NBER WORKING PAPERS SERIES

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Working Paper No. 3554

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
December 1990

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NBER Working Paper #3554
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ABSTRACT

The current debate in Western Europe centers on the relationship between economic and political integration. To address this problem, we construct a simple general equilibrium model in which the returns to trading are directly affected by the availability of a public good. In our model, heterogeneous agents choose both a club and a market to belong to. In the club, agents vote over the public good, are taxed to finance this good, and receive access to it when they trade. In the market, they are randomly matched with a partner. If a match occurs between traders of different clubs, they both suffer a transactions cost.

We show that, in general, the political boundaries established by the clubs can be distinct from market borders, leading to international trade between members of different clubs. Further, as the region develops, markets become wider (eventaully leading to a common market) and the desire to avoid transaction costs initially leads to political unification. At still higher levels of development, however, where transaction costs are less important, traders prefer the diversity offered by multiple clubs.

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Section 1: Introduction

Modern political economy uses a common framework to study economic and political systems, and emphasizes that one must understand both economic and political institutions to develop a successful theory of history (North (1979)). The debate over the formation of a European common market by 1992 clearly corroborates this view. There is little doubt that both the initial pressures towards accelerating economic integration, and more recent developments, emerged from a powerful alliance of economic and political motivations. Indeed the very scope of the economic changes considered has become increasingly entangled in political discussions, as countries express fears that economic integration might substantially infringe upon national sovereignty. Fundamentally, the discussion in the European Community is shaped by the belief that the creation of an integrated economic unit must eventually be accompanied by political unification. Investigating the validity of this belief is the central issue we address in this paper.

The belief that political unification must accompany economic integration is supported by two commonly held views. The first is that there are specific institutions which are uniformly "better" than others: they are more efficient in economic terms, and are preferred by all voters. One example is the Bundesbank (the German Central Bank), which is considered a preferred monetary institution. By eliminating barriers between countries, these better institutions will gradually permeate all of Europe, either through the forces of economic competition, or through migration of individuals and firms. The second view stresses that transaction costs are an inevitable source of economic losses when exchange occurs between traders belonging to different institutions; this view interprets transaction costs very broadly as including problem caused by different currencies, different legal systems, and even different languages. According to this view, as the volume of trade rises, more resources are wasted on transaction costs, and the pressure towards unification increases.

The model we construct in this paper is inspired by the latter view, and radically departs from the idea that economic integration should necessarily lead to unanimity in preferences over political institutions. We study a system of markets and political jurisdictions in which heterogeneity is maintained as markets become larger. The fundamental tension in the model is between transaction costs in a world of increasing trade, and diversification in political jurisdictions as a tool to accomodate heterogeneous preferences over public goods. Our results suggest that as markets become more integrated the trade – off between these two forces will lead to political integration, but only temporarily. Eventually, as a result of the increased profitability of trade in larger markets, transaction costs become less important, and a desire for political diversity reemerges.

We believe that our analysis is an initial step in defining more precisely the current European debate. More generally, this paper is a simple, but formal, general equilibrium example of the integrated approach to economic and political developments advocated by North and others.

Our starting point is a microeconomic specification of the trading relationship between two agents who are matched in a market and form a joint venture. Each agent is endowed with one unit of a good whose varieties are distributed uniformly along a line. When two agents match, their output depends on three factors: the combined productivity of the two agents' endowments, measured by their relative distance apart on the line; an exogenous productivity parameter; and the quantity of a trade-enhancing public good which is available to them. In introducing a public good which directly affects trading productivity, we depart somewhat from the standard literature, which has typically characterized public goods as entering directly into consumption (for example parks or clean air), or, less commonly, as entering into production. A primary interpretation of the trade-enhancing public good is that it refers to legal enforcement; as stressed by Milgrom, North, and Weingast (1990), Greif, Milgrom, and Weingast (1990), and Perotti and Modigliani (1990), a better legal system encourages contracting and investment by mitigating problems of moral hazard and opportunism. In the next section we discuss this interpretation more fully, and also suggest a number of alternative kinds of trade-enhancing public goods that are compatible with our formal structure.

Individuals in the society belong to two institutions, a market and a political institution which we call a "club". The club supplies to its members, and to them alone, the public good. In each club, the public good is determined by majority voting and is financed by lump sum taxes collected from the members of the club. In the market, each trader is matched with a partner randomly chosen from among the other individuals who have joined the same market. If he is matched with a "foreigner", that is somebody belonging to a different club, the partnership can choose which public good to use, but will suffer fixed transaction costs. Each agent chooses among the different clubs and the different markets which form in equilibrium, and neither clubs nor markets are allowed to exclude anyone.

In equilibrium, the size and composition of markets. and the size and composition of clubs, are simultaneously determined. Our objective is to study how the provision of the public good links political boundaries to market areas, and how this relationship changes as the region develops.

To achieve this goal, we proceed by stages. First, we characterize the model's equilibrium when all agents must belong to a single club. We show how market structure evolves in response to the exogenous productivity parameter, moving from multiple, local. relatively homogeneous trading pools to an integrated common market encompassing the whole spectrum of traders' types. In parallel, the supply of the public good rises, providing agents with the necessary institutional basis for profitable exchanges in the steadily enlarging markets. We also find that the transition towards successively larger markets always happens "too early" - i.e. at a value of the productivity parameter that is too low. causing a decline in expected per capita income. The agents who deviate from their local market and precipitate the collapse of the previous equilibrium are not representative: in the new market, the other traders need a much larger quantity of the public good, which they obtain (by voting) at too high a cost. It is only later, at higher productivity levels, that the wider market comes to be preferred by everybody.

As a second step, we derive the solution of the model when agents can not only choose which market to attend. but also which of two different clubs to join. We allow any group of individuals to form a market, but we focus exclusively on equilibria where each club consists of a connected set of members. This captures the empirical observation that the distribution of endowments tends to be relatively more homogeneous within a given country than across national borders. We reach two main conclusions in this part of the analysis. First, we show that an international market can emerge in equilibrium, even though it involves transaction costs that the agents could easily avoid by either changing markets or clubs, at zero cost. More precisely, international trade is defined to be trade between agents belonging to different clubs and having access to different public goods, and does not necessarily involve systematic differences in endowments. Thus our result is not related to the traditional "gains from trade" in the international trade literature, but stems from agents' desire to exploit foreigners' public good. Empirical analyses of international joint ventures would help to confirm the extent to which this effect is important empirically. Second, we study the pattern of migration between clubs as the productivity parameter rises and markets integrate. As long as productivity remains below a given threshold.

we find that transaction costs are the crucial factor determining the division between the two clubs. As markets integrate, more and more traders migrate towards one of the two clubs, thereby reducing their probability of being matched with a foreigner and incurring transaction costs. Numerical simulations discussed in the text show that eventually one of the two clubs disappears, and political union is acheived. However, at higher productivity, the importance of the fixed transaction costs declines, and even with a common market agents will again differentiate themselves into separate clubs, thereby helping to satisfy (though only partially) the heterogeneity in their preferences over the public good.

Finally, we compare the equilibria with one and two clubs in terms of an aggregate measure of welfare (expected income per capita), and of a referendum asking voters to choose between the two institutional set – ups. Both criteria confirm what the pattern of migration had lead us to expect. At low productivity, a single club is preferred. At high productivity, however, the importance of heterogeneity dominates the losses caused by the transaction costs, and diversification into two clubs is preferred. In addition, we find that political unification is achieved earlier through voting than through migration, since the referendum functions as a coordinating device.

Our approach to markets and clubs has its roots in several distinct literatures. The study of local public goods (Tiebout (1956); see also Bewley (1981)) emphasizes the sorting of individuals into different political jurisdictions on the basis of preferences for public goods, and argues that by "voting with their feet" citizens promote efficiency in the public sector. While we borrow from this literature the notion of freely forming political units which can provide different levels of a public good, we focus on tying this provision directly to trade. The closest work to ours in the local public goods tradition is probably the recent contributions by Wilson (1987a, 1987b). Wilson's work is different from ours in three ways: it relates local public goods to trade indirectly through general equilibrium price effects, whereas we are interested in the direct linkage; it assumes perfectly competitive markets, whereas we focus on matching and market extent; and it does not address the simultaneous and interrelated development of markets and political jurisdictions at different productivity levels, which is our central concern.

In defining the public good as excludable beyond the borders of the club, we are clearly borrowing from the public finance literature on clubs (Buchanan (1965); see also the survey in Scotchmer (1990)). However, by assuming free entry in both clubs and markets we avoid the problem of the correct pricing of the externality existing among the members of both institutions. This is the problem which is the focus of the literature on clubs.

In our central question, if not in methodology, we are close to the work of Milgrom, North, and Weingast (1990), and Greif, Milgrom, and Weingast (1990). These papers study the form and the evolution of legal enforcement as a necessary precondition for trade in the middle ages. Contrary to us, however, they focus on private enforcement of contracts and on the role of reputation, ignoring formal political institutions. In the period they consider, the Prince did not play a reliable part in legal enforcement. In addition, they do not analyze the formation of market areas.

Our work also relates to the theory of international trade. The original pure theory of trade does not include government, and focuses primarily on factor mobility and price determination. More recent contributions (see for example Krugman (1986) and Staiger and Tablellini (1987)) do include a government, but the government's role is usually to accomplish redistribution (through setting tariffs which alter relative prices) or to strategically compete with other governments. Thus the political structure rarely has a constructive relationship to economic trade in the sense we have in mind.

Related to international trade theory is the literature on "economic geography" pioneered by von Thunen (1966) and extended by Losch (1954) and Isard (1956). As discussed by Krugman (1990), this theory is confined to the study of market formation, usually in a geographic sense linked to transport costs; it has no government, and therefore no concern with the relationship between government and market areas.

Finally, our questions are related to the theory of regional integration in political science (see the collection of papers in Lindberg and Scheingold (1971)). More specifically, the approach we follow, where the question of political integration is linked to the development of wider market areas, is reminiscent of the "transactionalist" approach suggested by Deutsch (1968, 1969). According to Deutsch, agents who engage in increasing numbers of transactions across national borders develop a feeling of empathy towards their trading partners, and it is this feeling that constitutes the first essential nucleus of political transformation.

The remainder of the paper is organized as follows. The next section sets forth our assumptions. Section 3 analyzes the model when there is only one club. Section 4 presents the solution with two clubs. Section 5 compares the two institutional orders, section 6 concludes, and an appendix contains proofs and a number of additional results.

Section 2: Modeling Assumptions

2a: Joint Production

The economy we study consists of a large number of individuals, each of whom possesses an endowment of one unit. Individuals' endowments differ from one another in a way which can be formalized by arranging the endowments on a one dimensional continuum over the interval $[-1,1]$. We call this interval the variety space, and note that it has a total width of two. In all that follows the terminology "individual i" then refers to the individual possessing endowment $x_i, -1 \leq x_i \leq 1$.

An individual's endowment is not productive itself; rather it must be matched with someone else's endowment in a process of joint production. Further, each individual can form only one match, and each match must be between exactly two (and no more) individuals.

The matching technology which specifies the output of joint production lies at the heart of our analysis. When individuals i and j (representing endowments x_i and x_j) match, total output is twice

$$|x_i - x_j|(\beta d - |x_i - x_j|) \tag{1}$$

where β is a productivity parameter, and d is the quantity of trade-enhancing public good available to i and j. Both β and d play a central role in all that follows.

On one level, equation (1) may be viewed as specifying a particular functional form, which embodies a number of quantitative restrictions. However, it may also be viewed from a second, more abstract perspective. From this second perspective the equation has two important features: it represents one example of a general functional relationship involving the two traders and a third input – the public good; and it possesses certain qualitative features which reflect our perception of the economics of joint ventures.

Figure 1a displays a number of equation (1)'s qualitative and quantitative features. For a given value of βd, an individual of variety x_i has a pair of ideal trading partners, each at distance $\beta d/2$ from him.[1] Further, output is linear quadratic around these ideal partners. Thus if i is matched to a partner whose variety is very similar to his own x_i, the match output is close to zero, reflecting the small gains from trade available. Conversely, when i matches with a partner whose variety is further away than his ideal type, output again falls, reflecting the idea that the partners are too dissimilar to be able to successfully cooperate and prosper; output becomes zero at a distance βd from i (twice the ideal point) and then becomes negative for even more distant varieties. The negative output may be taken to refer to the presence of sunk costs in establishing the relationship (or learning one's partner's type), which are present in all matches but are normally outweighed by the match benefits.

The production relationship depends on the quantity of public good, d, in two important ways. First, notice that i's ideal partner is a function of d; as d increases, i's ideal partner moves further away from i's own variety x_i. Second, match output depends on d. When i is matched with his ideal partner output is equal to $\beta^2 d^2/4$, increasing quadratically in d. However, even when i is matched with some other partner, output increases in d. Both of these points are illustrated in figure 1b.

The public good which enters into the matching function may be of a number of different types, each designed to fulfill a different function in facilitating joint production. One interpretation is that the public good is legal enforcement of contracts. Thus a higher level of the public good corresponds to more complete, more accurate (for example, in determining liability), and faster enforcement. We expect improved enforcement to help mitigate problems of opportunism, familiar from the work of Williamson (1985) (see also the volume edited by Mowery (1988)); in turn, a reduced threat of opportunism encourages investment by both partners, and ultimately increases output.

While the public good may refer to legal enforcement broadly defined, it may also be more narrowly interpreted as a particular kind of law. A recent paper by Modigliani and Perotti (1990) emphasizes the importance of a credible and clear legal rule for financial transactions. They write

> Financial transactions are by their nature particularly sensitive to the legal framework in which they take place. Securities are nothing else than contractual legal claims representing liabilities of juridicial persons. Therefore, their value depends crucially on the enforcement of their stipulated rights, and more generally on the safeguards offered by the economic legislation that regulates their issuance, circulation, and participation in income and control.
> [p. 12]

To the extent that joint ventures reflect financial exchange or depend on external finance, these arguments apply.

Alternatively, the public good as legal enforcement might refer to patent protection. The partners will be more willing to share knowledge (which may in fact be one interpretation of their different varieties) and invest in research and development to the extent that their output (a new product) can be fairly divided and not expropriated by others outside the match.

Our model is too simple to incorporate all of the richness of these specific examples. We note, however, that the interpretation of d as legal enforcement is consistent with our assumption that the public good is more valuable the more disparate the partners' types: the more different the partners, the more difficult they will find it to monitor one another's actions (since these actions are less familiar to them), and therefore the greater potential role for enforceable legal committments.

A second interpretation of the public good is that it refers to the uniformity of standards (weights and measures) and regulations, facilitating compatibility of technologies and communication of "corporate cultures" between the two partners. Here we interpret the partners' varieties as their respective technologies and cultures, and interpret higher levels of the public good as representing an improvement in the standards' quality (reduced variability) and enhanced dissemination. North (1979) has discussed the role of the state historically in promulgating standards, and Farrel and Saloner (1986) and Katz and Shapiro (1985) provide an economic rationale for viewing such standardization as a public good. Just as with legal enforcement, the standards interpretation suggests that the public good will be more valuable in production the further apart (and therefore the less immediately compatible) are i's and j's varieties.

A third and last interpretation of the public good is that it provides insurance. Jackson (1989) presents a detailed description of the Japanese Keiretsu, clusters of Japanese firms which form mutual aid agreements, often centered around a major bank and representatives of several key industries. An important function of the keiretsu is to provide insurance to member firms, both financially and in terms of loyalty to one another's products. Possessing such insurance encourages the firms to take business risks, increasing expected output. We believe the keiretsu are a good example of the sort of "clubs" we describe in this paper, and that insurance can facilitate joint production; however, we should note that such insurance can also be useful to a single firm producing by itself, and not just in partnerships – in this sense insurance as a public good is less connected to our matching technology than the previous two examples. As with the earlier examples, this insurance public good is likely to be more valuable the more disparate the partners' varieties: if partners whose varieties are further apart have access to a technology characterized by a higher risk-higher return profile than relatively close partners, improved insurance coverage will allow them to increase their expected return relatively more.

2b: Clubs, Markets, Taxes, and Trades

Having described the matching function in some detail, we now go on to discuss the institutional features of our model. Each individual will choose to belong to two types of institutions: a club, in which he will participate in the vote to choose a level of the public good, receive that public good, and pay taxes; and a market, in which he will be randomly matched to a trading partner.

In our model a club's primary function is to provide a public good to its members. As in the original view of clubs introduced by Buchanan (1965) (see Scotchmer (1990) for a survey), an individual is entitled to use a club's public good only if he is a member of the club, or if he can attach himself to a member (we discuss this further below). Further. we assume that only one or at most two clubs can form, that every individual must belong to a club. and that all members of a club receive the same level of public good.

