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## ABSTRACT

Like a fixed exchange rate, a target zone system may be subject to speculative attacks when the reserves of the central bank are limited. This paper analyzes such speculative attacks and their implications; it shows that the recently developed "smooth pasting" model of target zones should be viewed as a special case that emerges only when reserves are sufficiently large. The paper then uses the target zone framework to resolve a seeming paradox in predicting speculative attacks on a gold standard, arguing that such a standard may best be viewed as the boundary between one-sided target zones.

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A target zone exchange rate regime may be defined as a system in which the monetary authorities attempt to set limits on the range of variation of the exchange rate, without literally attempting to hold it fixed. Such a target zone system was proposed by Williamson (1983) as an alternative to both fixed rates and free floating; the post-Louvre exchange rate arrangements are known to involve such target zones, although the exchange rate bands have never officially been announced. Actually, even so-called fixed rate systems are usually literally target zone systems, albeit with narrow bands; thus both Bretton Woods and the European Monetary System set bands for exchange rates (with the EMS bands wider), and even the gold standard was a sort of target zone system because of the non-trivial distance between the gold points.

In recent years a new literature has emerged that sheds an interesting light on the way in which a target zone regime operates. It was initially suggested (Krugman forthcoming) that exchange rate behavior inside the band be modelled by considering the equilibrium relationship between the exchange rate and a "fundamental" subject to random shocks; this led both to a rather simple formulation of the effects of a target zone on expectations and to a useful analogy between target zone analysis and options pricing. In particular, in tying down the ends of the fundamental-exchange rate relationship it turned out that one was led to a tangency condition analogous to the "smooth pasting" condition of options theory. Subsequent papers, notably Miller and Weller (1988, 1989), Froot and Obstfeld (1989a, 1989b), Flood and Garber (1989), and Svensson (1989, 1990) have followed up that analogy in greater detail.

In the course of further formalizing the target zone model, however, these papers have somewhat changed the focus of the discussion. The original discussion of target zones attempted to think about these zones as their advocates and administrators think of them -- namely, as bounds on the exchange rate. In the subsequent theoretical literature, however, it has been usual to note that in order to enforce these bounds it is necessary for the monetary authority to control the movement of some fundamental variable, such as the velocity-adjusted money supply, and to describe policy in terms of that variable rather than in terms of the exchange rate target per se. That is, the literature has shifted from an anlysis of target zones to an analysis of the exchange rate implications of limited variation in fundamentals -- which in turn implies a limited range of variation of the exchange rate.

Thhere were two reasons for this change of focus. First, it provides a clean way of excluding the possibility of perverse policies, such as buying one's currency when it rises to the top of its band. Since concern over the possible consequences of such perverse policies had been a focus of a number of papers (e.g. Flood and Garber 1983) clearing such perversities out of the way seemed important. Second, it allows the direct application of known results from other parts of economics and finance. And the change in focus does not change the economics, as long as we are willing to assume that the monetary authority really does have the ability to enforce the target zone.

That assumption, however, represents a serious limitation. An analysis of target zones based on the assumption that they will work, like an analysis of fixed exchange rates based on the assumption that they never collapse, is useful but excludes a lot of what we might be interested in. If we are interested in studying the feasibility of target zones, and the ways in which they may break down, we need to return to a model in which the monetary authority is attempting, possibly unsuccessfully, to keep the exchange rate within bounds, and in which the policy is therefore described in terms of exchange rates rather than fundamentals.

The purpose of this paper is to take a step toward reorienting the target zone literature back toward a description of policy as based on a band for the exchange rate rather than a band for fundamentals. We consider, in particular, a model in which the monetary authority uses unsterilized intervention to attempt to limit exchange rate movements, but has limited reserves. This formulation builds a link between the new target zone literature and the older literature on speculative attack (e.g. Krugman 1979 and Flood and Garber 1984). It also provides some new intuition on the still somewhat controversial use of the "smooth pasting" condition to tie down these models.

An analysis of target zones that are subject to possible speculative attack is also useful in thinking about a puzzle that arises when one tries to analyze speculative attacks under a gold standard. The "gold standard paradox", which arises in several recent attempts to model this exchange regime, appears to be resolvable if we think of a gold standard as a boundary between two imperfectly defensible target zones.

The paper is in six parts. The first part lays out the basic exchange rate model, and derives its behavior under a pure exchange rate float. The second part considers the effect of a one-sided exchange rate target in a situation in which the monetary authority has "small" reserves (in a sense that will become apparent). This analysis turns out not to yield a smooth pasting result, but instead to yield a result closer to that found in the earlier speculative attack literature. The third part then examines how the result changes as the monetary authority's reserves get larger, and shows how there is a transition to the now-familiar smooth pasting equilibrium.

The next three sections apply the analysis to the case of a an occasionally collapsing gold standard. The fourth section sets out a gold standard model, and demonstrates the existence of what we call the gold standard paradox; the fifth section offers one resolution of that paradox, while the final section uses the target zone analysis to offer an economically more plausible resolution.

## 1. The Basic Model

We consider a basic log-linear monetary model of the exchange rate. The exchange rate at any point in time is determined by

$$s = m + v + \gamma E[ds]/dt \qquad (1)$$

where s is the log of the price of foreign exchange, m the log of the money supply, v a money demand shock term (incorporating shifts in real income, velocity, etc.), and the last term captures the effect of expected depreciation.

