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LAST ONE OUT WINS: TRADE POLICY IN AN INTERNATIONAL EXIT GAME

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ABSTRACT

This paper examines the effect of government intervention on the order and timing of firm exit in an international industry with fixed costs and declined demand. A dynamic inconsistency problem arises when the government is unable to precommit to a path of policy: it always intervenes to prolong the viability of the firm located in its market, even when the firm's survival is not the socially optimal outcome. The effect of tariff intervention is in all cases to terminate market operation prematurely, and in many cases to reverse the order of firm exit. Intervention in the absence of precommittment is never first best, and actually reduces welfare relative to the free market equilibrium when the differential between firms' fixed costs is large.

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Introduction

The heavyweight motorcycle industry and others like it raise questions about the role of government intervention in declining industries. Schooled in the tenets of comparative advantage, economists are puzzled to see policymakers erecting high trade barriers to protect marginally profitable firms in industries where capacity outstrips declining demand.

This paper offers an explanation for the pervasiveness of protection in markets with high concentration and globally declining demand that hinges on the anticipation of monopoly rents accruing to the most persistent competitor. In an industry with high fixed costs and declining demand, a strategically disadvantaged firm stands to gain from protection that enables it to outlast its foreign rivals and earn monopoly rents. Although the survival of the disadvantaged home firm may be undesirable from a welfare point of view, the firm can induce its government to levy protective tariffs by continuing to commit its fixed costs, as long as the government cannot credibly precommit to refrain from levying tariffs in the future. Suboptimal policy results from a dynamic inconsistency problem: the firm knows that once it has committed its fixed costs the government's optimal policy will be a protective tariff, even when nonintervention and firm exit is preferred ex ante.

I formalise this explanation in an exit game between firms of different nationalities, in order to examine the effect of government intervention on the order and timing of firm exit and on welfare. Interestingly, I find that the government always intervenes to protect the firm located in its market, and the resulting level of protection is not first-best in terms of domestic welfare. The model predicts that firms in declining industries will receive protection long after they would exit in the absence of intervention, even when this is not welfare-maximising. The effect of tariff intervention is to terminate market operation prematurely in all cases, and to reverse the equilibrium order of exit in many cases.

With unilateral intervention in the market with the less competitive firm, reversal is more likely the smaller is the differential in fixed costs between the protected firm and its foreign rival and the more elastic is demand. The curtailment of production is more severe the more rapidly demand declines over time, and the more extreme is the reduction in foreign profits from the tariff. When there is reciprocal intervention, relative demand shares are also an important determinant of the strategic balance. In the presence of reciprocal tariffs, it is strategically advantageous to be located in the larger market. The presence of a rival interventionist government lessens a firm's commitment power vis-a-vis its own government, however, so that reversal of the free market order of exit is less likely than with unilateral intervention.

The game formulation I employ draws heavily on the industrial organisation literature on exit, and in particular on the model developed by Ghemawat and Nalebuff (1985).[1] Ghemawat and Nalebuff construct a model with complete information where firms differ in their capacity levels and the associated fixed costs. They show using backward induction that the firm with the smallest fixed cost prevails as a monopolist in the final periods. I develop a somewhat different model that preserves the central insight that low fixed costs are a strategic advantage in a market with declining demand.

Trade policy in declining industries is a subject that has largely been ignored in international trade theory, despite its importance in trade

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[1] There have been several extensions and refinements of the Ghemawat/ Nalebuff results. Whinston (1987) uses a discretised exit framework to consider the order of capacity retirement where one or more firms has multiple plants, and finds the smallest-leaves-last outcome does not go through in general. Both Whinston and Ghemawat and Nalebuff (1987) find that the subgame perfect equilibrium is the sequence of static Nash equilibria in the presence of continuous capacity reduction. Londregan (1986) elaborates the exit framework to incorporate the possibility of (costly) reentry and extends it to the industry growth phase; he finds that the smallest-leaves-last outcome obtains.

disputes. The models of infant or steady-state industries in which the trade literature has specialised shed little light in this area; their temporal framework is inappropriate to the analysis of decline. There is a literature that focuses on the important political dimension of protectionism in declining industries,[2] but equally important economic issues have not received comparable attention.

This paper also touches on the area of dynamic consistency in trade policy.[3] The question I am addressing is inherently dynamic: given a trajectory of demand that threatens one of the firms with imminent failure, when does the government intervene and to what effect? I find that the government's ability to influence the market outcome is undermined by its inability to precommit. As is common in such models, the policy that is ex ante optimal may not be feasible if the government cannot tie its hands, since the firms correctly anticipate the government's policy response in making their production decisions. If a firm commits its fixed costs before the government sets policy, the firm knows that the government will protect its investment ex post, even if this is not the ex ante optimal policy.

Take the case where there is one firm located in a domestic market and another in a foreign market. The inconsistency arises when the domestic firm stays in the market longer than is optimal from the perspective of domestic welfare because the anticipated cumulative private returns from staying in under protection exceed the social returns. It is particularly striking when domestic welfare is higher with the foreign firm exiting last but the home firm prevails instead because the government is unable to precommit to sufficiently low future tariffs to ensure foreign viability.

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[2] For example, Baldwin (1985), Cassing and Hillman (1986), and Hufbauer et al (1986).

[3] Recent work on dynamic consistency in trade policy includes Engel and Kletzer (1987), Matsuyama (1987), and Tornell (1989).

The paper proceeds as follows. Section I presents a simple one-period model to illustrate how the government's inability to precommit results in protection even when this is not the ex ante optimal policy. Section II develops a multiperiod exit framework, assuming that both firms export all of their production to a third country. This simplification is used to derive the smallest-leaves-last outcome, and to solve for the equilibrium outcome when government intervention is introduced. In Section III, the model is extended to include consumption at home. The equilibrium timing and order of exit under nonintervention and under tariff intervention are derived, and comparative statics are used to evaluate the sensitivity of the equilibrium to various parameter changes. Section IV presents a numerical simulation that compares the level of domestic welfare under varying degrees of government commitment, and tests the sensitivity of the welfare levels to parameter changes. Section V extends the model to consider reciprocal intervention by the foreign government. Section VI concludes.

I. One-Period Example

In an industry with sufficiently large fixed costs that only one firm is viable, trade protection benefits the firm in the protected market by giving it a strategic advantage vis-a-vis an unprotected rival. Such protection is not always optimal from the point of view of welfare in the protected market, however. If the firm commits its fixed costs before the government sets policy, the firm may be able to induce the government to erect protective barriers with the effect of maximising private returns to the firm rather than social returns.[4]

[4] The game is structured so that the government sets policy after the firms have made their participation decisions, in contrast to the assumption made in most of the imperfect competition trade literature that the government moves first. The timing structure assumed here is arguably more accurate in describing policymaking processes prevalent in the US where, for instance,

This is seen most clearly in a simple one-period production model. Suppose there is one foreign firm and one home firm, and demand is concentrated in a third market. The firms have identical constant marginal costs, and fixed costs that are larger for the home firm than for the foreign firm. The firms simultaneously choose whether to play "in" or "out". If a firm chooses to play in, it pays its fixed costs and then (simultaneously) chooses its output. When both firms play in there is a Cournot equilibrium, and there is monopoly output otherwise.

Consider a scenario in which the fixed cost difference is such that both firms are viable as monopolists, but only the foreign firm is viable as a duopolist. Then in the absence of intervention, the unique pure strategy equilibrium is foreign monopoly.

Now suppose the home government can levy an export subsidy, once firms have chosen whether to participate and paid their fixed costs. Define s as the subsidy level, and x(s) and y(s) as the home and foreign firms' profit-maximising quantities under the subsidy respectively, and define home and foreign fixed costs as k_h and k_f respectively. The government sets s to maximise welfare, which equals the home firm's profits less the cost of the subsidy:

$$(1.1) \qquad \max_{s} \quad W(s) = \{[P(x(s)+y(s))-c+s]x(s)-k_h\}-sx(s)$$

The subsidy revenues received by the home firm are exactly offset by the cost of the subsidy to the government, so that welfare is simply the unsubsidised profits of the home firm, incorporating the quantity responses of both firms to the subsidy. Concavity of the profit function and

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firms must demonstrate injury in order to be considered for protection by the USITC, and must have a contract in hand when applying for export credit subsidies. It can be shown that a similar suboptimality result obtains when the government moves first in the multiperiod game with domestic consumption, for certain specifications of the demand function.

downward-sloping reaction functions are sufficient to ensure that the optimal subsidy is always positive when both firms play in, and 0 otherwise. The imposition of a subsidy raises home welfare by inducing the home firm to raise output and the foreign firm to contract output by a greater amount. This is precisely the result derived by Brander and Spencer (1984) in the absence of fixed costs.[5]

The intuition for this is straightforward. Start by ignoring the fixed costs and consider the home and foreign firms' best response functions illustrated in figure 1.1. The home firm's best response to foreign quantities of y is given by the locus R(y), and R(x) gives the foreign firm's best response to home quantities of x. The intersection of the best response functions at N is the (symmetric) Cournot/Nash equilibrium. The profits associated with the Cournot equilibrium quantities are given by the isoprofit contours, π_h^n and π_f^n, for the home and foreign firms respectively. Profits are rising towards the axes, and are highest at the monopoly points π_h^m and π_f^m.

The imposition of a positive subsidy results in an outward shift of the home firm's best response function, from R(y) to $R_s(y)$, such that the equilibrium shifts to the point S, where home output is greater, and foreign output is lower. The profits of the home and foreign firms associated with the new equilibrium are given by the contours π_h^s and π_f^s respectively.

The subsidy is chosen to maximise welfare consistent with both firms choosing quantities along their respective best response functions. Figure 1.2 illustrates the optimal subsidy choice. The domestic isowelfare contours are simply the isoprofit contours of the home firm prior to the imposition of the subsidy, since domestic welfare is the domestic firm's profits net of the

[5] Eaton and Grossman (1986) show that the optimal policy is an export tax for Bertrand competition. In this case, the domestic firm's aftertax profits decline to 0 before welfare reaches 0, so that the domestic firm leaves earlier than is socially optimal. Reversal in the order of exit is less likely than under subsidised Cournot competition.

subsidy. The highest achievable level of domestic welfare along the foreign firm's best response function is at the tangency of the isowelfare contour W_s to $R(x)$, at the point S, which is the Stackelberg equilibrium with the home firm as leader. Thus, the optimal subsidy moves the equilibrium between the firms to the point S, raising welfare from W_n to W_s, and reducing foreign duopoly profits from π_f^n to π_f^s.

Now consider the firms' fixed costs. Recall that in the absence of subsidies both firms are viable as monopolists but only the foreign firm is viable as a duopolist. In figure 1.1, this implies that home's fixed costs, k_h, lie somewhere below π_h^n, in region H_1 or H_2, while k_f lies above π_f^n, in region F_1 or F_2. In figure 1.2, k_h lies between the isoprofit contour W_n and the point π_h^m, in region ω_1 or ω_2.

When the domestic government imposes the optimal subsidy, one of three equilibria results, depending on the relative sizes of the fixed costs. Refer again to figure 1.1. If k_f is in region F_1, between π_f^n and π_f^s, the foreign firm is not viable as a duopolist under the subsidy, and it will only play in if the home firm exits. If, on the other hand, k_f is in the region F_2, the foreign firm is viable as a duopolist under the subsidy and will play in regardless. Likewise, if k_h lies in the region H_2, the home firm is viable under the subsidy as a duopolist and will play in regardless. If its fixed costs lie in the region H_1, below π_h^s, the home firm is not viable as a duopolist even under the subsidy. The four possible outcomes are:

EQUILIBRIUM OUTCOMES

	$k_f \in F_1$	$k_f \in F_2$
$k_h \in H_1$	M_f or M_h	M_f
$k_h \in H_2$	M_h	D

where D is duopoly, M_h is home monopoly, and M_f is foreign monopoly.

In the equilibrium with home monopoly, the threat of subsidy intervention raises welfare, while in the equilibrium with foreign monopoly it has no effect. But in the duopoly equilibrium, even though the protected profits of the home firm exceed its fixed costs, the overall social surplus net of the fixed costs may be negative. Refer again to figure 1.2. When both firms are profitable under subsidised duopoly, k_f lies in region F_2, and k_h lies in ω_1 or ω_2 in figure 1.2. But in region ω_1, net social surplus under subsidised duopoly is negative. In this case, the government intervenes to secure positive rents for the home firm, even though the social returns are negative. As long as the government is unable ex ante to promise credibly not to intervene if the home firm decides to participate, it cannot avoid intervention that is potentially welfare-reducing.

