t, \theta) & \text{if } i_t = \text{operate or hold idle} \\ 0 & \text{if } i_t = \text{retire} \end{cases} \tag{13}$$

This is because if the current decision is to retire a kiln, future profits would be zero; if the decision is to operate or hold idle the expected future profits (which, for a given kiln, depends on future A, P, PE, PF and W) would be positive but the same (although the current profits do differ between these two decisions).

Using (13) and the profit function u(.) in (11), the value function (3) can now be written as (suppressing the subscript t)

$$V_t(x_t, \varepsilon_t, \theta) = \text{Max}\left\{ V_{0t}(x_t) + \varepsilon_{0t},\ V_{1t}(x_t) + \varepsilon_{1t},\ SV + \varepsilon_{2t} \right\}, \tag{3'}$$

where

$$V_{0t}(x_t) \equiv P_t.K_t - AVC_t.K_t - F_t + \beta EV_t(x_t, \theta)$$
$$V_{1t}(x_t) \equiv - F_t + \beta EV_t(x_t, \theta).$$

We have closed form choice probabilities by assuming that $q(\varepsilon|x)$ is given by a multivariate extreme value distribution:

---

unilaterally or jointly in their respective markets.

[21] See Rust (1988) for a proof.

$$\begin{aligned} p(\text{operate}\mid x_t,\theta) &= \exp(V_{0t}(x_t))/\,H_t \\ p(\text{hold idle}\mid x_t,\theta) &= \exp(V_{1t}(x_t))/\,H_t \\ p(\text{retire}\mid x_t,\theta) &= \exp(SV)/\,H_t, \end{aligned} \tag{14}$$

where $H_t \equiv \exp(V_{0t}) + \exp(V_{1t}) + \exp(SV)$ and $EV_t(x_t,\theta)$ is given by the unique solution to the functional equation:

$$EV_t(x_t,\theta) = \int \log\left(e^{V_{0t+1}(x_{t+1})} + e^{V_{1t+1}(x_{t+1})} + e^{SV}\right) p\left(x_{t+1}\mid x_t, i_t\right) dx_{t+1} \tag{15}$$

In (15), $p\left(x_{t+1}\mid x_t, i_t\right)$ is the law of motion of an observed state variable, where $x_t = (K_t,\ A_t,\ P_t,\ PE_t,\ PF_t,\ W_t)$. Its individual components are discussed below.

For a given kiln, K_t—the capacity—does not change over time.,

The age of a kiln, A_t, evolves in a deterministic manner:

$$p(A_{t+1}\mid A_t, i_t) = \begin{cases} 1 \text{ for } A_{t+1} = A_t + 1 \text{ if } i_t \neq \text{retire} \\ 1 \text{ for } A_{t+1} = A_t = \text{the absorbing state if } i_t = \text{retire} \\ \qquad\qquad\qquad\quad \text{or } A_t = 80 \text{ years} \\ 0 \text{ otherwise} \end{cases}$$

It would be ideal to estimate the price process $\{P_t, PE_t, PF_t, W_t\}$ jointly and without making any distributional assumptions. However, due to insufficient variation in our price data,[22] the joint estimation is simplified based on the following heuristics.

Since electricity and fuel markets are not specific to the cement industry their (real) prices are assumed to be determined in the respective

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[22] As noted in Section 3 the wage rate varies over time but not across states, while the other prices vary over time and across states but not much across the locations of kilns.

markets. Hence the transitions of their prices from t to t+1 are assumed to follow a univariate first order Markov process. These are denoted by $p(PE_{t+1}|PE_t)$ and $p(PF_{t+1}|PF_t)$ respectively.

Labor is highly unionized in the cement industry and hence the wage rate (W) and the cement price (P) are likely to be jointly determined. We assume that they follow a joint first order Markov process, denoted by $p(P_{t+1},W_{t+1}|P_t,W_t)$.

Hence in (15), *for each given value of capacity,* we have

$$p\left(x_{t+1}|\ x_t,i_t\right) = p(A_{t+1}|A_t,i_t).p(PE_{t+1}|PE_t).p(PF_{t+1}|PF_t).p(P_{t+1},W_{t+1}|P_t,W_t) \tag{16}$$

The actual estimation task begins with exp. (16). Notice that $p(A_{t+1}|A_t,i_t)$ does not contain any unknowns. The rest of the transition probabilities are nonparametrically estimated using the corresponding sample relative frequencies for the grid values of x_t at which $EV(x_t,\theta)$ was evaluated.[23]

Given the estimated $p(x_{t+1}|\ x_t,i_t)$, the estimates of h, d and SV are obtained by a maximum likelihood algorithm that consists of two loops, an

---

[23] Recently, the common lack of closed form value function for dynamic optimization problems has spurged many such grid approximation numerical solution routines. For instance, see the series of eleven articles in the *Journal of Business & Economic Statistics*, January 1990.

In our case, for each value of K in the data, the grid dimension of x_t used was 11520, which included 80 possible values for A_t and 144 values for $(P_t,\ PE_t,\ PF_t,\ W_t)$.

outer loop and an inner loop.[24,25,26]

It should be emphasized here that while in typical discrete choice models the parameters are identifiable only up to a positive scale, in this model we are able to obtain individual estimates by using the estimated AVC function. This can be seen by writing the instantaneous profit function up to a positive scale, say c, as

$$u(.) = \begin{cases} c.(P-\hat{AVC})K - c.h.K - c.d.A + \varepsilon_0 & \text{if } i = \text{operate} \\ - c.h.K - c.d.A + \varepsilon_1 & \text{if } i = \text{hold idle} \\ c.SV + \varepsilon_2 & \text{if } i = \text{retire} \end{cases} \qquad (11')$$

where $\hat{AVC}$ is already known and used as data.