Money demand is assume to follow a random walk with drift:

$$dv = \mu dt + \sigma dz \qquad (2)$$

As Miller and Weller (1989) have shown, more complex processes, notably autoregressive ones, can be incorporated into the analysis without changing the qualitative results. We stick with this process for simplicity.

The general solution to the model defined by (1) and (2) for a fixed money supply has by now become familiar. It takes the form

$$s = m + v + \gamma\mu + Ae^{\alpha_1 v} + Be^{\alpha_2 v} \qquad (3)$$

where $\alpha_1$ ,$\alpha_2$ are parameters that will be determined in a moment, and A and B are free parameters that need to be tied down by the economics of the situation.

To determine $\alpha_1$ and $\alpha_2$ , we first note that by applying Ito's Lemma we have

$$E[ds]/dt = \mu + \mu[\alpha_1 Ae^{\alpha_1 v} + \alpha_2 Be^{\alpha_2 v}] + \frac{\sigma^2}{2}[\alpha_1^2 Ae^{\alpha_1 v} + \alpha_2^2 Be^{\alpha_2 v}] \qquad (4)$$

Substituting (4) back into (1), and comparing it with (3), we find that the roots are

$$\alpha_1 = \frac{-\gamma\mu + \sqrt{\gamma^2\mu^2+2\gamma\sigma^2}}{\gamma\sigma^2} > 0$$
$$\alpha_2 = \frac{-\gamma\mu - \sqrt{\gamma^2\mu^2+2\gamma\sigma^2}}{\gamma\sigma^2} < 0 \tag{5}$$

We can now turn to the economic interpretation of (3). The first three terms in (3) evidently represent a sort of "fundamental" exchange rate: they reflect the combination of money supply, money demand, and the known drift in money demand. The other terms represent a deviation of the exchange rate from this fundamental value.

Suppose that the money supply were expected to remain unchanged at its initial level forever. Notice that v can take on any value. It seems reasonable to exclude solutions for the exchange rate that deviate arbitrarily far from the fundamental level when v takes on large positive or negative values. Thus under a pure float, in which the monetary authority is expected to remain passive whatever the exchange rate may do, we may assume A = B = 0. The exchange rate equation under a pure float is therefore

$$s = m + v + \gamma\mu \tag{6}$$

## 2. An Exchange Rate Target with "Small" Reserves

Now let us suppose that the monetary authority, instead of being passive, attempts to place an upper limit on the price of foreign exchange. Specifically, the monetary authority is willing to buy foreign exchange in an unsterilized intervention, up to the limit of its reserves, when the exchange rate goes above some level $s_{max}$.

Provided that these reserves are small enough (we will calculate the critical size below), this attempt will lead to a speculative attack in which the whole of the reserves are suddenly exhausted when the exchange rate reaches $s_{max}$.

We start by defining the initial money supply as the sum of reserves and domestic credit:

$$m = \ln(D+R) \tag{7}$$

Following the speculative attack, the money supply will fall to

$$m' = \ln(D) \tag{8}$$

Figure 1 illustrates the equilibrium before and after the speculative attack. After the attack, the exchange rate will be freely floating, with money supply m'; so the post-attack exchange rate equation is

$$s = m' + v + \gamma\mu \tag{9}$$

The attack will occur when v reaches the level at which the reduction in the money supply that results from the attack validates itself, by leading to the exchange rate $s_{max}$; this is shown in Figure 1 as point C, and corresponds to the level of v, v', such that

$$s_{max} = m' + v' + \gamma\mu \tag{10}$$

What about the exchange rate before the speculative attack? First, we note that since a regime change will be triggered if v goes above a certain level, we can no longer use a no-bubbles argument to require A=0 in equation (3). (Since this exchange rate target is one-sided, there is no lower limit on v, so we still must have B=0). The pre-attack exchange rate equation is therefore

$$s = m + v + \gamma\mu + Ae^{a_1 v} \tag{11}$$

To tie down A, we use the standard speculative attack argument: there must be no foreseeable jump in the exchange rate, so we must choose A so that s = $s_{max}$ when v = v'. It is apparent from Figure 1 that this requires A<0. The result is therefore that up until the attack the knowledge that the monetary authority will attempt to defend the currency will tend to hold down the price of foreign exchange. This may be seen in Figure 1 from the fact that the actual relationship between v and s before the attack lies everywhere below the free float relationship corresponding to the initial money supply m.

When reserves are small, then, the monetary authority fails in its effort to enforce an exchange rate target. The knowledge that it will try supports the currency; but eventually the target is overrun by a speculative attack. Notice that "smooth pasting" nowhere makes its appearance in this analysis. Indeed, the pre-attack schedule in Figure 1 is not tangent to the exchange rate target.

Our next step is to enlarge the monetary authority's reserves, and show that if these reserves are sufficiently large, a "smooth pasting" solution emerges.

## 3. A Target Zone With Large Reserves

A variety of alternative potential speculative attack scenarios can be generated by varying the parameter A in equation (11). In Figure 2 we show the curves traced out by increasingly negative values of A. A small absolute value of A corresponds to a speculative attack at $C_1$. A larger absolute value of A would produce an attack somewhere to the right of $C_1$, and this attack would consume more reserves because the implied fall in the money supply -- measured as the horizontal distance from the attack point to the free float locus -- would be larger.