II. Dynamic Exit Game with Production

This section develops a dynamic framework to analyse strategic interaction in a declining industry. The essential insight arrived at above survives in the more elaborate framework, albeit in a modified form. The dynamic structure introduces the possibility that a firm is willing to accept losses over some interval in the anticipation of earning monopoly rents after its rival exits. This leads to the striking possibility that the home firm remains in the market as a duopolist under the protective umbrella of subsidies long after it would have exited in the absence of intervention, without in the end securing the monopoly position. In this case, government intervention on the interval leading up to the home firm's exit is welfare-reducing. Even when intervention does succeed in reversing the order of exit, and raising net welfare overall, there is some interval over which instantaneous welfare under the subsidy is negative.

II. 1. Free Market Equilibrium

I start by describing the exit game between the firms in the absence of government intervention, and derive the result that the firm with the smaller fixed costs secures the monopoly rents, under the following assumptions:

A1 There is no consumption in the home market; the home firm exports all of its output.[6]

A2 Instantaneous demand at time t and price p is given by $Q(p,t)$, which has the properties necessary to ensure that profit functions are concave.

A3 The instantaneous demand function is separable in time and price:

(2.1) $$Q(p,t) = f(t)g(p) \implies p(Q,t) = g^{-1}(Q/f(t)) = P(Q/f(t))$$

A4 Demand is declining monotonically over time, i.e. $f'(t) < 0$, due to a change in taste or technology. The declining trajectory of demand is fully anticipated by firms at the start of the exit game.

A5 The time rate of discount is 0.[7]

A6 There are two firms that produce an identical good; firm h is located in the home market and firm f in the foreign market.

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[6] The inclusion of consumer surplus in the welfare function complicates the results, so it is useful to make a number of points cleanly before going on to the more elaborate model with domestic consumption in Section III.

[7] Alternatively, $f(t)$ could be thought of as incorporating a positive discount rate.

A7 Production entails a constant marginal cost, c, which is equal for both firms, and a fixed cost k_i, which is larger for the domestic firm, i.e. $k_h > k_f$.

A8 A sunk investment is required for entry, so that it is always better for firms to stay in than to leave and reenter.[8] Both home and foreign firms have already sunk their investments at the outset of the exit game.

A9 At the beginning of each period, the firms that have not yet exited simultaneously choose between exiting permanently and staying in. Any firm that chooses to stay in must pay its fixed cost, which can be thought of as a participation fee. The firms who have paid their fixed costs then choose output simultaneously, with full knowledge of their rival's participation decision. In other words, in period T (T=1,2,...) firms makes two decisions:

Ta Simultaneous choice of in or out. If a firm played out last period it must do so again. If a firm plays in, it pays its fixed cost.

Tb Simultaneous choice of quantities.

Thus, each firm has two strategic choices each period: its exit decision, and its level of output if it stays in. Given the timing structure of the stage game, the within-period equilibrium is Cournot-Nash when both firms have chosen in, and monopoly when only one has chosen in.[9] This leaves the exit decision as the sole significant strategic variable.

Given these assumptions, there is a unique equilibrium outcome in which the domestic firm exits the instant its duopoly profits become negative, and

[8] Londregan (1986) shows that it is always better for a firm not to leave than to leave and reenter, given any cost of reentering, however small.

[9] Benoit and Krishna (1987) show that uniqueness of the stage game equilibrium is sufficient to rule out the possibility of a collusive supergame equilibrium in a finite game context.

the foreign firm then monopolises the market until it is no longer profitable. This is the result Ghemawat and Nalebuff derive, starting with different assumptions on demand and technology.[10]

To show this, I start by solving for equilibrium profits under monopoly and duopoly. Instantaneous profits for firm i at time t can be written as:

$$(2.2) \quad \Pi(q_i,q_j,t) = \pi(q_i,q_j,t)-k_i = [P(Q/f(t))-c]q_i-k_i$$

where $\pi(q_i,q_j,t)$ is the instantaneous variable profit function, q_i is firm i's output, and Q is industry output ($Q = q_h+q_f$). The output level that solves the monopolist's profit maximisation problem at time t is $q_i^m(t)$:

$$(2.3) \quad q_i^m(t) = f(t)[c-P(q_i^m(t)/f(t))]/P' \qquad i=h,f$$

Notice that (2.4) is satisfied when the ratio $q_i^m(t)/f(t)$ is constant over time. Call home's optimal monopoly quantity normalised for the dynamic component of demand X, and that of foreign Y:

$$(2.4) \quad X = Y = q_i^m(t)/f(t) \qquad i=h,f$$

Then variable monopoly profits can be expressed in time-separable form as:

$$(2.5) \quad \pi(I,0)f(t) = \pi(I,0,t) = -\{[P(I)-c]^2/P'\}f(t) \qquad \text{for } I=Y,X$$

Similarly we can derive the firms' optimal quantities when both firms play in at time t from their first order conditions:

$$(2.6) \quad q_i^d(t) = f(t)[c-P(Q(t)/f(t))]/P' \qquad i=h,f$$

Again, firm i's first order conditions are satisfied by producing in constant proportion to the time component of demand. Call home's duopoly quantity normalised for time x, that of foreign y, and total duopoly output z:

$$(2.7) \quad x = y = q_i^d(t)/f(t) \qquad i=h \text{ for } x, f \text{ for } y$$

Then each firm's variable duopoly profits are separable into a time-invariant and a time-dependent component:

$$(2.8) \quad \pi(m,n)f(t) = \pi(q_i^d,q_j^d,t) = -\{[P(Q/f(t))-c]^2/P'\}f(t)$$

10 Ghemawat and Nalebuff assume the firms differ in their capacities, and that marginal revenue is always positive, so that it is always optimal to produce at full capacity if it is optimal to produce at all.

$i=h$ and $m=x$ when $j=f$ and $n=y$
$i=f$ and $m=y$ when $j=h$ and $n=x$

For notational simplicity, let π_i^d and π_i^m denote the time-invariant components of firm i's duopoly and monopoly variable profits respectively. Due to the equality of the marginal costs, the variable profits of the two firms are equal under each market structure in equilibrium, $\pi_h^b = \pi_f^b$ for b=m,d.

It is clear from equations (2.5) and (2.8) that the firms' profits decline monotonically with f(t). This property of the profit function makes it possible to specify each firm's critical point - the time at which instantaneous variable profits just cover the firm's fixed costs - under monopoly and duopoly. Define t_i^b as the critical points of firm i under market structure b:

$$(2.9) \qquad \{t_i^b = t \mid f(t_i^b)\pi_i^b = k_i\} \qquad \text{for } b=m,d$$

Since $k_h > k_f$ by assumption, it is straightforward to establish that the foreign firm's critical points exceed those of home in both duopoly and monopoly. It is also straightforward that each firm's monopoly critical point occurs later than its duopoly critical point, since $\pi_i^m > \pi_i^d$. The position of t_f^d relative to t_h^m is indeterminate.

Figure 2.1 shows the profits accruing to each firm from participating each period from t_h^d on as a monopolist and as a duopolist. Time t_h^m is defined by the intersection of the variable monopoly profit function, $\pi^m f(t)$, with the home fixed cost line, k_h, and t_f^m is defined by the intersection of $\pi^m f(t)$ with the foreign fixed cost line, k_f. Similarly, $\pi^d f(t)$ intersects k_h at t_h^d, and k_f at t_f^d. The area between $\pi^m f(t)$ and the fixed cost line, k_h, is the cumulative value of the home firm's monopoly profits, while that between $\pi^m f(t)$ and k_f is the cumulative value of foreign's monopoly profits, and likewise for duopoly. It is useful to define the cumulative value of the stream of profits of firm i on the interval $[t_o, t_1]$ for market structure b as:

$$(2.10) \quad V_i^b(t_o, t_1) = \int_{t_o}^{t_1} (\pi_i^b f(r) - k_i) dr \qquad \text{for } i=h,f \text{ and } b=m,d$$

The equilibrium of the game between the firms hinges on the order of the two pairs of critical points: the firm with the later critical point has the strategic advantage under each market structure. The equilibrium is derived through a process of backward induction.

Starting at any $t \geq t_f^m$, neither firm participates in equilibrium because neither is profitable in this region. Working backward to an instant before t_f^m, the home firm plays out since it is not profitable at any $t > t_h^m$, while the foreign firm plays in if it is in the market alone, since it earns positive profits. This argument holds back to an instant before t_h^m, the last instant at which home is viable as a monopolist. The value of foreign's profits on this interval, $V_f^m(t_h^m, t_f^m)$, is shown in figure 2.1 as the shaded area between the monopoly profit curve and k_f. Working back from time t_h^m, since the foreign firm anticipates monopoly profits of $V_f^m(t_h^m, t_f^m)$, it is willing to participate as a duopolist and make losses back as far as some point $\tilde{t}_f$, where its cumulative duopoly losses, $V_f^d(\tilde{t}_f, t_h^m)$, are just offset by the anticipated monopoly profits. $V_f^d(\tilde{t}_f, t_h^m)$ is shown as the shaded area below k_f in figure 2.1. The home firm, anticipating profits of at best 0 after t_h^m, cannot sustain duopoly losses on the interval $(\tilde{t}_f, t_h^m]$, so it is persuaded to leave at $\tilde{t}_f$. But now foreign anticipates receiving $V_f^m(\tilde{t}_f, t_f^m)$, and the same argument can be made working backward to some earlier point $\tilde{\tilde{t}}_f$ ... and so on back to t_h^d, the last instant at which home's duopoly profits are nonnegative. This establishes the unique subgame perfect equilibrium in which home exits at t_h^d, and foreign stays in as a monopolist between t_h^d and t_f^m.

II. 2. Intervention Equilibrium

Next I introduce domestic government intervention under the following assumptions:

A10 The domestic government can subsidise the domestic firm's exports at any level s per unit that it chooses.[11]

A11 The government chooses the level of s each period after firms have decided whether they will participate and paid their fixed costs, but before firms have chosen their output levels. Thus, the timing structure in period T (T=1,2,...) is as follows:

Ta Firms simultaneously choose in or out. If a firm played out last period it must do so again. If it plays in, it pays the fixed cost.

Tb Having observed firms' participation decisions, the government sets the level of s.

Tc Firms choose quantities simultaneously, taking into account the announced subsidy and their rival's participation decision.

Firms adjust their equilibrium quantities in response to the subsidy. In equilibrium, this yields normalised Cournot quantities under duopoly:

(2.11) $x(s) = q_h^d(s,t)/f(t) = [c-s-P(Q(s,t)/f(t))]/P'$

(2.12) $y(s) = q_f^d(s,t)/f(t) = [c-P(Q(s,t)/f(t))]/P'$

and the home firm adjusts its monopoly output for the subsidy yielding:

(2.13) $X(s) = q_h^m(s,t)/f(t) = [c-s-P(q_h^m(s,t)/f(t))]/P'$

The foreign firm produces at the free market monopoly level, Y, when it is in the market alone. Then $\pi_h^d(s)=\pi(x(s),y(s),s)$, $\pi_f^d(s)=\pi(y(s),x(s))$, $\pi_h^m(s)=\pi(X(s),0,s)$, and foreign monopoly profits are the same as in the free market equilibrium.

The domestic welfare function on [0,T] is the cumulative value of home profits less subsidy payments:

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[12] In the first four sections I focus on the effect of intervention by a single (and singlewilled) government. The effects of interaction between rival intervention governments will be addressed in Section V.

(2.14) $W(0,T) = \int_0^T [(\pi(m(s),n(s),s) - sm(s))f(r) - k_i]dr$

where $m(s) = \begin{matrix} x(s) \\ X(s) \end{matrix}$ for $n(s) = \begin{matrix} y(s) \\ 0 \end{matrix}$

Were the government able to precommit to a path of subsidies at the outset of the exit game, it would choose the path that maximises (2.14), taking into account the effect of the anticipated subsidies on both firms' exit decisions. However, given the game structure specified in assumption 11, the government always chooses the subsidy to maximise the instantaneous welfare function defined in equation (1.1). The logic of this is straightforward. The government cannot directly influence the exit decisions of the two firms, since it has no means of committing to a subsidy level before the firms have decided whether to commit their fixed costs each period. Thus, the best the government can do is to set the subsidy to maximise instantaneous welfare, taking into account the quantity responses of both firms but not their intertemporal participation constraints.