When only one club forms, all agents belong to it. However, when two clubs form. we must determine which individuals belong to which club. In studying this question of club configuration, we restrict the analysis to the case in which, in equilibrium, each club consists of a set of connected individuals. That is, two individuals belong to the same club only if they are connected to one another through a set of other individuals all of whom also belong to that club. In our simple economy inviduals are arranged along a line segment; hence a connected club consists of a single subsegment of agents. Since there are two clubs. they must therefore consist of adjacent line subsegments. As an example, the configuration $[-1, -\frac{1}{4}]$ and $[-\frac{1}{4}, 1]$ is a division into two connected clubs, while $[-1 - \frac{1}{2}] \cup [\frac{1}{3}, 1]$ and $-\frac{1}{2}, \frac{1}{3}]$ is not. We believe that the assumption of connectedness captures an important empirical fact: rarely does a country form consisting of disconnected islands of citizens. In what follows we show that such a configuration is stable (that is, that a disconnected individual will not deviate to join a connected club); but we cannot rule out other disconnected equilibria.

While our "clubs" are motivated by the general literature on club theory, they operate according to a number of specific rules which are somewhat different than the standard assumptions of club theory. In Buchanan's orginal work, a club good's availability is partially reduced by congestion; in this sense, it enjoys characteristics somewhere between those of purely private and purely public goods. To simplify our analysis, we assume that our club good suffers from complete congestion (a private good characteristic); that is, as the number of members in a club rises. expenditures must rise proportionately if a given level of club good is to be maintained. Although this aspect of our club good is similar to private goods, the good remains shared in the sense that all members of a club enjoy access to the same good. In terms of the earlier example of legal enforcement, assuming complete congestion captures the widespread view that law enforcement and a court system become slower and less effective as the caseload rises.

A second difference between our clubs and traditional club theory is that we assume the decision over the public good arises from a process of majority voting; this voting approach to clubs has also been discussed by Epple, Filimon, and Romer (1984). Related to this assumption, we assume that the club is free access – no entry fee is charged; instead, funds to finance the public good are levied as lump sum taxes on a per capita basis, with all club members paying the same amount. While we can imagine other ways of organizing the club, these assumptions best capture the sort of applications we have in mind.

For a given level of per capita taxes t, the amount of public good which a club can supply to each of its members is given by

$$d = t^{\alpha} \tag{2}$$

(Note that equation (2) implicitly incorporates the assumption of complete congestion.) The parameter α is a productivity parameter measuring the rate at which total tax revenues can be converted into the public good; a larger α indicates greater productivity.

Each market which forms is a random matching market in the spirit of Diamond (1982). As with clubs, the markets are free access – no entry fee is charged and no agent can be excluded from any market. Instead, individuals freely choose which market to attend based on their expectation of who else will attend that market. When only one market forms, individuals therefore have no choice and no ability to control who their partner may turn out to be. When two or more markets form, however, individuals can control the identity of their prospective partner by choosing their market. We recognize that the assumption of random matching within a market is strong. If we allowed individuals more control over their partner's identity, the model would take on a substantially different character, and might well lead to different results. However, we believe that addressing that issue would take us too far afield from our main focus on the interaction between clubs and markets.

When two individuals from different clubs attend the same market and are matched, several further issues arise. First, we must specify which public good (or what combination of public goods) enters the matching function given in equation (1). In all that follows, we will suppose that the public good used by the two partners is given by the maximum of the two public goods available to them. In the context of legal enforcement, we believe this is the best assumption to make: when partners from two different countries (or. in the world we envision, clubs) form a joint venture, they can usually choose to write the legal contract under either country's legal system (and in some cases even under a third country's system). However, in other contexts a different assumption might be more appropriate. For example, when the public good involves standards and compatibility, it might be better to use either the average or the minimum of the two public goods in (1). We cannot explore all of these different possible specifications in one paper. Our purpose here is simply to point out that many different assumptions about how the two public goods combine are possible, and. to the extent that the analysis depends on this assumption, it would be useful to be able to compare their implications.

Second. the traders suffer a transactions cost because they are familiar with different public good institutions and must expend time and effort exchanging information about one another's public goods and determining which public good should be used in their trading relationship. We denote the transactions cost by c; thus in a match between traders of different clubs, each trader's output is reduced by c. Our specification of transaction costs assumes that c is constant across all matches, and does not vary in proportion to the match output specified in equation (1). A number of our conclusions do depend on this assumption. and we discuss this further below.

Finally, we assume that in a match between traders of different clubs, the traders continue to divide the gains from trade equally, just as in the case of domestic trade between two individuals belonging to the same club. In extensions of the material presented in this paper, we have explored models in which the traders bargain over this division, with the trader from the better club earning a larger than 50% share of match output. However, we have found that introducing such bargaining does not substantially alter the qualitative features of our results, while considerably complicating the analysis. Hence we have chosen not to discuss bargaining in this paper.

Section 3: Single Jurisdiction Analysis

We begin our analysis by studying market formation and public good provision when there is a single political jurisdiction. For convenience, we first reproduce the assumptions of the model, and briefly discuss the model's structure. The main assumptions are:

1. Varieties x_i are distributed uniformly over $[-1, 1]$.
2. The output of a match between types x_i and x_j is $|x_i - x_j|(\beta d - |x_i - x_j|) - t$.
3. The public good is related to tax receipts by $d = t^\alpha$, $\alpha \in (0, \frac{1}{2})$.
4. The public good d is chosen by majority vote.
5. Traders attending the same market are randomly matched.
6. There is free entry into all markets.

The timing of the model is as follows. First, each agent learns his type (his position on the line). He then votes for the public good. Once the public good is decided upon, he chooses which market he wishes to attend, is matched with a partner, and trades.

Each individual must therefore solve two interrelated problems: he must decide how to vote over different public good alternatives: and must choose which market to enter. These decisions are interrelated because the public good enters directly into the trading relationship, so that he will prefer a higher level of the public good the more distant he expects his partner's type to be from his own type. Hence we must compute a perfect foresight Nash equilibrium in which the level of public good d^* is chosen with the subsequent division into markets correctly forecast by all agents.

3a: One Market

The simplest equilibrium arises when all traders participate in a single club and a single market. Figure 2a illustrates the market and club in this case - both coincide with the variety space. We present this example in some detail, because it is an important benchmark for comparison with later results, and because it illustrates very clearly a number of features of the model which continue to characterize the solution of the more complex cases presented next.

Concepually, the case of a single market and a single club corresponds to the formation of a large common market which coexists with a highly centralized political jurisdiction. Thus even though we begin with this scenario, it is most likely to arise, and most desirable, when productivity (as measured by β) is high, and therefore is most closely associated with late highly advanced stages of development.

When there is only one market, no single agent can deviate and establish a separate trading pool. Hence the only decision to be made is the choice of public good by voting. Each agent knows the extent of the market (the entire variety space) and can directly calculate the distribution of potential partners he faces. He uses this information to compute his preferences over different levels of d, and to determine his optimal or preferred level of d, which we denote $d_i^d = d(x_i)$ for the agent situated at position x_i.

The calculation of d_i^d proceeds in several steps. First, we integrate equation (1) over x_j between -1 and 1, deriving type x_i's expected output from the random matching:

$$Ey_i = \int_{-1}^{x_i} (x_i - x_j)[\beta d - (x_i - x_j)]dx_j + \int_{x_i}^{1} (x_j - x_i)[\beta d - (x_j - x_i)]dx_j - t$$

or

$$Ey_i = \frac{\beta d}{2}(1 + x_i^2) - x_i^2 - \frac{1}{3} - t \tag{3}$$

The term in parentheses is the expected distance between x_i and a random partner. It is increasing (and convex) in $|x_i|$: agents at the two extremes of the line have the highest potential productivity and the greatest need for the public good. Expected return, as a function of the agent's type, is therefore concave or convex depending on the public good provision. If there is a large amount of the public good ($\beta d > 2$), the high expected distance from a partner which characterizes agents at the two extremes of the market gives these agents the highest expected return; conversely, if the supply of the public good is low ($\beta d < 2$), expected return reaches a maximum at $x_i = 0$ (figure 2b).

Next we substitute equation (2) into equation (3) and differentiate with respect to d, to obtain each agent's optimal d:

$$d_i^d = [\frac{\alpha\beta}{2}(1 + x_i^2)]^{\frac{\alpha}{1-\alpha}} \tag{4}$$

Equation (4) illustrates how the differences in agents' relative positions in the market translates into differences in demands for the public good.

We note that for each x_i the demand for the public good is well-behaved and single-peaked around the optimal level in equation (4). Hence we can apply standard median voter results: the choice of the median voter will be supported by a majority against any other proposal (Downs (1957)). In our model the median voter is easy to identify. The preferences over d depend on expecte distance to one's partner. and are convex in x_i. symmetrical around $x_i = 0$, and attain a maximum at $|x_i| = 1$ (figure 2c); the median voters are those individuals at positions $x_i = +/-\frac{1}{2}$.

Setting x_i equal to the median voter's position in equation (4) gives us the supply of the public good in equilibrium:

$$d^* = [\frac{5\alpha\beta}{8}]^{\frac{\alpha}{1-\alpha}} \tag{5}$$

Finally, integrating the expected individual returns Ey_i (equation (3)) over x_i between -1 and 1, we obtain expected per capita income:

$$Ey = \frac{2}{3}(\beta d^* - 1) - t^* \tag{6}$$

where t^* is derived from assumption (3) above.

The model is now completely solved, as a function of the two exogenous parameters α and β. Given α, both the supply of the public good and expected per capita income are increasing in β – see figure 3.

As a final remark, note that because of the curvature in equation (3), the median voter's choice of d does not maximize expected per capita output. Since the expected distance from a partner is convex in x_i, mean expected distance is higher than the expected distance of the mean agent, and requires a higher level of the public good than the one preferred by the mean agent. But since the distribution of varieties is symmetric, the mean and the median voter coincide, and thus the level of public good preferred by the median voter is too low to maximize expected per capita output. Direct maximization of equation (6) with respect to d shows that the level of the public good which does maximize expected output is given by

$$d^{**} = [\frac{2\alpha\beta}{3}]^{\frac{alpha}{1-\alpha}} \tag{7}$$

which corresponds to the preferences of individuals positioned at $x_i = +/-\sqrt{1/3}$, lying to the outside of $+/-1/2$.

3b: Multiple Markets

A market consists of a pool of traders who are randomly matched to one another. While all traders may choose to belong to a single large market, this is not the only possible configuration. Instead, two or more distinct markets may form, with the interpretation that traders in each market are randomly matched amongst themselves but will not meet anybody from a different market. Since the distance of an individua's ideal partner increases with β, we expect that multiple markets will from when β is small, and that a smaller β will be associated with more markets. In this subsection we make this claim precise; throughout, we continue to assume that all traders belong to a single club which provides a common public good to all.

The major difficulty in characterizing the equilibrium with multiple markets lies in determining the composition of the various trading pools. It might seem that almost any configuration could be an equilibrium, with traders belonging to the same market coming from any possible number of segments of the variety space. However, this is not so, as the following proposition demonstrates:

> **Proposition 1:** If there is only one political jurisdiction, there is a unique equilibrium configuration allowing multiple markets. This configuration consists of n identical segments of adjacent traders forming n segregated markets, and is an equilibrium if and only if $\beta d \geq \frac{2}{n}(n > 1)$. (Proof in the Appendix.)

Proposition 1 establishes three points. First, it is not possible to sustain an equilibrium where agents belonging to non-adjacent segments all participate in the same market. Intuitively, if the public good is large enough to support trade between non-adjacent segments, the agents in the middle will always want to join the same market. Since we assume free entry, they cannot be excluded. Second, all trading pools must have identical width. And third, the number of possible pools is negatively correlated with β, verifying the intuition mentioned above. These last two features follow directly from the requirement that, in equilibrium, the temptation to deviate to a different market must be exactly zero for the trader at the border between two markets (the marginal trader), and negative for all others. Given the public good and the choice of market, each agent's expected output depends only on his expected distance from a random partner. It follows that if the marginal traders are indifferent and all trading pools are formed by contiguous segments, all pools must have identical width. In addition, since the temptation to deviate is highest for the marginal traders (because they are just indifferent between deviating and not), it must be that the public good is too low to support transactions with a higher expected distance than the one characterizing each market. In other words, in equilibrium expected output within each market falls as a trader's expected distance from his market's center increases, and is a minimum for the marginal trader. As it turns out, this last condition is equivalent to the requirement that $\beta d \leq \frac{2}{n}$; we discuss this further below.

The configuration specified in proposition 1 is intuitively pleasing. When the public good available is scarce, traders tend to remain in "neighborhood" markets, where the distance from an expected partner is low, but so is potential productivity. When the public good is more abundant, traders venture into wider markets (n must fall, and thus the size of each market must increase, as βd increases above the market n threshold $\frac{2}{n}$), and potential productivity is higher.

With n identical adjacent markets, the left-most pool of traders consists of types $[-1, (-1 + \frac{2}{n})]$, the next of types $[(-1 + \frac{2}{n}), (-1 + \frac{4}{n})]$, and so on. This configuration is displayed in figure 4. Note that all markets are equivalent, since each trader's output from a match depends only on the relative distance between him and his partner (hence, for example, match $(-1, -\frac{3}{4})$ is fully equivalent to match $(-\frac{1}{8}, \frac{1}{8})$). Thus we can compute expected output and preferences over the public good restricting attention to only one of the markets. For convenience, relabel the coordinates of this market so that it extends over $(-\frac{1}{n}, \frac{1}{n})$. Recomputing the integral in equation (3), we obtain:

$$Ey_i = \frac{\beta d}{2n}[1 + (nx_i)^2] - x_i^2 - \frac{1}{3n^2} - t \tag{8}$$

As anticipated, expected output is concave in x_i, and reaches a minimum at $x_i = +/- \frac{1}{n}$ (the marginal trader) when $\beta d < \frac{2}{n}$.

The computation of d^* for n markets becomes simple once it is recognized that a trader's preferences over d depend only on his relative position in the market he attends. Hence the median voters are now located at relative positions $+/- \frac{1}{2n}$ in each market. From equation (8), it follows that such traders' most preferred d is

$$d^* = [\frac{5\alpha\beta}{8n}]^{\frac{\alpha}{1-\alpha}} \tag{9}$$

The d^* associated with n markets is a factor $\frac{1}{n}^{\frac{\alpha}{1-\alpha}}$ lower than that associated with one market – the smaller width of each market reduces the demand for the public good.

Finally, expected per capita income is given by

$$Ey = \frac{2}{3n}(\beta d^* - \frac{1}{n}) - t^* \tag{10}$$

Notice that while βd^* cannot be larger than $\frac{2}{n}$, no restriction is placed on how small βd^* may be. Thus, for any α, multiple equilibria exist for most (smaller) values of β. In particular, one market is feasible for all β.[3] For β less than some β_2^{max}, two markets are also feasible: whether one or two markets form then depends on traders' beliefs. Figure 5 illustrates the various possible equilibria over the range $\beta \geq 1$, for $\alpha = \frac{1}{3}$.

Although a variety of market regimes may be possible for any particular β, these regimes differ in terms of the level of d^* and, more importantly, expected output. Figure 6 compares the one market and two market cases. Expected output per capita is higher in the two market case, whenever both regimes are feasible. Conversely, d^* is lower in the two market configuration. These findings are in fact quite general: comparing equations (5) and (9), and equations (6) and (10), one can show that whenever a number of different market regimes are feasible, they may be ranked in terms of d^* (with the higher n providing a lower d^* than any lower n) and expected output per capita (with the higher n providing a higher expected output).

To understand this result requires re-examining the conditions under which n markets are an equilibrium: For simplicity, we continue to focus on a comparison of two markets to one market. Two markets fail to be an equilibrium when β rises sufficiently high that some trader has an incentive to deviate and jump from his "home" market to the other market. The trader residing exactly on the border between the two markets (at zero) is always indifferent between them, since they are identical from his viewpoint. It is the varieties immediately next to him, who are "close" to the border, who are most likely to jump. Further, at the β at which these agents first choose to jump and upset the two market equilibrium the majority of other traders would prefer to remain in the two distinct markets. For these other individuals, located towards the edges of the variety space and far from the border between the two markets, traders in the other market, or in a common market (should that form), are too far away to be preferred partners (given β). Unfortunately, once the border people choose to jump, those next to them also find it worthwhile to jump, and a cascading process unravels the two market equilibrium, leaving the single market as the only viable configuration. Figure 7 depicts expected output for each variety at the level of β at which this unraveling occurs, illustrating the intuitive argument we have presented.

