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# ASSET RETURNS WITH TRANSACTIONS COST AND UNINSURED INDIVIDUAL RISK: A STAGE III EXERCISE

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**ABSTRACT**

We attempt to construct models that are consistent with the following features of asset returns and turnover in the post-war US economy: (i) The risk free real interest rate is very low. (ii) There is a large spread between returns on liquid assets (government debt and liquid accounts at depository institutions and money market funds) and stocks. (iii) Transaction velocities are much higher for liquid assets than for stocks. Specifically, we explore the extent to which incorporating an explicit motive for holding liquid assets can explain the above observations. We introduce a demand for liquid assets by adding uninsured individual risk together with differential costs of trading securities. We then parameterize a class of such models and compute the stationary equilibria. The simulations indicate that attempting to match the return data generates a ratio of liquid assets to income considerably below observed levels. We then explore some possible reasons for this discrepancy.

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## I. Introduction.

The secular average annual real return on Treasury Bills is less than one per cent. For stocks, it is about seven percent. These two facts have stimulated a lengthy discussion in the literature, beginning with Mehra and Prescott (1985). The issue is that it is difficult to generate these kinds of numbers using the standard intertemporal model of asset pricing (Lucas 1978). Reasonably parameterized versions tend to predict too low a risk premium and too high a risk free rate. These results lead Mehra and Prescott to conclude that it is not "reasonable to abstract from liquidity constraints, transactions costs and the like and to use a frictionless Arrow-Debreu economy to explain these observations."

A number of papers have attempted to save the frictionless framework. The strategies have included using alternative functional forms for individual preferences (Nason 1988, Constantinides 1988, Epstein and Zin 1987, Weil 1989) and for the stochastic processes that drive dividend and consumption behavior (Reitz 1988, Labadie 1989, Cecchetti, Lam and Mark, 1989). While these approaches have met with some limited success, they have almost exclusively focused on only one part of the puzzle: why the equity premium is large. Largely ignored has been the other: why the risk-free rate is so low. It is unclear whether it is desirable to separate the two questions; indeed, Mehra and Prescott conclude that resolving the latter is central to resolving the former.

In this paper, we develop and numerically simulate a model aimed at providing a joint explanation of the equity premium and the risk-free rate.[1] We follow Mehra and Prescott's suggestion and step outside the frictionless Arrow-Debreu economy. Our model relies on both incomplete securities markets and transactions costs. Individuals face idiosyncratic shocks to personal income. Markets for claims on personal income do not exist, by assumption. The absence of a complete set of contingent claims markets implies that individuals must self-insure, i.e., buy and sell assets to smooth consumption. Two kinds of securities are available, stocks and short-term government bonds (T-bills). One important distinction between the two is that, by assumption, stocks are costly to trade while T-bills are freely exchanged. One can think of T-bills as either being directly held by households or as being costlessly repackaged by an intermediary which in turn issues freely tradable securities to its depositors. A key premise is that intermediaries cannot similarly repackage stocks. In any event, regardless of whether they are directly or indirectly held by households, T-bills in our model have an edge over stocks as a vehicle for self-insurance.

Having non-traded individual income risks permits the model to generate a low risk-free rate. The equilibrium risk-free rate can potentially lie well below the rate of time preference. (See, for example, Bewley (no date), Bewley and Radner 1980, or Clarida 1987). Introducing costs of

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[1] That is, we roll up our sleeves and grind out some numbers (a "stage three exercise"). We forgo studying existence, uniqueness and optimality (stage one), nor do we present qualititative results based on analytical methods (stage two).

trading stocks in conjunction with uninsured individual risks enlarges the equity premium. The need for self-insurance motivates trade in securities. Costs of trading thus become relevant to pricing a security in equilibrium. The ease of exchanging T-bills implies that stocks must pay an added premium - a transactions/liquidity premium - to be competitive with bonds.

The model is consistent with two other facts that are anomalies in the context of the standard asset pricing model. The first fact relates to trading volume. Empirically, households turn over liquid assets (assets like savings accounts and money market deposits) at a much more rapid rate than stocks. This kind of behavior emerges in our model. Roughly speaking, individuals try to smooth income fluctuations by trading in T-bills (or assets backed by T-bills) and only use stocks as a last resort. The second fact relates to consumption behavior. Aggregate consumption is smooth in our framework, but individual consumption is highly variable due to the incompleteness in securities markets. This pattern in consumption seems consistent with the evidence.

Mankiw (1986) also appeals to uninsured individual risks to explain the equity premium. Our analysis differs in some important ways from his. First, we attempt to explain the risk-free rate puzzle as well, whereas Mankiw studies a framework where the risk-free rate is exogenous. Second, as Mankiw observes, his results rely on a very particular pattern of individual risk. Specifically, the burden of bad aggregate shocks is randomly concentrated on a subset of the population. If instead there is an analogous concentration of good aggregate shocks the resulting premium goes the wrong way. Our results instead rely on costs to individuals of trading in stocks. Third, we present numerical simulations of a fully specified heterogeneous agent, dynamic equilibrium economy as a means to judge the empirical siginificance of the imperfections we have introduced.

Work by Constantinides (1986) is also relevant. He studied a partial equilibrium economy with two assets, a stock and a riskless security, and with proportionate costs of trading the stock. His main conclusion was that the transactions costs had only a second order effect on pricing the securities. In addition to being a general equilibrium analysis, our framework differs by incorporating uninsured income risks. The effect is to enlarge the trading volume which permits a potentially greater role for transaction costs.

The rest of this paper is organised as follows. In section II we present an informal discussion of the nature and magnitude of the costs of trading stocks. Section III describes the formal framework, a variant of the Lucas asset pricing model, where individuals face uninsured idiosyncratic risks, there are restrictions on borrowing and trading stocks is costly. There is also an intuitive discussion of how transactions costs may impact on return spreads and of how small trading costs could generate a large spread. Section IV describes the algorithm for computing the solution to our heterogeneous agent economy. Here we borrow insights from Imrohoroglu (1988,1989), and Diaz-Gimenez and Prescott (1989) who studied related kinds of models. Section V discusses the parameterization of the model. In addition to the transactions costs, the unusual feature of the model is the presence of individual risk. We use panel data studies of annual earnings and hours variation to provide some guidance.

Because computing a rational expectations equilibrium with both aggregate and individual risk does not appear to be a feasible undertaking (see the discussion in section III) we restrict attention to the case of no aggregate dividend risk[2]. As a consequence of this simplification, any difference in the spread between stocks and bonds is attributable only to the frictions we have introduced, namely trading costs in conjunction with unisured individual risk. Thus, instead of trying to reproduce the observed spread, our strategy is to determine whether the model can generate a "transactions/liquidity" premium which is a significant fraction of the actual equity premium.

