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A SORTING MODEL OF LABOR CONTRACTS:  
IMPLICATIONS FOR LAYOFFS AND WAGE-TENURE PROFILES

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ABSTRACT

This paper analyzes a sorting model of labor contracts when workers have private information about their own productivities, and firms can test (monitor) workers. We show that sorting considerations alone generate steep wage-tenure profiles, high turnover rates of newly hired workers, and mandatory retirement rules.

We find that if test results are only informative to the testing firm, and hiring is costless, then all workers that fail the test are fired. When hiring is costly, we derive conditions under which the firm retains some (or all) workers that fail its test. We also derive conditions under which the firm tests some, but not all, of its workers.

In the second part of this paper, we consider the case when there are no hiring costs and there are many identical firms competing for the good type workers. We characterize the optimal contracts and show that competition for workers can lower total output. This is because competition can induce firms to increase the proportion of their workers that they test, but if the test is costly, this lowers output. Finally, we show that because a mandated minimum wage affects the probability of a firm testing its workers, an increase in the minimum wage can increase (or decrease) aggregate output.

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## I. Introduction

In this paper we derive equilibrium employment contracts when there are important informational asymmetries stemming from worker heterogeneity. We show that the use of tests and contingency wage contracts to resolve these informational asymmetries can explain the high layoff rates of newly hired workers, the increase in the variance of wages with seniority, the steep slope of wage-tenure profiles, and the use of mandatory retirement provisions in labor contracts.<sup>1</sup>

The large residuals in standard wage equations and large differences in the measured output of different workers on the same jobs suggest that unobserved differences among workers have important economic consequences. Firms often spend significant resources testing workers and monitoring newly hired employees as means of improving the quality of their work forces.

Tests can take a variety of forms, ranging from standardized written multiple choice exams to a probationary employment period during which a worker's performance is monitored. We focus on tests that monitor the performance of workers during some probationary period, however, the model is immediately applicable to instantaneous tests of job applicants.

Because workers have private (albeit imperfect) information about their own productivities, firms use contingency contracts to improve their pools of applicants and thus improve the quality of workers they hire contingent on test scores. These contracts may involve firing some workers.

Tests are valuable both because they directly reveal information and because they allow firms to offer contingency wage contracts that enable firms to avail themselves of the private information known by workers. Since tests

can be used to induce self-selection, the more heterogeneous is the labor force and the more knowledgeable are workers concerning their own productivities, the more likely is a firm to test workers. If the test is not very helpful for assigning workers to various jobs within a firm, and firms can inflict sufficiently large penalties on workers whose performance is low, then firms will test applicants randomly. Random testing with large penalties for low performance enables firms to sort workers according to their private information, while at the same time saving on the cost of testing.

We analyze a labor market in which firms are imperfectly informed about the characteristics of their workers, and workers are themselves imperfectly informed about their own abilities. By monitoring the performance of a worker during an initial apprenticeship period, a firm can gain additional information about the abilities of the worker and can improve the match between workers and jobs. However if the cost of monitoring workers outweighs the benefit from improved matches there is an efficiency loss from monitoring.

We allow firms to offer more comprehensive contracts than were permitted in previous papers.<sup>2</sup> A firm chooses a length of time to monitor workers, a probability of monitoring the worker, an application fee, a probability of firing workers whose observed output is low, wages during the testing period, and wages contingent on observed productivity. We derive conditions under which firms fire workers, and the determination of the length of the low-wage apprenticeship program, the accuracy with which firms test workers, as well as the various contingency wages firms pay. Previous research in this field has treated all these decisions, except the choice of contingency wages and, in a restricted sense, the accuracy of the test as exogenous.<sup>3</sup> Those artificial limits on the contracts that were available to firms made it difficult to interpret the results of those models.

On the other hand, allowing a rich set of contracts complicates our model. Consequently, we focus on the sorting features of the contracts and

ignore both learning on the job and the incentive effects of contracts. These omitted factors are likely to have important effects on the design of contracts in the real world. This paper should, therefore, be viewed as an attempt to see how much of observed labor market behavior can be explained solely by sorting considerations. The other gross simplification we make is to assume that workers are risk neutral.<sup>4</sup>

Our analysis also differs from previous work in this field by making the cost of testing a worker an increasing function of the length of the testing program, and by allowing information about the abilities of workers to improve their expected productivity (perhaps by improving the job assignment process within the firm).

Perhaps the most striking results we obtain concern conditions under which firms fire workers. If hiring is costless, then all workers that fail the firm's test will be fired in equilibrium. The assumption that hiring is costless is crucial here. If hiring is costly, and the costs per efficiency unit of labor from employing a failure were less than that of hiring a new worker, workers that fail the test may be retained by the firm.

We also derive the length of the testing period as well as the probability of a worker being tested in equilibrium.<sup>5</sup> As might be expected, when the direct benefits of testing outweigh the direct costs, all workers are tested. However, even when the direct costs outweigh the direct benefits of testing, there are circumstances under which some strictly positive fraction of workers are tested and conditions under which the length of the testing period is longer than is necessary to gain direct information about the productivities of workers. This is because a long low-wage testing period helps deter applications from type 2 workers, who are less likely to pass the test.

Another somewhat surprising result of the analysis is that for reasonable parameter values, competition among firms can increase the likelihood that a

worker will be tested, even when testing decreases aggregate output. The intuition here is that competition among firms bids up the wage paid to workers that fail the test. This makes it more difficult for firms to deter applications from type 2 workers. The firms respond by increasing the probability that they test a worker. The increased testing can impose a dead weight loss on society.

Finally, we show that even if there are many identical firms actively competing for workers (but with fixed output prices) each firm may earn positive profits in equilibrium. The reason for this last result is that an attempt to steal workers from other firms by offering a "better" employment contract could attract low productivity workers and thus make the higher wage firm worse off. This result depends on there being discrete types of workers. It does not hold if there is a continuum of types.

#### Relevant Literature

The labor market models that are most closely related to ours are sorting models such as Salop and Salop [1976], Guasch and Weiss [1980,1981,1982], Nalebuff and Scharfstein [1987], and MacLeod and Malcomson [1988]. In Salop and Salop, steep wage tenure profiles are used to deter applications from workers with high probabilities of quitting. In the Guasch and Weiss, and Nalebuff and Scharfstein models, tests, application fees and contingency wages are used to deter applications from workers with low (expected) productivity. In MacLeod and Malcomson [1988], a promotion hierarchy is used to prevent workers from shirking. While each of these papers offers an explanation of why the variance of wages would increase with tenure, none of them explain why inexperienced workers are more likely to be fired, nor do they derive the length of apprenticeship programs, the accuracy with which firms test workers, or the probability of a worker being fired.

### Structure of the Model

We shall investigate the equilibrium employment contracts for two different market structures: a) idiosyncratic firms in which test results for one firm are irrelevant for other firms vs. b) many identical firms competing for workers. In the latter case, test results are equally relevant to many firms. We shall refer to market structure (a) as the single firm case and (b) as the many firm case.

In each case the firm can combine a low-wage apprenticeship program with a test as means of improving the quality of their workers. We have chosen to emphasize the apprenticeship-test approach in which testing takes place over a nontrivial length of time for several reasons. First, there is strong evidence that much testing takes the form of observing the performance of workers on the job. Second, models in which all testing takes place before hiring seem at odds with evidence both that layoffs decline rapidly with tenure, and that at least some promotions and dismissals are based on job performance.<sup>6</sup> Third, sorting models with instantaneous tests require that workers incur significant costs to be tested.

On the other hand, apprenticeship-testing models are more complex than sorting models with instantaneous tests. This is because the cost to a worker of a low-wage apprenticeship varies with the worker's reservation wage, and because the firm must choose the length of the apprenticeship as well as the wage during the apprenticeship rather than simply choosing the application fee. In Section V, we show how our results change if minimum wage laws limited the use of low wages.

We make the following additional assumptions throughout: There are only two types of workers in the population: types 1 and 2. The productivity of type 2 is such that firms only wish to hire type 1 workers (the productivity

of type 2 workers could be a large negative number). Firms have a single pass-fail test available to them. Workers must work for the firm for at least  $\bar{m}$  periods for the firm to evaluate (test) them. The probability of a type  $i$  worker passing the test is denoted by  $p_i$ , where  $p_1 > p_2$ . Workers know their types, but not whether they would pass the test. Workers of the same type have the same alternative opportunities which generate a reservation wage of  $w_i$ , where  $w_1 \geq w_2$ . Firms incur a cost  $H \geq 0$  for each worker they hire.

All workers have the same work-life  $L$ , and the same liquid wealth  $S$  that can be used to pay an application fee. (We consider the case where  $S=0$ , which is equivalent to not allowing application fees.)

Workers are risk neutral. The instantaneous expected productivity of a type 1 worker while he is being monitored is  $Q_1^*$ , the expected productivity of an untested type 1 worker is  $Q_1$ , the productivities of tested type 1 workers that pass, or fail, the test are  $Q_1^p$  and  $Q_1^f$  respectively; where

$$Q_1^* < Q_1 < p_1 Q_1^p + (1 - p_1) Q_1^f \quad \text{and} \quad Q_1^f < Q_1 < Q_1^p.$$

The first inequality is intended to capture the cost of monitoring workers. We are assuming that cost is proportional to the length of the monitoring period. Most previous work on labor contracts with tests have assumed that tests are arbitrarily short, and that there is a fixed cost of testing a worker.<sup>7</sup>

The second inequality is intended to capture the efficiency gain from correctly assigning workers to jobs. We assume  $Q_1 > w_1$ , but do not restrict the sign of  $Q_1^f - w_1$ .

Firms choose the proportion of workers they test (monitor during some probationary period) and the proportion of tested workers that they retain. Workers know those proportions. Initially we do not allow firms to fire



untested workers. This restriction is consistent with court decisions on unjust dismissal (see Krueger [1989]).<sup>8</sup> We allow firms to commit to all other terms of their contract.

In Appendix B, we analyze the case in which firms can fire untested workers. There we also analyze the effect of restricting firms to only contracts that are enforceable in the sense that if the firm wishes (ex post) to renege on some terms of the contract the worker would wish to (and be able to) have these terms enforced, and, moreover, would be able to appeal to a third party to have those terms enforced. We shown that enforceability precludes tests lasting more than  $\bar{m}$  periods.<sup>9</sup>

The contract terms that a firm offers are:

- m: the length of the testing-probation program;  $m \geq \bar{m}$ , where  $\bar{m}$  is the minimum length testing period needed to measure the productivity of a worker. If the test lasts longer than  $\bar{m}$  periods no new information is revealed;
- z: the probability that a newly hired worker is placed in the testing-probation program;
- t: the probability that a worker who fails the test is fired;
- w: the wage paid an untested worker;
- $w^p$ : the wage paid a worker that passes the test;
- $w^f$ : the wage paid a worker that fails the test but is not fired;
- C: the application fee charged workers.

Firms can commit to a choice of z at the time the workers are hired. This commitment might be made by hiring a given number of supervisors, or by choosing a technology and distribution of jobs that generates a given level of z. In the absence of this commitment workers would disregard the announced value of z. Therefore, it is in the interest of the firm to choose technologies that commit it to a fixed value of z. This commitment increases

the set of feasible contracts available to the firm and hence increases expected profits.<sup>10</sup>

Firms know all the population parameters, such as  $p_1$ ,  $w_1$ ,  $S$ , but not the identity (type) of any particular new worker. To deter the type 2 workers from applying, it is optimal for the firm to pay each worker the legal minimum wage, which we set equal to zero, during the monitoring (apprenticeship) period, i.e.  $w^*=0$ .<sup>11</sup> This is because workers do not know that whether they are being monitored, and each worker's performance in the apprenticeship program is unknown until he completes the program.<sup>12</sup>

Without loss of generality we assume that workers who are fired are fired right after the program.

To deter the type 2 workers from applying, the firm charges the workers an application fee of  $S$  and pays it back (in expectation) in  $w^P$ , so that type 1 workers are not made worse off by the fee. Because  $p_1 > p_2$ , this tradeoff makes the contract less attractive to type 2 workers.

The organization of this paper is as follows. In Section II, we characterize the labor contract offered when test results are useful only to the testing firm. We analyze the cases of costless and costly hiring. In Section III, we analyze the case when there are many identical firms competing to hire type 1 workers. In Section IV, we compare the amount of testing in the one firm case versus that in the many firm case. In Section V, we consider the effect of raising the minimum wage on the optimal contracts. Section VI contains a summary.