Thus one interpretation of the finding that two markets give rise to higher expected output per capita is that they fail to be an equilibrium sooner (at a lower β – we use this terminology to stress the chronology of development we have in mind) than is desirable. As figure 6 makes clear, if two markets did remain an equilibrium "longer", the expected output per capita associated with this equilibrium would eventually be overtaken by that of the one market regime, which rises more rapidly in β. Overall, market structure is not optimal, either from the viewpoint of the majority of agents, or the viewpoint of maximizing expected output.

Section 4: Two Adjacent Jurisdictions

When we extend our analysis to two clubs, we must specify how traders move between the clubs, and how they combine the two public goods when "international" trade takes place.

Again, we reproduce here for convenience the assumptions that we add to the structure of the model:

7. There are no more than two clubs, and they are adjacent to one another.
8. A match between individuals at positions x_i and x_j produces output $|x_i - x_j|(\beta d - |x_i - x_j|) - t$, if the individuals belong to the same club, and output $|x_i - x_j|(\beta max(d_1, d_2) - |x_i - x_j|) - c - t$ otherwise.
9. There is free migration between clubs.

The timing of the model is as follows. First, each agent learns his type. He then chooses which club he will join to vote and be taxed. Next he votes for the public good. Finally, after the public goods have been determined, he chooses which market he wishes to attend and is matched with a partner.

This scenario differs from the model of section 3 in requiring each individual to choose both his club and his market (recall that in section 3 there was only 1 club and hence no choice amongst clubs). A perfect foresight Nash equilibrium must now specify the size and composition of markets, the size and composition of clubs, and the public goods d_1^* and d_2^*, subject to the requirements that: (i) d_1^* and d_2^* are indeed the outcomes of the elections (with each club taking as given the level of d chosen by the other club); (ii) each agent attends his preferred market (does not wish to deviate to the other market); and (iii) each agent prefers membership in the club to which he is assigned to membership in the alternative club.

4a: One Market

Once again, we start from the simplest case - one common market - and proceed later to the more complex analysis of multiple markets. As we will show, two clubs are desirable in some ways (relative to one club), but not in others. On the positive side, the existence of two clubs accomodates, at least in part, the heterogeneity in agents' preferences over the public good which derives from their differing positions in the market. On the negative side, the differentiation into two clubs creates transaction costs. The balance between these two effects determines when two clubs are feasible, and when they are socially desirable.

The first natural candidate for equilibrium is a symmetrical configuration in which both clubs provide the same amount of the public good. Such a symmetrical outcome requires that the median voters in the two clubs have identical preferences, and therefore that the clubs' memberships be mirror images of one another. Hence the only possible equilibrium partition in this case is a division at zero: if the two clubs provide the same d, they must have equal size. As it turns out, however, such a configuration is not a viable equilibrium. Given the other club's choice of d, it is possible to show that each club's members strictly prefer to either raise or lower their level of d; hence, in a referendum between the "symmetrical" d and this alternative "asymmetrical" d, the latter wins. We conclude that the symmetrical equilibrium is not possible, and do not discuss it further.

The alternative to a symmetrical equilibrium is an asymmetrical equilibrium in which one of the clubs supplies more of the public good than the other. Intuitively, such a configuration seems plausible: the club supplying more of the public good holds the promise of higher expected match output to its members, but at a higher cost in terms of taxes. It is not immediately clear, however, where the division between two such asymmetrical clubs will occur: to determine this, we rely on a formal analysis of the model.

Let us name club 2 as the club providing the higher level of the public good, and σ the division between the two clubs. All $x_i \leq \sigma$ belong to club 2, and all $x_i \geq \sigma$ to club 1. Consider the individual at position x_i in club 2. In all transactions, he will use d_2, and thus his preferences over the public good are identical to those derived under the assumption of a single club given in equation (4). We know the shape of this function, but we do not yet know where σ is. However, if σ is not too large, the agent in the middle of club 2 ($x_i = \frac{\sigma-1}{2}$) is the median voter: all voters to his left prefer a higher level of d_2, and all those to his right prefer a lower level. The reason why σ cannot be large for this result is that the demand for the public good by $x_i = \sigma$ must be lower than the demand by $x_i = \frac{\sigma-1}{2}$ for the latter to be the median voter. Evaluating the demands of these two individuals demonstrates that the condition on σ is $\sigma \leq \frac{1}{3}$. It turns out that in equilibrium σ is always negative (we verify this below); hence this condition is always satisfied. Computing expected output for individuals in club 2 as

$$Ey_i = \frac{\beta d_2}{2}(1 + x_i^2) - x_i^2 - \frac{1}{3} - t_2 - \frac{1-\sigma}{2}c \tag{11}$$

and substituting $x_i = \frac{\sigma - 1}{2}$, it follows that the equilibrium level of d_2 is

$$d_2^* = [\frac{\alpha\beta}{8}(4 + (\sigma - 1)^2)]^{\frac{\alpha}{1-\alpha}} \tag{12}$$

Now consider agents in club 1. With probability $\frac{1-\sigma}{2}$, each agent will be matched with a partner also from club 1, and will use d_1; with probability $\frac{1+\sigma}{2}$, the agent will meet a trader from club 2 and gain access to d_2. Thus the expected return for agents in club 1 is:

$$Ey_i = \frac{\beta d_1}{2}[x_i^2 + \frac{1+\sigma^2}{2} - x_i(1+\sigma)] + \frac{1+\sigma}{2}\beta d_2[x_i + \frac{1-\sigma}{2}] - x_i^2 - \frac{1}{3} - t_1 - \frac{1+\sigma}{2}c \tag{13}$$

The demand for the public good in club 1 is then

$$d_{1i}^d = [\frac{\alpha\beta}{2}(\frac{1+\sigma^2}{2} - x_i(1+\sigma) + x_i^2)]^{\frac{\alpha}{1-\alpha}} \tag{14}$$

for the agent positioned at x_i. Expression (14) is convex in x_i and symmetrical around its minimum at $x_i = \frac{1+\sigma}{2}$. The median voters in club 1 occupy positions $x_i = \frac{3+\sigma}{4}$ and $x_i = \frac{1+3\sigma}{4}$, and

$$d_1^* = [\frac{5\alpha\beta}{32}(1-\sigma)^2]^{\frac{\alpha}{1-\alpha}} \tag{15}$$

To complete the characterization on equilibrium, we need to determine σ. In equilibrium, no agent can wish to change clubs (the marginal agent at $x_i = \sigma$ is indifferent). Consider the individual at position x_i in club 2 ($x_i \in [-1, \sigma]$). If he remains in club 2, the individual's expected return is given by equation (11); if he switches to club 1, simple calculations show that his expected return will be:

$$Ey_i = \frac{\beta d_2}{2}[\frac{x_i^2 + (1+\sigma^2)}{2}] + \frac{1-\sigma}{2}\beta d_1(\frac{1+\sigma}{2} - x_i) - x_i^2 - \frac{1}{3} - t_1 - \frac{1+\sigma}{2}c \tag{16}$$

The temptation to deviate, T, is the difference between equations (16) and (11):

$$T = \frac{\beta}{2}(d_1 - d_2)[\frac{1-\sigma^2}{2} - x_i(1-\sigma)] - (t_1 - t_2) - \sigma c \tag{17}$$

Expression (17) is increasing in x_i; hence if the marginal agent is indifferent, everyone else will strictly prefer to remain in club 2. The equilibrium condition for σ is then simply that expression (17) equal zero for $x_i = \sigma$, or that

$$\frac{\beta}{4}(d_1 - d_2)(1-\sigma)^2 - (t_1 - t_2) - \sigma c = 0 \tag{18}$$

(This condition is identical if the temptation to deviate is approached from the point of view of individuals in club 1 deviating to club 2.) Note that if we substitute in equation (18) the solutions for d_1, d_2, and t_1, t_2, we find that for given α, σ depends only on the quantity $\frac{c}{\beta^{\frac{1}{1-\alpha}}}$. Thus, at higher β the larger public good d_2 becomes more attractive, but this effect is exactly offset by the higher taxes required to finance it. In this simple model, the two opposite forces offset each other, and the parameter β enters with the same exponent in the two first terms of equation (18). If transaction costs were zero, or if they were proportional to income, with no fixed term, the border between the two clubs would not be sensitive to changes in β. It is the decline in the relative importance of c at higher income levels, precisely captured by the ratio $\frac{c}{\beta}$, that determines the equilibrium division between the two political jurisdictions. This result carries over to the discussion of multiple markets.

In conclusion, the asymmetrical equilibrium is characterized by equations (11), (12), (13), (15), and (18), determining expected returns for all agents, the levels of the two public goods, and the partition into two clubs.

As the equilibrium conditions described above cannot be readily solved analytically, we have explored the properties of this model through numerical simulations. In all cases we have found that the equilibrium is valid: i.e., deviations by either club are unprofitable. Thus we have concluded that, over the range of parameter values we have explored, the model's solution does conform to the program outlined above.

Figure 8 illustrates the partition between the two clubs for different values of β, at the representative values $\alpha = \frac{1}{3}$ and $c = .05$. Two results are clearly visible from this figure. First, for β below a threshold β^{min}, two clubs are not feasible. Second, as β rises, the fraction of the population in the high public good club rises, and, as β becomes very large, σ approaches an asymptotic value σ^{max} strictly above -1; thus, for high enough values of β, the population in the high public good club tends to stabilize.

The intuition for these results is quite simple. The desire to belong to the high public good club is highest at the very edge of the variety space, where the expected distance of an agent's trading partner is largest. However, if β is too low, the desire to belong to such a high public good club is more than offset by the transaction costs which accompany a division into two clubs. As β rises, transaction costs become less important, and more agents join the high public good club. However, agents in the middle of the variety space never wish to join this club, since their need for the public good is too low to justify the higher tax levy. As we mentioned above, it can be proved analytically that with our specification σ^{max} will always be negative, implying that less than half of the population will ever belong to the high public good club.

Our requirement that the clubs be adjacent limits traders' ability to sort themselves according to their preferences over the public good. Still, the analysis does capture the intuition that as trade volumes increase and markets integrate there is no reason to expect the heterogeneity in traders preferences over the public good to disappear. The two clubs that are created (endogenously) in equilibrium are the tool (albeit imperfect) with which traders try to differentiate themselves. The fact that the two clubs do not merge into a single club as β rises suggests that the two clubs might be preferrable to a unique club, a question to which we return in section 5.

4b: Two Markets

With more than one market, the partition into different clubs and the configuration of market trading pools are determined simultaneously. The model becomes much richer, and allows us to raise new questions about the relationship between club membership and market configuration. We can now study whether the movement to economic integration will be accompanied by tendencies towards political unification. More generally, we can investigate whether the club and the market will normally coincide, or whether "international" trade between different political jurisdictions will emerge in equilibrium. We will say that a market is "international" if individuals from both clubs enter the market to trade; a market is "domestic" if it consists entirely of traders from a single club.

Once again, we must start our analysis by identifying how the two markets can form in equilibrium:

> **Proposition 2:** If there are two adjacent political jurisdictions, in equilibrium two separate markets can exist if and only if each market is formed by a segment of adjacent traders.

Proposition 2 is an extension of proposition 1 and is based on the same intuition; we omit its proof, which can easily be obtained by adapting the proof of proposition 1.

If each market can be formed only by adjacent traders, three configurations are possible, depicted in figure 9. In the first configuration, the club and the market coincide. This is a symmetrical equilibrium with no international trade. where the two jurisdictions (and the two markets) are identical. The division into two clubs is irrelevant in this case. and the equilibrium exactly reproduces the one club equilibrium discussed earlier. The other two configurations are asymmetrical, with one club providing a higher level of the public good than the other. In these cases the borders of the clubs and the borders of the markets do not coincide, and there is international trade. In one of these two configurations the international market consists of all traders in the low public good club and a subset of the traders belonging to the high public good club; the domestic market then consists of the remaining members of the high public good club. In the other configuration, just the opposite is true. Letting γ represent the border between the two markets, the first configuration corresponds to $\sigma > \gamma$. and the second to $\gamma > \sigma$.[4]

The first equilibrium described above. the symmetrical case. is clearly feasible. Since the two clubs (and markets) are identical, there is no temptation to migrate or to change markets. as long as βd is sufficiently low. Since jumping to the other market implies trading with people from a different club, transaction costs insure that the equilibrium with two markets can be sustained longer – i.e. at higher β – than it could be previously in the case of a single club. Given our earlier results that the transition to one market happens "too early". this implies that the transition to one common market will take place "later" and with a smaller drop in expected per capita income.

The two asymmetrical equilibria are more surprising and more complex. Agents enjoy completely free entry into both markets and clubs: still they choose to engage in international trade and incur the transaction costs that this entails. Intuitively, the desire to engage in such international trade arises from two effects: on the one hand. an individual may prefer the distribution of potential partners in the international market; and on the other hand, the individual has the opportunity to gain access to the foreign public good.

In this section we analyze in detail the asymmetrical equilibrium with $\sigma > \gamma$ (the case represented in the middle of figure 9): the other asymmetrical equilibrium is discussed in the appendix.

As before, let club 2 be the high public good club with jurisdiction over all agents between -1 and σ: club 1 then consists of all agents between σ and 1. The two markets comprise the intervals $[-1.\gamma]$ and $[\gamma.1]$. If $\sigma > \gamma$, the international market consists of two groups: members of club 2 positioned between γ and σ. and all members of club 1. To verify the conditions under which this configuration can be an equilibrium. we follow the usual procedured: we guess that the equilibrium has this configuration and derive the equations that must be satisfied if noone is to deviate.

Consider $x_i \in [\gamma, \sigma]$, i.e. x_i belonging to club 2 and attending the international market. In all transactions this agent uses d_2 (since he belongs to club 2 and $d_2 > d_1$). Since his partner is chosen randomly in the interval $[\gamma, 1]$, his expected return is

$$\frac{\beta d_2}{1-\gamma}[x_i^2 + \frac{1+\gamma^2}{2} - (1+\gamma)x_i] - x_i^2 - \frac{1-\gamma^3}{3(1-\gamma)} + x_i(1+\gamma) - \frac{1-\sigma}{1-\gamma}c - t_2 \quad (19)$$

where $\frac{1-\sigma}{1-\gamma}c$ is the expected transaction cost from trading with a partner in club 1.

Let us first verify that x_i will not want to change clubs. If he switched to club 1, with probability $\frac{1-\sigma}{1-\gamma}$ he would be matched with someone from club 1, would only have access to d_1, but would not suffer any transactions cost. With probability $\frac{\sigma-\gamma}{1-\gamma}$ he would be matched with someone from club 2, and would use d_2 and pay the transactions cost. Thus, his expected return would be:

$$\frac{1}{1-\gamma}[\beta d_1(\int_\sigma^1 (x_j - x_i)dx_j) + \beta d_2(\int_\gamma^\sigma |x_i - x_j|dx_j) - \int_\gamma^1 (x_i - x_j)^2 dx_j - (\sigma-\gamma)c] - t_1 \quad (20)$$

where $\frac{1}{1-\gamma}$ is the normalizing factor.

