NBER WORKING PAPER SERIES

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Michael W. L. Elsby
Axel Gottfries
Pawel M. Krolikowski
Gary Solon

Working Paper 33149
http://www.nber.org/papers/w33149

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
November 2024

We thank Mark Aguiar, Susanto Basu, John Grigsby, Joe Hazell, Rafael Lopes de Melo, Ben Moll, John Moore, Richard Rogerson, and Jonathan Thomas, as well as seminar participants at numerous institutions for helpful comments. All errors are our own. The views expressed are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Federal Reserve Bank of Cleveland, the Federal Reserve System, or the National Bureau of Economic Research.

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November 2024
JEL No. E24, E3, J3, J6

**ABSTRACT**

We present a model in which efficient long-term employment relationships are sustained by wage adjustments prompted by productivity shocks and outside job offers. These wage adjustments occur only sporadically, due to the presence of renegotiation costs. The model is amenable to analytical solution, yielding new insights for several labor market phenomena, including: (1) key features of empirical distributions of changes in pay among job stayers; (2) a near-"memorylessness" property in wage dynamics whereby hiring wages have only limited influence on later wages and allocation decisions; and (3) a crucial role for recruitment and retention bonuses in sustaining efficient employment relationships.

Michael W. L. Elsby
University of Edinburgh
School of Economics
31 Buccleuch Place
Edinburgh
EH8 9JT
United Kingdom
Mike.Elsby@ed.ac.uk

Axel Gottfries
University of Edinburgh
School of Economics
31 Buccleuch Place
Edinburgh
EH8 9JT
United Kingdom
axel.gottfries@gmail.com

Pawel M. Krolikowski
Federal Reserve Bank of Cleveland
1455 E 6th St.
Cleveland, OH 44114
pawelk@umich.edu

Gary Solon
University of Michigan
and NBER
gary.r.solon@gmail.com

A supplementary appendix is available at http://www.nber.org/data-appendix/w33149

This paper develops and analyzes a less restrictive and more realistic model of wage adjustment in long-term employment relationships. The relevant literature began with Becker's (1962) classic contribution, which articulated the central role of specific human capital (broadly defined[1]) in the durability of employment relationships. On the question of how wages evolve after the initial commitment to the relationship, Becker noted that preservation of efficient[2] relationships requires that, in present value terms, both the worker and employer receive positive shares of the *ex post* rent, but otherwise Becker treated the details of wage setting as indeterminate.

As wages are the price variable in the labor market, punting on how wages are set should be an unsatisfactory state of affairs for any economist that studies the labor market. Accordingly, over the six decades since Becker's work, several prominent contributions to the literature have imposed additional structure to close the model. With few exceptions, these have followed Becker in the presumption that workers and employers endeavor to preserve efficient relationships,[3] and typically have adopted one of two polar opposite approaches to wage adjustment. One approach (exemplified by Mortensen and Pissarides 1994) assumes continual wage renegotiation to maintain Nash-bargained shares of the rent. However, both casual observation and a large empirical literature on wage adjustment indicate that wages are not really renegotiated as frequently as this assumption suggests. The other approach (exemplified by MacLeod and Malcomson 1993) assumes that wages are adjusted only when participation constraints bind. Given the substantial specific human capital and associated *ex post* rent that make long-term employment relationships so prevalent, this assumption suggests that wage adjustment would occur only rarely. But again, casual observation and the evidence on wage adjustment indicate that, although wages are not adjusted continually, they are adjusted with considerable frequency.

In accordance with this reality, we develop a model that nests both of the above polar opposites as special cases, but permits realistically intermediate cases of sporadic wage

[1] Although Becker focused largely on on-the-job training, he emphasized (especially on pp. 17-18) that specific human capital arises from many sources, including search and hiring costs, employers' private information about workers' abilities, and idiosyncratic match quality.

[2] For brevity, and reflecting our focus, throughout we use the term "efficient" to refer to *bilateral efficiency* of employment relationships (as opposed to the efficiency of equilibrium labor market allocations).