It is immediately apparent, however, that one cannot in this way generate arbitrarily large speculative attacks. The reason is that the family of curves corresponding to different (negative) values of A all turn downward at some point, and for a sufficiently negative A the maximum of the curve lies below $s_{max}$. But it is not possible for the exchange rate pre-attack to lie on a locus that passes above $s_{max}$ before the attack takes place, since that would trigger the central bank's intervention.

The upshot is that the analysis of the previous section is valid only if the size of reserves is not too large; specifically, if the free float locus corresponding to the money supply that would follow elimination of all reserves does not lie to the right of the point $C_2$ in Figure 2.

If reserves are larger than this level, what must happen is that the pre-attack exchange rate equation is precisely that which leads to $C_2$ . That is, A must be chosen so that the exchange rate locus is tangent to the target. Smooth pasting therefore emerges, not as the general solution of this model, but as its solution when the central bank's reserves are sufficiently large.

We can derive the critical level of reserves as follows. First, the exchange rate locus must be flat at v':

$$ds/dv = 1 + \alpha_1 A e^{\alpha_1 v'} = 0 \tag{12}$$

It must also be true that the actual exchange rate at v' is precisely the target rate $s_{max}$:

$$s_{max} = m + v' + \gamma\mu + A e^{\alpha_1 v'} \tag{13}$$

Substituting (12) into (13), we find

$$s_{max} = m + v' + \gamma\mu - \frac{1}{\alpha_1} \tag{14}$$

But now notice that immdeiately following a speculative attack, the exchange rate must also be $s_{max}$:

$$s_{max} = m' + v' + \gamma\mu \tag{15}$$

From (14) and (15) we can therefore determine the change in the money supply that occurs at the maximum size speculative attack:

$$m' - m = -\frac{\mu}{\alpha_1} \tag{16}$$

But the change in the money supply in a speculative attack depends on the ratio of reserves to domestic credit:

$$m' - m = -\ln\left(\frac{D+R}{D}\right) = -\ln\left(1+\frac{R}{D}\right) \tag{17}$$

So the nature of the equilibrium changes from speculative attack to smooth pasting when

$$\frac{R}{D} > e^{\frac{1}{\alpha_1}} - 1 \tag{18}$$

When this criterion is met, the central bank is able to hold the line at $s_{max}$ with an infinitesimal intervention that slightly reduces the money supply, shifting the relationship between v and s down. If v then falls again, the exchange rate retreats down this new schedule; if v rises, another intervention must take place. These successive interventions would gradually shift the exchange rate schedule to the right. As long as the reserves remain sufficiently large, $s_{max}$ will act as a reflecting barrier for the exchange rate, which will sometimes rise to $s_{max}$, sometimes fall below it.

It is immediately apparent, however, that this process cannot go on indefinitely. When v is high, the monetary authority loses reserves; when it falls again, it does not regain them. So there is a gradual loss of reserves, which will gradually shift the exchange rate schedule to the right. Eventually the level of reserves will fall to the critical level where a speculative attack becomes possible; at that point, the next time that the exchange rate drifts up to the level $s_{max}$ there will be a full-scale speculative attack that eliminates all remaining reserves. In other words, a country that starts with large reserves will go through a "smooth pasting" phase where small interventions succeed in holding the line on the exchange rate; but there will be a gradual (albeit intermittent) drain on reserves, and as in conventional speculative attack models there will eventually be a crisis once reserves have dropped to a critical level.

This is not a very complicated analysis. Nonetheless, it makes several points that have been obscured in some of the recent literature on the subject.

First, it is clear from this model that looking at the case of bounded fundamentals is not equivalent to looking at the case of an exchange rate target. If we were to use the bounded fundamentals technique on this model, we would replace the idea of a target on s with that of an upper limit on m+v. This would correctly capture the notion of what happens as long as reserves are sufficiently large to achieve the smooth pasting solution; but it would miss both the case where initial reserves are small, and the logic of eventual crisis.

Second, in a related point, smooth pasting is not a general result of this model, the way it appears to be in the bounded fundamentals formulation. On the contrary, it is a special case that obtains when reserves are sufficiently large; otherwise the logic is that of speculative attack: the exchange rate is tied down by the requirement that there be no foreseeable jumps in the exchange rate.

Third, this model helps settle a controversy about the justification for the smooth pasting result. Some economists approaching the problem from the perspective of optimization models have questioned the use of the smooth pasting condition in ad hoc monetary models of this kind, arguing that a condition that arises from optimization is hard to justify when optimizing behavior is at best implicit. Those of us doing the ad hoc models have argued on the contrary that the condition can equally be seen as being implied by arbitrage.[1] In this model we see smooth pasting emerge as the limit of the "no foreseeable jumps" condition of a speculative attack model -- essentially an arbitrage condition -- when reserves are sufficiently large. We hope that this will settle the question of the validity of the smooth pasting solution, and also help give some more intution about why it works.

## 4. The Gold Standard Paradox

In the remainder of this paper we make use of the type of analysis developed in earlier sections to attack a particular problem that has been the subject of several recent papers, that of the role of speculative attacks under a gold standard system.

Several papers, notably Buiter (1989) and Grilli (1989), have analyzed the problem of speculative attack in a gold standard model. Grilli implements the model empirically as well. However, these models did not use the "smooth pasting" methodology, which had not yet been developed; as a result, they run into a serious conceptual paradox, which has not been widely recognized, but which undermines the logic of the analysis. What we will do in this part of the paper is demonstrate the existence of the "gold standard paradox", then show how it can be resolved by treating the gold standard as a boundary between two imperfectly sustainable target

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[1]Dumas and Delgado (1990) argue that since the tangency condition in these models does not arise from optimization, it really should not be called "smooth pasting". This seems a semantic point, and in any case the terminology has already become so common that it really cannot be undone.

zones.