Thus, in section VI we present results from simulations which explore the extent to which the model is capable of explaining (i) the observed low level of the riskless rate; (ii) a transactions/liquidity premium in the range of three percent - about half the equity premium; and (iii) the relative pattern of transactions velocities. A number of examples are studied, including ones which allow for costly borrowing. An important finding, however, is that the model predicts too low a ratio of liquid

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[2] See Kahn (1989) for an analysis of an overlapping generations economy with aggregate dividend risk and idiosyncratic individual risk. The overlapping generations framework permits some important simplifications for calculating an equilibrium. In Kahn's framework, which does not incorporate transactions costs, the equity premium is not large, suggesting that mixing individual risk only with aggregate dividend risk is not enough. Fisher (1990) does study the role of transactions costs and finds a significant effect; though he does not explicitly incorporate heterogenity and trade.

assets to income. The simulated values are between twenty and thirty percent of a rough benchmark number. At the same time, the simulated values of the stock to income ratios match the data reasonably well. We conclude that in the context of our model the equity premium puzzle can be restated as a puzzle as to why households have tended to hold such a large fraction of marketable wealth in the form of low yielding liquid assets. In section VII we offer some suggestions as to how possible extensions of our analysis might get at this issue, in addition to some other final remarks.

## II. Transactions Costs for Trading Stocks.

Statistics on trading volume are consistent with the notion that transactions costs matter. Stocks turn over much less frequently on average than, for example, do money market accounts. For stocks, the ratio of shares sold over a year to the average number of shares listed for the year is about .5. Further, a substantial fraction of the volume is accounted for by institutional traders which own about 50% of outstanding shares. Turnover by households, who own the other half, is virtually negligible. As a comparison, the equivalent turnover statistic for savings accounts is about 3 and for bank money market funds is about 7, indicating a substantially higher transactions velocity.

In practice there are three basic kinds of (pre-tax) costs involved in trading stocks: (i) brokerage commission costs; (ii) buy-sell spreads; (iii) time involved in acquiring knowledge and record keeping. At a deeper level, the existence of these costs reflects the informational frictions involved in trading heterogeneous assets like stocks. In addition, tax considerations are also likely to be a factor since capital gains levies are based on realization rather than accrual.

Brokerage costs have been declining due to deregulation, but are still consequential, particularly for small and medium size transactions. Commission rates for retail brokers are inversely related to the quantity of shares transacted. A schedule is provided in Figure 1 (taken from Sharpe, 1985, p.40). For shares priced at $40 (the average share price on the NYSE varied between $33 and $39 over the past six years) commission rates decline monotonically from 8 % to 2 % as the size of the trade rises from $1 to $4000. It then remains at about 2 % for trades up to $200,000. (There is typically also a minimum cost of $30 - $50.) Discount brokers charge thirty to seventy percent less but do not provide counseling or record keeping services. It does not appear that discount brokers are dislodging retail brokers.

Mutual funds provide an alternative to directly managing a portfolio, but still involve trading costs. There are two basic kinds of funds, load and no-load. Load funds, which are by far the most prevalent, charge an up front commission (typically) of five to eight per cent. The rate tends to vary positively with the riskiness of the portfolio. While the up front charge is steep, there is usually no extra charge for liquidation. No load funds do not charge an initial fee, but typically place restrictions on the speed at which the account can be liquidated[3]. One form this restriction may take, for example, is a steep charge (up to 8 %) for early withdrawal.

The bottom line is that whether individuals hold stocks directly or via mutual funds, they can lose considerably by frequently moving in and out of the market. Conventional wisdom dictates not to "churn".

Bid-ask spreads add to the cost of trading. For actively traded stocks of large companies, which constitute about 50 % of the market, the ratio of the spread to the price averages 0.52 % . This ratio rises as company size declines. It averages around one percent for the rest of the market, reaching as high as 6.55 % for a typical firm with assets under ten million dollars.

Finally, actively trading stocks requires time and expertise. Not much thought is required for exchanging safe, homogeneous securities like money-market deposits. Knowing which stocks to trade is a much more complex decision. Also, record keeping requirements are considerable. Survey data indicates that only about 25 % of households own stocks. (See Mankiw and Zeldes, 1989). This is consistent with the notion that managing a stock portfolio is neither costless nor effortless.

One last consideration is the frequency of the need to exchange the security. That is, even if the costs of a single transaction are small, the need to trade often can make the costs over a given time period large. This

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[3] There are however some no load funds which appear to have minimal costs or restrictions on trading. It is puzzling that these kinds of funds aren't more popular, and more generally, that the size of assets in no load funds is so small as compared to load funds.

consideration then will have a bearing on what kinds of securities to hold at the margin.

## III. The Basic Model

We consider a stationary, infinite horizon, pure exchange economy with no aggregate uncertainty. Time is discrete and is denoted by t which takes values 0,1,2,.... One kind of good exists, a nonstorable consumption good. There is a continuum of people of measure unity. Each person i has preferences over consumption given by

$$E_0\{\sum_{t=0}^{\infty}\beta^t U(c_t^i)\}, \quad 0 < \beta < 1 \tag{3.1}$$

where $c_t^i$ is consumption by i in period t, $\beta$ is the subjective discount factor and $E_0\{\cdot\}$ is the mathematical expectation conditioned on information at time zero.

Each period supplies of the perishable consumption good arrive from two kinds of sources. The first source is "capital". There exist $\bar{s}$ capital machines which costlessly produce output each period. The proceeds are distributed as dividends to shareholders who own the machines. There are $\bar{s}$ equity claims which are tradable and perfectly divisible. One claim entitles the owner to $1/\bar{s}$ percent of the total output from all the machines each period. We assume that the output per machine, d, is constant over time. The second kind of income is "labor". Each period, individual i receives an endowment of the consumption good, $y_t^i$, which obeys a stationary Markov chain. Further, fluctuations in labor income are independent across individuals. Thus per capita labor income is smooth, while individual labor income is highly variable. Moreover, while a market exists for claims on capital income, the same is not true for claims on labor income. Thus the variation in $y_t^i$ reflects uninsured individual risk. Later we demonstrate that the model can be easily reformulated so that this variation incorporates taste shocks as well as idiosyncratic income fluctuations.

