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CREDIBLE DISINFLATION WITH STAGGERED PRICE SETTING

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ABSTRACT

This paper determines the real effects of credible disinflation when price setting is staggered. The results are surprising: a fairly quick disinflation causes a boom. This finding suggests that nominal price rigidity alone does not explain why disinflation is costly in actual economies.

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I. INTRODUCTION

According to mainstream macroeconomics, disinflations are a major cause of recessions. Consensus explanations for the rise in U.S. and U.K. unemployment in the early 80s emphasize the Volcker and Thatcher disinflations. Tobin (1988) argues that six of nine postwar U.S. recessions resulted from disinflation, and Romer and Romer (1989) identify six disinflations and find that output fell each time. This paper and Ball (1990) ask whether the costs of disinflation can be explained. In particular, I examine two traditional explanations: nominal price rigidity and credibility problems.

This paper focuses on price rigidity. To isolate its effects, I assume full credibility: a disinflation is announced at its outset, and the announcement is believed. The effects of a credible disinflation are controversial. New Classical economists (Sargent, 1983) argue that a credible disinflation is costless. Keynesians (Taylor, 1983) argue that price rigidity — and in particular the staggered timing of wage and price adjustments — makes disinflation difficult even with full credibility. At best, output losses can be avoided only if disinflation is quite slow. This paper concludes that the New Classicals are mostly right, even if we make Keynesian assumptions. With staggered price setting, the Fed can disinflate fairly quickly (although not instantly) without reducing output. Indeed, my findings are even more positive than the New Classical position. Surprisingly, a quick disinflation causes a boom.

The view that staggered price setting impedes a credible disinflation arises largely from a misinterpretation of previous research. The problem is a confusion of levels and changes. Taylor (1979, 1980) and Blanchard (1983, 1986) show that staggered adjustment of individual prices produces "price level inertia": the price level adjusts slowly to a shock to the money supply, and the shock has large real effects. Disinflation, however, is a change in the growth rate of money, not a one-time shock to the level. Many economists apparently assume that the inertia result carries over from levels to growth rates — that staggering causes inflation to adjust slowly to a change in money growth.[1] But this view is not correct.

I use a conventional model of staggering to study a problem proposed by Phelps (1978) and Taylor (1983). Money growth and inflation start at a trend level that is expected to continue forever. At some point, the Fed announces a disinflation — a path for money growth that declines and eventually reaches zero. The public believes that the Fed will keep its promise, and it does. In this framework, fairly quick disinflations cause booms: output rises above the natural rate temporarily and never falls below the natural rate. For example, suppose that individual prices are adjusted once a year and inflation is initially 10%. A boom occurs if the Fed disinflates linearly over a year — if inflation is 5% after six months and zero after a year. Indeed, a boom occurs for any linear disinflation longer than .68 years. The explanation for this result, presented below, is subtle.

I do not take the finding of expansionary disinflations as a serious empirical prediction. Rather, it is a negative result: by itself, staggered price setting cannot explain why disinflations are costly. The next step is to

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[1]For example, Blanchard and Summers (1988) state that "on the ... 'Keynesian view,' even credible disinflation is likely to increase unemployment for some time, because of the inflationary momentum caused by overlapping price and wage decisions."

ask whether modifications of the model yield more plausible results. Ball (1990) relaxes the assumption of full credibility. In that paper, I argue that a combination of credibility problems and staggering explains the effects of disinflation.

Phelps (1978) and Taylor (1983) study models similar to mine but find that disinflation is more difficult. As detailed below, the differences in results have two sources. First, Phelps and Taylor consider only disinflation paths that keep output constant, and thus neglect quick paths that cause booms. Second, while I assume that a price is fixed between adjustments, Taylor specifies time-varying prices: a three-year labor contract sets different wages for each year. This assumption, a departure from Taylor's 1979 and 1980 papers, slows disinflation considerably.

The rest of the paper contains five sections. Section II presents the basic model and derives the behavior of the economy under steady inflation. Section III presents an example of a quick disinflation that causes a boom. Section IV derives more general conditions under which disinflations cause booms. Section V considers time-varying prices, and Section VI concludes.

II. THE MODEL

Part A of this section describes the basic model. The model is a simplification of the continuous time Taylor-Blanchard model in Ball, Mankiw, and Romer (1988). Part B derives the behavior of the economy under steady inflation, which sets the stage for the disinflation in Section III.

A. Assumptions

The economy contains a continuum of imperfectly competitive firms indexed by i and distributed uniformly on [0,1]. Firm i's profit-maximizing relative price is increasing in aggregate output:

$$(1) \qquad p_i^* = p + vy, \qquad 0<v<1, \quad p = \int_{i=0}^{1} p_i di ,$$

where p_i is the firm's nominal price, p is the aggregate price level, and y is aggregate output (all variables are in logs). Equation (1) can be derived from isoelastic demand and cost functions. Intuitively, an increase in aggregate spending raises a firm's profit-maximizing price by shifting out the demand curve that it faces.[2]

Money enters the model through a quantity equation for money demand:

$$(2) \qquad m - p = y ,$$

where m is the money stock.[3] Combining (1) and (2) yields

$$(3) \qquad p_i^* = vm + (1-v)p .$$

A firm's profit-maximizing nominal price is a weighted average of the money stock and the aggregate price level.

If firms adjusted prices continuously, each would set $p_i=p_i^*$ at every

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[2]To derive (1), let the demand equation facing firm i be $y_i = y - \varepsilon(p_i-p)$, $\varepsilon>1$ (demand depends on aggregate spending and the firm's relative price). Let the firm's log costs be $\gamma y_i + K$, $\gamma>1$, where K is a constant. For a convenient choice of K, these assumptions imply (1) with $v=(\gamma-1)/(1-\varepsilon+\varepsilon\gamma)$. For deeper microfoundations see Ball and Romer (1989), where (1) is derived from utility and production functions.