## II. Idiosyncratic Tests: Each Firm is a Monopsonist in the Hiring Market

We first consider the case in which only one firm hires workers and there is an excess supply of type 1 workers, when the firm offers the contract that

maximizes its profits subject to attracting type 1 workers and not attracting type 2. This case is equivalent to assuming that a firm's test results are irrelevant for other firms.

The firm's cost per efficiency unit of labor and the workers' utility function can be written in the following ways:

$$e = \frac{H + (L - m)[z p_1 w^P + z (1-p_1)(1-t)w_1 + (1-z)w_1] - S}{zmQ_1^* + (1 - z)mQ_1 + (L - m)[z p_1 Q_1^P + z(1-p_1)(1-t)Q_1^F + (1-z)Q_1]} \quad (1)$$

$$U_1 = z(L - m)[p_1 w^P + (1-p_1) w_1] + (1-z)(L-m)w_1 \quad (2)$$

$$U_2 = z(L - m)[p_2 w^P + (1-p_2) t w_2 + (1-p_2)(1 - t)w_1] + (1-z)(L-m)w_1 \quad (3)$$

We assume the firm's output is a concave function of the quantity of efficiency units of labor it employs, i.e., the inputs of different workers can be added together to form a measure of aggregate labor input.

It is a standard result in this literature that the firm will choose a contract to minimize (1) subject to (2) and (3) if its labor supply constraint is not binding at that contract and its production function is concave (see Weiss [1980]). Therefore, to attract type 1 workers while deterring type 2 workers from applying, the firm solves the following problem:

$$\min e \quad (4)$$

$$\text{s.t. } U_1 \geq w_1 L + S \quad (5)$$

$$U_2 \leq w_2 L + S \quad (6)$$

$$m \geq \bar{m} \quad (7)$$

we assume that the type 2 workers will not choose the contract if  $U_2 = w_2 L + S$ .

We shall refer to the contract that minimizes  $e$  subject to (5), (6), (7) as the optimizing contract. (It is, of course, not necessarily socially optimal.)

In the optimizing contract, (5) is binding. If a firm offers a contract that satisfies constraints (5) and (6), and (5) is not binding, then it can lower  $w^P$  without violating the constraints and thereby lower  $e$ . From  $U_1 = w_1 L + S$ , we have

$$w^P = \frac{w_1 L + S - (1-z)(L-m)w_1 - z(L-m)(1-p_1)w_1}{z(L-m)p_1} \quad (8)$$

Substituting (8) into (1) and (2), we have

$$e = \frac{H + w_1 L - zt(1-p_1)w_1(L-m)}{zmQ_1^* + (1-z)mQ_1 + (L-m)[z p_1 Q_1^P + z(1-p_1)(1-t)Q_1^f + (1-z)Q_1]} \quad (9)$$

$$\begin{aligned} U_2 &= z(L-m)(1-p_2) [ t w_2 + (1-t)w_1 ] + (1-z)(L-m) w_1 \\ &\quad + \frac{P_2}{P_1} [ w_1 L + S - (1-z)(L-m)w_1 - z(L-m)(1-p_1)w_1 ] \\ &= (L-m) w_1 + \frac{P_2}{P_1} (w_1 m + S) - z t (w_1 - w_2)(L-m)(1-p_2) \end{aligned} \quad (10)$$

If the following condition is satisfied for given  $m \geq \bar{m}$ ,

$$(L-m)w_1 + \frac{P_2}{P_1} [w_1 m + S] \leq w_2 L + S \quad (11)$$

then the type 2 workers are deterred for any non-negative  $t$  and positive  $z$  in the contract. That is, the type 2 workers could still be deterred from applying even if all failures are retained (i.e.  $t=0$ ), as long as the application fee  $S$  is large enough. If (11) does not hold,  $z$  and  $t$  must be positive for the contract to deter applications from the type 2 workers. In this case,

$$\left. \frac{dt}{dz} \right|_{U_2 = w_2 L + S} = - \frac{t}{z} \quad (12)$$

Since

$$\frac{\partial U_2}{\partial t} = -z(L-m)(1-p_2)(w_1 - w_2) < 0,$$

given any  $m \geq \bar{m}$ , any  $(z, t)$  combination in the shaded area in Figure 1 is feasible in deterring type 2 workers from applying.

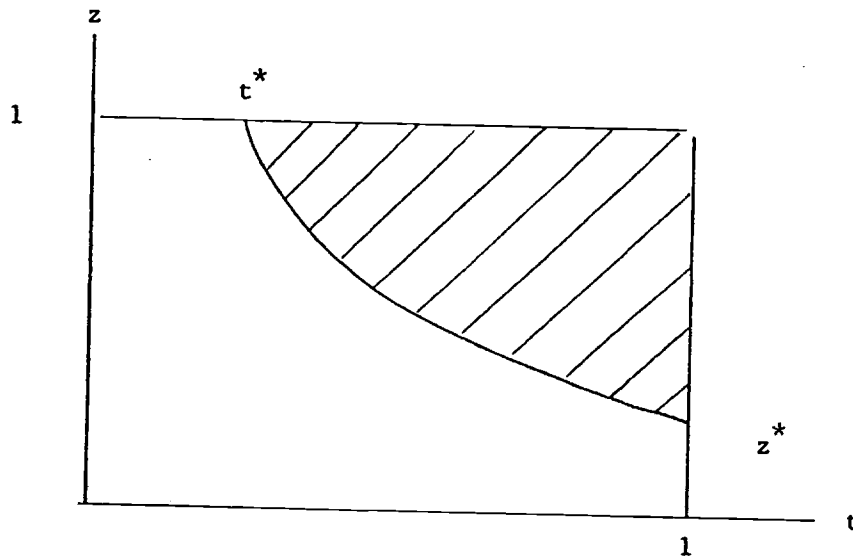


Figure 1. Feasible region for deterring type 2 workers given  $m$

We next characterize the profit maximizing contracts when  $H = 0$  and when  $H > 0$ .

### II.1 Hiring is Costless

In this case,  $H = 0$ , and we shall solve for  $z$ ,  $m$ , and  $t$ .

Let  $\bar{z}$  be the minimum  $z$  that satisfies  $U_2 \leq w_2L + S$  given  $t = 1$  and  $m = \bar{m}$ . Let  $\bar{m}$  denote the minimum value that satisfies  $U_2 \leq w_2L + S$  when  $t = 1$ ,  $z = 1$  and let  $m^*$  denote the limit of its minimum value when  $t = 1$ , as  $z \rightarrow 0$ .

Define

$$G(m) = (L - m)p_1(Q_1^P - Q_1) + m(Q_1^* - Q_1) \quad (13)$$

Thus  $G(m)$  is the gain (or loss if negative) in the labor input of the firm from testing a previously untested worker, if workers that fail the test were to be fired and replaced by untested workers for the rest of their work lives, and only type 1 workers are tested.<sup>13</sup> Define

$$g(m) = (L-m)(p_1Q_1^P + (1-p_1)Q_1^f - Q_1) + m(Q_1^* - Q_1) \quad (14)$$

Thus  $g(m)$  is the gain (or loss if negative) in the labor input of the firm from testing a previously untested worker, if workers that fail the test were to be retained and the test is of length  $m$ , and only type 1 workers were tested. We assume  $Q_1^f < Q_1 < Q_1^P$ ; therefore,  $g(m) < G(m)$ .

Our first result is that workers that fail the test are fired. Because hiring is costless and untested workers are paid the same wage as workers that fail the test ( $w_1$ ), firing failures and replacing them with untested workers costlessly improves the average productivity of the firm and helps the firm deter applications from the type 2 workers. We also show that firms may choose a testing period that is longer than  $\bar{m}$ ; i.e., the testing period may be chosen for its selection effects on job applicants. Since all workers receive  $w^*=0$  during the testing period, lengthening the period increases the cost of applying to the firm and deters applications from the type 2 workers. Finally, if  $G(\bar{m}) < 0$ , firms may randomly test workers.

### Theorem 1

If there is only one firm in the hiring market and  $H = 0$ , the contract is characterized by:

$t = 1$ , and

$$\text{if } \frac{w_1L + S}{w_2L + S} \geq \frac{p_1}{p_2}, \text{ then } m = \bar{m}, z = \begin{cases} 1, & \text{if } G(\bar{m}) \geq 0, \\ \bar{z}, & \text{if } G(\bar{m}) < 0; \end{cases}$$

$$\text{if } \frac{w_1 L + S}{w_2 L + S} < \frac{p_1}{p_2}, \text{ and}$$

(1) if  $G(\max(\bar{m}, \tilde{m})) \geq 0$ , then  $m = \max(\bar{m}, \tilde{m})$  and  $z = 1$ ;

(2) if  $G(\max(\bar{m}, \tilde{m})) < 0$ , then  $z \rightarrow 0$ , and if  $\bar{m} > m^*$ , then  $m = \bar{m}$ ,

otherwise  $m \rightarrow m^*$ .

### Proof

We first prove that for any  $m$ ,  $t=1$  in the profit maximizing contract.

Fixing  $m \geq \bar{m}$ , the firm minimizes  $e$  from (9) subject to  $U_2 \leq w_2 L + S$  from (10) for the contract.

Denoting the denominator of  $e$  in (9) as  $D$  for notational simplification, we have

$$\begin{aligned} \frac{\partial e}{\partial t} &= \frac{1}{D^2} z(1-p_1)w_1(L-m) \left\{ z[m(Q_1 - Q_1^*) - (L-m)(p_1 Q_1^P + (1-p_1)Q_1^f - Q_1)] - L(Q_1 - Q_1^f) \right\} \\ &\quad - \frac{1}{D^2} z(1-p_1)w_1(L-m) \left\{ -z g(m) - L(Q_1 - Q_1^f) \right\} \end{aligned} \quad (15)$$

$$\begin{aligned} \frac{\partial e}{\partial z} &= \frac{1}{D^2} w_1 L \left\{ m(Q_1 - Q_1^*) - (L-m)(p_1 Q_1^P + (1-p_1)Q_1^f - Q_1) - t(1-p_1)(L-m)(Q_1 - Q_1^f) \right\} \\ &\quad - \frac{1}{D^2} w_1 L \left\{ -g(m) - t(1-p_1)(L-m)(Q_1 - Q_1^f) \right\} \end{aligned} \quad (16)$$

and if (11) does not hold, (i.e. positive levels of  $z$  and  $t$  are required to deter type 2 workers)

$$\left. \frac{\partial e}{\partial z} \right|_{U_2 = w_2 L + S} = \frac{\partial e}{\partial z} + \frac{\partial e}{\partial t} \cdot \left. \frac{dt}{dz} \right|_{U_2 = w_2 L + S}$$

$$= \frac{1}{D^2} w_1 L \left\{ -g(m) - t(1-p_1)(L-m)(Q_1 - Q_1^f) \right\}$$

$$\begin{aligned}
& + \frac{1}{D^2} z(1-p_1) w_1(L-m) \left\{ -z g(m) - L(Q_1 - Q_1^f) \right\} \cdot \left(-\frac{t}{z}\right) \\
& - - \frac{w_1}{D^2} g(m) [L - (L-m)tz(1-p_1)] \tag{17}
\end{aligned}$$

If  $g(m) \geq -L(Q_1 - Q_1^f)$ , then  $\forall z \in (0, 1]$ ,  $\frac{\partial e}{\partial z} \leq 0$ . So given any  $z$ ,  $t=1$  is optimal.

If  $g(m) < -L(Q_1 - Q_1^f)$ , then  $\forall z \in (0, 1]$ ,  $\frac{\partial e}{\partial z} > 0$ . So either

a) Inequality (11) holds (i.e. the whole  $0 \leq t \leq 1$ ,  $0 \leq z \leq 1$  region is feasible in deterring type 2 workers from applying), so  $z=0$ . Thus  $t=1$  in the optimal contract because  $\partial e / \partial t < 0$  as  $z \rightarrow 0$ ; or

b) Inequality (11) does not hold. Then  $(z, t)$  in the optimal contract is on the boundary of  $U_2 \leq w_2L + S$ . Because  $g(m) < 0$  in this case,  $de/dz > 0$  along the boundary of  $U_2$ , and  $z = z^*$ ,  $t = 1$  in the optimal contract, where  $z^*$  is the minimum  $z$  that satisfies  $U_2 \leq w_2L + S$  given  $t=1$ .