There is a government sector which consumes g units per capita each period. It finances this activity with a per capita lump sum tax, $\tau$, and by issuing T-bills. The government budget constraint is given by

$$g + \bar{b}_t = \tau + \bar{b}_{t+1}/(1+r_t) \tag{3.2}$$

where $\bar{b}_t$ is the per capita quantity of T-bills at the beginning of period t in terms of market value and $r_t$ is the riskless interest rate from t to t+1.

Each period an individual decides how much to consume and the amounts of stock and T-bills to acquire. We assume that there are costs of trading stock that are proportionate to the value of the trade[4]. Let $\alpha_b$ be the per unit of value buying cost and $\alpha_s$ the per unit of value selling cost. An individual i's momentary budget constraint is then given by

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[4] In view of the discussion in section II, proportionate trading costs are a plausible approximation. It is not difficult to allow for fixed costs or decreasing marginal costs as depicted in figure 1. In fact, we consider the implications of fixed costs in section VI.

$$(3.3)\quad c_t^i + p_t(s_{t+1}^i - s_t^i) + b_{t+1}^i/(1+r_t) = y_t^i + s_t^i d + b_t^i - \tau$$
$$-\max\{\alpha_b p_t(s_{t+1}^i - s_t^i), \alpha_s p_t(s_t^i - s_{t+1}^i)\}$$

where $p_t$ is the period t price of equity.

Short sales of stock and borrowing are disallowed (later we relax the constraint on borrowing.) The following restrictions thus apply:

$$(3.4a)\qquad s_t^i \geq 0$$

$$(3.4b)\qquad b_t^i \geq 0$$

We restrict attention to steady states. Let F(s,b,y) be the joint cross-section distribution of stock holdings at the beginning of t, bond holdings at the beginning of t and labor income realization at t. That is,

(3.5) F(s,b,y) = fraction of people at the beginning of t for whom:

$$(s_t, b_t, y_t) \leq (s, b, y).$$

The Markov process describing the evolution of individual labor incomes is given by the following.

$$(3.6)\qquad Y(y', y) = \text{prob}[y_{t+1} \leq y' \mid y_t = y].$$

A steady state consists of a constant over time stock price p, a constant interest rate on bonds r, a constant per capita quantity of bonds

$\bar{b}$, and a cumulative distribution function F(s,b,y) which are consistent with individual optimization, the government budget constraint (3.2), and market clearing at each date.

A typical individual's dynamic optimization problem can be described in terms of the usual Bellman's equation of dynamic programming. The individual state vector is denoted $z_t^i$ and consists of $(s_t^i, b_t^i, y_t^i)$. We will use variables without primes to denote date t values and variables with primes to denote date t+1 values. Let $V(z^i)$ be the optimal value function for an individual. This must satisfy the Bellman equation,

$$V(z^i) = \max[U(c^i) + \beta E\{V((z^i)')|z^i\} \tag{3.7}$$

subject to (3.3) and (3.6).

The solution consists of decision rules for $(s^i)'$ and $(b^i)'$,

$$(s^i)' = s(z^i) \tag{3.8a}$$

$$(b^i)' = b(z^i). \tag{3.8b}$$

The above decision rules can be aggregated using F(.) to obtain the aggregate demand for stocks and bonds at the beginning of t+1. The aggregate supply of stocks is $\bar{s}$ and the aggregate supply of bonds is found from (3.2). The first requirement for a steady state is that the market for stocks and bonds clear. The second requirement is that the c.d.f. F(.) be consistent with individual optimization and market clearing. That is, when we use (3.8) and (3.6) together with F(.) to compute the distribution of (s′,b′,y′), the new distribution should coincide with F(.). This completes the description of the steady state.

We have abstracted from aggregate uncertainty because the general computational problem is quite formidable if, for example, dividends are stochastic. Asset prices will depend on the dividend shock as well as the beginning of period distribution of asset holdings. The distribution of asset holdings itself will be changing stochastically over time in response to dividend shocks. For the same reason, we have also assumed that government expenditures $g_t$ and per capita bonds $\bar{b}_t$ are constant over time, and that taxes $\tau^i_t$ are the same on all agents and constant over time. This enables us to look for a stationary equilibrium in which the interest rate $r_t$, the stock price $p_t$ and the cross section distribution of asset holdings and income $F_t(.)$ are all constant over time. The government budget constraint (3.2) simplifies to the following.

(3.9) $$g + r\bar{b}/(1+r) = \tau.$$

Another advantage of fixing dividends is that we can isolate the impact of the frictions we have introduced. Since there is no dividend risk, any spread between the returns on stocks and bonds is due only to the transaction costs operating in conjunction with the uninsured individual income risk.

In addition, we assume that the Markov process for income given by (3.6) is a finite Markov chain. We also assume that an agent can buy or sell each asset in discrete units only, where the unit is a small fraction of average income and that there is an upper bound to the quantity of stocks and bonds that can be held. These assumptions make the space of state vectors (s,b,y) for the agent's dynamic programming problem into a finite space. The agent's decision rules (3.8) together with (3.6) define a Markov chain on the finite space of vectors (s,b,y). The stationary cross section distribution F(.) can be obtained as the stationary probability distribution corresponding to the above Markov chain.

**Some Intuition on Rate of Return Spreads**

Here we provide some intuition for the role that transactions costs play in generating a spread between the returns to equity and government bonds. We begin by considering an individual's decision whether to buy or sell stocks. There will be two levels of income denoted $y_b(s,b)$ and $y_s(s,b)$, with $0 < y_s(a,b) < y_b(a,b)$ such that whenever income is below $y_s$, the individual sells stocks; when it is between $y_s$ and $y_b$, he holds; and when it is above $y_b$, he buys. Notice that in general, these regions will depend on the individual's initial holdings of stocks and bonds, s and b respectively.

Arbitrage requires that any individual buying both stocks and bonds at time t must be indifferent between acquiring either kind of asset at the margin. Therefore, for each person i in this position at t, the following Euler conditions must hold (where the i superscripts for agents are dropped for convenience):

(3.10a) $$U_c(c) = \beta(1+r)E\{U_c(c')\}$$

$$(3.10b)\qquad p(1+\alpha_b)U_c(c) = \beta\{\pi^b[d+(1+\alpha_b)p]E_b\{U_c(c')\} + \pi^s[d+(1-\alpha_s)p]E_s\{U_c(c')\} + \pi^h[d+p(1+\alpha_b)]E_h\{U_c(c')\}\}$$

where $\pi^b$, $\pi^s$ and $\pi^h$ are the probabilities the individual will be buying, selling or holding the stock next period; $E_b$, $E_s$ and $E_h$ are the expectations conditional on buying, selling or holding stocks next period[5].