[3]One approach to explaining the costs of disinflation is to assume that money demand depends on expected inflation as well as output. In this case, an announcement of disinflation causes a contractionary rise in money demand. While this effect may be important under very high inflation, it does not appear central to the costs of disinflation in the United States. In any case, the effect becomes irrelevant if m is defined more broadly as nominal GNP, making (2) a tautology. With this approach, disinflation means that policymakers choose a declining path for nominal GNP growth, adjusting money growth to offset any shifts in money demand.

instant. The equilibrium would be $p_i = p = m$ $\forall i$, $y = 0$. I assume, however, that a firm adjusts its price at intervals of length one (a normalization). The price is fixed between adjustments. Adjustments by different firms are staggered uniformly over time.

Previous work provides foundations for these assumptions. Ball, Mankiw, and Romer (1988) show that small costs of price adjustment lead firms to adjust infrequently. Ball and Cecchetti (1988) and Ball and Romer (1989) present explanations for staggered timing based on imperfect information and firm-specific shocks. Finally, Caballero (1989) shows that costs of gathering information lead firms to adjust at constant rather than "state-contingent" intervals. For simplicity, I do not make these foundations explicit in the current model.

A firm's instantaneous loss function is $(p_i - p_i^*)^2$ (a second-order approximation to the profit loss from price rigidity). Let x(t) be an individual price chosen at t for the interval [t,t+1). (I drop the i subscript because all firms adjusting at t choose the same price.) Ignoring discounting, a firm chooses x(t) to minimize its expected loss over [t,t+1):

$$(4) \qquad Z(t) = \int_{s=0}^{1} E_t(x(t) - p_i^*(t+s))^2 ds \, .$$

Minimizing (4) yields

$$(5) \qquad x(t) = \int_{s=0}^{1} E_t p_i^*(t+s) \, ds \, .$$

A firm's price is the average of its expected profit-maximizing prices over the interval when the price is in effect.

Finally, the definition of the aggregate price level and the assumption of uniform staggering imply

$$(6) \qquad p(t) = \int_{s=0}^{1} x(t-s)\,ds \ .$$

The price level is the average of prices set between t-1 and t.

B. Steady Inflation

Here I derive the behavior of the economy under steady inflation that is expected to last forever. Suppose that

$$(7) \qquad \begin{aligned} m(t) &= t \ ; \\ \dot{m}(t) &= 1 \ . \end{aligned}$$

That is, the money stock grows at rate one (another normalization). In this case, equations (2)-(6) imply

$$(8) \qquad \begin{aligned} p(t) &= t \ ; \\ y(t) &= 0 \ ; \\ x(t) &= t + 1/2 \ . \end{aligned}$$

The price level grows at the same rate as money, and output is constant at the natural rate. x(t) is the average of m(·) from t to t+1. Averaging x(·) from t-1 to t yields p(t), as required by (6).

III. DISINFLATION: AN EXAMPLE

This section presents an example of a fairly quick disinflation that, contrary to intuition, produces a boom. Part A describes the path of money growth, Part B proves that a boom occurs, and Part C discusses the result.

A. The Example

Assume that at all times $t<0$ the economy is in the steady inflation regime described by (7) and (8), which is expected to last forever. At $t=0$, the Fed announces a slowdown in money growth; the announcement is believed, and it is carried out without further surprises. The announced path for $\dot{m}(t)$ is

$$(9) \qquad \dot{m}(t) = 1 - t, \quad 0 \le t < 1; \qquad = 0, \quad t \ge 1.$$

This path and the fact that $m(0)=0$ determine the path for the money stock:

$$(10) \qquad m(t) = t - t^2/2, \quad 0 \le t < 1; \qquad = 1/2, \quad t \ge 1.$$

Figure 1 plots $\dot{m}(t)$ and $m(t)$. Money growth declines linearly and reaches zero after one unit of time. Again, a unit of time is the period for which individual prices are fixed. If prices are fixed for a year (a generous estimate), then money growth is cut in half in six months and reaches zero in a year. This is a quick disinflation; it would surely cause a recession in many real-world economies.

B. A Boom

Here I show that the disinflation path (9) implies a boom, defined as follows:

Definition: A disinflation causes a boom if $y(t) \ge 0$ $\forall t$ and $y(t)>0$ over a period of positive length.

A boom means that output rises above the natural rate for a non-negligible period and never falls below the natural rate.

The analysis is simplest in the limiting case in which the parameter v equals one. In this case, a firm's desired price p_i^* simplifies to the money stock m (see (3)). I first consider this case and then show that the result generalizes.

<u>The Case of $v=1$</u>: When $p_i^*=m$, one can derive the path for $p(t)$ from (5)-(6) and compare it to $m(t)$ to show that a boom occurs (see below). This approach proves tedious, however, and so I present an indirect but simpler proof. I first determine the path that individual prices $x(t)$ would have to follow for output to remain constant, given the path for money. I then show that firms in fact choose <u>lower</u> prices, which implies a boom.