Thus, the optimal policy choice each period is the same subsidy as was chosen in the one-period game to maximise (1.1): the equilibrium subsidy is time-invariant. Define s_b^* as the equilibrium subsidy under market structure b:[13]

(2.15) $s_d^* = (P-c)[(P')^2 - P''(P-c)]/(P')^2$

$s_m^* = 0$

In order to determine the order and timing of exit in the intervention equilibrium, it is necessary to adjust the duopoly critical points for the effect of the subsidy . Call firm i's subsidy-adjusted duopoly critical point, $\hat{s}_i^d$; at this point, the firm's instantaneous duopoly profits under the equilibrium subsidy go to 0. Each firm's duopoly critical point will lie at the intersection of the subsidy level which gives it zero profits at that

[13] s^* is the optimal instantaneous subsidy.

instant, and the equilibrium subsidy level, s^*_d. Define the subsidy level that gives firm i instantaneous profits of exactly 0 under duopoly at time t as:

$$(2.16) \qquad s^d_i(t) = \begin{cases} s \mid \pi^d_i(s)f(t) = k_i & \text{if such a value exists} \\ 0 & \text{otherwise} \end{cases} \qquad i=h,f$$

$s^d_i(t)$ is the subsidy level at which firm i's instantaneous participation constraint just binds at time t.

The paths of s^*_d, s^d_h, and s^d_f, and the associated critical points are illustrated in figure 2.2a, and the profits of the two firms under the equilibrium subsidy are shown directly below it in figure 2.2a. The home firm requires a rising level of subsidies over time in order to break even, while the foreign firm's zero-profit subsidy level declines over time. Notice that s^d_f equals 0 at t^d_f, while s^d_h equals 0 at t^d_h; the foreign and domestic firms break even as duopolists under a zero subsidy at their respective free market critical points. The intersection of the optimal subsidy with the home and foreign firms' zero-profit subsidies respectively defines the subsidy-adjusted critical points, $\hat{s}^d_h$ and $\hat{s}^d_f$. Call the time defined by the intersection of the two zero-profit constraint lines $\bar{\underline{s}}$. Up to $\bar{\underline{s}}$, both firms are viable as duopolists for any subsidy level between s^d_h and s^d_f; beyond $\bar{\underline{s}}$, the instantaneous incentive constraints are mutually exclusive.

The intervention equilibrium can be derived by a process of backward induction similar to the free market equilibrium. Again, the equilibrium hinges on the ordering of both pairs of critical points, adjusted for the subsidy. In equilibrium, subsidy intervention has no effect on the positions of the monopoly critical points, but its effect on the duopoly critical points is either to narrow the distance between them or reverse them altogether.

Consider first the case (N) where the optimal subsidy is sufficiently low relative to the intersection of the zero-profit subsidy levels that the gap between the duopoly critical points narrows but their order does not reverse. The subsidy functions for case N are shown in figure 2.2a, and the associated variable profit functions are shown in figure 2.2b. Starting an instant before t_h^m, home has anticipated profits of 0, while foreign anticipates monopoly profits of $V_f^m(t_h^m, t_f^m)$. Foreign is willing to accept duopoly losses up to this level, so home plays out. Working backwards in steps, the same reasoning applies back to $\hat{s}_h^d$, the last moment at which home is profitable as a duopolist. Thus, the home firm remains in as a duopolist until $\hat{s}_h^d$, well beyond its free market exit time, but in the end does not secure the monopoly position.

While the home firm earns higher profits than it would in the absence of intervention, domestic social surplus may be lower. The change in social surplus induced by the intervention policy is:

$$(2.17) \quad W(t_h^d, \hat{s}_f^d) = \int_{t_h^d}^{\hat{s}_h^d} ([\pi_h^d(s_d^*) - s_d^* x(s_d^*)]f(r) - k_h) \, dr$$

Since instantaneous welfare is lower than instantaneous profits at every instant by the amount of the subsidy payments, and the home firm does not leave until its subsidised profits go to 0 at $\hat{s}_h^d$, welfare is negative on some interval leading up to $\hat{s}_h^d$. By intervening on this interval, the government actually reduces welfare. Indeed, the losses on this interval may be sufficient to outweigh welfare gains on the interval immediately following t_h^d, so that welfare may be reduced overall, depending on the form of the demand function and the parameter values.

Welfare-reducing intervention is also possible in the case (RD) where the equilibrium subsidy lies above the intersection of the zero-profit constraints and the order of the duopoly critical points is reversed. The subsidy functions in case RD are shown in figure 2.3a, and the two possible configurations of the associated variable profit functions are shown in figures 2.3b and 2.3c.

Starting with figure 2.3b, an instant before t_h^m foreign is willing to sustain instantaneous duopoly losses in order to earn positive monopoly profits on (t_h^m, t_f^m), while home, with anticipated cumulative profits of 0, cannot credibly threaten to participate under duopoly. Working backwards, the same reasoning applies until an instant before $\hat{s}_h^d$, where home plays in because it is profitable under both duopoly and monopoly. Foreign continues to participate at this point, because anticipated monopoly profits of $V_f^m(\hat{s}_h^d, t_f^m)$ outweigh instantaneous duopoly losses. Working backwards, since $|V_f^d(s_d^*;\hat{s}_f^d,\hat{s}_h^d)| < V_f^m(\hat{s}_h^d, t_f^m)$, foreign can credibly commit to stay in as a duopolist until $\hat{s}_h^d$, where $V_i^d(s;t_o,t_1) = \int_{t_o}^{t_1}(\pi_i^d(s)f(r)-k_i)\ dr$. Thus, duopoly prevails until $\hat{s}_h^d$, when home drops out, and foreign monopolises the market thereafter.

Here, as above, home's participation as a duopolist is extended but it does not secure the monopoly position. The point at which the social surplus under the optimal subsidy goes to 0 is earlier than the point at which subsidised home profits go to 0, and government intervention on the interval between these points is welfare-reducing. As above, cumulative surplus over the interval $(t_h^d, \hat{s}_h^d)$ may be negative.

If, on the other hand, foreign's duopoly losses swamp anticipated monopoly profits before $\hat{s}_f^d$ is reached, as in figure 2.3c, foreign drops out at $\hat{s}_f^d$. In equilibrium, duopoly prevails until $\hat{s}_f^d$, and home monopoly prevails thereafter. Here the order of exit reverses and home earns monopoly profits on the interval $(\hat{s}_f^d, t_h^m)$. In this case, it is likely that the cumulative net effect of the policy is to raise welfare, although home welfare under the subsidy may still be negative over some interval leading up to $\hat{s}_f^d$ if $t_h^d < \hat{s}_f^d$.

Even in this case, where the cumulative net change in welfare is positive, intervention does not achieve the first best optimal. To see this, suppose that the government were able to commit irreversibly to a path of subsidies at time t_h^d. The optimal policy depends on the relative fixed costs of the two firms. If the foreign firm's cumulative monopoly profits on (t_h^m, t_f^m) are swamped by the cumulative value of its fixed costs over the interval (t_h^d, t_h^m), then the government optimises by threatening to impose a duopoly subsidy on (t_h^d, t_h^m) in excess of $s_f^d(t)$, and in equilibrium the foreign firm drops out at t_h^d, and home monopolises the market until t_h^m.

If instead foreign's monopoly profits on (t_h^m, t_f^m) swamp the cumulative value of its fixed costs on (t_h^d, t_h^m), foreign will stay in as a duopolist at any level of subsidy. The government optimises by precommitting to a path of subsidies that forces the home firm to leave when its continued operation is no longer socially productive. This entails the optimal subsidy s_d^* up to the time at which the social surplus from subsidised duopoly production goes to 0 - which is strictly earlier than $\hat{s}_h^d$ - and 0 thereafter.

Notice that the equilibrium order of exit is determinate in case N and in the free market game. This determinacy results from the uniform ordering of the duopoly and monopoly critical points: the firm that has the strategic advantage under both market structures always secures the monopoly position. Whenever the order of the pairs of critical points differs under monopoly and duopoly, however, as in case RD, each firm has an advantage under one market structure, and the equilibrium order of exit depends on the magnitude of each firm's cumulative anticipated monopoly profits on the final interval relative to the cumulative duopoly losses on the preceding interval. This rule helps to simplify the analysis in the remaining sections.

III. Dynamic Exit Game with Consumption at Home

In this section I elaborate the framework developed in Section II to include consumer surplus in home welfare, and tariff intervention. With consumption at home, there is a welfare tradeoff between domestic consumer surplus and the domestic firm's profits, which magnifies the potential conflict between the firm's interests and social welfare, and raises the value of government commitment power. In contrast to the pure production case, welfare may be higher with foreign monopoly in the final periods when there is consumption at home. Thus, the possibility of a net reduction in social surplus from intervention arises even when there is reversal, and vice versa.

Consumption in the home market introduces an intertemporal tradeoff and an intratemporal tradeoff into the welfare function, which make it difficult to determine the first-best order and timing of exit in general. The intertemporal tradeoff is between consumer surplus and tariff revenues from foreign monopoly output on (t_h^m, t_f^m) if home exits first, and domestic monopoly profits between the foreign exit time and t_h^m if instead foreign is forced out first. The intratemporal tradeoff is between the producer and consumer surplus and tariff revenues associated with duopoly production as against home monopoly production at each instant on (t_h^d, t_h^m). Because of these tradeoffs, the welfare-maximising order and timing of exit depend on the form of the demand function and the size of the fixed cost differential.[14] But without specifying the social optimum, I will show that the equilibrium policy does not in general attain it when the government is unable to precommit.

I proceed by deriving the equilibrium order and timing of exit under government intervention in the absence of commitment, and then use comparative statics to determine the effect of changes in key parameters on the equilibrium. I also show that government policy in the absence of commitment is not first best; the welfare implications will be explored in greater depth for a specific case in Section IV.

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[14] Although it is difficult to generalise about the welfare ordering over the three possible market structures at each point in time, the form of the demand function ensures that the welfare ordering changes monotonically over time.

III. 1. Unilateral Intervention

Start by modifying assumptions 1 and 10:

A1′ Demand is concentrated in the home market.[15]

A10′ The domestic government can levy a specific tariff, τ, on foreign imports at any nonnegative level. The tariff is the sole instrument.[16]

The free market equilibrium order and timing of exit are the same as those derived in Section II. The firms' decisions are invariant to the location of consumption in the absence of transport costs.

Under tariff intervention, the firms adjust their equilibrium outputs, yielding variable duopoly profits for home and foreign respectively:

$$(3.1)\quad \pi_h^d(\tau)f(t) = [P(z(\tau))-c]x(\tau)f(t) = f(t)[c-P(Q(\tau,t)/f(t))]^2/P'$$

$$(3.2)\quad \pi_f^d(\tau)f(t) = [P(z(\tau))-c-\tau]y(\tau)f(t) = f(t)[c+T-P(Q(\tau,t)/f(t))]^2/P'$$

and foreign variable monopoly profits:

$$(3.3)\quad \pi_f^m(\tau)f(t) = [P(Y(\tau))-c-\tau]Y(\tau)f(t) = f(t)[c+\tau-P(q_f^m(\tau,t)/f(t))]^2/P'$$

where $x(\tau)$, $y(\tau)$, and $z(\tau)$ are home, foreign, and total duopoly outputs, and $Y(\tau)$ is foreign's monopoly output under tariff τ. The home firm produces at the free market level when it is a monopolist, and receives the free market level of monopoly profits.

Social welfare on [0,T] is the cumulative value of consumer surplus, home profits, and tariff revenues from foreign imports:

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[15] A second market could be introduced under the assumptions of Section V, without changing the results. The critical assumption for the analysis in this section is that intervention is unilateral.

[16] If the government uses production subsidies instead, a similar suboptimality result obtains due to the lack of commitment power, and the possibility of reversal also arises, depending on the order of the critical points. However, under production subsidies, the effect on the market life can go either way.