We can now rewrite equation (3.10b) as follows.

$$(3.11)\qquad p(1+\alpha_b)U_c(c) = \beta[(d+p)E\{U_c(c')\}+\pi^b\alpha_b pE_b\{U_c(c')\} -\pi^s\alpha_s pE_s\{U_c(c')\}+\pi^h\alpha_b pE_h\{U_c(c')\}]$$

We can now combine equations (3.10a) and (3.11) to eliminate $U_c(c)$ and rewrite the resulting equation as follows.

$$(3.12)\qquad [(d+p)E\{U_c(c')\}+\pi^b\alpha_b pE_b\{U_c(c')\}-\pi^s\alpha_s pE_s\{U_c(c')\}+ \pi^h\alpha_b pE_h\{U_c(c')\}]/p(1+\alpha_b) = (1+r)E\{U_c(c')\}$$

The left side of equation (3.12) is the expected gain from buying a stock at the margin and the right side is the expected gain from buying a bond. The transactions costs affect the return to stocks in two basic ways. First, the individual must pay the up front proportionate transactions cost $\alpha^b$ when acquiring a stock; this implies the expected gain must be deflated

[5] Note that the Euler conditions for agents who are selling stocks or who are borrowing constrained and/or short sale constrained will be different from (3.18).

by the term $(1+\alpha_b)$. Second, the net payoff in the subsequent period depends on whether and how the individual will be adjusting his stockholdings at that time. In particular, the shadow value of a stock in the subsequent period is $(1+\alpha_b)p$ if he purchasing or holding stocks then; and it is $(1-\alpha_s)p$ conditional on selling. Intuitively, if there is a high likelihood that the individual will be buying stocks in the future, then an added advantage of buying today is the savings in avoiding transactions costs tomorrow. Conversely, if it is likely the individual will be selling in the future, a disadvantage of buying today is the high probability of having to incur selling costs tomorrow. The second, third, and fourth terms on the left side of equation (3.12) reflect how the probabilities of buying, holding or selling in the future, in conjunction with the transactions costs, influence the expected marginal gain from stocks.

Equation (3.12) can be further simplified to

$$(3.13) \qquad d/p - r = (1+r)\alpha_b - [\pi^b\alpha_b E_b\{U_c(c')\} - \pi^s\alpha_s E_s\{U_c(c')\} + \pi^h\alpha_b E_h\{U_c(c')\}]/E\{U_c(c')\}$$

Quite clearly, the transactions costs are responsible for the spread between the returns to stocks and bonds. The spread is increasing in $\alpha_b$, $\alpha_s$ and $\pi^s$, while it is decreasing in $1 - \pi^s$. Further, it is likely to be larger, the more risk averse the individual; this is because sales of stocks are likely when consumption is low, which makes the utility measure of the transactions costs of selling (relatively) high. Finally, the spread is likely to be higher the finer the trading interval. Note that the transaction cost parameters are invariant to the time interval, whereas the returns d/p and r shrink as the trading period gets shorter. Thus, the transactions costs become relatively more important as the frequency of trading increases.

As one example of applying (3.13) suppose that the individual is close to being risk neutral. In this case the spread between the returns on the stock and the bond is approximately equal to $\pi^s(\alpha_s+\alpha_b)$, the probability of selling times the roundtrip transaction cost. As the discussion of the magnitude of transaction costs in section II shows this spread in returns is not insignificant. If the period is a quarter, the roundtrip transaction cost is 7 percent and $\pi^s$ is 10 percent, then the spread is 0.7 percent per quarter or 2.8 percent per year.[6] Further, this spread does not take into account any aggregate riskiness in the dividends on the stock or risk aversion on the part of agents. It is possible that when these elements are added in we will have a fairly complete explanation for the equity premium.

In summary, the existence of trading costs for stocks in conjunction with the need to trade securities to smooth consumption can introduce a spread between stocks and bonds. Further, the incompleteness of markets for insurance implies a "low" riskless rate of interest in equilibrium. We verify these conjectures in section VI where we present some measures of the

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[6] Note that $\pi^s$ is the probability of next period selling the marginal unit of stock purchased this period, as opposed to all the stock purchased this period. Note also that $\pi^s_i$ will vary across individuals as a function of the individual state vector $z^i = (b^i, s^i, y^i)$.

kinds of magnitudes involved. Before presenting some results we need to discuss how the stationary equilibrium is computed and the model is parameterized. We do this in the next two sections.

### IV. Computation of Stationary Equilibrium

Generally speaking, the computational procedure involves first specifying values for asset returns and taxes, and then finding values for asset stocks and government purchases which support these returns in equilibrium. How successful the model is in explaining a particular configuration of asset returns then depends on how well the computed asset/income ratios and relative transaction velocities match with observed data.

The exact algorithm is as follows: We first specify

(i) the utility discount factor $\beta$ and the period utility function U(.),

(ii) the finite Markov chain for income with Y states and with the stationary transition probabilities

$$\pi^{y}(i,j) = \text{prob}[y_{t+1}=y_j \text{ given } y_t=y_i],\ i,j=1,2,\dots,Y \tag{4.1}$$

(iii) (a) returns on bonds r and on stocks $r_s = d/p$. (We always normalize p to unity.)

(b) taxes $\bar{\tau}$,

(iv) percentage transaction costs $\alpha_b$ and $\alpha_s$, which need not be equal.

(v) (a) finite grid for stocks with S values $\{s_i, i=1,2,..,S\}$ such that $\Delta s = (s_{i+1}-s_i)$ is a fraction $\gamma$ of average income. The largest value of stocks in the grid is adjusted to ensure that it is not a binding constraint on the equilibrium. That is, the stationary probability distribution is such that close to zero percent of the population is holding the largest quantity of stocks.

(b) finite grid for bonds with B values $\{b_i, i=1,2,..,B\}$ specified analogously to the grid for stocks.

The specified values for the asset returns and taxes are used in the individual's budget constraint (3.3). A person starts with some portfolio of stocks and bonds (in the grid), realizes some income and chooses an end of period portfolio of assets. This part of the problem is solved by discrete dynamic programming and leads to the portfolio decision rules denoted as follows.

(4.2) $$(s', b') = (\sigma_s, \sigma_b)(s, b, y) = \sigma(s, b, y)$$

The above decision rules can be used together with the transition probabilities for income $\pi^y$ to compute the transition probabilities for x = (s,b,y). Note that x can take on S×B×Y values. If we let $\pi^x$ denote the transition probabilities for x then the stationary probabilities $\theta^x$ can be computed as follows.