Constant output requires

$$(11) \qquad m(t) - p(t) = 0 \qquad \forall t ,$$

and thus

$$(12) \qquad \dot{m}(t) - \dot{p}(t) = 0 \qquad \forall t .$$

Equation (9) gives $\dot{m}(t)$ for $t\geq 0$. Differentiating (6) yields $\dot{p}(t)=x(t)-x(t-1)$. Substituting these expressions into (12) yields

$$(13) \qquad \begin{aligned} 1 - t - x(t) + x(t-1) &= 0 , \qquad 0\leq t<1 ; \\ -x(t) + x(t-1) &= 0 , \qquad t\geq 1 . \end{aligned}$$

For $0\leq t<1$, $x(t-1)$ is set during the steady inflation regime before time zero. Thus, using (8), $x(t-1)=t-1/2$ for $0\leq t<1$. Substituting this result into the first line of (13) and rearranging yields $x(t)=1/2$, $0\leq t<1$. This result and the second line imply $x(t)=1/2$, $t\geq 1$. Thus constant output requires

$$(14) \qquad x(t) = 1/2 , \qquad t\geq 0 .$$

What prices do firms actually set? Substituting $p_i^* = m$ into (5) yields

$$(15) \qquad x(t) = \int_{s=0}^{1} m(t+s)\,ds\ , \qquad t \geq 0\ ,$$

where I use the assumption of perfect foresight after time zero. x(t) is the average of money from t to t+1. Money is less than 1/2 for $0 \leq t < 1$ and equals 1/2 thereafter (see (10)). Thus x(t) is less than 1/2 for $0 \leq t < 1$ and equals 1/2 thereafter. For $0 \leq t < 1$, firms set prices below the level needed for constant output; thus there is a boom. More specifically, since prices set at t=1 last until t=2, the price level is too low, and there is a boom, until t=2:

$$(16) \qquad y(t) > 0\ , \qquad 0 < t < 2\ ;$$
$$y(t) = 0\ , \qquad t \geq 2\ .$$

<u>Generalization</u>: The following result implies that the simplifying assumption v=1 is inessential.

> <u>Lemma</u>: If a disinflation path causes a boom for v=1, then it causes a boom for all $v \in (0,1)$. Indeed, if there is a boom for v=1, then y(t) is greater for $v \in (0,1)$ than for v=1, $\forall t > 0$.

That is, relaxing the assumption of v=1 leads to a larger boom. Since output is higher for all t>0, it returns to the natural rate only asymptotically.

The Appendix proves the lemma. Intuitively, v=1 means that individual prices depend only on money, while v<1 means that prices also depend on the aggregate price level (see (3)). Since the price level is less than the money stock during a boom, individual prices are lower when v<1. In equilibrium, this produces a lower price level and a larger boom.

<u>C. Discussion</u>

Before explaining why disinflation causes a boom, I present another example that shows why our intuition suggests otherwise. Suppose that for $t<0$ the <u>level</u> of money is constant at one and expected to remain constant. At $t=0$, the Fed announces a decline in money:

$$(17) \qquad m(t) = 1 - t , \qquad 0 \le t < 1 ;$$
$$\qquad\qquad = 0 , \qquad t \ge 1 .$$

That is, $m(t)$ follows the path for $\dot{m}(t)$ in the previous example. This policy causes a recession (see Appendix). The intuition is familiar from previous work. At $t=0$, prices lasting until $t=1$ have been set equal to one, and this overhang of high prices causes a recession when money declines. In thinking informally, it is easy to imagine that a similar overhang causes a recession when money <u>growth</u> declines. In fact, moving up a derivative changes the results dramatically.

To see why disinflation causes a boom, let $v=1$ and substitute (5) into (6) to obtain

$$(18) \qquad p(t) = \int_{s=0}^{1} \int_{r=0}^{1} E_{t-s} m(t-s+r)\, dr\, ds .$$

For the moment, assume perfect foresight even for $t<0$. Dropping the expectations operator, (18) can be reduced to

$$(19) \qquad p(t) = \int_{s=0}^{1} (1-s) m(t-s)\, ds + \int_{s=0}^{1} (1-s) m(t+s)\, ds .$$

$p(t)$ is a weighted average of $m(\cdot)$ from $t-1$ to $t+1$ (this is a version of Taylor's [1980] forward- and backward-looking solution). Since $\dot{m}(t)$ declines between $t=0$ and $t=1$, $m(t)$ is concave over that interval. And since $m(\cdot)$ is concave, the average of $m(\cdot)$ from $t-1$ to $t+1$ is less than $m(t)$. Thus $p(t)<m(t)$ and $y(t)>0$. The concavity of $m(t)$ is the basic reason that disinflation causes a boom.

This argument assumes perfect foresight; in fact, there is a surprise at $t=0$. As intuition suggests, the surprise implies an overhang of prices set too high, which tends to cause a recession. Indeed, there _is_ a recession if disinflation is too fast (see Section IV). But the path in (9) is slow enough for the concavity of $m(\cdot)$ to outweigh the negative effects of the overhang.[4]

IV. DISINFLATION: GENERAL RESULTS

This section derives more general conditions under which disinflations cause booms. I also present quantitative results. Part A derives the behavior of prices for an arbitrary disinflation path. Part B uses this result to find the fastest linear disinflation that causes a boom. Part C shows that even faster paths cause booms if we relax linearity. Part D presents simple generalizations of these results. Finally, following Phelps (1978), Part E finds the disinflation path that keeps output constant.

A. The Behavior of Prices and Output

Assume again that the economy is in the steady inflation regime for $t<0$ and that a credible disinflation is announced at $t=0$. For $t>0$, I derive the paths of prices and output for an arbitrary path of money. Individual prices $x(t)$ are given by (8) for $t<0$ and by (5), without the expectations operator, for $t\geq 0$. Substituting these expressions into (6) yields an equation for the price level:

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[4]Why is the overhang of predetermined prices so easily overcome? Prices set before $t=0$ range as high as 1/2. Again, this implies a recession if the money stock is stabilized instantly at zero. But if money growth, while falling, remains positive for a short time, the level of money passes 1/2. At that point, predetermined prices are no longer a barrier to stabilizing money. In contrast, in the example of a fall in $m(t)$, prices set before $t=0$ are stuck at a level that money has fallen below permanently.

$$(20) \qquad p(t) = \int_{s=0}^{t} \int_{r=0}^{1} p_i^*(t-s+r)\,dr\,ds + \int_{s=t}^{1} (t-s+1/2)\,ds\,, \qquad 0 \le t < 1$$

$$= \int_{s=0}^{1} \int_{r=0}^{1} p_i^*(t-s+r)\,dr\,ds\,, \qquad t \ge 1\,.$$

This expression simplifies to

$$(21) \qquad p(t) = \int_{s=0}^{t} s p_i^*(s)\,ds + \int_{s=t}^{1} t p_i^*(s)\,ds$$

$$+ \int_{s=1}^{t+1} (1+t-s) p_i^*(s) + \frac{1}{2}(t-t^2)\,, \qquad 0 \le t < 1\,;$$

$$= \int_{s=0}^{1} (1-s) p_i^*(t-s)\,ds + \int_{s=0}^{1} (1-s) p_i^*(t+s)\,ds\,, \qquad t \ge 1\,.$$

The formula for p(t) changes at t=1, when the last prices set before t=0 expire.