---

[24] For given values of the parameters generated by the outer loop, at each iteration the inner loop computes the value function and its derivatives by backward induction. Using the value function and its derivatives the outer loop computes the likelihood function and its derivatives and generates new estimates of parameters to be given to the inner loop for the next iteration.

[25] A general problem with estimating dynamic models such as ours is that since the likelihood function in the sample is not globally concave for a positive value of the discount rate β, convergence may be quite difficult to obtain. The trick is to start at $\beta = 0$, maximize the likelihood (which is globally concave at $\beta = 0$) and use the converged values as initial values for maximizing the likelihood for slightly increased value of β from zero and so on. This is closely related to the homotopy method described in Zangwill and Garcia (1981). This method provides the maximum likelihood estimate of β by grid search but does not provide its standard errors, which is not a significant loss since estimating β is not our focus here.

The maximized likelihood value was obtained at $\beta = 0.9$ and the likelihood ratio statistic for the null $\beta = 0$ was equal to 19.8 and hence the null was rejected at 0.5% level of significance. This indicates that the dynamic model, $\beta > 0$, explains the data better than a static model, $\beta = 0$.

[26] Again, based on the work of Ahn and Manski, we expect that our estimates to converge at rate $\sqrt{N}$.

4.3. *Estimates of the Fixed Cost Parameters (h and d) and the Scrap Value (SV), and the Overall Fit of the Model*

The procedure outlined above generated the estimates of c, c.h, c.d and c.SV. Then c was eliminated to obtain (individual) estimates of h, d and SV. These estimates are reported in Tables 4 and 5.

In interpreting the significance of the estimates in Table 4, it should be recalled that the standard errors are underestimated, because these standard errors are obtained by assuming that the estimated AVC is the true function. Hence the significance of the estimates is evaluated accordingly.

The parameter c which indicates the significance of the estimated AVC function is positive and highly significant. Its t-ratio indicates that it would continue to be significant (at 5%) even if the true variance is 150 times higher than the one reported.

With regard to the holding cost parameter, c.h., the estimate is positive and would continue to be significant (at 5%) even if the true variance is 14 times higher than the one reported. The value of c and c.h imply that h--the holding cost--equals 0.0128 real dollars per unit of capacity (see Table 5). Since the minimum capacity of a kiln in the data is 42,000 tons/year and the maximum is 567,000 tons/year, it implies the fixed cost (K.h) to be between 538 and 7,258 real dollars per year (with 1967 as the base year). In current dollars, the fixed costs would of course be much higher.

The increase in fixed cost, d, as a kiln get older is surprisingly not significant (at 10%). It may indicate that with regular maintenance of the kilns, as indicated by the significance of the parameter h, age does not have a significant effect on the fixed cost.

Lastly, the net scrap value, SV, though significant (at 5%) in Table 4, would not continue to be so if the true variance is 1.4 times the one reported. Besides, as seen in Table 5, the value of SV (equal to 0.0953 real dollars per year) is not economically significant. Note that SV defines the scrap value, net of the cost of removal of the scrapped kiln which may be substantial. In the absence of secondary markets for kilns it is not surprising that the cost of removal matches up to the scrap value, as implied by the insignificance of SV.

Now using all the available estimates from steps 1 and 2 we can compare the model prediction to the data. One way is to compare the observed proportion of kilns operated, held idle and retired in each year to the respective mean probability (over kilns) predicted by the model for that year. These comparisons are given Figures 2, 3 and 4. It is seen in Figures 2 and 3 that the model tends to under predict utilization ("operated") and over predict idleness, but, except during 1975-1978, the predicted trends are close to those observed in the data. In Figure 4 we see that the model tracks retirement quite well until 1977 but fails to explain the "blips" in retirement in 1978 and 1980. Although the real electricity price rose steadily during 1978-80, it is unlikely to explain the blips since cement production is not electricity intensive. The model accordingly does not predict the blips. Given that cement production is fuel intensive, a more plausible explanation may lie in lags in learning about fuel prices by cement firms. The initial jump in the fuel price in 1974 may not have been regarded as a permanent phenomenon. But as the real fuel price stayed high until 1978, the industry may then have updated its long run expectation and thus reacted to such expectation. Since our empirical model does not capture well the effects of longer than one-period