(4.3) $$\theta^x(j) = \sum_i \theta^x(i)\pi^x(i,j)$$

Using the stationary probabilities $\theta^x$ we can compute the marginal probability distributions for stocks and bonds and thereby the implied per capita values of stocks and bonds $\bar{s}$ and $\bar{b}$. The per capita value of bonds is then used in the government budget constraint (3.9) to compute the implied value of g. As a check on the solution we also ascertain whether the marginal distributions for stocks and bonds tail off to zero at the upper ends of the respective grids. If so the grids are specified correctly in the sense that increasing the range at the upper ends will not affect the equilibrium values. We also check to ensure that government expenditures are non-negative.

We next use the stationary probabilities $\theta^x$ and the decision rules $\sigma(.)$ to compute the transactions velocities. Let TVS and TVB be the respective transactions velocities for stocks and bonds. These are computed as follows.

(4.4a) $$TVS = (1/2)\sum_x \theta^x |\sigma_s - s| / \bar{s}$$

(4.4b) $$TVB = (1/2)\sum_x \theta^x |\sigma_b - b| / \bar{b}$$

**V. Model Parameterization**

The values of some of the parameters depend on the period length. In what follows we report all parameter values as if the period is one year but in fact the values are adjusted in the appropriate fashion to reflect the period length chosen. We use a period length of one quarter. This seems to be a short enough time to allow for liquidity trading but long enough to permit some temporal aggregation in preferences. In the future, though, we hope to experiment with a shorter period length.

It may be worthwhile to indicate here how vastly the computational burden increases from shortening the period since this was a consideration in choosing the period length. The basic problem is that if we fix the units in which individuals can buy or sell assets as a percentage of their per period income then the number of grid points we must allow for assets rises sharply as the period shrinks. This occurs simply because asset/income ratios are greater the shorter the period length. As an example, we found that for a period length of one quarter it was desirable to have an asset grid of 1600 points (80 values for stocks and 20 for bonds), given that we allowed individuals to trade assets in amounts equal to ten percent of their (quarterly) income. Further, each person's dynamic programming problem involves three state variables (initial values of stocks and bonds plus current income) and two decision variables (end of period stocks and bonds). Since we posit a three state Markov chain for income, each value function iteration involves 4800 maximizations with each maximization being over 1600 grid points. Finally, computing the stationary probabilities $\theta^x$ from (4.3) involves a transition probability matrix of size 4800x4800[7].

If we shorten the period (say, to a month) or reduce the size of each transaction (say, to five percent of income) the computational burden increases enormously. The ratios of stock market value to period income and value of government bonds to period income go up resulting in many more grid points. In addition, shortening the period means a higher discount factor which will slow the convergence of value function iterations. These computational problems were sufficiently severe so that we started with a

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[7] Each run with a quarterly period uses about 30 minutes of C.P.U. time on the mainframe computer at the Minneapolis Fed.

period of one quarter.

We chose parameter values in the following fashion.

**Preferences:**

(5.1) $\beta = .96$ (annual)

(5.2a) $U(c,a) = (c^{1-a}-1)/(1-a),\ a\geq 0,\ a\neq 1$

$= \ln(c),\ a=1$

(5.2b) $a = 2$

**Income Process:**

We assume a three state Markov chain where the states are denoted u,l,h (and ordered the same way) and stand for unemployment, low employment and high employment, respectively. The low and high employment states are treated symmetrically so that the probability transition matrix is of the form:

(5.3)
$$\pi^y = \begin{array}{c} \\ (u) \\ (l) \\ (h) \end{array} \begin{array}{c} \begin{array}{ccc} (u) & (l) & (h) \end{array} \\ \begin{bmatrix} \pi_u & (1-\pi_u)/2 & (1-\pi_u)/2 \\ 1-\pi_e & \pi_e/2 & \pi_e/2 \\ 1-\pi_e & \pi_e/2 & \pi_e/2 \end{bmatrix} \end{array}$$

The numbers $\pi_u$ and $\pi_e$ are determined from the following considerations. Let $\theta_u$ be the fraction of people in the unemployment state in a stationary equilibrium and let $D_u$ be the duration of unemployment. It is easy to calculate that these are given by,

(5.4a) $\theta_u = (1-\pi_e)/[(1-\pi_e)+(1-\pi_u)]$

(5.4b) $D_u = 1/(1-\pi_u)$

We assume the following values for $\theta_u$ and $D_u$, chosen to roughly match the actual numbers, and use these in (5.4) to solve for the $\pi$'s.

(5.5a) $\theta_u = 0.05$

(5.5b) $D_u = 1.5$ quarters

These restrictions imply the following income probability transition matrix:

$$\pi^y = \begin{matrix} 0.34 & 0.33 & 0.33 \\ 0.035 & 0.4825 & 0.4825 \\ 0.035 & 0.4825 & 0.4825 \end{matrix} \qquad (5.6)$$

The (quarterly) income levels corresponding to the three states are chosen as follows. Let $\theta_e$ be the fraction of people in employment state l, also equal to the fraction of people in employment state h. (Thus, $\theta_e = (1-\theta_u)/2)$ . Let $\bar{y}$ be the average income and $y_e$ be the average income conditional on being employed. We normalize $\bar{y}$ to unity. We assume that income while employed can fluctuate up or down (relative to $y_e$) by 30 percent. In addition we assume that income in the unemployed state is 30 percent of average income while employed. Thus, we have

(5.7a) $\bar{y} = \theta_u y_u + \theta_e(y_l+y_h) = 1$

(5.7b) $y_u = 0.3y_e$

(5.7c) $y_l = 0.7y_e$

(5.7d) $y_h = 1.3y_e$

The above equations can be solved to obtain incomes in each state. The solutions follow.

(5.8) $y_u = 0.3100,\ y_l = 0.7254,\ y_h = 1.3470$

Our choice for the representation of the income process are based on the following considerations. We follow Diaz-Gimenez and Prescott (1989) by assuming that income while unemployed is equal to one third of mean income while employed, based on the fact that the ratio of the average manufacturing wage to the minimum wage is about three to one. (The argument presumes that the unemployed always have the option of working at minimum wage jobs). In addition, we have chosen to divide the employment state into two employment states to allow for variation in income while employed.