Solving the integrals in equation (20), and subtracting equation (19), we derive the temptation T_c to switch to club 1:

$$T_c = \beta\frac{1-\sigma}{1-\gamma}[\frac{1+\gamma^2}{2} - x_i](d_1 - d_2) + (t_2 - t_1) + \frac{1-2\sigma+\gamma}{1-\gamma}c \quad (21)$$

T_c is monotonically increasing in x_i: therefore if the individual on the border between the two clubs, at position σ, is indifferent between them, nobody else in the interval $[\gamma, \sigma]$ will want to deviate. Hence equilibrium requires only that T_c equal zero, or

$$\frac{\beta}{2}(1-\sigma)^2(d_1 - d_2) + (t_2 - t_1)(1-\gamma) + c(1 - 2\sigma + \gamma) = 0 \quad (22)$$

Equation (22) defines σ. The condition would be identical if we had derived it from the point of view of the agents in club 1, who reside between σ and 1.[5] In addition we must verify that in equilibrium no - one in club 2 in the interval $[-1, \gamma]$ wants to migrate to club 1. This requires

$$t_2 - t_1 < c \quad (23)$$

To characterize the border between the two markets, γ, we apply the same logic. If $x_i \in [\gamma, \sigma]$ were to jump to the club 2 domestic market, his partner would be chosen randomly in the interval $[-1, \sigma]$, the public good used in trading would always be d_2, and there would be no transaction costs. His expected return would thus be:

$$\beta d_2[x_i + \frac{1-\gamma}{2}] - x_i^2 - \frac{1+\gamma^3}{3(1+\gamma)} - x_i(1-\gamma) - t_2 \tag{24}$$

Subtracting equation (19) from (23) we obtain the temptation T_m to switch markets as

$$T_m = \frac{\beta d_2}{1-\gamma}[2x_i - \gamma - x_i^2] + \frac{2}{3}\gamma - 2x_i + \frac{1-\sigma}{1-\gamma}c \tag{25}$$

In equilibrium T_m must satisfy two requirements. First, it must reach a maximum at $x_i = \gamma$; this is equivalent to

$$\beta d_2 \leq 1 \tag{26}$$

Second, T_m must equal zero at $x_i = \gamma$, which is equivalent to

$$\beta d_2 \gamma - \frac{4}{3}\gamma + \frac{1-\sigma}{1-\gamma} = 0 \tag{27}$$

Equation (26) restricts the range of β values that can support this equilibrium.[6] Equation (27) defines the marginal trader γ. Again, the two conditions would be identical if we had derived them from the perspective of $x_i \in [-1, \gamma]$, i.e. x_i lying on the left side of the markets' border.

In addition, there must be no deviation to the domestic market in club 2 by any trader in club 1, i.e. by any $x_i \in [\sigma, 1]$. This requires

$$\frac{\beta d_2}{1-\gamma}\left[(1-\sigma)x_i + \frac{1-2\gamma+\sigma^2}{2}\right] - \frac{\beta d_1}{1-\gamma}(x_i^2 - (1+\sigma)x_i + \frac{1+\sigma^2}{2}) - 2x_i + \frac{2}{3}\gamma - c\frac{1-\sigma}{1-\gamma} \leq 0 \tag{28}$$

The final step in verifying the equilibrium is to solve for the median voters' choices of the two public goods. Let us first consider club 1. All agents in club 1 take part in the international market. Hence each of them has probability $\frac{1-\sigma}{1-\gamma}$ of being matched with someone from club 1, and probability $\frac{\sigma-\gamma}{1-\gamma}$ of being matched with someone from club 2. It is then straightforward to calculate expected returns, and to derive individual demands for the public good d_1 as a function of each agent's location:

$$d_{1i}^d = \left[\frac{\alpha\beta}{1-\gamma}(x_i^2 + \frac{1+\sigma^2}{2} - (1+\sigma)x_i)\right]^{\frac{\alpha}{1-\alpha}} \tag{29}$$

This function is convex in x_i and symmetric around a minimum at $x_i = \frac{1+\sigma}{2}$. The two median voters are $x_i = \frac{3+\sigma}{4}$ and $x_i = \frac{1+3\sigma}{4}$, and the outcome of the vote is

$$d_1^* = \left[\frac{5\alpha\beta(1-\sigma)^2}{16(1-\gamma)}\right]^{\frac{\alpha}{1-\alpha}} \tag{30}$$

The derivation of the equilibrium choice of the public good in club 2 is more complex: voters belong to two different markets, and their combined preferences do not exhibit the simple symmetry that allowed us to identify easily the median voters in all previous examples. From equation (19), we can derive the demand for the public good for $x_i \in [\gamma, \sigma]$ to be

$$d_{2i}^d = \left[\frac{\alpha\beta}{1-\gamma}\left(x_i^2 + \frac{(1+\gamma)^2}{2} - (1+\gamma)x_i\right)\right]^{\frac{\alpha}{1-\alpha}} \tag{31}$$

Similarly, we can differentiate expected returns for $x_i \in [-1, \gamma]$, and obtain the demand for the public good amongst those agents who trade in the domestic market:

$$d_{2i}^d = \left[\frac{\alpha\beta}{1+\gamma}\left(x_i^2 + \frac{1+\gamma^2}{2} + (1-\gamma)x_i\right)\right]^{\frac{\alpha}{1-\alpha}} \tag{32}$$

Equation (32) is convex in x_i reaches its minimum at the midpoint $x_i = \frac{1-\gamma}{2}$, and is symmetric around this point. Equation (31) is also convex in x_i and symmetric around its minimum, but the minimum is at $x_i = \frac{1+\gamma}{2}$, which may or may not belong to the relevant interval. In other words, while we know the shape of the public good demand for those agents who enter the domestic market, the demand for the public good amongst club 2 members who enter the international market is more complex: specifically, this second demand is monotonically decreasing in x_i if $\sigma < \frac{1+\gamma}{2}$, but not otherwise. In addition, the two functions in (31) and (32) are generally not equal at the border $x_i = \gamma$, which further complicates the identification of the median voters.

The identity of the median voters will depend on the relative sizes of the two markets and the two clubs. In broad terms, we can identify two distinct regimes. First, it is possible that the preferences of most individuals attending one of the markets are strictly larger than the preferences of individuals attending the other market; in this case, there would be a single median voter residing in one of the two markets. More interesting is the second regime, in which demands for the public good are comparable amongst the individuals in the two markets; in this case, the median voters do not all attend the same market. In all that follows we concentrate on this second regime, which turns out to be the relevant case for the parameter values which we examine. In general, this regime corresponds to a situation in which the two markets are of comparable width.

Within this second regime two configurations are possible. The first configuration arises when σ is not very large relative to $\frac{1+\gamma}{2}$. Then the demand for the public good near the border of club 2 is not very large, and there are three median voters: two in the domestic market, denoted positions z_1 and z_2, and one in the international market, denoted z_3. Then z_1, z_2, and z_3 are defined by the conditions

$$(1) d_2^* = d_{2i}^d(x_i = z_1) = d_{2i}^d(x_i = z_2) = d_{2i}^d(x_i = z_3), -1 < z_1 < z_2 < \gamma < z_3 < \sigma \quad (33)$$

$$(2)(z_3 - z_2) + (z_1 + 1) = \frac{1+\sigma}{2}$$

$$(3)(\gamma - z_2) = (z_1 + 1)$$

Each of these conditions has a simple justification. The first condition reiterates that these three individuals agree on the optimal level of d_2; note, however, that these demands are given by equation (32) for z_1 and z_2 and by equation (31) for z_3. The second condition states that these three individuals are the median voters in club 2, with 50% of the members of the club preferring a higher level of the public good than they do. Finally, the third condition exploits the symmetry of equation (32) to fix the relative positions of z_1 and z_2. The configuration described in conditions (33) is valid as long as the level of public good so determined is above what would be chosen by $x_i = \sigma$, the marginal member of club 2 in the international market.

In the second configuration σ is large relative to $\frac{1+\gamma}{2}$, and conditions (33) are not applicable. There are then four median voters, two in each market. They are characterized by:

$$(1) d_{2i}^d(x_i = z_1) = d_{2i}^d(x_i = z_2) = d_{2i}^d(x_i = z_3) = d_{2i}^d(x_i = z_4), \quad (34)$$

$$-1 < z_1 < z_2 < \gamma < z_3 < z_4 < \sigma$$

$$(2)(z_3 - z_2) + (z_1 + 1) + (\sigma - z_4) = \frac{1+\sigma}{2}$$

$$(3)(\gamma - z_2) = (z_1 + 1)$$

$$(4)\frac{1+\gamma}{2} - z_3 = z_4 - \frac{1+\gamma}{2}$$

The symmetry of equation (31) guarantees that z_3 and z_4 must lie at the same distance from $x_i = \frac{1+\gamma}{2}$, the minimum of the function - this is the content of condition (4).[7]

The equilibrium is now completely characterized. It is defined by equations (22), (26). (27). (30). (31). (32), and one of the two alternative sets of conditions (33) and (34). These equations determine the border between the clubs σ, the border between the markets γ, the acceptable range of β, and the public goods d_1^* and d_2^*. In addition, equations (23) and (28) must be satisfied.

To characterize these equilibrium conditions we have relied on extensive numerical simulations of the model. Figure 10 depicts a set of results from these simulations; in the figure, α and c are set at the representative values $\frac{1}{3}$ and .05, while β is allowed to vary from .5 to 1.25.

Figure 10 illustrates several qualitative features of the model's solution. First, the international market is always smaller than the domestic market, but as β increases the two markets approach equal size. Second, as β increases agents move into the high public good club, until, at high enough β, the low public good club completely disappears and we are back to the case of one club and two markets analyzed in section 3.

The results on the location of the markets division are easily explained. The border trader belongs to club 2 and used d_2^* in both markets. Since $\beta d_2^* \leq 1$, if the international market were larger than the domestic one, expected returns from trading in it would necessarily have to be concave in x_i. A smaller market, where the border trader would face smaller expected distance from his random partner, would then be preferred: the border trader would shift to the domestic market. Thus the international market cannot be larger. In addition, when meeting a foreigner in the international market the border trader suffers transaction costs. Therefore $\gamma = 0$ and equal markets cannot be an equilibrium either. The international market must be smaller, and γ must be positive, as long as there is any positive probability of meeting a foreign trader (as long as $\sigma < 1$). (Of course, the exact location of the border depends on c.) At higher β, agents are migrating to club 2, and the expected transaction costs of the border agent engaged in international trade are lower: γ must move towards zero, and reach it exactly when σ reaches one.

The migration between the two clubs is determined by the trade – off between transaction costs, heterogeneity in preferences, and the possibility of free – riding on the club providing the higher level of the public good. Consider the temptation to migrate as viewed by a border agent belonging to club 2, for given β. If he is very close to the edge of the market (σ close to zero), his relative position dictates a preference for club 2. However, the international market is dominated by traders from club 1, and the probability of incurring transaction costs is close to 1. If the transaction costs are not negligible, this second effect dominates the first, and the temptation to deviate to club 1 is positive. Thus σ cannot be close to zero. On the other hand, the argument applied to a border agent belonging to club 1 shows that σ cannot be close to one, if β is low and again transaction costs are important. From his point of view, transaction costs would be too high. Therefore for low β we expect σ to be somewhere near the middle of the market. Note that in this relative position, if transaction costs could be ignored, the border agent should be expected to prefer a lower public good, and therefore club 1.

At higher β, the importance of the transaction costs declines, and the preference for club 1 stemming from the location of the border agent would dominate. This case can no longer be an equilibrium. Equilibrium requires that club 1 be still smaller, so that transaction costs again make belonging to the very small club 1 too costly. The process continues at still higher β values, until eventually everybody has moved to club 2, and the partition into two jurisdictions has disappeared.

The results in figure 10 become still more meaningful if they are combined with the earlier discussion of two clubs and one market (figure 8). Figure 11 describes the relationship between these two regimes by graphing the size of club 1, as a function of β, over both regimes. First, consider values of β for which equilibrium consists of two markets. Within this range, when β is relatively small, agents are content to divide into 2 clubs and 2 markets. However, as β rises the productivity benefits of belonging to the high public good club begin to dominate the tax benefits of belonging to the low public good club, and club 1 disappears; this is the content of figure 10. Now consider further increases in β to the point at which a single common market emerges. At first, the agents continue to form only one club. Eventually, however, β becomes so large that heterogeneity in preferences over the public good reemerge, leading a second club to form. As β becomes very large, the division between the two clubs approaches an asymptote. This second chronology is the content of figure 8. Figure 11 combines these regimes together, and illustrates how the second club disappears during the transition to a common market, only to reemerge at a "later" stage of development.[8]

We close our discussion of the 2 club – 2 market case by noting that the qualitative nature of our results is relatively robust to changes in the parameters α and c. Of course, the values of these parameters clearly determine the exact position of the boundaries between markets and clubs. For example, if the transaction costs c were very low, the economy might settle towards the asymptotic division into two clubs before, and not after, the transition to a common market. However. the general conclusion we discussed above would not be affected: it would still be true that with lower productivity and smaller markets (where there are more than two of them) the transaction costs would play a larger weight and create pressure towards unification, while at higher β, and larger markets. the phenomenom would at a certain point reverse itself. Exactly at what value of β this reversal takes place depends on c and α, but it will always happen. In this sense, the calibration of our model is not important for the result. What is important is the lump sum nature of the transaction costs, so that their relative magnitude falls with respect to income as trade becomes more productive.

Section 5: Comparison Between One and Two Jurisdictions

In the previous discussion, the tendency towards political unification took the form of continuous migration to one of the two clubs. In this section. we study the question of political integration from a different perspective: we compare the equilibria with one and two clubs and ask which one yields the highest expected income per capita, and, most importantly. which one would be chosen by a majority of the agents in the economy if they were asked to vote over the two options. The first question is normative in character. In a model based on heterogeneity, expected per capita income before agents learn their type seems the appropriate welfare measure (Harsanyi (1955)). The second question is positive. Measures involving some degree of integration between different jurisdictions are political decisions, and with a democratic constitution. and ignoring the use of force, we expect them to be decided by majority rule, after voters learn their type. These positive and normative questions need not have the same answer, since the voting outcome strongly depends on distributional issues.

Differences in welfare depend on the equilibrium composition of the markets, and on the provision of public goods. The first point has been examined above. The second is described in figure 12, where the supply of public goods is plotted against β, in both the one and two market regions, and for both one and two clubs.

When β is lower, and two markets are an equilibrium. the public good voted in club 2 follows very closely the level of public good that would be chosen if there were only one club. This is surprising, since the two markets have very similar size in the two cases, and voters of club 2 expect to use d_2 in all transactions. Voters in club 1 only use d_1 when trading with other members of their club. The equilibrium level of d_1^* rises for a while with β, but then falls, eventually to zero, as the size of club 1 shrinks and the likelihood of using d_1 falls.[9]

The differentiation between the two clubs is more interesting for β values that support the one market equilibrium. Here, the role of the political division in allowing voters to sort themselves is clear. The two clubs provide public goods that differ substantially from what would be chosen if there were only one jurisdiction. On the one hand. club 2 is formed by traders at the very edge of the market, who desire a large amount of the public good. Voters in club 1, on the other hand, prefer a smaller public good, for two reasons: first, because of their position near the center of the market: and second, because they can free ride on club 2. Members of club 1 will use d_1 only when matched with someone else from club 1; and since all club 1 members are relatively close to one another, the need for the public good in these matches is small. If there were only one public good, a compromise would have to be struck between the different tastes of club 1 and 2 members, and the public good provided would be somewhere between d_1^* and d_2^*.

The total effect of market size, public good provision, and transaction costs on expected per capita income is depicted in figure 13. Expected per capita income is plotted against β, again over both the one and two market ranges, for both one and two clubs.

When two markets and two clubs are an equilibrium, productivity is low and transaction costs dominate the outcome. In addition, the two clubs are not very effective in sorting voters' types. Hence expected income is lower under 2 clubs than with 1 club, although the income levels associated with these two regimes approach one another as β rises and the smaller club disappears.

In contrast, when two clubs and one market are an equilibrium, productivity is high. transaction costs become increasingly irrelevant, and the role of the two clubs in sorting individuals leads to higher expected income under the 2 club regime. The difference in expected incomes rises with β. Thus welfare analysis confirms our previous statement that if wider markets are coupled with higher productivity, and if transaction costs are an important factor in evaluating the desirability of common institutions, then the importance of these transaction costs should decline as economic integration is achieved.

To evaluate the outcome of a referendum where voters are asked to choose between one and two clubs, we need to study how the difference in expected incomes is distributed.

Figures 14 and 15 show the comparison of expected individual incomes as a function of agents' types.

In figure 14. β is set equal to 1. and the equilibrium has two markets. It is clear that a majority of agents in both clubs would vote for political integration. The one club scenario is preferred by everyone, with the exception of the domestic traders in club 2 who occupy positions close to the border between the two markets. Since with two clubs the market border is translated (slightly) to the right ($\gamma > 0$), they are closer to the middle of the domestic market and better off under 2 clubs. The most ardent supporters of political integration are the individuals in club 2 positioned near the middle of the international market. For them. 2 clubs leads to a smaller market. the same level of taxes. and transaction costs. The members of club 1 prefer integration because it eliminates transaction costs (though it also results in higher taxes). While the specific distribution of voters' preferences depends on the value of β chosen. the overall conclusion that one club will defeat two clubs in a referendum is quite general. Further. this result holds true regardless of whether one or two clubs is the status quo point.

We conclude the discussion of two markets with two observations. First, voting reaches the same outcome as a normative decision based on expected per capita income. Second. a referendum process would result in political unification long before it could be achieved by the decentralized migration of individuals (compare figure 10 to 14). This is so even though we assume zero migration costs. This is an implication of the standard coordination problems with Nash equilibria: all agents in club 1 would be better off if they could move together, but if the others do not move each single agent (except for a few border people) prefers to remain. A vote between the two scenarios subjects everybody to the will of the majority, but is a way of solving the problem of coordination.