[3] This presumption of bilateral efficiency has been influential in the macroeconomics literature since Barro (1977), as well as the literatures on contracting (Thomas and Worrall 1988; MacLeod and Malcomson 1993), search and matching (Mortensen and Pissarides 1994), on-the-job search (Postel-Vinay and Robin 2002; Dey and Flinn 2005; Cahuc et al. 2006), and the large body of research these works have inspired.

adjustment prompted both by persistent idiosyncratic shocks to match productivity and the arrival of outside job offers due to on-the-job search. Following Mortensen and Pissarides, we assume that the employment relationship begins with an understanding about the intended rent-sharing proportions, but we embellish the model by introducing renegotiation costs. Because of these costs, wages are adjusted not continually, but intermittently when the rent shares get far enough out of line to motivate a renegotiation. Combined with the availability of costless wage adjustment by mutual consent of firm and worker, the model implies that all separations—both to unemployment, and from job to job—are bilaterally efficient.

The result is a parsimonious benchmark model of wage adjustment in efficient long-term employment relationships. An innovation of the model is its analytical tractability. Wages follow a *drunken walk*, adjusting minimally (by mutual consent) such that neither firm nor worker has a unilateral incentive to trigger a (costly) renegotiation. As a result, the decision of, say, the firm to initiate a renegotiation in the present depends on all the state-contingent renegotiation decisions of both firm and worker *in the future*. Our framework allows a novel analytical characterization of the division of match surplus between firm and worker, and thereby wage adjustment.

We show that our extended model succeeds in providing a coherent theoretical foundation for key features of the empirical distribution of year-to-year changes in job stayers' wages—a mass point at zero wage change, and a greater incidence of wage raises than wage cuts. By contrast, the polar extreme of the model in which wages are adjusted only as participation constraints bind implies wage changes that are far too rare relative to the data.[4] As another check on the approximate verisimilitude of our model, we show that the model also gives a good fit to the nonmonotonic empirical pattern of duration dependence in the job separation hazard rate noted by Farber (1999).

We recognize that our model's loosening of the restrictions in previous models comes at the cost of greater complexity. Is it worth it? It is if the model enables valuable new insights, and we will show that it does. We already have noted the model's success in accounting for the longevity of jobs and real-world wage adjustments. In addition, we

[4] This finding is in line with previous related work in which wages are assumed to be renegotiated only when participation constraints bind: Postel-Vinay and Turon (2010) find that year-to-year wage freezes account for 60 percent of their model-generated observations. Our model extends the work of Postel-Vinay and Turon by accommodating persistent (rather than i.i.d.) idiosyncratic shocks to match productivity. As they note, the latter has necessitated the use of numerical methods in other related work (Yamaguchi 2010; Lise et al. 2016). By contrast, our approach allows the option value of on-the-job search to be inferred analytically, facilitating a characterization of worker and firm values.

apply the model to reassess two high-impact conclusions from the macro/labor literature.

First, we reconsider Hall's (2005) influential result that rigid hiring wages could account for realistically large cyclical fluctuations in unemployment. Our model nests Hall's model as a special case that imposes the very strong restrictions of no shocks to match-specific productivity, no outside offers, and hence no impetus to wage renegotiation. In our model, which relaxes those unrealistic assumptions, subsequent (realistic) wage adjustments truncate the legacy of the hiring wage and thereby unravel Hall's influential result.[5] This has provocative implications for future work on cyclical unemployment fluctuations, suggesting they instead have their origin either in volatile labor demand (echoing the conclusions of Bils et al. 2023), or deviations from the efficient benchmark that has pervaded the literature—for example, due to asymmetric information (Hall and Lazear 1984; Perry and Solon 1985), costs associated with cutting wages (Bewley 1999; Bertheau et al. 2023; Davis and Krolikowski 2023), or impediments to matching outside offers (Mortensen 2003, section 5.1).[6]

Second, we reconsider the influential conjecture of Grigsby et al. (2021) that, because non-base pay (such as bonuses and commissions) is transitory and accounts for only a small fraction of overall compensation, it is unlikely to be allocatively important, and therefore attention should focus largely on base pay. Our model provides a natural motive for bonus pay, since it implies a limit to how much value firms credibly can deliver to workers through increases in base pay. Moreover, as in the empirical results of Grigsby et al., changes in total compensation are much more flexible than base pay changes in the model. But, echoing the old economic insight that marginal variation can be allocatively consequential, we illustrate that, in the context of our model, marginal adjustments afforded by recruitment and retention bonuses, despite being transitory and only a small proportion of overall compensation, are quantitatively important for maintaining efficient employment relationships.