The basic gold standard model may be presented as a two-country version of the model at the beginning of this paper. The exchange rate depends on the ratio of two countries' money supplies, a demand shock term, and the expected rate of depreciation:

$$s = m - m^* + v + \gamma \frac{E[ds]}{dt} \tag{19}$$

Each country's money supply consists of domestic credit plus reserves:

$$\begin{aligned} m &= \ln(D + R) \\ m^* &= \ln(D^* + R^*) \end{aligned} \tag{20}$$

Reserves, however, are now taken to consist of gold, which is in fixed world supply:

$$R + R^* = G \tag{21}$$

As before, we need to specify a process for the money demand term. In the gold standard literature, in contrast to the target zone literature, it has been standard to assume that money demand tends to return to some normal level -- a plausible assumption, and one consistent with the general stationarity that one assumes in the gold standard situation. So we have

$$dv = -\rho (v-\overline{v}) dt + \sigma dz \tag{22}$$

In order to analyze this model, we first find the exchange rate equation under a pure float. Let us simply guess that this equation takes the form

$$s = m - m^{*} + v + H(v-\overline{v}) \qquad (23)$$

If this is the true form of the equation, then it is straightforward to show that

$$E[ds]/dt = -\rho(1+H)(v-\overline{v}) \qquad (24)$$

(There are no terms resulting from second-order effects because of the assumed linearity). Substituting back into (23), we get the result

$$s = m - m^{*} + v - \frac{\gamma\rho}{1+\gamma\rho}(v-\overline{v}) \qquad (25)$$

Now let us turn to a fixed rate. Suppose that both countries peg their currencies in terms of gold, standing willing to buy or sell gold at prices that fix an implied exchange rate $s_{par}$. This regime will continue until one country or the other runs out of gold.

The seemingly obvious assumptions are that as long as the regime is in effect, there will be no expected change in exchange rates; and that when the regime collapses, the exchange rate reverts to a free float. It turns out, however, that this combination of assumptions gets us into big trouble. This is the gold standard paradox: it is not possible for the public both to expect zero change in the exchange rate while it is fixed, and to expect the post-collapse regime to be a free float.

To see this, first ask how reserves would appear to evolve if we in fact assume $E[ds]/dt = 0$. Then when v rises, gold will flow from the first country to the second; when it falls, it will flow in the other direction. Ignoring the possibility of speculative attack, this process could continue until the ratio of money supplies reaches either a maximum or minimum value. The maximum value of $m-m^*$ occurs when all gold has flowed out of the first country; at that point we have

$$m = \ln(D) \tag{26}$$

and

$$m^* = \ln(D+G) \tag{27}$$

Similarly, $m-m^*$ reaches a minimum when all the gold has flowed to the first country, so that

$$m = \ln(D+G) \tag{28}$$

and

$$m^* = \ln(D) \tag{29}$$

Now suppose for a moment that agents are naive, and do not realize that a regime change is in prospect. Then the gold standard would last until reserves of one country or another run out. Let's suppose that it is the first country that runs out of gold; it would run out at a level of v, $v_1$, determined by the condition

$$s_{par} = m - m^* + v_1 \tag{30}$$

Normally we would expect this to be a point of unusually low money demand in the country that runs out of gold, so that $v_1$ exceeds the "normal" money demand.

But suppose that we got to this point, and that with one country out of gold, the gold standard collapsed. Then the exchange rate regime would switch to one of floating; the exchange rate would now be determined by (25), so that we have

$$s = m - m^* + v_1 - \frac{\rho\gamma}{1+\rho\gamma}(v_1 - \bar{v}) \tag{31}$$

But there is something peculiar here: the exchange rate s implied by (31) is less than $s_{par}$ . That is, when a country runs out of gold, its currency <u>appreciates</u>.

This can't be right. However, one might think that the problem is that we have failed to allow for the possibility of speculative attack. It turns out, however, that this doesn't resolve the problem either.

The usual answer when an analysis of a fixed rate system leads to a predictable jump in the exchange rate is to look for a speculative attack that collapses the regime in such a way as to avoid the jump. As Flood and Garber (1984) pointed out, a speculative attack will occur when the "shadow price" of foreign exchange, defined as the price that would prevail following an attack, exceeds the fixed rate that is being defended. Thus in this model we would predict an attack as soon as the exchange rate predicted by the equation

$$s = m - m^* + v - \frac{\rho\gamma}{1+\rho\gamma}(v-\bar{v}) \tag{32}$$

exceeds $s_{par}$. However, we note that at $v_1$ the value of the shadow exchange rate is less than $s_{par}$ ; so there will not be a speculative attack at any lower value of v. But this means that a speculative attack, if it occurs, will happen only <u>after</u> the country would have run out of gold in the absence of a speculative attack. Again, this doesn't make sense. So we are back in the gold standard paradox.

Evidently, a wrong assumption has been made. What must be wrong is either the assumption that expected exchange rate change is zero under a fixed rate, or that following collapse the system reverts to a pure float. In the next two sections we offer two alternative resolutions of the paradox, each based on dropping one of these assumptions.