Therefore, in all cases,  $t = 1$ .

Substituting  $t = 1$  into (9) and (10), we have:

$$\min e = \frac{w_1L - z(L-m)(1-p_1) w_1}{zmQ_1^* + (1-z)LQ_1 + z(L-m)p_1 Q_1^P} \tag{18}$$

$$\text{s.t. } U_2 = (L-m) w_1 + \frac{P_2}{P_1} (w_1 m + S) - z (w_1 - w_2)(L-m)(1 - p_2) \leq w_2L + S \tag{19}$$

Substituting  $t = 1$  in (16), we have

$$\frac{\partial e}{\partial z} = \frac{1}{D^2} w_1 L \left\{ m(Q_1 - Q_1^*) - (L-m)p_1(Q_1^P - Q_1) \right\} - - \frac{w_1 L}{D^2} G(m) \tag{20}$$



From (18),

$$\begin{aligned} \frac{\partial e}{\partial m} &= \frac{1}{D^2} z w_1 L \left\{ p_1 Q_1^P + (1-p_1)(1-z)Q_1 - [p_1 + (1-p_1)(1-z)]Q_1^* \right\} \\ &= \frac{1}{D^2} z w_1 L \left\{ p_1(Q_1^P - Q_1^*) + (1-p_1)(1-z)(Q_1 - Q_1^*) \right\} > 0 \end{aligned}$$

To explore  $m$  and  $z$  in the optimal contracts, we consider the characteristics of  $U_2 = w_2 L + S$ .

From (10),

$$\frac{\partial U_2}{\partial z} = - (w_1 - w_2)(L-m)(1-p_2) < 0 \quad (21)$$

Therefore if  $U_2$  were drawn in  $(m, z)$  space where  $z$  is the vertical axis and  $m$  the horizontal, any  $(m, z)$  above  $U_2$  can deter applications from the type 2 workers.

Constraint (19) can be rewritten as:

$$z \geq \frac{w_1 \left(1 - \frac{p_2}{p_1}\right)}{(1-p_2)(w_1 - w_2)} + \frac{\frac{p_2}{p_1}(w_1 L + S) - (w_2 L + S)}{(L-m)(w_1 - w_2)(1-p_2)} \quad (22)$$

so

$$\left. \frac{dz}{dm} \right|_{U_2 = w_2 L + S} = \frac{\frac{p_2}{p_1}(w_1 L + S) - (w_2 L + S)}{(L-m)^2 (w_1 - w_2)(1-p_2)} \quad (23)$$

The slope of  $U_2$  depends on the sign of the numerator. Moreover, we shall show that it is concave in  $(m, z)$  space when it is downward sloping and convex when it is upward sloping:

$$\left. \frac{d^2 z}{dm^2} \right|_{U_2 = w_2 L + S} = 2 \cdot \frac{\frac{p_2}{p_1}(w_1 L + S) - (w_2 L + S)}{(L-m)^3 (w_1 - w_2)(1-p_2)} \quad (24)$$

Thus we have two cases:

CASE 1.

$$\frac{w_1 L + S}{w_2 L + S} \geq \frac{P_1}{P_2}$$

In this case,  $U_2 = w_2 L + S$  is upward sloping. Since  $\partial U_2 / \partial z < 0$  along boundary  $U_2 = w_2 L + S$ , any  $(m, z)$  above the boundary is feasible in deterring applications from type 2 workers.

From  $\partial e / \partial m > 0$ , we know that  $m = \bar{m}$ .

From  $\partial e / \partial z = -w_1 L \cdot G(m) / D^2$ , we know that

$$z = \begin{cases} 1, & \text{if } G(\bar{m}) \geq 0; \\ \bar{z}, & \text{if } G(\bar{m}) < 0, \end{cases}$$

where  $\bar{z}$  is the minimum  $z$  that satisfies  $U_2 \leq w_2 L + S$  at  $m = \bar{m}$ . If (11) holds for any  $m \geq \bar{m}$ , the whole region to the right of  $m = \bar{m}$  is feasible in deterring the type 2 workers and thus  $\bar{z}$  is equal to 0.

CASE 2.

$$\frac{w_1 L + S}{w_2 L + S} < \frac{P_1}{P_2}$$

In this case,  $U_2 = w_2 L + S$  is downward sloping. Recalling that  $\tilde{m}$  is the minimum  $m$  that satisfies  $U_2 \leq w_2 L + S$  given  $t = 1$  and  $z = 1$ ,  $\max(\bar{m}, \tilde{m})$  is the smallest  $m$  that is feasible in deterring type 2 workers.

If  $G(\max(\bar{m}, \tilde{m})) \geq 0$ , then  $m = \max(\bar{m}, \tilde{m})$ ,  $z = 1$ .

If  $G(\max(\bar{m}, \tilde{m})) < 0$ , then  $m = m^*$ ,  $z \rightarrow 0$ . This is because the contract in this case must be on the boundary of  $U_2 = w_2 L + S$  which has been shown to be concave. We show immediately below that the iso-profit (iso-efficiency wage) curves for the firm are convex, therefore  $(m, z)$  in the optimal contract

must be at one of the two extreme ends of  $U_2 = w_2L + S$ . As  $z \rightarrow 0$ ,  $m \rightarrow m^*$ ,  $e$  approaches its minimum for any  $m$  that satisfies  $G(m) \leq 0$ .

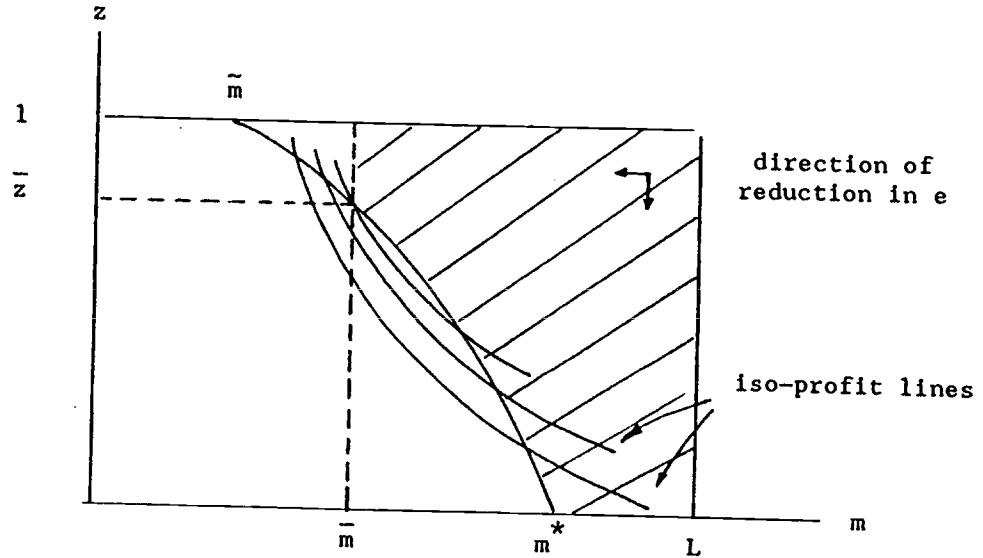


Figure 2. Feasible region and optimal contract in Case 2

Proof of Convexity of Iso-Profit Curves:

Solving for  $z$  from (18) while holding  $e$  fixed, we have

$$z = \frac{\frac{1}{e} w_1 L - Q_1 L}{L[p_1 Q_1^P + \frac{1}{e} (1-p_1) w_1 - Q_1] - m[p_1 Q_1^P + \frac{1}{e} (1-p_1) w_1 - Q_1^*]} \quad (25)$$

$$\frac{d^2 z}{dm^2} = \frac{\frac{1}{e} w_1 L - Q_1 L}{\left\{ L[p_1 Q_1^P + \frac{1}{e} (1-p_1) w_1 - Q_1] - m[p_1 Q_1^P + \frac{1}{e} (1-p_1) w_1 - Q_1^*] \right\}^3} \cdot 2 \left[ p_1 Q_1^P + \frac{1}{e} (1-p_1) w_1 - Q_1^* \right]^2$$

$$= \frac{1}{\left\{ L[p_1 Q_1^P + \frac{1}{e} (1-p_1) w_1 - Q_1] - m[p_1 Q_1^P + \frac{1}{e} (1-p_1) w_1 - Q_1^*] \right\}^2} \cdot z$$

$$\cdot 2 \left( [p_1 Q_1^P + \frac{1}{e} (1-p_1) w_1 - Q_1^*] \right)^2 \quad (26)$$

$\geq 0$

Therefore, we can conclude that all these curves are convex.

QED

In order to see the importance of the ordering of  $\frac{p_1}{p_2}$  and  $\frac{w_1 L + S}{w_2 L + S}$ ,

we rewrite  $U_1 = w_1 L + S$  and  $U_2 = w_2 L + S$  at  $t = 1$  as follows:

$$U_1 = (L - m) \left\{ z [p_1 w^P + (1-p_1) w_1] + (1-z) w_1 \right\} = w_1 L + S \quad (27)$$

$$U_2 = (L - m) \left\{ z [p_2 w^P + (1-p_2) w_2] + (1-z) w_1 \right\} = w_2 L + S \quad (28)$$

Consider a decrease in  $m$  accompanied by a decrease in  $w^P$  keeping  $U_1$

constant. This will reduce  $U_2$  if  $\frac{p_2}{p_1} \geq \frac{w_2 L + S}{w_1 L + S}$ . Thus, if  $(m, z)$  is

feasible in deterring type 2,  $(z, \bar{m})$  must also be able to deter them. Since  $\partial e / \partial m > 0$  along  $U_1 = w_1 L + S$ ,  $m = \bar{m}$  must be true in the optimal contract.

The selection of  $z$  will of course depend on the sign of  $G(\bar{m})$ .

If  $\frac{p_2}{p_1} < \frac{w_2 L + S}{w_1 L + S}$ , an increase in  $m$  accompanied by an increase in  $w^P$

keeping  $U_1$  constant will hurt the type 2 workers. However, the firm wants the minimum  $m$  that can deter the type 2 workers. At  $z = 1$ ,  $m$  achieves its minimum at  $\max(\bar{m}, \bar{m})$  in this case. If testing is profitable for the firm at this  $m$ ,  $z = 1$  must be the optimal selection. If testing is not profitable at this  $m$ , then testing is not profitable at any feasible value of  $m$ . In that case the firm minimizes  $e$  by letting  $z$  goes to zero (which is feasible in this case), and setting  $m$  at its value on the boundary of  $U_2 = w_2 L + S$  as  $z$  goes to zero.

(Note when we refer to testing being profitable or unprofitable at a given value of  $m$ , this is an informal way of indicating whether  $G(m) > 0$  or  $G(m) < 0$ .)

## II.2 Hiring is Costly

Now let hiring be costly and assume that the length of the apprenticeship program is fixed at  $m = \bar{m}$ . (We are implicitly assuming that at  $\bar{m}$  the results of the test are revealed to workers. Any worker that failed the test would then quit. Consequently w.l.o.g. we can set  $m = \bar{m}$ .)

Define the following ratios as the expected additional labor input per additional unit of labor cost incurred by respectively i) retaining one more worker who fails the firm's test; ii) hiring one more worker who is not going to be tested, and iii) hiring one more worker who is only retained if he passes the test.

$$\delta^f = \frac{Q_1^f}{w_1}, \quad \delta^u = \frac{Q_1 L}{H + w_1 L}, \quad \delta^p = \frac{(L-m)p_1 Q_1^p + m Q_1^*}{H + [(L-m)p_1 + m] w_1}.$$

We shall refer (loosely) to  $\delta^f$ ,  $\delta^u$  and  $\delta^p$  as the inverse of the expected cost per effective unit of labor from increasing the number of "failures", "untested", and "successful" workers, employed by the firm. The values of  $z$  and  $t$  in the optimal contract depend on the ordering of these numbers and the sign of  $g(m)$ . In particular, all workers who fail the test are fired,  $t=1$ , unless the workers who fail the test have the lowest cost per unit of labor,  $\delta^f > \max\{\delta^u, \delta^p\}$ , but even if  $\delta^f > \max\{\delta^u, \delta^p\}$ , it is possible for  $t=1$ , if this is needed to deter type 2. We also find that not all workers are tested ( $z < 1$ ) if either the untested have the lowest cost per effective unit of labor, or testing is wasteful,  $g(m) < 1$ .