In the next subsection, we provide further discussion of how our model compares to and extends beyond previous related analyses. Section 1 describes our model in detail; shows how the model can be embedded in a model of aggregate labor market equilibrium; and discusses how the model delivers a richer description of the "user cost of labor"

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[5] This echoes a conjecture of Mortensen and Nagypal (2007), who write of Hall's model that, "to maintain that the rigid wage is jointly rational, only small shocks can affect the employment relationship [...]. This assumption is greatly at odds with the extent of gross flows in the labor market that reflect the importance of idiosyncratic variability."

[6] For example, while evidence on offer matching is limited, the evidence available suggests only a modest propensity (Brown and Medoff 1996; Bewley 1999; Di Addario et al. 2023).

introduced by Kudlyak (2014) and revisited recently by Doniger (2021) and Bils et al. (2023). Section 2's quantitative exploration demonstrates that our model provides a good account of well-established facts about job duration and the distribution of year-to-year wage changes for job stayers. It also draws out the model's novel implications for reassessing the allocative roles of both hiring wages and non-base pay such as recruitment and retention bonuses. Section 3 shows how the model can be extended to an environment with variable non-zero inflation. Section 4 concludes with a summary and a discussion of the model's lessons for future work on the macroeconomics of the labor market.

**Related literature**. In addition to the work already cited, our paper relates to the following strands of literature.

First, our model echoes earlier work on long-term contracting. Most closely related is the model of MacLeod and Malcomson (1993) which emphasizes the role of renegotiations by mutual consent in sustaining efficient long-term relationships. Holden (1994) develops related ideas in a union setting. Our contribution is to provide a quantitative model in which wage adjustments are prompted both by idiosyncratic shocks and on-the-job search, and the implied adjustment thresholds can be characterized analytically.

More distant to our approach are models of dynamic contracting that emphasize insurance motives, starting with the classic contribution of Thomas and Worrall (1988). The optimal contract in their setting takes the form of the drunken walk described above, but the motivation for it originates instead from risk-sharing. Rudanko (2009) embeds these insights into a model of equilibrium unemployment dynamics with aggregate shocks. Balke and Lamadon (2022) extend these ideas to a model of directed search on the job in which worker search effort is noncontractible. Although their model generates rich wage dynamics, it does not exhibit inaction in wage adjustments, a central object of our study.

A second strand of related literature builds on Hall's (2005) early insights on the potential allocative effects of stickiness in hiring wages. Gertler and Trigari (2009) embed these ideas into a model of time-dependent staggered wage adjustment, and find that it gives rise to substantial unemployment fluctuations. By contrast, our focus is to study state-dependent wage adjustment prompted by plausible idiosyncratic shocks; we find instead that this does much to mute unemployment fluctuations.

In tandem, a growing empirical literature has sought to infer the extent of hiring wage stickiness from microdata (Pissarides 2009; Martins et al. 2012; Haefke et al. 2013; Gertler et al. 2020; Hazell and Taska 2020). A key message of our model, however, is that the degree of hiring wage stickiness may be less consequential for job creation, and thereby unemployment, in the presence of efficient *ex post* wage adjustments.

A third strand of related literature comprises recent work on nominal wage rigidity. Early empirical contributions by McLaughlin (1994), Card and Hyslop (1996), Kahn (1997), and Altonji and Devereux (1999) documented that distributions of year-to-year nominal wage changes for job stayers exhibit many instances of no wage change, more instances of wage increase, and a lower, though nontrivial, incidence of wage cuts. Since then, several researchers have sought to refine these survey-data based measures by using more-accurate administrative microdata (see the survey by Elsby and Solon 2019, as well as Jardim et al. 2019, Kurmann and McEntarfer 2019, and Grigsby et al. 2021). These latter measures are used to inform our quantitative analyses in what follows.

Our work is also related to a theoretical literature inspired by these facts. A common theme in these works is that they explore the allocative consequences of specific frictions that *impede* efficient wage adjustments. Elsby (2009) studies the wage-setting problem of a firm assumed to incur costs from wage cuts. Benigno and Ricci (2011) and Daly and Hobijn (2014) study New Keynesian models with spot labor markets in which wage cuts are assumed (stochastically) infeasible. Dupraz et al. (2022) extend these themes to a search and matching model with durable jobs. Fukui (2020) and Gottfries (2021) study the allocative role of rigid wages in models with on-the-job search in which firms are unable to respond to outside offers. Finally, like us, Blanco et al. (2022) study an environment with idiosyncratic shocks and long-term jobs, but their focus is instead on the role of constraints to wage adjustments in generating inefficient separations.