## 5. Resolving the Gold Standard Paradox: Randomization

In this section we maintain the assumption that following the exhaustion of either country's gold, the gold standard collapses permanently. That is, there is no prospect of a future reestablishment of $s_{par}$. This is the traditional assumption from the speculative attack literature: once the fixed rate collapses, there is a free float. As we will argue shortly, it is probably not the best assumption for a gold standard collapse. Nonetheless, it needs to be examined.

What we already know is that if there is a free float following the collapse, there must not be a zero expected rate of change of s before the collapse. Let us restate the reason why, in a way that will also indicate what must happen.

Figure 3 shows the situation near one end of the range in which the gold standard can be sustained. As in the previous section, $v_1$ is the value of the money demand term at which the first country would run out of gold given zero expected change in the exchange rate. We also show the "shadow exchange rate", however -- the value of s that would prevail under a free float following gold exhaustion. This rate is determined by

$$\hat{s} = \ln(D) - \ln(D^{*}+G) + v + \frac{\rho\gamma}{1+\rho\gamma}(v-\overline{v}) \qquad (33)$$

Because v is assumed to be above its normal level at $v_1$, the shadow exchange rate lies below the gold parity at this point. Thus if the system were in fact to collapse at that point, there would be a sudden appreciation of the currency of the country that has run out of gold.

Clearly this cannot happen. Aside from being peculiar, it implies a forseeable large capital gain for anyone who acquires a country's currency when that country has only small gold reserves left. Thus this perverse collapse is ruled out by the usual argument that there must be no forseeable jumps in s.

It is tempting to conclude that this solves the paradox. Since a sudden appreciation would happen if a country's reserves are exhausted, this provides an incentive for increased holdings of that currency; but by holding more of a country's currency, investors postpone the exhaustion of its gold. This is essentially the solution to the paradox proposed by Buiter and Grilli (1990): arbitrageurs absorb enough of a country's currency to prevent a collapse of the gold standard until the economy reaches the point where the shadow exchange rate coincides with the gold parity.

But on reflection this also can't be right. If it is known that there will be no collapse of the gold standard, and hence no fall in s, there is no incentive to hold additional currency. So we seem to be back in the paradox.

There is, however, a resolution, albiet a rather odd one. This is to suppose that once v has risen above $v_1$, there is a probability but not a certainty of collapse. Suppose that country 1 is almost but not quite out of gold -- a single ingot remains in the vaults. Now money demand falls. In order to avoid a certain appreciation of the currency, investors must be induced not to convert domestic money into foreign, requiring the country to sell that ingot and precipitating a collapse of the standard. That is, they must be persuaded to continue to hold the existing money stock. Each investor can be induced to do this if she perceives a probability $\pi$ per unit time that someone else will precipitate the collapse, leading to an expected appreciation of the currency equal to

$$E[ds]/dt = \pi(\hat{s}-s_{par}) \tag{34}$$

The value of $\pi$ that will just avert a certain collapse is determined implicitly by

$$s_{par} = m - m^{*} + v + \gamma\pi(\hat{s}-s_{par}) \tag{35}$$

or

$$\pi = \frac{s_{par}-m+m^{*}-v}{\gamma(\hat{s}-s_{par})} \tag{36}$$

Note that as v rises, the required value of $\pi$ also rises, both because the required rate of expected appreciation is higher and because the size of the jump that follows a collapse falls.

This is in effect a mixed strategy equilibrium. It can be sustained because each individual investor, given $\pi$, is just indifferent between selling a small amount of domestic currency and holding on; so any probability that he will actually make the conversion is consistent with maximization. How agents are actually induced to choose $\pi$ is unclear; but this is true of all mixed strategy solutions.

Thus this is a resolution of the gold standard paradox. It is, however, an economically implausible one. In the next section we consider an alternative assumption that leads to a much more reasonable-seeming resolution.

## 5. Resolving the Gold Standard Paradox: Redefining the Rules of the Game

The gold standard paradox can be rather neatly resolved if we make one assumption that is slightly different from the usual speculative attack setup. In a way this is cheating; but this solution is much more plausible than the mixed-strategy outcome.

The necessary assumption is the following: the central bank does not give up when it runs out of gold. Instead, it remains willing to buy gold at the par value, and thus to reinstate a gold standard if the opportunity arises.

An example may convey the essence of this assumption. Suppose that our two countries are America and Britain, and that they have established par values of gold of $35 and 7 pounds per ounce. If both countries have positive gold reserves, this will peg the dollar-pound exchange rate at 5. Suppose, however, that America has run out of gold. Then the exchange rate may float above this level -- say, at 7 dollars per pound. The price of gold will be set by the willingness of the British central bank to sell it, at 7 pounds per ounce.

What we will assume is that even though America has run out of gold, its central bank still remains willing to buy gold if the price falls to 35 dollars. (It would be willing to sell gold at that price also, but it doesn't have any to sell). With an exchange rate of 7, of course, the price of gold is $49, so there will be no current sales; but if the exchange rate falls (the dollar appreciates) to 5, gold purchases will commence.

Conversely, if Britain has run out of gold, the exchange rate will float at a level below 5; but if it rises to 5, Britain's central bank will again buy gold.

Consider what this implies. If the exchange rate is above 5, then everyone knows that if it falls to 5 America will buy gold and Britain sell it -- which means that America will increase its money supply and Britain reduce its money supply. This means that when America is out of gold, and the exchange rate is floating, the float is <u>not</u> free. Instead, there is in effect a one-sided target zone, in which there is a de facto commitment to support the pound with sterilized intervention if the dollar strengthens too much.