### Theorem 2

Suppose that each firm's test is idiosyncratic. Let  $t^*$  be the minimum  $t$

needed to deter type 2 when  $z = 1$  and let  $z^*$  be the minimum  $z$  needed to deter type 2 when  $t = 1$ .

(1) if  $\delta^P \geq \max(\delta^f, \delta^u)$ , then  $t = 1$ ,  $z = 1$ ;

(2) if  $\delta^u \geq \max(\delta^P, \delta^f)$ , then  $t = 1$ ,  $z = z^*$ ;

(3) if  $\delta^f \geq \max(\delta^P, \delta^u)$ , and

(3a)  $g(m) \leq 0$ , and (11) does not hold, then  $t = 1$ ,  $z = z^*$ ;

(3b)  $g(m) \leq 0$ , and (11) holds, then  $t = 0$ ,  $z \rightarrow 0$ ;

(3c)  $g(m) > 0$ , then  $t = t^*$ ,  $z = 1$ .<sup>14</sup>

In all these cases,  $w^P$  in the contract is determined by (8).

### Proof

Denoting the denominator in (9) as  $D$ , we have

$$\begin{aligned} \frac{\partial e}{\partial t} &= \frac{1}{D^2} \left\{ -z(L-m)(1-p_1)w_1 [LQ_1 + z g(m)] + z(1-p_1)(L-m)Q_1^f [H + w_1 L] \right\} \\ &= -\frac{1}{D^2} z(L-m)(1-p_1) \left\{ z w_1 g(m) - [Q_1^f (H + w_1 L) - w_1 L Q_1] \right\} \end{aligned} \quad (29)$$

$$\begin{aligned} \frac{\partial e}{\partial z} &= \frac{1}{D^2} \left\{ -t(L-m)(1-p_1)w_1 LQ_1 - [g(m) - t(1-p_1)(L-m)Q_1^f] [H + w_1 L] \right\} \\ &= \frac{1}{D^2} \left\{ t(L-m)(1-p_1) [Q_1^f (H + w_1 L) - w_1 L Q_1] - g(m) [H + w_1 L] \right\} \end{aligned} \quad (30)$$

If (11) holds, the feasible set of contracts that deter applications from type 2 workers is the whole set  $[0, 1] \times [0, 1]$  in  $(z, t)$  space. However, if (11) does not hold, the restriction imposed by  $U_2 \leq w_2 L + S$  is binding.

Therefore, we need to evaluate

$$\left. \frac{de}{dz} \right|_{U_2 = w_2 L + S} = \frac{\partial e}{\partial z} + \left. \frac{\partial e}{\partial t} \cdot \frac{dt}{dz} \right|_{U_2 = w_2 L + S}$$

$$\begin{aligned}
& - \frac{1}{D^2} \left\{ [Q_1^f(H + w_1L) - w_1LQ_1](1-p_1)(L-m)t - g(m)(H + w_1L) \right\} \\
& - \frac{1}{D^2} z^{(L-m)(1-p_1)} \left\{ z w_1g(m) - [Q_1^f(H + w_1L) - w_1LQ_1] \right\} \cdot \left(-\frac{t}{z}\right) \quad (31)
\end{aligned}$$

$$= - \frac{1}{D^2} g(m) \left\{ H + w_1L - (L-m)(1-p_1) zt w_1 \right\} \quad (32)$$

The term in braces in (32) is always positive, so the sign of  $de/dz$  along the incentive boundary of type 2 workers depends on the sign of  $g(m)$  only.

There are two cases:

$$\underline{\text{Case 1}} \quad w_1g(m) \geq Q_1^f(H + w_1L) - w_1LQ_1 \quad (\text{i.e. } \delta^P \geq \delta^f) \quad (33)$$

In this case, two situations can occur:

Case 1.1

$$(L-m)(1-p_1)[Q_1^f(H + w_1L) - w_1LQ_1] - g(m)[H + w_1L] \leq 0 \quad (\text{i.e. } \delta^P \geq \delta^u) \quad (34)$$

Under these conditions, if  $\delta^f \geq \delta^u$ , that is,

$$Q_1^f(H + w_1L) - w_1LQ_1 \geq 0 \quad (35)$$

then from (33) and (35), we have  $g(m) \geq 0$ , which with (34) imply  $\forall t \in [0, 1]$ ,

$$t(L-m)(1-p_1)[Q_1^f(H + w_1L) - w_1LQ_1] - g(m)[H + w_1L] \leq 0$$

i.e.,  $\partial e/\partial z \leq 0$ . Therefore,  $z = 1$  in the optimal contract.

But at  $z = 1$ , (33) implies  $\partial e/\partial t \leq 0$ . Thus  $t = 1$  must be true in the optimal contract.

If  $\delta^f \leq \delta^u$ , that is,

$$Q_1^f(H + w_1L) - w_1LQ_1 \leq 0 \quad (36)$$

then from (33), no matter whether  $g(m)$  is positive or negative,

$$z w_1 g(m) \geq Q_1^f(H + w_1 L) - w_1 L Q_1$$

$\forall z \in [0, 1]$ . Therefore,  $\partial e / \partial t \leq 0$ , which implies that  $t = 1$  in the optimal contract. But at  $t = 1$ ,  $\partial e / \partial z \leq 0$  from (34), which implies that  $z = 1$ .

Thus, in Case 1.1, in which  $\delta^P \geq \max(\delta^f, \delta^u)$ , we prove that  $t = 1$ ,  $z = 1$  must be true in the optimal contract, no matter (35) holds or not.

Case 1.2

$$(L-m)(1-p_1)[Q_1^f(H + w_1 L) - w_1 L Q_1] - g(m)[H + w_1 L] \geq 0 \quad (\text{i.e. } \delta^P \leq \delta^u) \quad (37)$$

In this case,  $g(m)$  must be non-positive and thus from (33) we have

$$Q_1^f(H + w_1 L) - w_1 L Q_1 \leq 0 \quad (\text{i.e. } \delta^f \leq \delta^u)$$

From this and (37), we have  $\forall t \in [0, 1]$ ,

$$t(L-m)(1-p_1)[Q_1^f(H + w_1 L) - w_1 L Q_1] - g(m)[H + w_1 L] \geq 0$$

which implies that  $\partial e / \partial z \geq 0$ . In the case when type 2 are deterred for all  $0 < t$ ,  $z < 1$ , (i.e. (11) holds),  $z \rightarrow 0$  in the optimal contract. In the case when (11) does not hold,  $(z, t)$  in the optimal contract must be on the boundary of  $U_2 = w_2 L + S$ . Since  $g(m) \leq 0$  in this case,  $de/dz$  along  $U_2 = w_2 L + S$  must be non-negative, which implies that  $z = z^*$ ,  $t = 1$  in the optimal contract. Thus, if we define  $z^* = 0$  in the case when (11) holds, we have the following result:

if  $\delta^u \geq \delta^P \geq \delta^f$ , then  $z = z^*$ ,  $t = 1$  in the optimal contract.

$$\text{Case 2} \quad w_1 g(m) < Q_1^f(H + w_1 L) - w_1 L Q_1 \quad (\text{i.e. } \delta^P < \delta^f) \quad (38)$$

Again, in this case, two situations can occur:

Case 2.1



$$(L-m)(1-p_1)[Q_1^f(H + w_1L) - w_1LQ_1] - g(m)[H + w_1L] \leq 0 \quad (\text{i.e. } \delta^P \geq \delta^u) \quad (39)$$

(38) and (39) imply

$$Q_1^f(H + w_1L) - w_1LQ_1 \geq 0 \quad (\text{i.e. } \delta^f \geq \delta^u) \quad (40)$$

and  $g(m) \geq 0$ , which together with (38) imply that

$$z w_1g(m) \leq Q_1^f(H + w_1L) - w_1LQ_1$$

Therefore,  $\partial e/\partial t \geq 0$ , and thus  $t = 0$  in the case when (11) holds and  $(z, t)$  on the boundary of  $U_2$  when (11) does not hold. Notice that at  $t = 0$ ,  $\partial e/\partial z$  has the opposite sign of  $g(m)$  from (30); if  $(z, t)$  is on the boundary, from (31),  $de/dz$  has the opposite sign of  $g(m)$  also. From this, we can conclude that  $z = 1$ ,  $t = t^*$  must be true in the optimal contract.

Therefore, we obtain the following result:

if  $\delta^f \geq \delta^P \geq \delta^u$ , then  $z = 1$ ,  $t = t^*$  hold in the optimal contract.

#### Case 2.2

$$(L-m)(1-p_1)[Q_1^f(H + w_1L) - w_1LQ_1] - g(m)[H + w_1L] > 0 \quad (\text{i.e. } \delta^P < \delta^u) \quad (41)$$

$$\text{If } \delta^f \geq \delta^u, \quad Q_1^f(H + w_1L) - w_1LQ_1 \geq 0 \quad (42)$$

$$\text{then } \forall z \in [0, 1], \quad z w_1g(m) \leq Q_1^f(H + w_1L) - w_1LQ_1$$

Therefore,  $\partial e/\partial t \geq 0$ , and thus  $t = 0$  in the case when (11) holds and  $(z, t)$  on the boundary of  $U_2$  when (11) does not hold. From (30) at  $t = 0$ ,  $\partial e/\partial z$  has the opposite sign of  $g(m)$ ; if  $(z, t)$  is on the boundary, from (31),  $de/dz$  has the opposite sign of  $g(m)$  also. Consequently if  $g(m) \geq 0$ , then  $z = 1$ ; if  $g(m) < 0$ , then  $z = z^*$ , where in the case when (11) holds,  $z^* = 0$ .

It follows that if  $\delta^f \geq \delta^u \geq \delta^P$ , then  $z = z^*$ ,  $t = 1$  if  $g(m) \leq 0$  and (11) does not hold;  $z = z^*$ ,  $t = 0$  if  $g(m) \leq 0$  and (11) holds;  $z = 1$ ,  $t = t^*$  if  $g(m) \geq 0$ .

$$\text{If } \delta^u \geq \delta^f, \text{ that is, } Q_1^f(H + w_1L) - w_1Q_1L \leq 0 \quad (43)$$

then we can conclude that  $g(m) < 0$  from (38), which also implies that  $\forall t \in [0, 1]$ ,

$$t(L-m)(1-p_1)[Q_1^f(H + w_1L) - w_1LQ_1] - g(m)[H + w_1L] > 0 \quad (44)$$

So  $\partial e/\partial z > 0$ .

In the case when (11) holds,  $z = 0$ , which implies that  $\partial e/\partial t = 0$ . So without loss of generality, we assume that  $t = 1$ . In the case when (11) does not hold,  $(z, t)$  is on the boundary of  $U_2 = w_2L + S$  and from  $g(m) < 0$ , we know that  $z = z^*$ ,  $t = 1$  hold in the optimal contract. So whether (11) holds or not,  $z = z^*$ ,  $t = 1$  hold in the optimal contract if  $\delta^u \geq \delta^f > \delta^P$ .

QED

The rough intuition for these results is as follows. In (1), the firm's cost per efficiency unit of labor is lowest if it tests all workers and only retains those who pass the test. Since this policy also deters type 2 the firm will do it. One way of interpreting (2), (3a) and (3b) is to note that in each of those cases  $g(m) < 0$  and  $\delta^P \leq \max(\delta^f, \delta^u)$ . Thus the firm tries to minimize the proportion of workers that it tests. If increasing  $t$  helps minimize  $z$  or if the cost per efficiency unit of labor from retaining workers who fail the test is greater than that of hiring a worker who will not be tested, then the firm will fire all the workers who fail the test; i.e., set  $t=1$  as in cases (2) and (3a). Otherwise the firm chooses  $t=0$  as in (3b).

In (3c) testing is profitable:  $g(m) > 0$ , and retaining workers that fail the test lowers the firm's cost per efficiency unit of labor. Consequently the firm tests all its workers, sets  $z=1$ , and minimizes the proportion of workers that fail its test who are fired, subject to the need to deter applications from type 2; i.e., sets  $t=t^*$ .

Similar to Theorem 1,  $t = 1$  must be true as hiring cost approaches zero:

Corollary

As  $H \rightarrow 0$ , and the firm is a monopsonist in the hiring market, then the firm will fire all workers that fail its test.