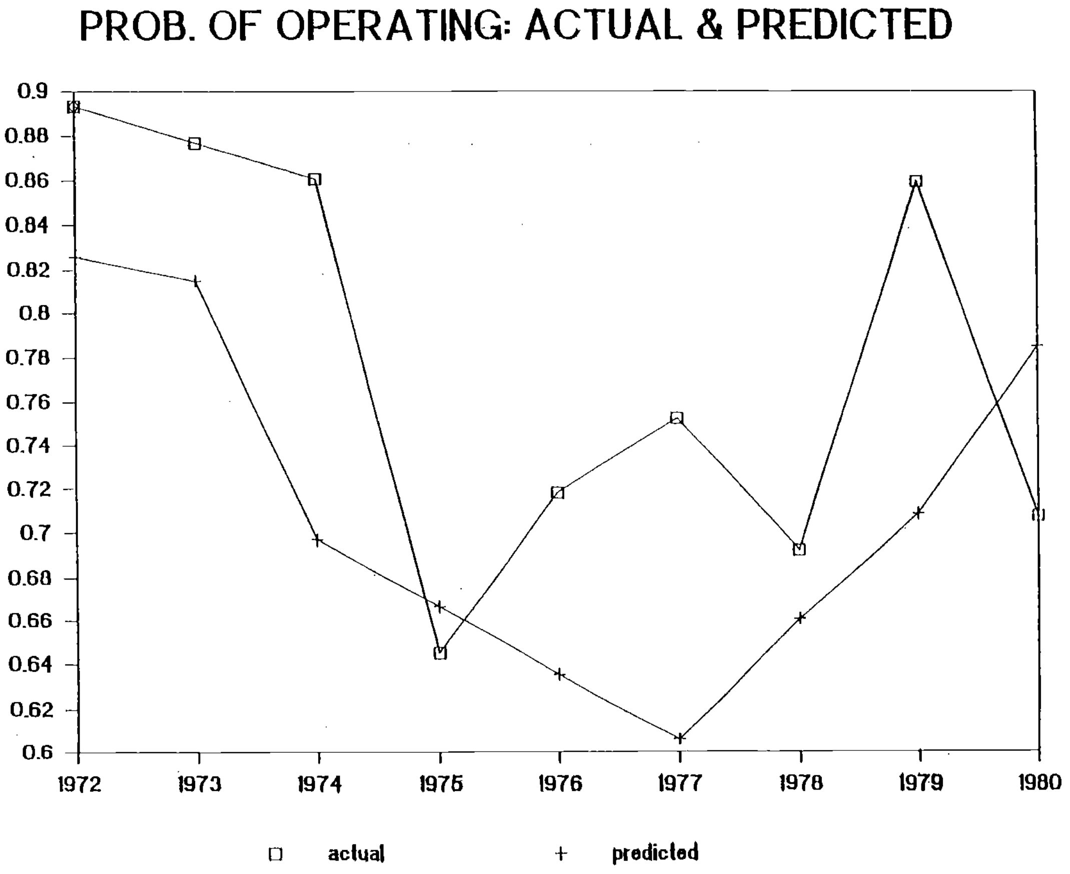
PROB. OF OPERATING: ACTUAL & PREDICTED
0.9
0.88
0.86
0.84
0.82
0.8
0.78
0.76
0.74
0.72
0.7
0.68
0.66
0.64
0.62
0.6
1972
1973
1974
1975
1976
1977
1978
1979
1980
□ actual
+ predicted

FIGURE 2

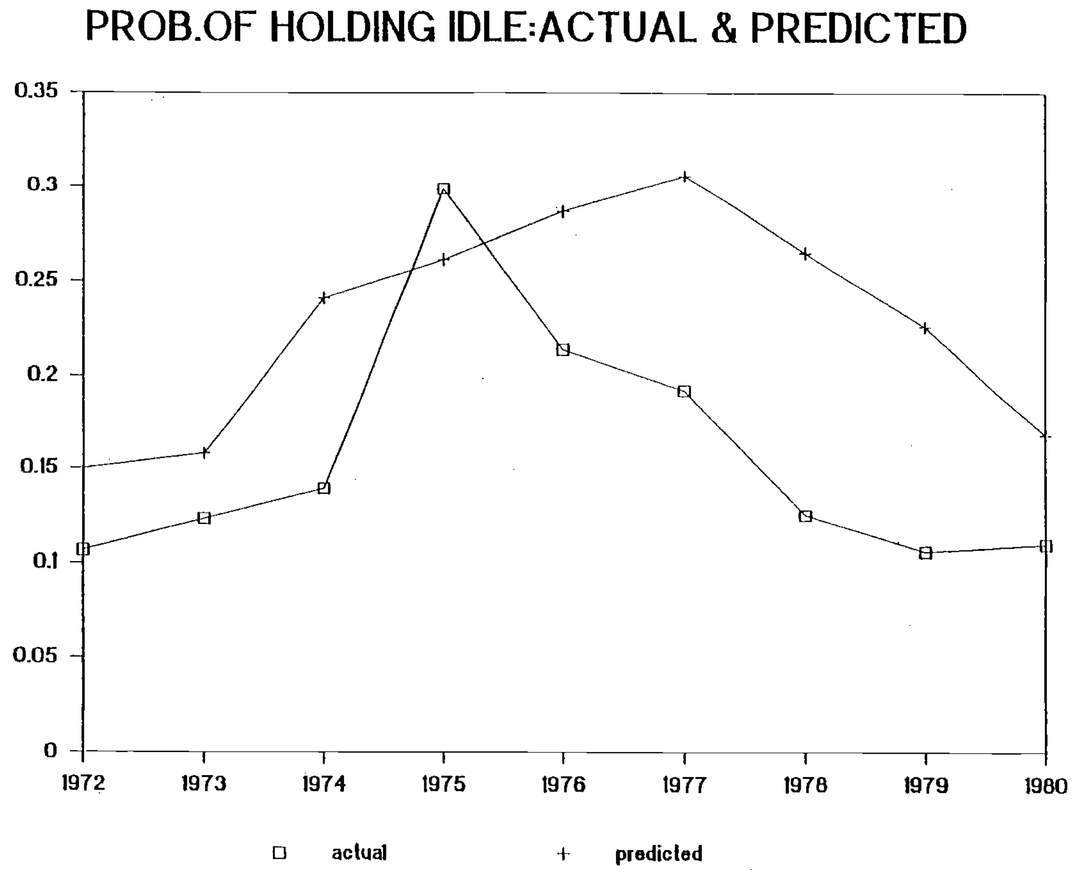
PROB.OF HOLDING IDLE:ACTUAL & PREDICTED
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
1972
1973
1974
1975
1976
1977
1978
1979
1980
□ actual
+ predicted