In figure 15, β is set equal to 2. and the equilibrium has one market. Agents in club 1 almost unanimously prefer two clubs. The closer they are to the middle of the market. the more they benefit from being able to choose a smaller public good, while if they are near the right-hand edge of the market they value the possibility of free riding on club 2. Only those members of club 1 close to the border with club 2 prefer one club: d_1^* is too low for their tastes and free riding on club 2 is of little use, since their most distant potential partners are in club 1. A majority of agents in club 2 prefer a common club. Unless they are extremely productive (at the edge of the market). the cost of financing the public good and the transaction costs are too high to make two clubs worthwhile. Once again, however, note that, if left to their own devices, they would not migrate to club 1; nor would they vote for a lower level of public good. At higher β values their preference for a common club becomes weaker, until eventually a majority prefers two clubs. With $\alpha = \frac{1}{3}$ and $c = .05$, however, this does not occur until β is close to 4.

Since most agents belong to club 1, it is not clear how much power the preferences of voters in club 2 really exert. If the status quo were two clubs, unification would be defeated by the majority in club 1. If instead the status quo were one club, a majority of the voters would vote for a political division, though not a majority of those agents who would subsequently end up in the smaller club 2.

Section 6: Conclusions

This paper has studied the contemporaneous formation of markets and political jurisdictions in a general equilibrium model where the return from private trade depends on a public good.

We have reached three main results. First of all we have shown that trade between different jurisdictions can occur in equilibrium, even with free entry in both markets and clubs, and with transaction costs reducing the return from such "international" trade. When productivity is low, and transaction costs are important, agents enter the international market either because their relative position in such a market is advantageous, or because they hope to free ride on the public good provided abroad. When productivity is high, a common market is the only equilibrium.

Second, as the relative importance of transaction costs declines, so does their role in shaping market structure and the composition of political clubs. At low but rising productivity, agents tend to progressively migrate towards one of the two clubs to mitigate the weight of the transaction costs. At higher productivity, on the contrary, traders differentiate themselves again on the basis of their different needs for the public good. Since these changes in productivity are accompanied by a movement to progressive economic integration, we expect a tendency towards political unity as markets become wider, counteracted eventually by a movement in the opposite direction.

Finally, this tendency is in line with social welfare. The value of different jurisdictions in sorting voters with heterogeneous tastes is higher at higher productivity, even if the volume of trade subject to transaction costs is also higher. If voters were asked to choose between one or two clubs, one club would be the observed outcome more often than it will be achieved by migration alone. However, this would again be reversed when productivity is sufficiently high.

Beside the general structure of the model, and the relationship linking preferences over the public good to agents' positions in each market, there are two fundamental assumptions that are responsible for our results. First of all, we have modeled agents' varieties as distributed along a line. This implies that when one common market forms at high productivity levels, the variance in traders' roles in the market, and thus in their tastes over the public good, rises. In turn this generates the need for differentiation that finds expression in the two clubs. If, for example, we had modeled agents' varieties as distributed around a circle, with one common market each agent would have been identical to any other, and preferences over the public good would have collapsed to complete unanimity around a single value. Clearly, one club would have been the final outcome. We believe that heterogeneity is a more appropriate characterization of traders' preferences, and that it is likely to increase, and not disappear in a world of wider markets and more sophisticated and specialized technologies. More generally, it would be useful to extend the model to two dimensions, which seems a more accurate representation of real world market and club divisions.

Second, we have assumed that transaction costs are fixed costs. Thus if productivity and income rise, their relevance tends to fall even if transactions increase. Fixed costs seems to correctly describe a number of important examples, including learning the functioning of foreign regulations and laws, standards, currency, or language. However, it is possible that the nature of our results would change if transaction costs were partly fixed and partly proportional to trading output.

Assuming that partners can use the best of the two public goods available to them has biased our results in favor of two clubs, by avoiding the underprovision of public goods that would take place if neither club completely internalized the markets' needs. However, we do not think that alternative assumptions would alter our conclusions on the relatively superior desirability of multiple clubs at higher productivity levels.

Finally and independently of our specific results, we want to stress again the general motivation behind this model. Its purpose was to provide an example where the structure of economic transactions and political affiliations is decided simultaneously by heterogeneous agents, and is liable to evolve as development proceeds.

Figure 1

The Production Technology

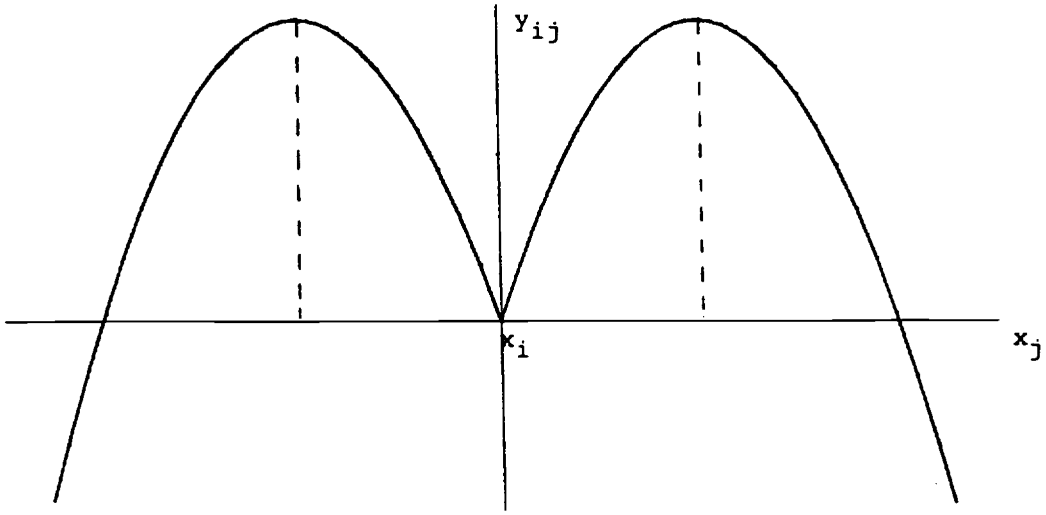


Figure 1a: Output as a function of x_i's partner

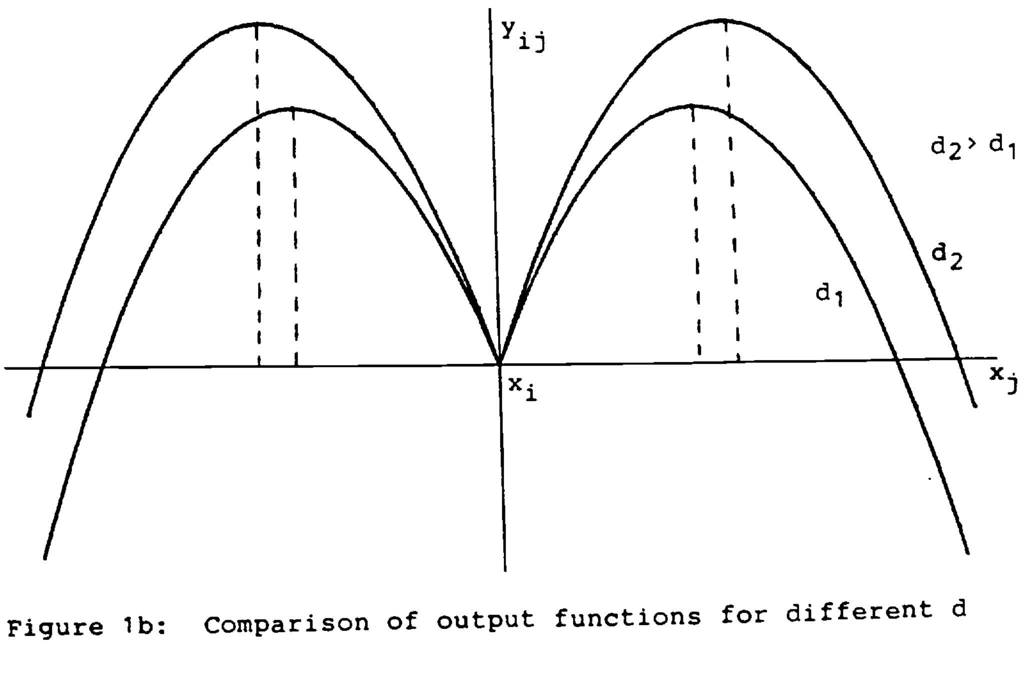


Figure 1b: Comparison of output functions for different d

Figure 2

One Club, One Market
Agents' Characteristics

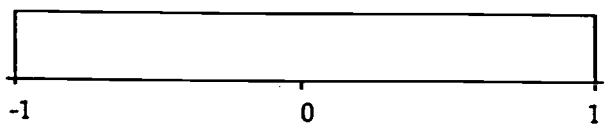


Figure 2a: Distribution of Varieties.

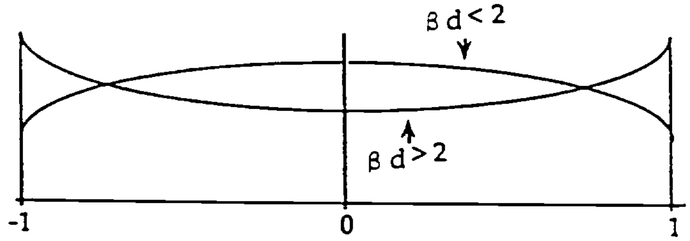


Figure 2b: Expected gains from trade.

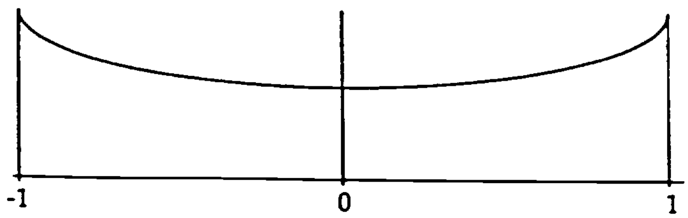


Figure 2c: Demand for the public good.

