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THE RESURGENCE OF INVENTORY RESEARCH: WHAT HAVE WE LEARNED?

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ABSTRACT

Recent empirical and theoretical research on business inventories is surveyed and critically evaluated. While most inventory research has had macroeconomic motivations, we focus on its microtheoretic basis and on potential conflicts between theory and evidence.

The paper asks two principal questions. First, how can inventories, which are allegedly used by firms to stabilize production, nonetheless be a destabilizing factor at the macroeconomic level? Second, why, if firms are following the production-smoothing model, is production more variable than sales in many industries? We suggest that the so-called (S,s) model may help answer both questions.

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I. INTRODUCTION

After a long period of dormancy, research on inventories has undergone something of a renaissance in recent years. Yet a student can still study undergraduate and graduate economics for years in the best universities without hearing much about inventories in macroeconomics and without ever hearing the subject mentioned in microeconomics.

This omission would not be cause for concern if inventories were a detail of minor economic significance or little intrinsic interest. After all, economists do not dwell on the fact that GNP is higher between 9 a.m. and 5 p.m. than between 9 p.m. and 5 a.m.. But simple observations suggest that inventory behavior is considerably more important than that. At the macro level, we have known for a long time (but periodically forgotten) that inventory movements are dominant features of business cycles. In a purely arithmetical sense, Table I shows that the drop in inventory investment has accounted for 87% of the drop in GNP during the average post war recession in the U.S..[1] At the micro level, we know that firms vary a great deal in the levels and types of inventories they hold and devote much time and effort to inventory management. It would be curious indeed if inventory movements were details of such little importance that economists could safely ignore them.

We think this most unlikely. In contrast, we suspect that a better understanding of inventory behavior is essential to achieving a better understanding of both the macroeconomics of business cycles and the microeconomics of the firm. In the following pages, we endeavor to show why, to give some indication of what has been learned about inventories over the last decade or so, and to set forth some of the important unanswered questions for future research.

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[1]If the extreme observations (1960 and 1980) are omitted, the average drops to 77%. The dates in the table are not official NBER dates because, on NBER dating, GNP rose from peak to trough in the 1960–1961 recession.

TABLE I

INVENTORY INVESTMENT AND POSTWAR RECESSIONS

GNP Peak to Trough	Change in Real GNP [1]	Change in Inventory Investment [1]	Change in Inventory Investment As a Percentage of Change in Real GNP
1948:4-1949:4	-22.2	-28.2	127%
1953:2-1954:2	-43.7	-18.4	42%
1957:3-1958:1	-55.4	-21.7	39%
1960:1-1960:4	-17.5	-40.6	232%
1969:3-1970:4	-19.4	-28.2	145% [2]
1973:4-1975:1	-120.1	-78.1	65%
1980:1-1980:2	-76.4	-1.8	2%
1981:3-1982:3	-110.1	-45.1	41% [3]
		Average:	87%

(1) Billions of 1982 Dollars

(2) 72% if trough is 1970:2

(3) 90% if trough is 1982:4

The Intellectual Heritage

Though inventories were much discussed in the pre–Keynesian literature on business cycles, as for example in Hawtrey (1928), modern interest in inventory behavior derives from the work of Lundberg (1937) and Metzler (1941) who demonstrated that an inventory–accelerator mechanism can produce cycles in simple Keynesian models. Empirical and theoretical aspects of inventory behavior were hot topics from the early 1950s until the early 1960s. Abramovitz's (1950) seminal empirical work documented the overwhelming importance of inventories in business cycles in the interwar period. A series of papers in operations research beginning with Arrow, Harris, and Marschak (1951) and ending with Scarf (1960) developed the mathematics of the (S,s) model of optimal inventory holdings, which had been introduced decades before. Karlin (1958) and others used stochastic dynamic programming techniques to analyze inventory behavior in models with convex costs. A book by Holt, Modigliani, Muth, and Simon (1960), written as a manual for plant managers, became instead an economic classic by introducing and popularizing the linear–quadratic approach to optimization. Finally, Lovell's (1961) empirical estimates of inventory stock–adjustment models became part of the Keynesian econometric modeling tradition.

These developments in the world of ideas took place against the backdrop of periodic business cycles that looked much like the inventory cycles Metzler had written about a decade or two earlier. And no one seemed to notice the tension that was developing between the macroeconomic and microeconomic views of inventories. Macroeconomists, following Metzler, Abramovitz, and Lovell, thought of inventory behavior as a destabilizing factor. In theory, the inventory accelerator created cycles that otherwise might not exist; in practice, GNP was more volatile than final sales (GNP less inventory investment). Yet the prevailing micro theory, following Holt et al., viewed inventories as a stabilizing factor — a buffer stock that cost–minimizing firms could use to smooth production in the face of fluctuating sales. Could something that was stabilizing at the micro level actually destabilize the macroeconomy? It was a fascinating question that was left unanswered.

Instead, and somewhat inexplicably, interest in inventories dried up in the 1960s. Perhaps it was because scholars, failing to note the abovementioned micro–macro tension, mistakenly concluded that the central intellectual problems were solved. More likely, the long economic expansion of the 1960s reduced macroeconomic interest in inventory movements, which are, after all, mainly important during recessions and around cyclical turning points. Whatever the reason, there was precious little research on inventory behavior by economists between about 1962 and about 1975.[2]

All this started to change after 1975 when economists came to the startling realization that even a cyclical contraction as severe as that of 1973–1975 could be predominantly an inventory cycle. At roughly the same time, Feldstein and Auerbach (1976) called attention to some difficulties with the traditional stock–adjustment model. Although these problems had been pointed out before,[3] they were not widely known outside a small group of inventory specialists; nor was it appreciated just how devastating they were. How, Feldstein and Auerbach asked, could firms take months or even years to adjust to sales shocks when peak to trough movements in inventory levels in most industries amounted to only a few days' production? It was a good question. A few years later, Blinder (1981) and Blanchard (1983) deepened the mystery by showing that the well–known fact that GNP is more variable than final sales carries over to industry–level

[2]Noteworthy exceptions include the theoretical work done by Zabel (1970,1972) and the empirical applications of the linear–quadratic approach done by Childs (1967), Belsley (1969) and Hay (1970).

[3]See, e.g., Carlson and Wehrs (1974) and Orr (1967).

data as well: In the vast majority of industries, production is more variable than sales. If firms use inventories to smooth production, why is production more variable than sales? Another good question.

By the early 1980s, then, economists knew once again something they had known in the 1950s: that inventory investment was of first–order importance in business cycles. But they were also beginning to suspect that both the standard theoretical model of inventory behavior, the production smoothing/buffer stock model, and its empirical counterpart, the stock adjustment model, were in deep trouble. This paper focuses on developments since that realization.

II. FACTS TO BE EXPLAINED

If we are to know what questions to ask of inventory theory, it helps to begin with the facts. At the end of 1989, U.S. businesses held over $1 trillion worth of inventories. That is a sizable sum— over $4200 for every living American. Put somewhat differently, the cost of holding all those inventories, even using a low 10% estimate for carrying costs, exceeded the dividend payments made by all U.S. non–financial corporations. Furthermore, contrary to popular belief, inventories are not leaner now than they were decades ago. Despite the alleged revolution in inventory practices brought about by computerization, Figure 1 shows that the economy–wide ratio of real inventories to real sales has been trendless for 40 years. Inventory stocks amounted to 3.3 months' sales in 1949, 3.3 in 1959, 3.4 in 1969, 3.4 in 1979, and 3.1 in 1989.[4]

Who holds the inventories? At the end of 1989, over 87% of nonfarm inventories

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[4]A linear regression actually shows a trivial (and insignificant) upward trend. The inventory–sales ratio in nominal terms, however, has declined, which is rather puzzling.

Inventory Sales Ratio
1947-1988

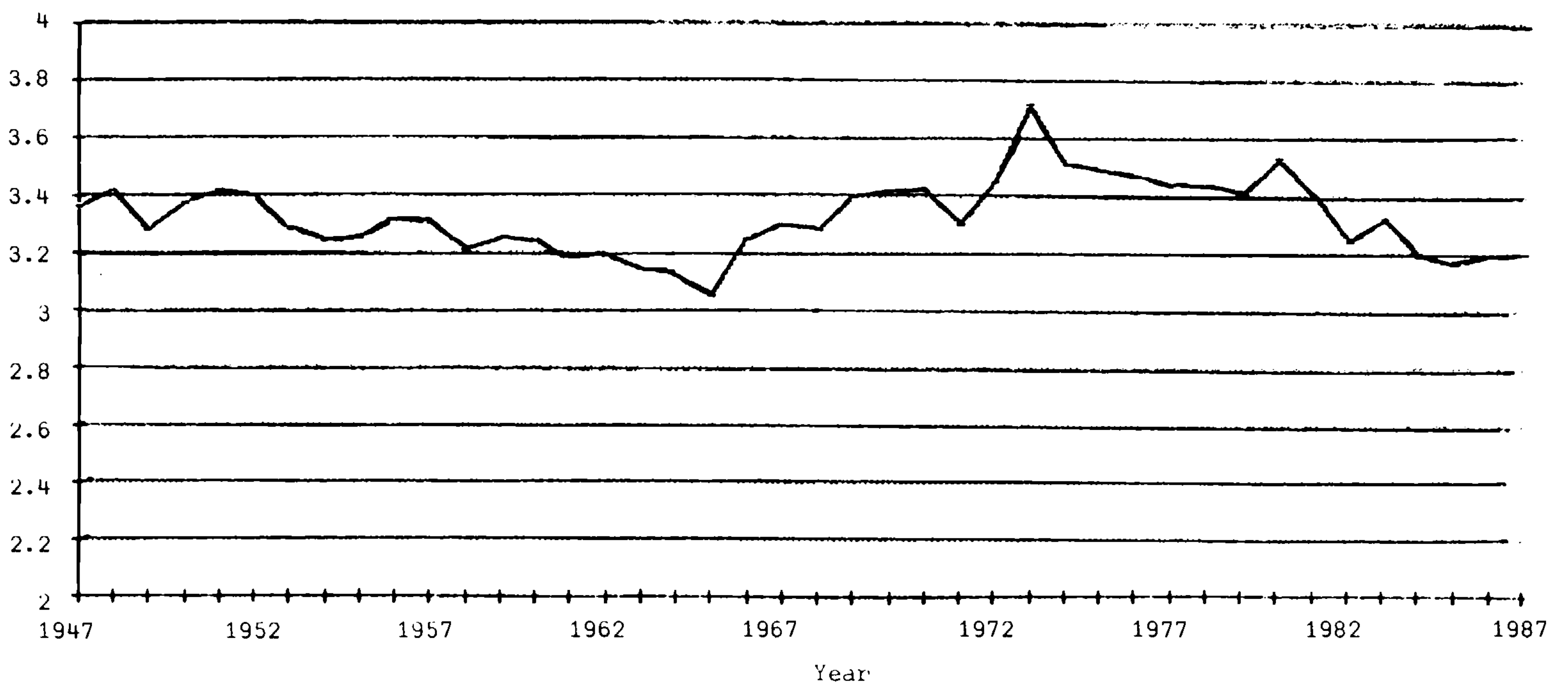

Figure 1

were held in the manufacturing and trade sectors of the economy.[5] It is thus worth examining closely the basic facts about manufacturing and trade inventories—especially since it was here that certain puzzling and provocative facts sparked the resurgence of interest in inventories.

Table II reports some basic facts on manufacturing and trade inventories broken down by sector and, within manufacturing, by stage of fabrication. The data are monthly, seasonally adjusted, measured in 1982 dollars, and cover the period 1959:1 through 1986:10.[6] Statistics are reported for manufacturing, wholesale trade, and retail trade. Within manufacturing, inventories are broken down into finished goods, work in progress, and materials and supplies. Wholesale and retail trade inventories are almost entirely finished goods.

Table II–A reports means and variances for the inventory data.[7] Focusing on the five major components of inventory stocks— the three components of manufacturing inventories plus the two trade components, we see that each accounts for (very) roughly a

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[5]Farm inventories are about 7.5% of total inventories so that nonfarm inventories dominate the total.

[6]See Hinrichs and Eckman (1981) for a description of the deflation procedure. The inventory stocks were also adjusted to place inventory stocks and shipments in comparable units. This is needed because inventories are valued at cost rather than market. See Blinder and Holtz–Eakin (1983) and West (1983) for the appropriate adjustments.

[7]The means are calculated with undetrended data. The variances and correlations are calculated with data which were detrended as follows: The logarithmic level of each variable was regressed on a constant, time, an OPEC variable, which is another time trend that begins with the value one in 1973:10, and a measurement dummy that is one for all values from 1967:1 onward. The OPEC variable was entered to see whether a change in trend occurred after the first OPEC shock. The measurement dummy was included to adjust for data revisions which extended back only to 1967:1. Each regression was estimated by a maximum likelihood procedure that allowed for a second–order process in the error term. The detrended variable was constructed by subtracting the antilogarithm of the fitted values of the above regression from the actual values of each variable. Both the OPEC variable and the measurement dummy were generally very small and played little role in the detrending procedure.