By contrast, the impetus behind our study is instead to evaluate a parsimonious model in which wages *can* be adjusted when necessary to sustain efficient relationships. We see the two approaches as complementary: One interpretation of our finding that the latter view cannot account for unemployment fluctuations is that it motivates further study of frictions that impede efficient wage adjustments. We further hope that progress to this end will, in turn, be aided by the analytical methods we develop in this paper.

Finally, our study relates to recent work that has begun to explore the macroeconomic implications of flexibility in components of pay. Broer et al. (2023) study labor contracts which specify a base wage, plus "marginal" wages that vary with *ex post* hours worked. They show that the availability of marginal wages moderates labor market fluctuations relative to a fully rigid base wage, but that rigidity of the contract with respect to shocks amplifies labor market fluctuations relative to a neoclassical benchmark. Gaur et al. (2022) study incentive pay motivated by the presence of worker moral hazard. Optimal incentive pay in their model offsets changes in workers' incentives to supply effort, and so incentive pay flexibility is *irrelevant* for labor fluctuations, by the envelope theorem. Along with our findings on the allocative role of recruitment and retention compensation, these works underscore the importance of the microeconomic origins of non-base pay flexibility in shaping macroeconomic outcomes.

# 1. A model of wage adjustment

In this section, we devise a new model of wage determination that delivers intermittent wage adjustment as an equilibrium outcome. The model nests a class of canonical models that share the property that wages adjust to preserve bilateral efficiency. The model further admits an analytical characterization of an array of outcomes, in particular the endogenous thresholds for wage adjustment. The section closes by demonstrating how the wage protocol can be inserted straightforwardly into canonical models of aggregate labor market equilibrium, allowing an analysis of its implications for unemployment dynamics.

## 1.1 Environment

Time is continuous and the horizon infinite. The labor market is comprised by workers and firms, with production organized in worker-firm matches.

Workers are risk neutral, and occupy one of two labor force states: unemployment and employment. While unemployed, workers receive a flow payoff $b$, and realize job offers at rate $\lambda$. While employed, workers receive a flow wage $w$, and realize job offers at rate $s\lambda$; $s$ thus indexes the relative efficiency of on-the-job search. For now we treat the arrival rate $\lambda$ as a parameter but endogenize it in section 1.5.

A new worker-firm match generates an initial flow product for the firm $x_0$. Thereafter, the idiosyncratic flow product $x$ evolves according to a geometric Brownian motion,

$$\mathrm{d}x = \mu x \mathrm{d}t + \sigma x \mathrm{d}z, \tag{1}$$

where $\mathrm{d}z$ is the increment to a standard Brownian motion. Matches are destroyed exogenously at rate $\delta$, and endogenously subject to the free disposal of worker and firm.

We impose three further assumptions. First, to ensure existence of a stationary distribution of productivity $x$ across matches, we assume $\mu < \delta$.

Second, to economize on notation, and in the interest of parsimony, we confine ourselves in the main text to the case in which $\mu = \sigma^2/2$, so that $\ln x$ is a driftless Brownian motion. The Appendix provides an extended version of the model with general $\mu$, thereby encompassing the possibility of specific human capital accumulation; our main results, however, are essentially unchanged in this case.

Third, we assume zero inflation, so that payoffs can be read as both real and nominal. In section 3, we show how to modify the baseline model to accommodate inflation.

Productivity $x$ is realized at the beginning of each $\mathrm{d}t$ period, after which firm and worker may choose to separate. If the match continues, outside job offers are realized. Wage determination—the focus of our analysis—then proceeds as follows.