Proof

As  $H \rightarrow 0$ ,  $\delta^u > \delta^f$ . The only possible situations are (1) and (2) in the Theorem, which implies that  $t = 1$  in the optimal contract.

QED

### III. Many Identical Firms Hiring

In this section, we assume that two or more identical firms are competing for type 1 workers. If the firms are making zero profits, then the payoffs of firms and workers are unaffected by whether workers that fail the firm's test and untested workers are fired or retained. The reason is that other firms will offer both failures and untested workers wages equal to their expected productivities. The original employer chooses the best contract to deter type 2 workers from applying for jobs. Since  $p_2 < p_1$ , it is obvious that a firm would not pay failures more than the value of their output. Firms also will not want to pay the untested more than  $Q_1$ , since that is equivalent to reducing the application fee and making it more difficult to deter applications from type 2 workers. Hence given our assumptions of constant returns to scale, and restricting our analysis to cases when firms earn zero profits, w.l.o.g. we can assume that when many firms compete for type 1 workers the equilibrium contracts entail firing all failures, and retaining the untested workers.<sup>15,16</sup>

In order to attract type 1 workers and deter type 2 workers from applying for the job, firms offer contracts that maximize type 1's utility while keeping the firms' profits non-negative subject to deterring type 2.

We express the efficiency wage of the firms and the utilities of the workers as:

$$e = \frac{(1-z)(L-m)Q_1 + z(L-m)p_1w^P - C}{zmQ_1^* + (1-z)LQ_1 + z(L-m)p_1Q_1^P} \quad (45)$$

$$U_1 = (1-z)(L-m)Q_1 + z(L-m)[p_1w^P + (1-p_1)\bar{w}_1] + S - C \quad (46)$$

$$U_2 = (1-z)(L-m)Q_1 + z(L-m)[p_2w^P + (1-p_2)\bar{w}_2] + S - C \quad (47)$$

where  $\bar{w}_1 = \max(Q_1^f, w_1)$  and  $\bar{w}_2 = \max(Q_1^f, w_2)$  are respectively the opportunity wages of type 1 and type 2 workers when they are identified as type 1 failures.

$$\begin{array}{ll} \text{Firms maximize} & U_1 \\ \text{s.t.} & e \leq 1 \end{array} \quad (48)$$

$$U_2 \leq w_2L + S \quad (49)$$

$$C \leq S \quad (50)$$

As in Section II, firms charge each applicant an application fee  $C = S$ , the maximum of the applicants' wealth, since firms want to deter type 2 workers from applying. Following footnote 15, we set  $e=1$ . Therefore,

$$w^P = \frac{zmQ_1^* + (1-z)mQ_1 + z(L-m)p_1Q_1^P + S}{z(L-m)p_1} \quad (51)$$

Substituting for  $w^P$  in (46), we have

$$U_1 = (1-z)LQ_1 + zmQ_1^* + z(L-m)[p_1Q_1^P + (1-p_1)\bar{w}_1] + S \quad (52)$$

$$\text{Let } G(m) = mQ_1^* + (L-m)[p_1Q_1^P + (1-p_1)\bar{w}_1] - LQ_1 \quad (53)$$

be the gain (or loss if negative) in the labor input of the firm from testing a previously untested worker, when workers that fail the test are fired, the test is of length  $m$ , and only type 1 workers are tested. Substituting for  $G(m)$ ,

$$U_1 = LQ_1 + zG(m) + S \quad (54)$$

$$\frac{\partial U_1}{\partial z} = G(m) \quad (55)$$

$$\frac{\partial U_1}{\partial m} = zQ_1^* - z[p_1Q_1^P + (1-p_1)\bar{w}_1] < 0 \quad (56)$$

Substituting  $w^P$  from (52) in (48), we have

$$U_2 = (1-z)(L-m)Q_1 + z(L-m) \left[ (1-p_2)\bar{w}_2 + p_2 \frac{zmQ_1^* + (1-z)mQ_1 + z(L-m)p_1Q_1^P + S}{z(L-m)p_1} \right] \\ = (1-z)(L-m)Q_1 + z(L-m)(1-p_2)\bar{w}_2 + \frac{p_2}{p_1} \left[ zmQ_1^* + (1-z)mQ_1 + z(L-m)p_1Q_1^P + S \right] \quad (57)$$

So

$$\frac{\partial U_2}{\partial m} = -(1-z)Q_1 - z(1-p_2)\bar{w}_2 + \frac{p_2}{p_1} \left[ zQ_1^* + (1-z)Q_1 - z p_1Q_1^P \right] \\ = -(1-z) \left( 1 - \frac{p_2}{p_1} \right) Q_1 - z \frac{p_2}{p_1} \left[ p_1Q_1^P + \frac{(1-p_2)p_1}{p_2} \bar{w}_2 - Q_1^* \right] \quad (58)$$

Since  $(1-p_2)p_1/p_2 > 1 - p_1$ , the term in square brackets above is positive. Therefore,  $\partial U_2/\partial m < 0$ , i.e., an increase in  $m$  hurts both type 1 and type 2 workers. Differentiating  $U_2$  with respect to  $z$ , we have

$$\frac{\partial U_2}{\partial z} = -(L-m)Q_1 + (L-m)(1-p_2)\bar{w}_2 + \frac{p_2}{p_1} \left[ mQ_1^* - mQ_1 + (L-m)p_1Q_1^P \right]$$

$$- (L-m) [ p_2 Q_1^P + (1-p_2) \bar{w}_2 - Q_1 ] - \frac{P_2}{P_1} (Q_1 - Q_1^*)^m \quad (59)$$

Notice that for  $m < L$   $\partial U_2 / \partial z$  can be either positive or negative. That is, increasing the probability of a worker being tested can either increase or decrease type 2 workers' utility. It is easy to see that it can hurt type 2 workers. The intuition for why it can help them comes from (51). If  $Q_1^P$  is sufficiently large,  $p_2 w^P$  will exceed  $Q_1$ . In that case, type 2 workers prefer to be in the apprenticeship program than to be paid  $Q_1$  with certainty. On the other hand from (51) we also see that increasing  $z$  decreases  $w^P$ . If the first effect outweighs the second,  $\partial U_2 / \partial z$  will be positive, and type 2 workers can only be deterred by increasing the length of the testing period (increasing  $m$ ). This has the effect of implicitly increasing the application fee.

From (57),  $U_2 \leq w_2 L + S$  can be written as

$$m \geq \frac{z L (p_2 Q_1^P + (1-p_2) \bar{w}_2 - Q_1) + \left[ L(Q_1 - w_2) - \left(1 - \frac{P_2}{P_1}\right) S \right]}{z \left[ p_2 Q_1^P + (1-p_2) \bar{w}_2 - \frac{P_2}{P_1} Q_1^* - \left(1 - \frac{P_2}{P_1}\right) Q_1 \right] + \left(1 - \frac{P_2}{P_1}\right) Q_1} \quad (60)$$

$U_2 = w_2 L + S$  could be upward or downward sloping in  $(z, m)$  space. From (59), we know that it is downward sloping if

$$p_2 Q_1^P + (1-p_2) \bar{w}_2 \leq Q_1 \quad (61)$$

since  $\partial U_2 / \partial z$  is negative in this case and  $\partial U_2 / \partial m$  is always negative. In the case when (61) does not hold,  $U_2 = w_2 L + S$  is downward sloping if and only if

$$L \left[ p_2 Q_1^P + (1-p_2) \bar{w}_2 - Q_1 \right] \left(1 - \frac{P_2}{P_1}\right) Q_1 \leq \left[ L(Q_1 - w_2) - \left(1 - \frac{P_2}{P_1}\right) S \right] \cdot \left[ p_2 Q_1^P + (1-p_2) \bar{w}_2 - \frac{P_2}{P_1} Q_1^* - \left(1 - \frac{P_2}{P_1}\right) Q_1 \right] \quad (62)$$

which is equivalent to

$$\begin{aligned} & \left[ p_2 Q_1^P + (1 - p_2) \bar{w}_2 - Q_1 \right] \left[ L \left( 1 - \frac{p_2}{p_1} \right) Q_1 - L (Q_1 - w_2) + \left( 1 - \frac{p_2}{p_1} \right) S \right] \\ & \leq \left[ L(Q_1 - w_2) - \left( 1 - \frac{p_2}{p_1} \right) S \right] \frac{p_2}{p_1} (Q_1 - Q_1^*) \end{aligned} \quad (63)$$

$$\text{If } L \left( 1 - \frac{p_2}{p_1} \right) Q_1 \leq L (Q_1 - w_2) + \left( 1 - \frac{p_2}{p_1} \right) S \quad (64)$$

then the left-hand-side of (63) is negative while the right-hand-side is positive, so (63) could not hold. If (64) does not hold, i.e.,

$$\frac{w_2 L + S}{Q_1 L + S} > \frac{p_2}{p_1} \quad (65)$$

(63) will hold if and only if

$$p_2 Q_1^P + (1 - p_2) \bar{w}_2 \leq Q_1 + \frac{\left[ L(Q_1 - w_2) - \left( 1 - \frac{p_2}{p_1} \right) S \right] \frac{p_2}{p_1} (Q_1 - Q_1^*)}{(w_2 L + S) - \frac{p_2}{p_1} (Q_1 L + S)} \quad (66)$$

Therefore, we obtain the following lemma:

Lemma

Given that firms are offering contracts that break even,  $U_2 = w_2 L + S$  is downward sloping if and only if either (61) holds or (65) and (66) hold. In all other cases,  $U_2 = w_2 L + S$  is upward sloping.

When  $U_2 = w_2 L + S$  is downward sloping,  $\partial U_2 / \partial z < 0$ , i.e., holding  $m$  fixed, an increase in  $z$  will hurt the type 2 workers. Since this can happen only when  $p_2 Q_1^P + (1 - p_2) \bar{w}_2$  is low relative to  $Q_1$ , type 2 workers would prefer

being untested and paid  $Q_1$ . This is because an untested worker will get  $Q_1$  after the apprenticeship program while a tested type 2 worker will get  $p_2 w^P + (1-p_2)\bar{w}_2$ . As  $w^P$  has to satisfy the budget constraint of the firms, it is related to  $Q_1^P$ . When  $p_2 Q_1^P + (1-p_2)\bar{w}_2$  is large, type 2 workers prefer being tested. When  $p_2 Q_1^P + (1-p_2)\bar{w}_2$  is small, type 2 workers prefer being untested.

Condition (65) determines whether  $z \rightarrow 0$ ,  $m = L$  is feasible in deterring type 2 workers. If (65) holds, then such deterrence is feasible. If (65) does not hold, then such a contract will not deter the type 2 workers from applying for the job.

Let  $\bar{z}$  be the minimum  $z$  that satisfies  $U_2 \leq w_2 L + S$  given  $m = \bar{m}$ . Recall that  $\bar{m}$  and  $m^*$  are the minimum values of  $m$  that satisfy  $U_2 \leq w_2 L + S$  when  $z=1$  and  $z \rightarrow 0$ , respectively.

### Theorem 3

Suppose  $U_2 = w_2 L + S$  is upward sloping in  $(z, m)$  space; then

- (1) if  $G(\max\{m^*, \bar{m}\}) \leq 0$ ,  $m = \max\{m^*, \bar{m}\}$ ,  $z = 0$ ;
- (2) if  $G(\max\{m^*, \bar{m}\}) > 0$ ,  $m \in [\max\{m^*, \bar{m}\}, \bar{m}]$ ,  $z \in [\bar{z}, 1]$ .

Suppose  $U_2 = w_2 L + S$  is downward sloping in  $(z, m)$  space; then

- (3) if  $G(\max\{\bar{m}, \bar{m}\}) > 0$ ,  $m = \max\{\bar{m}, \bar{m}\}$ ,  $z = 1$ ;
- (4) if  $G(\max\{\bar{m}, \bar{m}\}) \leq 0$  and (65) holds,  $m = \max\{m^*, \bar{m}\}$ ,  $z = 0$ ;
- (5) if  $G(\max\{\bar{m}, \bar{m}\}) \leq 0$  and (65) does not hold,  $m \in [\bar{m}, L]$ ,  $z \in [z^*, \bar{z}]$ .