Our income process implies a standard deviation of earnings relative to trend of slightly more than 30 % for quarterly income and slightly more than 15 % for annual income. The latter falls within the ballbark of estimates for the variation of annual earnings. (We have been unable to locate measures of the variation in quarterly earnings). Kydland (1984) calculates that the standard deviation of annual hours worked for employed prime age males from the Panel Study on Income Dynamics (PSID) is about 15 percent. Since wages are mildly procyclical variations in income while employed would be at least that much. Abowd and Card (1987) using data from the PSID as well as the National Longitudinal Survey (NLS) of Men 45-49, report that the standard deviations of percent changes in real earnings and annual hours are about 40 percent and 35 percent, respectively. If deviations of real earnings from trend are serially uncorrelated, then the above figures suggest that the standard deviation of real earnings relative to trend for employed prime age males is about 28 percent. However, deviations of real earnings from trend are likely to be positively serially correlated which would result in an even larger figure for the standard deviation of real earnings relative to trend. If the serial correlation coefficient exceeds one half then real earnings relative to trend will be even more variable than real earnings growth[8].

We, therefore, feel that our income process matches up to conservative estimates of the variation in annual earnings. Unfortunately we could not find analagous numbers to match up the quarterly variation. We chose to make the quarterly percentage variation about twice the annual percentage variation by postulating that quarterly fluctuations of income about trend while employed are i.i.d. This assumption may be a reasonable way to

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[8] Let $W_t$ be real earnings, $\sigma(w)$ be the standard deviation of real earnings relative to trend and $\sigma(g_w)$ be the standard deviation of real earnings growth. Suppose $W_t$ can be represented as: $(\text{Trend})_t(1+\varepsilon_t)$ where $\varepsilon_t = \rho\varepsilon_{t-1}+u_t$, and $u_t$ is i.i.d. with mean zero and standard deviation $\sigma_u$. Then it is easy to calculate that, $\sigma(w) = \sigma_u/\sqrt{(1-\rho^2)}$ and $\sigma(g_w) = \sigma_u\sqrt{2/(1+\rho)}$. If $\rho$ exceeds 1/2 then $\sigma(w)$ will exceed $\sigma(g_w)$.

approximate the quarterly idiosyncratic risk faced by individuals, since this risk includes factors in addition to income variation from which we have abstracted These other factors consist primarily of taste shocks and uninsured components of accidents. It is worth noting that we can easily modify our model to incorporate taste shocks. Under this reformulation, the idiosyncratic risk is interpretable as arising from income as well as preference shocks. For example, the utility function in our model can be respecified as $U(c^*+\varepsilon)$ where $\varepsilon$ represents taste shocks and $c^*$ is consumption. As can be seen from the budget constraint (3.3) this formulation is equivalent to one where consumption is taken to be $c = c^*+\varepsilon$ and income is taken to be $y+\varepsilon$. Thus, fluctuations in effective income are partly due to taste shocks and hence larger.

**Transactions costs**

Based on the discussion in section II we experiment with several different values for the transactions costs parameters. Initially we set the buying and selling costs the same though later we plan to experiment with differing values for these two parameters. We choose,

(5.7) $$\alpha_b = \alpha_s \in \{0.02, 0.035, 0.05\}$$

**Asset Returns and Asset/Income Ratios**

We pick the following values for asset returns and taxes.

(5.8a) $$r = 0$$

(5.8b) $$r_s = d/p = 0.03 \text{ (annual)}$$

(5.8c) $\tau = 0$

Following Labadie (1989, p.289) we calculate the average annual real return on 90 day government Treasury bills from 1949 to 1978 to be about zero. Her figure for the average annual real return on S&P 500 over the same period is 7.7 percent (standard deviation = 7.03 percent). Since we certainly do not wish to claim that transactions costs are the sole explanation for the observed return differential we set ourselves the more modest goal of explaining a 3 percent return differential[9]. This explains our choice of $r_s$ in (5.8b). Finally, we set taxes at zero. This allows for some simplification, since at r = 0, the implied value of g is also zero.

Finally, as suggested earlier, an important consideration for judging the model is ascertaining how well the computed asset/income ratios match up with observed values. We use the following numbers as benchmarks for the latter:

(5.9a) $\bar{s}/y = .65$

(5.9b) $\bar{b}/y = .35$

The first number is the average of the ratio of household ownership of

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[9] Labadie (1989) argues that by using a continuous state space generalization of the Mehra and Prescott (1985) model it is possible to obtain an equity premium close to three percent. A simple monetary version of the same model can produce a premium of close to six percent. However, the risk free real rate that she obtains is over three and a half percent which seems too high. While we do not mean to imply that one can simply add what we get to her premium, the results are suggestive that factoring in transactions costs can close the gap.

tradable equity (including both direct holdings and mutual funds) to national income over the period 1964-80. It is probably worth adding that this number varied considerably over the time period, ranging from about .4 to 1.1. Also the ratio tended to decline steadily over the period. The second number is a rough measure of the ratio of household liquid assets to national income over the same period. We included in the numerator household holdings of liquid securities which bore approximately the same return as T-bills: specifically, the sum of savings accounts at depository institutions, time deposits with a maturity of a year or less, money market accounts and direct holdings of marketable government securities. (Recall that a working hypothesis of our model is that T-bills which households do not hold are held by intermediaries which in turn issue liquid claims to households.) The liquid asset/income ratio did vary over the period, but not nearly as much as the stock/income ratio.

## VI. Results

In this section we describe the results of computations based on our model. These are organised in the form of examples. The stock price is always normalized to unity and the buying and selling costs are taken to be the same and denoted by $\alpha$. As discussed earlier, we consider a period length of one quarter. However, the numbers we report are converted to annual values (when relevant).

**Example 1**

$\beta = 0.96$, $a = 2$, $r = 0$, $r_s = 0.03$, $\tau = 0$.

$\gamma$ (step size) = 0.1×average income

| | | | | |
|---|---|---|---|---|
| | $\alpha$ = | 0.02 | 0.035 | 0.05 |
| solutions: | $\bar{s}/y$ = | 0.69 | 0.65 | 0.60 |
| | $\bar{b}/y$ = | 0.07 | 0.10 | 0.12 |
| | TVS = | 0.08 | 0.07 | 0.06 |
| | TVB = | 1.44 | 1.15 | 0.99 |

Note that since r and $\tau$ have been set to zero the computed value of g is also zero. The marginal probability distributions for stocks and bonds are given in Table 1 at the end. These show that the probabilities do tail off to zero at the upper ends of the respective supports.

Several features of the example are worth noting. First, while the ratio of stocks to income matches up reasonably well, the ratio of liquid assets to income is way too low. It appears to be off by a factor of between three and five, depending on the transactions cost. The relative transactions velocities, however, seem reasonable. This particular example leads to liquid assets circulating about sixteen times more rapidly than stocks. It is true that the absolute transaction velocities are too low. However, this is probably in large part due to the fact that the period length is so long. Clearly, if a year is divided into N periods, then the transactions velocity can never exceed N/year. Also we have abstracted from other reasons for trading securities besides liquidity trading.

