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LONG-RUN COMPARATIVE STATICS

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ABSTRACT

What are the long-run effects of permanent changes to the economy? We characterize long-run comparative statics for a broad class of models in terms of expenditure shares, substitution elasticities, and capital supply elasticities. Our key insight is that long-run analysis can be performed using an as-if static economy where capital is treated as an intermediate input subject to endogenous markups. These markups, which measure deviations from the Golden Rule of savings, equal the ratio of capital income to investment. This reframing yields a surprising result: long-run consumption responses follow second-best principles even in efficient economies. In particular, reallocations have first-order effects since the envelope theorem does not apply. Furthermore, sales alone do not summarize industries' importance for long-run consumption. To show how these points matter in practice, we develop a quantitative model of the world economy to study how markups, tariffs, and productivities affect long-run consumption. The model features input-output linkages, imperfectly elastic capital supply, heterogeneous returns, and endogenous net foreign asset positions. We find large negative first-order effects of tariffs and markups, even when initial tariffs and markups are zero. We also find that the productivities of industries upstream of investment goods have substantially larger long-run consumption effects than their sales shares would suggest.

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1 Introduction

What are the long-run effects of permanent changes to the economy? For example, how do consumption levels respond to permanent changes in tariffs, taxes, or industry productivities, taking into account adjustments to capital and other accumulable factors?

This paper characterizes comparative statics of balanced growth paths (BGPs) in terms of primitives: expenditure shares, elasticities of substitution in preferences and production, and the elasticity of savings with respect to returns. Our results are valid for a large class of models, allowing for multiple countries, potentially with overlapping generations of households facing uninsurable idiosyncratic risk, arbitrary substitution patterns, an arbitrary number of perishable and capital goods, factor endowments, input-output networks, as well as an arbitrary pattern of tax-like distortions.

We use our characterization to study how long-run consumption responds to changes in productivities and distortions such as markups and tariffs. A rich literature, dating back to Harberger (1964) for distortions and Hulten (1978) for productivities, has examined how these forces affect output and TFP using envelope conditions.[1] This envelope approach has enabled researchers to derive general results without relying on functional form assumptions. Our paper extends this tradition to the study of long-run consumption in environments with capital accumulation.[2]

We do this by establishing an equivalence between dynamic and static economies. Specifically, we show that BGP prices and quantities form an equilibrium of an as-if static economy. In this equivalent static economy, capital goods are intermediate inputs produced from investment goods and sold at a markup. The as-if markup for a capital good i captures deviations from the Golden Rule of savings, and is given by $\mu_i = \frac{r_i+\delta_i}{g+\delta_i}$, where r_i is the rate of return, δ_i is its depreciation rate, and g is the economy's growth rate.

The equivalence follows from two observations. First, sustaining a unit of capital on a BGP requires $g + \delta_i$ units of investment. This relationship between BGP capital stocks and investments map to production functions of capital in the as-if static economy. Second, the rental price of capital services is proportional to $r_i + \delta_i$. The ratios of rental rates to capital production costs $g + \delta_i$ map to the as-if markups. These as-if markups equal one

[1]Some papers in this tradition include, for example, Foerster et al. (2011), Gabaix (2011), Acemoglu et al. (2012), Di Giovanni et al. (2014), and Baqaee and Farhi (2019) for efficient economies. For distorted economies, examples include Basu and Fernald (2002), Jones (2011), Petrin and Levinsohn (2012), Bigio and La'O (2016), Liu (2019), Baqaee and Farhi (2020), Buera and Trachter (2024) and Dávila and Schaab (2023).

[2]Our method can also be used to characterize long-run changes in GDP, which includes changes to both consumption and investment. We focus on long-run consumption because, as noted in Kuznets (1941), consumption goods are the only true final goods, with investment goods being intertemporal intermediate inputs. For a recent in-depth discussion on this point, see Barro (2021).

at the Golden Rule and – crucially for empirical work – can be measured as the ratio of capital income to investment, with as-if markups exceeding one whenever capital income exceeds investment.[3]

Our equivalence means that to study long-run consumption, we can employ tools developed for the analysis of distorted static economies.[4] In particular, when analyzing the effects of distortions and productivities, there are dynamic counterparts to concepts such Hulten's theorem, first-order reallocation effects in the presence of distortions, and the distinction between cost and revenue weights of producers.

1. **Hulten's theorem for long-run consumption.** If the economy operates at the Golden Rule, then all as-if markups are equal to one and the equivalent static economy is efficient.[5] This has two implications. First, since the envelope theorem applies to long-run consumption, changes in distortions have no first-order effects. Second, for changes in productivities, a version of Hulten's theorem applies: the response of long-run consumption to a producer's productivity equals that producer's sales relative to aggregate consumption. To a first order, nothing else matters.[6] Note that these consumption elasticities are larger than traditional Domar weights, which are defined as sales relative to GDP. This amplification reflects that capital is reproducible in the long run.[7]

2. **Reallocation effects.** If capital stocks are below their Golden Rule levels, then as-if markups are above one, and the equivalent static economy is inefficient. Since the envelope theorem does not apply to long-run consumption, reallocations have first-order effects. This implies that changes in distortions, which act through reallocations, also have first order effects. It also creates an extra channel through which productivities can affect long-run consumption beyond mechanical effects.

   Just as in static distorted economies, reallocation effects are positive if resources

[3]This is the same criterion as the one introduced by Abel et al. (1989) for determining whether the capital stock is below its Golden Rule value.

[4]The distorted static economy is an analytical tool for studying long-run outcomes. It does not imply that the dynamic equilibrium is necessarily Pareto inefficient. When there exists a single representative agent, so that time-0 welfare is unambiguously defined, traditional envelope approaches still apply, using consumption goods indexed by dates. If the first welfare theorem holds, then an intertemporal version of Hulten's theorem applies, with welfare effects being equal to the discounted value of sales relative to the net present value of consumption. For the case with distortions, Basu et al. (2022) show that traditional measures of TFP still have a time-0 welfare interpretation.

[5]This assumes that prices are equal to marginal costs in other markets as well.

[6]In particular, the result does not depend on functional forms, preferences, or input-output structure.

[7]In contrast, for the effect on TFP, which holds the quantity of capital and labor constant, Hulten (1978) still applies regardless of whether the economy is at the Golden Rule, with the effect on TFP given by standard Domar weights. See footnote 22 for further discussion.

move towards the production of goods that are underproduced. In our case, this means that changes that reallocate resources towards the production of investment goods boost long-run consumption to a first-order. For example, a tax on capital reduces long-run consumption by redistributing resources away from investment, with an effect that scales with the distance from the Golden Rule. We show that a sufficient statistic for the effect of a reallocation is how much it changes the labor share. Reductions in the labor share indicate positive reallocation, since it means that more resources are being allocated to the production of capital.

3. **Revenue vs cost weights.** If the economy is not at the Golden Rule, consumption elasticities do not equal sales weights even when resource reallocations are absent. The reason is that if a producer charges a markup, their suppliers are not paid their full marginal revenue product, and so sales underestimate shadow values. To correct for this, productivity effects need to be calculated using *cost-based* Domar weights, which are based on an input-output matrix where input shares are recorded relative to costs, rather than to revenues. Cost weights exceed revenue weights for industries that supply inputs to industries charging markups. In our case, this means industries that are directly or indirectly supplying inputs to the production of investment goods, such as construction and machinery industries. Thus, productivity shocks to such industries have an outsized effect on long-run consumption compared to their sales.

To show how these points matter in practice, we study the long-run consumption effects of markups, tariffs, and productivities using a calibrated dynamic model of the world economy. The model features a rich international input-output structure, as in Costinot and Rodriguez-Clare (2014), and overlapping generations of households in each country that accumulate capital subject to undiversifiable idiosyncratic investment risks, as in Angeletos and Panousi (2011). The model delivers closed-form solutions for household asset demand. Endogenous risk premia clear physical capital markets within each country, while the risk-free rate is pinned down by market clearing in the world bond market.

The model belongs to the class covered by our theoretical results, which means that we can express its long-run comparative statics in terms of expenditure shares, capital wedges, and trade and capital supply elasticities.[8] We calibrate our model using expenditure shares from the World-Input Output Database (Timmer et al., 2015), augmented

[8]Given production functions that are nested CES, our characterization can also be used to derive non-linear effects of large shocks by iterating the first-order effects.

with investment flows data from Ding (2022). We match capital wedges to long-run averages of capital income relative to investment rates. To obtain capital supply elasticities, we calibrate our household model to target NFA holdings, capital stocks by country, risk premia by industry, as well as an aggregate asset demand elasticity from Auclert et al. (2021).

To study the effects of markups, we consider an experiment that raises all markups uniformly. We find large first-order effects on long-run consumption: a 10 percentage point increase in the aggregate markup reduces long-run global consumption by 7.7 percent. The mechanism operates through capital accumulation: markups raise the price of investment goods relative to labor, depressing capital formation. This reduction in capital generates a first-order consumption effect through its interaction with Golden Rule wedges. Notably, when capital is fixed or the economy begins at the Golden Rule, these first-order effects vanish entirely.

Our second experiment examines the first-order effects of a uniform increase in tariffs. Similar to markups, tariffs reduce capital accumulation, which affects consumption through its interaction with the Golden Rule wedges. We develop a formula in the style of Harberger (1971) that decomposes country-level consumption changes into effects from capital accumulation, terms-of-trade effects, and changes in the current account. We find that for all countries, the capital adjustment effect dominates, with a magnitude more than ten times larger than conventional terms of trade effects. Moreover, unlike terms-of-trade effects, the capital effects do not cancel out globally. As a result, global consumption losses are first-order even in the absence of initial tariffs. The magnitude is substantial: a 10 percentage point increase in tariffs reduces global consumption by 1.4 percent.

For both markups and tariffs, the results are stronger when there are high substitution elasticities between capital and labor, or high elasticities of savings with respect to rates of return. The reason is that since consumption falls in line with the capital reduction that comes from higher investment good prices, the consumption reduction is bigger when capital is more substitutable with labor, or when reductions in capital demand are not mitigated by reductions in rates of return.

Finally, we study how important the productivities of different industries are for long-run consumption. In line with our theoretical findings, industries upstream of investment have a disproportionate impact on these outcomes. For example, while sales in construction and machinery are only 15.4% and 5.4% of global consumption, the elasticity of long-run consumption with respect to their productivities are 40.6% and 13.5% respectively. When there is a unit elasticity between labor and capital so that reallocation effects are limited, the cost-based Domar weights of industries provide a very good approximation of the full quantitative effect.

Related Literature. Our paper is related to a long tradition on the treatment of investment and capital in national income accounting. One recurring theme in this literature is that since consumption is the only true final good, investment goods should be viewed as intertemporal intermediate inputs (Kuznets, 1941; Hulten, 1979), prompting suggestions for different ways of netting out investment costs from GDP, to be consistent with the treatment of other intermediates (Weitzman, 1976; Barro, 2021). Our paper provides a precise sense in which capital goods are equivalent to intermediates: balanced growth paths in dynamic economies with capital are also equilibria of static economies where capital goods are intermediates. We also show how the equivalence between capital goods and intermediates is helpful in characterizing and interpreting long-run comparative statics.

Our paper is also related to Foerster et al. (2022) and Ding (2022) who study balanced-growth paths and steady states of multi-sector models. Foerster et al. (2022) work with a closed-economy Cobb-Douglas model with an infinitely-lived representative agent. Our analysis relaxes these assumptions by having an open economy, arbitrary elasticity structure, imperfectly elastic capital supply, and heterogeneous returns across sectors. Ding (2022) constructs investment flow tables for the world economy, and uses this data to study the gains from trade relative to autarky allowing for adjustments in capital. We relax the assumption of infinitely-lived agents, no growth, financial autarky, and homogeneous returns across capital goods. We also characterize the response of the economy to a different set of counterfactuals.

Our paper is also related to quantitative dynamic disaggregated and international general equilibrium models, pioneered by Long and Plosser (1983) and Backus et al. (1992). Some recent contributions include Alvarez (2017), Kehoe et al. (2018), Ravikumar et al. (2019), Dix-Carneiro et al. (2023), Lyon and Waugh (2019), Vom Lehn and Winberry (2022), and Kleinman et al. (2023).[9] We complement this literature by providing analytical characterizations for balanced-growth outcomes. Second, in contrast to our paper, this literature tends to work with infinitely-lived representative agents which results in a capital supply curve that is infinitely elastic and a rate of return that is fully pinned down by preferences and the growth rate.

In terms of methodology, we draw on tools from Baqaee and Farhi (2020) and Baqaee and Farhi (2024). They consider static production networks with exogenous wedges and

[9] In this paper, we abstract from fixed costs and entry/exit decisions of firms, for example, as in Hopenhayn (1992), Melitz (2003) . Alessandria et al. (2021) review this literature as it pertains to international trade. Barkai and Panageas (2021) study how the distribution of the types of entering firms affects long-run consumption near the Golden Rule. Although we do not explicitly study this class of models, we provide an example of how our results can be extended to models with firm entry in Section 2.

exogenous trade imbalances. We extend these frameworks to account for capital accumulation. In our paper, wedges and trade imbalances are determined endogenously by equilibrium in capital markets.

Our approach to modelling trade imbalances is based on the intertemporal approach to the current account from international macroeconomics (Obstfeld and Rogoff, 1995). By using a model where demand for savings is not infinitely elastic in steady-state, we are able to solve for the long-run outcomes without having to solve transition dynamics. By allowing for financial frictions, we can relate global imbalances in the current account to financial development, similar to Caballero et al. (2008) and Mendoza et al. (2009).

The structure of the rest of the paper is as follows. Section 2 presents the equivalence result between balanced growth paths and distorted static economies. Section 3 provides the full characterization of long-run comparative statics and illustrates the result using the neoclassical growth model as an example. Sections 4-6 develop the quantitative model and present the results for changes in markups, tariffs, and productivities. Section 7 concludes.

2 Equivalence between BGPs and Static Economies

We first establish an equivalence between BGPs of dynamic economies and equilibria of static economies. Our results apply to a wide class of macroeconomic models. The class of models is implicitly defined by a set of necessary conditions that the balanced growth variables must satisfy. Any model satisfying these conditions falls within the class. We first prove a characterization result for competitive closed economies without taxes, before extending the analysis to incorporate international trade, taxes and markups.

2.1 Equivalent Static Economy

Consider a class of closed, competitive, economies, defined by the following equations holding along a balanced growth path.

Production and profit maximization:

$$Y_i = A_i F_i \left[\{A_f L_{if}\}_{f \in F}, \{Y_{ij}\}_{j \in N}, \{K_{ij}\}_{j \in N} \right], \tag{1}$$

$$\max_{\{Y_i, L_{if}, Y_{ij}, K_{ij}\}} \pi_i = p_i Y_i - \sum_{f \in F} w_f L_{if} - \sum_{j \in N} p_j Y_{ij} - \sum_{j \in N} R_j K_{ij}. \tag{2}$$

User cost of capital:

$$R_j = (r_j + \delta_j) p_j. \tag{3}$$

Consumption:

$$\{C_1, \ldots, C_N\} \in \arg\max_{\{C_i\}} U(C_1, \ldots, C_N) \quad \text{s.t.} \quad \sum_{i \in N} p_i C_i \leq \sum_{f \in F} w_f L_f + \sum_{j \in N} (r_j - g) B_j. \tag{4}$$

Market clearing:

$$Y_i = C_i + X_i + \sum_{j \in N} Y_{ji}, \quad X_i = (g + \delta_i) K_i, \quad \sum_{i \in N} L_{if} \leq L_f, \quad \sum_{j \in N} K_{ij} \leq K_i, \quad B_j = p_j K_j. \tag{5}$$

Equation (1) states that outputs, Y_i, have to be a constant returns to scale function of labor inputs. L_{if}, intermediate inputs, Y_{ij}, and capital services, K_{ij}.[10] Equation (2) requires that producer choices maximize profits taking prices as given, with rental prices $R_j = (r_j + \delta_j) p_j$ on capital j's services. Consumption quantities should maximize utility given aggregate consumer spending, which equals labor income and asset income, minus the savings needed to grow asset holdings at the economy's growth rate g.[11] Finally, markets should clear: for each good i, prsoduction equals consumption, investment, and intermediate input use; investment maintains the capital stock accounting for depreciation and growth; factor and capital service markets clear; and asset holdings equal the value of capital stocks.

These equations do not fully characterize the equilibrium of any specific dynamic model.[12] However, the equations can still be used to derive necessary features of balanced growth paths. In particular, suppose that we have a BGP of a model that belongs to our class, featuring some vector of equilibrium returns $\{r_i\}$, and equilibrium quantities and prices that satisfy the equations above. Based on this, we can derive the following proposition.

Proposition 1 (Equivalence Between BGP and Distorted Static Economies)**.** *Consider a BGP equilibrium with returns $\{r_i\}$. Its prices and wages $\{p_i, w_f\}$ and quantities $\{Y_i, C_i, K_{ij}, L_{if}, Y_{ij}\}$ are also the equilibrium of a static economy where:*

1. *The production functions of goods and the preferences of the representative household are the same as in the dynamic economy.*

[10]We assume production functions have constant returns to scale without loss of generality, since decreasing returns can always be captured via fixed factor endowments.

[11]We assume that this growth takes the form of growth in the efficiency units of primary factors (i.e. labor). This can either take the form of growth in the amount of factors L_f, or in the growth of factor-augmenting efficiency shifters A_f.

[12]In particular, the equations omit how returns are determined, and in cases where the variables represent aggregates, the equations do not pin down underlying micro-level details. For example, in Section 4, we consider a model where capital stocks and consumption levels are aggregated from a heterogeneous agent model with idiosyncratic capital risk.

2. *Capital goods are intermediate inputs produced with a linear technology from investment goods*

$$K_i = A_{K_i} X_i,$$

with productivity shifter $A_{K_i} = 1/(g + \delta_i)$.

3. *Capital goods are sold at a markup*

$$\mu_i = \frac{r_i + \delta_i}{g + \delta_i}$$

with profits distributed to households.

On a strictly mathematical level, condition 2 in the proposition follows from treating the balanced-growth path relationship between investment and the capital stock as a production technology. Given this, the marginal cost of producing K_i, in terms of the value of investment goods, is $(g + \delta_i)p_i$. Condition 3 follows from noting that the rental price faced by firms for capital services is $(r_i + \delta_i)p_i$ — hence, it is as if capital services are being provided at a markup over marginal cost equal to $(r_i + \delta_i)/(g + \delta_i)$. Accordingly, μ_i is equal to the compensation of capital relative to its investment. This shows that for long-run outcomes, capital services are subject to an as-if markup unless either $r_i = g$, which is the Golden Rule for savings, or $\delta_i = \infty$, in which case capital is not durable, and thus acts like a regular intermediate input.

The economic intuition is that, along the BGP, capital effectively functions as an intermediate input — it is a produced factor built from investment goods and used in production. However, due to discounting, the long-run consumption value of capital services exceeds their production costs. This wedge emerges because the capital goods in use today were produced in the past while the investment goods produced today will be used in the future. Thus, with positive discounting, the net present value of new investments can only be zero if the marginal product of installed capital is higher than its replacement cost. The proposition shows that this gap between marginal product and marginal cost is mathematically equivalent to a markup in a static setting.[13]

2.2 Extensions of Basic Framework

Proposition 1 can be extended to allow for multiple countries, tax-like distortions that cause deviations from marginal-cost pricing, and intangible capital.

[13]However, importantly, because the markup arises from discounting rather than market power, taxes, or other distortions, it does not necessarily indicate dynamic inefficiency. The isomorphism is instead a positive statement about the mathematical structure of balanced growth allocations.