TABLE II

A. BASIC STATISTICS [1,2]

INVENTORY COMPONENT	MEAN: INVENTORY STOCKS	PERCENT OF TOTAL	MEAN: INVENTORY INVESTMENT	PERCENT OF TOTAL	VARIANCE OF INVENTORY INVESTMENT	PERCENT OF TOTAL
MANUFACTURING & TRADE	622.3	100.0	1.57	100.0	63.7	100.0
MANUFACTURING	378.6	60.8	0.78	49.7	29.5	46.3
FINISHED GOODS	103.6	16.6	0.20	12.7	3.6	5.7
WORK IN PROGRESS	120.5	19.4	0.32	20.4	6.1	9.6
MATERIALS AND SUPPLIES	154.6	24.8	0.26	16.6	12.9	20.3
ALL COVARIANCE TERMS	NA	NA	NA	NA	6.9	10.8
WHOLESALE TRADE	106.2	17.1	0.37	23.6	5.7	8.9
RETAIL TRADE	137.5	22.1	0.42	26.8	15.5	24.3
ALL COVARIANCE TERMS	NA	NA	NA	NA	13.0	20.4

B. CONTEMPORANEOUS CORRELATIONS-INVENTORY INVESTMENT [2]

	MANUFAC. & TRADE	MANUFAC-TURING	FINISHED GOODS	WORK IN PROGRESS	MATERIALS & SUPPLIES	WHOLESALE TRADE	RETAIL TRADE
MANUFACTURING & TRADE	1.00	0.80	0.41	0.52	0.63	0.55	0.59
MANUFACTURING		1.00	0.52	0.65	0.79	0.26	0.08
MAN.- FINISHED GOODS			1.00	0.19	0.12	0.16	0.02
MAN.-WORK IN PROGRESS				1.00	0.19	0.11	0.10
MAN.-MATERIALS AND SUPPLIES					1.00	0.23	0.04
WHOLESALE TRADE						1.00	0.15
RETAIL TRADE							1.00

(1)The means of inventory stocks and inventory investment are measured in billions of 1982 dollars.

(2)Sample Period: 1959.3--1986.10

fifth of average inventory levels. Looking at inventory investment, however, a somewhat different picture emerges. Two facts are striking. First, investment in manufacturers' finished goods inventories is the smallest component of total inventory investment. For example, in a typical month, retail inventories grew more than twice as much as manufacturers' inventories of finished goods. More importantly, using the variance of detrended inventory investment as a measure of volatility, finished goods inventories held by manufacturers is the least volatile component of total inventory investment. Despite this lack of importance in business fluctuations, manufacturers' inventories of finished goods have received the lion's share of attention in both theoretical and empirical work. Most researchers seem to have barked up the wrong tree.

Second, retail inventories and materials and supplies held by manufacturers are by far the two most volatile components of inventory investment. Indeed, these two together play an exceedingly influential role in movements in total inventory investment. This is of particular interest to theorists because the decisions of manufacturers to hold inventories of materials and supplies and of retailers to hold inventories of finished goods are quite similar. In each case, the key decision variable is deliveries— of materials and supplies in one case and of finished goods in the other. If fixed costs are incurred in placing orders or receiving deliveries, then models that stress "bunching" rather than "smoothing" of deliveries can be applied fruitfully in both cases. Yet the literature concentrates on inventory models based on production smoothing.

Table II–B presents contemporaneous correlations among the various components of detrended inventory investment. Again focusing on the five major components, the correlations are surprisingly low, never exceeding .23. Each major component of inventory investment seems to have a life of its own, which is discouraging from the viewpoint of finding a single, unified theory to explain inventory investment.

Tables III and IV report on the variances and covariances of output, sales, and inventory investment. In each case, the focus is on the identity:

$$Y_t = X_t + \Delta N_t$$

where Y_t is real output, X_t is real shipments, and ΔN_t is real inventory investment. Two measures of output, corresponding to two different measures of inventory investment, have been widely used in the literature. Y_1 adds investment in finished goods inventories to shipments to obtain output; Y_2 adds investment in work in progress inventories as well.[8] Table III reports the facts for the major sectors of the economy that hold inventories. Table IV presents the facts for two–digit industries in manufacturing.

Two findings stand out. First, the variance of production exceeds the variance of shipments or sales in virtually all cases. This is true for the major sectors of the economy that hold inventories and for almost all the two–digit manufacturing industries, no matter how output is measured. Facts such as these cast doubt on the basic idea of the production smoothing model, namely, that firms smooth output in the face of fluctuating sales.

An alternative explanation of this finding is that the Department of Commerce data are inaccurate. Recently, Ghali (1987), Krane and Braun (1989), and Fair (1989) find that production is less variable than sales in a few industries in which physical unit data are available. Blanchard (1983), on the other hand, who also works with physical unit data, finds that production varies more than sales in the automobile industry. These findings do raise questions about the accuracy of the Commerce Department data, but so far the contrary findings are for precious few industries and for limited sample periods and hence

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[8] The appropriate definition of output is ambiguous in a multi–stage production process. Most investigators have used Y_1, which appears to be the natural definition. However, the Commerce Department uses Y_2 in constructing the price indexes it uses to deflate inventory stocks. Hence the identity in the text holds only when N is defined to include work in progress. See Blinder (1986b), p. 434n.

TABLE III
PRODUCTION, SALES AND INVENTORY INVESTMENT
VARIANCES AND CORRELATIONS BY SECTOR [1]

SECTOR [2]	VAR(Y1)	VAR(Y2)	VAR(X)	VAR(Y1)/VAR(X)	VAR(Y2)/VAR(X)	COR(X,ΔN1)	COR(X,ΔN2)
MANUFACTURING	559.3	615.7	545.3	1.03	1.13	0.46	0.38
DURABLES	305.2	351.2	297.1	1.03	1.18	0.48	0.42
NONDURABLES	58.1	59.1	56.3	1.03	1.05	0.12	0.05
WHOLESALE TRADE	182.4	182.4	166.5	1.10	1.10	0.18	0.18
DURABLES	83.1	83.1	72.3	1.15	1.15	0.26	0.26
NONDURABLES	34.1	34.1	30.9	1.10	1.10	0.004	0.004
RETAIL TRADE	171.6	171.6	136.6	1.26	1.26	0.09	0.09
DURABLES	71.1	71.1	54.8	1.30	1.30	0.06	0.06
NONDURABLES	16.2	16.2	12.6	1.29	1.29	0.06	0.06

(1) Notation: X = Real Shipments
In manufacturing, Y1 = Real output = X + ΔN1 where N1 = Finished goods inventories
Y2 = Real Output = X + ΔN2 where N2 = N1 + Work in progress inventories
In wholesale and retail trade, Y1 = Y2 = X + ΔN where N = Total inventories

(2) Sample Period: MANUFACTURING AND RETAIL TRADE 1959.3--1986.10
WHOLESALE TRADE 1967.3--1986.10 (The sample period for wholesale trade is limited by a lack of data on shipments prior to 1967.)

TABLE IV
PRODUCTION, SALES AND INVENTORY INVESTMENT
VARIANCES AND CORRELATIONS BY INDUSTRY[1]

	VAR(Y1)	VAR(Y2)	VAR(X)	VAR(Y1) / VAR(X)	VAR(Y2) / VAR(X)	COR(X,ΔN1)	COR(X,ΔN2)
DURABLE GOODS INDUSTRIES[2]							
PRIMARY METALS	16.4	16.5	16.6	0.99	0.99	-0.02	-0.11
FABRICATED METALS	8.7	9.2	8.5	1.02	1.08	0.20	0.09
ELECTRICAL MACHINERY	5.25	6.3	5.1	1.03	1.24	0.34	0.30
NON-ELECT. MACHINERY	11.07	14.0	9.9	1.12	1.41	0.44	0.39
TRANSPORTATION EQUIP.	39.71	46.0	39.1	1.02	1.18	0.19	0.22
LUMBER	0.69	0.74	0.66	1.05	1.12	0.06	0.06
FURNITURE & FIXTURES	0.18	0.20	0.17	1.06	1.18	0.24	0.09
STONE, CLAY & GLASS	0.7	0.72	0.65	1.08	1.11	0.24	0.07
INSTRUMENTS	0.5	0.62	0.54	0.93	1.15	0.15	0.11
OTHER DURABLES	0.22	0.24	0.19	1.16	1.26	0.02	0.04
NONDURABLE GOODS INDUSTRIES[2]							
FOOD	1.97	2.00	1.6	1.23	1.25	-0.08	0.05
TOBACCO	0.11	0.13	0.08	1.38	1.63	-0.01	-0.02
TEXTILES	0.46	0.49	0.43	1.07	1.14	0.22	0.02
APPAREL	0.55	0.62	0.45	1.22	1.38	0.15	0.09
LEATHER	0.058	0.06	0.05	1.16	1.20	0.10	0.05
PAPER	0.71	0.73	0.73	0.97	1.00	0.19	-0.04
PRINTING & PUBLISH.	0.66	0.72	0.62	1.06	1.16	0.12	-0.02
CHEMICALS	4.85	5.0	4.7	1.03	1.06	0.16	-0.05
PETROLEUM	12.29	12.5	12.0	1.02	1.04	0.01	0.02
RUBBER & PLASTICS	0.91	0.95	0.85	1.07	1.12	0.24	0.06

(1) Notation: X = Real Shipments
Y1 = Real Output = X + ΔN1 where N1 = Finished goods inventories
Y2 = Real Output = X + ΔN2 where N2 = N1 + Work in Progress Inventories

(2) Sample Period: 1959.3--1986.10

are not compelling as a general finding, especially when retail and wholesale trade as well as manufacturing are considered.[9]

Second, sales and inventory investment are more frequently positively than negatively correlated. This is true even with the most narrow definition of inventory investment, namely, investment in finished goods inventories; and it raises questions about whether inventories act as a buffer stock. A word of caution is in order here, however. What is relevant in evaluating whether inventories serve as a buffer stock is the correlation between the unanticipated components of sales and inventory investment, not the total levels. But empirical studies that have tried to investigate the correlation between inventories and various measures of unanticipated sales have found relatively few negative correlations.[10]

Thus, we take the basic stylized facts to be explained as these:

(1) production tends to be more variable than sales in most industries;

(2) sales and inventory investment normally are not negatively correlated;

(3) the most volatile components of inventory investment are retail inventories and manufacturers' inventories of raw materials and supplies.

III. MICROECONOMIC THEORIES OF INVENTORY BEHAVIOR

A microeconomic theory of inventory behavior begins by specifying a reason why firms hold inventories. Many have been suggested. Inventories can be held for display

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[9]For example, among the seven manufacturing industries studied by Fair, three are in primary metals, which is the exceptional sector in Table IV, while production is more variable than sales in two others. Hence only two small industries— cement and (by some measures) tires— contradict the findings of Tables III and IV.

In related work, Miron and Zeldes (1987) show that two measures of output, namely, sales plus the change in inventories, which is the measure we use, and the industrial production index, have very different time series properties. A possible (but not the only) explanation of this finding is that the inventory data are inaccurate.

[10]See e.g., Blinder (1986a,b) and Haltiwanger and Maccini (1989).

purposes, as unavoidable "pipeline" inventories, to improve production scheduling, to smooth production in the face of fluctuating sales, to minimize stockout costs, to speculate on or hedge against price movements, to reduce purchasing costs by buying in quantity, to shorten delivery lags, and so on. It is clear that no single model can hope to explain the rich variety of inventory behavior; an explanation that is plausible for one industry or type of inventory may be implausible for another.

Any abstract theory of inventory behavior must simplify and generalize. Probably, that means focusing on just one motive for holding inventories. But which one? As we have already observed, economists have singled out for attention the production smoothing/buffer stock motive, which would seem to apply most naturally to manufacturers' inventories of finished goods. The basic idea behind this model is simple and intuitively appealing. If production cost functions are convex and sales vary over time, a cost–minimizing strategy will smooth production relative to sales. Alternatively, an explicit cost of changing production from period to period will have the same effect. If, in addition, sales are stochastic, inventories will serve as a buffer stock as well.

Several of the facts enumerated in Section II suggest that the production smoothing/buffer stock model may not be the best choice. First, manufacturers' finished goods account for less than 13% of manufacturing and trade inventory investment. Furthermore, they are the least volatile type of inventory investment, accounting for under 6% of the variance (see Table II–A). Second, production seems to be more variable than sales in most industries, which seems odd if firms are really trying to smooth production (see Tables III and IV). Third, the aggregate ratio of inventories to sales shows no downward trend (see Figure 1), which casts serious doubt on buffer stock theories of inventory behavior since computerization should have reduced the need for inventories as buffers.

Nonetheless, the production smoothing/buffer stock model has played the leading role in both the theoretical and empirical literature. So, if we are to understand the evolution of recent thought on inventories, we must begin our discussion there.

A. THE PRODUCTION SMOOTHING/BUFFER STOCK MODEL

1. THE BASIC MODEL

Production smoothing arises from variable demand and convex costs[11]. A buffer stock motive arises from stochastic demand. We start with the simplest possible model that incorporates these essential ingredients, show why it fails to fit the facts, and then add complications designed to bring theory and data into closer conformity.