(3.4) $W(0,T) = \int_0^T [f(r)(u(o(\tau))-P(o(\tau))n(\tau) + \pi(m(\tau),n(\tau))+\tau n(\tau))-k_h]dr$

where $m(\tau) = \begin{matrix} x(\tau) \\ X(t) \\ 0 \end{matrix}$ for $n(\tau) = \begin{matrix} y(\tau) \\ 0 \\ Y(\tau) \end{matrix}$; and $o(\tau) = m(\tau)+n(\tau)$

As in the pure production case, however, the government is unable to influence the participation decisions of the two firms directly because it lacks the power to precommit. Thus, the optimal policy is to set tariffs to maximise instantaneous welfare, so that in equilibrium monopoly and duopoly tariffs are time-invariant. The equilibrium tariff is τ_d^* under duopoly and τ_m^* under monopoly:

(3.5) $\tau_d^* = (3P''(P-c)-4(P')^2 \pm [16(P')^4-3(P'(P-c))^2-12(P')^2P'(P-c)]^{1/2})/2P''$

(3.6) $\tau_m^* = (P''(P-c)-(P')^2 \pm (P')^2[(P')^2-P''P-c)]^{1/2})/P''$

As above, the order and timing of exit in the intervention equilibrium depend on the order of the critical points adjusted for the tariffs. Call firm i's tariff-adjusted critical point under market structure b $\hat{t}_f^b$. It lies at the intersection of the equilibrium tariff level and the tariff level for which firm i's instantaneous participation constraint just binds under market structure b at that instant. Define $\tau_i^b(t)$ as the level of tariffs that gives firm i profits of exactly 0 under market structure b at time t:

(3.7) $\tau_i^b(t) = \begin{cases} \tau \mid f(t)\pi_i^b(\tau)=k_i & \text{if such a value exists} \\ 0 & \text{otherwise} \end{cases}$

There are five relevant tariff levels in all:

TARIFFS	Monopoly	Duopoly
Home zero-profit		$\tau_h^d(t)$
Foreign zero-profit	$\tau_f^m(t)$	$\tau_f^d(t)$
Equilibrium	τ_m^*	τ_d^*

Figure 3.1 illustrates the relationship between the tariff levels and the critical points. The assumptions made above on demand imply that the home

firm requires a rising level of tariff protection in order to break even over time, while the foreign firm's monopoly and duopoly zero-profit tariff levels must decline over time. $\tau_f^m(t)$ lies everywhere above $\tau_f^d(t)$ because foreign's monopoly profits are higher than its duopoly profits by a constant factor. The zero-profit tariff levels intersect the x-axis at the free market critical points of the associated firms because these are the points at which the firms break even under a tariff of zero. Home's tariff-adjusted duopoly critical point, $\hat{t}_h^d$, lies at the intersection of the equilibrium duopoly tariff, $\tau_d^\star$, with home's zero-profit duopoly tariff, $\tau_h^d(t)$; the home firm just breaks even under $\tau_d^\star$ at this point. Similarly, the intersection of $\tau_f^d(t)$ with $\tau_d^\star$ defines the tariff-adjusted foreign duopoly point, $\hat{t}_f^d$, and the intersection of $\tau_f^m(t)$ and $\tau_m^\star$ defines the tariff-adjusted foreign monopoly critical point, $\hat{t}_f^m$. Note that the instantaneous incentive constraints are compatible up to the time defined by the intersection of the two zero-profit constraint lines, $\bar{t}$, and mutually exclusive beyond it.

There are four possible configurations of the tariff-adjusted critical points, which are illustrated in figures 3.1 through 3.4. The effect of the tariffs may be strong enough to reverse both duopoly and monopoly critical points (case RB), as in figure 3.1. The tariffs may reverse the order of only one pair of critical points and narrow the gap between the second pair; monopoly reversal (case RM) is shown in figure 3.2, and duopoly reversal (case RD), is shown in figure 3.3. Lastly, the tariffs may narrow both pairs of critical points without reversing either (case N), as in figure 3.4.

CASES	$\hat{t}_f^m < t_h^m$	$t_h^m < \hat{t}_f^m$
$\hat{t}_f^d < \hat{t}_h^d$	RB	RD
$\hat{t}_h^d < \hat{t}_f^d$	RM	N

As was established in the pure production game of Section II, the equilibrium order and timing of exit hinge on whether the equilibrium tariffs reverse the order of either or both pairs of critical points. When the order of the critical points is the same under both market structures, the firm that is profitable longer prevails. Thus, if the tariffs reverse the order of both pairs of critical points (RB), the order of exit in equilibrium also reverses. If the tariffs do not reverse the order of either pair (N), the foreign firm maintains the monopoly position, but duopoly is prolonged, and the foreign firm abandons its monopoly position earlier. When the tariff reverses the order of only one pair (RM and RD), either outcome is possible, depending on the relative magnitudes of each firm's cumulative monopoly rents in the final period relative to the cumulative duopoly losses leading up to it.

There are thus two equilibria in total. In both, the effect of intervention is to terminate the life of the market prematurely and to prolong the life of the home firm relative to the free market equilibrium. In one, the order of exit is reversed, while in the other it is maintained.

To see this, consider case RB shown in figure 3.1, in which both pairs are reversed. Between $\hat{t}_f^m$ and t_h^m, home has a positive value to remaining in as a monopolist, while foreign anticipates losses if it plays in. An instant before $\hat{t}_f^m$, home can credibly threaten to play in because its instantaneous duopoly losses will be offset by anticipated cumulative monopoly profits of $V_h^m(\hat{t}_f^m, t_h^m)$. With anticipated profits of 0, foreign cannot credibly threaten to take duopoly losses, so it must play out. Working backwards, the same reasoning applies back to $\hat{t}_f^d$, where the optimal tariff first satisfies the foreign zero-profit constraint. Thus, the intervention equilibrium entails reversal of the order of exit: duopoly persists until foreign leaves at $\hat{t}_f^d$, followed by home monopoly until t_h^m.

The same result is possible in case RM, in figure 3.2, where only the monopoly critical points are reversed, depending on the relative values of the two firms. Working backward from $\hat{t}_h^m$, home is profitable as a monopolist and foreign is not under the tariff. Thus, foreign stays out, and home earns monopoly profits. This reasoning works back as far as $\hat{t}_f^d$, where foreign is profitable under both market structures, and thus plays in. Continuing backwards, if the home firm's anticipated monopoly profits, $V_h^m(\hat{t}_f^d, t_h^m)$, swamp duopoly losses on the interval leading up to it, $V_h^d(\tau_d^*; \hat{t}_h^d, \hat{t}_f^d)$, then home will stay in as a duopolist on $(\hat{t}_h^d, \hat{t}_f^d)$ despite losses, foreign is forced out at $\hat{t}_f^d$, and home then monopolises the market until t_h^m. The order of exit is reversed.

However, there is a second possibility in case RM, where $V_h^m(\hat{t}_f^d, t_h^m) < |V_h^d(\tau_d^*; \hat{t}_h^d, \hat{t}_f^d)|$. In this case, home cannot absorb duopoly losses on $(\hat{t}_h^d, \hat{t}_f^d)$, since they swamp its anticipated monopoly profits on $(\hat{t}_f^d, t_h^m)$. Thus, home drops out at $\hat{t}_h^d$, and foreign monopolises the market until $\hat{t}_f^m$.

The same two equilibrium outcomes are possible in case RD, where only the duopoly critical points are reversed. The arguments that were used to determine the equilibria in case RD in the pure production game of Section II apply here, with the sole difference that the foreign firm's monopoly critical point is reduced under the tariff. Thus, if foreign's duopoly losses, $V_f^d(\tau_d^*; \hat{t}_f^d, \hat{t}_h^d)$, swamp anticipated monopoly profits of $V_f^m(\tau_m^*; \hat{t}_h^d, \hat{t}_f^m)$, foreign must drop out at $\hat{t}_f^d$, the order of exit is reversed, and home monopolises the market on $(\hat{t}_f^d, t_h^m)$. If instead $|V_f^d(\tau_d^*, \hat{t}_f^d, \hat{t}_h^d)| > V_f^m(\tau_m^*, \hat{t}_h^d, \hat{t}_f^m)$, foreign is able to absorb duopoly losses on $(\hat{t}_f^d, \hat{t}_h^d)$, in anticipation of monopoly profits on $(\hat{t}_f^d, \hat{t}_f^m)$. In equilibrium, home drops out at $\hat{t}_h^d$, and foreign monopoly prevails until $\hat{t}_f^m$. Here, the order of exit is preserved, but home stays in later and foreign leaves earlier than in the free market equilibrium.

The no reversal outcome, with home exit at $\hat{t}_h^d$ and foreign monopoly until $\hat{t}_f^m$, is the sole equilibrium in case N, shown in figure 3.4. Since neither pair of critical points is reversed, the free market order of exit is preserved, but home leaves later and foreign earlier than in the free market equilibrium.

To summarise, there are two equilibrium outcomes. In one the order of exit is reversed: the foreign firm exits at $\hat{t}_f^d$ and home secures the monopoly position until t_h^m. In the second, home stays in as a duopolist until $\hat{t}_h^d$ and foreign prevails as a monopolist on $(\hat{t}_h^d, \hat{t}_f^m)$. In both equilibria the life of the market is truncated relative to the free market equilibrium, and home stays in beyond its free market exit time under the tariff's protective umbrella. In the no reversal equilibrium, duopoly production is extended, while in the reversal equilibrium, it may be either extended or curtailed.

III.2. Comparative Statics

Clearly, the likelihood of reversal is greater the more are foreign's critical points foreshortened by the tariffs relative to those of home. In this section, I turn to comparative statics to examine the effect of various parameters on the relative positions of the firms' critical points, and thereby on the order and timing of exit.

There are two sources of asymmetry in the firms' critical points and, by extension, in their strategic positions: the differential between the firms' fixed costs and the (specific) tariff, which acts like an increase in foreign's marginal cost relative to home. A reduction in the difference between k_h and k_f, or an increase in the tariff shortens foreign's critical points relative to home, and thereby diminishes foreign's strategic advantage.

Changes in the demand-side parameters push both firms' critical points in the same direction, so that the relative effect on the critical points depends only on their initial positions and the shape of the demand function. An increase in the constant in either the instantaneous demand function or the time component of demand prolongs the viability of both firms. An increase in the elasticity of the demand function associated with a decrease in the slope of either the inverse instantaneous demand function or the time component of demand lengthens the productive lives of both firms.

Starting with the production parameters, consider an increase in the difference between the firms' fixed costs. Total differentiation of the home and foreign profit functions about the critical points yields:

(3.8) $d\hat{t}_h^u/d(k_h-k_f) = dk_h/[d(k_h-k_f)\pi_h^u f'(\hat{t}_h^u) < 0$ $\quad$ u=m,d

(3.9) $d\hat{t}_f^u/d(k_h-k_f) = dk_f/[d(k_h-k_f)\pi_f^u f'(\hat{t}_f^u) > 0$ $\quad$ u=m,d

where $\hat{t}_h^m = t_h^m$. An increase in the fixed cost differential extends foreign's tariff-adjusted critical points relative to those of home, strengthening the strategic position of foreign, and making reversal less likely.

Turning to an increase in the level of the tariff, differentiation of the profit functions about the critical points yields:

(3.10) $dt_h^u/d\tau = -f(t_h^u)\pi_{h\tau}^u(\tau)/((f'(t)\pi_h^u)$ $\quad$ $= 0$ for u=m; > 0 for u=d

(3.11) $d\hat{t}_f^u/d\tau = -f(\hat{t}_f^u)\pi_{f\tau}^u(\tau)/((f'(\hat{t}_f^u)\pi_f^u(\tau)) < 0$ $\quad$ for u=m,d

An increase in the tariff works precisely like an increase in foreign's marginal cost relative to home. Thus, under monopoly, an increase in the tariff decreases the foreign critical point and leaves the home monopoly critical point unchanged, weakening the strategic position of the foreign firm. Under duopoly, the effect on the critical points is more extreme, since the tariff raises home profits as well as reducing foreign profits:

Changes in the parameters of the instantaneous demand function affect the two critical points of both firms in the same direction, but in different degrees, depending on their initial positions and the shape of the time component of demand. With linear demand of the form $P(Q/f(t))=a-bQ/f(t)$, a decrease in the slope and an increase in the intercept raise the critical monopoly and duopoly times of both firms. Differentiating with respect to a yields:

(3.12) $d\hat{t}_i^u/da = -2f(\hat{t}_i^u)/((f'(\hat{t}_i^u)(a-c)) > 0$ $\quad$ i=h,f; u=m,d

A shift upward in the level of demand lengthens the productive lives of both firms under both market structures. The effect on the relative positions of the firms' critical points depends on the elasticity of f(t) and on the initial critical points.

Differentiating with respect to b yields:

$$(3.13) \quad d\hat{t}_i^u/db = f(\hat{t}_i^u)/(bf'(\hat{t}_i^u)) < 0 \qquad i=h,f;\ u=m,d$$

which are equal when the initial critical points are equal. Thus, an increase in the slope of the (inverse) demand function (a decrease in the elasticity of instantaneous demand) shortens the viability of both firms under both market structures. Again the effect of the change on the differential between the two firms' critical points under each market structure depends solely on the shape of f(t) and on the initial critical times.

Changes in the time component of demand also move the critical points of both firms in the same direction, but to differing degrees, depending on the initial positions of the critical points. Upward shifts in the time component of demand raise the firms' tariff-adjusted critical points, while increases in the slope of the time component shorten the firms' viable production periods. Take $f(t) = A+e^{-rt}$. Then an increase in A raises the critical points by an amount that depends on their initial positions and the discount rate:

$$(3.14) \quad d\hat{t}_i^u/dA = \exp(r\hat{t}_i^u)/r > 0 \qquad i=h,f;\ u=m,d$$

The effect is more extreme the larger the initial critical point. An increase in r lowers the critical points by an amount that depends on their initial positions:

$$(3.15) \quad d\hat{t}_i^u/dr = -\hat{t}_i^u/r < 0 \qquad i=h,f;\ u=m,d$$

As demand declines more rapidly over time, the monopoly and duopoly critical points fall, and by more the higher the initial critical point.