Open economy. To apply Proposition 1 to an open economy, suppose there are multiple household types indexed by origin country c. We partition goods and primary factor endowments between countries, and, modify each country's budget constraint so that it earns primary factor income from factors located within its borders. To allow for international borrowing and lending, we introduce an additional asset in zero net supply. Finally, we modify the asset market clearing condition so that the value of each capital stock is equal to the sum of asset holdings by all household types.

Along a balanced growth path, aggregate consumption in country c equals primary factor income plus free cash flows. Along the BGP, this can be written ass

$$\sum_i p_i C_{ci} = \sum_{f \in F_c} w_f L_f + \sum_j (r_j - g) B_{cj} = \sum_{f \in F_c} w_f L_f + \Pi_c,$$

where F_c denotes the set of primary factors in country c, holdings of asset j by residents of country c is B_{cj}, where $j = 0$ denotes the bond, and Π_c is free cash flows from assets adjusted by growth. Denote the share of cash flows earned by each household by $\pi_c = \Pi_c / \sum_{c'} \Pi_{c'}$.

With these modifications, the following proposition holds.

Proposition 2 (Extension to Open Dynamic Economies)**.** *Consider a BGP equilibrium with returns $\{r_i\}$ and distribution of cash flows $\{\pi_c\}$ — its prices and wages $\{p_i, w_f\}$ and quantities $\{Y_i, C_i, K_{ij}, L_{if}, Y_{ij}\}$ are also the equilibrium of a static economy where: (1) production functions of goods and the preferences of households in each country are the same as in the dynamic economy; (2) capital goods are intermediates produced with linear technology from investment $K_i = A_{K_i} X_i$, with productivity shifter $A_{K_i} = 1/(g + \delta_i)$; (3) Capital goods are sold at a markup $\mu_i = (r_i + \delta_i)/(g + \delta_i)$; (4) profits from markups are distributed to households in accordance to $\{\pi_c\}$.*

The main difference relative to Proposition 1 is that we must account for the fact that in the equivalent static economy, profit income from markups must be consistent with the portfolio holdings and returns on the BGP in the dynamic economy.

Distortions. Proposition 1 assumes that the dynamic economy is competitive without any taxes or markups. We can capture deviations from this benchmark via reduced-form wedges, as in Restuccia and Rogerson (2008) or Hsieh and Klenow (2009). Following Baqaee and Farhi (2020), and without loss of generality, we represent such wedges as implicit output taxes. Suppose the tax on good i is τ_i.

Proposition 3 (Extension to Distorted Dynamic Economies)**.** *Consider a BGP equilibrium with returns $\{r_i\}$ and taxes $\{\tau_i\}$ — its prices and wages $\{p_i, w_f\}$ and quantities $\{Y_i, C_{ci}, K_{ij}, L_{if}, Y_{ij}\}$*

are also the equilibrium of a static economy where: (1) production functions of goods and the preferences of households in each country are the same as in the dynamic economy; (2) capital goods are intermediates produced with linear technology from investment $K_i = A_{K_i} X_i$, with productivity shifter $A_{K_i} = 1/(g+\delta_i)$; (3) capital goods are sold at a markup $\mu_i = (r_i + \delta_i)/(g+\delta_i)$; (4) Goods are taxed according to $\{\tau_i\}$, and tax revenues and profits are distributed to households.

Non-physical capital. In this paper, we interpret capital stocks K as physical capital. However, our results can also be applied to models with intangible capital, such as those with investments into firm creation, provided that intangible investment and capital stocks satisfy equations (1)-(5). In particular, production functions should have constant returns to scale in intangible capital and other inputs, and there should be a linear relationship between the investments and stocks of intangible capital on the balanced growth path. Appendix A provides an explicit example where we consider a model with costly firm entry and exogenous firm exit, in the style of Hopenhayn (1992) and Melitz (2003). We show that the steady state of this model is an equilibrium of a static economy where entry is distorted by a profit tax $\frac{r}{r+\delta}$ that reflects the deviation from the Golden Rule.

2.3 Equilibrium Response of Returns

Proposition 1 takes returns $\{r_i\}$ as given.[14] To pin down returns, we rely on asset market clearing, and on the fact that asset holdings B_i on a balanced growth path have to be consistent with households' savings decisions.

To model household savings, we assume that there is a capital supply correspondence $\mathcal{A}_i(\mathbf{r})$, such that

$$\mathcal{A}_i(\boldsymbol{r}) = \frac{B_i}{\sum_f w_f L_f}(\boldsymbol{r}), \qquad i \in K \tag{6}$$

holds on a balanced growth path.[15] The correspondence $\mathcal{A}_i(\mathbf{r})$ summarizes the role of household savings across different models of accumulation. For example, in Aiyagari (1994), the capital supply for each r is an integral over the ergodic distribution of assets implied by that r. In overlapping generations models, capital supply is instead given by the integral across age groups of the asset holdings coming out of an optimal savings decision given r. In a neoclassical growth model, capital supply is infinitely elastic at some fixed r determined by preferences and the growth rate. We normalize asset holdings by total labor income since in many standard models, steady-state asset accumulation is

[14]The open-economy version, Proposition 2, also takes the distribution of free-cash flows, $\{\pi_c\}$, as given.

[15]Formally, we add an additional restriction on our class of models by requiring that for a model to be part of the class, there must be a correspondence $\mathcal{A}_i(\boldsymbol{r})$ such that (6) hold on a BGP.

homothetic in labor income, which means that asset holdings normalized by labor income only depend on returns (see Auclert and Rognlie, 2018).[16]

Combining the capital supply correspondence with asset market clearing, we obtain the following equation for returns on the BGP:

$$\mathcal{A}_i(\boldsymbol{r}) = \frac{p_i K_i}{\sum_f w_f L_f}(\Theta, \mu(\boldsymbol{r})), \tag{7}$$

where Θ is some exogenous parameter of interest. The right-hand side is the capital demand in the economy, and is given by the ratio of the value of capital stocks to primary factor income. Capital demand is a property of the as-if static economy, depending on the parameter Θ as well as the markups μ, which in turn depend on the vector of returns $\boldsymbol{r}$ via Proposition 1. The condition states that the rate of return vector $\mathbf{r}$ is such that capital demand and supply are equal for each capital asset.[17]

Examples of capital supply mappings.

1. *Neoclassical growth model.* With a representative infinitely-lived household maximizing discounted utility with discount rate ρ, no population growth, labor-augmenting growth rate g, and inverse elasticity of intertemporal substitution θ, the only return consistent with consumer optimality is $r = \rho + \theta g$. In this case, capital supply, $\mathcal{A}$, is given by

$$\mathcal{A}(r) = \begin{cases} -\frac{1}{r-g} & \text{if } r < \rho + \sigma g \\ \left[-\frac{1}{r-g}, \infty\right) & \text{if } r = \rho + \sigma g \\ \varnothing & \text{if } r > \rho + \sigma g \end{cases}. \tag{8}$$

2. *Aiyagari model.* With incomplete markets and borrowing constraints, the capital supply equation $\mathcal{A}(r)$ is the steady state aggregate desired asset holdings. As shown by Aiyagari (1994), this is an upward sloping function of r which asymptotes to infinity for some value of r less than $\rho + \theta g$.

[16]Our assumption that $\mathcal{A}$ only depends on $\mathbf{r}$ and household parameters simplifies exposition and can be relaxed.

[17]Note that the formal definition has as many equations as capital goods. In cases where assets are perfectly substitutable from the perspective of households, the effective number of equations is smaller. See Section 3 for a discussion, and the quantitative model in Section 4 for an example.

3 Long-Run Comparative Statics

In this section, we analyze balanced growth path comparative statics. Our main result is that these can be performed in terms of comparative statics in the equivalent static economy characterized in Proposition 1 together with the asset supply function in (7). We first describe the nature of our experiment and the main result. We then provide a primer on comparative statics in distorted economies based on Baqaee and Farhi (2020) and apply the framework to some simple economies.

Consider an economy in the class of models studied in Section 2 with a vector of parameters $\Theta \in \mathbb{R}^K$. The parameter vector includes industry TFP levels, factor-augmenting technology shifters, and taxes on producers. In the case of an open economy, it also includes iceberg trade costs and tariffs.

Assume that the economy is initially at a BGP with prices and quantities X_0 for some Θ_0. Consider the effect of permanent shocks $\Delta\Theta$ to the parameter vector. Assuming that balanced growth paths are locally unique and attractive, the economy eventually converges to a new balanced growth path in the neighborhood of X_0. Define the long-run effect of $\Delta\Theta$ as the change between the initial and terminal BGP, as illustrated in Figure 1. This procedure defines a function, denoted by $X^{BGP}(\Theta)$, mapping parameters to long-run prices and quantities.[18]

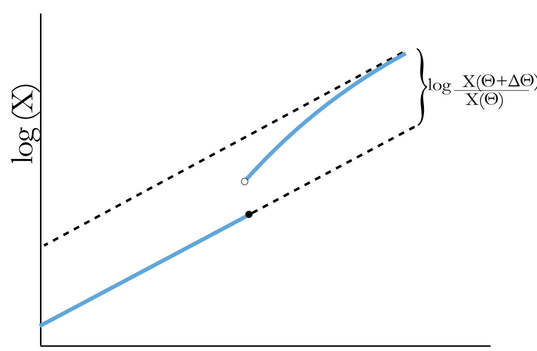

Figure 1: Response of BGP equilibrium to a permanent change in some parameter Θ.

3.1 Comparative Statics via Equivalent Static Economy

We are interested in expressing the derivatives of X^{BGP} with respect to Θ in terms of the equivalent static economy and the capital supply function $\mathcal{A}(\mathbf{r})$. To this end, we introduce

[18]If a BGP equilibrium exists with parameter values Θ_0 and outcomes $X^{BGP}(\Theta_0)$, then we can guarantee the existence of locally isolated BGP equilibria in the neighborhood of Θ_0 by the inverse function theorem. However, we do not prove that such equilibria necessarily are locally attractive.

the function $X^{static}(\Theta, \mu[\mathbf{r}])$, which maps parameter values Θ and as-if markups on capital μ into prices and quantities of the static economy described in Proposition 1. We assume that the solution to the static economy is locally unique in Θ and $\mathbf{r}$. By Proposition 1, X^{BGP} and X^{static} are related by the identity

$$X^{BGP}(\Theta) = X^{static}(\Theta, \mu[\mathbf{r}(\Theta)]), \tag{9}$$

where $\mathbf{r}(\Theta)$ is the vector of returns at the BGP associated with Θ.[19]

Equation (9) lets us express the derivative of outcomes along the BGP in terms of the derivative of outcomes in the equivalent static economy, up to changes in the as-if markup μ. To pin down changes in μ, we use the asset market clearing condition (7). Let the capital demand function be the value of each capital stock relative to non-capital income in the equivalent static economy with parameters Θ and markups μ: $\mathcal{K}_i^{static}[\Theta, \mu(\boldsymbol{r})] = \frac{p_i K_i}{\sum_f w_f L_f}(\Theta, \mu(\boldsymbol{r}))$. Then the asset market clearing equation can be rewritten as:

$$\mathcal{K}^{static}(\Theta, \mu[\mathbf{r}(\Theta)]) = \mathcal{A}[\mathbf{r}(\Theta)]. \tag{10}$$

Totally differentiating (9) and (10), we obtain the following proposition.

Proposition 4 (Long-Run Comparative Statics). *Provided that the functions X^{BGP}, X^{static}, and $\mathcal{A}$ are differentiable, the effect of a shock that does not affect capital supply satisfies:*

$$\begin{aligned} \frac{dX^{BGP}}{d\Theta} &= \frac{\partial X^{static}}{\partial \Theta} + \sum_i \frac{\partial X^{static}}{\partial \mu_i} \frac{d\mu_i}{d\Theta}, \\ \frac{d\boldsymbol{\mu}}{d\Theta} &= \frac{1}{g+\boldsymbol{\delta}} \frac{d\boldsymbol{r}}{d\Theta} = \frac{1}{g+\boldsymbol{\delta}} \left(\epsilon_r^s + \epsilon_r^d\right)^{-1} \frac{\partial \log \mathcal{K}^{static}}{\partial \Theta}, \end{aligned} \tag{11}$$

where $\epsilon_r^s = \frac{d \log \mathcal{A}}{d\mathbf{r}}$ is the semi-elasticity matrix of capital supply and $\epsilon_r^d = -\frac{\partial \log \mathcal{K}^{static}}{d\boldsymbol{r}}$ is the semi-elasticity matrix of capital demand.

The proposition expresses balanced growth comparative statics in terms of comparative statics in the equivalent static economy, $\partial X^{static}/\partial\Theta$, $\partial X^{static}/\partial\mu_i$, $\partial \log \mathcal{K}^{static}/\partial\theta$, and ϵ_r^d, together with semi-elasticities of capital supply, ϵ_r^s. Since the elasticities of outcomes in the equivalent static economy can be characterized in terms of expenditure shares and substitution elasticities (e.g. Baqaee and Farhi, 2024), the proposition provides a complete

[19] To see why (9) is true, note that $X^{BGP}(\Theta)$ is a balanced growth path with rates of return $\mathbf{r}(\Theta)$, and that the implication of Proposition 1 is precisely that such balanced growth paths are equilibria of static economies with the same Θ and as-if markups associated with $\mathbf{r}(\Theta)$.

characterization of balanced growth comparative statics in terms of expenditure shares, substitution elasticities, and capital supply elasticities.

The first expression in Proposition 4 shows that the derivative of the balanced growth path consists of a direct effect of Θ, and an indirect effect operating through changes in as-if markups. The direct effect is given by the effect of changing Θ in the equivalent static economy, keeping as-if markups constant. The indirect effect is given by the effect of changing markups in the static economy, keeping Θ constant, times the change in markups.

The next expression shows how as-if markups, μ, change in response to changes in Θ, with the first equality relating changes in μ to changes in $\mathbf{r}$, and the second unpacking the changes in $\mathbf{r}$. The relation between $\frac{d\boldsymbol{\mu}}{d\Theta}$ and $\frac{d\boldsymbol{r}}{d\Theta}$ follows directly from $\mu_i = \frac{r_i+\delta_i}{g+\delta_i}$, while the expression for $\frac{d\boldsymbol{r}}{d\Theta}$ comes from differentiating the asset market clearing condition (10) with respect to Θ.[20]

The intuition for the change in $\mathbf{r}$ is that the shock to Θ induces a change in capital demand $\mathcal{K}^{static}$, which forces an adjustment in returns. The extent of adjustment depends on the elasticity of both capital demand and supply, with adjustments being small if the sizes of capital demand and supply elasticity are high. The neoclassical limit is when $\epsilon_r^d = \infty$, in which case rates do not respond to capital demand shocks. In the one asset case, both ϵ_r^s and ϵ_r^d are typically positive, and so positive shocks to capital demand tend to increase returns and vice versa. The matrix algebra in Proposition 4 generalizes this intuition to the case with multiple assets and returns.

Perfectly substitutable assets. Proposition 4 assumes that capital supply is a differentiable mapping from returns to portfolio vectors. This assumption may not hold when households view different assets as perfectly substitutable. For instance, if households treat all assets as identical, capital supply becomes non-differentiable: it jumps when returns diverge, and when returns are equal ($r_i = r$), it is a correspondence rather than a function, encompassing all asset holdings consistent with the desired level of total wealth.

Even with perfect substitutable assets, a version of Proposition 4 remains valid. However, it requires using a reduced set of market clearing conditions – one for each block of perfectly substitutable assets – along with non-arbitrage conditions for returns within each block. For example, when households view assets as identical, we only need one market clearing condition (equating the total capital stock to desired aggregate wealth) and one non-arbitrage condition (requiring equal returns across assets). The quantitative

[20] This expression assume that there are no shocks to δ_i, something we maintain in our quantitative analyis. If δ_i is being shocked, $\frac{d\mu_i}{d\Theta}$ has an extra term $-\frac{d\delta_i/d\Theta}{(g+\delta_i)^2}$.

model in Section 4 provides an example with multiple blocks and heterogeneous returns.

Infinitely elastic capital supply. Similarly, the restriction that capital supply, $\mathcal{A}(\boldsymbol{r})$, must be differentiable rules out infinitely elastic long-run capital supply, such as in the standard neoclassical growth model. In these setups, there is typically only a single rate of return that is consistent with balanced growth. In this case, Proposition 4 is true setting $\frac{d\boldsymbol{\mu}}{d\Theta} = 0$. We could also consider cases where there are shocks to capital supply, like capital taxes, $\boldsymbol{\tau}^k$, or changes in time preferences, ρ. In these cases, Proposition 4 is true replacing $\frac{dr}{d\Theta} = \frac{d\boldsymbol{\tau}^k}{d\Theta} + \frac{d\rho}{d\Theta}$.[21]

Open economy. When there are multiple countries, Proposition 2 establishes that the equivalent static economy is not only characterized by its as-if markups, but also by the distribution of free cash-flows π_c. In this case, Proposition 4 needs to be modified in two ways. The first equation of Proposition 4 needs to be adjusted to account for changes in the distribution of profit income in the static economy:

$$\frac{dX^{BGP}}{d\Theta} = \frac{\partial X^{static}}{\partial\Theta} + \sum_i \frac{\partial X^{static}}{\partial \mu_i}\frac{d\mu_i}{d\Theta} + \sum_c \frac{\partial X^{static}}{\partial \pi^c}\frac{d\pi_c}{d\Theta}.$$

Next, the asset market clearing condition, (10), for each capital stock i must be adjusted to sum over capital supply from all countries:

$$p_i K_i(\Theta, \boldsymbol{\mu}, \boldsymbol{\pi}) = \sum_c \left[\sum_{f\in\mathcal{F}_c} w_f L_f\right] \mathcal{A}_{c,i}(\boldsymbol{r}), \tag{12}$$

where the left-hand side is the value of the capital stock, determined by the static model, and $\mathcal{A}_{c,i}$ is capital supply from country c for capital good i relative to primary factor income in country c. Totally differentiating (12) gives the multi-country generalization of (11), omitted for brevity.

Finally, the distribution of free cash-flows must be consistent with asset holdings at the new balanced growth path:

$$\pi_c = \frac{\sum_{i\in K}(r_i - g)\left(\sum_{f\in\mathcal{F}_c} w_f L_f\right)\mathcal{A}_{c,i}(\boldsymbol{r})}{\sum_{c'}\sum_{i'\in K}(r_{i'} - g)\left(\sum_{f' in \mathcal{F}_{c'}} w_{f'} L_{f'}\right)\mathcal{A}_{c',i'}(\boldsymbol{r})}. \tag{13}$$

Together, Equations (12) and (13) pin down returns, $\boldsymbol{r}$, and the distribution of free cash

[21] In Appendix C.1, where we provide our quantitative model, we formally incorporate capital taxes into the equilibrium equations.

flows, $\boldsymbol{\pi}$, along the BGP. In the equivalent static economy, this determines the as-if markups, and the distribution of profit income from those markups across households in different countries. An economy with a single country is one where (13) is redundant, since all free cash flows must accrue to the sole household.

Distortions. Proposition 4 assumes that the dynamic economy is competitive — there are no implicit or explicit taxes in production. If there are wedges in the dynamic economy, denote the revenues they generate by T. Assume that tax revenues are rebated to households in proportion to their labor income. In this case, (6) must be modified so that the capital supply function satisfies

$$\mathcal{A}_i(\boldsymbol{r}) = \frac{B_i}{\sum_f w_f L_f + T}(\boldsymbol{r}) \qquad i \in K \tag{14}$$

instead. This is because desired asset holdings now scale proportionally to the sum of labor and tax revenues. Given this modification, we must also modify the definition of capital demand: $\mathcal{K}_i^{static}[\Theta, \mu(\boldsymbol{r})] = \frac{p_i K_i}{\sum_f w_f L_f + T}(\Theta, \mu(\boldsymbol{r}))$. Given these modifications, the rest of Proposition 4 applies without any other changes.