In the absence of an outside offer, wage adjustment may occur through two channels. First, the wage can be adjusted, at zero cost, subject to mutual consent of firm and worker. Second, either party may initiate a (re)negotiation of the wage. Importantly, the latter is not always costless: If the employee (firm) *unilaterally* initiates a renegotiation, there is an initial probability $\Delta_e$ ($\Delta_f$) that the relationship breaks down and is destroyed.[7] By contrast, if both worker and firm initiate a negotiation, there is no breakdown risk. Absent breakdown, wages are then determined according to a Nash bargain over the total match surplus, with worker bargaining power $\beta \in (0,1)$. If neither channel of wage adjustment is exercised, the wage remains unchanged.[8]

In the presence of an outside offer, the worker chooses whether to exercise the offer. If the offer is not exercised, wage determination proceeds as above. If the offer is exercised, wages are determined according to a variation of the bargaining protocol proposed by Dey and Flinn (2005), and Cahuc et al. (2006). As there, the worker can use the total surplus of the unsuccessful match as their outside option when bargaining. If the outside match is more productive, then both the outside firm and the worker initiate a renegotiation, and the worker captures an additional share $\beta$ of the difference in surplus between the two matches. If, however, the outside match is less productive, the bargain is initiated *unilaterally* by the worker, and breakdown risk again applies as above. An implication is that the worker quits if and only if the outside firm has a greater match surplus.

Upon completion of wage setting and worker turnover decisions, production takes place and wages are paid in the relevant firm, and the period concludes.

The environment we consider provides a minimal set of ingredients to capture the stylized facts of wage adjustment in a framework with efficient long-term jobs:[9] Worker-

[7] This has the tractable implication that the match either ends in the event breakdown occurs, or continues with its surplus unimpaired. As will become clear, an implication is that separations are bilaterally efficient.

[8] That the wage remains unchanged in the absence of renegotiation (as opposed to some other default path) is often justified by invoking Malcomson's (1997) argument that it is the outcome of renegotiation by *mutual consent* (see, e.g., Hall 2005; Postel-Vinay and Turon 2010). Malcomson, in turn, argues that renegotiation by mutual consent has several appealing properties: In a European context, it is often enshrined in law; in a US context, it is a natural way to make enforceable contracts that avoid possible charges of a breach in the presence of exceptions to employment *at will* (see, e.g., Krueger 1991; Autor et al. 2006); and, echoing our motivation, it naturally implies that wages change only intermittently, as in the data.

[9] By the same token, the model of course abstracts from additional ingredients in the interest of parsimony. For example, incorporating risk aversion, although realistic, would complicate the problem by rendering

firm matches are subject to persistent idiosyncratic shocks and the arrival of outside offers that together drive changes in wages, while costs of renegotiating the wage in the form of breakdown risk shape the frequency of these wage changes.

A key innovation of the model is thus breakdown risk, as captured by $\Delta_e$ and $\Delta_f$. Breakdown risk appeals to us as a tractable way of modeling renegotiation costs, but we also think of it as a proxy for other renegotiation costs, such as time, effort, and stress, or indeed any resources lost when one party forces a renegotiation against the other party's will. These renegotiation costs play an important role in the model, by accommodating intermediate degrees of wage adjustment between the two polar cases studied in the literature, which we later find to be essential for the model to generate empirically realistic wage adjustments.

To anticipate this, note first that the presence of breakdown risk nests canonical approaches to wage determination. Models that invoke *ex post* wage bargaining correspond to the case in which unilateral threats to renegotiate are fully credible, $\Delta_e = \Delta_f = 0$. Thus, the search and matching models of Pissarides (1985) and Mortensen and Pissarides (1994) are accommodated, respectively, by the special cases $\sigma = s = \Delta_e = \Delta_f = 0$, and $\sigma > 0 = s = \Delta_e = \Delta_f$. Alternatively, models in which renegotiation of an existing agreement may occur only by mutual consent of both firm and worker correspond to the case in which unilateral threats to initiate a renegotiation are not credible, $\Delta_e = \Delta_f = 1$ (MacLeod and Malcomson 1993). Thus, Hall's (2005) model of entry wage rigidity is accommodated by the special case in which $\sigma = s = 0$ and $\Delta_e = \Delta_f = 1$. Finally, the environment nests canonical models in which renegotiation is prompted by the receipt of outside offers generated by on-the-job search. The sequential auctions model of Postel-Vinay and Robin (2002) corresponds to the special case $s > 0 = \sigma = \beta$ and $\Delta_f = 1$, whereas the model of Dey and Flinn (2005) and Cahuc et al. (2006) corresponds to $s > 0 = \sigma = \Delta_e$ and $\Delta_f = 1$.