Figure 3

One Club, One Market Equilibrium

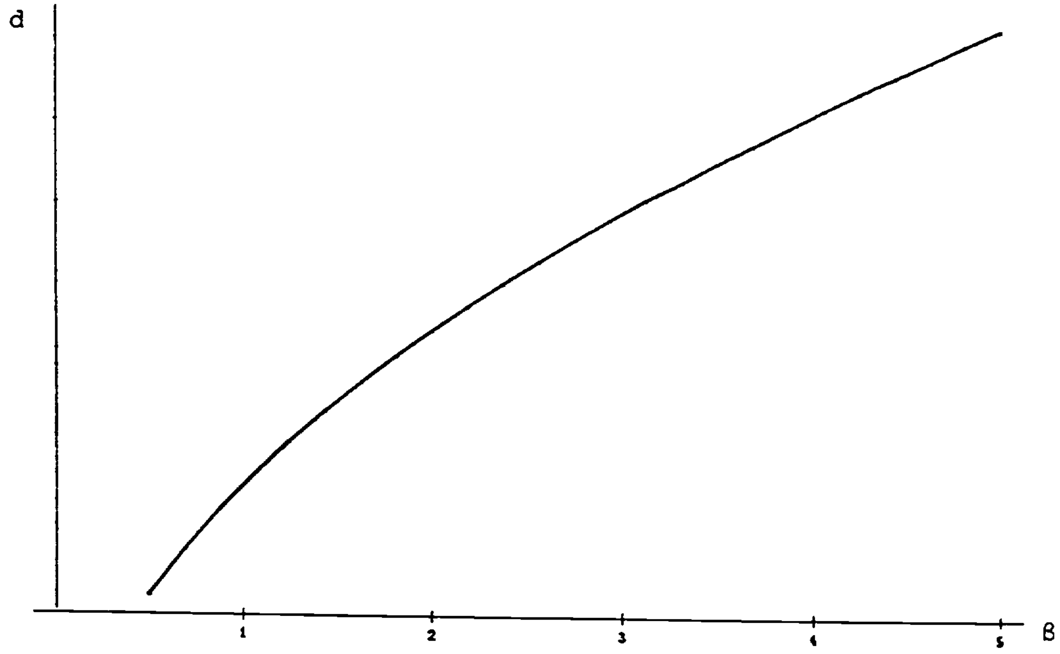


Figure 3a: Provision of the Public Good

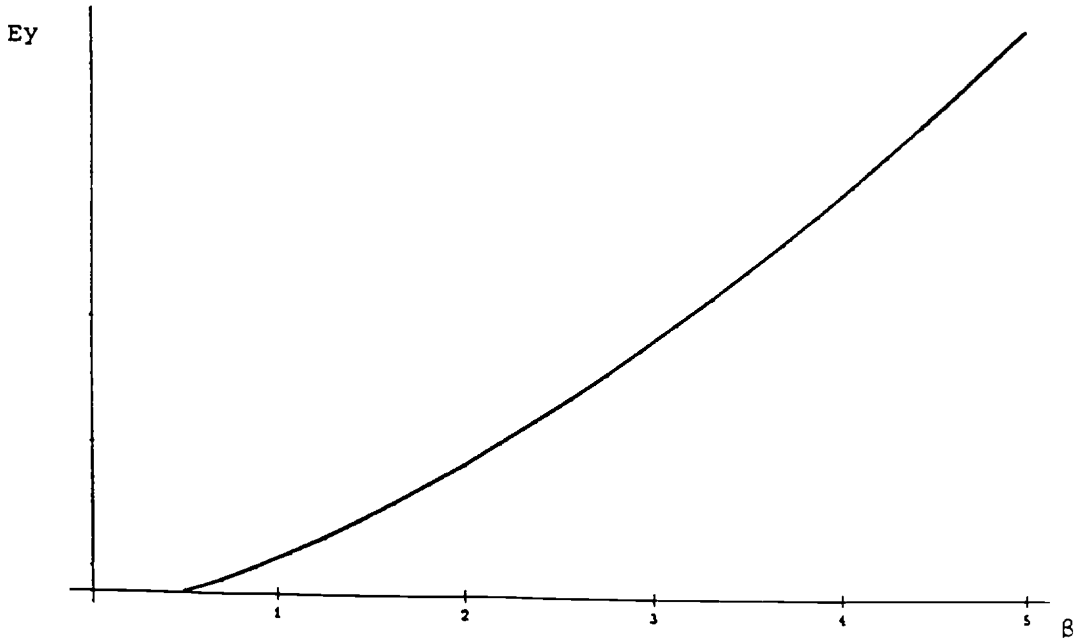


Figure 3b: Expected Income per Capita

Figure 4

One Club, n Markets

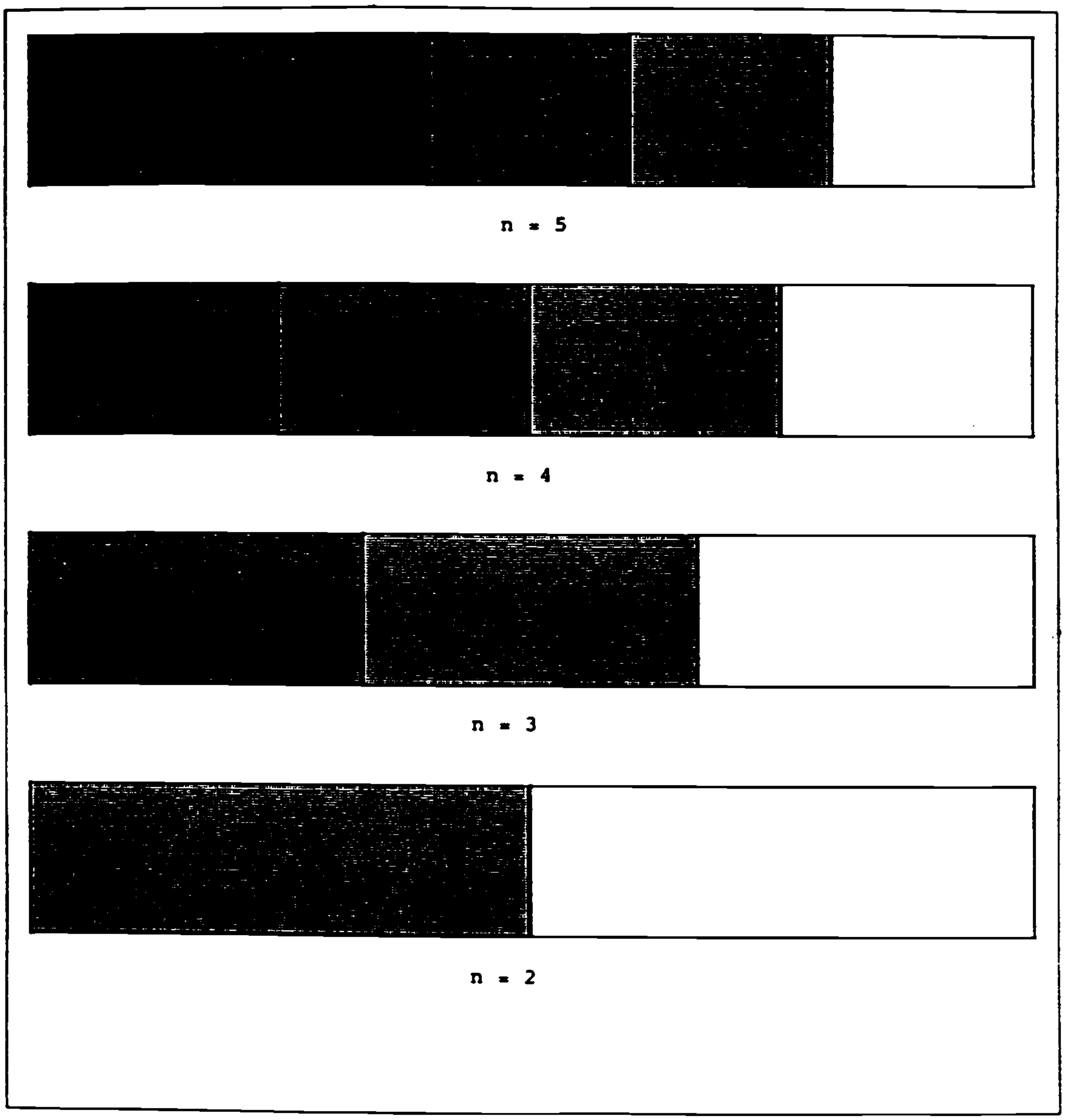


Figure 5

One Club, Multiple Markets
Transition Points
$\alpha = 1/3$

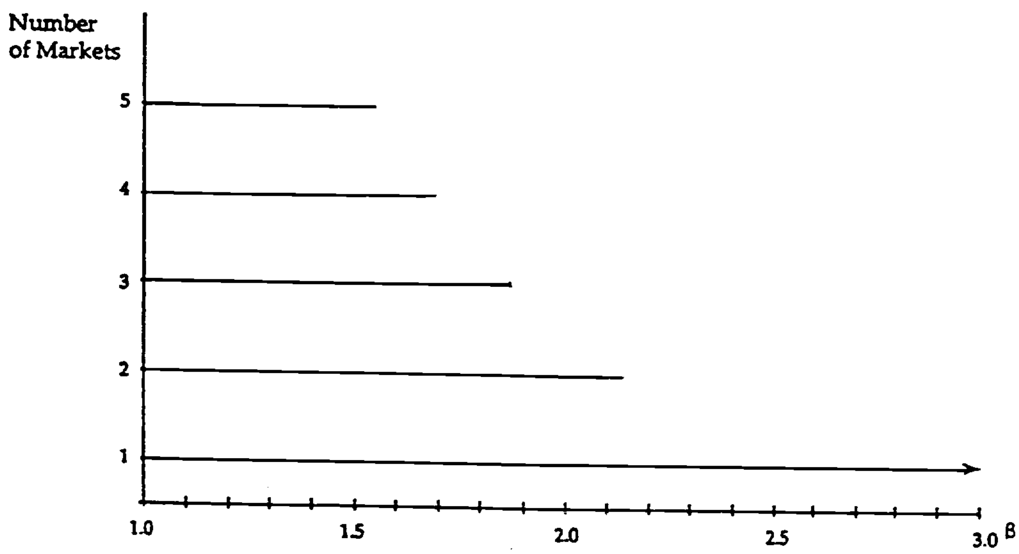


Figure 6

One Club
Comparison Between One and Two Markets

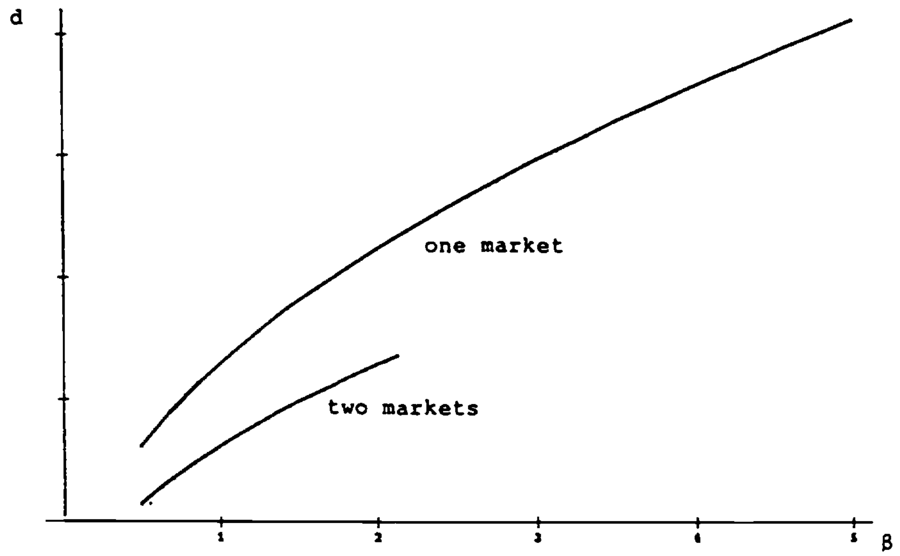


Figure 6a : Provision of the Public Good

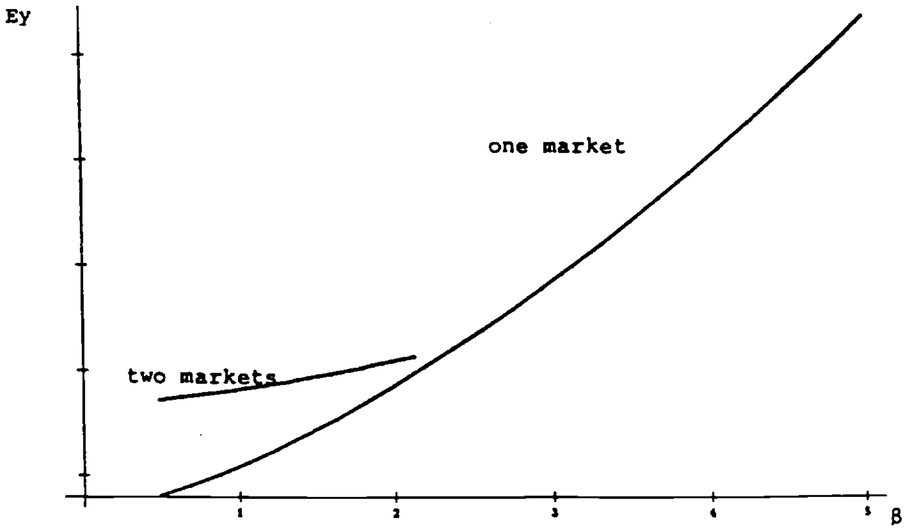


Figure 6b : Expected Income per Capita

Figure 7

One Club
Transition between Two and One Market

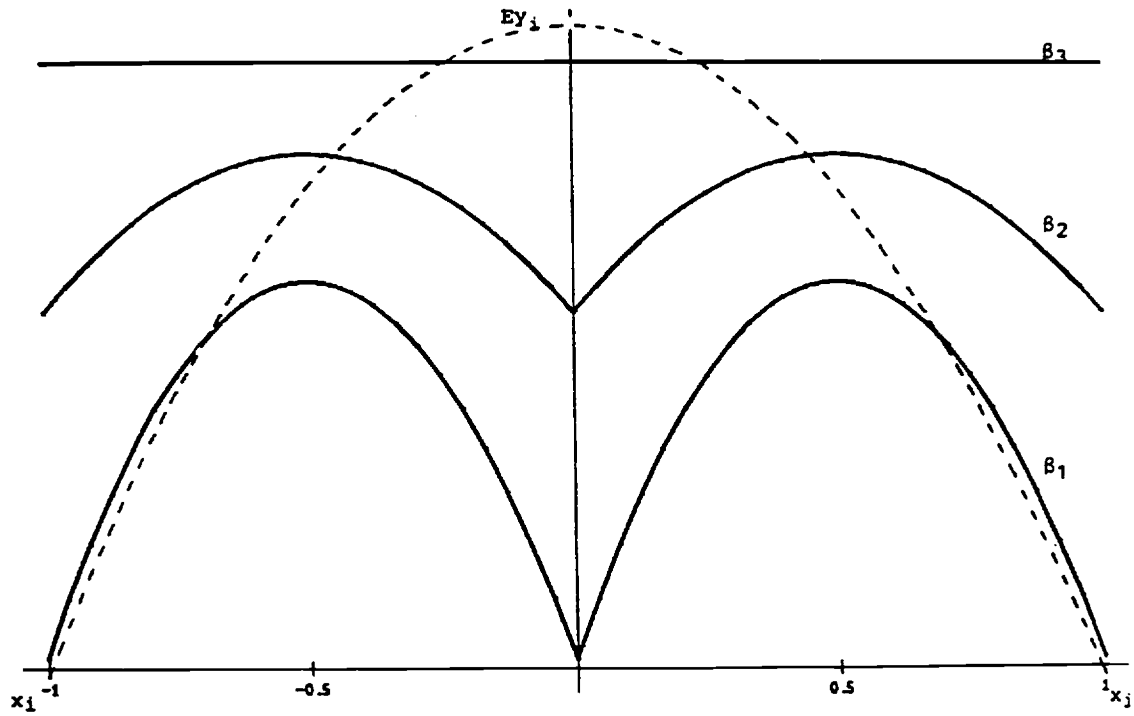


Figure 8

Two Clubs, One Market

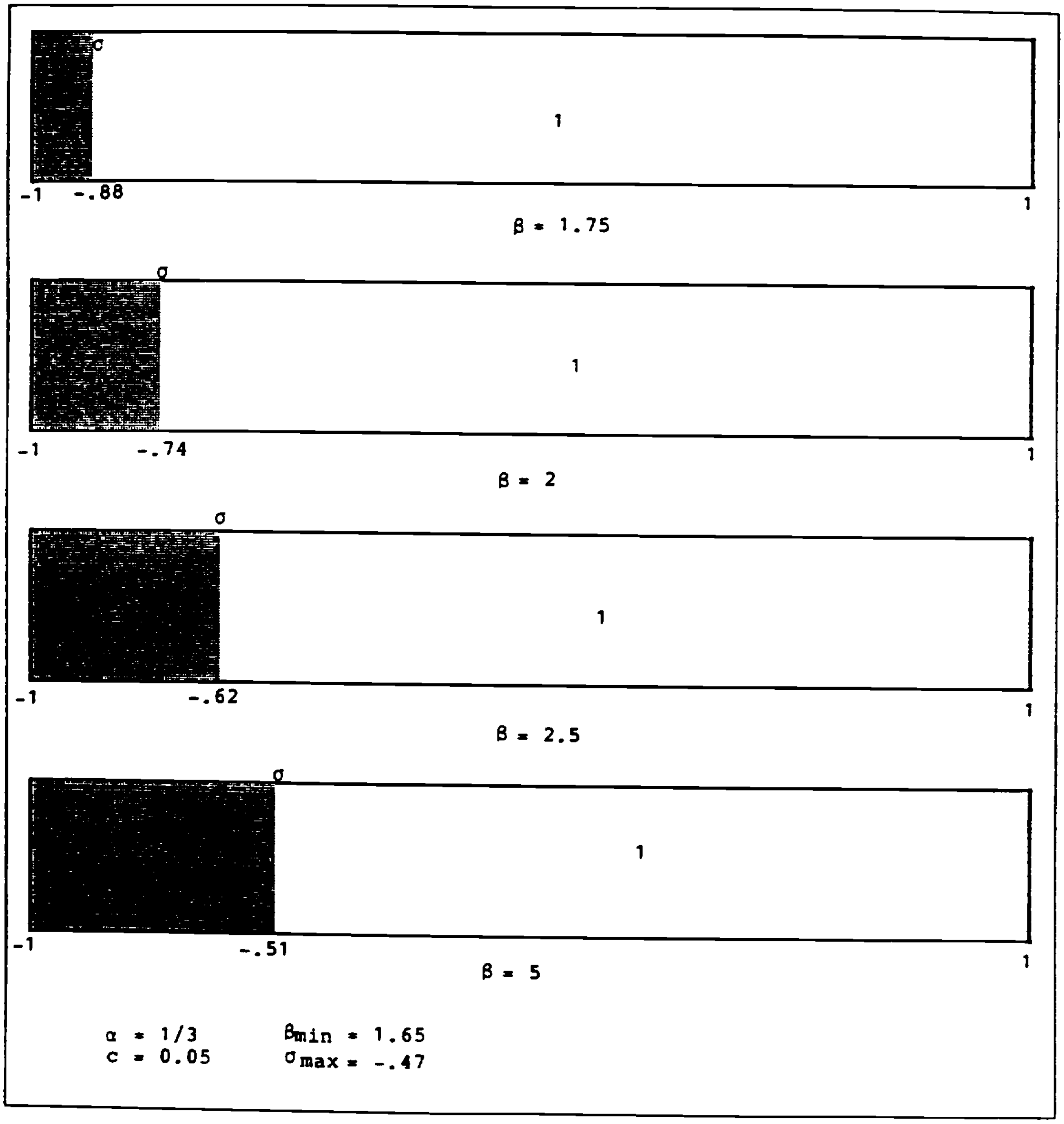