In this example we considered a three percent spread between stocks and bonds. The next example considers the sensitivity of the results to small adjustments in the spread: first, down to 2.6%, then up to 3.4%.

**Example 2**

Except for the return on stocks, the rest of the parameters are the same as in example 1. The results are as follows:

$r_s = 2.6$

| | | | | |
|---|---|---|---|---|
| | $\alpha$ = | .02 | 0.035 | 0.05 |
| solutions: | $\bar{s}/y$ = | 0.60 | 0.54 | 0.49 |
| | $\bar{b}/y$ = | 0.07 | 0.11 | 0.14 |
| | TVS = | 0.09 | 0.07 | 0.06 |
| | TVB = | 1.40 | 1.08 | 0.91 |

$r_s = 3.4$

| | | | | |
|---|---|---|---|---|
| | $\alpha$ = | 0.02 | 0.035 | 0.05 |
| solutions: | $\bar{s}/y$ = | 0.83 | 0.79 | 0.75 |
| | $\bar{b}/y$ = | 0.04 | 0.08 | 0.11 |
| | TVS = | 0.10 | 0.07 | 0.05 |
| | TVB = | 2.00 | 1.37 | 1.08 |

The marginal probability distributions for stocks and bonds for this example show that the probabilities do tail off to zero at the upper ends of the respective supports, though we do not report them due to the similarity with example 1. Even at the 2.6% spread the average quantity of liquid assets is too low.

In the next example we allow for costly borrowing.

**Example 3 (Costly Borrowing)**

We assume that the parameters are the same as in example 1, except that we also allow for negative values in the grid for bonds so that individuals are permitted to borrow. However, there is a transaction cost associated with borrowing (but not with lending) which is a fixed percentage of the amount borrowed. This percentage borrowing cost is denoted by $\delta$ and is chosen to be 0.02. This number implies an annual spread between the consumer loan rate and the risk free rate of 8%, which is reasonable given historical data on conumer loan rates (the historical difference between the credit card rate and the risk free rate is larger than 8%).[10] In addition we impose a credit limit on consumer loans equal to 40% of quarterly income, so that the lower support of the grid on bonds now extends to -.4. The results follow.

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[10] The costs of borrowing are somewhat lower for individuals who own large amounts of stock and can pledge the stock as collateral. For example, the spread between the loan rate and the risk free rate is about five and a half percent for a collateralized loan under ten thousand dollars and declines to about two and a half percent for a collateralized loan over one hundred thousand dollars. There are also minimum income and margin requirements which add to the effective costs. Thus, except on very large loans, it seems that even weathy stockholders face a nontrivial gap between borrowing and lending rates.

$r_s = .03$

| | | | |
|---|---|---|---|
| $\alpha$ = | 0.02 | 0.035 | 0.05 |
| $\bar{s}/y$ = | 0.61 | 0.61 | 0.56 |
| $\bar{b}/y$ = | 0.05 | 0.04 | 0.07 |
| TVS = | 0.08 | 0.06 | 0.05 |
| TVB = | * | * | * |
| LA/y = | 0.06 | 0.06 | 0.09 |
| TVLA = | 1.48 | 1.32 | 1.10 |

In the above table, LA and TVLA stand for the quantity of liquid assets and the transactions velocity of liquid assets, respectively. Note that the supply of liquid assets now consists of the sum of government bonds and consumer loans, i.e., non-negative holdings of private bonds. (As mentioned earlier, think of private intermediaries as holding these securities as assets and issuing liquid liabilities to consumers). It is interesting to note that the stock/income ratio now becomes less sensitive to the transactions cost. This occurs since individuals have borrowing as an alternative to smoothing consumption, making the need for a distress sale of stocks less likely. As with the other examples, however, the ratio of liquid assets to income is too low. Finally, it is worth observing that the credit limit is binding for only a very small fraction of the population, as the marginal distributions for asset holdings recorded in Table 2 indicate[11].

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[11] See Huggett (1989) for a related analysis with only inside lending and borrowing, and where borrowing is costless (i.e., individuals can borrow at

In the next example we consider the impact of fixed costs with and without some borrowing.

**Example 4 (Fixed cost)**

We assume that the parameters are the same as in example 1 except that we allow for a fixed cost of transacting in stocks in addition to the constant marginal cost represented by $\alpha$. Figure 1 clearly implies that fixed costs are relevant. The fixed cost is assumed to be 1 percent of average quarterly income which is consistent with the schedule depicted in Figure 1. At first, we do not permit any borrowing.

| | | $r_s$ = 3.0 (No borrowing) | | |
|---|---|---|---|---|
| | $\alpha$ = | 0.02 | 0.035 | 0.05 |
| solutions: | $\bar{s}/y$ = | 0.63 | 0.58 | 0.53 |
| | $\bar{b}/y$ = | 0.12 | 0.15 | 0.17 |
| | TVS = | 0.07 | 0.06 | 0.05 |
| | TVB = | 1.19 | 0.98 | 0.85 |

Next, we also allow for borrowing where the parameters are the same as in example 3.

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the riskless rate). With costless borrowing and a large credit limit, the risk-free rate gets close to the rate of time preference, which suggests that some kind of frictions in borrowing may be needed to explain a low riskless rate. See also Mehrling (1989).

$r_s$ = .03 (with costly borrowing)

| | | | |
|---|---|---|---|
| $\alpha$ = | 0.02 | 0.035 | 0.05 |
| $\bar{s}/y$ = | 0.58 | 0.53 | 0.49 |
| $\bar{b}/y$ = | 0.07 | 0.09 | 0.12 |
| TVS = | 0.07 | 0.05 | 0.05 |
| TVB = | * | * | * |
| LA/y = | 0.09 | 0.11 | 0.13 |
| TVLA = | 1.42 | 1.16 | 0.99 |

In comparing the above examples with examples 1 and 3, respectively, we find that the fixed cost increases the ratio of bonds (liquid assets) to income and reduces the ratio of stocks to income and the transactions velocities of stocks as well as bonds. The relative transactions velocity of bonds (liquid assets) to stocks is not much affected. More importantly, the higher ratio of bonds (liquid assets) to income still falls too short of the target.