3.2 Comparative Statics in Distorted Static Economies

Proposition 4 reduces long-run dynamic comparative statics to comparative statics in static distorted economies. One advantage of this reduction is that the theory of comparative statics in distorted static economies is well-developed. Here, we present a primer on notation and results in distorted static economies, based on Baqaee and Farhi (2020), together with results applying to economies with capital in particular.

Primer on distorted static economies. Consider a static economy with a set of commodities and production functions for each commodity. Partition the commodities into three subsets: a set C of consumption bundles, a set M of intermediates, and a set F of primary factor endowments. All production and consumption functions are constant returns to scale. The consumption bundles in C are distinguished from the commodities in M in not being used in the production of any other commodity.

Let μ_i denote the ratio of the price of i to its marginal cost. Assume, without loss of generality, that markups are equal to unity for every consumption and factor commodity. Let Ω and $\tilde{\Omega}$ denote the revenue-based and cost-based input-output matrices, which have dimension $(C + M + F)^2$. The ijth element of Ω is equal to expenditures by i on input j relative to all expenditures going to i. The ijth element of $\tilde{\Omega}$ is equal to expenditures by i on input j relative to i's total expenditures. Let μ be a diagonal matrix whose iith element measures the markup on i. The revenue-based and cost-based input-output matrices are related to each other via

$$\Omega = \mu^{-1}\tilde{\Omega}.$$

Define the revenue- and cost-based Leontief inverses by

$$\Psi = (I - \Omega)^{-1}, \qquad \tilde{\Psi} = (I - \tilde{\Omega})^{-1}.$$

Define Φ to be the $(C + M + F) \times 1$ vector whose first C elements are final expenditures relative to total final expenditures, with the remaining terms equal to zero. Define the Domar weight for each $i \in C + M + F$, denoted λ_i, to be expenditures on i divided by total final expenditures. Accounting identities imply that

$$\lambda' = \Phi'\Psi.$$

Intuitively, the Domar weight of i measures the average direct and indirect expenditures of final goods on i. Define the cost-based Domar weights to be

$$\tilde{\lambda}' = \Phi'\tilde{\Psi}.$$

The cost-based Domar weight of i measures the average direct and indirect exposures of final goods to i, measured using costs rather than revenues.

Equivalent static economy to a BGP. When applied to study the BGP of the dynamic economy described in Proposition 1, capital goods, denoted by the set K, are added to the set of non-durable intermediates, denoted by the set N, so that $M = N + K$. The IO matrices take the following form:

$$\Omega_{ij} = \begin{cases} \frac{p_i C_{ci}}{\sum_i p_{i'} C_{ci'}} & i \in C \\ \frac{p_j Y_{ij}}{p_i Y_i} & i, j \in N \\ \frac{R_j K_{ij}}{p_i Y_i} & i \in N, j \in K \\ \frac{w_j L_{ij}}{p_i Y_i} & i \in N, j \in F \\ \frac{p_j X_{ij}}{R_i K_i} & i \in K, j \in N \\ 0 & \text{otherwise} \end{cases}, \qquad \tilde{\Omega}_{ij} = \begin{cases} \frac{p_i C_{ci}}{\sum_i p_{i'} C_{ci'}} & i \in C \\ \frac{p_j Y_{ij}}{p_i Y_i} & i, j \in N \\ \frac{R_j K_{ij}}{p_i Y_i} & i \in N, j \in K \\ \frac{w_j L_{ij}}{p_i Y_i} & i \in N, j \in F \\ \frac{p_j X_{ij}}{\sum_{j'} p_i X_{ij'}} & i \in K, j \in N \\ 0 & \text{otherwise} \end{cases}.$$

We assume that there are no wedges other than those associated with capital goods, so the only difference between Ω and $\tilde{\Omega}$ is in the treatment of capital services — total expenditures on capital good i is equal to rental payments, $R_i p_i K_i$, whereas total expenditures by i equal investment costs, $\sum_j p_i X_{ij}$. The ratio of these two numbers is the Golden Rule wedge on i, which we call the as-if markup on capital good i.

If there is a single country and consumption bundle, then $\Phi_c = [1, 0, \ldots, 0]$. If there are multiple countries, then we treat the consumption bundle consumed by households in each country as a different final good. Therefore, the final expenditure share vector, Φ, is equal to consumption expenditures by country c relative to global consumption:

$$\Phi_c = \frac{\sum_i p_i C_{ci}}{\sum_{c'} \sum_{i'} p_{i'} C_{c'i'}} \mathbf{1}_{[c \in C]},$$

where C_{ci} denotes consumption of good i by households in country c. Since investment is not part of final demand, the Domar weight of each i in the equivalent static economy is equal to the sales of i divided by total nominal consumption (not total nominal GDP).

Finally, define changes in real consumption (in response to changes in primitives) to be the share-weighted change in the quantity of final consumption bundles:

$$d \log C = \sum_c \Phi_c d \log C_c.$$

This is definition is consistent with how real consumption is measured in national income accounts.

Results for economies with capital. With this mapping, we can now apply results for static distorted economies to characterize how outcomes respond to shocks along the BGP. The following proposition decomposes the response of long-run consumption into a technology and a reallocation effect.

Proposition 5 (Real Consumption Response)**.** *The change in long-run aggregate real consumption, in response to productivity changes, $d \log A$, or wedge changes, $d \log \mu$, is*

$$d \log C = \underbrace{\sum_{i \in N} \tilde{\lambda}_i d \log A_i}_{\textit{technology}} \underbrace{- \sum_i \tilde{\lambda}_i d \log \mu_i - \sum_{i \in F} \tilde{\lambda}_i d \log \lambda_i}_{\textit{reallocation}}.$$

The first set of summands is the mechanical result of technology shocks, holding fixed the allocation of resources. The remaining terms capture reallocation effects, which are nonzero because long-run consumption is not maximized at the initial equilibrium. There are two important observations: (1) the mechanical effect of technology shocks are captured by $\tilde{\lambda}_i$, which, while straightforward to compute using input-output tables, do not equal Domar weights; (2) the mechanical effect of technology shocks can be amplified or mitigated by the reallocations they induce. Reallocation effects are positive whenever resources are redirected towards more capital intensive (high-markup) activities. A sufficient statistic for reallocation effects is how much labor shares are reduced, $-\sum_{i \in F} \tilde{\lambda}_i d \log \lambda_i$, netting out mechanical reductions caused by changes in markups, $\sum_i \tilde{\lambda}_i d \log \mu_i$.

A noteworthy special case is when $\mu_i = 1$ for every i, which obtains if either $r_i = g$ or $\delta_i = \infty$. In these cases, reallocation effects are zero, revenue- and cost-based Domar weights coincide, and we recover a result reminiscent of Hulten (1978).

Corollary 1 (Real Consumption Response at Golden Rule)**.** *Suppose that either $r_i = g$ or $\delta_i = \infty$ for every capital good i. The long-run change in aggregate real consumption, in response to productivity changes, $d \log A$, or wedge changes, $d \log \mu$, is*

$$d \log C = \sum_{i \in N} \tilde{\lambda}_i d \log A_i = \sum_{i \in N} \lambda_i d \log A_i.$$

When $\mu_i = 1$ for every capital good, the elasticity of long-run consumption to productivity shocks is simply sales divided by consumption. Furthermore, wedges, like capital taxes, markups, and tariffs, have no effects on long-run consumption.[22]

We can also repurpose Harberger (1964)'s classic formula for measuring economic waste, derived for closed static economies, to study how long-run consumption responds to changes in wedges in open dynamic economies.

Proposition 6 (Harberger's Formula For Dynamic Open Economies)**.** *The change in long-run aggregate real consumption, in response to changes in wedges, $d \log \mu$, is*

$$d \log C = \sum_{i \in K} \frac{R_i p_i K_i}{\sum_{c'} \sum_{i'} p_{i'} C_{c'i'}} \left[\frac{r_i - g}{r_i + \delta} \right] d \log K_i. \tag{15}$$

Furthermore, for each country c, the change in long-run real consumption is

$$d \log C_c = \sum_{i \in K_c} \frac{R_i p_i K_i}{P_c C_c} \left[\frac{r_i - g}{r_i + \delta} \right] d \log K_i + \sum_{i \in \mathcal{N}} \frac{NX_{ci}}{P_c C_c} d \log p_i - \frac{d[\sum_{i \in N} NX_{ci}]}{P_c C_c}, \tag{16}$$

[22] It is worth pointing out that Corollary 1 is not literally Hulten's theorem since it applies to long-run consumption, rather than GDP. Indeed, Hulten's theorem holds in our environment for GDP regardless of whether or not the economy is at the Golden-Rule. Define GDP to be the sum of all consumption and investment expenditures. Then, the long-run response of real GDP to shocks is

$$d \log \text{Real GDP} = \sum_i \frac{p_i Y_i}{GDP} d \log A_i + \sum_i \frac{R_i p_i K_i}{GDP} d \log K_i,$$

where the change in the capital stock is determined by the equilibrium of the as-if static economy, following Proposition 4.

where K_c are capital goods in country c, and NX_{ci} are the net exports of good i.

The first equality in (15) expresses the change in aggregate consumption in terms of Domar weights, as-if markups, and changes in quantities: if quantities rise for goods with high as-if markups, then consumption rises. The as-if markups coincide with Golden rule wedges on each capital good, the quantities are the capital stocks, and the Domar weights are capital compensations relative to consumption.

Equation (16) generalizes this expression to study long-run consumption of individual countries. The first summand is the same Harberger-"rectangles" formula as in (15), but applied at the country-level. The second summand is the terms-of-trade effect. The final term captures changes in net factor payments from abroad. The second and third terms are zero-sum, whereas the first summand need not add up to zero at the world level (unless the world is at the Golden Rule, in which case the first term is zero for every country).

We use Propositions 5 and 6 and Corollary 1 to decompose and gain intuition for our quantitative results in Sections 5 and 6. But before describing the quantitative model, we first apply these results to the neoclassical growth model to build more intuition.

3.3 Example: Neoclassical Growth Model

To illustrate how to apply these results, consider a version of the neoclassical growth model, represented using the notation and results of the previous subsection. We show how the economy can be represented using cost-based and revenue-based input-output matrices, and use Proposition 4 and Proposition 5 to derive comparative statics with respect to permanent productivity shocks, markups, and capital taxes. We then apply Proposition 5 and Proposition 6 to interpret the economic logic of these counterfactuals. The results illustrate how well-known counterfactuals in the neoclassical growth model can be viewed as special cases of a more general theory for long-run counterfactuals.

Setup. The production and accumulation blocks of the economy are given by

$$Y = A_Y F[K, A_L L], \quad C = A_C Y_C, \quad X = A_X Y_X, \quad Y = Y_C + Y_X, \quad \dot{K} = -\delta K + X.$$

Final output is produced using capital and labor, which is used to produce consumption and investment goods. The terms A_Y and A_L are Hicks-neutral and labor-augmenting technology terms, with A_L assumed to assume grow at a constant rate g.[23] The produc-

[23]For simplicity, we assume there is no population growth, but none of the arguments hinge on this assumption.

tivity of consumption and investment goods is given by the productivity terms A_X and A_C. To apply Proposition 4, we assume that the long-run capital supply function is infinitely elastic, with long-run equilibrium in capital markets requiring that

$$r = \rho + \gamma g + \tau^k,$$

where ρ is the discount rate, γ is the inverse of the intertemporal elasticity of substitution, and τ^k is a wealth tax on capital.

Representation. Proposition 2 applies to this economy, so its balanced growth path is an equilibrium of a distorted static economy, which can be represented using the notation in Section 3.2. Writing $\alpha = \frac{\partial \log F}{\partial \log K}$ for the output elasticity with respect to capital, and ordering commodities as consumption, output, capital, and labor, we obtain the following:

$$\Phi' = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}, \tag{17}$$

$$\Omega = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & 1-\alpha \\ 0 & \left(\frac{r+\delta}{g+\delta}\right)^{-1} (\equiv \mu^{-1}) & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad \tilde{\Omega} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & 1-\alpha \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{18}$$

$$\Psi = \begin{bmatrix} 1 & \frac{1}{1-\alpha/\mu} & \frac{\alpha}{1-\alpha/\mu} & \frac{1-\alpha}{1-\alpha/\mu} \\ 0 & \frac{1}{1-\alpha/\mu} & \frac{\alpha}{1-\alpha/\mu} & \frac{1-\alpha}{1-\alpha/\mu} \\ 0 & \frac{1/\mu}{1-\alpha/\mu} & \frac{1}{1-\alpha/\mu} & \frac{(1-\alpha)/\mu}{1-\alpha/\mu} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad \tilde{\Psi} = \begin{bmatrix} 1 & \frac{1}{1-\alpha} & \frac{\alpha}{1-\alpha} & 1 \\ 0 & \frac{1}{1-\alpha} & \frac{\alpha}{1-\alpha} & 1 \\ 0 & \frac{1}{1-\alpha} & \frac{1}{1-\alpha} & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{19}$$

$$\lambda' = \Phi'\Psi = \begin{pmatrix} 1 & \frac{1}{1-\alpha/\mu} & \frac{\alpha}{1-\alpha/\mu} & \frac{1-\alpha}{1-\alpha/\mu} \end{pmatrix}, \quad \tilde{\lambda}' = \Phi'\Psi = \begin{pmatrix} 1 & \frac{1}{1-\alpha} & \frac{\alpha}{1-\alpha} & 1 \end{pmatrix}. \tag{20}$$

The vector Φ represents the share of final expenditure on different commodities, with all weight on the consumption good. The revenue-based and cost-based input-output matrices Ω and $\tilde{\Omega}$ have identical first two rows, reflecting that there are no markups in the consumption and raw output sectors. The first row shows that 100% of consumption good costs come from Y, while Y's cost shares are α for capital and $1-\alpha$ for labor. In $\tilde{\Omega}$, the third row indicates that all costs for producing capital goods come from sector Y, while the third row of Ω shows that only a fraction $\frac{g+\delta}{r+\delta}$ of capital sector rental income goes to the producer of investment goods, with the remainder being net capital income. The difference between $\tilde{\Omega}$ and Ω reflects the as-if markup μ – while raw inputs constitute 100% of investment costs, they do not account for 100% of rental revenue. The final row in both matrices represents labor, which requires no inputs.

The revenue-based Leontief inverse, Ψ, captures how much each dollar spent on that row's commodity contributes to the revenues of other commodities, while each row of the cost-based $\tilde{\Psi}$ captures how much of the costs of that row's commodity are accounted for by different inputs. Note that the term $1 - \alpha/\mu$ that consistently shows up in Ψ represents the consumption share of GDP, since $\alpha/\mu = \frac{\alpha}{r+\delta}(g + \delta)$ is the investment share of GDP. Since all final demand is in the consumption good sector, our primary interest is in the first rows of each matrix, which equal the Domar weights λ and $\tilde{\lambda}$.

The revenue-based Domar weights λ are the sales of different sectors relative to final consumption. The first element shows that consumption sales are, unsurprisingly, 100% of final consumption. Since consumption-to-GDP is $1 - \alpha/\mu$, the subsequent three elements follow from dividing GDP, capital income, and labor income by total consumption. Moreover, when as-if markups are positive, the cost-based Domar weights $\tilde{\lambda}$ are weakly larger than revenue-based Domar weights. Consider, for example, the last element in λ and $\tilde{\lambda}$, which represents the Domar weight of labor. While the cost-based weight of labor is 1 (consistent with all costs scaling linearly with the price of labor), its revenue-based weight is less than one if $\mu > 1$, since consumption in this case is partly financed by net capital income (i.e., profits in the equivalent static economy).

Effect of productivity shocks with Cobb-Douglas production. We are interested in the long-run effect on consumption from permanently increasing the productivities A_C, A_Y, A_X, and A_L. For this, we use Proposition 5, which states the effect of a productivity increase for a good is given by its cost-based Domar weight, plus a reallocation term that depends on the change in primary factor shares.

We first consider the case when F is a Cobb-Douglas production function with capital share α. In this case, the labor share, λ_4, does not respond to changes in productivity. Hence, we recover

$$\begin{pmatrix} \frac{\partial \log C}{\partial \log A_C} & \frac{\partial \log C}{\partial \log A_Y} & \frac{\partial \log C}{\partial \log A_X} & \frac{\partial \log C}{\partial \log A_L} \end{pmatrix} = \tilde{\lambda}' = \begin{pmatrix} 1 & \frac{1}{1-\alpha} & \frac{\alpha}{1-\alpha} & 1 \end{pmatrix}.$$

Note that these formulas contain the classic formulas for capital amplification in the neoclassical model: the unit elasticity with respect to labor-augmenting technology A_L and the amplified response $1/(1 - \alpha)$ to Hicks-neutral productivity A_Y. Our analysis shows how these familiar expressions can be derived from a general principle: with Cobb-

Douglas production, long-run consumption responses equal cost-based Domar weights in the equivalent static economy.[24]

Note that using cost-based Domar weights is essential. Consider, for example, that labor-augmenting productivity and consumption-sector productivity are equally powerful in increasing long-run consumption. This is true despite labor income being smaller than consumption, and thus having a lower Domar weight (regardless of the denominator used). The reason is that labor, being upstream of capital, has an importance for long-run consumption that is understated by its share of income. Intuitively, since labor's output is used in the future, discounting means that its current value understates its impact on steady-state consumption. In the equivalent static economy, this effect is captured by the as-if markup. Notably, this effect disappears when $r = g$ and we are at the Golden Rule. At the Golden Rule, the investment share of GDP is α and there is no net capital income. Therefore, consumption equals labor income, and the discrepancy between revenue-based and cost-based Domar weights disappears.

Effect of productivity shocks with non-unitary elasticity of substitution. Next, we consider the effect of permanent productivity shocks when the production function for output F has an elasticity of substitution $\theta \neq 1$ between capital and labor. In this case, the reallocation effect in Proposition 5 can be non-zero, since the labor share of the economy can change in response to productivity shocks. The effect of consumption and labor productivity changes are the same as before since they do not change relative prices of capital and effective labor. However, the effects of changing raw output and investment productivities A_Y and A_X change, and are given by: x

$$\frac{\partial \log C}{\partial \log A_Y} = \frac{1}{1-\alpha} + \lambda_3(\theta - 1)\frac{r-g}{r+\delta},$$
$$\frac{\partial \log C}{\partial \log A_X} = \frac{\alpha}{1-\alpha} + \lambda_3(\theta - 1)\frac{r-g}{r+\delta},$$

where $\lambda_3 = \frac{\alpha}{1-\alpha/\mu}$ is the revenue-based Domar weight of capital. The first terms are the same as in the Cobb-Douglas economy while the second terms capture the reallocation effect. These effects are non-zero if we have a joint deviation from both Cobb-Douglas and the Golden Rule. For example, if capital and labor are complements, $\theta < 1$, there is a negative reallocation effect, as improvements in A_Y and A_X reduce the capital share of the economy, which reduces long-run consumption in line with the size of the deviation

[24]Since the long-run effect of A_L and A_Y are both scaled by $1/(1-\alpha)$, one may conjecture that the long-run response of any productivity shock should simply scale its initial impact on TFP by $1/(1-\alpha)$. This conjecture is false: the long-run effects of consumption and labor productivity shocks are equal, despite consumption having a higher revenue share and thus a larger initial TFP impact.

from the Golden rule.