To permit closed–form solutions, the demand function is specified as linear with a stochastic intercept:

$$(1)\quad x_t = \delta_0 - \delta p_t + \epsilon_t,$$

and both production costs and inventory holding costs are assumed to be quadratic:

$$(2)\quad C(y_t) = c_1 y_t + (1/2c) y_t^2$$

$$(3)\quad B(N_t) = b_1(N_t - K_t) + \frac{b}{2}(N_t - K_t)^2$$

Here K_t is the level of inventories that minimizes inventory holding costs; we take K_t to be zero at first. Notice that stochastic elements of cost are omitted at first; we bring them in later.

The decision rules implied by maximizing the expected present discounted value of profits depend on the nature of the demand shocks.[12] We allow for two kinds, corresponding

[11] Several authors, e.g., Blanchard (1983), add a cost of changing the level of production to production costs. This creates an additional incentive for firms to smooth output over and above the incentive inherent in rising marginal cost of production. Such costs, however, are generally rationalized by appealing to costs of hiring and firing workers, etc.. These are essentially costs of changing inputs, not output. We take up this subject later when we discuss quasi–fixed factors of production.

[12] The model is a slight adaptation of Blinder (1986b). Details on the solution can be found there and in Blinder (1982).

to two possible meanings of the word "unanticipated":

$$(4)\quad \epsilon_t = \epsilon_t^1 + \epsilon_t^2.$$

The first shock, ϵ_t^1, is a surprise only to the econometrician, who has less information than the firm; it is known to the firm before it must decide on this period's production and prices. The second, ϵ_t^2, is truly unanticipated; it is unknown to the firm when it sets y_t and p_t. Hence, by definition, ϵ_t^2, cannot affect output and must come entirely out of inventories. At first, we assume that both demand shocks are serially independent; later we consider serial correlation.

With i.i.d. demand shocks, the solutions for production, sales, and inventory investment take simple forms which bring out the central features of the production smoothing model:

$$(5)\quad y_t = \bar{y} + \beta\lambda(\bar{N} - N_t) + \tfrac{1}{2}\beta\lambda\epsilon_t^1$$

$$(6)\quad x_t = \bar{x} - (1-\beta)\lambda(\bar{N} - N_t) + \tfrac{1}{2}[1-(1-\beta)\lambda]\epsilon_t^1 + \epsilon_t^2$$

$$(7)\quad N_{t+1} - N_t = \lambda(\bar{N} - N_t) - \tfrac{1}{2}(1-\lambda)\epsilon_t^1 - \epsilon_t^2.$$

Here $\bar{y}$, $\bar{x}$, and $\bar{N}$ are the nonstochastic steady-state solutions, and β and λ are proper fractions that depend on the parameters b, c, and δ and on the real rate of interest.

The interpretation of these equations is straightforward. If the firm suddenly acquires an additional unit of inventories, it will reduce production by $\beta\lambda < 1$, raise sales by $(1-\beta)\lambda < 1$, and carry $1-\lambda$ of a unit as additional inventory into the next period. The parameter λ is thus naturally interpreted as the "speed of adjustment" to inventory disequilibrium. Similarly, a unit rise in demand which the firm sees <u>before</u> making its production plan (ϵ_t^1) will be divided among higher output, higher sales, and lower inventories. These are the basic production smoothing results. Finally, a sales shock that is truly unanticipated by the firm (ϵ_t^2) must come entirely out of inventories. This is the basic buffer stock result.

A glance at (5) – (7) shows heuristically why the simplest version of the production smoothing model implies:

(8) $\text{var}(y) < \text{var}(x)$

(9) $\text{cov}\,(x, \Delta N) < 0.$

Ignore the inventory terms on the righthand side of (5) – (7) and focus on the coefficients of the shocks.[13] Both shocks contribute more to var(x) than to var(y) because $1-\lambda(1-\beta) > \beta\lambda$; hence inequality (8) is implied. Inequality (9) holds because the coefficients of the shocks have opposite signs in (6) and (7). The intuition is also clear. Inequality (8) just says that firms succeed in smoothing production. Inequality (9) says (approximately) that positive sales shocks reduce inventories so that inventories perform a buffer stock role.

Finally, both the steady–state values of inventories and the speed of adjustment depend on the real rate of interest. This observation serves as the basis for expecting a negative response of inventory investment to changes in interest rates. But the functional dependence of inventory investment on interest rates is complex.

2. EMPIRICAL ISSUES

Although the simple version of the production smoothing/buffer stock model produces intuitively plausible predictions and rests on seemingly weak assumptions, it has encountered empirical problems. To understand the nature of these problems, and to discuss various ways to reconcile the theory with the facts, we take up three issues: the degree to which production smoothing takes place; the performance of the stock–adjustment model; and the response of inventory investment to interest rates.

a. Production Smoothing

The stylized facts of Section II already suggest that the simplest version of the

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[13]This is why the argument is only heuristic; a full solution must solve for N_t as a function of lagged shocks. This involves quite a bit of algebra but does not change the conclusions stated in the text.

production smoothing/buffer stock model is in trouble because the two key predictions of the theory— inequalities (8) and (9)—are frequently violated in the data. Contrary to the theory, production is often more variable than sales, and inventory investment and sales are generally positively correlated.

Stylized facts, of course, do not constitute formal tests of any theory. Unfortunately, the theory does not fare any better in formal tests. When West (1986) generalized inequality (8) into a clever variance bounds test of the model, he found that production is excessively volatile relative to sales under a variety of assumptions. Miron and Zeldes (1988) showed that seasonal variation in production conforms closely to seasonal variation in sales, so that inventories play little role in smoothing seasonal fluctuations. This is an important result because firms might be expected to forecast regular seasonal fluctuations rationally; hence the test can reasonably be viewed as a test of the production–smoothing model rather than as a joint test of the model and rational expectations. To minimize the need to impose strong restrictions about market structure and industry demand, Eichenbaum (1989) tested the production–smoothing model using only the necessary condition for cost minimization. He found that the over–identifying restrictions implied by the model were overwhelmingly rejected. Furthermore, the key buffer–stock prediction of the theory has also been rejected in recent econometric investigations of inventory investment. The response of inventory investment to various measures of unanticipated sales tends to be either positive, or, when it is negative, well below unity in absolute value (see Blinder (1986a) and Haltiwanger and Maccini (1989)). Clearly, the implications of the simplest version of the model do not accord with the facts.[14]

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[14]On the other hand, Ghali (1987), Braun and Krane (1989), and Fair (1989) report results supportive of production smoothing for a few selected industries.

These discrepancies between theory and fact ought to be of interest well beyond the narrow confines of inventory theory, for the notion of production smoothing rests on convex costs; and convex costs constitute one of the main pillars of neoclassical economic theory. Without it, there is little reason to expect supply curves to be upward sloping. Can it be that typical real–world cost functions are concave instead of convex? Blinder (1986b) mentions this as one possible rationalization of the findings on inventory behavior. Ramey (1988) has undertaken a formal test of the idea and offers evidence that marginal cost functions are decreasing (in fact, steeply decreasing) rather than increasing for several two–digit manufacturing industries. If this is true on a widespread basis, it would be easy to understand why firms bunch rather than smooth production. But, at the same time, the main foundation of the law of supply would crumble.

There are less radical explanations of the key facts (other than errors in the data). One obvious candidate is cost shocks. Suppose (2) is modified to allow for an additive shock to marginal cost:

$$(2') \quad C(y_t) = (c_1 + \Gamma_t)y_t + (1/2c)y_t^2,$$

where Γ_t is an i.i.d. cost shock. Suppose further that this cost shock is seen by firms <u>before</u> they choose production. Then, in the absence of demand shocks, the solutions become:

$$(5') \quad y_t = \bar{y} + \beta\lambda(\bar{N} - N_t) - c(1-\beta\lambda)\Gamma_t$$

$$(6') \quad x_t = \bar{x} - (1-\beta)\lambda(\bar{N} - N_t) - c(1-\beta)\lambda\Gamma_t$$

$$(7') \quad N_{t+1} - N_t = \lambda(\bar{N} - N_t) - c(1-\lambda)\Gamma_t.$$

Clearly, cost shocks tend to make output more variable than sales and induce a positive covariance between inventory change and sales. The intuition is clear. If a firm is faced with cost shocks rather than demand shocks, inventories allow it to profit from intertemporal substitution by raising production and building inventories when costs are unusually low and cutting production and depleting inventories when costs are unusually high.

Empirically, however, cost shocks seem to provide an incomplete explanation for the disturbing facts. Some authors, e.g., Blinder (1986a) and Maccini and Rossana (1981, 1984), have found evidence that cost shocks in the form of raw material prices, but not wage rates, affect inventory investment. Others have allowed for the possibility of various types of cost shocks in direct tests of the production smoothing model. Miron and Zeldes (1988) find no evidence that various measures of real input prices explain the behavior of production. Eichenbaum (1989) does find evidence to support a model that includes an unobserved, but serially correlated, technology shock. Thus, cost shocks seem to work only when they are unobserved "technology shocks", not when they are observed factor prices.[15] This suggests that, while cost shocks may play some role in rationalizing the facts, they are not plausible as a complete explanation. Something else is needed.

A simple and empirically appealing candidate is to allow demand shocks to be serially correlated. If, for example, ϵ_t^1 is AR(1) with serial correlation parameter ρ, then (5) – (7) become:

$$(5'')\quad y_t = \bar{y} + \beta\lambda(\bar{N} - N_t) + \frac{1}{2}\frac{\beta\lambda}{1-\theta\rho}\epsilon_t^1$$
$$(6'')\quad x_t = \bar{x} - (1-\beta)\lambda(\bar{N} - N_t) + \frac{1}{2}[1 - \frac{(1-\beta)\lambda}{1-\theta\rho}]\epsilon_t^1 + \epsilon_t^2$$
$$(7'')\quad N_{t+1} - N_t = \lambda(\bar{N} - N_t) - \frac{1}{2}(1 - \frac{\lambda}{1-\theta\rho})\epsilon_t^1 - \epsilon_t^2,$$

where $\theta = \frac{1-\lambda}{1+r} < 1$ and r is the real rate of interest. The only coefficients that change are those of ϵ_t^1 in each equation. Simple manipulations of these coefficients show that serial correlation: (a) brings the coefficients of ϵ_t^1 in (5") and (6") closer together, which raises the ratio var(y)/var(x) toward unity; and (b) makes it possible for the coefficients of ϵ_t^1 in

[15] The above work was done with data for the manufacturing sector. West (1988c) finds that unobserved, serially correlated cost shocks dominate movements in aggregate GNP.

(6") and (7") to have the same sign, which could lead to a zero or positive covariance between x and ΔN.

The intuition behind these results is clear: more permanent demand shocks give firms less reason to smooth production. Moreover, higher order stochastic processes for ϵ_t, in which sales are rationally anticipated to "build" before they decline, enhance prospects that the relevant inequalities can be reversed. Blinder (1986b) shows that, in the presence of highly serially correlated demand shocks, even rather small cost shocks are enough to reverse both inequalities (8) and (9). However, ϵ_t^2 shocks continue to point toward the empirically falsified inequalities (8) and (9). So, to bring the model's predictions into greater conformity with the data, we also need an auxiliary assumption that most demand shocks are known to firms before they commit themselves to production plans. We do not know whether this is true.

In short, cost shocks combined with highly serially correlated demand shocks can explain why the variance of production exceeds that of sales and why inventory investment and sales are positively correlated. In this sense, the concept of convex production costs can be saved. But, empirically, it is not clear that this is a convincing explanation.

b. Stock Adjustment Models

Since Lovell (1961), the following stock–adjustment model has been widely used empirical work on inventory investment:

$$(10)\qquad N_{t+1} - N_t = \pi_1(N_t^* - N_t) - \pi_2(x_t - E_t(x_t))$$

$$0 < \pi_1 \leq 1 \qquad 0 < \pi_2 \leq 1$$

where

$$(11)\qquad N_t^* = \eta_0 + \eta_1 E_t(x_t) + \eta_2 E_t(\Gamma_t) + \eta_3 E_t(r_t)$$

The first term in (10) is planned or anticipated inventory investment, which is proportional to the gap between the "desired" and actual stocks of inventories. Here π_1 is the "adjustment speed" and N_t^* is the desired stock, which is typically related to current expected sales, $E_t(x_t)$, current expected costs, $E_t(\Gamma_t)$, such as wage rates, raw material prices, and energy prices, and current expected real interest rates, $E_t(r_t)$. The second term in (10) is <u>unanticipated</u> inventory investment, which captures the extent to which inventories buffer sales surprises. The closer is π_2 to unity, the greater the degree to which inventories buffer demand shocks.