Figure 9

Two Clubs, Two Markets
Equilibrium Configurations

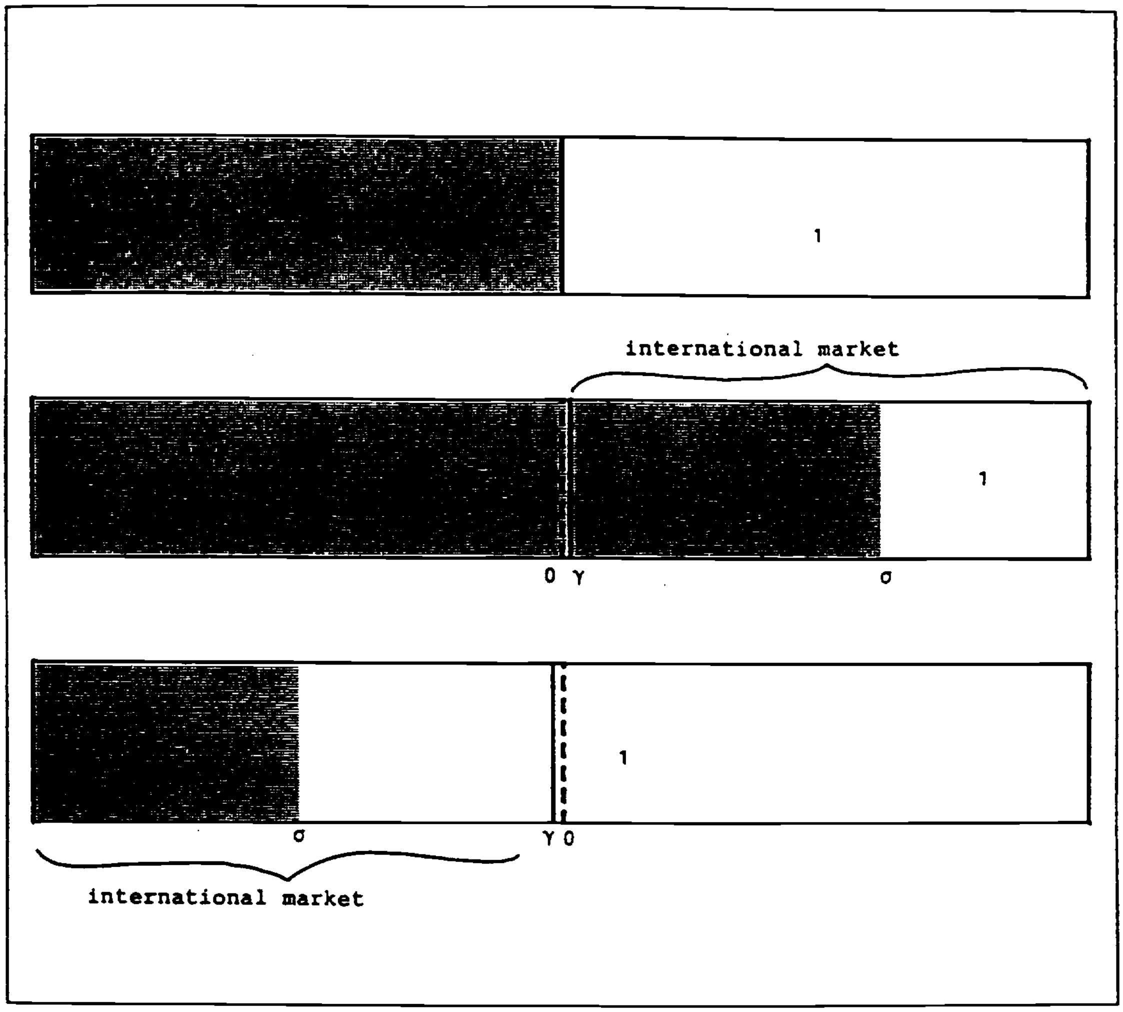


Figure 10

Two Clubs, Two Markets $\sigma > \gamma$

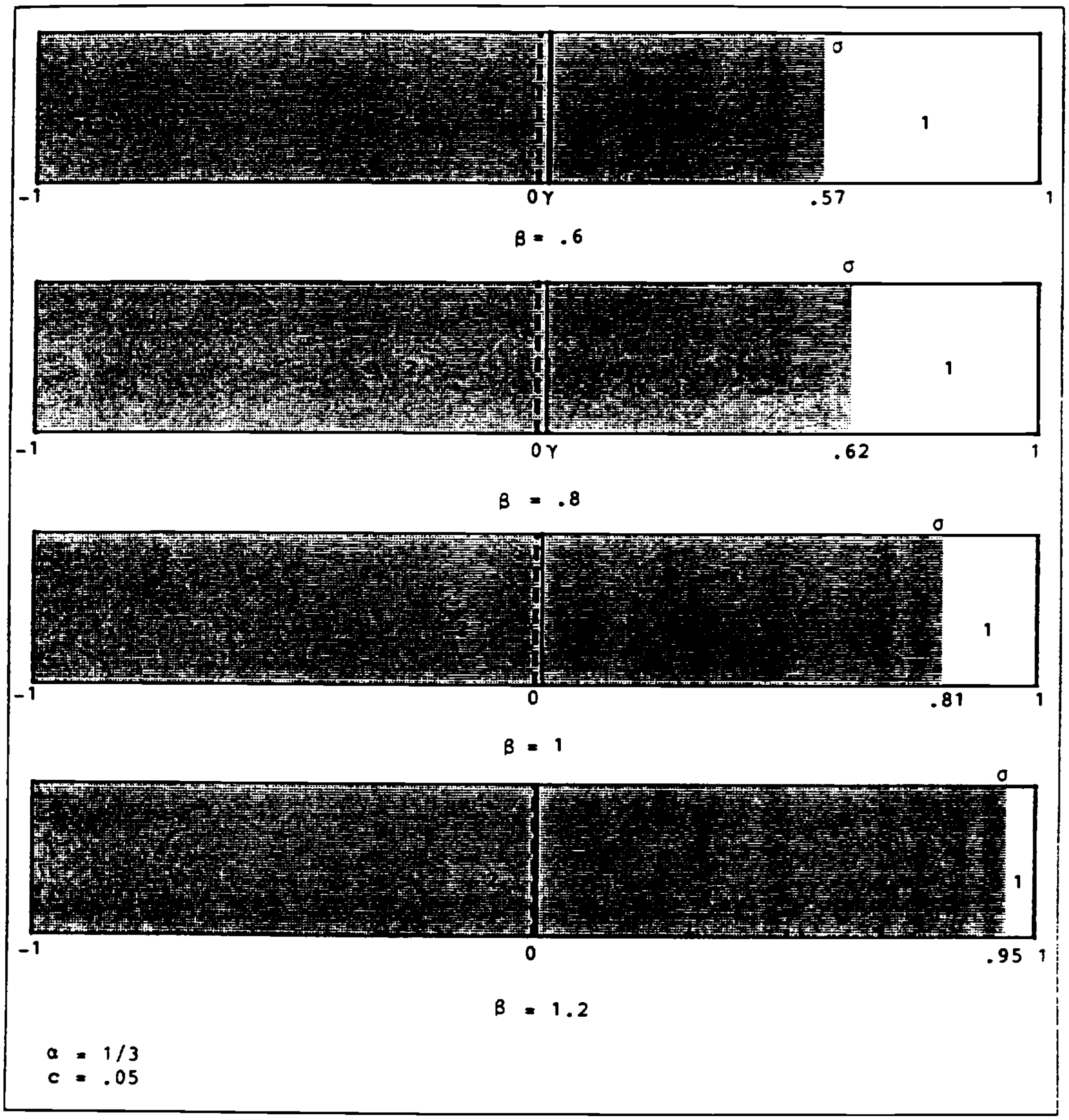


Figure 11

Equilibria with Two Clubs
Size of the Small Club
$\alpha = 1/3$, $c = 0.05$

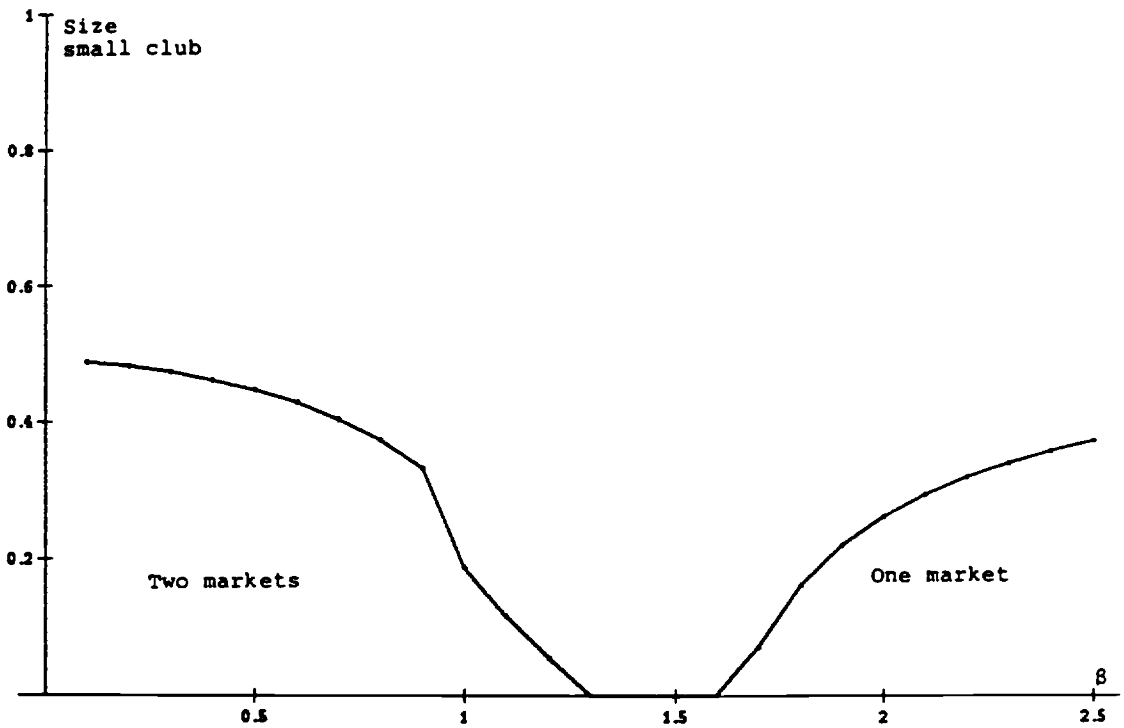


Figure 12

Provision of the Public Goods
Comparison Between One and Two Clubs
$\alpha = 1/3$, $c = 0.05$

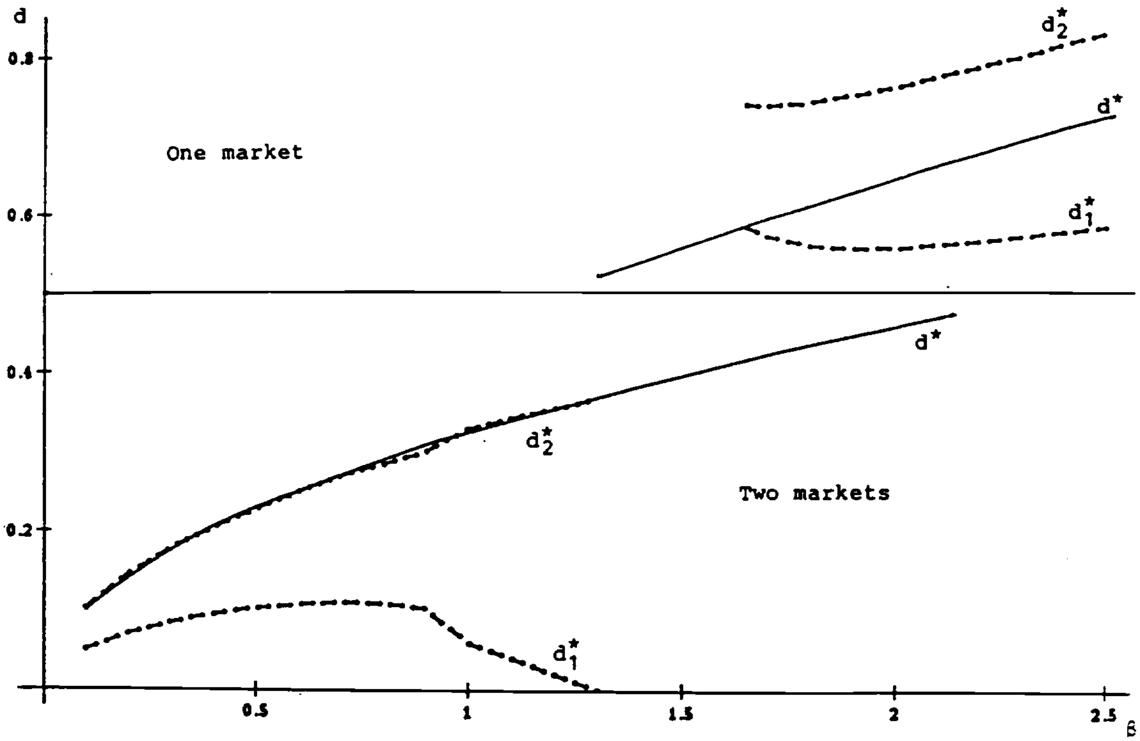
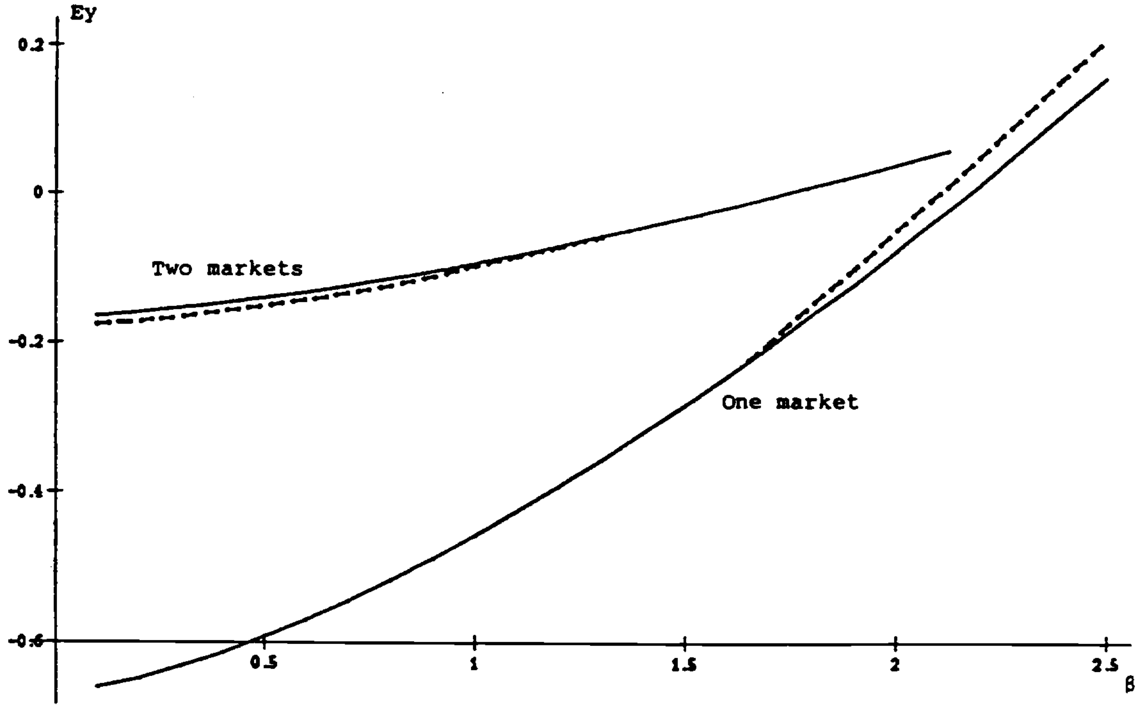


Figure 13

Expected Per Capita Income
$\alpha = 1/3$, $c = 0.05$



———————— One club

- - - - - - - - - Two clubs

Figure 14

Two Markets
Expected Individual Income
$\alpha = 1/3$, $c = 0.05$, $\beta = 1$

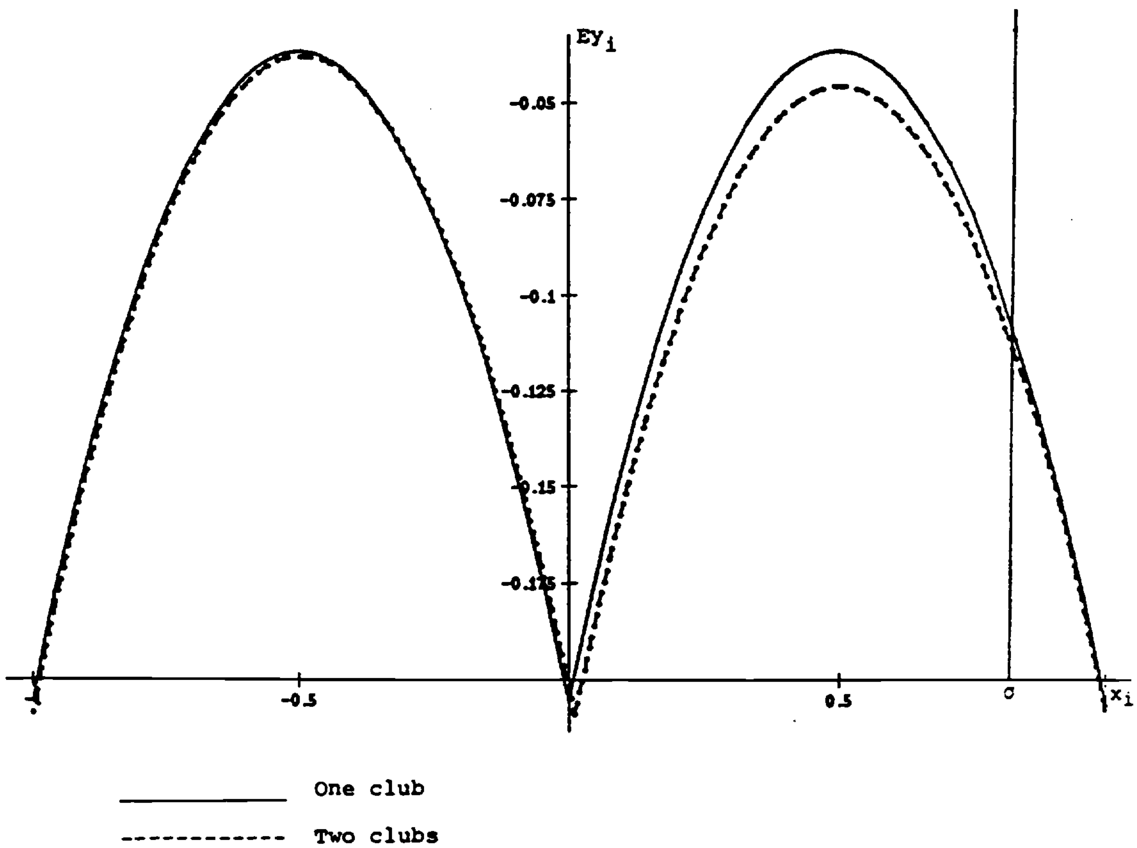
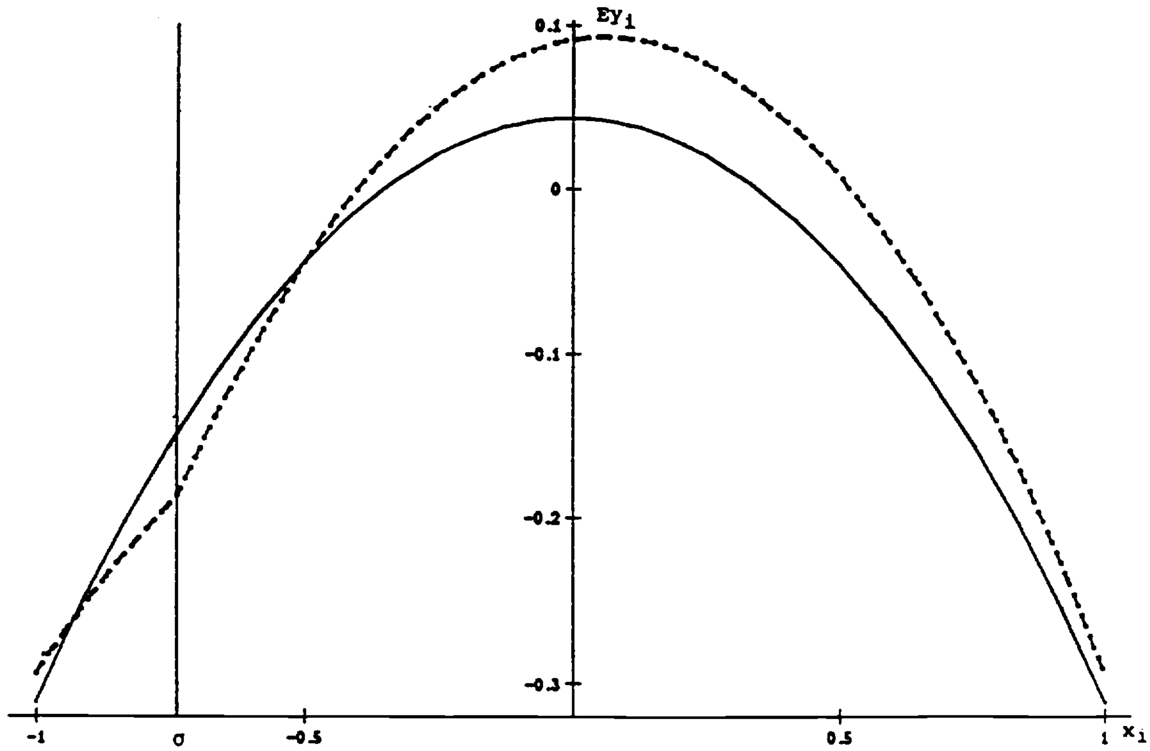