## VII. Conclusion

Our goal was to explore whether allowing for an explicit demand for liquidity could help resolve the risk-free rate and equity premium puzzles. We motivated a household demand for liquid assets by introducing uninsured individual risks in conjunction with costs of trading equity. While the simulated model did well on some grounds - explaining the relative transactions velocities of stocks and liquid assets and the ratio of stocks to income - it predicted too low a ratio of liquid assets to income. In our view the asset return puzzles should be thought of in this way: why is it that household demand for low yielding liquid assets has been historically so high?

Closer inspection of the data indicates that a substantial fraction of liquid assets are held by a group of households who own relatively little stock and, relatedly, that the ownership of stock is heavily concentrated. For example, Avery, Elliehausen and Kennickell (1988) estimate that in 1963 the bottom 90% of the wealth distribution held 53% of the total quantity of liquid assets, but only 9% of the equity. Conversely, the top 1% held over 60 % of the equity but only 10 % of liquid assets. These figures suggest that one possible way of adjusting our model to resolve the "liquid assets" puzzle is to allow for additional heterogeneity, in the form of stock-holders versus non-stockholders[12]. One approach to endogenizing the segmentation might be to allow for declining average costs of trading stock and for borrowing costs to vary inversely with collateralizable wealth.

Another possible factor is that the only motive for holding liquid assets in our framework involves precautionary considerations. We ignore transactions motives. Certainly a component of household holdings of savings and money market accounts stems from transactions needs. Subject to computational considerations and some of the usual issues in introducing

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[12] Mankiw and Zeldes (1989) also emphasize the importance of distinguishing between individuals who regularly hold stock and those not inclined to do so, though for somewhat different reasons.

money, one could modify our framework to allow for transactions demands. (We would also need to introduce a small cost of trading liquid assets other than money.)

There are some other extensions of our analysis which would be desirable. On the theoretical side, our model does not endogenize the absence of insurance markets, limited nature of financial markets, limitations on borrowing and short selling or for that matter government policy. Endogenizing limitations on insurance and borrowing along the lines of Phelan and Townsend (1989) is one possibility. It seems more difficult to endogenize costs of trading equity. Finally, we would like to allow for aggregate dividend risk, but this task appears to be quite formidable.

Typical Commission Rates for Selected Transactions; Dollar Commission as a Percentage of the Value of the Order

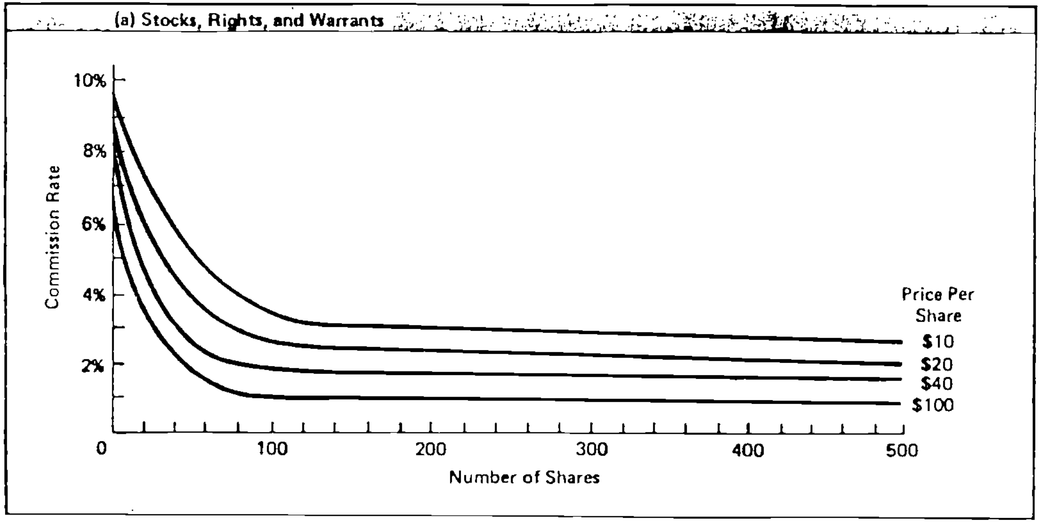


FIGURE 1

(FROM SHARPE, 1985, p.40)

**Table 1**

(marginal pdf's for stocks and bonds for example 1)

Stocks go from 0 to 8.4 in steps of 0.1

| α\s | 0 | .1-1.4 | 1.5-2.8 | 2.9-4.2 | 4.3-5.6 | 5.7-7.0 | 7.1-8.4 |
|---|---|---|---|---|---|---|---|
| .02 | .002 | 0.15 | 0.47 | 0.23 | 0.10 | 0.05 | 0.01 |
| .035 | .002 | 0.17 | 0.49 | 0.21 | 0.09 | 0.03 | 0.0 |
| .05 | .004 | 0.21 | 0.49 | 0.20 | 0.08 | 0.02 | 0.0 |

Bonds go from 0 to 2.4 in steps of 0.1

| α\b | 0 | .1-.3 | 0.4-0.6 | 0.7-0.9 | 1.0-1.2 | 1.3-2.4 |
|---|---|---|---|---|---|---|
| .02 | .34 | 0.29 | 0.36 | 0.01 | 0.0 | 0.0 |
| .035 | .25 | 0.23 | 0.32 | 0.20 | 0.0 | 0.0 |
| .05 | .20 | 0.18 | 0.27 | 0.23 | 0.12 | 0.0 |

**Table 2**

(marginal pdf's for stocks and bonds for example 3)

Stocks go from 0 to 8.4 in steps of 0.1

| α\s | 0 | .1-1.4 | 1.5-2.8 | 2.9-4.2 | 4.3-5.6 | 5.7-7.0 | 7.1-8.4 |
|---|---|---|---|---|---|---|---|
| .02 | .011 | .19 | .47 | 0.18 | 0.08 | 0.03 | 0.0 |
| .035 | .005 | .22 | .47 | 0.19 | 0.09 | 0.02 | 0.0 |
| .05 | .004 | .26 | .46 | 0.20 | 0.07 | 0.01 | 0.0 |

Bonds go from 0 to 2.4 in steps of 0.1

| α\b | -.4 | -.3--.1 | .0-0.2 | 0.3-0.5 | 0.6-0.8 | 0.9-1.1 | 1.2-2.0 |
|---|---|---|---|---|---|---|---|
| .02 | .03 | 0.09 | 0.38 | 0.29 | 0.20 | 0.0 | 0.0 |
| .035 | .10 | 0.17 | 0.28 | 0.25 | 0.19 | 0.01 | 0.0 |
| .05 | .10 | 0.13 | 0.23 | 0.21 | 0.19 | 0.12 | 0.02 |

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