A second useful implication of breakdown risk is that it extends these canonical models to accommodate intermediate degrees of wage adjustment in the presence of idiosyncratic shocks. Central distinctions between canonical approaches are polar assumptions on the credibility of unilateral threats to renegotiate, and the presence of idiosyncratic shocks. Our generalized environment allows the credibility of these threats to be varied flexibly via the renegotiation costs $\Delta_e$ and $\Delta_f$. Combined with idiosyncratic shocks to productivity $x$, this gives rise to a first channel of sporadic equilibrium wage adjustment.

To see how, consider the case in which an outside offer is absent. Denoting the match

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utility nontransferable, without an obvious gain in explanatory power. For similar reasons, we also abstract from general human capital accumulation, as well as permanent differences in firm productivity.

surplus by $S$, the firm (respectively, worker) can guarantee an expected surplus equal to $(1-\beta)(1-\Delta_f)S$ (respectively, $\beta(1-\Delta_e)S$) by unilaterally triggering a renegotiation. The latter is costly to the match, however, as it risks breakdown. Instead, (costly) off-equilibrium unilateral threats sustain (costless) equilibrium wage adjustment by mutual consent. Suppose that, upon realization of $x$, the worker's surplus under the existing agreement falls below $\beta(1-\Delta_e)S$. Rather than countenance the risk of breakdown $\Delta_e$, the firm will accede to a wage increase by mutual consent that restores the worker surplus to $\beta(1-\Delta_e)S$. Symmetrically, whenever the firm's surplus under the existing agreement falls below $(1-\beta)(1-\Delta_f)S$, the worker will agree to a wage cut by mutual consent that restores the firm's surplus to $(1-\beta)(1-\Delta_f)S$. Otherwise, both worker and firm surpluses lie above their respective thresholds, neither party has a credible unilateral threat to initiate a renegotiation, and the wage remains unchanged.

The case in which an outside offer is present is more straightforward. Since the employee always has the option not to exercise an outside offer, the only case in which the bargain is initiated unilaterally is when the employee does so. This will arise in the case in which the outside firm knows that the worker will not quit, and the current firm knows renegotiation will only raise the worker's compensation. In this case, the current firm accedes to an increase in compensation, by mutual consent, that renders the employee indifferent to unilateral initiation of a bargain. Per the bargaining protocol of Dey and Flinn (2005) and Cahuc et al. (2006), the worker receives $S(x_0)+\beta(1-\Delta_e)[S(x)-S(x_0)]$. Otherwise, in the case in which the outside firm is more productive, both the outside firm and the worker initiate renegotiation, and the worker receives $S(x)+\beta[S(x_0)-S(x)]$.

The protocol thus links the frequency of wage adjustment both to the credibility of unilateral threats to renegotiate (the $\Delta$s), and the arrival of outside job offers (at rate $s\lambda$).

A final virtue of the model is that it is amenable to analytical solution. Key to tractability is the observation that the wage protocol preserves bilateral efficiency. The availability of costless wage adjustment by mutual consent ensures that the costs of unilaterally triggered renegotiation are never realized on the equilibrium path, and that matches separate only if the total match surplus $S$ is extinguished. A consequence is that solution of the model can be decoupled into an optimal stopping problem that determines the durability of matches, and a novel wage determination problem, the solution of which is a key contribution of this paper. We now describe each of these in turn.

## 1.2 Surplus, separations and tenure

Bilateral efficiency implies that total match surplus is independent of wage determination.

We thus denote the total surplus of a match with current productivity $x$ by $S(x)$. Separations then occur whenever the match surplus reaches zero.

Define a separation threshold $x_l$. Then, for all $x \in (x_l, \infty)$, $S(x)$ satisfies

$$(r+\delta)S(x) = x - rU + \beta s\lambda \max\{S(x_0) - S(x), 0\} + \mu x S_x + \frac{1}{2}\sigma^2 x^2 S_{xx}. \tag{2}$$

The flow surplus of the match is given by flow output $x$, less the annuitized value of unemployment to the worker $rU$. The match then faces capital gains from two sources. First, at rate $s\lambda$ the worker receives an offer from an outside firm of productivity $x_0$. Since, upon job-to-job transition, the worker captures the entirety of the surplus of her previous match, $S(x)$, plus a share $\beta$ of the difference, $S(x_0) - S(x)$, on-the-job search is surplus enhancing. Second, there are idiosyncratic shocks to match productivity, which evolve according to the stochastic law of motion (1). Ito's lemma implies the form in (2).