Recall that $p_i^*(t)$ equals vm(t)+(1−v)p(t). When v=1, (21) gives a closed-form solution for p(t) in terms of m(t). When v<1, p(t) is defined only implicitly, but one can solve for p(t) and y(t) numerically. One can also establish qualitative properties of the paths using the results for v=1 and the lemma in Section III. Figure 2 presents the output path for a linear disinflation in one unit of time (equation (9)). I consider v=1 and v=1/4. Starting at t=0, output rises, peaks at t=1, and then falls back to the natural rate. As discussed above, the boom is larger and longer-lived for v=1/4 than for

$v=1$.[5]

B. The Fastest Linear Disinflation

Here I continue to assume that disinflation is linear and ask how fast it can be while causing a boom. Consider a disinflation completed in k units of time:

$$(22)\qquad \dot{m}(t) = 1 - t/k\,, \qquad 0 \le t < k\,;$$
$$\qquad\qquad\quad = 0\,, \qquad\qquad t \ge k\,.$$

For a given k, substituting the implied path for m(t) into (21) defines the paths of prices and output. The Appendix shows that a boom occurs for all v if and only if $k \ge .68$. If the Fed slows money growth too fast, the overhang of prices set before the announcement causes a recession. But, if prices are fixed for a year, the Fed can reach zero money growth in about eight months while causing a boom.[6]

C. The Fastest Disinflation

Here I relax the assumption that disinflation is linear. Assuming only that $\dot{m}(t)$ is non-increasing, I find the boom-inducing path that stabilizes the money

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[5]For $v=1/4$, the path of prices is derived as follows. p(t) converges to the final level of money, which is 1/2 in this example. Thus I set $p(t)=1/2$ for $t \ge 25$ (the results are not sensitive to raising this bound). For $0 \le t < 25$, I guess an initial path for p(t), substitute the guess into (21), and integrate numerically to obtain a new p(t). Then I iterate this procedure until p(t) converges. This approach yields a unique solution for p(t) because (21) is a contraction mapping (see Appendix).

[6].68 is the smallest k that produces a boom for $v=1$. Therefore, by the Lemma in IIIB, .68 is the smallest k that produces a boom for all v. For particular v's, the minimum k (derived numerically) can be even lower; the smallest k for $v=1/4$ is .62. Disinflation can be faster still if a boom is defined as a path that raises output on average (rather than for all t). With this definition, the minimum k is .46 for $v=1$ and .50 for $v=1/4$. For all v, $k=0$ unambiguously causes a recession: $y(t) \le 0$ $\forall t$ with strict inequality over (0,1).

stock in the shortest interval.[7] The Appendix shows that the fastest path is of the form

$$(23) \qquad \dot{m}(t) = 1, \quad 0 \le t < T; \qquad = 0, \quad t \ge T,$$

where T is a constant. This path is "cold turkey" with a delay: the Fed announces at $t=0$ that $\dot{m}(t)$ will drop discontinuously at $t=T$. The sharp drop in $\dot{m}(t)$ makes a boom more likely by increasing the concavity of $m(t)$.

The Appendix shows specifically that the fastest path is (23) with $T=.37$. The Fed can stabilize money after only 37% of prices have expired. This result strengthens the conclusion that staggered price setting does not prevent quick disinflation.[8]

D. Simple Generalizations

The assumption that initial money growth is one is just a normalization. Disinflation can occur as quickly when annual money growth is initially 20% as when it is 5%.[9] One can show that final money growth is also irrelevant: disinflation from 10% to a final level of 5% takes the same time as disinflation from 10% to zero. (Disinflation that ends at 5% is equivalent to disinflation that starts 5% lower and ends at zero, plus an unchanging 5% trend. The added

[7]If one allows $\dot{m}(t)$ to increase, the money stock can be stabilized instantly. This requires that $m(t)$ jump discretely at $t=0$ and then remain constant ($\dot{m}(0) \rightarrow \infty$). The initial jump in money offsets the overhang of predetermined prices.

[8]Once again, the results are even stronger for particular values of v; for $v=1/4$, the minimum T is .32. With the weaker definition of a boom (see note 6), the minimum T is .21 for $v=1$ and .25 for $v=1/4$.

[9]This result assumes that the interval between price adjustments is fixed. If firms choose the interval optimally given initial inflation and an adjustment cost, the interval is shorter at high inflation (Ball, Mankiw, and Romer). In this case, disinflation can be faster when initial inflation is high.

trend is neutral.)

An implication of these results is that disinflation, once begun, should be sustained. Suppose that money growth follows

$$(24) \qquad \dot{m}(t) = 1 - t, \qquad 0 \le t < 1/2 ;$$
$$= 1/2 , \qquad t \ge 1/2 .$$

Here the Fed starts to carry out a linear disinflation with k=1 but stops halfway, keeping money growth permanently at 1/2. In a sense this disinflation is milder than one that continues to zero, but its effects are less benign. By the irrelevance of final money growth, (24) has the same qualitative effects as a complete disinflation with k=1/2; thus, since 1/2<.68, output falls below zero. Intuitively, the prospect that disinflation will end raises the prices set during disinflation, causing a recession. The need to sustain disinflation is important when credibility problems are added to the model (Ball, 1990).