### Proof

When  $U_2 = w_2 L + S$  is upward sloping,  $\max\{m^*, \bar{m}\}$  is the minimum value of  $m$  that is feasible in deterring applications from type 2 workers. If for this  $m$ ,  $G(m) \leq 0$ , then  $z \rightarrow 0$  is optimal. If for this  $m$ ,  $G(m) > 0$ , then  $(z, m)$  lies on  $U_2$  between  $\max\{m^*, \bar{m}\}$  and  $\bar{m}$ , and the corresponding  $z$  between  $\bar{z}$  and 1.



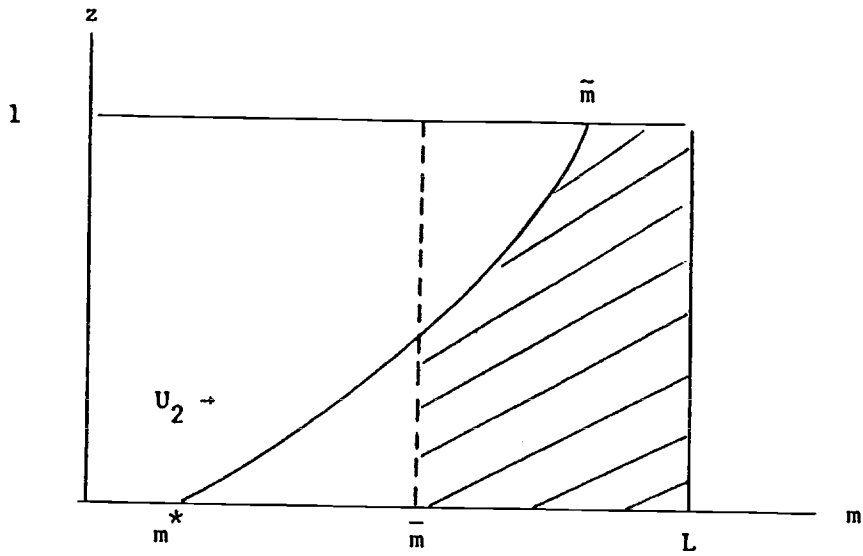


Figure 3. Optimal contract when  $U_2 = w_2L + S$  is upward sloping

When  $U_2 = w_2L + S$  is downward sloping,  $\max(\tilde{m}, \bar{m})$  is the minimum  $m$  that is feasible in deterring applications from type 2 workers.

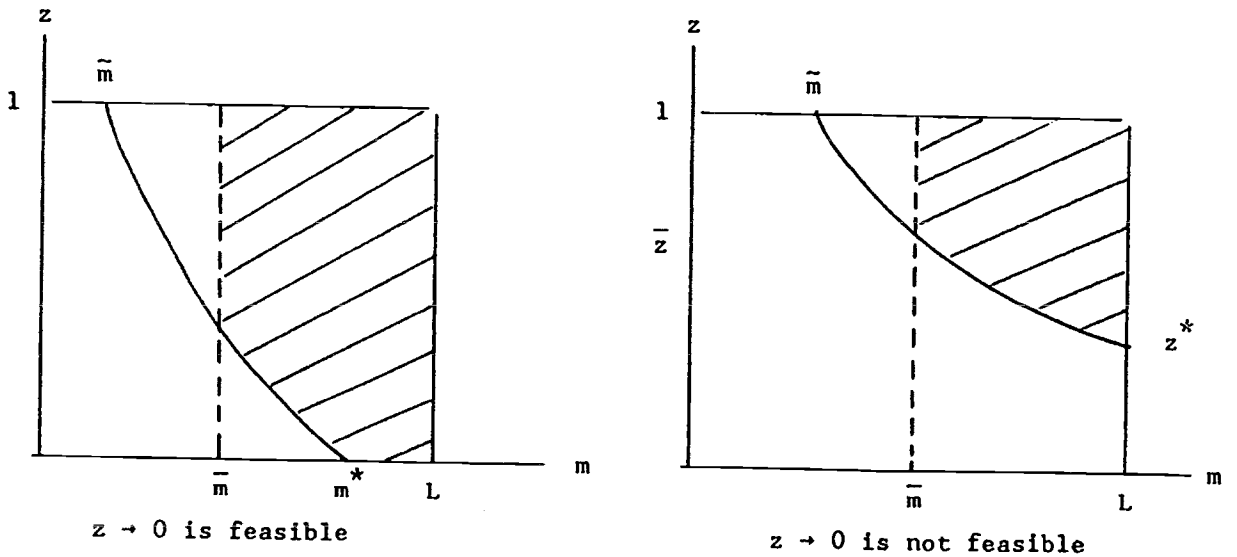


Figure 4. Optimal contract when  $U_2 = w_2L + S$  is downward sloping

If for this  $m$ ,  $G(m) > 0$ , then  $z = 1$  at this  $m$  will be optimal. If for this  $m$ ,  $G(m) < 0$ , however, then the result depends on whether  $z \rightarrow 0$  is

feasible in deterring type 2 workers or not. If it is feasible, then  $z \rightarrow 0$ ,  $m \rightarrow \max(m^*, \bar{m})$  is the optimal. This is because  $z \rightarrow 0$  will always maximize the utility of type 1 from (54), since  $G(m) < 0$ . If it is not feasible, then the optimal  $m$  will lie on the boundary between  $\bar{m}$  and  $L$  and the corresponding  $z$  is between  $[z^*, \bar{z}]$ .

QED

#### IV. Testing: One Firm Case vs. Many Firm Case

In many models competition among firms for workers improves the allocation of resources, and thus increases total output. In our model of adverse selection, this is not necessarily true. Comparing (2) in Theorem 1 with (5) in Theorem 3, we see that there are conditions under which the cost of testing outweighs its benefit:  $G(m) < 0$  and  $Q_1^f < w_1$ , and competitive firms test a strictly positive proportion of their applicants while a single, idiosyncratic firm would not. This is because when firms compete for type 1 workers they bid up wages, increasing the proportion of workers that a firm has to monitor to deter the type 2 workers. As we show in the following example, the cost of the test may be such that the total output of the workers is reduced.

##### Example

Let  $Q_1 = 10$ ,  $Q_1^p = 20$ ,  $Q_1^f = 0$ ,  $Q_1^* = 0$ ,  $p_1 = 0.5$ ,  $p_2 = 0$ ,  $L = 1$ ,  $m = 0.25$ ,  
 $w_1 = 2$ ,  $w_2 = 1$ ,  $S = 0$ .

It is easy to check from (13) and (14) that  $G(m) < 0$ ,  $g(m) < 0$ , which means that going through the apprenticeship program will lower the total output of the type 1 workers.

In the many firm case, firms' profit is equal to zero because of competition. In order to maximize type 1 workers' utility,  $z$  should be as

small as possible. From (57) and (49), the minimum  $z$  that can still deter the type 2 workers for applying is 0.963.

In the one firm case, dismissed workers receive their reservation wage. Because  $Q_1 > w^1 > w^2 > Q_1^f$ , firing untested workers has a greater deterrence effect in the one firm case than in the many firm case. In the one firm case,  $z \rightarrow 0$  and  $w^p \rightarrow \infty$ , and  $z \cdot w^p$  is just enough to attract the type 1 workers while deterring the type 2 workers, when the untested are paid a wage of zero during the testing period and  $w_1$  afterwards.

In the one firm case expected output per employed type 1 workers is 10. In the many firm case it is  $.037(10) + (.963)(.75)[(.5)20 + (.5)2] = 8.315$ . (Note that in the many firm case we include the output of self employed type 1 workers. In neither case are we including the effects of industry structure on levels of aggregate employment.)

#### V. Effect of Minimum Wage Legislation

Suppose that there is a minimum wage law requiring all workers to be paid at least  $w^*$ . We shall initially assume that the minimum wage is less than  $\min(w_2, Q_1^f)$  so that it only affects wages during the apprenticeship program. Obviously, no firm will pay more than this. Holding the length of the training program fixed, a higher wage is equivalent to a reduction in the application fee which firms can charge. Thus, the effect of an increase in the mandatory minimum wage is the same as a decrease in the application fee  $S$ . This makes it more difficult for firms to deter the type 2 workers.

First consider the many firm case. When firms make zero profit due to competition in the hiring market, and the incentive constraint for type 2 workers is binding, an increase in the minimum wage decreases the utility of type 1 workers, this is because type 2 workers are more difficult to deter.

Notice that the total output and the utility of the type 1 workers are moving in the same direction in this case, a raise in the minimum wage will decrease total output.<sup>17</sup>

Turning to the single firm case, since the firm could have paid the higher wage mandated by the minimum wage law and chose not to, and since the selection constraints for both types of workers are binding, increasing the minimum wage must increase the firm's cost per efficiency unit of labor. As the firm has to test more to deter the type 2 workers, the aggregate output will decrease.

The situation is considerably more complex if the increase in the minimum wage limits the employment opportunities of workers that fail the test. In the many firm case this will occur if the minimum wage is increased from a value below  $Q_1^f$  to a value above  $Q_1^f$ , so that rather than being employed at a wage of  $Q_1^f$ , workers that fail a firm's test become self employed at wages of  $w_1$  or  $w_2$  depending on their type. Since  $w_1 < w_2$  this effect will help deter applications from type 2 workers. Consequently, an increase in the minimum wage law could either tighten or loosen the conditions for deterring applications from type 2 workers. This ambiguity causes a corresponding ambiguity in the effect of the minimum wage law on the proportion of workers that are tested. If an increase in the minimum wage increases the penalty from being fired, firms will be able to decrease the proportion of workers that they test while still deterring type 2 workers from applying. In that case if  $G(m)$  is negative, and testing is socially wasteful, an increase in the minimum wage law could increase total output. The gain from lowering  $z$  would outweighs the loss in output from the type 1 workers being self employed rather than employed at a firm paying them  $Q_1^f$ .

In the special case in which  $w_2 \ll w_1 = Q_1^f = Q_1^*$  and testing is socially wasteful, an arbitrarily small increase in the minimum wage from slightly

below  $Q_1^f$  to slightly above it will increase output. This is because the effect of the increase minimum wage on the wage paid during the apprenticeship would be small, and there is no loss in output from forcing the type 1 workers into self employment. On the other hand forcing the type 2 workers who fail the test into self employment would have a large effect on the deterrence of type 2 workers. Consequently the optimal contract would entail a smaller value of  $z$ , and consequently an increase in aggregate output. Note that this anomalous result is due to the "beneficial" effects of forcing workers who fail a test into self employment.

## VI. Summary

We have characterized the labor contracts offered by firms when workers have information about their own productivity that is not available to firms, and firms can administer tests to workers. These tests are imperfect. They take the form of an apprenticeship or training period during which the performance of the worker may be monitored.

In order to discourage applications from workers with bad private information about their own productivity, firms will pay workers that pass their tests more than the expected value of their output, and pay workers less than the expected value of their outputs during the testing period. Thus the results are consistent with steep wage tenure profiles, and the use of mandatory retirement rules. The reason for mandatory retirement in this model is similar to that in Lazear (1979). Because senior workers are paid more than the value of their output, they would not quit when the value of their output falls below the opportunity cost of their time. Either mandatory retirement, or wage cuts around the time when retirement is efficient, increases the total return from the employment relationship.

If firms are idiosyncratic, so that a worker's productivity with the firm that is testing her is uncorrelated with her productivity elsewhere, it may be the case that some workers who fail the firm's test are retained. For workers that fail the test to be retained it is necessary that the firm's cost per efficiency unit of labor be lower from retaining those workers than its cost per efficiency unit of labor from replacing them with new hires. (The latter cost is calculated as the minimum of the cost of an untested worker and a tested worker, ignoring the incentive effects of testing.) Obviously this is quite a strong condition. For *all* workers to be retained another condition must be satisfied:  $g(m) < 0$ , so that the cost of testing a worker must be greater than the efficiency gain from identifying the low productivity workers, i.e.,  $g(m) < 0$ . Note, that these are only some of the necessary conditions for workers to be retained. The other necessary conditions, presented in Theorem 2, are somewhat more complicated. Since even the necessary conditions discussed immediately above are unlikely to be satisfied, either version of our model could explain the high dismissal rates of newly hired workers.

## Appendix A

### Conditions under which Firms Make Positive Profit

In this appendix, we consider the case when the zero-profit constraint (48) for firms' profit is not binding in an optimal contract; i.e., (48) is not binding when firms maximize  $U_1$  subject to (49). In this case, firms make positive profits, since any contract that offers more utility to type 1 workers will at the same time attract type 2 workers. Because of the competition for the type 1 workers, firms must not offer a less attractive contract to type 1 workers, since otherwise no type 1 workers will apply for the job.