FIGURE 3

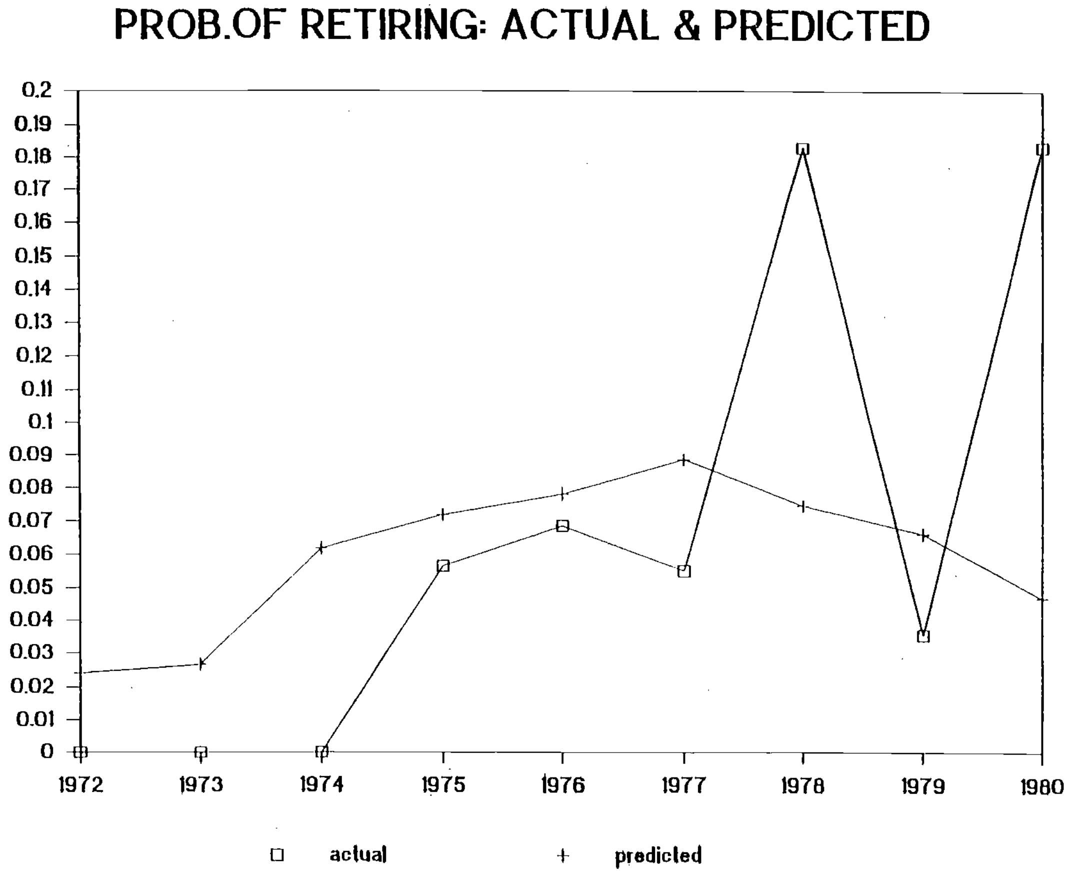
PROB.OF RETIRING: ACTUAL & PREDICTED
0.2
0.19
0.18
0.17
0.16
0.15
0.14
0.13
0.12
0.11
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
1972
1973
1974
1975
1976
1977
1978
1979
1980
□ actual
+ predicted

FIGURE 4

lag in prices it is not able to explain the blips in retirement.

These are all qualitative statements on the fitness of the model. The overall statistical fit can be evaluated by the χ^2 statistic, obtained by the formula (see Rao (1973, ch. 6)):

$$\sum_i \sum_t (p_{dit} - p_{mit})^2 / p_{mit},$$

where

i = operate, hold idle or retire

t = 1972, 1973,.., 1980

p_{dit} = observed proportion of kilns with the ith choice in the data (given in Table 2)

p_{mit} = mean (over kilns) of the ith choice probability in year t predicted by our model.

Its value equals 1.1214. In our model, the χ^2 statistic has 17 degrees of freedom (the number of cells (27) - the number of estimated parameters (9) - 1). Hence the null hypothesis that our model is true cannot be rejected at 0.5% level of significance. Thus it may concluded that statistically our model describes the data well.

We now use the structural estimates obtained to simulate the response of the probabilities of operating, idling and retiring a kiln to changes in output and input prices.

5. Sensitivity Analysis

The model permits a number of interesting simulations with respect to changes in prices, and age and size (capacity) of the kiln at various possible combinations. For the sake of brevity, I shall however present only a few. It may be remarked here that the qualitative impacts of a change in size on the decisions to operate, hold idle or retire are not obvious from the theoretical model. Nor are the qualitative impacts of changes in age or prices on the decisions to hold idle and retire as they are similar alternatives to operating the capital in the current period. In general however, the qualitative as well as the quantitative impacts are of interest here.