E. Disinflation with y(t)=0

Following Phelps (1978), I now find a disinflation path that keeps output constant, avoiding either a boom or a recession. One can show that no constant-output path stabilizes the money stock in finite time, but a unique path does so asymptotically. The path for money growth, derived numerically, is presented in Figure 3. The path is rather odd. Money growth drops discretely to .71 at t=0, then declines steadily and becomes negative — the level of money falls. Money growth jumps up at t=1, and then oscillates around zero. The oscillations die out asymptotically, but become small by t=1.5. The oscillations are needed for constant output because monotonically decreasing money growth implies a concave path for money, causing a boom.[10]

Phelps and others focus on the constant-output path because they assume that alternative paths cause recessions.[11] In fact, more natural paths cause booms while completely stabilizing money before t=1. Thus there is no reason to prefer the constant-output path.

V. TIME-VARYING PRICES

This section considers a variant of my model in which, as in Fischer (1977), a firm chooses a time-varying path for prices rather than a single fixed price. The main examples of time-varying prices in actual economies are wages in three-year labor contracts. Time-varying prices make disinflation more difficult. To avoid a recession, money growth must stay constant from t=0 to t=1, when it can drop to zero. The difference between fixed and time-varying prices explains most of the difference between my results and Taylor (1983).

A. The Result

Assume that a price setter chooses a continuous path of prices over the next unit of time. Otherwise, the model is unchanged. Let $x(\tau,t)$ be a price set at τ for $t \in [\tau,\tau+1)$. The price level is the average of prices set for t over the last unit interval:

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[10]Phelps proves uniqueness of the constant-output path in a discrete time staggering model. The result carries over to my model, which is a limiting case of Phelps's. Phelps derives explicit price level paths when prices are fixed for two, three, or four periods; for three and four periods, the results are similar to Figure 3. In my model, the constant-output path is the solution to (21) with v=0. The solution is found by iterating (21) after an initial guess. I assume p(t)=p(50) for t≥50 and update p(50) at each iteration.

[11]See, for example, Phelps's argument that "gradualist" disinflation causes a recession (pp. 796-7).

$$(25) \qquad p(t) = \int_{s=0}^{1} x(t-s,t)\,ds \; .$$

A firm sets its prices equal to expected optimal prices:

$$(26) \qquad x(\tau,t) = E_\tau p_i^*(t) = E_\tau[vm(t) + (1-v)p(t)] \; .$$

For $\tau<0$, $E_\tau m(t)=E_\tau p(t)=t$, since inflation is expected to continue at rate one. For $\tau\geq 0$, $E_\tau m(t)=m(t)$ and $E_\tau p(t)=p(t)$. Substituting these results and (26) into (25) yields

$$(27) \qquad \begin{aligned} p(t) &= (1-t)t + t[vm(t) + (1-v)p(t)] \; , && 0\leq t<1 \; ; \\ &= vm(t) + (1-v)p(t) \; , && t\geq 1 \; . \end{aligned}$$

The solutions for prices and output are

$$(28) \qquad \begin{aligned} p(t) &= \frac{t[1-t+vm(t)]}{1-t+vt} \; , && 0\leq t<1 \; ; \\ &= m(t) \; , && t\geq 1 \; . \end{aligned}$$

$$(29) \qquad \begin{aligned} y(t) &= (m(t)-t)\left(\frac{1-t}{1-t+vt}\right) \; , && 0\leq t<1 \; ; \\ &= 0 \; , && t\geq 1 \; . \end{aligned}$$

For $t<1$, output is zero if $m(t)=t$ — if money growth remains at one — and negative if money growth falls. For $t\geq 1$, output is zero regardless of $m(t)$. Thus the fastest disinflation without recession is cold turkey at $t=1$. Costless disinflation is much slower than with fixed prices, which permit linear disinflations that start immediately or cold turkey at $t=.37$. And no disinflation causes a boom.

To understand these results, recall the reason for booms with fixed prices: $m(t)$ is concave, and prices are averages of past and future money. With time-variation, prices for t depend only on money at t; firms need not average over money at different times. Thus the concavity of $m(t)$ is not expansionary, and there is no positive effect of disinflation to offset the overhang of predetermined prices. To avoid a recession, disinflation must wait until all prices set before $t=0$ expire.[12]

These results reinforce the point that shocks to the level of money and changes in money growth are contractionary under different circumstances. A fall in the level of money causes a longer recession with fixed prices than with time-varying prices (i.e. Taylor's 1980 model generates more persistence than Fischer's model). In contrast, disinflation is more likely to cause a recession with time-varying prices.

B. Comparison to Taylor (1983)

Taylor's 1983 paper studies disinflation in a model based on actual U.S. labor contracts. Taylor finds that, to avoid a recession, money growth must be almost constant for several years and then can drop rapidly. The main sources of these results are the facts that contracts last three years and wages are time-varying — firms set different wages for each year. Time-varying wages are a crucial departure from Taylor's 1979 and 1980 papers. As in my model, time-variation delays disinflation considerably.

Even on his assumptions, Taylor's results are a bit misleading because, like Phelps, he focuses on the disinflation path that keeps output constant. With freely time-varying prices, this path is the fastest one that avoids a recession.

[12]A further reason that disinflation is more difficult in this case is that the overhang of predetermined prices is worse. There is an overhang not just of high prices, but of prices required to grow. Specifically, with fixed prices the highest prices set before $t=0$ equal 1/2. In the current model, prices set just before $t=0$ grow to one as $t\to1$.