The reverse is also true: when Britain has run out of gold, the float is in effect a target zone with a commitment to support the dollar with sterilized intervention if the pound strengthens to its par value.

This tells us that the par value implied by the prices at which each currency is pegged to gold may be regarded as a boundary between two one-sided target zones. In the lower zone, in which America has all the gold -- which we will call the A-zone -- the dollar-pound exchange rate is held below its free-float locus by the prospect of US gold sales and British gold purchases if the pound rises too much. In the B-zone, where Britain has all the gold, the rate is correspondingly held above its free-float locus.

If the world's gold stock is large enough, the picture looks like Figure 4, which plots the exchange rate against v. (We will describe the case with insufficient gold backing for the world's currencies below). The 45-degree lines represent the free-float loci -- that is, the 45-degree line on the left represents how s would vary with v if America had all the gold and there was no prospect of future intervention, the 45-degree line on the right the corresponding case with all gold in British hands. The actual relationship in the A-zone, however, is that which we have already seen for a one-sided target zone with large reserves: a curve that lies below the free-float locus and is tangent to the par value line at some value $v_A$. Similarly, in the B-zone the relationship between v and s lies above the free-float locus and is tangent to the par value line at $v_B$.

The relationship between v and s is therefore indicated by the curve on the left up to $v_A$; the par value is sustained between $v_A$ and $v_B$; and s follows the curve on the right for v greater than $v_B$. Outside the range where the par value is sustained, the prospect of a return to the gold standard either supports or depresses the exchange rate.

What happens if v starts within the range where the par value can be sustained, then drifts out of that range, say to $v_B$? The answer is that as long as we are on the "flat", there will be a gradual American loss of gold. When $v_B$ is reached, however, there will be a speculative attack that leads to a discrete American loss of its remaining gold. The reason is that the post-attack v-s relationship is flat -- so autoregression contributes nothing to E[ds]/dt -- and convex, so that the variance term makes E[ds]/dt positive. That is, when the gold standard collapses the expected rate of dollar depreciation immediately goes from zero to some positive number, reducing relative American money demand -- even if v is expected to fall. Similarly, if v drops to the bottom of the range there will be a speculative attack that leads to a discrete British loss of its remaining gold.

The reason why the currency of the country that runs out of gold is expected to depreciate immediately following the gold exhaustion is somehat ironic: it is the result of the expectation that the country will try to buy gold if its currency should subsequently appreciate to the par value, which therefore depresses its value under the float.

What happens if America has no gold, and v drifts back into the range in which the par value is enforced? The answer is that there is a speculative run <u>into</u> the dollar, leading to a discrete gain in reserves at British expense.

It may be useful to illustrate this model of the gold standard more explicitly. Let us therefore consider a special case: that where v follows a random walk, i.e., where there is no expected autoregression. In this case there need not, of course, be a gold standard paradox, but it does allow an explicit calculation of the relationship between v and s under our assumptions.

We begin by noting that when v follows a random walk with no drift, the two roots in the solution sum to zero. Thus the basic exchange rate equation may be written

$$s = m - m^{*} + v + Ae^{\alpha v} + Be^{-\alpha v} \tag{37}$$

where $\alpha$ may be calculated using the methods of section 1.

There are now two de facto target zones: the "A-zone" in which America has all the gold, and the "B-zone" in which Britain has all the gold. The relative money supplies in these zones are therefore as follows: in the A-zone,

$$m-m^{*} = \ln\left(\frac{D+G}{D^{*}}\right) \tag{38}$$

while in the B-zone

$$m-m^{*} = \ln\left(\frac{D}{D^{*}+G}\right) \tag{39}$$

To calculate $v_A$, we first note that since in the A-zone v is unbounded below, we must have B=0, and must choose a value of A such that the exchange rate reaches its par value at $v_A$:

$$s_{par} = \ln\left(\frac{D+G}{D^{*}}\right) + v_A + Ae^{\alpha v_A} \tag{40}$$

Also, the curve must be flat at $v_A$:

$$\frac{ds}{dv} = 1 + \alpha A e^{\alpha v_A} = 0 \tag{41}$$

Putting these together, we find that

$$v_A = s_{par} - \ln\left(\frac{D+G}{D^*}\right) + \frac{1}{\alpha} \tag{42}$$

A similar calculation shows that

$$v_B = s_{par} - \ln\left(\frac{D}{D^*+G}\right) - \frac{1}{\alpha} \tag{43}$$

What is the significance of the term $1/\alpha$? It is the horizontal distance from each end of the gold standard range to the corresponding free float locus. It therefore measures the extent to which the target zone aspect of the exchange regime when one country has run out of gold leads to a collapse of the gold standard before the gold would have run out under a perfectly credible system. And $1/\alpha$ also measures the change in the log of the ratio of national money supplies that occurs when there is a speculative attack.

This example illustrates how a gold standard with limited gold reserves may be modelled as a boundary between two target zones. However, the example also reveals a problem. As drawn in Figure 4, we show $v_B>v_A$, so that there is a range in which the par value can be maintained. But there is no guarantee that this is true. We note that

$$v_B-v_A = \ln\left(\frac{D^*+G}{D}\right) + \ln\left(\frac{D+G}{D^*}\right) - \frac{2}{\alpha} \qquad (44)$$

This will be positive only if gold reserves G are large enough relative to the world money supply. When gold reserves are sufficient, we get the story illustrated in Figure 4. But what if they aren't sufficient?