We shall investigate conditions under which  $e < 1$ . To derive necessary and sufficient conditions for  $e < 1$ , we first characterize the contract that maximizes  $U_1$  subject to  $U_2 \leq w_2L + S$ . We then present conditions under which that contract would lead to  $e < 1$ . We first show that if  $e < 1$  in the optimal contract it must be that  $z = 1$ . We then maximize  $U_1$  subject to (48) - (50).

It is easy to see that (49) is binding in the solution, since an increasing in  $w^P$  will increase both type 1 and type 2 workers' utilities. Therefore, if (49) is not binding,  $w^P$  can be increased a bit while keeping other terms unchanged and type 1 workers are better off from this.

From  $U_2 = w_2L + S$  in (47) letting  $C = S$ , we have

$$w^P = \frac{w_2L + S - (1-z)mQ_1 - z(L-m)(1-p_2)\bar{w}_2}{z(L-m)p_2} \quad (a1)$$

Substituting (a1) into  $U_1$  in (46), we have

$$U_1 = (1-z)(L-m)Q_1 + \frac{P_1}{P_2} [w_2L+S - (1-z)(L-m)Q_1 - z(L-m)(1-p_2)\bar{w}_2] + (1-p_1)\bar{w}_1z(L-m) \quad (a2)$$

$$\begin{aligned} \frac{dU_1}{dz} &= - (L-m)Q_1 + \frac{P_1}{P_2} [(L-m)Q_1 - (L-m)(1-p_2)\bar{w}_2] + (1-p_1)\bar{w}_1(L-m) \\ &\geq \frac{P_1 - P_2}{P_2} (L-m)Q_1 + \left[ (1-p_1) - \frac{P_1}{P_2} (1-p_2) \right] \bar{w}_1(L-m) \end{aligned} \quad (a3)$$

$$\begin{aligned} &= \frac{P_1 - P_2}{P_2} (L-m)(Q_1 - \bar{w}_1) \\ &\geq 0 \end{aligned} \quad (a4)$$

(a3) holds because  $\bar{w}_1 \geq \bar{w}_2$ ; (a4) holds because of our assumption  $Q_1 \geq w_1$ , and thus  $Q_1 \geq \bar{w}_1$ . So  $z = 1$  will maximize  $U_1$  subject to type 2 workers' incentive constraint. The intuition is that the slopes of  $U_1$  and  $U_2$  differ in  $w^P, z$  space.  $U_1$  is flatter because  $p_1 > p_2$  and  $\bar{w}_1 > \bar{w}_2$ . Increase in  $w^P$  and  $z$  along indifference curve of type 2 benefits type 1. Therefore, if  $e < 1$ ,  $z = 1$ . Substituting for  $z = 1$  in (a2), we have

$$U_1 = \frac{P_1}{P_2} [w_2L+S - (L-m)(1-p_2)\bar{w}_2] + (1-p_1)\bar{w}_1(L-m) \quad (a5)$$

If  $p_2(1-p_1)\bar{w}_1 > p_1(1-p_2)\bar{w}_2$ , then  $m = \bar{m}$  will maximize  $U_1$ . Let  $\bar{w}^P$  be the  $w^P$  that corresponding to  $m = \bar{m}$ ,  $z = 1$  in (a1). In this case,  $e < 1$  if and only if

$$\frac{(L - \bar{m}) p_1 \bar{w}^P - S}{\bar{m}Q_1^* + (L - \bar{m}) p_1 Q_1^P} < 1 \quad (a6)$$

If  $p_2(1-p_1)\bar{w}_1 \leq p_1(1-p_2)\bar{w}_2$ , then  $m \rightarrow L$  will maximize  $U_1$ . In this case,  $w^P \rightarrow \infty$ . However,  $(L-m) \cdot w^P$  is determined by  $U_2 = w_2L + S$  given  $z = 1$ :



$$(L-m) \cdot w^P = \frac{1}{p_2} (w_2 L + S) \quad (a7)$$

In this case,  $e < 1$  if and only if

$$e = \frac{\frac{p_1}{p_2} (w_2 L + S) - S}{LQ_1^*} < 1 \quad (a8)$$

To see how likely it is for the equilibrium to be characterized by positive profits ( $e < 1$ ), let us consider the case where  $S=0$  and  $Q_1^f > w_1$ , so  $p_2(1-p_1)\bar{w}_1 < p_1(1-p_2)\bar{w}_2$  and  $p_1 w_2 / p_2 Q_1^* < 1$  becomes relevant condition for positive profit equilibrium. For  $w_2 / Q_1^*$  sufficiently small, this condition is always satisfied so that firms earn positive profit.

This result, however, is an artifact of our assumption that there is a finite number of types. If the distribution of types is atomless, positive profit equilibrium do not exist.

## Appendix B

### Optimal Contracts When Firms Can Fire Untested Workers

In this appendix, we consider the case when firms can commit themselves to fire some proportion of their untested workers. In the many firm case, this option has no effect, since the untested workers are paid  $Q_1$  whether they are fired or not. In the one firm case, this option affects the optimal contract.

Let  $\varphi$  be the proportion of untested workers that are fired. The firm's efficiency wage and the workers' utility functions are now expressed as:

$$e = \frac{H + (L - m) [z p_1 w^P + z (1-p_1)(1-t)w_1 + (1-z)(1-\varphi)w_1] - F}{zmQ_1^* + (1-z)mQ_1 + (L-m) [z p_1 Q_1^P + z(1-p_1)(1-t)Q_1^f + (1-z)(1-\varphi)Q_1]} \quad (b1)$$

$$U_1 = z(L - m) [p_1 w^P + (1-p_1) w_1] + (1-z)(L-m)w_1 \quad (b2)$$

$$U_2 = z(L - m) [p_2 w^P + (1-p_2) t w_2 + (1-p_2)(1 - t)w_1] + (1-z)(L-m) [ (1-\varphi)w_1 + \varphi w_2] \quad (b3)$$

Once again, the optimization problem of the firm is:

$$\min e \quad (b4)$$

$$\text{s.t. } U_1 \geq w_1 L + S \quad (b5)$$

$$U_2 \leq w_2 L + S \quad (b6)$$

We assume that the type 2 workers will not choose the contract if  $U_2 - w_2 L + S$ .

(b5) must be binding in the optimal contract. If we get a contract that

satisfies constraints (b5), (b6), while (b5) is not binding, then we can lower  $w^P$  without violating the constraints and therefore  $e$  is lower.

Proposition

The firm does not fire any untested workers unless it fires all workers that failed its test.

Proof Type 1 untested workers produce more than type 1 failures:  $Q_1 > Q_1^f$ , but type 1 failures are paid the same wage as the untested. Also, type 2 workers have a higher probability of being a failure if tested. If  $\varphi > 0$  when  $t < 1$ , then an increase in  $t$  accompany with a decrease in  $\varphi$  while keeping the total percentage of workers fired  $(1-z)(1-\varphi) + z(1-p_1)(1-t)$  unchanged will decrease  $e$  and  $U_2$  and keep  $U_1$  unchanged.

QED

We now characterize the optimal contract, when the parameter values are such that all workers that fail the firm's test are fired, i.e.  $t = 1$ .<sup>18</sup>

From  $U_1 = w_1L + S$ , we solve for  $w^P$  and substitute it into (b1) and (b3),

$$w^P = \frac{w_1L + H - (1-z)(L-m)w_1 - z(L-m)(1-p_1)w_1}{z(L-m)p_1} \quad (b7)$$

$$e = \frac{H + w_1L - (L-m)w_1 [z(1-p_1) + (1-z)\varphi]}{zmQ_1^* + (1-z)mQ_1 + (L-m)[z p_1 Q_1^P + (1-z)(1-\varphi)Q_1]} \quad (b8)$$

$$U_2 = z(L-m)[p_2w_1 + (1-p_2)w_2] + (1-z)(L-m)\{(1-\varphi)w_1 + \varphi w_2\} + \frac{P_2}{P_1} [S + mw_1] \quad (b9)$$

Thus (b6) can be rewritten as follows:

$$\varphi \geq \frac{(1-z)(L-m)w_1 + z(L-m)[p_2w_1 + (1-p_2)w_2] + \frac{P_2}{P_1} (S + mw_1) - w_2L - S}{(1-z)(L-m)(w_1 - w_2)}$$

$$= 1 - p_2 + \frac{K}{(1-z)(L-m)(w_1 - w_2)} \quad (b10)$$

which defines the parameter of  $K$  and the set of feasible contracts in  $(m, \varphi)$  space, and on the boundary, i.e.,  $U_2 = w_2L + S$ , we have

$$\varphi = 1 - p_2 + \frac{K}{(1-z)(L-m)(w_1 - w_2)}$$

$$\left. \frac{d\varphi}{dz} \right|_{U_2 = w_2L + S} = \frac{K}{(1-z)^2(L-m)(w_1 - w_2)} = \frac{\varphi - (1 - p_2)}{(1 - z)} \quad (b11)$$

where  $K$  is defined in (b10).

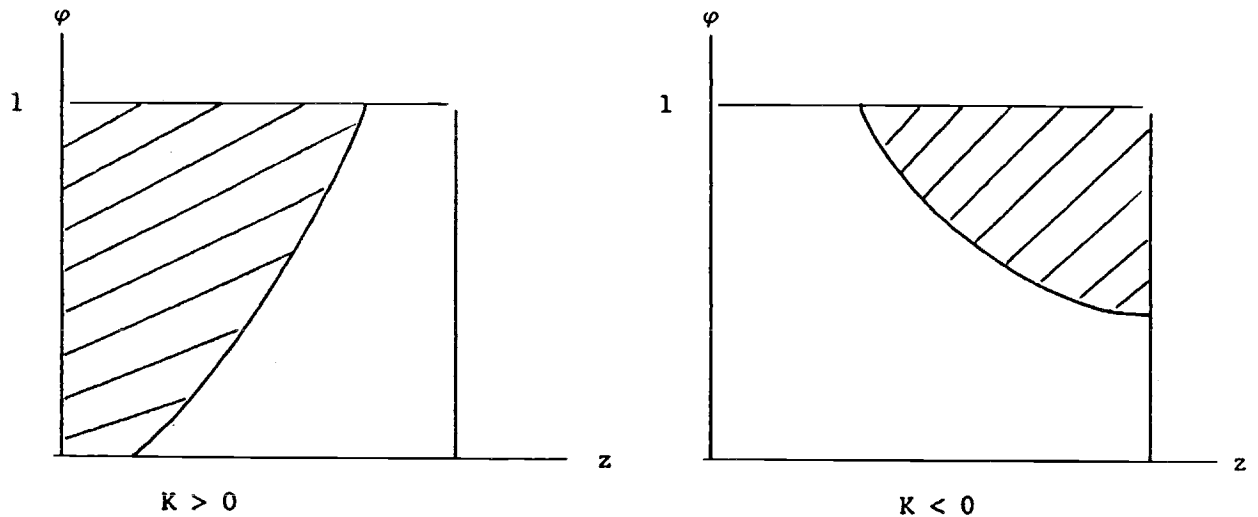


Figure 5. The set of feasible contracts

$$\text{Let } \Psi = H[G(m) + (p_1 - p_2)(L - m)Q_1] + w_1 G(m) \frac{(1 - \frac{p_2}{p_1})(S + mw_1)}{w_1 - w_2}$$

Theorem 4

For parameter values such that all workers that fail the firm's test are

fired (i.e.  $t = 1$ ), the firm's optimization problem has the following solutions:

Case 1  $K > 0$

If  $w_1 G(m) \geq HQ_1$ , then  $\varphi = 1$ ,  $z = z^*$ ;

If  $w_1 G(m) < HQ_1$ , then

$\varphi = \varphi^*$ ,  $z \rightarrow 0$  if  $\Psi \leq 0$ ;

$\varphi = 1$ ,  $z = z^*$  if  $\Psi > 0$ .

Case 2  $K \leq 0$

If  $w_1 G(m) \geq HQ_1$ , then  $\varphi = 1$ ,  $z = 1$ ;

If  $w_1 G(m) < HQ_1$ , then

$\varphi = \varphi^*$ ,  $z \rightarrow 0$  if  $\Psi \leq 0$ ;

$\varphi = 0$ , if  $\Psi > 0$ .