But Taylor assumes some fixity: the model is quarterly, and wages are fixed for a year. Taylor would find faster paths if he allowed booms, although the difference might not be large. (At a minimum, the length of disinflation could be reduced from four years to three.)

Overall, Taylor's and my results suggest that three-year contracts with time-varying wages are a serious impediment to disinflation. Rigidity in output prices is a weaker barrier, because prices are usually adjusted at shorter intervals and fixed between adjustments. A possible conclusion is that labor contracts are the basic reason that disinflation is costly. There are, however, well-known problems with using these contracts to explain monetary nonneutrality (Mankiw, 1990). Wages in long-term contracts may not be allocative, and the prediction of countercyclical real wages appears false. Most important, three-year contracts are rare outside the North American union sector. More than 80% of U.S. wages and almost all wages in Europe are set for a year or less with no time-variation. Thus most wage setting fits the assumptions that make disinflation easy.

VI. CONCLUSION

New Keynesian macroeconomics explains the real effects of monetary policy with nominal price rigidity. A central result is that staggered price setting slows the adjustment of prices to a fall in the money stock, so that output falls substantially. In the postwar United States, however, monetary contractions have been slowdowns in money growth — disinflations — rather than changes in the level of money. This paper shows that staggered price setting is not sufficient to explain the costs of disinflation. With staggered adjustment and credible policy, quick disinflations cause <u>booms</u>.

This result does not fit the U.S. experience of costly disinflations. Thus the next step is to ask whether modifications of the model produce more plausible results. Following Sargent (1983), a natural approach is to relax the assumption that policy announcements are credible. Firms may not believe a promise of disinflation, because the Fed can boost output by reneging. Thus when policymakers do disinflate, money growth is unexpectedly low and output falls.

A sequel to this paper (Ball, 1990) examines this argument. There are two points. First, credibility problems alone — like staggering alone — cannot explain the effects of disinflation. Simple credibility stories imply that announced disinflations cause recessions when policymakers are tougher than expected, but booms when disinflation is milder than expected. With rational expectations, the <u>average</u> output effect of announced disinflation is zero. In contrast, I argue that disinflations in actual economies systematically reduce output.

Second, the costs of disinflation <u>can</u> be explained by a <u>combination</u> of credibility problems and staggered price setting. As discussed in Section IV, disinflation under staggering is costless only if firms know that it will be sustained. If disinflation begins but firms fear that policy will ease, they do not reduce their price increases enough to match the current fall in money growth. In this case, an announced disinflation can reduce output on average even if it would cause a boom with full credibility.

APPENDIX

A. Proof of Lemma

Here I prove the lemma in Section III: if a disinflation path causes a boom for $v=1$, then it causes a larger boom for $v\epsilon(0,1)$. I first show that the general expression for $p(t)$, (21), defines a contraction mapping. Then I use the properties of contraction mappings to establish the lemma.

For a given path $m(t)$, (21) defines a mapping from the space of price paths to itself. Denote this mapping by $Gp(t)$. Inspection of (21) shows that G satisfies the monotonicity assumption of Blackwell's Theorem (Stokey and Lucas, 1989, p. 54). G also satisfies the discounting assumption since, for a constant a, $G(p(t)+a)\leq G(p(t))+(1-v)a$ and $v\epsilon(0,1)$. Thus (21) is a contraction mapping.

By the Contraction Mapping Theorem (Stokey and Lucas, p. 50), there exists a unique solution for $p(t)$. Starting with an arbitrary guess for $p(t)$ and repeatedly applying the mapping G produces a sequence of $p(t)$'s that converges to the solution. Start with the guess $p(t)=m(t)$. For $v\epsilon(0,1)$, this guess implies $p_i^*(t)=m(t)$. Setting $p_i^*(t)=m(t)$ in (21) is equivalent to assuming $v=1$. Thus $Gm(t)$ for $v\epsilon(0,1)$ equals the solution for $p(t)$ when $v=1$. By the assumption that there is a boom when $v=1$, $Gm(t)\leq m(t)$ $\forall t$ with strict inequality over a nonnegligible interval.

Since G is monotonic, if $Gp(t)\leq p(t)$ with strict inequality over a nonnegligible interval, then $G(Gp(t))\leq Gp(t)$ with strict inequality over a nonnegligible interval. Indeed, (21) implies that the range of strict inequality becomes larger: if $Gp(t)<p(t)$ for $t\epsilon[a,b]$, then $G(Gp(t))<Gp(t)$ for $t\epsilon[\max(a-1,0),b+1]$. (This follows from the fact that $p(t)$ depends on $p(\cdot)$ from

max(0,t-1) to t+1.) Along with the relation between Gm(t) and m(t), these results imply that the sequence produced by repeatedly applying G to m(t) converges to a solution for p(t) with $p(t)<Gm(t)\ \forall t>0$. That is, the solution for p(t) when $v\epsilon(0,1)$ lies below the solution when v=1, which implies a larger boom.

B. A Decline in m(t)

Here I consider the example in which the Fed reduces the level rather than the growth rate of money (Section IIIC). The money stock equals one for t<0 and is expected to remain constant, but at t=0 the Fed announces (17). I find the path that x(t) would have to follow to keep output constant and show that firms in fact choose higher prices, causing a recession. I again assume v=1 (one can show that the result generalizes).

Constant output requires $\dot{p}(t)=\dot{m}(t)$. Differentiating (6) and (17) yields

$$(A1)\qquad x(t) - x(t-1) = -1\ ,\qquad 0\le t<1\ ;$$
$$= 0\ ,\qquad t\ge 1\ .$$

For t<1, x(t-1)=1, since x(t-1) was set when m was expected to remain at one. Substituting this fact into (A1) yields

$$(A2)\qquad x(t) = 0\ ,\qquad t\ge 0\ .$$

For v=1, the actual path of x(t) is defined by substituting (17) into (5). x(t) is positive for t<1 and zero for t≥1. Thus actual prices are higher than the prices needed for constant output.