On reflection, the story is apparent: it is illustrated In Figure 5. The par exchange rate $s_{par}$ still represents the boundary between two target zone regimes, but the loci in each regime no longer "smooth paste" to the par value. Instead, they smooth paste to each other at some critical value of v. Whenever v crosses that value, there is a speculative attack that transfers all of the gold from America to Britain or vice versa. The central banks are trying to enforce a gold parity, but one or the other is always failing.

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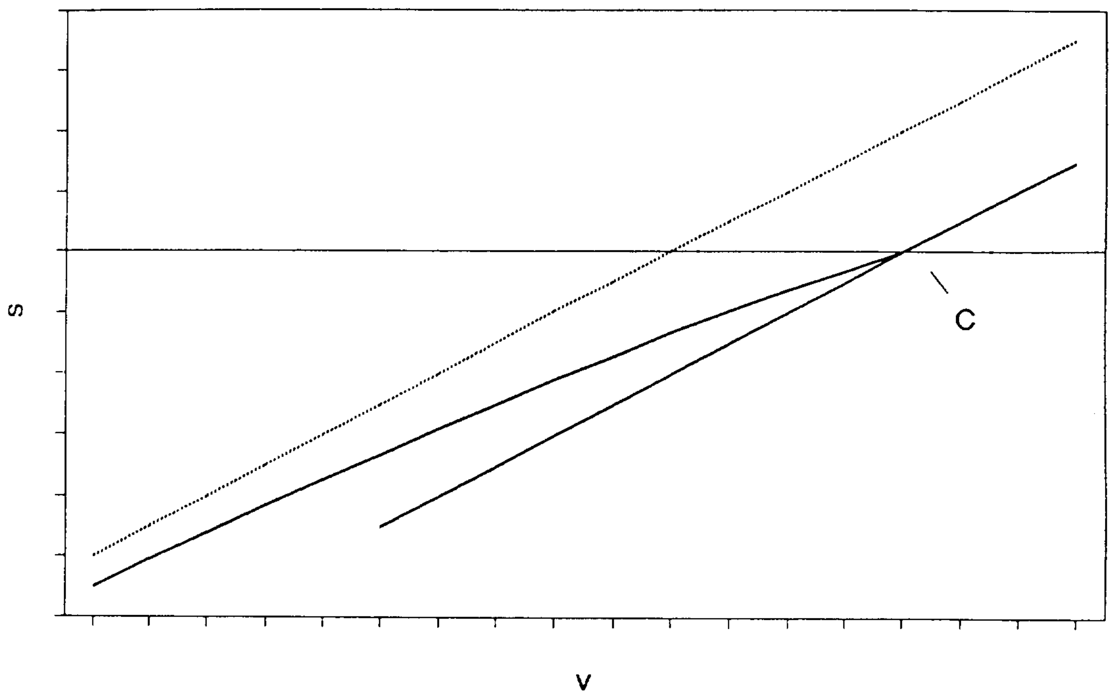
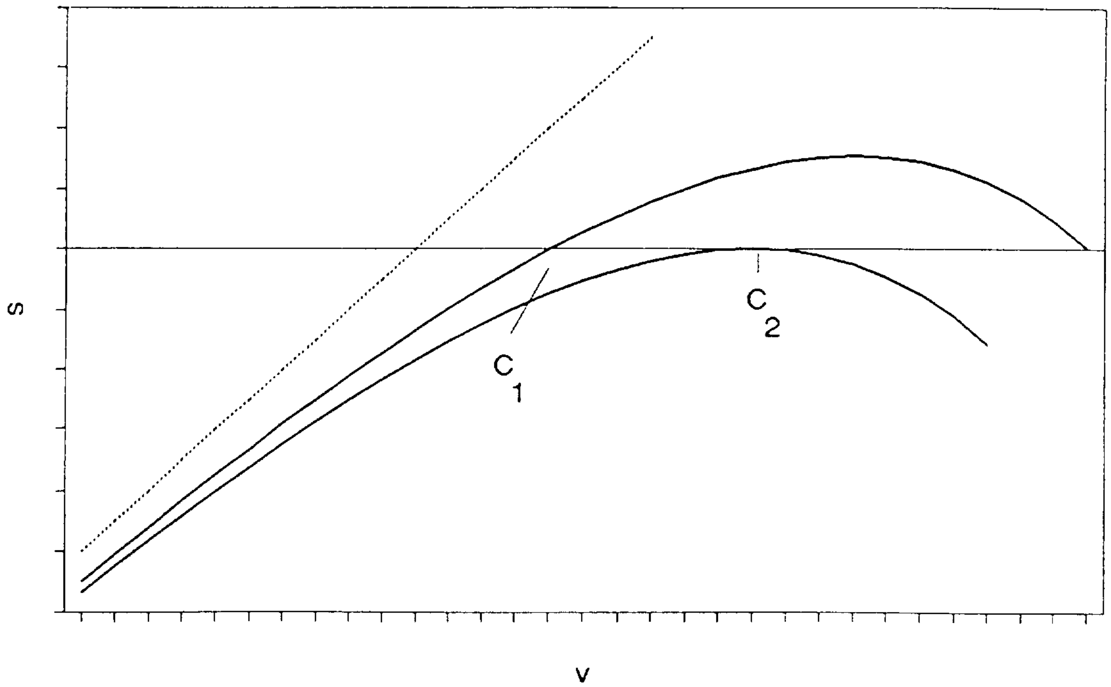
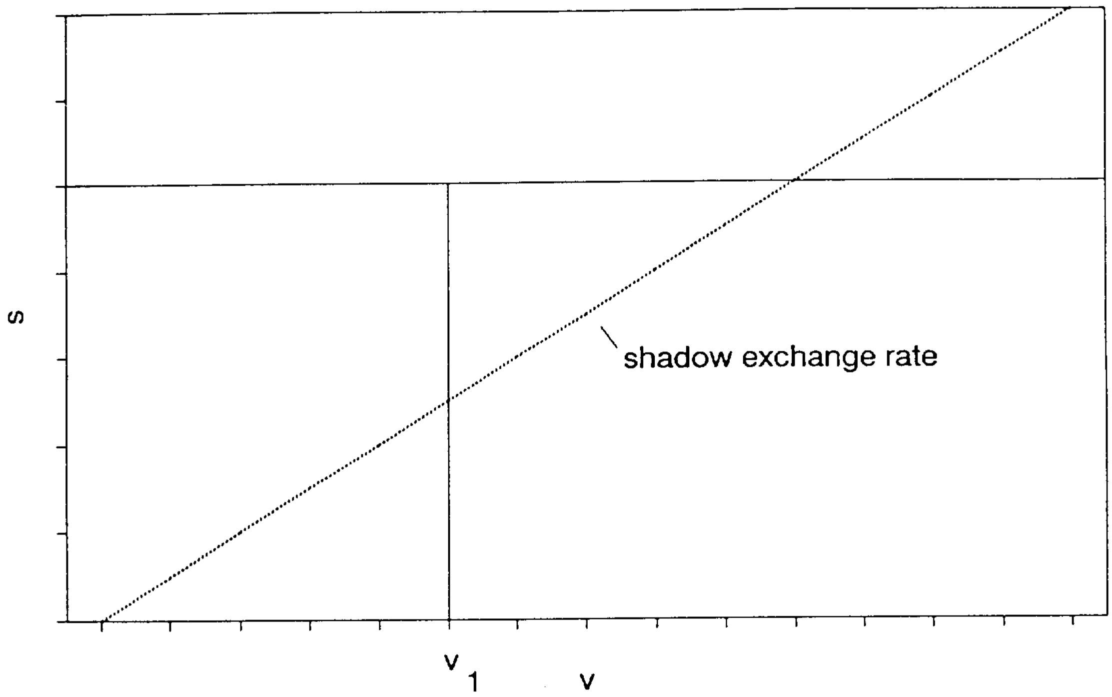
# FIGURE 1