III.3. Suboptimality of Policy

In the pure production equilibrium of Section II, it is clear that reversal is the welfare-maximising outcome, since welfare includes only producer surplus. When intervention in the absence of commitment fails to achieve reversal, it may actually reduce welfare. In contrast, with the addition of consumer surplus, welfare may be higher when the free market order of exit is maintained, due to the value of consumer surplus and tariff revenues under foreign monopoly on the interval $(t_h^m, t_f^m]$. This is more likely the more gradually demand declines over time and the greater the fixed cost differential between the two firms. But here again, just as in the pure production case, government intervention in the absence of commitment power may not achieve the welfare-maximising order of exit and does not achieve the domestic welfare-maximising timing of exit.

Although there is no general expression for the first-best path of tariffs that maximises $W(t_h^d, \infty)$, it is straightforward to establish that the no commitment equilibrium is not generally first best by showing that the government could do better under full commitment. This is most obvious in the equilibrium where the imposition of tariffs curtails the monopoly exit time of the foreign firm without reversing the free market order of exit. Here the government could raise welfare simply by precommitting to a tariff of $\tau_f^m < \tau_m^*$ that allows the foreign firm to continue producing between $\hat{t}_f^m$ and t_f^m.

Higher welfare can also be achieved in the equilibrium where the tariffs reverse the order of exit, if the cost of keeping the foreign firm in under a prohibitive tariff on $(\hat{t}_f^d, t_h^m]$, $\int_{\hat{t}_f^d}^{t_h^m} k_f$, is offset by the value of foreign monopoly profits on $(t_h^m, t_f^m]$, $V_f^m(t_h^m, t_f^m)$. The foreign firm can be persuaded to stay in under a prohibitive tariff on $(\hat{t}_f^d, t_h^m]$, with the promise of a path of tariffs below the lesser of τ_m^* and $\tau_f^m(t)$ on $(t_h^m, t_f^m]$ that gives it sufficient monopoly profits to offset its earlier duopoly losses. The prohibitive tariff results in welfare equal to that in the no commitment equilibrium up through t_h^m, and additional surplus is generated by foreign production on $(t_h^m, t_f^m]$.

Thus, in both equilibria, the no commitment policy achieves lower welfare than that achievable with full government precommitment power. Comparison of no commitment equilibrium welfare with first best welfare requires specification of the utility function and the parameter values. In the next section, I compare first best policies that maximise welfare over the entire horizon with equilibrium no commotment policies for a specific case with linear demand.

IV. Numerical Simulation

This section presents results from a numerical simulation that measures the effects of intervention on welfare under varying degrees of government commitment. The simulation compares welfare in the no commitment intervention equilibrium, the free market equilibrium, and the full commitment intervention equilibrium. It examines the sensitivity of the three welfare levels to changes in several key parameters for the case with consumption at home. It also examines the sensitivity of welfare to changes in the fixed cost differential in the pure production case, because of the centrality of this parameter in the analysis in Sections II and III.

The simulation suggests that the welfare loss from the government's inability to precommit is high. For a range of parameters it is preferable to avoid intervening altogether in the absence of commitment power. Welfare in the no commitment intervention equilibrium is a distant second and sometimes third best for all parameter ranges, with one exception. As the fixed cost differential goes to 0, welfare under no commitment approaches that under full commitment, since both entail reversal in the order of exit.

In the range of parameter values in which no commitment intervention actually reduces welfare, the fixed cost differential is sufficiently large that the free market order of exit is preferred, but not large enough that the protected home firm drops out. Given consumption in the home market, the first best full commitment policy preserves the free market order of exit for a broad range of parameter values, except as the fixed cost differential approaches 0. In contrast, the no commitment policy always reverses the free market order of exit, except as the fixed cost differential gets very large. This corresponds to the case RD in figure 3.3. In the pure production case, no commitment intervention similarly reduces welfare when the fixed cost differential exceeds a critical level, but in this instance the reduction occurs because intervention prolongs the life of the domestic firm without reversing the order of exit. This corresponds to the case RD in figure 2.3.

In addition, the periods of both duopoly and monopoly production are significantly reduced relative to the free market equilibrium under no commitment intervention for a broad range of parameter values. In contrast, under full commitment the period of duopoly production is prolonged and that of monopoly production is preserved in all cases.

The simulation characterises the firms' value functions as follows:

(4.1) $V_i^u(\tau;t_0,t_1)=\int_{t_0}^{t_1}(f(r)\alpha\pi_i^u(\tau)-k_i)dr$ for $i=h,f$ and $u=m,d$

where the instantaneous profit functions for the two firms are given in equations (3.1), (3.2), and (3.3) above, and α can be interpreted as a population size parameter that multiplies a unit demand function. The instantaneous demand function is assumed to be linear:

(4.2) $P(M) = a-bM$

The time component of demand is assumed exponential:

(4.3) $f(t) = A+e^{-rt}$

The welfare function is given in equation (3.4) above, adjusted for the population parameter. The equations for the optimal instantaneous tariffs, τ_d^* and τ_m^*, are given in equations (3.5) and (3.6), and those for the zero-profit tariffs $\tau_f^m(t)$, $\tau_f^d(t)$, and $\tau_h^d(t)$ are given in equation (3.7).

The parameter values for the base case are:

$a=100 \quad b=4 \quad c=15 \quad k_h=30 \quad k_f=10 \quad \alpha=1 \quad A=.022 \quad r=.3$

yielding critical points:

$t_h^d = 6.9 \quad \hat{t}_h^d = 9.3 \quad t_f^d = 11.9 \quad \hat{t}_f^d = 2.8$

$t_h^m = 11.4 \quad \hat{t}_h^m = 11.4 \quad t_f^m = 29.5 \quad \hat{t}_f^m = 11.9$

The critical points are shown in figure 4.1. Table 4.1 summarises welfare levels, the equilibrium order of exit, and exit times for changes in each of the parameter values under the three regimes.

In the base case, the free market order of exit entails duopoly until $\hat{t}_h^d$, followed by foreign monopoly until $t_f^m=29.5$, yielding total welfare over the interval of 741. In the no commitment equilibrium, the duopoly critical points are reversed, while the monopoly critical points maintain their free market order (case RD). In equilibrium, foreign monopoly profits on $(\hat{t}_h^d, \hat{t}_f^m)$ are swamped by its duopoly losses on $(\hat{t}_f^d, \hat{t}_h^d)$, so that foreign drops out at $\hat{t}_f^d=2.8$, and home prevails as a monopolist until $t_h^m=11.4$. Total welfare is 749, slightly above the free market level.

The first-best full commitment path of tariffs is derived by optimising over the three market structures on the interval (t_h^d, ∞), taking into account the incentive constraints of the two firms. It entails duopoly under τ_f^d up to $\bar{t}$; followed by duopoly under τ_h^d up to the point where cumulative foreign duopoly losses just offset its anticipated monopoly profits; followed by foreign monopoly under τ_m^* up to $\hat{t}_f^m$; and foreign monopoly under τ_f^m until $t_f^m=29.5$. The home firm is protected as long as is feasible without sacrificing surplus from the foreign firm's production in the later periods by making the foreign firm just indifferent between exiting and staying in over the interval. Thus, in equilibrium, duopoly lasts until $\hat{t}_h^d=9.3$, substantially longer than in the free market equilibrium, but the free market order of exit and market life are preserved. Total welfare is 928, roughly one-quarter higher than under no commitment.

The equilibrium path of tariffs under full commitment and no commitment is shown in figure 4.2. The no commitment equilibrium path is flat at the optimal instantaneous tariff level until the exit of the foreign firm, while the first best path is highly irregular (and somewhat implausible).

Qualitatively, the results are robust for a broad range of parameter values. Table 4.1 compares the equilibrium order of exit under the three regimes in the base case for changes in each of the parameter values. In all cases, the home firm leaves at t_h^d, and foreign prevails as a monopolist until t_f^m in the free market equilibrium. In all cases, there is reversal under no commitment intervention, such that the foreign firm leaves first at $\hat{t}_f^d$, and the home firm prevails as a monopolist until t_h^m. And, strikingly, the full commitment equilibrium maintains the free market order of exit in all but one case: the home firm leaves first at $\hat{t}_h^d$, long after its free market exit time, followed by foreign monopoly until t_f^m. The sole exception is where the fixed cost differential becomes very small; here, the full commitment equilibrium entails home monopoly following foreign exit at $\bar{t}$.

The duopoly and monopoly terminal points for each of the parameter values under the three regimes are illustrated in figure 4.3. Note that under no commitment intervention both duopoly and monopoly production are truncated relative to the free market outcome, while under full commitment the period of duopoly production is extended, and the period of monopoly production is preserved, with the one exception noted above. The truncation of the market under no commitment relative to the free market and full commitment is particularly strong for a low discount rate.

Figure 4.4 compares welfare levels under the three regimes for changes in the parameter values. The welfare differential between the full commitment and no commitment intervention equilibria is similar in magnitude to that between the no commitment intervention and free market equilibria for many of the parameter changes. However, a decrease in the time rate of discount raises the value of full commitment relative to no commitment disproportionately. This is because the consumer surplus associated with foreign monopoly production on (t_h^m, t_f^m) rises as demand declines more gradually over time, so that the value to keeping the foreign firm in is high. Similarly, as the ratio of home to foreign fixed costs rises from 1.5 to 3 in the base case, the value of full commitment rises, while the welfare gain from no commitment intervention over nonintervention goes to 0.

The sensitivity of welfare to changes in the fixed cost differential is worth examining in greater depth. Figure 4.5 shows the sensitivity of the welfare differential between full commitment and no commitment intervention (FC-NC), and between no commitment intervention and the free market (NC-FM) to increases in the size of the fixed cost differential.

When the firms' fixed costs are very similar, the welfare gain from intervention is high regardless of the government's commitment power because reversal in the order of exit is welfare-enhancing and easily achieved. The reduction in market life from reversal and the corresponding consumer surplus loss are small, while home monopoly profits are high. As the size of the fixed cost differential rises, the increase in welfare from no commitment intervention over nonintervention falls, eventually going below 0, while the value of commitment rises significantly. Between 1.5 and 4.5, the fixed cost differential is sufficiently large that the first-best policy is to retain the foreign firm as a monopolist in the final periods, but it is not large enough that the home firm is induced to drop out in the no commitment equilibrium. Notice that in the region between 3.2 and 4.5, no commitment intervention actually reduces welfare. As the ratio of home to foreign fixed costs approaches 4.5, the net welfare gain from no commitment intervention rises, and the additional benefit to full commitment declines. In this region, the fixed cost differential is sufficiently large that the home firm is forced out in the no commitment equilibrium, and the no reversal outcome is first best.

There is a similar switch in the pure production case, although here reversal is always welfare-maximising, so that no commitment intervention reduces welfare whenever the fixed cost differential is sufficiently large that reversal is not attained. The welfare differentials are shown in figure 4.6. When the fixed cost ratio is below 1.7, intervention under no commitment reverses the order of exit, so that the home firm secures the monopoly profits. The welfare gain over the free market equilibrium is high, and there is no additional value to commitment power. As the fixed cost ratio rises past 1.7, and the home firm's subsidy-adjusted duopoly critical point declines relative to foreign, the anticipated cumulative monopoly profits of the foreign firm offset its duopoly losses under the no commitment subsidy level on the shortened interval, so that the foreign firm is able to stay in and secure the monopoly position. Thus, for a fixed cost ratio above 1.7, no commitment intervention lowers welfare, and by an increasing amount. The value of full commitment is rising in this range because the fixed cost differential is still sufficiently small that a government with full commitment could force the foreign firm out with a prohibitive subsidy. The value of full commitment begins to fall when the fixed cost ratio reaches 2.5, because the home firm's monopoly exit time is sufficiently early that the foreign firm is viable under any subsidy level.

In sum, the simulation makes clear that the home firm receives more protection than is socially optimal in the absence of government commitment power. In the home consumption case, the effect of the protection is to secure the monopoly rents for the home firm at the expense of higher social surplus associated with foreign monopoly in the final periods, when the fixed cost differential is large but not enormous. In the pure production case, the effect of no commitment intervention may be to reduce welfare when the fixed cost differential is sufficiently large that reversal is not achieved.

V. Exit with Reciprocal Intervention

So far the analysis has focused on unilateral intervention, but an essential feature of trade disputes in many declining industries is that they take place in a multimarket setting with intervention by more than one government. Accordingly, this section considers the effect of reciprocal intervention in a two-market setting, where there is production, consumption, and a potentially interventionist government in each market.