Effect of capital taxes. Next, we consider the effect on steady-state consumption of permanently increasing the capital tax τ^k. Given the specification of capital supply in the neoclassical growth model, this change in capital tax translates one-to-one into a higher interest rate $dr = d\tau^k$. Given that this change does not alter any productivity parameters, it is equal to a markup shock $d\mu = \frac{d\tau^k}{g+\delta}$, and we can use proposition Proposition 6, which gives the effect on long-run consumption from markup shocks.

Proposition 6 can be rewritten as

$$d\log C = \left[\frac{R_i p_i K_i - p_i X_i}{P_c C_c}\right] d\log K, \tag{21}$$

which expresses the effect on final consumption in terms of the deviation between capital income and investment relative to consumption, times the change in the capital stock. Furthermore, from the equivalent static economy, an increase in capital taxes is equivalent to an increase in the markup on capital goods. The change in the capital stock is given by

$$d\log K = -\frac{\theta}{1-\alpha} d\log(r+\delta),$$

giving us

$$d\log C = -\left[\frac{R_i p_i K_i - p_i X_i}{P_c C_c}\right] \times \frac{\theta}{1-\alpha}\frac{d\tau^k}{r+\delta}$$

The expression shows that the losses from capital taxation scales in both the distance from the Golden Rule and the elasticity of substitution between capital and labor. This highlights the mathematical analogy between long-run dynamic losses from capital taxation, and the losses from markups in static economy, which also scale in the initial markup and the degree of substitutability (see, e.g., Baqaee and Farhi, 2020).

Effect of output taxes. Last, consider a sales tax on the output good, Y, denoted by $1+\tau$. Revenues from this tax are rebated to the household as a lump sum. We consider how the introduction of a tax affects long-run returns and consumption, starting at $\tau = 0$. From the equivalent static economy, the semi-elasticity of capital demand to the tax, holding fixed returns, is

$$-\frac{\partial \log \mathcal{K}}{\partial \tau} = \frac{-\theta}{1-\alpha}.$$

For illustration, suppose that the capital supply curve $\mathcal{A}(r)$ is an upward sloping function with finite elasticity, as in Aiyagari (1994) or Angeletos (2007). Denote the semi-elasticity of the capital supply curve with respect to the rate of return by $\epsilon^d = \partial \log \mathcal{A}/\partial r > 0$. Recall that the capital demand function is defined to be $\mathcal{K} = pK/(wL + \tau pY)$ in the equivalent static economy. Let ϵ^s denote the (negative) semi-elasticity of capital demand.[25]

Hence, Proposition 4 implies that the change in as-if markups, is equal to

$$d\mu = \frac{1}{g+\delta}\frac{dr}{d\tau} = \frac{1}{g+\delta}\left[\frac{1}{\epsilon^s + \epsilon^d}\right]\frac{-\theta}{1-\alpha}d\tau.$$

For a reasonable calibration, the tax weakly lowers the equilibrium rate of return. In the neoclassical benchmark, where capital supply is infinitely elastic, the rate does not respond since $\epsilon^s = \infty$.

Putting this together, the reduction in capital implied by the tax is

$$d\log K = -\frac{\theta}{1-\alpha}\left[d\tau + \frac{1}{g+\delta}dr\right],$$

where reductions in the rate of return mitigate reductions in the capital stock. Accordingly, Proposition 6 implies that the reduction in long-run consumption is

$$d\log C = -\left[\frac{R_i p_i K_i - p_i X_i}{P_c C_c}\right]\frac{\theta}{1-\alpha}\left[d\tau + \frac{1}{g+\delta}dr\right].$$

Hence, losses are increasing the elasticity of substitution between labor and capital, and declining in the elasticity of capital supply.

4 A Quantitative Model of the World Economy

To study these effects beyond simple examples, we set up and calibrate a dynamic general equilibrium model of the world economy. The model features a rich input-output structure with trade as in Costinot and Rodriguez-Clare (2014), overlapping generations of households in each country that accumulate capital subject to undiversifiable idiosyncratic investment risks, as in Angeletos and Panousi (2011). Sections 5 and 6 study the quantitative effects to changes in distortions and productivities using this model. The main text provides a high-level summary of the model with details given in the appendix.

4.1 Production Side

There are C countries, with each country producing differentiated varieties of the same set of products. We index producers by two indices: (c, i), where $c \in C$ is the country of

[25] From the equivalent static economy, this is $\epsilon^S = -\frac{\partial \log \mathcal{K}}{\partial r} = \left[\frac{\theta-\alpha}{1-\alpha}\right]\frac{1}{r+\delta}$.

origin and $i \in N$ is the industry-type of the producer (e.g. agriculture, mining, and so on).

The production function of industry i in country c is a Cobb-Douglas composite of labor, capital, and intermediate inputs from other industries.[26] Intermediate inputs from industry type j purchased by (c, i) are aggregated using an Armington CES aggregator with elasticity θ across different origins (c', j). Following common terminology in the trade literature, we refer to $\theta - 1$ as the (micro) trade elasticity.

Each country has three factor endowments: low-, medium-, and high-skill labor, with F_c denoting the set of primary factors in country c. Each industry uses a different mix of these labor types using a Cobb-Douglas aggregator. There is uniform labor-augmenting productivity growth equal to g_A in every country to ensure the existence of a BGP.

We assume that each industry in each country, (c, i), has a specialized capital stock produced by a specialized investment goods producing industry. All other goods in the economy are perishable (infinite depreciation). This means that the number of capital stocks in each country is equal to the number of industries in each country. Denote the set of capital goods in country c by K_c. The investment good of (c, i) is a Cobb-Douglas composite of inputs from different industries. Just as for perishable goods, investment inputs from industry type j from different origin countries are combined using an Armington CES aggregator with elasticity θ.

4.2 Household Savings and Aggregate Capital

Capital supply is given by a household block featuring a perpetual youth overlapping generation structure as in Blanchard (1985), with a constant death rate $\nu_c > 0$ by country and a constant growth rate of newborns $g_L \geq 0$ in every country. Below, we derive the key relationships that need to hold on a balanced growth path (see appendix for more details).

Household accumulation. Each household can invest in a global risk-free bond with return r, or in a domestic capital good i with average return r_i. Investment in a capital good is associated with idiosyncratic risk, and each household can only be active in a single domestic capital good, which they choose at birth.

On a balanced growth path, a household active in capital good i starts life with zero net worth and solves a savings problem where they choose the path by age of consumption $c(s)$, net worth $n(s)$, and capital $k(s)$ subject to a natural borrowing limit and a non-

[26] In our experiments, we also consider an extension where labor and capital is aggregated using a CES aggregator with elasticity σ_{KL}.

negativity constraint on capital:

$$V_i = \max_{n(s),c(s),b(s),k_i(s)} \mathbb{E} \int_0^\infty e^{-(\rho_c+\nu_c)s} \frac{c_i(s)^{1-1/\gamma}}{1-1/\gamma} dz,$$

subject to

$$dn(s) = \left\{ \sum_f w_f \ell_f (1+T_c) - p_c c(s) + (\nu_c - g_A) n(s) + b(s) r + k(s) r_i \right\} dt + \sigma_i p_i k_i(s) dZ,$$

$$n(s) = b(s) + k(s) > -\frac{\sum_f w_f \ell_f}{r + \nu_c - g_A}, \qquad k(s) \geq 0, \qquad n(0) = 0.$$

Individual labor income is $\sum_f w_f \ell_f$, and expected capital income is $b(s)r + k(s)r_i$, while the term $\nu_c - g_A$ term captures a mortality bonus from an actuarially fair annuity and a correction term to capture that $n(s)$ is wealth normalized by labor productivity. As in Angeletos (2007), return on capital is subject to a Brownian risk σ_i, which captures the idiosyncratic volatility that a household is not able to diversify due to financial frictions.[27]

For each household, we define effective wealth at age s as their net worth plus the present value of their labor income

$$\omega(s) = n(s) + \frac{(1+T_c) \sum_f w_f \ell_f}{r + \nu_c - g_A},$$

where income is discounted at $r + \nu_c$, with the minus g_A-term reflecting that labor income grows at a rate g_A over the life-cycle on a balanced growth path. The scalar T_c is lump sum payments from taxes, which are rebated to households in proportion to their labor income. Hence, effective wealth at birth is proportional to labor income plus payments from taxes.

Given this definition, the problem can be reformulated as one of choosing a path of effective wealth $\omega(s)$, consumption $c(s)$, and a share of effective wealth invested in risky assets subject to

$$d\omega(s) = \omega(s)[r + \nu_c - g_A + \phi(s)(r_i - r) - p_c c(s)]dt + \sigma_i \omega(s) \phi(s) dZ.$$

In this formulation, the problem is equivalent to a classic Merton (1969) portfolio problem. Thus, the optimal solution, which is constant over time, features a share of effective

[27] In the appendix, we show that this can be microfounded in the style of Di Tella (2017) in terms of a friction where capital owners can steal capital by misreporting their idiosyncratic shocks, in which case σ_i is the product of intrinsic volatility and a parameter that depends on the quality of financial institutions.

wealth allocated to the risky asset, a share to the riskless asset, and the remainder for consumption. The following proposition summarizes some important features of the solution.

Proposition 7 (Household Savings Behavior)**.** *Let $S_i = (r_i - r)/\sigma_i$ be the Sharpe ratio associated with capital i. The solution for households active in capital good i satisfies the following.*

- *At all ages, a fixed share of effective wealth $\phi_i = \gamma S_i \sigma_i^{-1}$ is invested in the risky asset k_i, and a fixed share is consumed.*
- *The expected effective wealth of surviving households in country c grows at a constant rate $g_{\omega,i} = \gamma\left([r-\rho_c] + \frac{\gamma+1}{2}S_i^2\right)$.*
- *Expected utility at birth is $U_0 = \frac{\omega_0^{1-1/\gamma}}{1-1/\gamma}[(r+\nu_c) - \gamma[r-\rho] - \gamma(\gamma-1)\frac{S_i^2}{2}]^{-1/\gamma}$, where $\omega_0 = \frac{\sum_f w_f \ell_f}{r+\nu_c-g_A}$ is the level of effective wealth at birth.*

Aggregate capital supply and asset market clearing. To obtain total capital supply, we aggregate holdings of k_i over all households active in i, and solve for the share of households going into different lines of capital.

First, we note that on a BGP, the total labor supply of households born at time t is given by $L_{0,f}(t) = e^{g_L t}(\nu_c + g_L)L_f$.[28] Hence, for a cohort of age s that entered capital good i, their productivity-normalized total holdings of capital are

$$K_i(s) = \phi_i e^{(g_{\omega,i}-\nu_c-g_A-g_L)s}\frac{(1+T_c)(\nu_c+g_L)\times\sum_f w_f L_f}{r+\nu_c-g_A}.$$

The formula follows from capital holdings being proportional to effective wealth. For the age-s cohort, effective wealth at birth was $e^{-(g_A+g_L)s}\frac{(1+T_c)\sum_f w_f L_f}{r+\nu_c-g_A}$, and the term $e^{(g_{\omega,i}-\nu_c)s}$ captures the cohort's subsequent growth of wealth, adjusting for mortality. Integrating over s and using the risky share ϕ_i from Proposition 7, the supply of K_i is

$$K_i = \pi_i^K \frac{\gamma S_i}{\sigma_i}\frac{\nu_c+g_L}{\nu_c+g_A+g_L-g_{\omega,c}}\frac{(1+T_c)\sum_f w_f L_f}{r+\nu_c-g_A},$$

where π_i^K is the share of households in country c that enters i.

To solve for π_i^K, note that attained utility is strictly increasing in S_i (by Proposition 7), so indifference across domestic capital lines requires that Sharpe ratios are equalized within

[28] On a BGP, $L_{0,f}(t)$ has to grow at rate g_L. The constant ensures that $\int_{-\infty}^{t} e^{g_L s}L_{0,f}(s)ds = L_f(t)$. Note that total labor supply L_f is the product of population and ℓ_f.

a country. Thus, there exists a country-level Sharpe ratio S_c such that

$$r_i = r + \sigma_i S_c,$$

for every capital good in country c. That is, within every country, returns on individual capital goods satisfy a Merton-style pricing formula. Furthermore, identical Sharpe ratios imply identical growth rates of wealth $g_{\omega,i} = g_{\omega,c}$. Since households are indifferent across capital goods, the shares π_i^K are pinned down by capital demand. The following proposition summarizes the asset market equilibrium.

Proposition 8 (Asset Market Clearing)**.** *Along the balanced growth path, there exist country-specific Sharpe ratios S_c, such that returns by sector satisfy $r_i = r + \sigma_i S_c$. The risk prices S_c in each country and the risk-free return r in the world satisfy*

$$\sum_{i\in K_c}(r_i - r)p_i K_i = \gamma S_c^2 \frac{\nu_c + g_L}{\nu_c + g - g_{\omega,c}} \frac{(1+T_c)\sum_{f\in F_c} w_f L_f}{r + \nu_c - g_A}, \qquad c \in C,$$

$$\sum_{c\in C}\sum_{i\in K_c} p_i K_i = \sum_{c\in C} \frac{g_{\omega,c} - g_A}{\nu_c + g - g_{\omega,c}} \frac{(1+T_c)\sum_{f\in F_c} w_f L_f}{r + \nu_c - g_A},$$

where $g_{\omega,c} = \gamma\left([r-\rho_c] + \frac{\gamma+1}{2}S_c^2\right)$.

Returns, r_i, within each country are functions of the global risk free rate, r, and country-level Sharpe ratios, S_c. The first displayed equation in Proposition 8 provides C conditions, one for each country, pinning down the Sharpe ratios by requiring that households desired risky portfolio holdings in each country are equal to the quantity of capital demanded by firms. The second expression requires that risk-free holdings sum to zero, which is equivalent to requiring that total net worth in the world equals the total capital stock. By combining Proposition 8 with the static as-if economy, which pins down capital demand as a function of returns, we can solve for all BGP prices and quantities.

4.3 Linearized Model

To solve for comparative statics, we log-linearize the BGP equilibrium conditions. As noted by Baqaee and Farhi (2020), changes in biased productivity shifters and wedges can be reduced to changes in TFPs and output wedges using relabeling and the introduction of virtual industries.[29] Hence, to simplify notation, let $d\log A$ and $d\log\tau$ represent the vector of shocks to technologies and wedges of interest.

[29] This isomorphism does not extend to the case when we shock idiosyncratic capital risk σ_i or capital income taxes τ_c^k because these perturb the capital supply equation.

Appendix C presents the full set of linearized equations. Through repeated substitution into the labor and asset market clearing constraints, we obtain a system of the following form

$$\Xi \begin{pmatrix} d\log \lambda_f \\ dS_c \\ dr \end{pmatrix} + \Gamma \begin{pmatrix} d\log A \\ d\log \tau \end{pmatrix} = \mathbf{0}_{F+C+1},$$

where $d\log \lambda_f$ are changes in labor shares (or labor prices, since quantity of labor is fixed), dS_c are changes in the Sharpe ratios, and dr is the change in the risk-free rate. This system determines relative wages and rates of return by combining labor and asset market clearing conditions, with the matrices Ξ and Γ only depending on expenditure shares, elasticities of substitution, and elasticities of capital supply. We can express all other endogenous variables in terms of these prices and returns.

4.4 Calibration Strategy

To calibrate the model, we must specify the matrices Ξ and Γ. We do this by constructing a benchmark economy using a set of externally calibrated parameters and constraints on outcomes, which are provided in Table 1. Based on these, it is possible to derive all other outcomes and parameters needed for our comparative statics.[30] The targets are expressed in the static distorted economy notation of Section 3, exploiting our equivalence results.

Risk-free rate, growth rate, and depreciation rates. The risk-free rate, population growth, and technological growth rate are based on long-term yields on US treasuries, GDP-weighted working age population growth rates, and per capita growth in US real GDP (Penn World Table, UN Population Prospects, and FRED). Our results are not very sensitive to the precise values of these parameters since the Golden Rule wedge is not defined relative to the risk-free rate. We use industry-specific depreciation rates from the Bureau of Economic Analysis.

Cost-based input output matrix. The matrix of cost shares combine information from the World Input-Output Database (WIOD) (Timmer et al., 2015) and investment flow tables from Ding (2022). Ordering goods in terms of C consumption goods, N regular good, K

[30]While our targets are sufficient to determine the outcomes relevant for comparative statics, they do not uniquely pin down all outcomes and parameters (such as prices, iceberg costs, TFP levels, and CES share parameters). While the exact parameter values are irrelevant for comparative statics, we establish the consistency of our targets by showing that there exists an assignment of parameters such that the outcomes are a BGP. See the discussion below and appendix for details.

Table 1: Calibration targets

Variable	Description	Value
r	Risk-free rate	0.025
g_L	Population growth rate	0.004
g_A	Labor-augmenting technology	0.02
δ_i	Depreciation by industry	varies by industry
$\tilde{\Omega}$	Cost shares	varies by industry
μ_i	As-if markup	varies by industry
$\theta - 1$	Armington trade elasticity	4
B_c	Net foreign assets	varies by country
γ	Intertemporal elasticity of sub.	0.5
ρ_c	Impatience parameter	varies by country
ν_c	Mortality rate	varies by country
S_c	Sharpe ratio	varies by country

capital goods, and F labor inputs, the structure of $\tilde{\Omega}$ is:

$$\tilde{\Omega} = \left(\begin{array}{c|c|c} \tilde{\Omega}^{CN} & \mathbf{0} & \mathbf{0} \\ \hline \tilde{\Omega}^{NN} & \tilde{\Omega}^{NK} & \tilde{\Omega}^{NF} \\ \hline \tilde{\Omega}^{KN} & \mathbf{0} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} & \mathbf{0} \end{array}\right),$$

where $\tilde{\Omega}^{NK} = \text{diag}(\tilde{\Omega}^{K})$ is a diagonal matrix, reflecting that each good has a dedicated capital good used in production.

We match $\tilde{\Omega}^{CN}$ to the expenditure shares on different goods in final consumption, $\tilde{\Omega}^{NN}$ to the world input-output matrix, and $\tilde{\Omega}^{NF}$ to the labor compensation shares of different industries, all taken from WIOD. For the capital input coefficients $\tilde{\Omega}^{KN}$, we set $\tilde{\Omega}^{NK}_{nn}$ equal to the industry's capital compensation relative to gross output.[31] Last, for $\tilde{\Omega}^{KN}$, we use the investment flows table, from Ding (2022), to obtain the distribution of investment spending of each country-industry combination across different goods.

[31]Note that by attributing all capital compensation to the rental rate of physical capital, we are assuming that the dynamic economy is perfectly competitive.

Capital wedges and rates of return. To calibrate the capital wedges, we equate $\mu_k = \frac{r_k+\delta_k}{g+\delta_k}$ for capital good k to the ratio of capital compensation to investment for its associated industry on the balanced growth path. Hence, industries with high profits relative to their investment levels are presumed to have some combination of low depreciation rate δ_i or high returns r_i. The intuition is that such profits can only occur if an industry has either a large capital stock relative to investment – implying a low δ_i – or high returns on that capital stock – implying a high r_i. To separately identify r_i, we solve for rates of return as $r_i = (g + \delta_i)\mu_i - \delta_i$, using BEA data on depreciation by industry to obtain δ_i.

Trade elasticity and net foreign assets. The Armington trade elasticity, $\theta - 1$ is set equal to 4, similar to Simonovska and Waugh (2014).The net foreign asset positions are from the External Wealth of Nations Database (Lane and Milesi-Ferretti, 2018).