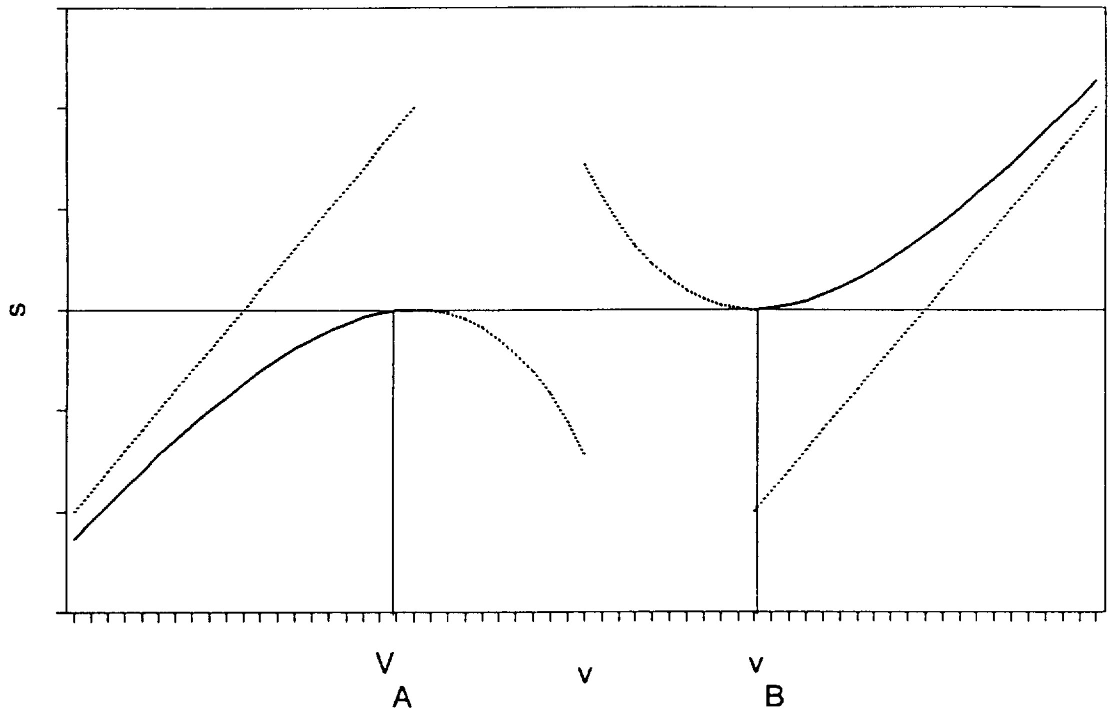
# FIGURE 2



# FIGURE 3



# FIGURE 4



# FIGURE 5