The theory we have just outlined can rationalize something close to this empirical model. Assume that the firm minimizes expected discounted costs, including both production and inventory holding costs as specified in (2) and (3) above, and faces exogenous sales and cost shocks. Then the inventory investment relationship that emerges from the model is:

$$(12) \qquad N_{t+1} - N_t = \hat{\lambda}(N_t^* - N_t) - \epsilon_t^2$$

where $\hat{\lambda}$ depends on b,c and r. Further

$$(13) \qquad N_t^* = (\hat{\lambda})^{-1}[-(1-\hat{\lambda})E_t(x_t) + \hat{\lambda}\sum_{i=1}^{\infty}(\phi(1-\hat{\lambda}))^i E_t(x_{t+i})$$
$$-c(1-\hat{\lambda})\Gamma_t + c\hat{\lambda}\sum_{i=1}^{\infty}(\phi(1-\hat{\lambda}))^i E_t(\Gamma_{t+i}) + \hat{N}]$$

where $0 < \hat{\lambda} < 1$, and ϕ is the discount factor.[16] If sales are endogenous and the firm maximizes expected discounted profits, then inventory investment can again be written as (12) with the adjustment parameter, $\hat{\lambda}$, now depending on δ as well and with $\frac{1}{2}E_t(\epsilon_{t+i}^1)$ replacing $E_t(x_{t+i})$. Note that ϵ_t^1, the demand shock that is known to the firm when it makes price and output decisions, may be interpreted as a shift variable which enters the firm's demand curve, e.g., real income.

Clearly, the production smoothing/buffer stock model does rationalize certain key

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[16]The inventory investment equation and the specification of the desired stocks are derived by solving the second-order difference equation for inventories that emerges from the optimality conditions.

features of the stock adjustment model. For one, the speed of adjustment is less than unity. Further, it is easy to show that the adjustment speed is slower the greater is the incentive to smooth production, i.e., the smaller is c in (2). For another, when output and price are set before demand is known, so that inventories alone buffer demand shocks, the model predicts that $\pi_2 = 1$.

Some substantive differences between the theoretical and the empirical models arise with the specification of the desired stock. One is that expected sales is almost always used as the relevant expected demand variable in empirical work, even when the theory is based on profit maximization, in which case sales are endogenous.

A second is that the desired stock should depend on future as well as current expected sales and costs. This distinction is important because current and future expected sales have opposite signs in (13). An increase in $E_t(x_t)$ reduces N_t^* but an increase in $E_t(x_{t+i})$ raises N_t^* for all $i > 0$. Hence, if $E_t(x_t)$ and $E_t(\Gamma_t)$ alone are included in emprical models, they must be regarded as capturing the net effects, which will be reasonable only if current and future expected sales and costs are highly correlated.

How has the stock adjustment model performed empirically? Not very well. A major difficulty is that estimated adjustment speeds are extremely low, often less than 10% per month. This is implausible, especially when, as pointed out by Feldstein and Auerbach (1976), even the widest swings in inventory stocks amount to no more than a few days of production.

A natural reaction to this state of affairs is to argue that the estimates of the adjustment speed must suffer from econometric biases. One obvious potential source of such bias is omitted variables. Until recently, most studies of inventory investment omitted cost variables. If such variables are correlated with inventories, then π_1 may be biased downward. Some influence of real raw material prices, but not real wage rates, has in fact been found. But substantially higher adjustment speeds have not been achieved.[17]

Another potential source of a bias is the estimation procedure. In a model with a lagged dependent variable and a serially correlated error term, an inapproriate estimation procedure may give rise to adjustment speeds that are biased downwards. Maccini and Rossana (1984) argued that previous studies of inventory investment had failed to correct properly for serial correlation. Using Hatanaka's two–step procedure, they found very fast adjustment speeds coupled with high serial correlation in the error terms. Blinder (1986a), however, countered that the Hatanaka method tends to settle on a local rather than a global minimum of the sum of squared errors and that the global minimum normally has slow adjustment and low serial correlation. Subsequently, work by Hall and Rossana (1989) suggests that the difference in results may have more to do with differences in model specification than with differences in estimation methods. In any case, an appropriate adjustment for serial correlation seems not to resolve the problem.

Still another possibility is aggregation bias—either over time or over firms. Christiano and Eichenbaum (1987) observed that, if firms make decisions at intervals shorter than the interval over which the data are sampled, then empirical work may turn up estimates of adjustment speeds that are too low. Using estimation procedures that take time aggregation into account, they have had some limited success in producing more plausible adjustment speeds. Seitz (1988), using survey data for individual firms in Germany, found that aggregation over firms led to substantially slower estimated adjustment speeds.[18] These are promising avenues that need to be explored further.

Unless aggregation biases prove to be very important, it is difficult to attribute the

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[17]See Maccini and Rossana (1981, 1984), Blinder (1986a), and Irvine (1988) for empirical investigations of this question.

[18]Blinder (1986a) had observed the same phenomenon at a coarser level of aggregation in U.S. data.

low adjustment speeds to statistical biases. Yet theory strains to explain slow adjustment, requiring steep marginal cost or marginal revenue schedules or a flat marginal inventory holding cost schedule to do it. But the first of these implies a strong incentive to smooth production, which does not seem to be present. In short, the puzzle remains.

Another troubling feature of empirical work with stock adjustment models is that estimates of π_2, which measures the degree to which inventories serve as a buffer stock, are frequently quite low, generally statistically insignificant, and sometimes even of the wrong sign. This suggests that finished goods inventories play little role as a buffer for demand shocks. Blinder (1986b) suggests that the low values of π_2 may mean that the bulk of demand shocks are known to firms when they make decisions but are treated as unexpected by the econometrician. This, however, is a difficult point to establish empirically.

Finally, consider estimates of the determinants of desired stocks. The estimates of the parameter, η_1, which captures the response of inventories to changes in expected demand, tend to be very small, bouncing from positive to negative values in sign. This may be due to the fact that the variable that is used as a measure of expected demand is a blend of current and future expected sales which have opposite effects on inventories (see equation (13)), producing on net small coefficients with little systematic influence on inventories.

c. Inventory Investment and Interest Rates

The financial press and statements by business people are replete with assertions that higher interest rates induce firms to cut inventory holdings. Although it is often not clear whether real or nominal interest rates are being referred to, there is perceived to be an inverse relationship between interest rates and inventory investment. Consistent with this, intertemporal price speculation is the first idea that pops into the minds of many economists when they think about inventory behavior. The central idea is absolutely simple — so simple, in fact, that it is hard (for economists, anyway) to imagine how it could be wrong.

Consider a firm that owns a storable commodity worth p_t today. It expects each unit to be worth Ep_{t+1} next period. Suppose the commodity depreciates at the nonstochastic rate μ and can be carried forward for one period at a cost of h per unit (incurred at the start of the period). If the firm is risk–neutral and can borrow or lend freely at nominal interest rate i, its intertemporal choice is simple. A unit can be sold today for p_t or carried forward for one period to net the firm:

$$\frac{(1-\mu)E_t p_{t+1}}{1+i} - h \tag{14}$$

The firm should therefore sell nothing today if (14) exceeds p_t and sell everything if (14) is less than p_t.[19]

The observable implications of this model could hardly be further from reality. The model implies that the firm's demand for inventories should be infinitely elastic with respect to the real rate of interest. Yet little influence of real interest rates on inventory investment can be found empirically.

It is easy to escape the unhappy conclusion that the interest elasticity should be infinite. Just (a) make the marginal storage cost, h, a rising function of the amount stored, N, as in the production smoothing model; or (b) introduce imperfect competition, i.e., a downward–sloping demand curve for the firm, as in equation (1) above; or (c) introduce risk aversion by maximizing utility rather than wealth, which would amend (14) by attaching marginal utility weights to the prices. None of these modifications, however,

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[19] If firms behave this way, competitive equilibrium obviously enforces the arbitrage equation:

$$p_t = \frac{(1-\mu)Ep_{t+1}}{1+i} - h.$$

When $h = \mu = 0$, this is the well–known Hotelling rule.

disturbs the basic conclusion that the interest sensitivity of inventory investment is negative— which seems to be an inescapable feature of any intertemporal optimization model of inventories.

To test for this sensitivity empirically in the linear–quadratic version of the theory, however, is difficult because the real interest rate enters the solution for inventories in a highly nonlinear fashion—in the discount rate for expected future sales or costs, and as a determinant of both the adjustment speed and the steady state value of inventories (see (12) and (13) above). One approach that has been used to deal with this problem is to assume the real interest rate takes on a particular known value to facilitate the identification of other parameters and to ease estimation.[20] This eliminates by assumption any influence of interest rates on inventory investment and thus throws out a potentially important channel through which monetary policy may operate.

An alternative is to work with a specification of desired stocks, as in (11), which includes expected real interest rates as a determinant of N_t^* and thus of inventory investment. This crude but workable approximation to the role the real interest rate plays in the theory has been pursued by numerous investigators. Unfortunately, it generally fails to uncover an effect of real interest rates on inventory investment, especially that of finished goods in manufacturing.[21]

Why? Perhaps including expected real interest rates as a determinant of desired stocks is simply too crude an approximation to the theoretically correct influence. If so,

[20] See, e.g., Blanchard (1983) or Eichenbaum (1984). Years ago, economists argued that real interest rates were in fact constant. The experience of the late seventies and early eighties, however, appears to have shattered this argument.

[21] See Akhtar (1983) for references to the literature through the early eighties. The only recent work that finds an effect of interest rates on finished goods inventory investment is Ramey (1989) who treats finished goods inventories as a factor of production, which seems doubtful on theoretical grounds, and finds only weak evidence in a few two–digit industries.

the theory with a varying discount factor may need to be carried all the way through to the estimation stage. Or perhaps empirical measures of real rates fail to capture the effects of tighter or looser credit conditions faced by firms when borrowing to finance the holding of inventories.[22] Or, finally, the nominal rate and the expected rate of inflation may enter the relevant estimating equation separately due to tax considerations, the prevalence of FIFO pricing practices, etc.[23] Whatever the reason, the question of why inventory investment seems to be insensitive to changes in real interest rates remains open and important.

3. SOME ADDITIONAL RESCUE ATTEMPTS

Over the years, the basic production smoothing/buffer stock model has been extended in a number of directions at least in part to resolve the empirical puzzles mentioned above. We now explore the key features and implications of several of these extensions: quasi–fixed factors of production, stockouts, labor contracts, and strategic behavior.

a. Quasi–Fixed Factors of Production

Several authors have introduced quasi–fixed factors of production into inventory models.[24] Factors are quasi–fixed when there are (strictly convex) adjustment costs to changing them. Some examples are employment when there are costs of hiring and firing workers, and materials and supplies or intermediate goods inventories when there are costs of acquiring or disposing of such goods more quickly. Such costs give decisions on the relevant factor inputs a crucial intertemporal aspect, creating an interaction between

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[22]See Greenwald and Stiglitz (1988) for an analysis of the effects of imperfect information and associated finance constraints on production and, potentially, inventory decisions of firms.

[23]See Blinder (1981) and Irvine (1981a) for discussion of this point.

[24]See, e.g., Eichenbaum (1983), and Maccini (1984) for theoretical work and, e.g., Eichenbaum (1984), Maccini and Rossana (1984), Rossana (1990), and Topel (1982) for empirical work.

"investment" in these factors and in finished goods inventories. The empirical implication of this interaction is that the basic stock–adjustment equation, (10), must be modified to:

$$(15) \qquad N_{t+1} - N_t = \pi_1(N_t^* - N_t) - \pi_2(x_t - E(x_t)) + \pi_3(M_t^* - M_t)$$

where M_t and M_t^* are the actual and "desired" stocks of the quasi–fixed factor and $\pi_3 < 0$.[25] The reason for the negative sign for π_3 is the following: if the firm finds itself with an excess stock of the quasi–fixed factor, then the adjustment costs induce the firm to eliminate the excess stock gradually by producing more output and thus adding to its stock of finished goods inventories. The prime motivation for including quasi–fixed factors into inventory investment equations was the conjecture that the low estimated adjustment speeds might be due to statistical biases arising from their omission.

Unfortunately, while some evidence exists for the interaction between finished goods inventories and certain quasi–fixed factors (significant estimates of π_3), including actual stocks of quasi–fixed factors in empirical inventory investment equations has had little effect on the size of adjustment speeds (estimates of π_1).[26] Furthermore, adjustment costs attached to quasi–fixed factors tend to smooth fluctuations in these factors and thus to reduce the variability of output. This clearly does not help to bring the model into closer conformity with the fact that production tends to be more variable than sales.

b. Stockouts

One extension of the basic model that does help to resolve the empirical puzzles is

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[25]A similar effect arises from including costs of changing output in the firm's cost structure, as indicated above. Such costs render output a state variable and result in a similar modification of the basic stock adjustment equation, with lagged y replacing M in (15). Costs of changing output, however, are usually rationalized on the basis of adjustment costs of changing factors of production. Such costs are costs of changing inputs, not output, and thus we have focused our discussion on quasi–fixed factors of production.