The inclusion of a second market and reciprocal intervention introduces a direct link between the distribution of and each government's ability to influence the firms' relative strategic positions. Accordingly, the analysis assumes that the instruments and the commitment power of both governments are the same, in order to focus attention on the effect of differences in demand shares.

Several interesting conclusions emerge. The equilibrium order of exit depends significantly on the relative demand shares in each market, as well as on the firms' fixed cost differential. When tariffs are imposed reciprocally, a large share of demand in a firm's export market relative to its own market weakens its strategic position; the tariffs are used by the rival government to extract rents from the firm in return for access to the demand in the export market. In equilibrium, both governments always intervene, and, as in the single market case, the effect of tariff intervention is always to truncate the life of the market. It may also reverse the free market order of exit, but reversal is less likely to occur under reciprocal than unilateral intervention, all else equal. This is because the government's dynamic inconsistency problem caused by its lack of commitment power vis-a-vis the local firm is lessened by the countervailing intervention of the second government.

When the governments intervene using export subsidies, relative demand shares continue to play a strategic role, but it is opposite to that under tariffs. Since export subsidies provide privileged access to the export market, a large relative demand share in the export market confers a strategic advantage. Here again, the equilibrium order of exit may be reversed under reciprocal intervention, but it is less likely than under unilateral intervention. When there is reversal, the life of the market is truncated, and the free market life is preserved otherwise.

V. 1. Free Market Equilibrium

First the free market equilibrium with two markets is derived under the following assumptions:

A1" Demand is divided between the home and foreign markets. The demand function in each market is multiplicative in the level of demand:

(5.1) $Q^i(p,t) = \alpha^i f(t)g(p) \text{ ----> } p^i(Q^i,t) = g^{-1}(Q^i/\alpha^i f(t)) = P(Q^i/\alpha^i f(t))$,

where $i=h,f$, and α^i denotes the level of demand in market i.

A12 There are no transport costs for the firms, but the marginal cost to consumers of reselling the good in the other market is prohibitive.[17]

By assumption 12, there is no interdependence between the two markets in the

17 Assumption 12 implies there is perfect arbitrage on the production side and perfect segmentation on the consumption side. This can be interpreted as a significant sunk investment in market-specific distribution networks.

output decisions of each firm. The sole interdependence is in the determination of the firms' exit times. Thus, the firms choose quantities to maximise profits in each market separately.

The multiplicative form of the demand function ensures that the profit-maximising output choice can be normalised for the market level of demand, α^i, in addition to the normalisation for time that was used in the single market case. Call x the normalised equilibrium duopoly output produced by a firm for sale in its own market. Thus, home's normalised equilibrium duopoly output for the home market, and foreign's normalised equilibrium duopoly output for the foreign market are:

(5.2) $$x = q_i^{di}/(\alpha^i f(t)) = [c - P(Q^i/(\alpha^i f(t)))]/P' \qquad \text{for } i = h,f$$

where q_i^{dj} denotes firm i's duopoly sales in market j. Similar expressions define the normalised equilibrium levels of output produced for export by each firm under duopoly, y; total output in each market under duopoly, z; output produced for local consumption under monopoly, X; and output produced for export under monopoly, Y.

The level of profits is then multiplicatively separable into three parts: a normalised instantaneous unit profit function, a market size parameter, and the time component of demand. Firm i's normalised duopoly profits in its own market are:

(5.3) $$\pi_i^{di} = \pi(x,y) = \pi(q_i^{di}, q_j^{di}, t)/(\alpha^i f(t)) \qquad \text{for } i=h \text{ when } j=f \text{ and conversely}$$

and normalised duopoly profits in firm i's export market are:

(5.4) $$\pi_i^{dj} = \pi(y,x) = \pi(q_i^{dj}, q_j^{dj}, t)/(\alpha^j f(t)) \qquad \text{for } i=h \text{ when } j=f \text{ and conversely}$$

Similar expressions define normalised monopoly profits in firm i's own market:

(5.5) $$\pi_i^{mi} = \pi(X,0) = \pi(q_i^{mi}, 0, t)/(\alpha^i f(t)) \qquad \text{for } i=h \text{ when } j=f \text{ and conversely}$$

and normalised monopoly profits in firm i's export market:

(5.6) $$\pi_i^{mj} = \pi(Y,0) = \pi(q_i^{mj}, 0, t)/(\alpha^j f(t)) \qquad \text{for } i=h \text{ when } j=f \text{ and conversely}$$

Firm i's total variable profits at time t are $(\alpha^i \pi_i^{ui} + \alpha^j \pi_i^{uj}) f(t)$, for $u=m,d$.

Perfect arbitrage on the production side ensures that the variable monopoly profits of the two firms are equal in the free market equilibrium, and likewise for their variable duopoly profits. Firm i's critical point under market structure b, T_i^b, is defined by the time at which its joint variable profits from both markets just cover its fixed costs. As in the single market exit game described in Section III, the sole determinant of the free market equilibrium order of exit is the firms' relative fixed costs. Given $k_h > k_f$, home's critical points precede those of foreign under both monopoly and duopoly. Thus, in equilibrium, home leaves first at T_h^d, and foreign monopolises the market until monopoly profits go to 0 at T_f^m.

V. 2. Intervention with Tariffs

Next a government is introduced in each country. As in assumption 10', each government may levy a specific tariff, τ, on imports. The governments are assumed to choose tariffs simultaneously each period, after the firms' participation decisions but before their output decisions.

These assumptions are sufficient to solve for the tariff-adjusted critical points of the two firms. I proceed by defining the equilibrium profit and tariff levels. Solving for profit-maximising quantities under reciprocal intervention yields two normalised variable profit levels under duopoly - the profits of a firm in its own market where it is protected by a tariff, $\pi_+^d(\tau)$, and the firm's profits in its export market, where it pays a tariff, $\pi_-^d(\tau)$:

(5.7) $$\pi_+^d(\tau) = [P(z(\tau))-c]x(\tau) - (c-P(z(\tau))^2/P'$$

(5.8) $$\pi_-^d(\tau) = [P(z(\tau))-c-\tau]y(\tau) - (c+\tau-P(z(\tau))^2/P'$$

Similarly, a monopolist exporting to a protected market earns normalised variable profits of $\pi_-^m(\tau)$:

(5.9) $$\pi_-^m(\tau) = [P(Y(\tau))-c-\tau]Y(\tau) - (c+\tau-P(Y(\tau))^2/P'$$

and a monopolist selling to its own market earns normalised variable profits of $\pi_{+}^{m}(\tau)$, which are just its free market profits:

The welfare function for each country is the same as (3.4), multiplied by the level of demand, with the addition of profits from the second market. Again, since the governments have no precommitment power, and cannot influence the exit decisions of the two firms directly, they set tariffs to maximise instantaneous welfare rather than welfare over the entire horizon. The equilibrium tariffs, in each market are the same as those defined in equations (3.5) and (3.6), τ_{d}^{*} and τ_{m}^{*}; they are invariant to the addition of a second market, due to the multiplicative nature of demand and the independence of the firms' output choice in each market.

The normalisation of the profit functions and the equality of the tariff levels simplify the comparison of the critical points. Differentiation of the monopoly profit functions with respect to τ establishes that:

(5.11) $\quad \pi_{+}^{m}(\tau_{m}^{*}) > \pi_{-}^{m}(\tau_{m}^{*})$

Assuming that the good is a strategic substitute so that reaction functions are downward sloping, differentiation of duopoly profits yields:

(5.12) $\quad \pi_{+}^{d}(\tau_{d}^{*}) > \pi_{-}^{d}(\tau_{d}^{*})$

Recall that whenever the order of both pairs of tariff-adjusted critical points is the same (as in cases RB and N), the firm with the lower critical points exits first at its duopoly critical point, and the remaining firm prevails as a monopolist from that point until its monopoly exit time. When the order of the pairs differs (cases RD and RM), the equilibrium order of exit depends on the cumulative value of duopoly losses relative to anticipated monopoly gains, as well as on the relative positions of the critical points.

Call firm i's critical point adjusted for tariffs under market structure u, $\hat{T}_{i}^{u}$. The monopoly critical points are determined by the equations:

(5.13) $\quad f(\hat{T}_{h}^{m})[\alpha^{h}\pi_{+}^{m}(\tau_{m}^{*})+\alpha^{f}\pi_{-}^{m}(\tau_{m}^{*})] = k_{h}$

$$f(\hat{T}_f^m)[\alpha^f \pi_+^m(\tau_m^*)+\alpha^h \pi_-^m(\tau_m^*)] = k_f$$

Notice that in the two-market case, there are two key parameters determining the order of the critical points: the relative demand shares and the fixed cost differential. To focus on the relative demand shares, take the case where the two firms are symmetric in their fixed costs. With equal fixed costs, the sign of $\hat{T}_h^m$-$\hat{T}_f^m$ is the same as that of α^h-α^f. If, for instance, $\alpha^h<\alpha^f$, then by (5.11) and (5.13):

(5.14) $k_h/f(\hat{T}_h^m) = [\alpha^h \pi_+^m(\tau_m^*)+\alpha^f \pi_-^m(\tau_m^*)] < [\alpha^f \pi_+^m(\tau_m^*)]+\alpha^h \pi_-^m(\tau_m^*) = k_f/f(\hat{T}_f^m)$

which implies $\hat{T}_h^m < \hat{T}_f^m$, since fixed costs are equal, and f(t) is a decreasing function. The same relationship between the relative demand shares and the order of exit times obtains under duopoly when fixed costs are equal. Briefly, if $\alpha^h<\alpha^f$, then from equation (5.12) we have:

(5.15) $k_h/f(\hat{T}_h^d) = [\alpha^h \pi_+^d(\tau_d^*)+\alpha^f \pi_-^d(\tau_d^*)] < [\alpha^h \pi_-^d(\tau_d^*)+\alpha^f \pi_+^d(\tau_d^*)] = k_f/f(\hat{T}_f^d)$

which in turn implies $\hat{T}_h^d<\hat{T}_f^d$.

Under reciprocal intervention the firm in the market with the larger demand has larger monopoly and duopoly variable profits. Given equal fixed costs, the firm in the market with the larger demand is viable longer under both market structures, and therefore secures the monopoly position in the restricted trade equilibrium. The privileged access to the larger market confers a strategic advantage.

When both the fixed cost wedge and the demand shares work to the advantage of the same firm, the outcome is similarly determinate and favourable to that firm. Again the firm in the larger market prevails, and again the free market order of exit is preserved. This outcome also obtains when demand shares are equal and the fixed costs alone differ. With equal demand shares, the profits of the two firms are affected symmetrically by the tariffs, so that fixed costs are the sole determinant of strategic advantage.

In the case where the larger fixed cost firm is located in the country with larger demand, the advantage on the demand side may outweigh the disadvantage on the cost side, so that in equilibrium the order of one or both pairs of the critical points is reversed. Whether the order of each pair of critical points is reversed depends on the relative magnitudes of the difference in the fixed costs, demand shares, and profits of the protected and the tariff-paying firms in a complicated way. The reversal of one pair of critical points by the imposition of tariffs does not necessarily imply the reversal of the other pair, so that the equilibrium order of exit may also depend on the cumulative value of duopoly losses as against monopoly profits. Thus, the effect of the tariffs could fit any of the four cases described in Section III (RM, RD, RM, and N), and either of the two associated equilibria (reversal and no reversal) is possible.

Thus, in the presence of reciprocal intervention, the effect of an increase in the relative demand share in a firm's own market is to strengthen the firm's strategic position by raising its critical points relative to its rival. The intuition is quite simple. In a state of excess capacity, the viability of a firm depends on access to demand. The government, by discriminating among firms in granting access to the demand in its market, confers a strategic advantage on the firm in its market. Loosely, when tariffs are imposed reciprocally, the firm in the country with larger demand benefits relatively more from the restrictions.

The introduction of a second interventionist government also tends to lessen the likelihood of reversal of the free market order of exit, since the profit-shifting effect of tariffs in one market is at least partially offset by the tariffs in the other market. Only when the larger concentration of demand coincides with the location of the larger firm does the possibility of reversal arise. And even then, reversal is less likely than under unilateral intervention, all else equal, because the relative benefits of protection are smaller when intervention is reciprocal.

Whether or not there is reversal of the order of exit, it is true in any reciprocal tariff equilibrium that the monopoly exit time is curtailed relative to the free trade time, since by equation (5.11), both firms' monopoly profits are lower:

$$(5.16) \qquad \alpha^i\pi_+^m(\tau)+\alpha^j\pi_-^m(\tau)<\pi_+^m(\tau)(\alpha^i+\alpha^j)=\pi^m(0)(\alpha^i+\alpha^j) \qquad \longrightarrow \hat{T}_i^m<T_i^m$$

for i=h when j=f, and the reverse. As in the single-market case, equilibrium policy is suboptimal relative to that which would prevail with full government precommitment. At minimum, both governments could do better by precommitting to the zero-profit tariff level on the segment between the rival firm's monopoly restricted and free-market exit times.