Figure 15

One Market
Expected Individual Income
$\alpha = 1/3$, $c = 0.05$, $\beta = 2$



One club

Two clubs

Appendix

Proof of Proposition 1

We first prove that such an equilibrium exists, and then show that it is unique.

(a) Existence. A division in multiple markets is an equilibrium if no trader wants to enter a different market than he is assigned to. Consider the configuration described in the proposition. Traders at the border between two markets must, by continuity, be indifferent between them. Since d is common to all, this condition is satisfied if all segments of adjacent traders forming separate markets are identical. Traders inside the borders cannot be indifferent (since their market position is not symmetrical to the position they would find themselves occupying in any other market) and must strictly prefer their own trading pool. Consider type x_i belonging to market $[-\frac{3}{n}, -\frac{1}{n}]$, and evaluate a possible jump to the neighboring market $[-\frac{1}{n}, \frac{1}{n}]$. For x_i, the gain from staying in his local market (the difference in expected returns) is given by

$$\frac{\beta dnx_i^2}{2} + 3\beta dx_i + \frac{5\beta d}{2n} - \frac{4}{n^2} - \frac{4x_i}{n} \qquad (A1)$$

This expression is convex in x_i and has a minimum at $x_i = -\frac{1}{n}$ if and only if $\beta d \leq \frac{2}{n}$. Thus if this condition is satisfied, the trader at the border $x_i = -\frac{1}{n}$ is the one with the least incentive to stay in the local market. But since he is indifferent, when $\beta d \leq \frac{2}{n}$ no-one will jump.

Two points are worth noting. The temptation to jump to a neighborhood market arises at lower levels of d than the temptation to join a market further away. Second, $\beta d \leq \frac{2}{n}$ is the condition for concavity in x_i of the expected return within each market.

(b) Uniqueness. To prove that this equilibrium is unique, we need to show that the other possible configurations cannot be sustained. We do this through two lemmas.

Lemma 1: There cannot be an equilibrium where one market is composed of a segment of adjacent traders but is surrounded on both sides by traders participating in a second market.

Proof: Consider a scenario where one market is formed by traders belonging to the interval $[-\gamma+\alpha, \gamma+\alpha]$ and a second market is formed by traders from $[-1, -\gamma+\alpha] \cup [\gamma+\alpha, 1]$, where $\gamma < 1$ and $\alpha < 1-\gamma$. Comparing expected returns, we see that trader $x_i \in [\gamma+\alpha, 1]$ will stay in the second market if and only if the following condition is satisfied:

$$\frac{3}{2}\beta d(x_i - 1)^2 + 3\alpha\beta d - (1-\gamma^2) + 3\alpha(\alpha - 2x_i) \geq 0, x_i \in [\gamma+\alpha, 1] \qquad (A2)$$

If $\alpha \geq 0$, the left-hand-side of equation (A2) is decreasing in x_i. Since the condition holds with equality for the border trader $x_i = \gamma + \alpha$, it must be violated for all $x_i \in (\gamma + \alpha, 1]$, implying that all traders in this interval will want to join the inner market.

If $\alpha < 0$, exactly the same argument holds for all $x_i \in [-1, -\gamma + \alpha)$. The conclusion would not be altered if the outer market were to include other non-adjacent segments of traders.

Lemma 2: There cannot be an equilibrium where two markets are composed of alternating segments of traders.

Proof: Consider a scenario where one market is formed by traders belonging to $[-1, -a] \cup [0, a]$ and a second is formed by traders from $[-a, 0] \cup [a, 1]$, where $a < 1$. Following the usual procedure for comparing expected returns, we can establish that trader $x_i \in [0, a]$ will not deviate if and only if:

$$3d[x_i^2 + 2x_i(1 - 2a)] - 2x_i(1 - 2a^2) \geq 0 \qquad (A3)$$

This condition has to hold with equality at the two borders $x_i = 0$ and $x_i = a$. But since the left-hand-side of equation (A3) is convex over the entire interval this implies that the condition must be violated for all $x_i \in (0, a)$, implying that all traders in this interval would want to join the other market. The symmetry imposed in this example simplifies the notation but is irrelevant to the proof. Again, the conclusion would not be altered by allowing more than two disjoint segments of traders in each market, and/or a larger number of markets.

Two Clubs and Two Markets, Equilibrium with $\gamma > \sigma$

We report here the equations characterizing the asymmetrical equilibrium with two clubs and two markets, when $\gamma > \sigma$. In this case, the international market is formed by all traders belonging to the high public good club, club 2, and by a fraction of the traders in club 1.

The equilibrium conditions are derived following the same logic described in the paper for the case $\sigma > \gamma$.

Considering first the determination of the equilibrium border σ between the two clubs, we calculate the temptation to deviate to club 2 for any x_i in the interval $[\sigma, \gamma]$. This tempation T_c is given by

$$T_c = \frac{\beta}{1+\gamma}\left[\frac{\sigma^2 + \gamma^2}{2} - x_i(\sigma + \gamma) + x_i^2\right](d_1 - d_2) + (t_2 - t_1) + \frac{\gamma - 2\sigma - 1}{1+\gamma}c \qquad (A4)$$

T_c is symmetric and convex in x_i. Thus $T_c = 0$ at the border is necessary and sufficient to guarrantee that no agent will want to change club:

$$\frac{\beta}{2}(\gamma - \sigma)^2(d_1 - d_2) + (t_2 - t_1)(1 + \gamma) + c(\gamma - 2\sigma - 1) = 0 \qquad (A5)$$

Equation (A5) defines σ. Again, the condition would be identical if derived from the viewpoint of agents in club 2 between -1 and σ. In addition, we must verify that in equilibrium no-one in club 1 in the interval $[\gamma, 1]$ wants to migrate to club 2. This requires

$$\frac{\beta}{2}(1 - \gamma)(d_2 - d_1) - (t_2 - t_1) - c \leq 0 \qquad (A6)$$

To characterize the border between the two markets, γ, we derive the temptation to switch from the international to the domestic market for $x_i \in [\gamma, \sigma]$:

$$T_m = \frac{\beta d_1}{1+\gamma}\left[\frac{1 + 2\gamma - \sigma^2}{2} - x_i(1 - \sigma) - x_i^2\right] - \frac{2}{3}\gamma + 2x_i + \frac{1+\sigma}{1+\gamma}c \qquad (A7)$$

$$-\frac{\beta d_2}{1+\gamma}\left[(1 + \sigma)x_i + \frac{1 - \sigma^2}{2}\right]$$

In equilibrium T_m must reach a maximum at $x_i = \gamma$ and be equal to zero at that point. The first condition amounts to:

$$\beta d_2(1 + \sigma) + \beta d_1(1 - \sigma + 2\gamma) \leq 2(1 + \gamma) \qquad (A8)$$

and the second requires

$$\frac{\beta d_1}{2}(1 - \sigma^2 + 2\gamma\sigma - 2\gamma^2) - \frac{\beta d_2}{2}(1 - \sigma^2 + 2\gamma\sigma + 2\gamma) + \frac{4}{3}\gamma(1 + \gamma) + c(1 + \sigma) = 0 \quad (A9)$$

As before, equation (A8) restricts the range of β values that can support this equilibrium, while equation (A9) defines the marginal trader γ. The two conditions would be identical if we had derived them from the perspective of $x_i \in [\gamma, 1]$.

In addition, there must be no deviation to the domestic market in club 1 by any trader in club 2. This requires

$$\frac{\beta d_2}{1-\gamma}(\gamma - 2\gamma x_i - x_i^2) - \frac{2}{3}\gamma + 2x_i - \frac{1+\sigma}{1+\gamma}c \leq 0$$

for all $x_i \in [-1, \sigma]$.

In club 2, the demand for the public good for each voter x_i is

$$d_{2i} = [\frac{\alpha\beta}{1-\gamma}(x_i^2 + \frac{\gamma^2+1}{2} + (1-\gamma)x_i)]^{\frac{\alpha}{1-\alpha}} \qquad (A10)$$

This function is convex in x_i and symmetric around a minimum at $x_i = \frac{\gamma-1}{2}$. The identity of the median voter x_{2m} depends on the relationship between γ and σ. Precisely:

$$x_{2m} = \frac{\sigma+2\gamma-1}{4} \text{ or } \frac{2\gamma-3-\sigma}{4}, if\ \gamma-\sigma < \frac{1+\sigma}{2}$$

$$\frac{\bar{\sigma}-1}{2}, otherwise$$

In club 1, the demand for the public good depends on the market voters belong to. If they belong to the domestic market, $x_i \in [\gamma, 1]$, their preferences are

$$d_{1i} = [\frac{\alpha\beta}{1-\gamma}(x_i^2 + \frac{1+\gamma^2}{2} - (1+\gamma)x_i)]^{\frac{\alpha}{1-\alpha}} \qquad (A11)$$

for $x_i \in [\gamma, 1]$, while if they belong to the international market, $x_i \in [\sigma, \gamma]$, their preferences are

$$d_{1i} = [\frac{\alpha\beta}{1+\gamma}(x_i^2 + \frac{\sigma^2+\gamma^2}{2} - (\sigma+\gamma)x_i)]^{\frac{\alpha}{1-\alpha}} \qquad (A12)$$

The identification of the median voter is once again not immediate, and depends on the relative width of the two market areas. In our numerical simulations, however, the only equilibrium we could find was such that the median voter always belonged to the domestic market, with all voters in the international market wanting a smaller amount of the public good. In other words, the mass of traders from club 1 participating in the international market was always sufficiently small. In this case $x_{1m} = \frac{3+\sigma}{4}$ or $\frac{4\gamma+1-\sigma}{4}$, and $d_1^* \geq d_{1i}(x_i = \sigma)$. This second condition states that the identification of the median voter is correct as long as the public good he prefers is larger than the preferred choice of the trader in the international market with the highest need for d_1, i.e. $x_i = \sigma (or \gamma)$.

The equilibrium is now completely characterized. We have run numerical simulations for several parameter values. With $\alpha = \frac{1}{3}$ and $c = .05$, we have found that this configuration can be an equilibrium for $\beta \leq 1.6$. Within this range, both σ and γ are quite insensitive to changes in β, with σ always remaining very close to -0.5 and γ remaining just slightly to the left of 0. As β rises, σ rises from $-.51$ (for $\beta = .1$) to $-.497$ (for $\beta = 1$), and then falls back to .51 (for $\beta = 1.6$). The market border γ moves monotonically with

β, falling from $-.019$ (for $\beta = .1$) to $-.033$ (for $\beta = 1.6$). Thus, the international market is again just slightly smaller than the domestic market, and shrinks a little at higher β. Club 1, the low public good club, is much larger than club 2, with 75% of the total population belonging to it; further, this division is quite stable in β. When β rises above 1.6, the growth of the domestic market in club 1 is such that d_1^* rises above d_2^*, and the equilibrium collapses to the case discussed in the text of the paper (indeed to its mirror image). But since that configuration could not support an equilibrium with two separate clubs for $\beta \geq 1.3$, it follows that for $\beta \geq 1.6$, we can only have a single club. Note that the two clubs can thus be an equilibrium at higher values of β if $\gamma > \sigma$, but that this does not alter the need for political unification before one common market can be achieved.

Both expected per capita income and its changes with β are remarkably similar to the values obtained when $\sigma > \gamma$, as discussed in the text. Both this measure of welfare, and direct voting by the agents on the choice between one or two clubs, are similar to the results presented in the text.

Footnotes

This paper was completed while Alessandra Casella was a National Fellow at the Hoover Institution. We are grateful to George Akerlof, David Baron, Raquel Fernandez, and Barry Weingast for comments, as well as to seminar participants at the Yale Conference on Fiscal Aspects of European Integration held during March 1990, the Ohlin Conference on Political Economy held during June 1990, the Lothian Foundation Conference on European Currency Union held during September 1990, the NBER 1990 Summer Institute on International Economics. the University of California at Berkeley, the University of California at San Diego, the University of Quebec at Montreal, the Univeristy of Texas at Austin. and the Hoover Institution.

[1]Of course, if x_i is towards the edge of the $[-1, 1]$ strip and βd is sufficiently high, one (or both) of these ideal partners may not exist.

[2]Note that for very high β an agent prefers to be matched with someone at the farthest edge of the space.

[3]No markets forming and no trade is also always possible, but of little interest.

[4]In theory, asymmetrical equilibria with no international trade are also possible: each club then coincides with a market, but the two clubs (and the two markets) do not have the same size, and do not provide the same public good. In our model, however, this can arise only when α is very close to 1/2, and even then the maximum asymmetry that can be supported is quite small. To be more precise, we have found that the small club must always be at least 86% of the size of the large club, approaching this limit as α approaches 1/2. Equilibria with international trade where the two clubs provide the same public good cannot exist, since agents will then all migrate to one club to avoid transaction costs.

[5]Note that we are using the fact that any single deviation to club 1 would leave the two public goods unaffected.

[6]Note that since the two markets have different sizes, equation (25) is not equivalent to requiring concavity in expected returns. From the viewpoint of a member of club 2, this would require $\beta d_2 \leq 1 - \gamma$ if he belonged to the interval $[\gamma, \sigma]$ and $\beta d_2 \leq 1 + \gamma$ if he belonged to $[-1, \gamma]$.

[7]For a small range of β values, the two sets of conditions (31) and (32) can both apply, and two equilibria exist. We will ignore this in what follows, both because the relevant range of β is indeed small, and because this does not affect in any way our conclusions. We will assume that the median voters are determined by (32) as soon as these conditions are possible.

[8]In the other possible scenario with 2 clubs, two markets, and international trade ($\gamma > \sigma$), as β rises and economic unification takes place the transition to political unification again occurs, and the conclusion in the text remains applicable. However, as long as two clubs are an equilibrium, the border between them remains almost unchanged, with a discrete jump when β reaches a critical threshold value.

[9]The "blip" in d_1^* and d_2^* in correspondence to $\beta = 1$ comes from the shift in regime in the identification of the median voters, from conditions (31) to (32).

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