C. The Fastest Linear Disinflation

This section derives the output path for the general linear disinflation, (22). I then find the smallest length k that produces a boom — the smallest k for which minimum output is non-negative. I assume v=1 to find the smallest k that causes a boom for all v.

The path of money implied by (22) is

$$(A3) \qquad m(t) = t - t^2/2k\,, \qquad 0 \le t < k\,;$$
$$\qquad\qquad = k/2\,, \qquad t \ge k\,.$$

Assume $k \le 1$ (one can show that a boom occurs for any $k > 1$). Substituting (A3) into (21) and integrating yields the path of the price level:

$$(A4) \qquad p(t) = -\frac{tk^2}{6} - \frac{t^3}{6} + \frac{t^4}{24k} + \frac{t^2 k}{4} + \frac{tk}{2} + \frac{t}{2} - \frac{t^2}{2}\,, \qquad 0 \le t < k\,;$$
$$= -\frac{k^3}{24} + \frac{tk}{2} + \frac{t}{2} - \frac{t^2}{2}\,, \qquad k \le t < 1\,.$$

Subtracting (A4) from (A3) yields $y(t)$ for $0 \le t < 1$. A similar calculation shows that $y(t) \ge 0\ \forall t \ge 1$.

The first derivative of $y(t)$ is zero at one point in $(0,k)$ and one point in $(k,1)$. The second derivative is negative over $(0,k)$ and positive over $(k,1)$. Finally, $y(t)$ is continuous. These facts imply that, for given k, $y(t)$ is minimized at either zero or the critical point in $(k,1)$. This critical point is

$$(A5) \qquad t^* = \frac{k+1}{2}\,.$$

$y(0)$ equals zero. Combining (A5) with (A3) and (A4) yields $y(t^*)$:

$$(A6) \qquad y(t^*) = \frac{k^3}{24} - \frac{k^2}{8} + \frac{k}{4} - \frac{1}{8}\,.$$

Output is nonnegative for all t iff $y(t^*)$ is nonnegative. $y(t^*)$ is nonnegative iff $k \ge .68$. Thus $k \ge .68$ implies a boom.

D. The Fastest Disinflation

Here I show that the fastest boom-inducing disinflation is delayed cold turkey, (23), with $T = .37$. First I assume (23) and show that .37 is the smallest T that causes a boom. Then I show that any path besides (23) that reaches zero by t=.37 causes a recession.

Assume (23) with T<1 (any T≥1 implies a boom). Output is nonnegative for t≤T (because money growth has not yet fallen) and for t≥1 (because prices set before t=0 have expired). For t∈(T,1), m(t)=T. Substituting this expression into (21) and setting v=1 yields

$$(A7) \qquad p(t) = \frac{t}{2} + Tt - \frac{t^2}{2} - \frac{T^3}{6}, \qquad T \le t < 1 .$$

Subtracting (A7) from T yields y(t) for t∈(T,1). y(t) is minimized at

$$(A8) \qquad t^* = \min(T+\frac{1}{2},\ 1) .$$

The minimum output level is

$$(A9) \qquad y(t^*) = \frac{T^3}{6} - \frac{T^2}{2} + \frac{T}{2} - \frac{1}{8}, \qquad 0 \le T < 1/2 ;$$

$$= \frac{T^3}{6}, \qquad 1/2 \le T < 1 .$$

This expression is nonnegative for T≥.37.

Now consider any non-increasing path for $\dot{m}(t)$ that differs from (23) and stabilizes the money stock by t=.37. Let $\bar{m}$ be the level of money for t≥.37. $\bar{m}$<.37 by the assumptions that the path differs from (23) and money growth does not rise. I show that output falls below zero; specifically, output is negative at t=.87, the point when output just equals zero for (23) with T=.37.

y(.87) equals $\bar{m}$-p(.87). For given $\bar{m}$, p(.87) is increasing in m(t) for t∈(0,.37). The smallest possible m(t) for t∈(0,.37) is

$$(A10) \qquad m(t) = \frac{\bar{m}t}{.37}, \qquad 0 \le t < .37$$

(if money fell below this path, money growth would have to rise to produce $m(.37)=\bar{m}$). Thus, for given $\bar{m}$, (A10) produces the smallest possible $p(.87)$ and largest $y(.87)$. Substituting (A10) into (21) yields $p(.87)=.057+.847\bar{m}$. For $\bar{m}<.37$, this implies $y(.87)<0$.

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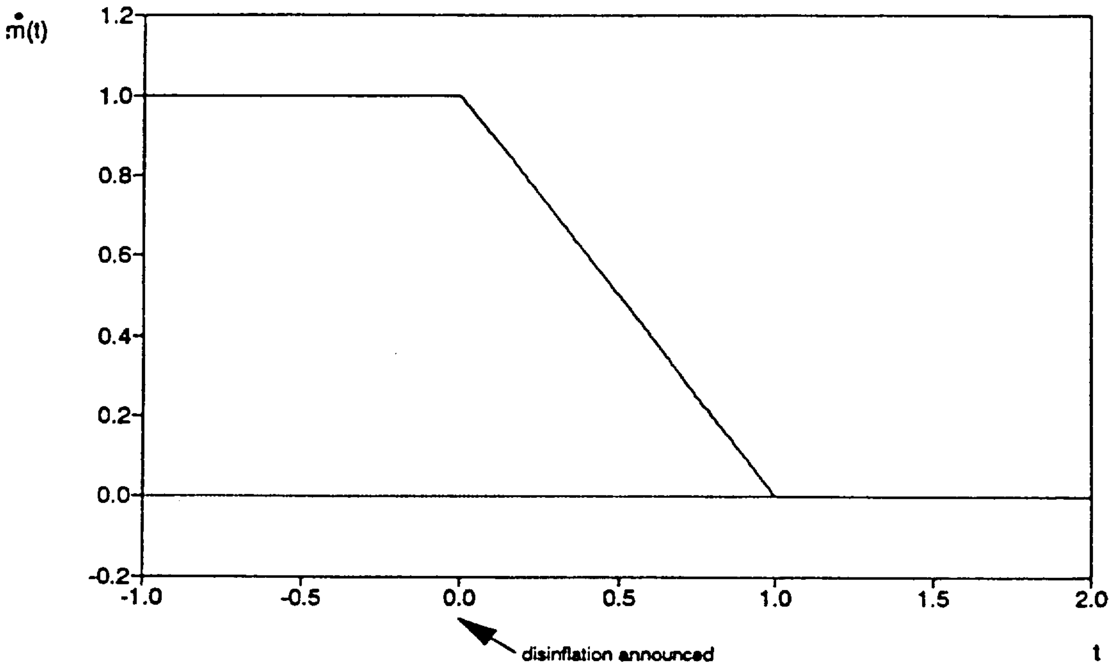
Figure 1
A Disinflation