[26]See, e.g., Maccini and Rossana (1984), and Blinder (1986a).

to allow for the possibility that the firm may be caught out of stock when an order arrives and thereby suffer a loss in sales. Stockouts were incorporated in a rough–and–ready way into the original linear–quadratic model of Holt et. al., by thinking of inventory–holding costs as the sum of two components. One is the cost of carrying inventories, which rises monotonically with inventories. The other is the expected cost of stocking out which, for any given level of expected sales, falls with inventories. Define K_t as the value of inventories that minimizes the sum of these two costs. Up to now, we have assumed that K_t is zero. Suppose, instead, that K_t is proportional to sales: $K_t = \alpha x_t$. Now an increase in sales raises desired inventory accumulation because it raises K_t. This tends to increase the variability of output relative to sales and also to impart a positive covariance to inventories and sales.

As several authors (e.g., Kahn (1987)) have pointed out, however, the treatment of stockouts in the linear–quadratic model suffers from two deficiencies. One is that negative inventories are permitted. Any orders that the firm cannot serve are assumed to be backlogged and result in sales and revenue in the period in which they occur, which seems to contradict the very notion of stockouts and the associated lost sales. The other deficiency is that the specification of the determinants of the target stock, K_t, is ad hoc.

A rigorous treatment of stockouts requires that the model incorporate the idea that a firm loses sales if it is caught out of stock when demand is high.[27] To do this, we must distinguish between demand, q_t, and sales, x_t. With stockouts, sales are:

$$x_t = \min(q_t, N_t + y_t)$$

where $N_t + y_t$ is the amount of goods available for sale in period t. If $q_t > N_t + y_t$ stockouts occur, and the unsatisfied demand is assumed to be lost forever.

[27]Zabel (1972, 1988) in particular has drawn out the implications of the so–called lost sales model. See also Abel (1985), Haltiwanger and Maccini (1988a, 1988b) and Kahn (1987) for recent work.

A little analysis shows that expected sales are the difference between expected demand and expected stockouts. Hence, in contrast with the linear–quadratic model, stockouts affect revenue, not costs, and are more complex in that expected stockouts depend on production as well as sales. Nevertheless, it remains true that higher initial inventories, N_t, reduce expected stockouts and thus raise expected revenue—which corresponds to the downward–sloping component of the firm's inventory holding cost function in the ad hoc linear–quadratic approach.

Stockouts alone seem to do little to explain the troubling facts. Imposing a non–negativity constraint on inventories and allowing for lost sales in a model with i.i.d. demand shocks does not raise the variance of production above that of sales (See Zabel (1988) or Kahn (1987)). Further, stockouts provide little help in explaining the low adjustment speeds or the interest insensitivity of inventory investment.

Recently, however, Kahn (1987) has modified the standard lost sales model to help explain the puzzling variance inequality. The idea is that a firm that stocks out in one period backlogs some of the excess demand, and the backlogged orders become a component of demand in the next period. Specifically, total demand in a period, $\hat{q}_t$, is:

$$\hat{q}_t = q_t + a[\hat{q}_{t-1} - (N_{t-1} + y_{t-1})] \qquad 0 < a < 1$$

where q_t is now "new" demand, and $a[\hat{q}_{t-1} - (N_{t-1} + y_{t-1})]$ is the portion of last period's excess demand which was backlogged. Kahn shows that under these circumstances the variance of production can exceed the variance of sales.[28] Essentially, the reason is that sales are smoothed because the ability to backlog demand enables the firm to shift sales away from periods when demand is high, and current production is made more volatile

[28]Kahn (1987) also shows that a rigorous treatment of stockouts together with serial correlation in demand, which we considered above, will do the trick. Alternatively, stockouts combined with small cost shocks may be enough to explain the pertinent facts. For empirical evidence on the stockout motive, see Kahn (1988).

because it responds to the backlogged excess demand of previous periods.[29]

c. Labor Contracts

Another extension that helps to explain the facts is implicit labor contracts, which were originally developed by Azariadis (1975), Baily (1974), and others.[30] Multiperiod labor contracts attach a labor force to the firm. If firms face significant hiring and firing costs and workers face search and mobility costs, the attached labor force will exhibit quite a bit of inertia. But, firms can vary the utilization of the attached labor force, and thus employment, through temporary layoffs and recalls that absorb unanticipated demand shocks. In effect, the firm now has both a stock of inventories and a stock of workers which can be used to "buffer" demand shocks.

This theoretical innovation helps explain the stylized facts in two ways. First, if the firm's cost structure dictates reliance on temporary layoffs and recalls rather than inventories to buffer demand shocks, then employment and production may be more variable relative to sales.[31] As long as costs are convex, this factor by itself will never raise the variance of production above that of sales. But small cost shocks may be enough to accomplish that.

Second, temporary layoffs can explain why finished goods inventories appear not to be used as buffer stocks. If labor contracts make temporary layoffs and recalls a relatively low cost alternative to firms, then adjustments in the labor force, not inventories, may do most of the buffering of demand shocks. Hence, π_2 in (10) may be quite small. Some

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[29]West (1988b) offers evidence for seven production–to–order industries that shipments are in fact strongly smoothed relative to new orders but that production is not smoothed relative to shipments.

[30]See Topel (1982) and Haltiwanger and Maccini (1988a, 1988b) for studies that incorporate labor contracts into inventory models.

[31]This is unlike models that attach adjustment costs to employment rather than to the attached labor force.

evidence in support of this position is presented in Haltiwanger and Maccini (1988c, 1989).

d. Strategic Behavior

With the surge in recent years in the application of game theory to economic problems, it is not surprising that strategic ideas have been used to try to understand the puzzling behavior of inventories. In particular, Rotemberg and Saloner (1989) develop a model of a duopoly in which inventories act as a strategic variable which deters each firm from deviating from an implicit collusive arrangement. They show that under certain conditions each firm holds inventories to serve as an "arsenal" to use to "punish" its rival if it deviates from the sustainable, symmetric equilibrium. High inventories play this role by signalling that, if the rival deviates from the cooperative equilibrium, the firm will be able to respond with a large increase in sales. This makes collusion more attractive to a potential deviator.

While the model is special in many respects, it does suggest that thinking of inventories as a strategic variable may help to unravel some of the empirical puzzles. In particular, Rotemberg and Saloner show that, under certain conditions, the model can explain the empirical finding that inventory investment and sales are positively correlated. In their model, an upward shift in industry demand raises both equilibrium sales and the incentive for a firm to deviate from the equilibrium. To counter this threat, each duopolist may raise its inventory holdings to increase its capability to punish its rival. Hence, inventories rise with sales.

Inventories are more apt to be used for strategic reasons in industries that are susceptible to collusive arrangements. Consistent with this implication, Rotemberg and Saloner provide some empirical evidence that the positive association between inventories and sales is stronger in more concentrated industries.

4. TAKING STOCK

An enormous amount of research has used variants of the production smoothing/buffer stock model to analyze the holding of finished goods inventories by manufacturers. The theory rests on seemingly weak assumptions but has encountered rough seas in empirical work. It has difficulty explaining why production is more variable than sales in many industries and why inventory investment and sales are positively correlated—though the feat can be achieved with some combination of cost shocks, serial correlation in demand, stockouts, labor contracts, or strategic considerations. The theory's empirical performance in its stock–adjustment form has been distinctly disappointing, producing implausibly low adjustment speeds and a lack of sensitivity of inventory investment to changes in interest rates.

A serious question must be raised at this stage of research: Is it productive to devote so many resources to the application of the production smoothing/ buffer stock model to the analysis of manufacturers' inventories of finished goods? We indicated in Section II that manufacturer's inventories of finished goods are the least volatile component of inventory investment. Much of the research is thus applying a model that does not fit the data terribly well, no matter how much it is manipulated, to an area—manufacturers' inventories of finished goods— that is relatively unimportant. If we are to achieve a better understanding of movements in aggregate inventory investment, it seems wise to pay more attention to other components of inventories, in particular, to retail inventories and to manufacturers' inventories of non–finished goods and materials and supplies. To do this, models other than the production smoothing/buffer stock model probably will need to be considered. The most prominent alternative is the (S,s) model.

B. THE (S,s) MODEL OF INVENTORY BEHAVIOR

If short–run marginal costs are increasing, inventories should be used to smooth production. However, this is not necessarily the only, nor even the most natural, technological assumption in all, nor even in most, applications. Recall, for example, the finding in Section II that manufacturers' inventories of finished goods and work in progress account for just 22% of the variance of monthly inventory investment; the remaining 78% comes from retail and wholesale inventories and from manufacturers' holdings of materials and supplies.[32] The productive activities in this last group consist basically of moving goods, not making them. Here the presumption that production is subject to rising marginal costs simply has no persuasive rationale.

When a manufacturer acquires raw materials and supplies for subsequent use, goods are transported from one factory or warehouse to another. When wholesalers and retailers purchase goods from manufacturers, products are taken off one shelf, transformed in some way (perhaps as little as moving and unpacking), and put on another shelf. Who really believes that such transport activities are subject to rising marginal costs, e.g., that the first dozen shirts a department store orders from a manufacturer cost less than the 15th dozen? In fact, the opposite presumption is more plausible in most cases: Beyond the fixed costs of handling an order, marginal costs are probably either constant or declining.

The technological assumption that acquisition costs consist of a fixed cost plus a constant marginal cost leads to the so-called (S,s) model of inventory behavior. Under this strategy, the firm optimally picks some number, s, below which it will not let its inventories fall. Whenever stocks reach this lower trigger point, it places an order large enough to restore inventories to some upper limit, S, also selected optimally.[33] The quantity $S - s$ is called the optimal lot size and depends on such variables as the fixed cost,

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[32]The 22%–78% split ignores all covariance terms. For a full accounting, see Table II–A.

[33]If time is discrete rather than continuous, or if there are delivery lags, inventories might fall below s and the purchase needed to restore inventories to S might therefore differ from S–s.

the purchase price, the probability distribution of sales, and the rate of interest.[34] If these variables change through time, then the optimal policy is an (S_t, s_t) rule with time–varying trigger points.

The economic behavior implied by the (S,s) model differs greatly from that of the production smoothing model, at least at the level of the individual firm. For example, a firm following an (S,s) strategy has neither an "optimal level" of inventory nor a "speed of adjustment." Instead, it has an optimal range. Whenever its lower trigger point is hit, it adjusts immediately; otherwise, it does not adjust at all.

The (S,s) idea is an old one, dating back at least to Harris (1915). Modern interest in (S,s) was sparked by the pioneering paper of Arrow, Harris, and Marschak (1951), which was followed by a series of papers culminating in Scarf's (1960) demonstration that (S,s) behavior is optimal under quite general conditions as long as the cost of acquiring goods is precisely:

$$C(y) = A + cy \quad \text{if } y > 0 \qquad (16)$$
$$= 0 \quad \text{if } y = 0,$$

where y is the purchase quantity and A and c are positive constants.[35]

Introspection suggests that the cost function (16) is much more relevant to the activities of wholesalers, retailers, and manufacturers holding raw materials than the convex cost functions we associate with manufacturing processes. More important, the (S,s) or two–bin strategy seems to be widely used in industry. We know this from anecdotal evidence (e.g., we see it practiced in stores) and from the fact that reference

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[34]Closed–form solutions for s and S are generally hard to come by. See Hadley and Whitin (1963) for examples.

[35]Bar–Ilan (1985) established that (S,s) behavior is optimal with even more general cost functions if time is continuous. The key ingredient is the presence of fixed costs.

books for purchasing agents and retail management texts concentrate on (S,s) strategies.[36]

Despite this, the standard stock–adjustment model has been the organizing framework for most of the empirical work that has been done on retail inventories, wholesale inventories, and manufacturers' inventories of materials and supplies.[37] Not surprisingly, this research has run into the usual litany of difficulties: implausibly low adjustment speeds, insignificant effects of unanticipated sales, etc..

Table III shows that the puzzling inequality var(y) > var(x) holds even more strongly in trade than it does in manufacturing, and that cov(x,ΔN) is small but positive in retailing and wholesaling. It is easy to understand why the (S,s) model feels at home with the stylized facts that pose such difficulties for the production smoothing model. In the inventory identity:

$$y = x + \Delta N,$$

y now denotes deliveries and x denotes usage for manufacturers' raw materials or sales for retailers and wholesalers. In an (S,s) model, a large realization of x will automatically make cov(x,ΔN) < 0 for any firm that does <u>not</u> hit its lower trigger point. Firms that hit their lower s barriers, however, will replenish inventories, thereby making cov(x,ΔN) <u>positive</u> for them. So we should not be surprised to find cov(x,ΔN) ≈ 0 in the aggregate. If that is so, then var(y) must necessarily exceed var(x). It is precisely this reasoning that first led Blinder (1981) to offer (S,s) as an explanation of the stylized facts for retailers.

The critical problem with applying the (S,s) model to the data economists generally have — which are aggregated over time, products, and firms — is that the model does not lend itself easily to aggregation. Because firms react to the same sales shock differently depending on where they are in their (S,s) range, the market does not behave like a blowup

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[36]See Mosser (1986, p. 52, fn.1) for a summary of the business literature.