Over the sample period the variations in cement, fuel and electricity prices much exceed the variation in the wage rate. Hence the model is simulated with respect to these prices individually while keeping other prices, the wage rate and age and size of the kiln fixed at their respective sample means.

In Figures 5 through 9 we graph how the probabilities of operating, holding idle and retiring a cement kiln change as the cement price, fuel price, electricity price, kiln size and kiln age vary. In Figure 5, as the (real) cement price varies through the range of $6.25/ton to $35/ton, the probability of operating increases from 39.4% to 61.8%, that of holding idle decreases from 51.7% to 33.8% and that of retiring decreases from 9% to 4%; thus changes in the cement price mostly affect the choice between operating and holding idle.

In Figure 6, as the (real) fuel price varies through the range $0.25/mbtu to $2.3/mbtu, the probability of operating decreases from 52.8% to 28.2%, that of holding idle increases from 42% to 55% and that of retiring increases from 5% to 16.8%. Thus each decision is quite sensitive to fuel price changes, which is consistent with the observed behavior reported in Table 2.

In Figure 7 the electricity price ranges from $1.27/mbtu to $12.14/mbtu. Correspondingly, the probability of operating decreases from 51% to 44%, that of holding idle increases from 43.4% to 48% and that of

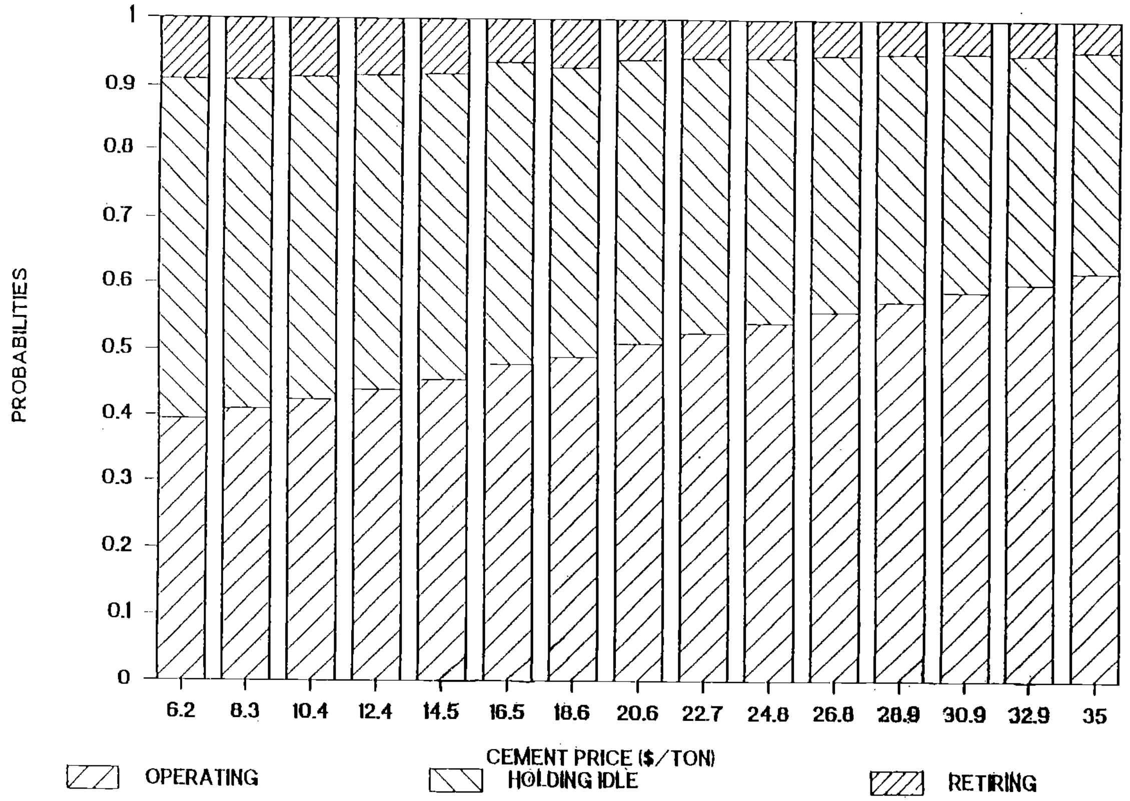
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
PROBABILITIES
6.2
8.3
10.4
12.4
14.5
16.5
18.6
20.6
22.7
24.8
26.8
28.9
30.9
32.9
35
CEMENT PRICE ($/TON)
OPERATING
HOLDING IDLE
RETIRING