Proof

$$\begin{aligned} \frac{\partial e}{\partial \varphi} &= \frac{1}{D^2} \left\{ - (1-z)(L-m)w_1 \left[ zmQ_1^* + (1-z)mQ_1 + (1-z)(L-m)Q_1 + z(L-m)p_1Q_1^P \right] \right. \\ &\quad \left. + (1-z)(L-m)Q_1 \left[ H + w_1L - z(L-m)w_1(1-p_1) \right] \right\} \\ &= - \frac{(1-z)(L-m)}{D^2} \left\{ zw_1 \left[ (L-m)(p_1Q_1^P + (1-p_1)Q_1) + mQ_1 - LQ_1 \right] - H Q_1 \right\} \\ &= - \frac{(1-z)(L-m)}{D^2} \left\{ zw_1 G(m) - H Q_1 \right\} \end{aligned} \quad (b12)$$

Similarly, we have

$$\begin{aligned} \frac{\partial e}{\partial z} &= \frac{1}{D^2} \left\{ \left[ \varphi(L-m)w_1 - (L-m)w_1(1-p_1) \right] \left[ mQ_1 + (1-\varphi)(L-m)Q_1 \right] \right. \\ &\quad \left. - \left[ mQ_1^* - mQ_1 - (1-\varphi)(L-m)Q_1 + (L-m)p_1Q_1^P \right] \left[ A + w_1L - \varphi(L-m)w_1 \right] \right\} \end{aligned} \quad (b13)$$

$$\begin{aligned}
& - - \frac{1}{D^2} \left\{ w_1 [L - \varphi(L-m)] G(m) + H [mQ_1^* + (L-m)p_1 Q_1^P - [L-\varphi(L-m)] Q_1] \right\} \\
& - - \frac{1}{D^2} \left\{ (L - \varphi(L-m)) [w_1 G(m) - HQ_1] + H [mQ_1^* + (L-m)p_1 Q_1^P] \right\} \quad (b14)
\end{aligned}$$

Along  $U_2 = w_2 L + S$ , the change of  $e$  is given by:

$$\begin{aligned}
\frac{de}{dz} &= \frac{\partial e}{\partial z} + \frac{\partial e}{\partial \varphi} \frac{d\varphi}{dz} \Big|_{U_2 = w_2 L + S} \\
& - - \frac{1}{D^2} \left\{ (L - \varphi(L-m)) [w_1 G(m) - HQ_1] + H [mQ_1^* + (L-m)p_1 Q_1^P] \right\} \\
& - - \frac{(1-z)(L-m)}{D^2} \left\{ zw_1 G(m) - HQ_1 \right\} \frac{\varphi - (1-p_2)}{(1-z)} \\
& - - \frac{1}{D^2} \left\{ (L - \varphi(L-m)) [w_1 G(m) - HQ_1] + H [mQ_1^* + (L-m)p_1 Q_1^P] \right. \\
& \quad \left. + (L-m) [\varphi - (1-p_2)] [zw_1 G(m) - HQ_1] \right\} \\
& - - \frac{1}{D^2} \left\{ H [G(m) + (p_1 - p_2)(L-m)Q_1] + w_1 G(m) [L - \varphi(L-m) + z [\varphi - (1-p_2)](L-m)] \right\}
\end{aligned}$$

But along  $U_2 = w_2 L + S$ , from (b10), we have

$$\begin{aligned}
& L - \varphi(L-m) + z [\varphi - (1-p_2)](L-m) \\
& - L - (1-p_2)(L-m) - \frac{K}{(1-z)(w_1 - w_2)} + z \frac{K}{(1-z)(w_1 - w_2)} \\
& - L - (1-p_2)(L-m) - \frac{K}{w_1 - w_2}
\end{aligned}$$

$$\begin{aligned}
& - L - (1-p_2)(L-m) - \frac{(L-m)p_2(w_1 - w_2) + \frac{P_2}{P_1}(S + mw_1) - S - mw_2}{w_1 - w_2} \\
& - m - \frac{\frac{P_2}{P_1}(S + mw_1) - S - mw_2}{w_1 - w_2} \\
& - \left(1 - \frac{P_2}{P_1}\right) \frac{S + m w_1}{w_1 - w_2} > 0 . \tag{b15}
\end{aligned}$$

Therefore,

$$\begin{aligned}
\frac{de}{dz} &= -\frac{1}{D^2} \left\{ H [G(m) + (p_1 - p_2)(L-m)Q_1] + w_1 G(m) \frac{\left(1 - \frac{P_2}{P_1}\right)(S + mw_1)}{w_1 - w_2} \right\} \tag{b16} \\
&= -\frac{1}{D^2} \Psi
\end{aligned}$$

1) Suppose  $K > 0$ , then  $U_2 = w_2 + S$  is upward sloping in  $(z, \varphi)$  space.

If  $w_1 G(m) \geq HQ_1 > 0$ , then  $\partial e / \partial z < 0$ , which implies that  $(\varphi, z)$  in any optimal contract is on the boundary.  $G(m) > 0$  implies  $de/dz < 0$ , which in return implies that  $\varphi = 1$ ,  $z = z^*$  is optimal, where  $z^*$  can be solved by letting  $\varphi = 1$  in  $U_2 = w_2 + S$ .

If  $w_1 G(m) < HQ_1$ , then  $\partial e / \partial \varphi > 0$ , which implies that  $(\varphi, z)$  is on the boundary. However, which  $(\varphi, z)$  is optimal depends on the sign of  $de/dz$ :

a)  $de/dz \geq 0$ , i.e.,  $\Psi \leq 0$

the optimal contract is  $z \rightarrow 0$ ,  $\varphi = \varphi^*$ , where  $\varphi^*$  can be solved by letting  $z = 0$  in  $U_2 = w_2 + S$ .

b)  $de/dz < 0$ , i.e.,  $\Psi > 0$

the optimal contract is  $z = z^*$ ,  $\varphi = 1$ .

2) Suppose  $K \leq 0$ , then  $U_2 = w_2 + S$  is downward sloping in  $(z, \varphi)$  space.

If  $w_1 G(m) \geq HQ_1 > 0$ , then  $\partial e / \partial z < 0$ , which implies that  $z = 1$ . At  $z = 1$ ,  $\partial e / \partial \varphi \leq 0$ , implying  $\varphi = 1$ . So  $\varphi = 1, z = 1$  is the optimal solution.

If  $w_1 G(m) < HQ_1$ , the  $\partial e / \partial \varphi > 0$ , which implies that  $\varphi = 0$  or  $(\varphi, z)$  on the boundary if  $\varphi \neq 0$ .

a)  $de/dz \leq 0$ , i.e.,  $\Psi \geq 0$ , then  $\varphi = 0$ .

b)  $de/dz > 0$ , i.e.,  $\Psi < 0$ , then it must be that  $\partial e / \partial z > 0$ , so  $\varphi = \varphi^*, z \rightarrow 0$ .



## ENDNOTES

<sup>1</sup> Topel [1990] and Kletzer [1989] provide convincing evidence that the return to seniority is large and significantly above what is found by Altonji and Shakotko [1987], Abraham and Farber [1987], Marshall and Zarkin [1987], and Topel [1986]. Lang [1987] had shown that, except for Marshall and Zarkin [1987], the latter estimates were estimates of the lower bound of the returns to tenure.

<sup>2</sup> See for example, Nalebuff and Scharfstein [1987], Stiglitz [1975] and Guasch and Weiss [1980].

<sup>3</sup> Nalebuff and Scharfstein [1987], Stiglitz [1975] and Guasch and Weiss [1981] discuss conditions under which the proportion of workers tested goes to zero.

<sup>4</sup> While risk neutrality seems more reasonable than the other common assumption that firms can impose infinitely large penalties (in utility terms) on workers, we do not believe workers are risk neutral. This assumption was made to make the analysis tractable.

<sup>5</sup> Previous research only derived conditions under which a positive portion of workers are tested. Those models also assumed that utility was unbounded from below. (See Nalebuff and Scharfstein [1987] or Guasch and Weiss [1981].)

<sup>6</sup> See Gibbons and Katz [1990] for evidence that workers whose performance is low are more likely to be laid off for "economic" reasons.

<sup>7</sup> See for example, Burdett and Mortensen [1981], Nalebuff and Scharfstein [1987], and Guasch and Weiss [1980, 1982]. The instantaneous test analyzed in those models can be considered as special costs of our more general testing technology. To analyze a short but costly test we fix the length of the monitoring period,  $m$ , at an arbitrarily small number and choose  $Q_1^*$  such that  $m(Q_1 - Q_1^*)$  represents the fixed cost of testing.

<sup>8</sup> An alternate justification for not allowing firms to fire untested workers is to assume that firms can (costlessly) prove that a worker failed the test, but cannot prove that a worker was not tested. Hence, contracts that allowed untested workers to be fired could not preclude firms from firing workers that passed the test. As we shall see, firms wish to be able to commit themselves to retaining those workers; hence they will not offer contracts that involve firing the untested workers.

On the other hand we assume that firms will not renege on terms of their contracts in ways that would hurt workers that they intend to continue employing, even if those contract terms cannot be enforced. The justification for that assumption is that a firm could not prevent those workers from retaliating against the firm. Thus we assume firms can commit to the wages they promise.

<sup>9</sup> See Stiglitz-Weiss [1983] for a definition of enforceability or Dewatripont [1989] for a definition of renegotiation proofness. In our context, they impose the same restrictions on contracts.

<sup>10</sup> If workers cannot observe the firm's choice of  $z$ , or if that choice cannot be verified by a third party, then the only contracts that are credible are ones that require the firm to choose the value of  $z$  that is profit maximizing given the other terms in the contract. One of the advantages of unionization is that it lowers the cost (to a given worker) of observing  $z$ , and provides a means of enforcing contracts that require firms to monitor a fixed proportion of their workers. It is in the firm's interest to be able to

make these commitments: that can lower its cost per efficiency unit of labor. Hence the presence of a labor union can increase firm profits.

11 Absent minimum wage laws, firms won't choose  $w^* < 0$  because the profit maximizing contracts sets  $C = S$  so that workers have no additional means to pay for their "training".

12 This can be proved in a simple way. In order to deter the type 2 workers from applying, it is better for the firm to pay every worker zero before he knows his status and to compensate him by increasing  $w^P$ , the wage the firm pays to workers who pass the test. Since  $p_1 > p_2$ , an increase in  $w^P$  which exactly compensates type 1 workers for their lower wages during the testing period would make type 2 workers worse off. This would help the firms to separate type 1 workers from the type 2.

13 In the many firm case, if workers that are fired are paid  $Q_1^f$  by other firms then there is an additional social gain of  $zt(Q_1^f - w_1)$  that we are ignoring when we refer to  $G(m)$  as the "benefit" from testing.

14 Note that  $\delta^f > \delta^P > \delta^u$  implies  $g(m) > 0$ , so that case (3c) subsumes  $\delta^f > \delta^P > \delta^u$ .

15 We are implicitly assuming that the necessary conditions for a positive profit equilibrium are not satisfied. These conditions are derived in Appendix A. Since positive profit equilibria do not exist if there are a continuum of types of workers with no mass point; it seems reasonable to force attention on the case when  $e=1$ .

16 Going beyond our model, there is a further reason for believing that the failures would be fired. If any other firms have information about a firm's worker that is not known by the firm, then paying workers a wage equal to their expected productivity causes "winner's curse" problems. Other firms would bid away the best of its failures. This problem can be avoided by firing those workers. The winner's curse problem is far less likely arise for workers that pass the test, since they are paid more than the expected value of their output.

17 If the incentive constraint of type 2 workers is not binding then small changes in the minimum wage have no effect. They will be fully offset by a reduction in  $w^P$ . In the special case where firms make positive profits, so  $z = 1$  and  $t = 1$ , an increase in the minimum wage would actually lower the utility of type 1 workers and increase firm profit. This is because a rise in  $w^*$  that is offset by a fall in  $w^P$  that keep the expected income of type 1 workers unchanged raises the expected income of type 2. Thus the fall in  $w^P$  must be disproportionately large relative to the increase in  $w^*$ .

18 If  $t < 1$ , then  $\varphi = 0$  and the problem is unchanged from the analysis in the body of the paper.

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