Asset demand parameters. In addition to the externally calibrated IES $\gamma = 0.5$, in line with microeconomic evidence, we need Sharpe ratios S_c, patience parameters ρ_c, and death rates ν_c by country. To ensure asset market clearing and consistency with our previously calibrated b_c, we target net foreign asset positions and the total risk exposure $\sum_{k\in\mathcal{K}^c} \frac{(r_i-r)p_iK_i}{r_i+\delta_i}$ by country. In addition, we target the semi-elasticity of capital supply with respect to changes in the rate of return, setting it equal to 18 as in Auclert et al. (2021). Last, given estimates of S_c and r_i, the idiosyncratic risk exposure satisfies $\sigma_i = \frac{r_i-r}{S_c}$.

As we show in the appendix, the restrictions discussed so far pin down all the terms relevant to apply our comparative static formulas. They do not, however, uniquely pin down all structural parameters. For instance, our fomulas do not require taking a stance on whether home bias is rationalized via iceberg costs or differences in tastes.[32]

4.5 Calibration Results

We apply our calibration strategy to a world economy consisting of sixteen regions: the United States of America, Australia, Brazil, Canada, China, the United Kingdom, India, Indonesia, Japan, Mexico, South Korea, Russia, Turkey, Taiwan, the European Union, and an aggregated "Rest of World" region.

Table 2 summarizes some of the results of our calibration exercise for the four largest

[32]To establish the existence of at least one balanced growth path that can rationalize our calibration targets, we show that you can assign values to the remaining parameters such that outcomes form a balanced growth path given the full set of parameters. The argument is spelled out in the appendix, with the key step being to find TFPs, production share parameters, and prices so that $\tilde{\Omega}$ is consistent with firm optimization. This can be done in multiple equivalent ways: one simple way is to set all TFPs and prices to unity, and equating share parameters in the cost functions to the rows of $\tilde{\Omega}$.

Table 2: Summary of steady-state calibrated values for the four largest economies

Parameter	**Description**	**USA**	**CHN**	**EU**	**JPN**
$\bar{r}_c$	Average return on capital	0.124	0.052	0.156	0.119
$\bar{\mu}_c$	Harmonic average wedge on capital	2.372	1.799	2.443	2.175
S_c	Sharpe ratio	0.231	0.125	0.252	0.234
ν_c	Mortality rate	0.083	0.063	0.094	0.075
ρ_c	Discount rate	-0.021	-0.051	-0.014	-0.020
b_c/GDP_c	NFA relative to local GDP	-0.186	0.101	-0.136	0.345
NX_c/GDP_c	Trade balance relative to local GDP	-0.000	0.000	-0.000	0.000
χ_c	Ratio of total to human wealth	1.363	1.549	1.314	1.415
$g_{\omega c}$	Wealth growth given survival	0.043	0.044	0.043	0.043

economies in the model. In our calibration, the average risky return on capital is reasonably high in all countries except China. Recall that the risky return for each capital type i satisfies $(r_i + \delta_i)/(g + \delta_i) = \mu_i$. Hence, the higher is the Golden Rule wedge for capital stock i, the higher is the return on capital i. The Golden Rule wedge, in turn, is given by the ratio of capital compensation to investment. Hence, r_i and μ_i tend to be higher in countries where capital compensation, measured as value-added minus labor compensation, is larger than investment. Relatedly, Table 2 shows that the average (harmonic) implicit markup on capital services in our calibration is quite high (the average tends to be above 2). This suggests that the initial equilibrium is far from maximizing long-run consumption, and so reallocation effects can potentially play a big role in equilibrium responses. Sharpe ratios are around 0.2, but lower for China due its lower returns.

Because our benchmark model has an overlapping generations structure, and agents have finite lives, the effective discount rate is the sum of the mortality and discount rate (which must always be positive in order for utility to be bounded). China has the lowest effective discount rate of all countries, since it invests most heavily — whereas the EU has one of the highest effective discount rates.

In our calibration, the US, the EU and the UK are net borrowers, whereas China and Japan are net savers (this simply matches the net foreign asset positions of these countries). Since we assume that the economy is on a balanced growth path, this implies that the US, the EU, and the UK must run small trade surpluses, whereas China and Japan run small trade deficits.[33]

[33] This is counterfactual because, for some countries, net foreign asset positions and trade balance have the same sign in the data (e.g. the US has negative net foreign assets and runs a trade deficit). However, although our calibrated trade imbalances sometimes have the wrong sign, this has very small effects on the calibrated size of the country as measured by Φ_c since trade imbalances are small relative to consumption.

In the next two sections, we consider the effect of some salient distortions (markup wedges and tariffs) and technology changes (industry-level TFP).

5 Effect of Markups and Tariffs

In this section, we study the long-run consumption effects of increasing some salient distortions. We revisit classic questions from two different literatures: the effect of markups and the effect of tariffs.

Our main finding is that markups and taxes cause large first-order losses in long-run consumption. These losses stem almost entirely from the adjustment in the capital stock and the presence of the initial Golden Rule wedge. That is, the effects disappear if either the elasticity of capital to wedges is zero (which happens if either capital demand or supply are perfectly inelastic) or if the Golden Rule wedge is zero (which happens if capital is non-durable or if the rate of return is equal to the growth rate). Lastly, parameters that do not exert a significant influence on either capital supply or demand, like the trade elasticity in our model, are relatively unimportant to a first-order.

5.1 Markups

Table 3 shows the first-order effects on long-run global consumption from a uniform increase in markups of goods-producing industries in every country. We assume that profits generated by the markup are rebated lump sum to local households in proportion to their labor income. All results are expressed as semi-elasticities of long-run consumption with respect to markups, so that a value of -1.0 means that a net markup of 1% lowers log long-run consumption by 0.01.

The first row shows that consumption effects are very powerful: a 10% increase in markups reduces long-run global consumption by 7.7%. As Proposition 6 illustrates, the reduction in global consumption is driven by reductions in capital stocks. Intuitively, the increase in markups raise the price of investment goods relative to labor, which reduces capital demand and investment. Since capital is below its long-run consumption maximizing level, this reduction in investment depresses long-run consumption.

To further understand the economic forces, Table 3 also displays how this semi-elasticity changes as we vary some of our modelling choices. When the capital supply elasticity is raised to infinity, as it would be with an infinitely-lived representative agent, then long-run consumption losses are almost doubled. This is because infinitely elastic capital supply eliminates a mitigating force: in the benchmark, falling investment demand reduces

Table 3: Change in long-run consumption due to increase in markups

Scenario	Description	Global Consumption
Benchmark	Baseline calibration in Section 4.5	-0.770
Rep. agent	Baseline calibration holding returns and current accounts constant	-1.293
Static	Investment treated as a final expenditure and capital treated as an endowment	0.000
$\sigma_{KL} = 1.2$	Higher elasticity of substitution between capital and labor	-0.878
$\sigma_{KL} = 0.6$	Lower elasticity of substitution between capital and labor	-0.425
$\theta = 1$	Benchmark calibration, but trade elasticities equal to zero ($\theta - 1 = 0$)	-0.763
$\delta = \infty$	All depreciation rates set to infinity. Implies that all as-if markups equal 1, and that capital is treated as an intermediate	0.000

the rates of return, lowers the user cost of capital, and partially offsets higher investment prices. Without this offset, capital falls by more, and hence consumption falls by more. This effect is consistent with a general intuition from the misallocation literature that reallocation effects are more important when the elasticity of quantities to distortions is high (in this case, the elasticity of capital supplied). By contrast, when we lower the capital supply elasticity to zero, as it would be in a static model, losses from an increase in markups go to zero. This is also consistent with Proposition 6, since in this case, the quantity of capital does not adjust and there are no losses.

Similarly, altering the elasticity of capital demand affects losses. For example, when the elasticity of substitution between capital and labor is 1.2, instead of 1.0, as in Karabarbounis and Neiman (2013), then losses from markups are magnified. In contrast, when the elasticity of substitution between capital and labor is lower than the benchmark, for example 0.6 as in Antras (2004), the losses are substantially smaller. The losses are roughly linear in the elasticity of substitution between labor and capital. This also mirrors the intuition from the misallocation literature where losses from wedges are proportional to elasticities of substitution (see, e.g. Baqaee and Farhi, 2020).

In contrast, the trade elasticity does not have an important effect on how either capital supply or capital demand responds to higher investment prices. Hence, its value is relatively unimportant for the overall consumption losses from markups.

Finally, holding elasticities constant, losses also decline if the Golden Rule wedge were smaller. For example, if we set $\delta = \infty$, capital becomes non-durable and acts like a regular intermediate input. In this case, the Golden Rule wedge is zero and hence long-run aggregate consumption is maximized in the initial equilibrium. Accordingly, changes in markups have no first-order effect due to the envelope theorem.

5.2 Tariffs

We now consider the effect of tariffs. Suppose every country imposes a uniform tariff on all imports, with each region's tax revenues rebated to its local households. Table 4 shows the first-order effects on long-run consumption: the rows display results for a selection of countries and the columns show the decomposition in Proposition 6. The last row shows the average consumption effect (i.e. world aggregate consumption). As before, all results are expressed as semi-elasticities of long-run consumption with respect to tariffs. Results for the full set of countries is in the appendix.

Table 4: Decomposition of consumption changes via Proposition 6 for selected regions

Country	$d \log C_c$	**Harberger**	**Terms of trade**	**Δ Current account**
United States	-0.106	-0.098	-0.007	-0.001
Canada	-0.314	-0.271	-0.041	-0.002
China	-0.105	-0.120	0.011	0.004
United Kingdom	-0.229	-0.185	-0.044	-0.000
India	-0.216	-0.212	-0.002	-0.002
Japan	-0.111	-0.110	-0.005	0.003
Mexico	-0.669	-0.672	0.010	-0.008
European Union	-0.083	-0.087	0.006	-0.001
Rest of the World	-0.185	-0.215	0.027	0.003
Global	-0.137	-0.137	0.000	-0.000

The first column show the effects in our benchmark specification. Generally, the effects are large: for example, a 10% increase in tariffs reduces long-run global consumption by 1.37%. The effects tend to be smaller for larger and more closed economies (e.g. Japan and the United States). The remaining columns, which sum to give the total effect, are the the changes in the Harberger "misallocation" rectangles, changes in the terms of trade, and the changes in the current account. Table 4 shows that the primary driver of first-order losses are due to the reductions in the capital stocks. The other two terms, which are a primary focus in much of the trade literature, are comparatively unimportant for this long-run consumption experiment.

Intuitively, the effect comes from tariffs raising the price of investment goods relative to labor, similar to markups. This reduces capital demand and investment and depresses long-run consumption. The heterogeneous effects across countries are driven by two types of heterogeneity: (1) investment goods are differentially reliant on foreign imports in different countries, for example, Mexican investment goods are heavily reliant on imports compared to investment goods in the United States; (2) there is heterogeneity in capital wedges across countries, and countries with larger capital wedges will see larger reductions, conditional on the adjustment in the capital stock. For example, Chinese capital wedges are lower than the capital wedges in the United Kingdom, since in China, the level of capital compensation is closer to investment.

Table 5: Change in long-run consumption due to increase in tariffs

Selected regions	**Benchmark**	**Rep. agent**	**Static**	$\sigma_{KL} = 0.6$	$\sigma_{KL} = 1.2$	$\theta = 1$	$\delta = \infty$
United States	-0.106	-0.127	0.006	-0.049	-0.124	-0.136	-0.023
European Union	-0.083	-0.107	0.006	-0.030	-0.101	-0.116	-0.005
China	-0.105	-0.119	-0.022	-0.042	-0.130	-0.127	-0.000
Japan	-0.111	-0.124	0.008	-0.050	-0.132	-0.151	-0.018
Canada	-0.314	-0.397	-0.074	-0.198	-0.348	-0.143	-0.077
Global	-0.137	-0.190	0.000	-0.073	-0.158	-0.128	0.000

To further understand the economic forces, Table 5 displays the reduction in long-run consumption under alternative calibrations of the model. The first column is the benchmark model. The second column makes capital supply infinitely elastic, as in representative agent models. This makes the negative effects signficantly larger: for a 10% tariff, global long-run consumption declines by -1.9% instead of -1.4%. The intuition here is similar to the one for markups. The third column considers a static version of the model where capital is treated as an endowment (so that capital supply is inelastic). In this case, global consumption does not respond to a first order to tariffs, due to the envelope theorem. Instead, tariffs are purely redistributive to a first order. Similarly, as we raise the elasticity of substitution between capital and labor, which makes capital demand more elastic, the losses from tariffs are greater or smaller, similar to the markup exercise.

Interestingly, lowering all trade elasticities to zero ($\theta - 1 = 0$) affects global and even country-level losses only mildly. This is because the negative effect of the tariffs primarily work through capital-labor substitution rather than through domestic-foreign substitution. Thus, what matters is how much tariffs raise investment prices relative to wages, which depends directly on capital goods' import content, but less on the trade elasticity.[34] Finally, the last column show that all these effects disappear when capital depreciates instantly. In this case, capital is like a regular intermediate input and there are no capital wedges.

6 Effects of Productivity Shocks

Next, we study how permanent changes to industry-level productivity affect long-run consumption, with a focus on how capital accumulation shapes the relative importance of different industries. While sales shares are often used to gauge industries' importance for TFP following Hulten (1978), it is well-known that with capital accumulation, output and TFP effects are not the same (see e.g., Hulten, 1979 and Foerster et al., 2022). We use our model to examine this question in a global setting, focusing on long-run consumption and leveraging the theoretical framework developed in Section 3 to interpret the results.

6.1 Consumption Elasticities, Sales Shares, and Cost Shares

Table 6 shows the elasticity of global consumption to global productivity shocks to different industries. For reference, we include information on each industry's sales as a share of GDP and consumption, as well as the cost-based Domar weight $\tilde{\lambda}_i$. Our main finding is that industries upstream of investment are much more important for long-run consumption than would be suggested by their sales shares. Instead of sales shares, long-run consumption elasticities are instead extremely well approximated by the cost-based Domar weights in the equivalent static economy. Cost-based Domar weights exceed sales shares when there are high cumulative wedges between a sector and its final use. Thus, sectors that are upstream of investment, directly or indirectly, receive relatively high cost-based Domar weights due to the capital wedges between them and final consumption. This deviation between sales share and cost-based Domar weight only arises due to the Golden Rule wedges on capital. If the economy were calibrated to operate at the golden rule, then global consumption elasticities equal sales shares, provided these are expressed as ratios of sales to final consumption.

While large industries are generally more important than small industries, some indus-

[34] However, the trade elasticity is important for the second order effect of tariffs, since they regulate how quickly the foreign content of capital goods falls as tariffs go higher.

Table 6: Long-run global consumption effect of TFP shocks for selection of industries

Sector	$\frac{\partial \log C}{\partial \log A_i}$	$\frac{\text{Sales}_i}{\text{GDP}}$	Sales weights $\frac{\text{Sales}_i}{\text{C}}(=\lambda_i)$	Sales weights $\frac{\left(\frac{\partial \log C}{\partial \log A_i}\right)}{\lambda_i}$	Cost weights $\tilde{\lambda}_i$	Cost weights $\frac{\left(\frac{\partial \log C}{\partial \log A_i}\right)}{\tilde{\lambda}_i}$
Agriculture	0.111	0.080	0.098	1.131	0.111	1.003
Mining	0.112	0.050	0.061	1.830	0.108	1.042
Food	0.126	0.098	0.121	1.046	0.126	1.000
Chemicals, Pharma	0.102	0.061	0.075	1.353	0.101	1.009
Basic and Fab Metals	0.201	0.073	0.090	2.228	0.197	1.019
Machinery	0.136	0.043	0.053	2.570	0.134	1.011
Electrical Eqmt	0.188	0.069	0.085	2.212	0.185	1.016
Transport Eqmt	0.183	0.070	0.086	2.142	0.180	1.017
Energy, Gas, Water	0.079	0.048	0.059	1.342	0.078	1.006
Construction	0.407	0.118	0.145	2.810	0.399	1.021
WRTR*	0.536	0.301	0.371	1.445	0.530	1.010
Finance	0.173	0.111	0.137	1.260	0.172	1.002
Real Estate	0.188	0.132	0.162	1.160	0.188	0.998
Prof. Services	0.291	0.146	0.180	1.613	0.292	0.996
Health	0.109	0.088	0.108	1.009	0.109	1.000

*WRTR: Wholesale, Retail, Transportation, Repair

tries are dramatically more important than suggested by their sales shares. For example, sales in construction and machinery are 12.4% and 4.4% of global GDP, but their long-run consumption elasticities are 40.6% and 13.5% respectively. In contrast, sales in health care are almost as large as those in construction – 8.7% of global GDP – but the sector's consumption elasticity is only 11%.

How should this be interpreted economically? One intuition is that amplification simply reflects standard intermediate input amplification. In the long-run, capital acts as an intermediate input, suggesting that consumption might be a more appropriate denominator than GDP, analogous to sales-to-GDP rather than sales-to-gross output being the right weight for static comparative statics. This intuition would be valid at the Golden Rule, but not in general. For example, while sales-to-consumption is very close to the consumption elasticity for some industries like health care, it is much smaller than consumption elasticities for others, like construction and machinery.

Instead, long-run consumption elasticities are almost perfectly approximated by the cost-based Domar weight $\tilde{\lambda}$ in the equivalent static economy. Recall that these weights are obtained by constructing Domar weights from the network using cost shares rather than revenue shares. In particular, in the context of our economy with capital, it means assigning the full capital share of costs to investment sectors, even though capital rental costs typically exceed the value of investment. Cost-based weights exceed sales shares in line with the cumulative size of wedges between a sector and final consumption. Since goods upstream of investment have to pass through the capital wedge on its way to final consumption, this explains why investment goods like machinery and construction have high cost-based Domar weights. In contrast, sectors like health care that are almost exclusively used for final consumption have cost-based shares that are similar to sales shares.

6.2 Role of Capital-Labor Substitutability

We next analyze how capital-labor substitutability shapes the effect of productivity shocks. The key mechanism is through determining the scope for reallocation. Proposition 5 shows that global consumption responses are given by cost-based Domar weights plus a reallocation effect. In Table 6, the close correspondence between consumption effects and cost-based Domar weights indicates limited reallocation effects. A key reason is the Cobb-Douglas production function, which generally limits reallocation from productivity shocks, since input savings from higher productivity are perfectly matched by higher demand (the neoclassical growth model in Section 3.3 provides a worked-out example).[35]

To explore the role of capital-labor substitutability, we vary the elasticity of substitution between capital and labor. To show how the role of substitutability depends on the position of the industry in the investment network, we consider the effect both for health care and for construction. We also consider the cases with infinitely elastic asset demand and the $\delta = \infty$ case when capital fully depreciates. Table 7 shows the resulting long-run consumption elasticities, with Panel A showing the effect for health care, and Panel B for construction. The rows vary capital-labor substitutability, σ_{KL}, and the columns vary the calibration. The first column provides the cost-based Domar weight $\tilde{\lambda}$ for reference.

Panel A shows that the elasticity of substitution matters little for productivity shocks to health. Intuitively, since health is not upstream of investment, shocking its productivity does little to induce reallocation between capital and labor, because the shock does little to change the relative price between investment goods and labor. Moreover, the absence of a capital demand shock makes the difference between imperfectly elastic asset demand

[35] In an economy where all elasticities of substitution are equal to one, it can be shown that reallocation effects due to productivity shocks are precisely zero. In our benchmark economy, there is no such theorem due to the non-unitary trade elasticity, but in practice, these reallocation forcesÂ areÂ small.