[37]See, for example, Irvine (1981a,b) and Ramey (1989).

of any representative firm. For example, consider a retailer with S = 10 and s = 2 and sales of five units this period. If it starts the period with inventories of 7 or less, it will place an order for S − s = 8 new units this period. But if it starts with more than 7 units in stock, it will order nothing. Thus it is clear that the total volume of orders resulting from any given level of aggregate sales depends on the distribution of initial inventories across firms. If the initial distribution of inventories is skewed toward s, orders will react strongly. If, on the other hand, it is skewed toward S, the reaction will be muted.

For a long time, the seeming impossibility of aggregating the (S,s) model was thought to be an insuperable barrier to confronting it with data. But first Blinder (1981) and then Caplin (1985) showed — in two quite different ways — that the barrier could be overcome. Both authors deal with single–product firms with identical values of S and s — analogous to the usual assumption that all firms have the same cost function. Firms differ in that they have different opening inventory stocks and get different realizations of sales.

Caplin's (1985) approach begins by observing that an (S,s) policy makes inventories a Markov process: Only the opening inventory stock and this period's sales affect the closing inventory stock. The stationary distribution of this stochastic process is the uniform distribution in which all inventory levels between s and S are equally likely, just as common sense suggests. The mean of this steady–state distribution, (S+s)/2, is obviously constant over time. Less obviously, Caplin shows that the expected value of N <u>conditional on sales</u> is equal to the same unconditional mean, and is therefore itself constant. Thus:

$$E(\Delta N) = E(y - x) = 0,$$

from which it follows that:

$$y - x = e, \qquad (17)$$

where e is an error term with zero mean. From (17), it follows trivially that var(y) > var(x) and $cov(\Delta N,x) = cov(e,x) = 0$. These are roughly the two puzzling stylized facts of

Section II.

Caplin's results are surely elegant and insightful. But they seem to be of limited practical use because they pertain only to the (unconditional) stationary distribution of inventories. Somewhat surprisingly, Mosser (1987, 1988) nonetheless offers empirical evidence that supports the model for both retailers and wholesalers. Her most direct test is of the proposition that x and ΔN should have zero covariance, which she can reject in only one of nine retail sectors and five of eighteen wholesale sectors. So perhaps Caplin's results have more empirical applicability than it seems at first glance.

Blinder's (1981) approach, by contrast, requires no steady–state assumption; but it does embody a crucial linear approximation. He begins by writing the (S,s) rule as a rule for orders:

$$\begin{aligned} y &= S - s && \text{if } N - x \leq s \\ &= 0 && \text{if } N - x > s, \end{aligned}$$

where N is initial inventory, x is sales, and y is orders. Taking the expectation of y over the density of $Q \equiv N - x$ gives the market–wide expected value of orders:

$$\bar{y}_t = \int_{s-x^*}^{s} (S-s) f_t(Q_t) dQ_t = (S-s) F_t(s) \tag{18}$$

where x^* is the largest possible value of x.[38] Equation (18) displays the aggregation problem clearly: The average level of orders depends not just on the average levels of sales and opening inventories, but also on their distributions.

Blinder makes (18) empirical by linearization. For example, consider some parameter z that changes S while affecting neither s nor the initial distribution function F(Q). By direct differentiation of (18):

$$\frac{\partial \bar{y}}{\partial z} = \frac{\partial S}{\partial z} F_t(s). \tag{19}$$

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[38]Blinder (1981) actually assumes discrete rather than continuous time, and hence obtains slightly different formulas. The continuous–time case, used here, is simpler.

In principle, derivatives like this depend on the shape of the distribution function $F_t(Q)$ and hence should vary over time. Blinder ignores this time dependence and treats them instead as regression coefficients to be estimated. This seems neither more nor less objectionable than linearizations that are routinely done in other contexts.

It is sometimes argued that the (S,s) model, once aggregated, yields behavior identical to the stock adjustment model. This is not true, even though Blinder (1981, p. 475) shows that an aggregated (S,s) model can be manipulated to yield an equation which relates average inventory investment to average sales and average initial inventories. Specifically:

$$N_{t+1} - N_t = \alpha_1 E_t(x_t) + \alpha_2(x_t - E_t(x_t)) + \alpha_3 N_t + \text{error term}$$

where the α_i parameters are combinations of derivatives like (19) that depend, among other things, on the initial density function of $Q = N - x$. This certainly resembles a stock–adjustment equation; compare it with (10) above with N_t^* a function of expected sales. However, the coefficient of opening inventories (α_3) has nothing to do with the "speed of adjustment" and might even exceed unity; and the error term is complicated and correlated with N_t.

In addition, the model's dynamics differ dramatically from those of the stock adjustment model. As Blinder (1981) illustrates in a numerical example, following any temporary sales shock, an economy consisting of many firms following (S,s) policies will ultimately settle down into a steady state with a uniform distribution of stocks across firms, just as Caplin's result dictates. But, the adjustment process may take a long time and, in the interim, inventory stocks and flows will follow complex paths that echo past events in ways that bear little resemblance to the stock adjustment model.

Can we cope with such complexity empirically and test the model rigorously? The estimation of (S,s) models has barely begun; and there may be better aggregation techniques than either Blinder or Caplin has thought of — methods, for example, that deal with the time–varying nature of S and s. So we are a long way from knowing the answer to this question. It would be premature to declare now that the (S,s) model is the solution to the micro/macro paradox mentioned at the start of this essay. But we do know a few things. First, the (S,s) model can be derived from optimizing behavior under plausible conditions. Second, (S,s)–type strategies are apparently in wide use in industry. And third, the (S,s) model is consistent with the stylized facts of inventory behavior, at least in a broad sense. All this seems to constitute a good start

IV. THE MACROECONOMICS OF INVENTORIES

The discussion up to now has been almost entirely microeconomic. Yet the reader may have noticed that most of the authors whose work has been cited probably consider themselves macroeconomists. This curious state of affairs stems from data like those in Table I, which shows the dominant role of inventory investment in postwar U.S. recessions. Such data have apparently captured the attention of (at least some) macroeconomists while being ignored by microeconomists.

There is nothing unusual about the data in Table I. Abramovitz (1950) observed some 40 years ago that precipitous declines in inventory investment were a major feature of U.S. recessions in the period between the two world wars. Wilkinson (1989) found that inventory movements have played an even larger role in cyclical fluctuations in most other members of the Group of Seven than in the U.S. Furthermore, West (1988a) has pointed out that aggregate output seems to be more volatile than final sales in most of these countries.

Such empirical evidence suggests that inventories play a major role in the propagation of recessions.[39] Yet inventories are assigned a bit part, at best, in most economic theories of the business cycle. They are either tacked on as an afterthought or, more typically, omitted entirely. In our view, the neglect of inventories is a serious omission—not just for the sake of descriptive realism, but because inventories have important implications for the behavior of a wide variety of macro models. In fact, adding inventories to a macro model often changes its implications substantially, as we now show.

A. INVENTORIES IN KEYNESIAN MODELS

Early in the post–Keynesian era, Erik Lundberg (1937) and Lloyd Metzler (1941) realized that deviations of actual from expected sales would push inventory stocks out of equilibrium and that firms' subsequent efforts to restore inventory equilibrium could lead to cyclical behavior. Metzler constructed a series of formal models in which inventory dynamics provide a natural explanation for business cycles but can, for certain parameter values, lead to dynamic instability.

Since the details of Metzler's original model look quaintly idiosyncratic to the modern eye, we begin our investigation of the macroeconomics of inventories with a modernized version that is true to the spirit, though not the letter, of his work. The key assumption is that intended inventory accumulation follows a stock–adjustment equation with desired inventories a linear function of expected sales:

$$\text{Intended } N_{t+1} - N_t = \lambda[\theta + \alpha x_t^e - N_t], \qquad 0 \leq \lambda \leq 1.$$

This is, in fact, a hallmark of macroeconomic work on inventories: Despite the fact that aggregate output is more variable than sales, inventory holdings are almost always motivated by production–smoothing. Assume further that sales (aggregate demand) are linearly related to income (production), that sales expectations are adaptive, and that aggregate output is the sum of expected sales plus intended inventory accumulation. Close

[39]This does not mean that "inventory shocks" cause business cycles.

the model by appending the standard inventory identity. Manipulation of such a model yields a second-order difference equation for output:

$$(20)\quad y_t = Ay_{t-1} - By_{t-2} + C$$

where:

$$A = 1-\gamma + 1-\lambda(1-b) + b\gamma(1+\lambda\alpha) > 0$$

$$B = (1-\gamma)(1-\lambda(1-b)) + b\gamma(1+\lambda\alpha) > 0$$

$$C = \gamma\lambda a > 0.$$

and where b is the marginal propensity to spend and γ is the adaptation parameter for expectations.

As is well-known, second-order equations like (20) can produce cycles, whereas simpler first-order Keynesian models without inventories imply monotonic convergence to the steady state. Furthermore, we know today that AR(2) processes like (20) with $A > 1$, $B < 1$ and $A - B < 1$ characterize GNP data rather well.[40] The coefficients in (20) satisfy these restrictions for a wide variety of reasonable parameter values. Thus the simple Metzler model takes us remarkably far toward explaining actual GNP fluctuations.

However, Metzler noted that the inventory accelerator can make an otherwise stable model dynamically unstable. To see this, consider the case where the characteristic equation of (20) has complex roots, so that cycles arise. It is well known that the parameter B controls the amplitude of the cycle. A glance at the formula for B shows that it is less than unity when $\alpha=0$; so the model is stable in the absence of an inventory accelerator. But, B is an increasing function of α; so stronger inventory accelerators lead to more pronounced cycles. It is even possible that α could be large enough to make $B > 1$ and therefore render the inventory cycle explosive. Thus, even though inventories are held

[40]This statement is not meant to take a position on whether there is a unit root in GNP. Even if there is, an AR(2) like (20) gives a fairly good account of the data as long as $A - B$ is near one.

to smooth production, they can interact with the Keynesian multiplier to destabilize the macroeconomy. This goes some way towards resolving the micro–macro paradox mentioned at the start of this paper.

Metzler's model lacks most of the characteristic features of contemporary macro models. If IS and LM equations and an aggregate supply curve are added, a variety of new implications emerge. (See Blinder (1980)). Inventory investment is countercyclical in the very short run but predominately procyclical over the business cycle. The dynamics also generate periods of stagflation in which prices are rising while output is falling—a point made earlier by Maccini (1976). Most interestingly, if inventories shift the demand for labor, then real wages may be procyclical, which is an empirical regularity that standard Keynesian models have difficulty explaining.

Thus, putting inventories into even a simple Keynesian macro model leads to a model that can produce cycles and helps to explain important empirical phenomena. As we said, inventories matter.

B. INVENTORIES IN NEW CLASSICAL MONETARY MODELS

Both the modernized Metzler model and Blinder (1980) employ adaptive expectations, an assumption that has been widely criticized. This assumption is not crucial, however; stochastic models with rational expectations behave quite similarly. Blinder and Fischer (1981) is a case in point. Their model uses IS and LM equations, a Lucas supply function with a role for inventories:

$$(21)\quad y_t = k_t + \delta(p_t - {}_{t-1}p_t) + \varphi(N_t^* - N_t) + e_t$$

and a stock–adjustment equation of the following form:

$$(22)\quad N_{t+1} - N_t = \lambda(N_t^* - N_t) - \phi(p_t - {}_{t-1}p_t) + u_t,$$

where N^* is the desired inventory stock and e and u are stochastic errors. Equations (21) and (22) say that an inventory shortfall leads to both higher output and intended inventory accumulation while a price surprise leads to higher output and inventory decumulation. Blinder and Fischer (1981) finesse the dynamic instability problem noted by Metzler by assuming that N^* does not depend on (actual or expected) sales and that the rational expectations solution selects the stable root. Like most new classical models, the Blinder–Fischer model lacks an explicit labor market and hence is mum on the question of real wages. However, its other implications are much like the Keynesian model with inventories.

Blinder and Fischer use their model to make two points that are fairly obvious, but are also fairly important to the debate over new classical models. First, inventories are a natural source of persistence. Second, if N^* depends on the real interest rate, anticipated changes in money are not neutral.[41] Thus, if you take a new classical model with no persistence and neutral money and add the production–smoothing motive for holding inventories, you get a model with strong persistence and nonneutral money. The former is theoretically obvious (Metzler knew it!), but probably of great empirical importance since the other sources of persistence discussed in the literature probably cannot account for the economy's short–run dynamics.[42] The latter is theoretically interesting, but probably of little empirical importance.

C. INVENTORIES AND PRICE STICKINESS

Deriving price stickiness and/or asymmetries in price adjustment from rigorous microfoundations has been a longstanding intellectual challenge. In the 1980s, a series of papers including Reagan (1982) and Blinder (1982) suggested that inventories might help unravel the puzzle. The intuitive idea is that inventory stocks enable firms to meet part of any change in demand by altering inventory holdings, rather than by changing prices.

[41]The latter requires interest–sensitive money demand and a real balance effect on spending.

[42]For a general discussion of the persistence problem in rational expectations models, see Gordon (1981).

When demand surges, supply from current production is augmented by inventory liquidation, which reduces the price increase needed to clear the market. Conversely, when final demand is weak, firms augment demand by building inventories, which props up prices.