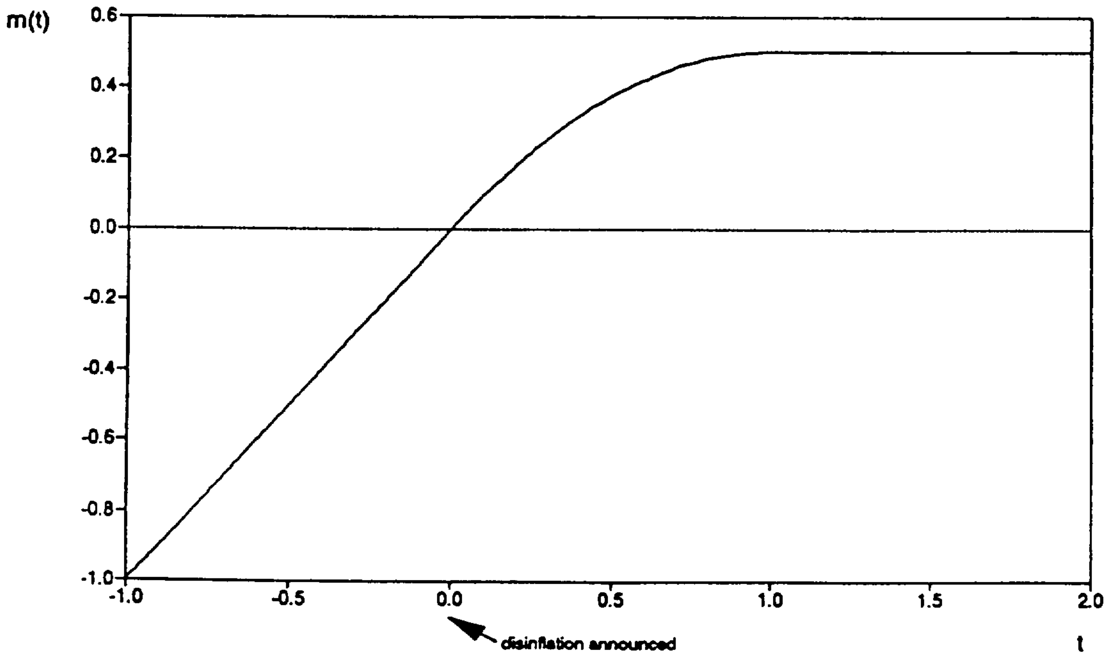


Figure 2
Output During a Disinflation

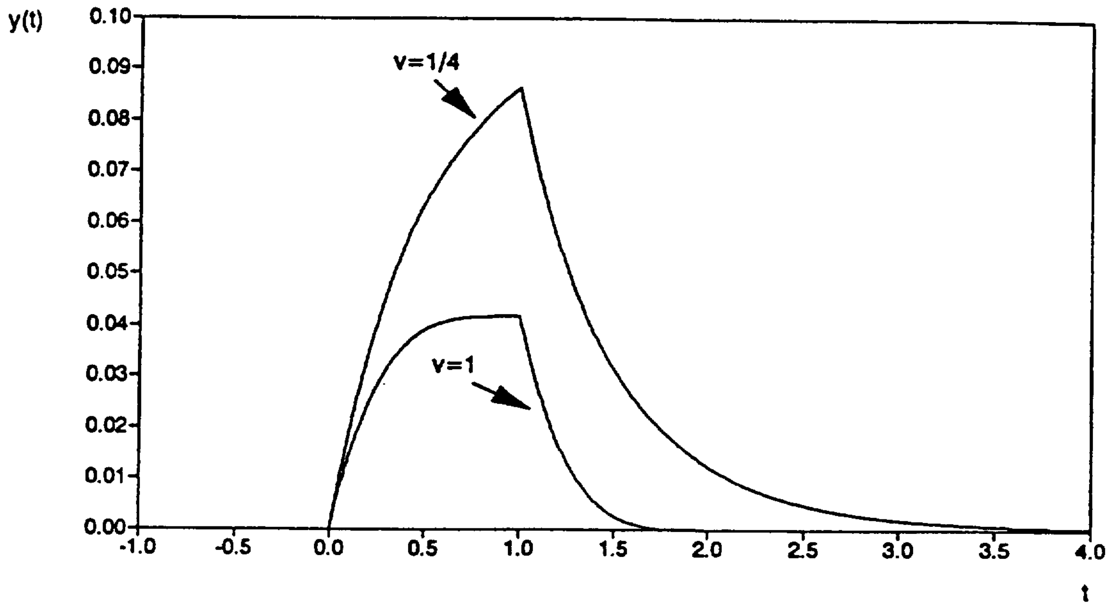


Figure 3
Disinflation with y(t)=0