FIGURE 5

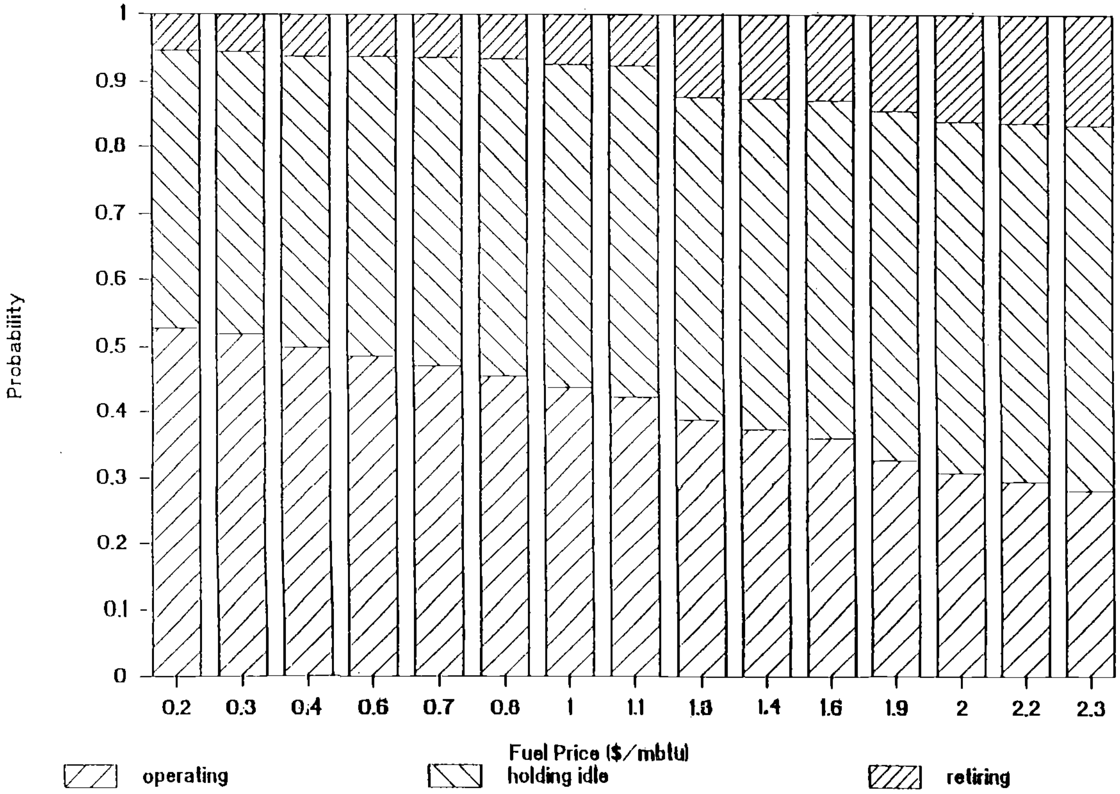
Probability
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.2
0.3
0.4
0.6
0.7
0.8
1
1.1
1.3
1.4
1.6
1.9
2
2.2
2.3
Fuel Price ($/mbtu)
operating
holding idle
retiring

FIGURE 6

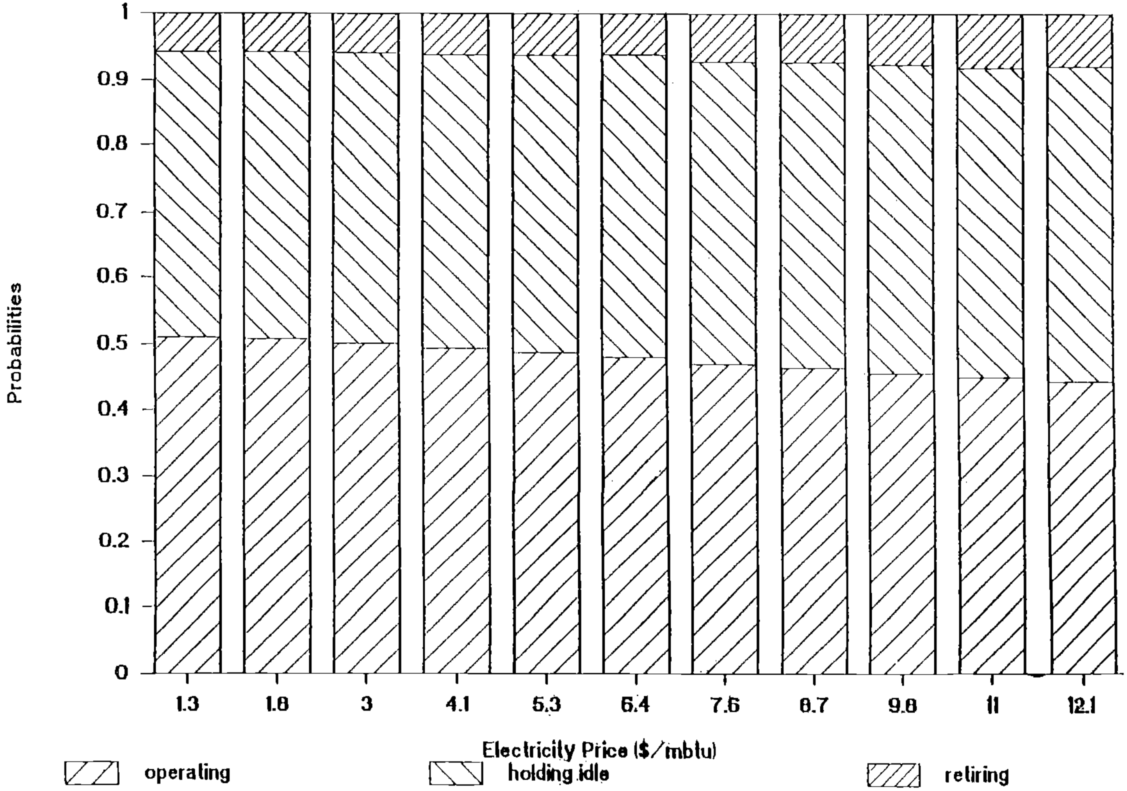
Probabilities
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1.3
1.8
3
4.1
5.3
6.4
7.6
8.7
9.8
11
12.1
Electricity Price ($/mbtu)
operating
holding idle
retiring

FIGURE 7

retiring increases from 5.6% to 7.8%. Hence the effects are the most on the decision to operate and least on the decision to retire.