Asymmetries in the reaction of prices arise if there are stockouts.[43] When demand falls, firms can mute price increases by accumulating inventories. But, when demand increases, firms may encounter stockouts and be forced to raise prices rather than disgorge inventories. The conclusion is that prices may be more sticky downward than upward, as often assumed in Keynesian models.

There is one important loose thread in these arguments, however. Like most explanations of price rigidity based on maximizing behavior, the models implicitly deal with real, not nominal, rigidity. What is sticky is the price relative to some numeraire. Thus the model provides no reason for nominal rigidity. And it is nominal rigidity that is needed to make Keynesian macroeconomics work.

Thanks to some recent work by Ball and Romer (1987), however, this criticism is not as devastating as it first appears. Although they do not have an inventory application in mind, Ball and Romer show that small menu costs coupled with a significant real rigidity can lead to a substantial degree of nominal rigidity. Thus, if the previous arguments provide a satisfactory explanation for relative price rigidity, menu costs can turn it into an explanation of nominal price rigidity.

D. DISEQUILIBRIUM MODELS

In the 1970s, disequilibrium models based on the short–side rule or "min condition" became a popular mode of macroeconomic analysis.[44] A key feature of these models is that

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[43]This is the main point of Reagan (1982) and of Amihud and Mendelson (1983).

[44]Important work in this area includes that of Barro and Grossman (1976) and Malinvaud (1977). See Drazen (1980) for a survey. Inventories have been introduced into these

excess demand or supply in one market "spills over" into other markets. As Blinder (1980) and Honkapohja and Ito (1980) pointed out, however, the presence of inventories limits the extent of spillovers.

Consider, for example, the case of excess supply in the goods market. In the standard disequilibrium model, this excess supply spills over into the labor market, because firms that cannot sell all the output they want reduce output and employment. But, if output is storable, firms will reduce output and employment less and accumulate the unsold output as inventories. Thus inventories mute the spillover of excess supply in the goods market into excess supply in the labor market.

Alternatively, suppose there is excess demand in the goods market so that workers are unable to purchase all the goods they want at prevailing prices. In standard disequilibrium analysis, they react by reducing their labor supply. Excess demand in the goods market thereby spills over onto the labor market, and a rise in demand will actua[illegible]Xa <u>lower</u> output. However, if firms hold inventories, then excess demand in the goods market can be met out of inventories, unless the excess demand is so substantial that it leads to widespread stockouts. Once again, the spillover effect is muted, and perhaps even eliminated, and the negative "supply multiplier" is avoided.

E. REAL BUSINESS CYCLE MODELS

In recent years, real business cycle models have been used to analyze movements in output and related aggregates. In these models of competitive markets which always clear, serially correlated fluctuations in output are due primarily to productivity shocks which arise from technological innovations, the weather, and the like. Inventories have played a significant role in the development of at least one strain of these models.

In the influential paper of Kydland and Prescott (1982), inventories of finished

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models by Benassey (1982), Green and Laffont (1981), and Eckalbar (1985).

goods enter the production function with the standard neoclassical properties: positive and diminishing marginal products.[45] In their "calibration" analysis, Kydland and Prescott find that their model causes inventories to smooth production and generates a negative correlation between inventories and output. The theory we outlined in Section III.A.1 shows why. As we have repeatedly emphasized, however, these predictions run counter to the facts.[46]

Several authors have tried to reconcile the Kydland–Prescott model with the observed behavior of inventories. Bain (1985) constructed a model in which inventories are disaggregated into work in progress held by manufacturers and final goods held by retailers. This extension permits manufacturing production to vary more than retail sales. Christiano (1988) modified the Kydland–Prescott model so that agents must precommit to employment and capital decisions before taste and technology shocks are revealed, while inventory decisions are made later and therefore play a buffer stock role. This extension enables the model to explain the high volatility of inventory investment relative to output over the cycle, but coexists uneasily with the empirical difficulty of detecting a buffer–stock role for inventories.

Unfortunately, the rationale for inventories in these models is weak. Why should inventories of finished goods be an argument of the production function with the usual neoclassical properties? Kydland and Prescott suggest that firms can economize on the use of labor resources attached to restocking, make larger production runs thereby avoiding machine downtime, etc. These arguments are not only vague but actually suggest

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[45]Ramey (1989) has undertaken empirical work which tests this idea for finished goods inventories—without much success. Cooper and Haltiwanger (1989) introduce inventories in a more conventional way into a multi–sector version of real business cycle models.

[46]To obtain more appealing results, Kydland and Prescott add the stock of capital goods projects under construction—which can be interpreted as a goods in process inventory—to finished goods to get a broader concept of inventories.

non–convexities which violate the neoclassical properties that are assumed about the production function.

The work of Bain, however, does suggest a promising new direction: to pursue a multi–sector approach which distinguishes between the production and distribution of goods.[47] Two recent papers push this idea in an interesting direction. Cooper and Haltiwanger (1988) analyze a Robinson Crusoe economy in which a single agent manages a sector that produces a final good for consumption together with two sectors that produce intermediate goods. An appealing interpretation is to think of the final goods sector as retailing and the intermediate goods sectors as manufacturing. The authors show that, if it is very costly for manufacturers to hold inventories, then a non–convex technology (and thus bunching) in the retail sector will induce production bunching in the manufacturing sector, even though the production technology of the latter is convex. Given the plausibility of (S,s)–type technologies in retailing, but not in manufacturing, this theoretical result has evident empirical appeal.

In a similar vein, Lovell (1988) builds a multi–sector simulation model in which firms hold inventories of both raw materials and finished goods. The raw materials are guided by (S,s) rules while the finished goods are guided by conventional flexible accelerators. He finds that production is more variable than final demand due to the influence of the (S,s) policies followed by firms in managing raw materials inventories. If firms adjust the trigger points in their (S,s) rules frequently due to changes in economic variables, then a tremendous liquidation of stocks takes place when the cycle reaches a cyclical peak, and vice–versa at a cyclical trough.

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[47]This style of research harks back to the early work of Lovell (1962) and Foster (1963) who explored the stability of multi–sector models.

V. WHERE DO WE GO FROM HERE?

At present, there seem to be two well–developed microeconomic models of inventory behavior — production smoothing and (S, s) — with a number of variants and wrinkles on each.

The production smoothing model makes a seemingly weak assumption about technology: that production costs are convex with zero marginal cost at zero output (or that there is a convex cost of changing production). When economists introspect, that seems to be a plausible characterization of technology for many manufacturing processes. And, apparently for that reason, it has become the standard model for manufacturers' inventories of finished goods — the most extensively studied (but least important) type of inventory. It would be nice to have better evidence than that. The (S,s) model also makes a simple assumption about technology, but a different one: that marginal cost is not zero at zero. Instead, some finite fixed cost must be paid to produce anything. This sort of technology leads to production bunching rather than to production smoothing and seems to apply most naturally to purchases by retailers, wholesalers, and manufacturers buying raw materials and supplies.

The linear–quadratic version of the production smoothing model is easy to aggregate because it leads to linear decision rules. Hence a micro model can readily be turned into a macro model based on a "representative firm." (S,s) models, by contrast, are notoriously difficult to aggregate because they have no concept of a representative agent. However, there are approaches to aggregation that lead to empirically implementable models; and other approaches can probably be invented.

Production smoothing has not only been the model of choice of almost all theorists who have tried to model inventory behavior, but also underlies the stock-adjustment model, which dominates econometric work on inventories. However, something resembling an (S,s) policy appears to be widely used in business.

The production–smoothing model has trouble coping with at least three salient facts about inventory behavior: first, production is more variable than sales in many industries; second, inventory investment and output do not covary negatively; third, estimated adjustment speeds are implausibly slow. The theory can be spruced up in various ways to make it more compatible with the first two facts. But some of these fixups look like epicycles designed to patch up what amounts to a Ptolemaic theory.[48] The (S,s) approach, by contrast, needs no such fixups, for it coexists naturally with the basic facts.

We have mentioned several times in this essay the micro–macro paradox: Inventories, which are supposed to stabilize production at the micro level, apparently destabilize output at the micro level. The (S,s) model sees no paradox because it views inventories as a way to make production more variable than sales even at the micro level. The production–smoothing model resolves the paradox through the interaction of the Metzlerian inventory accelerator (the idea that desired inventory stocks rise with sales) and the Keynesian multiplier. Neoclassical models with inventories have a hard time coping with this problem.

Of course, production smoothing and (S,s) are not the only games in town. There may be other models — such as stockouts — which work better than either, both theoretically and empirically. Alternatively, multi–sector models where some sectors are driven by production smoothing policies and others by (S,s) policies show a good deal of promise. But, if forced to choose between the two, the weight of the evidence to date seems to point strongly in the direction of (S,s). And this despite the fact that the contest is one sided: There has been vastly more work done within the production–smoothing

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[48]An exception is cost shocks, which are natural explanations of the first two facts. However, many authors, including the present ones, have a hard time believing that cost shocks dominate demand shocks.

framework than within the (S,s) framework.

(S,s) models are lately being applied to problems as diverse as purchases of consumer durables, price setting, portfolio choices, and industrial entry and exit.[49] Given this burgeoning interest in (S,s)–type reasoning in a wide variety of areas, it would indeed be strange if the (S,s) model were forsaken in the area in which it originated: inventory behavior.

[49]See Bar–Ilan and Blinder (1989), Caplin and Leahy (1989), Constantinides (1987), Grossman and Laroque (1987), and Dixit (1989)

References

Abel, Andrew B., "Inventories, Stockouts, and Production Smoothing," Review of Economic Studies, (April 1985) pp. 283–94.

Abramovitz, Moses, Inventories and Business Cycles, New York: National Bureau of Economic Research, 1950.

Akhtar, M. A., "Effects of Interest Rates and Inflation on Aggregate Inventory Investment in the United States," American Economic Review 73, (June 1983), pp. 319–328.

Amihud, Yakov and Haim Mendelson, "Price Smoothing and Inventory," Review of Economic Studies L (January 1983), pp. 87–98.

Arrow, Kenneth, T. B. Harris, and Jacob Marschak, "Optimal Inventory Policy," Econometrica, Vol. 19, No. 3 (July 1951), pp. 250–272.

Azariadis, Costas, "Implicit Contracts and Underemployment Equilibria," Journal of Political Economy LXXXIII (December 1975), pp. 1183–1202.

Baily, Martin N., "Wages and Employment under Uncertain Demand" Review of Economic Studies, XLI (January 1974) pp. 37–50.

Bain, Ian, "A Theory of the Cyclical Movements of Inventory Stocks", University of Minnesota mimeo, December 1985.

Ball, Laurence and David Romer, "Real Rigidities and the Non–Neutrality of Money," NBER Working Paper No. 2476, December 1987.

Bar–Ilan, Avner, "Optimal Decision Rules and Fixed Transactions Costs," Princeton University, Ph.D. Dissertation, 1985.

Bar–Ilan, Avner and Alan S. Blinder, "Consumer Durables and the Optimality of Usually Doing Nothing", mimeo, Princeton University, 1989.

Barro, Robert and Herschel I. Grossman, Money, Employment and Inflation, (Cambridge: Cambridge University Press), 1976.

Belsley, David A., Industry Production Behavior: The Order Stock Distinction, Amsterdam: North Holland, 1969.

Benassy , Jean–Pascal, The Economics of Market Disequilibrium, (New York: Academic Press), 1982.

Blanchard, Oliver J., "The Production and Inventory Behavior of the American Automobile Industry," Journal of Political Economy, Vol. 91, No. 3 (June 1983), pp. 365–400.

Blinder, Alan S., "Inventories in the Keynesian Macro Model," Kyklos, Vol. 33, No. 4, 1980, pp. 585–614.

Blinder, Alan S., "Retail Inventory Behavior and Business Fluctuations," Brookings Papers on Economic Activity, No. 2, 1981, pp. 443–505.

Blinder, Alan S., "Inventories and Sticky Prices: More on the Microfoundations of Macroeconomics," American Economic Review,Vol. 72, No. 3 (June 1982), pp. 334–349.

Blinder, Alan S., "More on the Speed of Adjustment in Inventory Models,"Journal of Money, Credit, and Banking, Vol. 18, No. 3 (August 1986a), pp. 355–365.

Blinder, Alan S., "Can the Production Smoothing Model of Inventory Behavior Be Saved?," Quarterly Journal of Economics, Vol. 101, No. 3 (August 1986b), pp. 431–453.

Blinder, Alan S. and Stanley Fischer, "Inventories, Rational Expectations, and the Business Cycle," Journal of Monetary Economics, Vol. 8, No. 3 (November 1981), pp. 277–304.

Blinder, Alan and Douglas Holtz–Eakin, "Constant Dollar Manufacturers' Inventories: A Note," manuscript, Columbia University, 1983.

Caplin, Andrew S., "The Variability of Aggregate Demand with (S,s) Inventory Policies," Econometrica, Vol. 53, No. 6 (November 1985), pp. 1395–1409.

Caplin, Andrew S. and John Leahy, "Monetary Effectiveness with Two–Sided (S,s) Policies" mimeo, Princeton University, May, 1989.