V. 3. Intervention with Export Subsidies

If both governments intervene through export subsidies rather than import tariffs, a large demand share in a firm's export market confers strategic advantage. As with tariffs, there may or may not be reversal with export subsidies, depending on whether the demand share advantage reinforces or countervails the fixed cost advantage. When there is reversal, the market life may be truncated relative to the free market equilibrium.

Just as for import tariffs, I solve for normalised quantities and profits in equilibrium under export subsidies. This yields two normalised variable profit levels under duopoly: the profits of a firm in its own market when imports from the rival firm are subsidised, $\pi_-^d(s)$, and the subsidised profits of a firm in its export market, $\pi_+^d(s)$. Under monopoly, a firm selling to its own market earns the normalised free market level of profits, $\pi_-^m(s)=\pi^m(0)$, and profits from subsidised exports are $\pi_+^m(s)$.

Again, since the governments cannot affect the firms' exit decisions directly, they set subsidies to maximise instantaneous welfare each period.

The equilibrium subsidies are those derived in equation (2.15), s_d^* under duopoly and 0 under monopoly. Due to the stationarity of the subsidies and the multiplicative nature of demand, the optimal duopoly subsidy is the same in both markets. Assuming the good is a strategic substitute, differentiation of the duopoly profit functions with respect to s establishes that:

$$(5.12') \qquad \pi_+^d(s_d^*) > \pi_-^d(s_d^*)$$

Again, it is useful to isolate the effect of demand shares on the firms' critical points. Since the equilibrium monopoly subsidy is 0, the free market order of the monopoly critical points is maintained. Define $\hat{S}_i^d$ as firm i's duopoly critical point adjusted for the equilibrium subsidy. Suppose k_h=k_f. Then the monopoly critical points are equal, and the order of the duopoly critical points is determined by the equation:

$$(5.15') \qquad f(\hat{S}_h^d)[\alpha^h\pi_-^d(s_d^*)+\alpha^f\pi_+^d(s_d^*)] - k_h = k_f - f(\hat{S}_f^d)[\alpha^f\pi_-^d(s_d^*)+\alpha^h\pi_+^d(s_d^*)]$$

Together equations (5.12') and (5.15') imply that the positions of the duopoly critical points are inversely related to the relative demand shares.

In direct contrast to tariff intervention, strategic advantage is conferred by a large relative share of demand in the export market when export subsidies are imposed reciprocally. This is because export subsidies give privileged access to demand in the export market, in contrast to tariffs, which give privileged access to local demand. Thus, when the firm with smaller fixed costs is located in the market with a smaller demand share, neither pair of critical points is reversed (case N), and the free market order of exit and terminal time are preserved.

When the firm with the smaller fixed cost is located in the market with a larger demand share, there are two possible cases (N and RD). If the demand disadvantage outweighs the fixed cost advantage, the subsidies reverse the duopoly critical points (RD). The order of exit will be reversed if the value of the cumulative monopoly profits of the small fixed cost firm on the interval following its rival's duopoly critical point is swamped by its cumulative duopoly losses on the interval between the two firms' duopoly critical points. The free market order of exit is preserved otherwise. Where the subsidies narrow the gap between the duopoly critical points without reversing them (case N), the free market order of exit and terminal time are preserved.

VI. Conclusion

In this paper I have offered an explanation for the proclivity of policymakers to protect marginally profitable firms in markets with declining demand and excess capacity. In an industry where fixed costs are significant and declining demand ensures that eventually only one firm will be viable, the anticipation of monopoly rents stimulates protectionist pressures from the disadvantaged, high fixed cost firm. As long as the government cannot credibly precommit to allow the disadvantaged firm to fail in the future, the firm will persist, confident that if it does so the government will find it optimal to erect protective barriers when the future date arrives.

I model this phenomenon in a duopoly exit game, and find that the government always intervenes to protect its firm in equilibrium, and that the equilibrium policy is not first-best. In the most transparent case, where firms export to a third market, unilateral intervention may reduce welfare by prolonging the duopoly tenure of the strategically disadvantaged home firm without in the end securing the monopoly position.

When consumption is concentrated in the home market, the path of tariffs chosen by the government in the absence of precommitment power is similarly suboptimal relative to that under full precommitment, and may reduce welfare. The effect of the tariffs is to reverse the equilibrium order of exit in many cases, and to truncate the period during which production takes place and prolong the home firm's productive life in all cases. Reversal is more likely the smaller is the fixed cost differential, and truncation is more severe the more rapidly demand is declining and the more extreme is the reduction in foreign profits from the optimal tariff, all else equal.

When there is reciprocal intervention, the precommitment problem is overlaid by the strategic interaction of the governments. The net effect is that both governments always intervene in equilibrium, and the market declines faster than it does in the free market equilibrium. However, reversal of the free market order of exit is less likely than under unilateral intervention, all else equal, because the presence of countervailing intervention lessens each domestic firm's commitment power vis-a-vis its own government. The introduction of a second government highlights the importanceof relative demand shares in determining the order of exit, in addition to the fixed cost differential. With reciprocal tariffs, it is strategically advantageous to be located in the larger market, since tariffs give privileged access to own market demand, while with export subsidies the reverse is true.