Table 7: Industry TFP increase with different capital-labor elasticities

Panel A: Health Sector				
	$\tilde{\lambda}$	Benchmark	Rep. agent	$\delta = \infty$
$\sigma_{KL} = 0.6$	0.109	0.108	0.108	0.109
$\sigma_{KL} = 1.0$	0.109	0.109	0.109	0.109
$\sigma_{KL} = 1.2$	0.109	0.109	0.109	0.109
Panel B: Machinery Sector				
	$\tilde{\lambda}$	Benchmark	Rep. agent	$\delta = \infty$
$\sigma_{KL} = 0.6$	0.134	0.095	0.084	0.134
$\sigma_{KL} = 1.0$	0.134	0.136	0.136	0.134
$\sigma_{KL} = 1.2$	0.134	0.149	0.163	0.134

and the representative agent case irrelevant.

Panel B shows that the logic is quite different for the construction sector. Here, the consumption elasticity falls from 13.6% in the Cobb-Douglas case to 9.5% in the case with $\sigma_{KL} = 0.6$ and rises to 14.9% when $\sigma_{KL} = 1.2$. Intuitively, since construction is upstream of investment, increasing its productivity when $\sigma_{KL} < 1$ leads to a fall in the capital share of the economy. Thus, resources are allocated away from investment, which, due to the capital wedge, lowers long-run consumption. The reverse is true when $\sigma_{KL} > 1$.

The representative agent column shows that these reallocation effects are strengthened when asset demand is infinitely elastic. This occurs because return adjustments mitigate the reallocation effect by lowering rates of return when $\sigma_{KL} < 1$ and raising them when $\sigma_{KL} > 1$. Note also that perfectly elastic asset demand has a minimal impact in the benchmark Cobb-Douglas case. This is because with Cobb-Douglas production, there is little change in r anyhow since an increase in investment productivity does not constitute a capital demand shock from the perspective of the asset market clearing condition. The reason is that the falling quantity of capital is almost perfectly offset by an increase in its price, leaving the desired value of firms' capital stock largely unchanged at fixed returns.

Last, the $\delta = \infty$ column shows that σ_{KL} does not matter when capital is a regular intermediate input. The reason is that reallocation only matters when long-run consumption is not maximized. Otherwise, the envelope theorem precludes any first-order reallocation effects, and we are back to the Hulten-style result of Corollary 1, where consumption effects are given by sales shares, independent of network structure and elasticities.

7 Conclusion

In this paper, we provide a framework for analyzing long-run comparative statics of dynamic disaggregated economies by representing them as distorted static disaggregated economies. This representation allows us to use tools from the static literature to study these models, and provides intuition about which model features matter for understanding long-run comparative statics. The recurring theme is that long-run consumption responses can be understood through second-best principles, even in efficient economies.

Although we focus on small shocks and first-order approximations in this paper, our methods can also be used to study nonlinear effects of shocks. For example, Baqaee and Malmberg (2025) use the methods in this paper to study the long-run consequences of sanctions on Russia nonlinearly. Similarly, while we focus on tangible capital accumulation in this paper, similar arguments and methods apply to the accumulation of non-physical and intangible capital as well, and this is an important avenue for future work.

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Appendix

A Appendix to Section 2

A.1 Proof of Proposition 1

An equilibrium of a static economy as described in the proposition is any set of quantities $\{Y_i, Y_{ij}, K_{ij}, L_{if}, C_i\}$, prices $\{p_i, R_i, w_f\}$, markup wedges $\{\mu_i\}$ and transfers T that satisfy:

$$
\begin{aligned}
Y_i &= A_i F_i[\{L_{if}\}, \{Y_{ij}\}, \{K_{ij}\}] \\
Y_i, \{L_{if}\}, \{Y_{ij}\}, \{K_{ij}\} &\in \arg\max p_i Y_i - \sum_f w_f L_{if} - \sum_j p_j Y_{ij} - \sum_j R_j K_{ij} \\
K_i, X_i &\in \arg\max \mu_i^{-1} R_i K_i - (g+\delta_i) p_i X_i \\
C_1, \ldots, C_N &\in \arg\max \ \mathcal{U}(C_1, \ldots, C_N) \quad s.t. \quad \sum_i p_i C_i \le \sum_f w_f L_f + T \\
\sum_i L_{if} &\le L_f \\
K_i &\le X_i / (g+\delta_i) \\
C_i + X_i + \sum_j Y_{ji} &\le Y_i \\
T &= \sum_i \left(1 - \frac{1}{\mu_i}\right) R_i K_i, \\
\mu_i &= \frac{r_i + \delta_i}{g + \delta_i},
\end{aligned}
$$

where p_i is the price of good i, and R_i is the price of the capital good associated with i.

Suppose now that we have a set of quantities and prices $\mathcal{X}$ that form a BGP with returns $\{r_i\}$. We want to check that this constitutes a static equilibrium of the desired form. We do this by setting $R_i = p_i(r_i + \delta_i)$, $\mu_i = \frac{r_i+\delta_i}{g+\delta_i}$, $T = \sum_j (r_j - g) B_j$ and checking that the equilibrium conditions hold under the assumption that $\mathcal{X}$ is a BGP.

The production function, market clearing conditions, capital user cost, and consumer optimization in the static economy coincide exactly with those on the BGP given our definitions of R_j and T, so they hold given our assumption that $\mathcal{X}$ is a BGP. Furthermore $K_i, X_i \in \arg\max \mu_i^{-1} R_i K_i - (g+\delta_i) p_i X_i$ since $\mu_i R_i = g + \delta_i$. Last, we have that on a BGP, it holds that

$$
T = \sum_i (r_i - g) B_i
$$

$$= \sum_i (r_i - g) p_j K_i$$
$$= \sum_i \frac{r_i - g}{r_i + \delta_i} R_i K_i$$
$$= \sum_i \left(1 - \frac{1}{\mu_i}\right) R_i K_i,$$

where the second line uses asset market clearing, the third line the definition of R_j, and the last line the definition of μ_i. Hence, the BGP forms an equilibrium of the static model.

A.2 Proof of Proposition 3

This can be proved almost exactly as Proposition 1. The one difference lies in the distribution of free cash-flow. Given a balanced growth path, there are associated asset holdings by country B_{ci}, which means that the balanced growth path has an associated allocation of free cash flows

$$\pi_c = \frac{\sum_j B_{cj}(r_j - g)}{\sum_j \sum_c B_{cj}(r_j - g)}.$$

In the equivalent static economy where profits are distributed according to π_c, transfers going to households in country c need to satisfy

$$T_c = \pi_c \sum_i \left(1 - \frac{1}{\mu_i}\right) R_i K_i. \quad (22)$$

By substituting in asset market clearing conditions $p_i K_i = B_i$ and the expressions for R_i, μ_i, and π_c, it can be verified that

$$T_c = \sum_i (r_i - g) B_i.$$

Hence, the balanced growth path is an equilibrium of a distorted static economy where the profit distribution rule is given by (22).

A.3 Proof of Proposition 3

For the case with initial linear taxes, most of the proof of Proposition 1 is unaffected: we still use the BGP to construct $R_i = p_i(r_i + \delta_i)$ and $\mu_i = \frac{r_i + \delta_i}{g + \delta_i}$ and verify that the equilibrium equations of the static economy are satisfied by the BGP prices and quantities. The BGP equations for production, factor market clearing, and asset market clearing are the same as before. The two differences are the profit maximization problems of producers and the budget constraint of households, which now are

$$Y_i, \{L_{if}\}, \{Y_{ij}\}, \{K_{ij}\} \in \arg\max (1-\tau_i) p_i Y_i - \sum_f w_f L_{if} - \sum_j p_j Y_{ij} - \sum_j (r_j + \delta_j) K_{ij}$$

$$\sum_i p_i C_i \leq \sum_f w_f L_f + \sum_i (r_i - g) B_i + T.$$

Furthermore, transfers satisfy $T = \sum_i \tau_i p_i Y_i$. These are the equilibrium equations in a static economy with taxes, capital goods produced as in Proposition 1 and a markup wedge $\mu_i = \frac{r_j+\delta_j}{g+\delta_j}$ on capital goods, so a BGP satisfies those equations, and thus forms a static equilibrium.

A.4 Non-Physical Capital

To illustrate how the results can be extended to a model with non-physical capital, we analyze a model where firm entry costs operate as a form of capital. The model features one with a single homogeneous output good and a continuum of perfectly competitive firms. Firms with productivity z have production function $y(z) = z\ell^{\eta}$, where $\eta \in (0, 1)$ captures diminishing returns and l is the number of workers hired for production. Firms produce a homogeneous good sold at price p. To start a firm, entrepreneurs pay a sunk entry cost, w/z_e, where z_e is the productivity of the entry technology. Firms exit at an exogenous rate δ and the discount rate is r.

We write ℓ, y, π for labor, output, and profit per firm, M for the steady-state measure of firms, and $Y \equiv My$, $M\ell = L_Y$, $M\delta/z_e = L_E$ for steady-state output, production labor, and entry labor. Aggregate production satisfies

$$Y = zM^{1-\eta} L_Y^{\eta} \equiv zF[M, L_Y],$$

with wages and per-firm per-period profits satisfying

$$w = zF_{L_Y}$$
$$\pi = zF_M.$$

Free entry implies that

$$w/z_e = \int_0^{\infty} e^{-(r+\delta)t} \pi = \frac{\pi}{r+\delta}.$$

Furthermore, the steady-state measure of firms need to satisfy

$$M\delta = z_e L_e,$$

where L_e is the amount of labor allocated to entry. This can be used to obtain the following set of steady-state equations

$$\begin{aligned}
Y &= zF[M, L_Y], \\
Y, M, L_Y &\in \arg\max Y - wL_Y - \pi M, \\
M\pi &= L_e w \frac{r+\delta}{\delta} \iff M\pi\left(1 - \frac{r}{r+\delta}\right) = L_e w \\
C &= wL + M\pi - L_e w \\
L_e + L_y &\leq L.
\end{aligned}$$

We see that the equations are precisely as though there is an investment flow of value $L_e w$, a capital stock of value $\frac{L_e w}{\delta}$ rented out at $r + \delta$. We obtain the following proposition.

Proposition 9 (Isomorphism with firm entry and exit)**.** *Steady-state prices and quantities in the economy described above form an equilibrium of an equivalent static economy where the production functions of goods are the same as in the dynamic economy; entry costs in the static model are $c_e^{static} = \delta / z_e$; profits are taxed at rate $\tau = \frac{r}{r+\delta}$ with tax revenues distributed to households.*

B Appendix to Section 3

B.1 Proof of Proposition 4

Proposition 1 states that balanced growth paths are also equilibria of static economies. Assuming that $X^{BGP}(\Theta)$ and $X^{static}(\Theta, \mu)$ are differentiable functions in a neighborhood of some initial balanced growth path, this implies that

$$X^{BGP}(\Theta) = X^{static}\left[\Theta, \mu(X^{BGP}(\Theta))\right],$$

in that neighborhood, where we can write μ as a function of X^{BGP} since it can be calculated directly from BGP objects. Differentiating the system with respect to Θ implies that

$$\frac{dX^{BGP}}{d\Theta} = \frac{\partial X^{static}}{\partial \Theta} + \sum_i \frac{\partial X^{static}}{\partial \mu_i} \frac{d\mu_i}{d\Theta}$$

as in the proposition, where $\frac{dX^{BGP}}{d\Theta}$ and $\frac{\partial X^{static}}{\partial\Theta}$ are Jacobian matrices, and $\frac{\partial X^{static}}{\partial\mu_i}$ is a vector with the derivative of all components of the static equilibrium with respect to μ_i. To derive $\frac{d\boldsymbol{\mu}}{d\Theta}$, we note that it follows from $\mu_i = \frac{r_i+\delta_i}{g+\delta_i}$ and differentiating the asset market clearing condition $\mathcal{K}(\Theta, \mu[\mathbf{r}(\Theta)]) = \mathcal{A}[\mathbf{r}(\Theta)]$ with respect to $\mathbf{r}$.

B.2 Proof of Proposition 5

The proposition is implied by Theorem 1 in Baqaee and Farhi (2020), which characterizes output responses to shocks to productivities and wedges in distorted static economy. The theorem applies to our case due to Proposition 4, from which we know that the change in consumption with respect to productivities and wedges is given by the same comparative statics in a distorted static economy.

B.3 Proof of Corollary 1

Given Proposition 5, Corollary 1 follows by same argument as Corollary 1 follows from Theorem 1 in Baqaee and Farhi (2020).

B.4 Proof of Proposition 6

Define permanent domestic product (PDP) along the balanced growth path to be GDP minus investment:

$$PDP_c = \underbrace{\sum_{i\in N_c} p_i Y_i - \sum_{i\in N_c}\sum_{j\in N} p_j Y_{ij}}_{\text{GDP of country } c} - \underbrace{\sum_{i\in K_c} p_i X_i}_{\text{investment of country c}}$$

Define net permanent output of each good $i \in N$ produced by country c to be

$$q_{ci} = Y_i \mathbf{1}\left[i \in \mathcal{N}_c\right] - \sum_{j\in\mathcal{N}_c} Y_{ji} - X_i$$

Note that

$$PDP_c = \sum_{i\in\mathcal{N}} p_i q_{ci}.$$

Define growth in real PDP, denoted by $d\log Y_c$, to be

$$d\log Y_c = \sum_{i\in\mathcal{N}} \frac{p_i}{PDP_c} dq_{ci}.$$

Similarly, define the implicit PDP deflator $d\log P^{Y}$ to be

$$d\log Y_c = \sum_{i\in\mathcal{N}} \frac{q_i}{PDP_c} dp_i.$$

Define net exports of each good i to be

$$NX_{ci} = p_i\left[\sum_{j\notin N_c} Y_{ji} + \sum_{c'\neq c} C_{c'i} + \sum_{j\notin K_c} Y_{ji}\right] - p_i\left[\sum_{j\in N_c} Y_{ji} + C_{c'i} + \sum_{j\in K_c} Y_{ji}\right].$$

Define

$$NX_c = \sum_i NX_{ci}.$$

Note that

$$PDP_c = P_c C_c + NX_c.$$

Hence,

$$d\log\left[P_c C_c\right] = \frac{PDP_c}{P_c C_c} d\log\left[PDP_c\right] - \frac{1}{P_c C_c} d\left[NX_c\right]$$
$$d\log C_c = \frac{PDP_c}{P_c C_c} d\log Y_c + \frac{PDP_c}{P_c C_c} d\log P^{Y} - d\log P_c - \frac{1}{P_c C_c} d\left[NX_c\right]$$

Writing this out gives

$$\begin{aligned} d\log C_c &= \frac{PDP_c}{P_c C_c} d\log Y_c + \frac{PDP_c}{P_c C_c}\sum_{i\in\mathcal{N}} \frac{\left[q_i - c_{ci}\right]}{PDP_c} dp_i - \frac{1}{P_c C_c} d\left[NX_c\right] \\ &= \frac{PDP_c}{P_c C_c} d\log Y_c + \sum_{i\in\mathcal{N}} \frac{\left[p_i q_i - p_i c_{ci}\right]}{P_c C_c} d\log p_i - \frac{1}{P_c C_c} d\left[NX_c\right] \\ &= \frac{PDP_c}{P_c C_c} d\log Y_c + \sum_{i\in\mathcal{N}} \frac{NX_{ci}}{P_c C_c} d\log p_i - \frac{1}{P_c C_c} d\left[NX_c\right]. \end{aligned}$$

The result follows if we can show that

$$\frac{PDP_c}{P_c C_c} d\log Y_c = \sum_{i\in K_c} \frac{R_i p_i K_i - p_i X_i}{P_c C_c} d\log K_i.$$

From the resource constraints:

$$\frac{dq_{ci}}{q_{ci}} = \frac{Y_i}{q_{ci}} d\log Y_i \mathbf{1}\left[i\in N_c\right] - \sum_{j\in N_c} \frac{Y_{ji}}{q_{ci}} d\log Y_{ji} - \frac{X_i}{q_{ci}} d\log X_i,$$

and from technologies (using that $\tilde{\Omega}_{n,\cdot} = \Omega_{n,\cdot}$ for $n \in N$ since the only initial wedge is on capital):

$$\left(d \log Y_j - \sum_{f \in F_c} \Omega_{jf} d \log L_{jf} - \Omega_{j\hat{k}(j)} d \log K_j\right) = \sum_{i \in N} \Omega_{ji} d \log Y_{ji},$$

where $\hat{k}(j)$ is the capital good associated with j. Last, along the balanced growth path, we have:

$$d \log K_j = d \log X_j.$$

Hence, unpacking the definition of $d \log Y$ and substituting, we get:

$$\begin{aligned}
d \log Y_c &= \sum_{i \in N} \frac{p_i}{PDP_c} [dq_{ci}] = \sum_{i \in N} \frac{p_i}{PDP_c} \left[dY_i \mathbf{1}\left[i \in \mathcal{N}_c\right] - \sum_{j \in \mathcal{N}_c} dY_{ji} - dX_i \right] \\
&= \left[\sum_{i \in N_c} \frac{p_i Y_i}{PDP_c} d \log Y_i - \sum_{i \in N} \frac{p_i}{PDP_c} \sum_{j \in \mathcal{N}_c} dY_{ji} - \sum_{i \in N_c} \frac{p_i X_i}{PDP_c} d \log X_i \right] \\
&= \left[\sum_{i \in N_c} \frac{p_i Y_i}{PDP_c} d \log Y_i - \sum_{i \in \mathcal{N}_c} \sum_{j \in N} \frac{p_j Y_{ij}}{PDP_c} d \log Y_{ij} - \sum_{i \in N_c} \frac{p_i X_i}{PDP_c} d \log X_i \right] \\
&= \left[\sum_{i \in N_c} \frac{p_i Y_i}{PDP_c} d \log Y_i - \sum_{i \in \mathcal{N}_c} \frac{p_i Y_i}{PDP_c} \sum_{j \in N} \Omega_{ij} d \log Y_{ij} - \sum_{i \in N_c} \frac{p_i X_i}{PDP_c} d \log K_i \right] \\
&= \left[\sum_{i \in N_c} \frac{p_i Y_i}{PDP_c} d \log Y_i - \sum_{i \in \mathcal{N}_c} \frac{p_i Y_i}{PDP_c} \left[d \log Y_i - \sum_{f \in F_c} \Omega_{if} d \log L_{if} - \Omega_{i\hat{k}(i)} d \log K_i \right] \right. \\
&\quad \left. - \sum_{i \in N_c} \frac{p_i X_i}{PDP_c} d \log K_i \right] \\
&= \left[\sum_{i \in \mathcal{N}_c} \frac{p_i Y_i}{PDP_c} \left[\sum_{f \in F_c} \Omega_{jf} d \log L_{jf} + \Omega_{i\hat{k}(i)} d \log K_i \right] - \sum_{i \in N_c} \frac{p_i X_i}{PDP_c} d \log K_i \right] \\
&= \left[\sum_{i \in K_c} \frac{R_i p_i K_i - p_i X_i}{PDP_c} d \log K_i \right].
\end{aligned}$$

which completes the proof.

C Appendix to Section 4

C.1 Model characterization

Here we present the full model without imposing the assumption of a balanced growth path, and also include the explicit aggregation decisions that link household decisions to aggregates. For generality, we also include a capital tax τ^c that is not part of the model in the main paper.

Indexing. We write C, N, K, and F for the set of countries, types of perishable industries, types of capital goods, and factors. There is one consumption industry per country. The full set of industries consists of a collection $\{(c,i) : c \in C, i \in N\}$ giving pairs of countries and industry types (we use the index (c,c) with $c \in C$ to index quantities associated with c's consumption good). There is one capital good (c,k) associated with each country-industry pair (c,i). We write $\hat{k}(i)$ for the element in K associated with $i \in N$. We also write F^c for the set of factors located in country c.