Comparing the sensitivity with respect to these prices, it is seen that the decisions are most sensitive to fuel price changes, next to changes in the cement price and least to changes in the electricity price. It is also found (but not shown) that the impacts these price changes are more pronounced for bigger and/or older kilns.

In Figure 8 shows the marginal impact of kiln size (while the kiln age and all the prices are kept at their sample means). The probability of operating hardly changes with size. The probability of retiring increases and almost "crowds out" the probability of holding idle as the size increases.

Finally, in Figure 9 we see that the age of a kiln has relatively little impact on the decision to hold idle. But it has significant impact on the decisions to operate and retire.

6. Concluding Remarks

This paper has analyzed a micro econometric model of capital utilization and retirement choice. At any given point of time, a firm has a discrete decision problem with respect to an existing piece of capital stock: operate, hold idle or retire. The paper has obtained <u>structural</u> estimates of the parameters of such discrete decision rules in the context of cement kilns.

A two-step estimation procedure was involved. Estimation of some of the relevant parameters obtained previously constitutes step 1. Conditional on these estimates, the estimation of the remaining parameters (step 2) is done in this paper by solving a stochastic dynamic programming

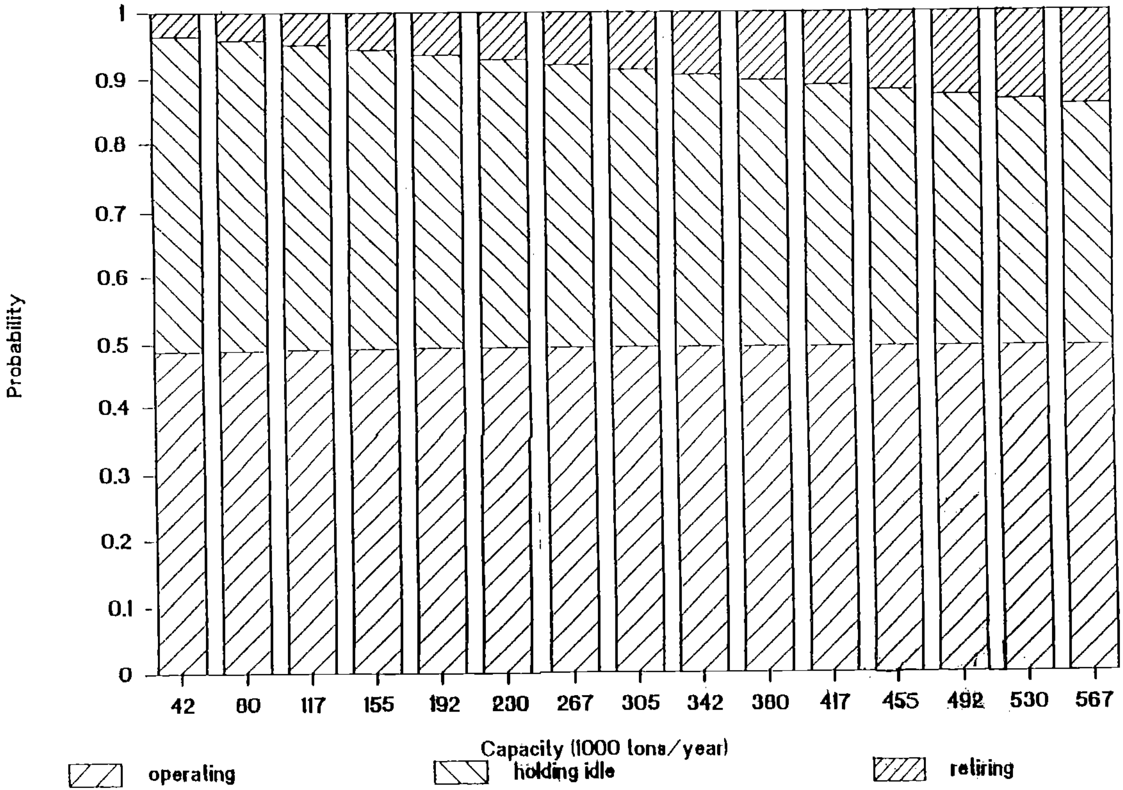
Probability
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
42
80
117
155
192
230
267
305
342
380
417
455
492
530
567
Capacity (1000 tons/year)
operating
holding idle
retiring