Carlson, John A. and William Wehrs, "Aggregate Inventory Behavior," in G. Horwich and P. A. Samuelson (eds.), Trade, Stability and Macroeconomics: Essays in Honor of Lloyd A. Metzler, New York: Acadmic Press, pp. 311–332, 1974.

Childs, G., Unfilled Orders and Inventories (Amsterdam: NorthHolland Publishing Company), 1967.

Christiano, Lawrence J., "Why Does Inventory Investment Fluctuate So Much?," Journal of Monetary Economics, Vol. 21 No. 2, (March, 1988), pp. 247–280.

Christiano, Lawrence J. and Martin S. Eichenbaum, "Temporal Aggregation and Structural Inference in Macroeconomics," CarnegieRochester Conference Series on Public Policy, No. 26, pp. 63–130, 1987.

Constantinedes, George M., "Capital Market Equilibrium with Transactions Costs", Journal of Political Economy, 94 (August, 1986), pp. 842–862.

Cooper, Russell and John Haltiwanger, "Macroeconomic Implications of Production unching: Factor Demand Linkages," University of Maryland mimeo, (October 1988).

Cooper, Russell and John Haltiwanger, "Inventories and the Propogation of Sectoral Shocks," American Economic Review, 80 (March, 1990), pp.170–190. 2425, (November 1987).

Dixit, Avinash, "Entry and Exit Decisions under Uncertainty", Journal of Political Economy, 97 (June, 1989), pp. 620–638.

Drazen, Allen, "Recent Developments in Macroeconomic Disequilibrium Theory," Econometrica, Vol. 48, No. 2 (March 1980), pp. 283–306.

Eckalbar, John C., "Inventory Fluctuations in a Disequilibrium Macro Model", The Economic Journal, vol. 95 (December 1985), pp. 976–991.

Eichenbaum, Martin S., "A Rational Expectations Equilibrium Model of Inventories of Finished Goods and Employment," Journal of Monetary Economics, Vol. 12, (August 1983), pp. 259–278.

Eichenbaum, M. S.,, "Rational Expectations and the Smoothing Properties of Inventories of Finished Goods", Journal of Monetary Economics, Vol. 14, pp. 71–96, 1984.

Eichenbaum, Martin S., "Some Empirical Evidence on the Production Level and Production Cost Smoothing Models of Inventory Investment," NBER Working Paper #2523, (February 1988).

Fair, Ray C., "The Production Smoothing Model is Alive and Well," Journal of Monetary Economics, Vol. 24, No. 3 (November, 1989), pp.353–370.

Feldstein, Martin and Alan Auerbach, "Inventory Behavior in Durable Goods Manufacturing: The Target–Adjustment Model," Brookings Papers on Economic Activity, No. 2, pp. 351–396, 1976.

Foster, Edward, "Sales Forecasts and the Inventory Cycle," Econometrica 31, (July 1963), pp. 400–421.

Ghali, Moheb A., "Seasonality, Aggregation, and the Testing of the Production Smoothing Hypothesis," The American Economic Review, Vol. 77, No. 3 (June 1987), pp. 464–469.

Gordon, R. J., "Output Fluctuations and Gradual Price Adjustment," Journal of Economic Literature, (June 1981).

Green–Laffont, Jerry and Jean–Jacques Laffont, "Disequilibrium Dynamics with Inventories and Anticipatory Price–Setting", European Economic Review, vol. 16 (1981), pp. 199–221.

Greenwald, Bruce and Joseph E. Stiglitz, "Imperfect Information, Finance Constraints, and Business Fluctuations, in M. Kohn & S. C. Tsiang (eds.), Finance Constraints, Expectations, and Macreconomics, Oxford, Clarendon Press, pp. 103–140, 1988.

Grossman, Sanford and Guy Laroque, "Asset Pricing and Optimal Portfolio Choice in the Presence of Illiquid Consumption Goods", NBER Working Paper No. 2369, August 1987.

Hadley, G. and T. M. Whitin, Analysis of Inventory Systems, Englewood Cliffs, NJ, Prentice-Hall, Inc., 1963.

Hall, Alastair and Robert Rossana, "On Estimating the Speed of Adjustment in Partial Adjustment Models"North Carolina State mimeo, May, 1989.

Haltiwanger, John C. and Louis J. Maccini, "Inventories, Orders, Temporary and Permanent Layoffs: An Econometric Analysis," forthcoming in the Carnegie-Rochester Conference Series on Public Policy, 1989.

Haltiwanger, John C. and Louis J. Maccini,, "A Model of Inventory and Layoff Behavior Under Uncertainty," Economic Journal, (September 1988a), pp. 731-745.

Haltiwanger, John C. and Louis J. Maccini, "Inventories, Multi-period Implicit Contracts, and the Dynamic Behavior of the Firm Under Uncertainty," The Johns Hopkins University, mimeo, 1988b.

Haltiwanger, John C. and Louis J. Maccini, "The Time Series Properties of Aggregate Orders, Inventories and Labor Inputs", in A. Chikan and M. Lovell (eds.), The Economics of Inventory Management, Amsterdam: Elsevier Science Publishers, 1988c.

Harris, F., Operations and Cost, (Factory Management Series), Chicago, A. W. Shaw and Co., 1915

Hawtrey, R. G., Trade and Credit, (London), 1928.

Hay, G., "Production, Price and Inventory Theory," American Economic Review Vol. 60, No. 4 (September 1970), pp. 531-545.

Hinrichs, John C. and Anthony D. Eckman, "Constant Dollar Manufacturing Inventories," Survey of Current Business 61, (November 1981) pp. 16-23.

Holt, Charles C., Franco Modigliani, John F. Muth, and Herbert Simon, Planning Production, Inventories, and Work Force, Englewood Cliffs, New Jersey: Prentice-Hall, 1960.

Honkapohja, Seppo and Takatoshi Ito, "Inventory Dynamics in a Simple Disequilibrium Macroeconomic Model", Scandanavian Journal of Economics, vol. 82, (1980), pp. 184-198.

Irvine, F. Owen, "Retail Inventory Investment and the Cost of Capital," American Economic Review Vol. 70, No. 4 (September 1981a), pp. 633-648.

Irvine, F. Owen, "Merchant Wholesaler Inventory Investment and the Cost of Capital," American Economic Review 71, (May 1981b), pp.23-29.

Irvine, F. Owen, "Specification Errors and the Stock–adjustment Model: Why Speeds of Adjustment are Too Slow in Inventory Equations", in A. Chikan and M. Lovell (eds.), The Economics of Inventory Management, Amsterdam: Elsevier Science Publishers, 1988.

Kahn, James A., "Inventories and the Volatility of Production," American Economic Review, Vol. 77, No. 4 (September 1987), pp. 667–679.

Kahn, James A., "Why is Production More Volatile than Sales? Theory and Evidence on the Stockout–Avoidance Motive for InventoryHolding," (December 1988), University of Rochester Working Paper No. 168.

Karlin, S., "Optimal Inventory Policy for the ArrowHarrisMarschak Dynamic Model," in Arrow, K., S. Karlin and H. Scarf (eds.) Studies in the Mathematical Theory of Inventory and Production. Stanford: Stanford University Press, 1958.

Kydland, Finn E. and Edward C. Prescott, "Time to Build and Aggregate Fluctuations," Econometrica Vol. 50, (November 1982), pp. 1345–1369.

Krane, Spencer and Steven Braun, "Production Smoothing: Evidence from Physical Product Data," Board of Governors of the Federal Reserve System, (June, 1989).

Lovell, Michael C., "Manufacturers' Inventories, Sales Expectations, and the Acceleration Principle," Econometrica, Vol. 29, No. 3 (July 1961), pp. 293–314.

Lovell, Michael C., "Buffer Stocks, Sales Expectations and Stability: A Multi–sector Analysis of the Inventory Cycle," Econometrica 30, (April 1962), pp. 267–296.

Lovell, Michael C., "Simulating Aggregate Inventory Behavior, "Wesleyan University mimeo, 1988.

Lundberg, Erik, Studies in the Theory of Economic Expansion, (London), 1937

Maccini, Louis J., "An Aggregate Dynamic Model of Short–Run Price and Output Behavior", Quarterly Journal of Economics, 90, pp. 177–196, 1976.

Maccini, Louis J., "The Interrelationship Between Price and Output Decisions and Investment Decisions: Microfoundations and Aggregate Implications", Journal of Monetary Economics, (January 1984), Vol. 13, No. 1, pp. 41–65.

Maccini, Louis J. and Robert J. Rossana, "Investment in Finished Goods Inventories: An Analysis of Adjustment Speeds," The American Economic Review, Vol. 71, No. 2, (May 1981), pp. 218–236.

Maccini, Louis J. and Robert J. Rossana, "Joint Production, Quasi–Fixed Factors of Production, and Investment in Finished Goods Inventories," Journal of Money, Credit and Banking, Vol. 16, No. 2 (May 1984), pp. 218–236.

Malinvaud, Edmond, The Theory of Unemployment Reconsidered (Oxford: Basil Blackwell), 1977.

Metzler, Lloyd A., "The Nature and Stability of Inventory Cycles," Review of Economic Statistics, Vol. 23 (August 1941), pp. 113–129.

Miron, Jeffrey A. and Stephen P. Zeldes, "Production, Sales, and the Change in Inventories: An Identity that Doesn't Add Up," University of Pennsylvania, Wharton School, mimeo, 1987.

Miron, Jeffrey A. and Stephen P. Zeldes, "Seasonality, Cost Shocks, and the Production Smoothing Model of Inventories," Econometrica, Vol. 56, No. 4 (July 1988), pp. 877–908.

Mosser, Patricia C., Essays on Inventory Behavior and Excess Volatility, unpublished Ph.D. dissertation, MIT, 1986.

Mosser, Patricia C., "Empirical Tests of the (S,s) Model for Merchant Wholesalers", in A. Chikan and M. Lovell (eds.), The Economics of Inventory Management, Amsterdam: Elsevier Science Publishers, 1988.

Mosser, Patricia C., "Trade Inventories and (S,s)," Columbia University mimeo, 1987.

Orr, L., "A Comment on Sales Anticipations and Inventory Investment," International Economic Review, October 1967, pp. 368–373.

Ramey, Valerie A., "Inventories as Factors of Production and Economic Fluctuations", American Economic Review, Vol. 79 No. 3, (June 1989), pp. 338–354.

Ramey, Valerie A., "Nonconvex Costs and the Behavior of Inven tories," University of California at San Diego, mimeo, 1988.

Reagan, Patricia B., "Inventory and Price Behavior", Review of Economic Studies, Vol. 49, (January 1982), pp. 137–142.

Rossana, R. J., "Interrelated Demands for Buffer Stocks and Productive Inputs: Estimation for Two–Digit Manufacturing Industries", Review of Economics and Statistics, 72 February 1990, pp. 19–29.

Rotemberg, Julio and G. Saloner, "The Cyclical Behavior of In ventories," Quarterly Journal of Economics, February 1989.

Scarf, Herbert, "The Optimality of (S,s) Policies in a Dynamic Inventory Problem," in Mathematical Methods in the Social Sciences, Arrow, Karlin, and Suppes (eds.), Palo Alto: Stanford University Press, 1960.

Seitz, Helmut, "Still More on the Seed of Adjustment in Inventory Models: A Lesson in Aggregation," University of Manheim Discussion Paper, 1988.

Topel, Robert H., "Inventories, Layoffs, and the Short–Run Demand for Labor," American Economic Review, Vol. 72, No. 4 (September 1982), pp. 769–787.

West, Kenneth D., "A Note on the Econometric Use of Constant Dollar Inventory Series," Economic Letters, Vol. 13, pp. 337–341, 1983.

West, Kenneth D., "A Variance Bounds Test of the Linear Quadratic Inventory Model," Journal of Political Economy, Vol. 94, No. 2 (April 1986), pp. 374–401.

West, Kenneth D., "Evidence from Seven Countries on Whether Inventories Smooth Aggregate Output," NBER Working Paper #2264, 1988a.

West, Kenneth D., "Order Backlogs and Production Smoothing," in A. Chikan and M. Lovell (eds.), The Economics of Inventory Management, Amsterdam: Elsevier Science Publishers, 1988b.

West, Kenneth D., "The Sources of Fluctuations in Aggregate Inventories and GNP," Princeton University mimeo, 1988c.

Wilkinson, Maurice, "Aggregate Inventory Behavior in Large European Economies," European Economic Review, Vol. 33 (1989), pp. 181–194.

Zabel, Edward, "Monopoly and Uncertainty", Review of Economic Studies, Vol. 37, No. 110, (April 1970), pp. 73–87.

Zabel, Edward, "Multiperiod Monopoly Under Uncertainty," Journal of Economic Theory, Vol. 5, No. 3 (December 1972), pp. 524–536.

Zabel, Edward, "Price Smoothing in a Monopolistic Market," International Economic Review (June 1986), pp. 349–364.

Zabel, Edward, "Price, Output and Inventory Behavior with a General Demand Structure" in A. Chikan and M. Lovell (eds.), The Economics of Inventory Management, Amsterdam: Elsevier Science Publishers, 1988b.

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