Production and profit maximization. The production functions for consumption goods, other perishable goods, and investment goods are given by

$$C_c(t) = A_{cc} \prod_{j \in N} Y_{cc,j}(t)^{\tilde{\Omega}_{cc,j}} \quad \forall c \in C \tag{23}$$

$$Y_{ci}(t) = A_{ci} \prod_{j \in N} Y_{ci,j}(t)^{\tilde{\Omega}_{ci,j}} \left[\alpha_{ci,L}^{\frac{1}{\sigma_{KL}}} L_{ci}(t)^{\frac{\sigma_{KL}-1}{\sigma_{KL}}} + \alpha_{ci,K}^{\frac{1}{\sigma_{KL}}} \left(K_{ci}(t)^{\tilde{\Omega}_{ci,K}} \right)^{\frac{\sigma_{KL}-1}{\sigma_{KL}}} \right]^{\frac{\sigma_{KL}}{\sigma_{KL}-1}} \forall c \in C, \forall i \in N \tag{24}$$

$$L_{ci}(t) = A_f e^{g_A t} \prod_{f \in F_c} [L_{ci,f}(t)]^{\frac{\tilde{\Omega}_{ci,f}}{\sum_{f' \in F_c} \tilde{\Omega}_{ci,f'}}} \qquad \forall c \in C, \forall i \in N \tag{25}$$

$$X_{ck}(t) = A_{ck} \prod_{j \in N} Y_{ck,j}(t)^{\tilde{\Omega}_{ck,j}} \quad \forall c \in C, \forall k \in K. \tag{26}$$

For all three goods, there is a Cobb-Douglas aggregates of intermediates from different types of industries. For regular goods, there is also a CES aggregator of labor inputs and capital from that industry's capital good, with labor being a Cobb-Douglas aggregate of different labor types, with factor-augmenting productivity growth on labor input that is common across labor types and countries. Note that the expression K_{ci} here refers to a quantity of a capital good and not the set of capital goods. For all three expressions, we have $\sum_j \tilde{\Omega}_{ci,j} + \sum_f \tilde{\Omega}_{ci,f} + \tilde{\Omega}_{ci,K} = 1$, with $\tilde{\Omega}_{ci,f}$ and $\tilde{\Omega}_{ci,K}$ interpreted as zeros in the case of consumption and investment good production. We also have $\alpha_{ci,L} + \alpha_{ci,K} = 1$ and $\alpha_{ci,L} = \sum_f \tilde{\Omega}_{ci,f}$

The terms $Y_{cc,j}(t), Y_{ci,j}(t), Y_{ck,j}(t)$ are Armington aggregates associated with the industry type j. They are given by

$$Y_{ci,j}(t) = \left[\sum_{c'} W_{ci,jc'}^{\frac{1}{\theta}} Y_{ci,jc'}(t)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta-1}{\theta}}, \qquad i \in \{c\} \cup N \cup K, j \in N \tag{27}$$

where $\sum_{c'} W_{ci,jc'} = 1$ and c' stands for origin countries. The indexing for i captures that the same aggregation is performed for consumption goods, regular goods, and capital goods. The indexing over j captures that there are no cross-country Armington aggregates of consumption and capital goods.

Producers allocate expenditure across different countries to minimize costs:

$$\{Y_{ci,c'j}(t)\} \in \arg\min \sum_{c'} (1 + t_{c,jc'}) p_{c'j}(t) Y_{ci,c'j}(t) \quad s.t. \quad (27) \qquad \forall c \in C \quad i, j \in N$$

subject to (27), where $t_{c,jc'}$ is an import tariff levied by country i on good j from country c'. Similarly, labor is chosen to minimize cost given the desired labor aggregate

$$\{L_{ci,f}(t)\} \in \arg\min \sum_{f \in F_c} w_f(t) L_{ci,f}(t) \quad s.t. \quad (25) \qquad \forall c \in C, i \in N.$$

Writing $p_{ci,j}(t)$ and $w_{ci}(t)$ for the unit cost for industry (c,i) associated with buying a unit of inputs from industry j and a unit of labor input, the choices of producers maximize profits

$$\{C_c(t), Y_{cc,j}(t)\} \in \arg\max p_c(t) C_c(t) - \sum_{j \in N} p_{cc,j}(t) Y_{cc,j}(t) \quad \forall c \in C \tag{28}$$

$$\{Y_{ci}(t), Y_{ci,j}(t), L_{ci}(t), K_{ci}(t)\} \in \arg\max (1 - \tau_{ci}) p_{ci}(t) Y_{ci}(t) \tag{29}$$

$$- \sum_{j \in N} p_{ci,j}(t) Y_{ci,j}(t) - w_{ci}(t) L_{ci}(t) - R_{ci}(t) K_{ci}(t) \quad \forall c \in C, \forall i \in N \tag{30}$$

$$\{X_{ck}(t), Y_{ck,j}(t)\} \in \arg\max (1 - \tau_{ck}) p_{ck}(t) X_{ck}(t) - \sum_{j \in N} p_{ck,j}(t) Y_{ck,j}(t) \quad \forall c \in C, \forall k \in K, \tag{31}$$

where τ_{ci}, τ_{ck} are output taxes associated with regular goods and investment goods.

Households. A household born at time t_b chooses which capital good to enter, solving the problem:

$$V_c(t_b) = \max_{i \in N_c} V_i(t_b),$$

where $V_i(t_b)$ is the expected utility of operating capital good i, and it is determined by a lifecycle problem where households choose a path of net worth $n(t)$, consumption $c(t)$, capital $k(t)$, and bonds $b(t)$ to maximize expected utility

$$V_i(t_b) = \max_{n(t),c(t),k(t),b(t)} \mathbb{E} \int_{t_b}^{\infty} e^{-\nu_c(t-t_b)} e^{-\rho_c(t-t_b)} \frac{c(t_b,t)^{1-1/\gamma}}{1-1/\gamma} dt, \tag{32}$$

The households choose subject to the following constraints

$$dn(t) = \left\{ (1+T_c(t)) \sum_{f \in F} w_f(t)\ell_f + \nu_c n(t) + n(t)[r + \phi(t)(r_i(t) - r)](1-\tau^c) - p_c(t)c(t) \right\} dt \tag{33}$$

$$+ \sigma_i n(t)\phi(t) dZ_i, \tag{34}$$

$$\phi(t) = \frac{p_i(t)k(t)}{n(t)} \tag{35}$$

$$r_i(t) = R_i(t) - \delta_i \tag{36}$$

$$n(t) = b(t) + p_i(t)k(t) \tag{37}$$

$$-w_c\bar{b} \leq n(t), \tag{38}$$

$$n(t_b) = 0. \tag{39}$$

In the evolution equation (34), $\sum_{f \in F} w_f(t)\ell_f$ is primary factor income, where ℓ_f is the households' endowment of factor f, and $T_c(t)$ is the transfers in country c as a share of labor income. We assume that labor ℓ_f whenever the household is alive. Households have access to an actuarially fair annuity and optimally choose to annuitize all their wealth. Thus, capital income consists of $r(t)b(t)$ in interest payments on bonds and $\nu_c n(t)$ of survival benefits from the annuity.[36] In addition, the household allocates $p_i(t)k(t)$ to their idiosyncratic capital line, earning a net return $R_i(t) - \delta_i$, where $R_i(t)$ is the rental rate of capital i and δ_i is its depreciation rate. All capital income is subject to a capital tax τ^c that is common across industries in a country. [37]

The net return on capital generally exceeds the risk-free rate r, but comes at the cost of exposure to idiosyncratic risk, given by $\sigma_i p_i(t)k(t)dZ_i$. Here, σ_i captures the volatility of idiosyncratic risk in industry i, and dZ_i is an increment of the standard Brownian motion.

[36] Formally, the household contracts with a financial intermediary to obtain a flow payment conditional on survival in return for giving up all assets upon death, including their individual capital line, which can be resold at a price $p_i k_i(t)$.

[37] The common rental rate is consistent with the setup of industry-specific capital stocks consisting of perfectly substitutable varieties.

This formulation, following Di Tella (2017), can be microfounded in terms of information frictions between the entrepreneur and the investor. We assume that either all capital goods are risky, or that none are. We assume that the capital tax does not reduce the risk faced by the entrpreneur, consistent with an interpretation where the entrepreneur has to be exposed to a sufficient amount of risk in their post-tax income.

The choice across industries yields stochastic processes for the household variables. For any given variable x, we write $x_i(t_b, t)$ for the stochastic process associated with a household born at time t_b and operating in industry i. To streamline notation, we extend the stochastic processes to the full range of t by assuming that they take value 0 before birth and after death.

Financial intermediaries. We assume that annuities and claims on capital lines are intermediated by a financial institution. On the annuity side, the intermediary sells annuities that provide flow payments to living households in exchange for their assets upon death. In terms of claims of capital lines, the intermediary issues derivatives which promise payments to households after bad shocks in return for payments after good shocks. The intermediary does not hold any assets or make any profits. For annuities, this follows from free entry which implies that annuities are actuarially fair, making the intermediary a mere conduit for redistributing between surviving and dying households. On the capital side, zero profits follow from free entry and the fact that the intermediary is risk-neutral with respect to household shocks, meaning that the derivatives have no upfront value and yield no expected profits.

We assume that risk-free bonds are in zero net supply and that financial markets are integrated, implying a market clearing condition

$$\sum_{i \in \mathcal{N}} B_i(t) = 0, \tag{40}$$

where financial integration is captured by summing across all industries in the world.[38]

Demographics and aggregation. For each $t_b \in (-\infty, \infty)$, there is an exogenous number of births $L_{0,c}(t_b) = e^{g_L t_b} L_{0,c}$ in country c. To obtain aggregate variables, we integrate household outcomes across cohorts:

$$L_f(t) = \int_{-\infty}^{\infty} L_c^0(t_b) \mathbb{E} \ell_f dt_b, \qquad f \in F, \tag{41}$$

[38] If there is financial autarky, this equilibrium condition is replaced by one for each country, with the sum running over $\mathcal{N}_c$ for each country c.

$$K_{ci}(t) = \int_{-\infty}^{\infty} L_c^0(t_b)\varphi_{ci}(t_b)\mathbb{E}k_{ci}(t_b,t)dt_b, \quad c \in C.i \in N, \tag{42}$$

$$B_i(t) = \int_{-\infty}^{\infty} L_c^0(t_b)\varphi_{ci}(t_b)\mathbb{E}b_i[t_b,t]dt_b, \quad i \in N, \tag{43}$$

$$C_c(t) = \int_{-\infty}^{\infty} L_c^0(t_b)\sum_{i \in K_c}\varphi_{ci}(t_b)\mathbb{E}c_i[t_b,t]dt_b, \quad c \in C. \tag{44}$$

The expectations are taken over realizations of cohort-t_b's processes, which are stochastic due to mortality risk, as well as due to idiosyncratic risk to capital accumulation. The terms $\varphi_{ci}(t_b)$ are the shares of households born at time t_b in country c that operates capital line i. Further, the convention of treating stochastic processes as zero before birth and after death implies that the survival probability is implicitly accounted for in the expectations. It also implies that there are no contributions to aggregates from cohorts born after time t.

Resource constraints and market clearing. In addition to the bond market clearing condition, the resource constraints are given by

$$\begin{aligned}
C_c(t) &= Y_{cc}(t), \qquad c \in C \\
Y_{ci}(t) &= \sum_{c' \in C, j' \in C+N+K} Y_{c'j',ci}(t), \qquad c \in C, i \in N \\
\dot{K}_{c,i}(t) &= -\delta_{k(i)}K_{c,i}(t) + X_{c,k(i)}(t) \qquad c \in C, i \in N \\
L_f(t) &= \sum_{i \in N} L_{ci,f}(t), \qquad c \in C, f \in F^c.
\end{aligned}$$

Furthermore, transfer income to households need to be consistent with tax income for the government:

$$T_c(t) = \frac{\sum_{i \in N_c}\tau_{ci}p_{ci}(t)Y_{ci}(t) + \sum_{i,j,c'} t_{i,jc'}p_{jc'}(t)Y_{ci,jc'}(t) + \tau^c[r(t)B_c(t) + \sum_{i \in K^c} p_{ci}(t)K_{ci}(t)r_i(t)]}{\sum_{f \in F_c} w_f(t)L_f(t)},$$

where the size of the transfer is expressed relative to factor income in that country.

Equilibrium. Given taxes and tariffs, an equilibrium of the model consists of prices, quantities, transfers, household values and decision functions, as well as the shares of households $\varphi_{ci}(t_b)$ that enter different capital lines. They satisfy the following properties.

1. Given prices, aggregate quantities are consistent with profit maximization, capital accumulation equations, resource constraints, and bond market clearing.

2. For each cohort t_b, the households' value and decision functions solve their opti-

mization problem (32)-(39) given prices, with the implied stochastic processes being consistent with aggregate quantities, (41)-(44).

3. For each t_b, all industries with strictly positive entry offer the same expected utility to newborn households. That is, if $\varphi_{ci}(t_b), \varphi_{ci'}(t_b) > 0$, then

$$V_{ci}(t_b) = V_{ci'}(t_b).$$

Balanced growth path. Balanced growth paths are equilibria with the following properties:

1. Constant risk-free rate r, rental rates $\{R_i\}$, and good prices $\{p_i\}$.

2. Constant shares φ_{ci} of households in every country entering each industry.

3. Primary factor prices $w_f(t)$ grow at a constant common rate g_A.

4. Consumption, output, intermediate inputs, and capital stocks grow at a common constant rate $g \equiv g_L + g_A$.

C.2 Proof of Proposition 7

Consider the household problem when they face a wage profile $w_f(t) = e^{g_A t} w_f$ and constant rates of returns $r(t) = r$, $r_i(t) = r_i$, and fixed prices $p_c(t) = p_c$, $p_i(t) = p_i$. Furthermore, write $y = \sum_f w_f \ell_f (1 + T_c)$ for total factor income. Define $\hat{n}(t) = e^{-g_A t} n(t)$ and $\hat{c}(t) = e^{-g_A t} c(t)$ to be normalized levels of net worth and consumption. Then, the problem can be expressed as

$$\max \int_0^\infty e^{-\tilde{\rho}_c t} \frac{\hat{c}(t)^{1-1/\gamma}}{1-1/\gamma}$$

where $\tilde{\rho}_c = \rho_c + \nu_c - g_A(1 - 1/\gamma)$, subject to

$$\begin{aligned} d\hat{n}(t) &= \{y - p_c \hat{c}(t) + \nu_c \hat{n} + \hat{n}(t)\left[r - g_A + \phi(t)(r_i - r)\right]\} dt + \sigma_i \phi(t) \hat{n}(t) dZ_i \\ \hat{n}(t) &\geq -\frac{y}{r + \nu_c - g_A}, \\ \phi(t) &\geq 0. \end{aligned}$$

Defining effective wealth as

$$\hat{\omega}(t) = \hat{n}(t) + \frac{y}{r + \nu_c - g_A},$$

we obtain

$$d\hat{\omega}(t) = [(r + \nu_c - g_A)\hat{\omega}(t) + \phi(t)(r_i - r)\hat{\omega}(t) - p_c c(t)]dt + \sigma_i \phi(t)\hat{\omega}(t)dZ$$

This is a Merton portfolio problem with a risk-free return $r + \nu_c - g_A$ and a risky return $r_i + \nu_c - g_A$. The solution is given by allocating a constant share of effective wealth to the risky asset:

$$\phi_i \equiv \gamma \times \frac{r_i - r}{\sigma_i^2} = \gamma S_i \sigma_i^{-1},$$

where $S_i \equiv \frac{r_i - r}{\sigma_i}$ is the Sharpe ratio. The household also consumes a constant share of effective wealth

$$p_c c(t) = \xi \hat{\omega}(t) \qquad \xi \equiv \gamma \times \left(\tilde{\rho} - (1 - 1/\gamma)\left(\frac{\gamma(r_i - r)^2}{2\sigma_i^2} + r + \nu_c - g_A\right)\right)$$

Substituting this expression into $p_c c(t)$, and using that the expected growth of the Brownian term is 0, we obtain that the expected growth rate of $\hat{\omega}(t)$ is

$$\mathbb{E}d\hat{\omega}(t) = \hat{\omega}(t)\left[-g_A + \gamma(r - \rho_c) + \frac{\gamma(\gamma+1)}{2}S_i^2\right].$$

This mean that non-normalized wealth grows as $\gamma(r - \rho_c) + \frac{\gamma(\gamma+1)}{2}S_i^2$, as stated in the proposition.

To derive the expected utility at birth, we note that flow utility at time t is given by

$$\frac{\xi e^{-\tilde{\rho}t}\hat{\omega}(t)^{1-1/\gamma}}{1 - 1/\gamma}.$$

Using the linearity of expectation, the expectation of the integral over $\frac{\xi e^{-\tilde{\rho}t}\hat{\omega}(t)^{1-1/\gamma}}{1-1/\gamma}$ equals the integral over $\frac{\xi e^{-\tilde{\rho}t}\mathbb{E}\hat{\omega}(t)^{1-1/\gamma}}{1-1/\gamma}$. Using standard Ito algebra, we can derive a stochastic differential equation for $\hat{\omega}(t)^{1-1/\gamma}$ in terms of the drift and diffusion of $\hat{\omega}$, which lets us solve for the growth rate of expected utility $\mathbb{E}\hat{\omega}(t)^{1-1/\gamma}$, and thus for expected utility at birth.

C.3 Proof of Proposition 8

First, we note that since attained utility only depends on industry properties through S_i, these must be equalized within a country when all capital goods are active. This implies there exists an S_c such that

$$r_i = r + \sigma_i S_c \quad \forall i \in K^c$$

as stated in the proposition. Moreover, since the growth rate of effective wealth among survivors also depends only on the Sharpe ratio, there exists a country-specific growth rate $g_{\omega,c}$ of effective wealth as well.

To derive the remaining parts of the proposition, we first observe that the total flow $L_{0,f}$ of new units of factor f satisfies

$$L_{0,f} = L_f(g_L + \nu_c), \qquad f \in F_c$$

reflecting that the inflow of new factors must compensate for deaths and maintain growth at rate g_L. Furthermore, the normalized amount of effective wealth in the economy is the integral over historical cohorts, yielding

$$\begin{aligned}\mathcal{W}_c &= \int_0^\infty \sum_f \frac{[e^{-g_L t} L_{0,f}][e^{-g_A t} w_f](1 + T_c)}{r + \nu_c - g_A} e^{(g_{\omega,c} - \nu_c)t} \\ &= \frac{\nu_c + g_L}{\nu_c + g_L + g_A - g_{\omega,c}} \frac{\sum_f L_f w_f (1 + T_c)}{r + \nu_c - g_A},\end{aligned}$$

where the term $e^{-g_A t}$ reflects that the nominal value of wages started out at a lower level in the past.

Using that the share of risky assets in industry i is $\phi_i = \gamma \frac{S_i}{\sigma_i}$, market clearing for capital implies that

$$\sigma_i p_i K_i = \varphi_{c,i} \gamma S_c \mathcal{W}_c \qquad \forall i \in K^c,$$

where $\varphi_{c,i}$ is the share of households in country c investing in industry i. Using $\sigma_i = (r_i - r)/S_c$ and summing over $i \in K^c$ yields

$$\sum_{i \in K^c} \frac{(r_i - r) p_i K_i}{S_c} = \gamma S_c \mathcal{W}_c$$

Substituting in $\mathcal{W}_c$ and moving S_c to the right-hand side gives us

$$\sum_i (r_i - r) p_i K_i = \gamma S_c^2 \frac{\nu_c + g_L}{\nu_c + g_L + g_A - g_{\omega,c}} \frac{\sum_f w_f L_f (1 + T_c)}{r + \nu_c - g_A}$$

as required.