FIGURE 8

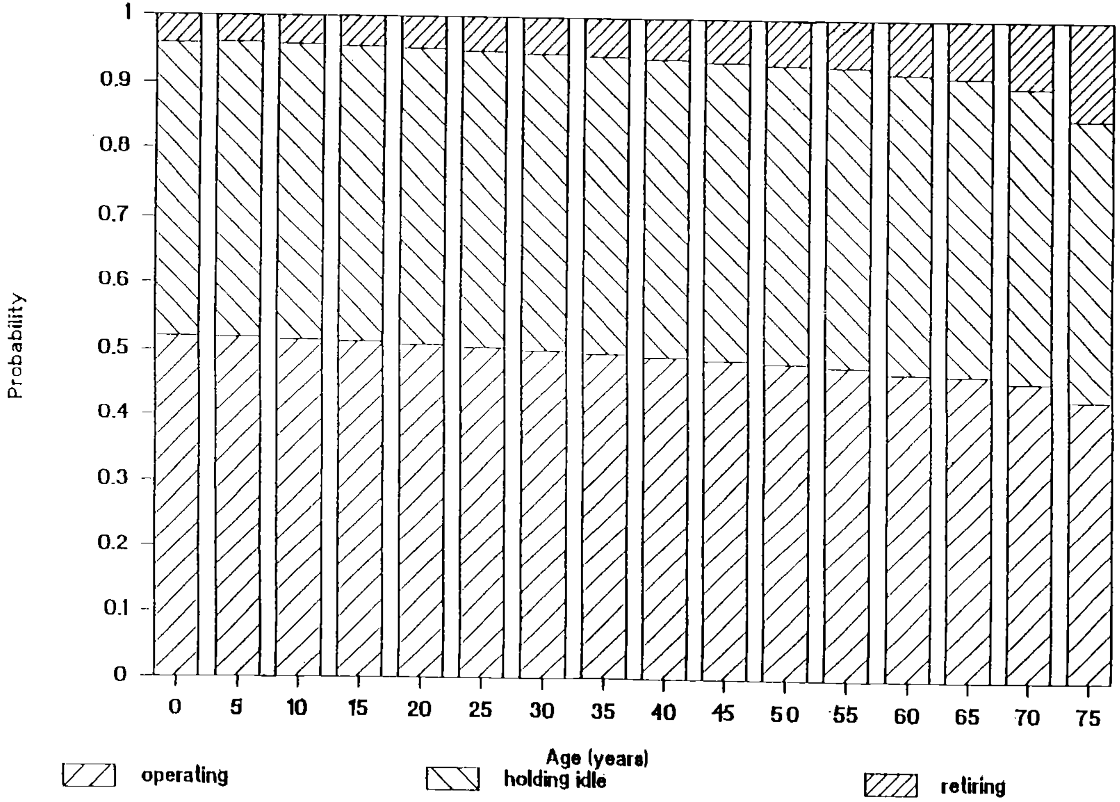
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Probability
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
Age (years)
operating
holding idle
retiring

FIGURE 9

model. A method similar to Rust (1988) was used, which solved the dynamic programming model numerically at each iteration of maximization of a likelihood function. In contrast to the discrete choice dynamic programming models in the existing literature that obtain estimates unique only up to a positive scale, our two-step procedure enabled us to obtain individual parameters of the decision rules, though the standard errors of the step 2 estimates are biased.

Given the estimates, the probabilities of operating, holding idle and retiring a cement kiln that would be predicted by our model were computed. These predicted probabilities compare well with the observed proportions in the data in accordance with the chi-square test. Finally, various simulations were conducted to determine the nature of variations in the predicted probabilities due to changes in cement price, fuel price, kiln size and kiln age.

Although the estimates per se are specific to the cement industry, the general methodology of the paper that deals with generic features such as discreteness or lumpiness of choices toward existing capital at the micro level, size and age structure of capital, stochastic processes of exogenous variables in the decision making, numerical solution of the underlying dynamic programming model etc. transcends the particular application highlighted in the paper and can be applicable, with suitable modifications, to other micro econometric studies of choice behavior on the use of capital.

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TABLE 1:

Frequency Distribution of Utilization Rates

Annual Utilization Rate (aur)	Proportion of Kilns (Out of 987)
$aur = 0$	.21
$0 < aur \le .2$	.01
$.2 < aur \le .4$	.02
$.4 < aur \le .6$	.02
$.6 < aur \le .8$	.09
$.8 < aur \le .99$	.19
$aur = 1.0$	.46

TABLE 2:

Summary Statistics

Year	Proportion of Kilns Operated	Proportion of Kilns Idled	Proportion of Kilns Retired	Mean Age of Kilns in yrs	Mean Cement Price* in $/ ton	Mean Fuel Price* in $/ mbtu	Mean Electr. Price* in $/ mbtu	Mean Wage Rate* in $/ hour
1972	0.89	0.11	0.00	31	15.64	0.33	3.07	3.93
1973	0.88	0.12	0.00	32	15.75	0.33	3.08	3.95
1974	0.86	0.14	0.00	32	17.31	0.50	3.81	3.82
1975	0.64	0.30	0.06	33	18.70	0.64	4.33	3.80
1976	0.72	0.21	0.07	33	19.29	0.63	4.36	4.13
1977	0.75	0.19	0.06	34	19.33	0.63	4.65	4.21
1978	0.69	0.13	0.18	35	20.16	0.61	4.79	4.23
1979	0.86	0.10	0.04	31	21.12	0.63	4.53	4.21
1980	0.71	0.11	0.18	31	20.31	0.56	5.02	4.10

*These are real prices, deflated by the CPI with 1967 as the base year

TABLE 3:

Estimates of the Unobserved Cost Parameter, and Electricity, Fuel and Labor Coefficients

	α	AVEI	AVFI_0	δ	AVLI
Estimate	0.4965	0.3680	4.5623	0.0087	0.5744
Standard Error	(0.0847)	(0.0136)	(0.4380)	(0.0005)	(0.3385)

TABLE 4:

Estimation of the Fixed Cost Parameters and The Scrap Value

	c	c.h	c.d	c.SV
Estimate	33.5966	0.4292	0.0009	3.2018
Standard Error	(1.6640)	(0.0689)	(0.0026)	(1.6221)
t-ratio	20.1903	6.2293	0.3462	1.9739

$\chi^2 = 1.1214$

TABLE 5:

Rescaled Estimates

	h	d	SV
Estimate	0.0128	0.00003	0.0953