FIGURE 1,1

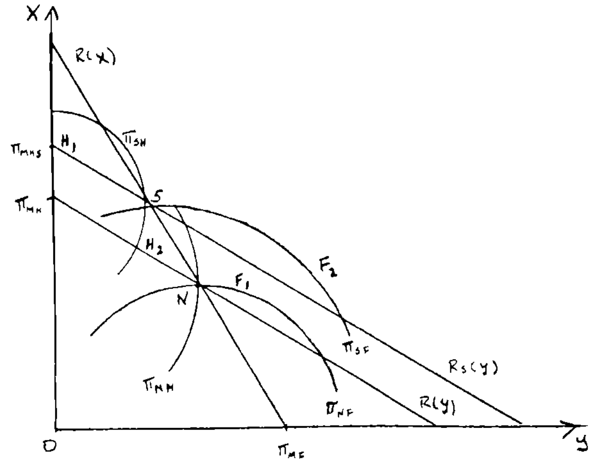


FIGURE 1.2

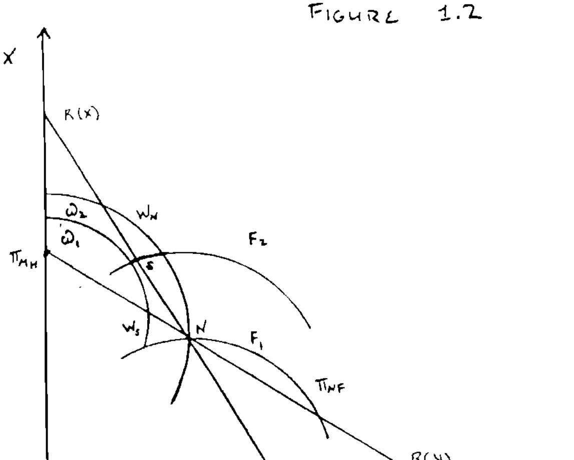


FIGURE 2.1

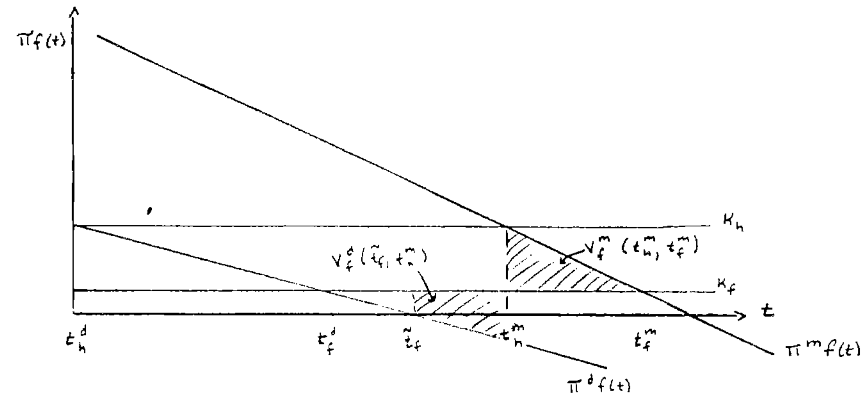


FIGURE 2.2a

N

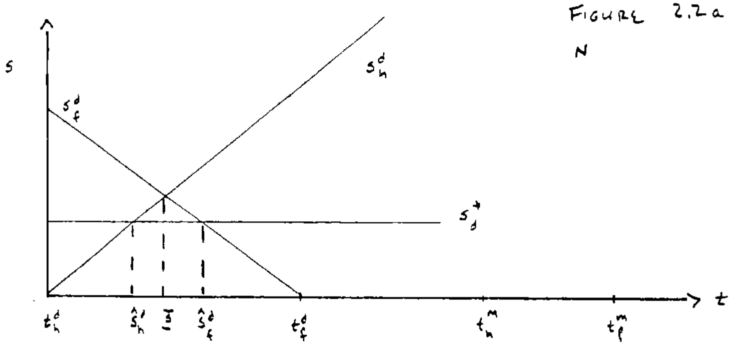


FIGURE 2.2b

N

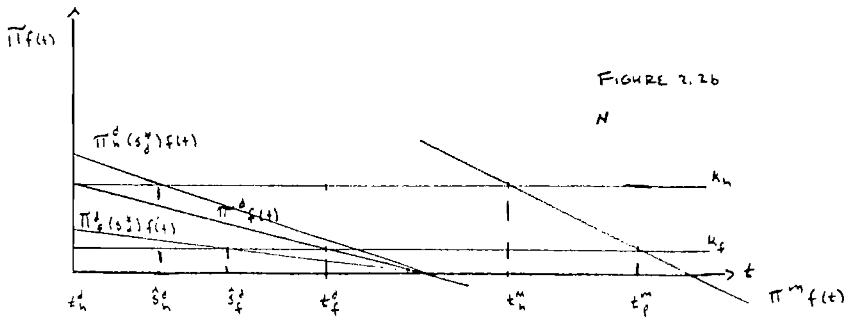


FIGURE 2.3a

RD

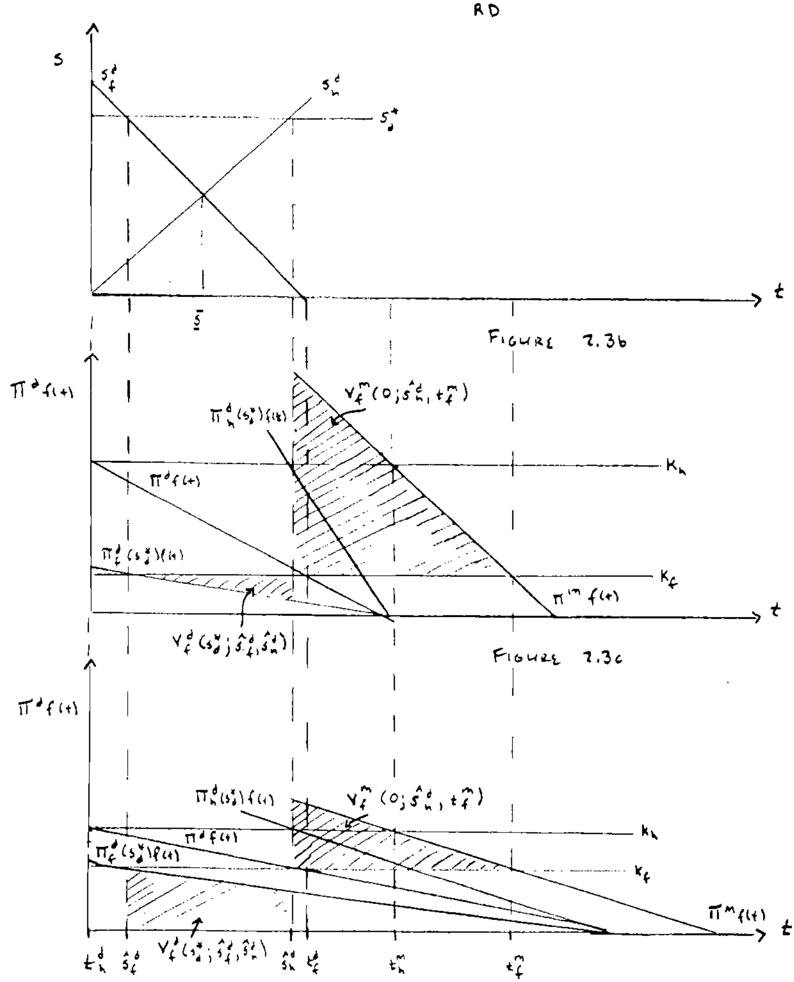


FIGURE 2.3b

FIGURE 2.3c

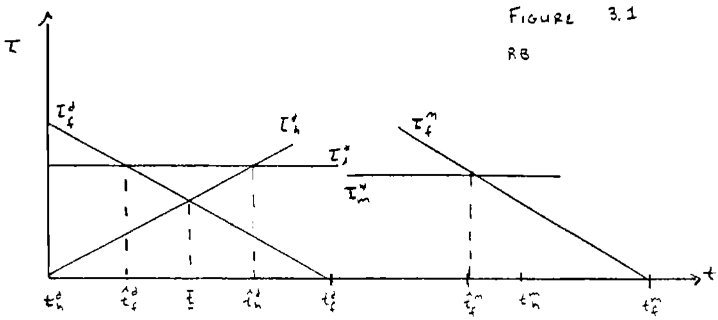


FIGURE 3.1
RB

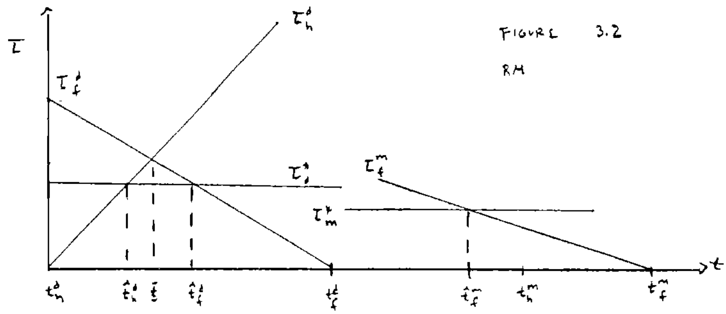


FIGURE 3.2
RM

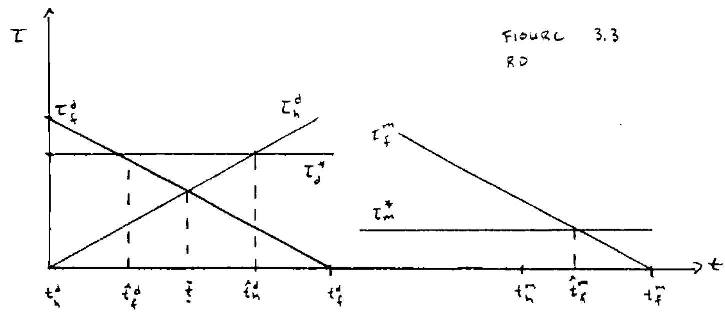


FIGURE 3.3
RD

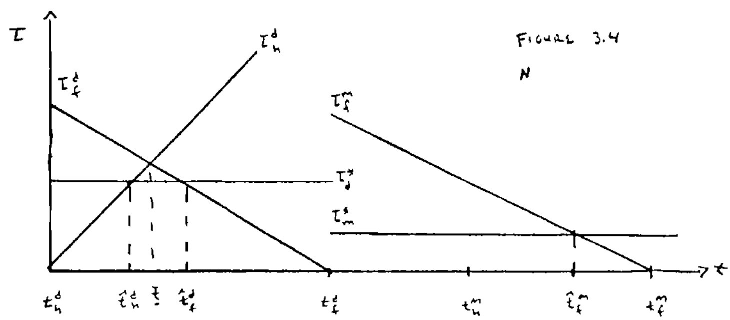


FIGURE 3.4
N

Table 4.1

SENSITIVITY ANALYSIS

Parameter	Change in Value	Welfare			Last one in			Terminal times					
								Duopoly			Monopoly		
		FM	NC	FC	FM	NC	FC	FM	NC	FC	FM	NC	FC
Base case		740	749	928	F	H	F	6.9	2.8	9.4	29.5	10.4	29.5
Kh-Kf	-15.0	875	936	949	F	F	F	9.8	2.8	10.4	29.5	15.0	15.0
A	-0.007	647	722	820	F	H	F	6.7	2.8	9.0	16.5	9.9	16.5
r	-0.1	1111	1123	1393	F	H	F	10.3	4.3	14.1	44.2	15.6	44.2
a	25.0	627	707	794	F	H	F	4.3	0.4	6.5	12.7	7.3	12.7
b	1.0	664	731	841	F	H	F	6.0	2.1	8.4	17.2	9.3	17.2
alpha	-0.25	656	727	832	F	H	F	5.8	1.8	8.2	16.3	9.0	16.3
c	7.5	669	734	848	F	H	F	6.2	2.2	8.6	17.9	9.5	17.9

Base Case Parameters

Kh-Kf	20.0
A	0.022
r	0.3
a	100.0
b	4.0
alpha	1.0
c	15.0

Key

FM: Free Market
NC: No Commitment
FC: Full Commitment
F: Foreign
H: Home

Figure 4.1

ZERO PROFIT AND OPTIMAL INSTANTANEOUS TARIFFS

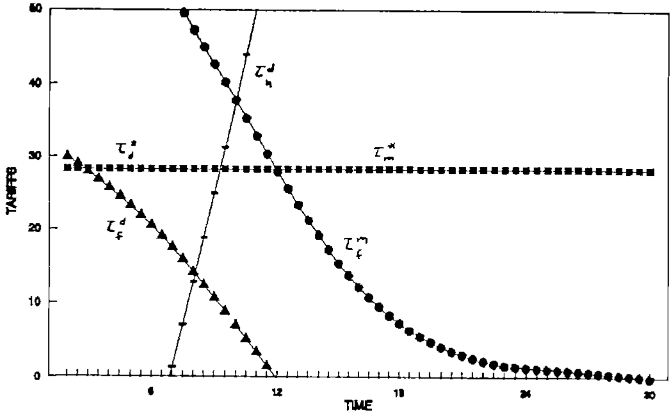
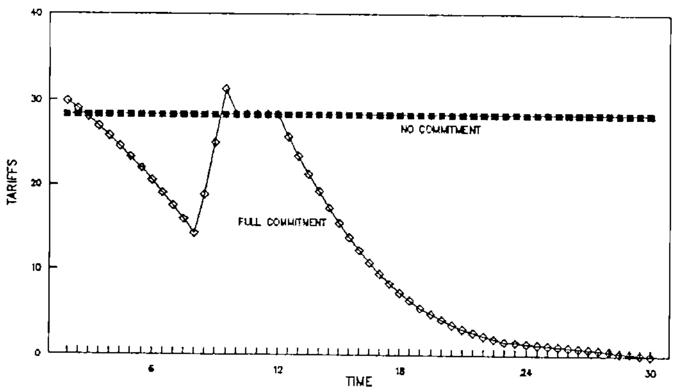


Figure 4.2

EQUILIBRIUM TARIFFS
UNDER NO COMMITMENT AND FULL COMMITMENT



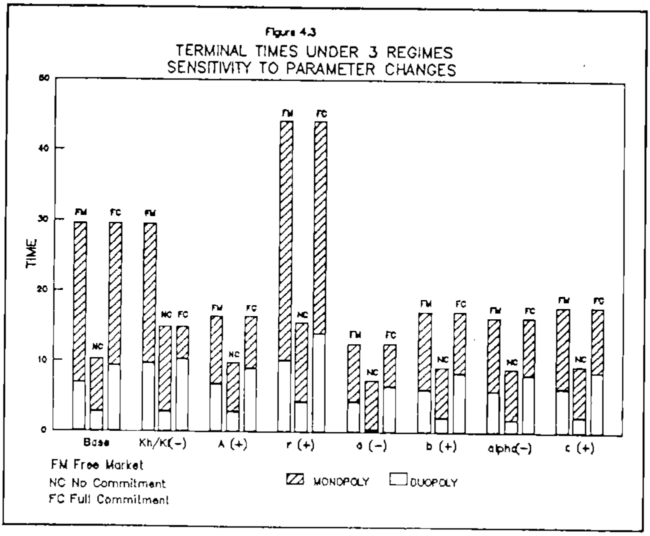


Figure 4.3
TERMINAL TIMES UNDER 3 REGIMES
SENSITIVITY TO PARAMETER CHANGES
TIME
0
10
20
30
40
50
FM
NC
FC
Base
Kh/K(-)
A (+)
r (+)
a (-)
b (+)
alpha(-)
c (+)
FM Free Market
NC No Commitment
FC Full Commitment
MONOPOLY
DUOPOLY

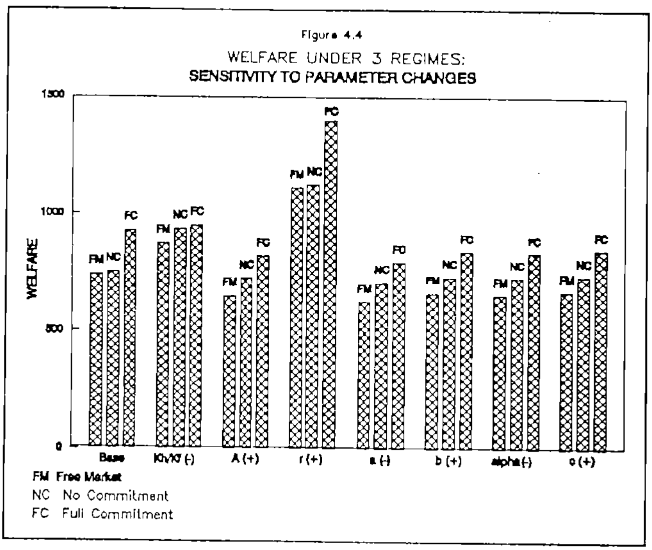


Figure 4.4
WELFARE UNDER 3 REGIMES:
SENSITIVITY TO PARAMETER CHANGES
WELFARE
0
500
1000
1500
FM
NC
FC
Base
Kh/K(-)
A (+)
r (+)
a (-)
b (+)
alpha(-)
c (+)
FM Free Market
NC No Commitment
FC Full Commitment

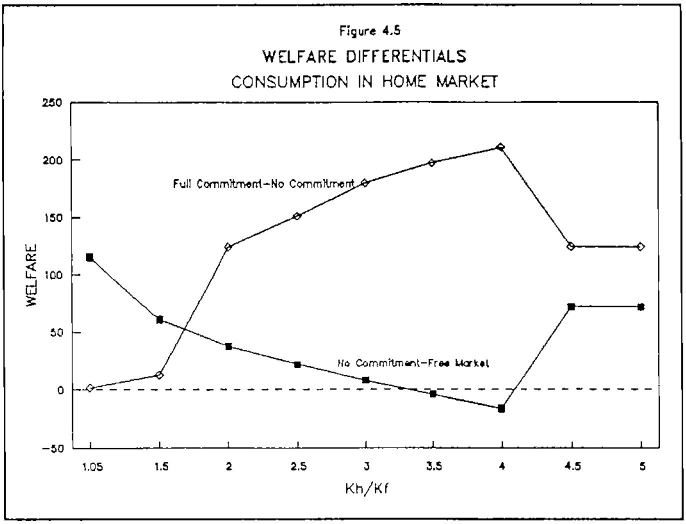


Figure 4.5
WELFARE DIFFERENTIALS
CONSUMPTION IN HOME MARKET
WELFARE
250
200
150
100
50
0
-50
Full Commitment-No Commitment
No Commitment-Free Market
1.05
1.5
2
2.5
3
3.5
4
4.5
5
Kh/Kf

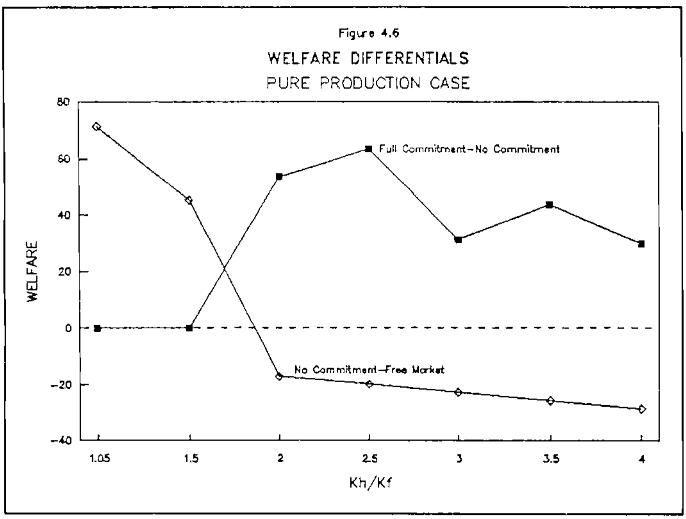


Figure 4.6
WELFARE DIFFERENTIALS
PURE PRODUCTION CASE
WELFARE
80
60
40
20
0
-20
-40
Full Commitment-No Commitment
No Commitment-Free Market
1.05
1.5
2
2.5
3
3.5
4
Kh/Kf

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