Finally, given that $\sum_c B_c(t) = 0$, the value of all capital assets must equal total effective wealth minus effective wealth from labor. This gives us

$$\begin{aligned}\sum_{c\in C}\sum_{i\in K} p_{ci}K_{ci} &= \sum_{c}\left[\mathcal{W}_c - \sum_{f\in F^c}\frac{(1+T_c)\sum_{f\in F_c} w_f L_f}{r+\nu-g_A}\right] \\ &= \sum_{c\in C}\left[\frac{\nu_c+g_L}{\nu_c+g_L+g_A-g_{\omega,c}}-1\right]\frac{\sum_f L_f w_f(1+T_c)}{r+\nu_c-g_A} \\ &= \sum_{c\in C}\frac{g_{\omega,c}-g_A}{\nu_c+g_L+g_A-g_{\omega,c}}\frac{\sum_f L_f w_f(1+T_c)}{r+\nu_c-g_A},\end{aligned}$$

which concludes the proof.

C.4 Balanced growth equations

The following equations determine the BGP equilibrium using the input-output notation in Section 3. Without loss of generality, following Baqaee and Farhi (2017), we relabel the input-output matrix so that each CES aggregator is treated as a separate producer (this simplifies notation). This means that we also drop the notation that indexes producers in terms of countries and type of industry. Instead, we use a single index to denote every CES aggregate in the model. We assume that the Armington CES nests are located in the destination country. We write $t_{i,j}$ for a bilateral tax on nest i's purchases of j, $t_{i,j}$, with the assumption that tax revenues are rebated to destination households. Allowing for $t_{i,j}$ nests the tariffs in the main model. Furthermore, we allow for reduced-form output wedges, $\tilde{\mu}$, which behave like markups and whose revenues are rebated to origin households.[39] The allocational consequences of such wedges can be constructed from tariffs and output taxes in our model description. CES share parameters are denoted using overlines, with shares being zero for goods not in that nest (for example, in an Armington nest, all the shares but the ones associated with that input bundle are zero).

Capital Supply Equations

- Wealth growth conditional on survival
  $g_{\omega,c} = \gamma \times \left[(1-\tau_c^k)r - \rho + \left(\frac{\gamma+1}{2}\right)(1-\tau_c^k)^2 S_c^2\right]$
- Ratio of total wealth to human wealth
  $\chi_c = \frac{\nu_c+g_L}{\nu_c+g-g_\omega^c}$

[39] Allowing for such output wedges does not require extending the framework in Appendix C.1, because their effect is equivalent to that of output taxes.

- Desired financial wealth of households in country c
  $W_c = \frac{\left(\sum_{f \in F_c} \lambda_f (1+T_c)\right)}{(1-\tau_c^k)r + \nu_c - g_A} \left[\chi_c - 1\right]$

Production Block

- Growth rate
  $g = g_L + g_A$
- Tax and markup revenues relative to labor income
  $T_c(t) = \frac{\tau_c^k \left[r b_c(t) + \sum_{i \in K_c} \frac{r_i}{r_i+\delta_i} \lambda_i\right] + \sum_{i \in N_c} \left[1 - \frac{1}{\tilde{\mu}_i}\right] \lambda_i + \sum_{j \in N_c + K_c} \sum_{(ic') \in N+K} \frac{t_{j,i}}{1+t_{j,i}} \lambda_{jc} \Omega_{jc,ic'}}{\sum_{f \in F_c} \lambda_f}$
- Country consumption
  $\Phi_c = \sum_{f \in F_c} \lambda_f (1 + T_c) + \sum_{i \in K_c} \lambda_i \left[1 - \frac{1}{\mu_i}\right] + (r - g) b_c$
- Goods prices
  $p_n = \frac{\tilde{\mu}_n}{A_n} \left(\sum_{j \in N} \bar{\Omega}_{nj} [(1 + t_{nj}) p_j]^{1-\theta_n} + \sum_{j \in F} \bar{\Omega}_{nj} w_j^{1-\theta_n} + \sum_{j \in K} \bar{\Omega}_{nj} R_j^{1-\theta_n}\right)^{\frac{1}{1-\theta_n}} \qquad n \in C + N$
- Capital user costs
  $R_n = \frac{\mu_n(\delta_n + g)}{A_n} \left(\sum_{j \in N} \bar{\Omega}_{nj} [(1 + t_{nj}) p_j]^{1-\theta_n}\right)^{\frac{1}{1-\theta_n}}, \qquad n \in K.$
- Effective markups
  $\mu_m = \frac{r_n + \delta_n}{g + \delta_n} \mathbf{1}(m \in K) + \tilde{\mu}_m \mathbf{1}(m \notin K).$
- Goods, labor, and capital services market clearing
  $\lambda_i = \sum_{c' \in C} \Phi_{c'} \frac{\Omega_{c',i}}{1+t_{c',i}} + \sum_{j \in N+K} \lambda_j \frac{1}{1+t_{j,i}} \Omega_{j,i}.$

Capital Market Clearing and Distribution of Free Cash Flows

- Physical capital market clearing by country[40]
  $\sum_{i \in K_c} \sigma_i \frac{\lambda_i}{r_i + \delta_i} = W_c \frac{\chi_c}{\chi_c - 1} \gamma S_c$
- Bond market clearing
  $\sum_{i \in K} \frac{\lambda_i}{r_i + \delta_i} = \sum_{c \in C} W_c$
- No arbitrage within country
  $r_i = r + \frac{\sigma_i S_c}{(1-\tau_k)}, \qquad i \in K_c$

[40]Recall that if i is a capital good, then λ_i is the compensation of capital i relative to world consumption, hence $\lambda_i / (r_i + \delta_i)$ is the value of the capital stock.

- Net foreign assets
  $b_c = W_c - \sum_{i \in K_c} \frac{\lambda_i}{r_i + \delta_i}$

Invertibility of BGP system. To see that there exists a set of share and productivity parameters such that our calibration targets constitute a balanced growth path, we consider the case when all TFPs are 1 except for investment goods that have productivity $\mu_n(g + \delta_n)$. Furthermore, we set the values of the share parameters $\bar{\Omega}$ to the observed cost shares for each nest. In that case, the equations for goods prices and capital costs are satisfied with prices $p_n = 1$ for all $n \in N + C$ and $R_n = 1$ for all $n \in K$.

C.5 Linearized solution

Here are the linearized equations. It is conducted around a balanced growth path with no initial taxes.

Wealth growth conditional on survival:

$$dg_{\omega,c} \equiv \gamma \times \left[dr + 2\left(\frac{\gamma+1}{2}\right) dS_c - d\tau_c^k \left(r + 2\left(\frac{\gamma+1}{2}\right) S_c^2 \right) \right]$$

Ratio of total wealth to human wealth:

$$d\chi_c = \frac{d\nu_c}{\nu_c + g - g_\omega^c} + \chi_c \frac{dg_{\omega,c} - d\nu_c}{\nu_c + g - g_\omega^c}$$

Desired financial wealth of households in country c:

$$dW_c = \frac{\sum_{f \in F_c} \lambda_f}{r + \nu_c - g_A} \left[\frac{d\left(\sum_{f \in F_c} \lambda_f\right)}{\sum_{f \in F_c} \lambda_f} + d\chi_c + \frac{r\, d\tau_c^k}{r + \nu_c - g_A} - \frac{dr}{r + \nu_c - g_A} + dT_c \right]$$

Tax revenues relative to labor income:

$$dT_c = \frac{d\tau_c^k \left[rb_c + \sum_{i \in K_c} \frac{r_i}{r_i + \delta_i} \lambda_i \right] + \sum_{i \in N_c} \frac{\lambda_i}{\tilde{\mu}_i^2} d\tilde{\mu}_i + \sum_{j \in N_c + K_c} \sum_{i \in N+K} \left[dt_{j,i} \lambda_j \Omega_{j,i} \right]}{\sum_{f \in F_c} \lambda_f}$$

Country consumption:

$$d\Phi_c = \sum_{f \in F_c} d\lambda_f + dT_c \sum_{f \in F_c} \lambda_f + \sum_{i \in K_c} \left[d\lambda_i \left(1 - \frac{1}{\mu_i} \right) + \lambda_i \frac{d \log \mu_i}{\mu_i} \right] + d[(r-g)b_c]$$

Goods prices:

$$d \log p_n = d \log \frac{\tilde{\mu}_n}{A_n} + \sum_{j \in N} \tilde{\Omega}_{n,j} [dt_{nj} + d \log p_j] + \sum_{m \in K} \tilde{\Omega}_{n,m} d \log R_m + \sum_{f \in F_c} \tilde{\Omega}_{n,f} d \log w_f$$

Capital user costs:

$$d \log R_k = d \log \left(\frac{\mu_{k'}}{A_{k'}(\delta_{k'} + g)} \right) + \sum_{j \in N} \tilde{\Omega}_{k,j} [d \log p_j + dt_{n,j}] \quad k \in K$$

Effective markups:

$$d\mu_m = \left[\frac{dr_m + d\delta_m}{g + \delta_m} - \mu_m \frac{d\delta_m}{g + \delta_m} \right] \mathbf{1}(m \in K) + d\tilde{\mu}_m \mathbf{1}(m \notin K)$$

Goods, labor, and capital services demand:

$$d\lambda_i = \sum_{c' \in C} [d\Phi_{c'} \Omega_{c',i} + \Phi_{c'} (d\Omega_{c',i} - dt_{c',i} \Omega_{c',i})] + \sum_{j \in N+K} \left[d\lambda_j \Omega_{j,i} + \lambda_j (d\Omega_{j,i} - dt_{ji} \Omega_{j,i}) \right]$$

Change in cost shares:

$$d\tilde{\Omega}_{ij} = (1 - \theta_i) \tilde{\Omega}_{ij} \left[d \log p_j + dt_{ij} - \sum_k \tilde{\Omega}_{ik} (d \log p_k + dt_{ik}) \right]$$

$$d\Omega_{ij} = -d \log \mu_i \Omega_{ij} + \mu_i^{-1} d\tilde{\Omega}_{ij}$$

Labor market clearing:

$$d \log w_f = d \log \lambda_f \qquad f \in F$$

Asset market clearing:

$$\sum_{i \in K_c} \left[d\sigma_i \frac{\lambda_i}{r_i + \delta_i} + \sigma_i d \left(\frac{\lambda_i}{r_i + \delta_i} \right) \right] = \gamma W_c \frac{\chi_c}{\chi_c - 1} S_c \times \left[d \log W_c + d \log \left(\frac{\chi_c}{\chi_c - 1} \right) + d \log S_c \right]$$

$$\sum_{i \in K} \left[\frac{d\lambda_i}{r_i + \delta_i} - \frac{\lambda_i}{r_i + \delta_i} \frac{dr_i + d\delta_i}{r_i + \delta_i} \right] = \sum_{c \in C} dW_c$$

No arbitrage within country:

$$dr_i = dr + d\sigma_i S_c + \sigma_i dS_c + (r_i - r) d\tau_k, \qquad i \in K_c$$

Net foreign assets:

$$db_c = dW_c - \sum_{i \in K_c} \left[\frac{d\lambda_i}{r_i + \delta_i} - \frac{\lambda_i}{r_i + \delta_i} \frac{dr_i + d\delta_i}{r_i + \delta_i} \right]$$

C.6 Details on calibration

The main part of the calibration is using the World Input Output Database augmented with investment flow data from Ding (2022) to calibrate the cost-share matrix $\tilde{\Omega}$, the capital wedges μ, and thus the revenue-share matrix Ω.

We calibrate $\tilde{\Omega}$ using two primary data sources: the World Input Output Database (WIOD) (and the associated Socio-Economic Accounts) for consumption spending, intermediate input use, and labor inputs, and investment flow data from Ding (2022) for investment spending. To ensure consistency, we aggregate WIOD sectors to match the sectoral classification in Ding (2022), yielding 27 sectors.

The matrix $\tilde{\Omega}$ consists of submatrices giving the cost shares for consumption goods $\tilde{\Omega}_{C,N}$ in terms of perishable goods from different countries, cost shares of perishable goods $\tilde{\Omega}_{C,N}, \tilde{\Omega}_{C,K}, \tilde{\Omega}_{C,F}$ in terms of intermediate inputs, capital goods, and labor inputs, and cost shares of investment goods $\tilde{\Omega}_{K,N}$. All other submatrices are zero.

Consumption shares $\tilde{\Omega}_{C,N}$. Since households only consume perishable goods, the consumption share submatrix $\tilde{\Omega}_{C,N}$ has dimensions $C \times N$, with each row c containing the shares of country c's consumption spending across all country-industry pairs (c', j') that produce perishable goods. The elements of this matrix are given by

$$\tilde{\Omega}_{c,c'j'} = \begin{cases} \frac{X^C_{c,c'j'}}{X^C_c} & \text{if } j' \in N \\ 0 & \text{otherwise} \end{cases}$$

where $X^C_{c,c'j'}$ is the dollar value spent by country c on consumption goods from country c', industry j', and $X^C_c = \sum_{c'j'} X^C_{c,c'j'}$ is aggregate consumption in country c. We define consumption as the sum of household, government, and non-profit consumption.

Intermediate input shares $\tilde{\Omega}_{N,N}$. For the submatrix $\tilde{\Omega}_{N,N}$ giving intermediate input shares of perishable goods on other perishable goods (c, i), we have

$$\tilde{\Omega}_{ci,c'j'} = \frac{X_{ci,c'j'}}{GO_{ci}} \quad c' \in C \quad j' \in N$$

The input coefficients on origin-industry pairs is the intermediate input spending taken from the WIOD on that pair, divided by gross output.

Capital cost shares $\tilde{\Omega}_{N,K}$. For capital cost shares $\tilde{\Omega}_{N,K}$, we have

$$\tilde{\Omega}_{ci,c'k'} = \begin{cases} \frac{GOS_{ci}}{GO_{ci}} & \text{if } c' \in C \text{ and } k' = \hat{k}(i) \\ 0 & \text{otherwise} \end{cases}$$

where $\hat{k}(i)$ denotes the capital good associated with industry i, and $GOS_{ci} \equiv GO_{ci} - \sum_{c' \in C, j' \in N} X_{ci,c'j'} - \sum_{f \in F} X^L_{ci,f}$ is the gross operating surplus. This expression states that the capital cost share for (c, i) is only positive for the capital good associated with that industry, which is $(c, k(i))$. For this good, the cost share is the ratio of gross operating surplus of the industry relative to its gross output. This assumption captures that we assume no pure profits beyond capital rents, which means that the full operating surplus reflects rental payments on capital.

Labor cost shares $\tilde{\Omega}_{N,F}$. For labor, we have

$$\tilde{\Omega}_{ci,c'f} = \begin{cases} \frac{X^L_{ci,f}}{GO_{ci}} & c' \in C, f \in N^c, \\ 0 & \text{Otherwise.} \end{cases}$$

The condition $c' \in C, f \in N^c$ captures that countries only use labor inputs from their own country. For these labor inputs, the share are given by labor compensation of (c, i) on labor type f taken from the Socio-Economic Accounts, with labor types being low-skilled, medium-skilled, and high-skilled. For these, cost shares are defined relative to gross output.

Investment cost shares $\tilde{\Omega}_{K,N}$. The submatrix $\tilde{\Omega}_{K,N}$ gives the cost shares of different investment goods (c, k) on different inputs (c, i).

$$\tilde{\Omega}_{ck,c'i'} = \frac{X^{Inv}_{c\hat{i}(k),c'i'}}{X^{Inv}_{c\hat{i}(k)}},$$

where $\hat{i}(k)$ is the regular good associated with investment good k, $X^{Inv}_{c\hat{i}(k),c'i'}$ is the investment spending of industry $(c, \hat{i}(k))$ on (c', i') in the data of Ding (2022), and $X^{Inv}_{c\hat{i}(k)} = \sum_{c'i'} X^{Inv}_{c\hat{i}(k),c'i'}$ is the total investment of $(c, \hat{i}(k))$ in the database.

Aggregating $\tilde{\Omega}$ over time. The WIOD data is annual while the data in Ding (2022) is only from 1997. For all submatrices but $\tilde{\Omega}_{K,N}$, we take averages from 1995 to 2009. For $\tilde{\Omega}_{K,N}$, we take the 1997 values.

Calibrating μ. For each industry-country pair, we calculate $\mu_{c,\hat{k}(i)}$ as the ratio of gross fixed capital formation to gross output in industry (c, i), averaged over 1995-2009.

Depreciation rates We map BEA industry-specific depreciation rates to WIOD sectors. For each sector and year, we calculate the depreciation rate as the total value of depreciation divided by the total value of the capital stock of the sector. The resulting depreciation rate δ_i for $i \in N$ is used for all countries.

Revenue-based input output matrix. From $\tilde{\Omega}$ and μ, we obtain the revenue-based input output matrix from $\Omega_{ij} = \frac{1}{\mu_i}\tilde{\Omega}_{ij}$. This matrix is identical to $\tilde{\Omega}$, apart from the rows associated with investment goods being deflated by $\frac{r_k+\delta_k}{g+\delta_k}$. In particular, this means that the input-output matrix rows associated with investment generally sum to less than 1, reflecting that not all rental payments to capital goods end up as investment good spending.

Revenue shares. From Ω, we define the total requirement matrix $\Psi = (I - \Omega)^{-1} = I + \Omega + \Omega^2 + \cdots$, with Ψ_{ij} capturing the share of spending on i that ends up in j, directly and indirectly through the input-output network. The revenue of each i relative to world consumption satisfies

$$\lambda_i = \sum_{c' \in C} \Phi_{c'} \Psi_{c',i}, \tag{45}$$

where $\Phi_{c'}$ is the share of world consumption in country c', where i indexes perishable goods, capital goods, and labor.

Consumption weights by country. The previous results express λ_i up to consumption shares Φ_c. To solve for these shares, we note that they need to satisfy

$$\Phi_c = \sum_{f \in F_c} \lambda_f + \sum_{k \in K_c} \lambda_k \left(1 - \frac{1}{\mu_k}\right) + (r - g) b_c, \tag{46}$$

where b_c is the ratio of net foreign assets (bonds in our model) relative to world consumption. This equation states that consumption in a country equals its factor income plus net income from domestic capital plus earnings on net foreign assets. Since labor and net capital income can be expressed in terms of Φ_c using (45), and $\sum_c \Phi_c = 1$, equation (46) can be solved for Φ_c as a function of net factor payments $(r - g)b_c$. To calibrate the latter, we use our earlier calibration for r and g, and set b_c equal to net foreign asset positions relative to global consumption, with net foreign asset positions taken from the External Wealth of Nations Database, and global consumption from WIOD.[41]

D Appendix to Section 5

[41] We do not calibrate Φ_c according to country-level consumptions from WIOD because this would result in an inconsistency between trade balance and net foreign assets. Nevertheless, the Φ_c we do calibrate to are similar to those implied by the WIOD. See footnote 33 for more information.

Table 8: Decomposition of consumption changes according to Proposition 6 for all regions

Country	$d \log C_c$	**Harberger**	**Terms of trade**	**Δ Current account**
United States	-0.106	-0.098	-0.007	-0.001
Canada	-0.314	-0.271	-0.041	-0.002
China	-0.105	-0.120	0.011	0.004
United Kingdom	-0.229	-0.185	-0.044	-0.000
India	-0.216	-0.212	-0.002	-0.002
Japan	-0.111	-0.110	-0.005	0.003
Mexico	-0.669	-0.672	0.010	-0.008
European Union	-0.083	-0.087	0.006	-0.001
Rest of the World	-0.185	-0.215	0.027	0.003
Global	-0.137	-0.137	0.000	-0.000