The definition of the separation threshold $x_l$, and optimality, imply the following value-matching and smooth-pasting conditions,

$$S(x_l) \equiv 0, \quad S'(x_l) = 0, \quad S(x_0^-) = S(x_0^+), \quad \text{and} \quad S'(x_0^-) = S'(x_0^+). \tag{3}$$

Applying these boundary conditions to (2) yields a solution for the total surplus.

**Proposition 1** *(i) The total match surplus is given by*

$$S(x) = \begin{cases} \dfrac{x}{r+\delta-\mu+\beta s\lambda} - \dfrac{rU - \beta s\lambda S(x_0)}{r+\delta+\beta s\lambda} + \mathcal{S}_1 x^{-\sqrt{\frac{r+\delta+\beta s\lambda}{\mu}}} + \mathcal{S}_2 x^{\sqrt{\frac{r+\delta+\beta s\lambda}{\mu}}} & \text{if} \quad x < x_0, \\ \dfrac{x}{r+\delta-\mu} - \dfrac{rU}{r+\delta} + S_1 x^{-\sqrt{\frac{r+\delta}{\mu}}} & \text{if} \quad x \geq x_0, \end{cases} \tag{4}$$

*for all $x \in (x_l, \infty)$. The coefficients $\mathcal{S}_1$, $\mathcal{S}_2$ and $S_1$, and the separation boundary $x_l$ can then be recovered from the boundary conditions (3).*

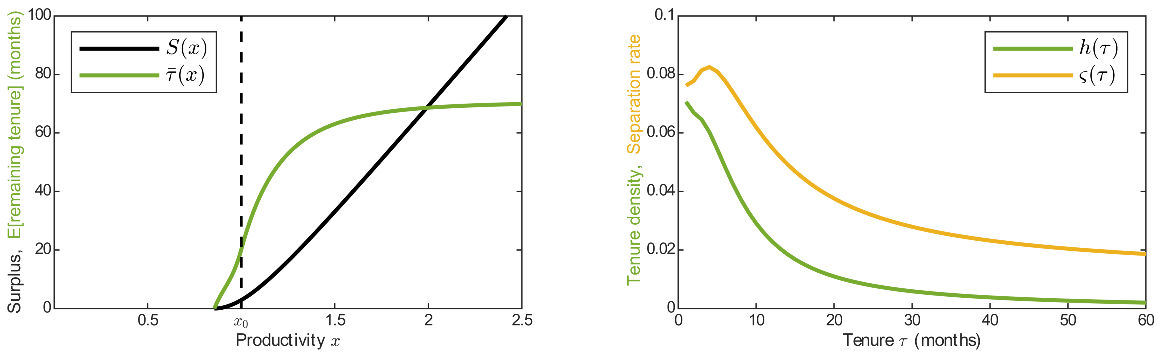
*(ii) Holding fixed the separation boundary $x_l$ and the surplus of a new match $S(x_0)$, expected remaining tenure at each productivity level can be recovered from*

$$\bar{\tau}(x) = -S_{rU}(x)|_{r=0,\beta=1}. \tag{5}$$

Proposition 1 provides an analytical general solution for the total surplus up to a set of constants. Given these, the boundary conditions (3) then comprise a system of nonlinear equations in the constants and $x_l$. The affine terms in (4) represent the value of the match in the absence of the option to separate, and absent the possibility of switching between states in which contacted workers quit ($x < x_0$), or are retained ($x \geq x_0$). The nonlinear terms capture the values of these future prospects. Figure 1A illustrates.

Figure 1. Match surplus, tenure and separations

A. Surplus $S(x)$ and expected tenure $\bar{\tau}(x)$ B. Density $h(\tau)$ and separation rate $\varsigma(\tau)$




Notes. Parameter values are based on the model calibrated as described in Table 1.

Proposition 1 further reveals that the solution for the match surplus $S(x)$ can be used to recover a solution for expected remaining tenure at each productivity level, $\bar{\tau}(x)$. Intuitively, holding fixed $x_l$ and $S(x_0)$, a marginal decrease in the annuitized value of unemployment $rU$ raises current match surplus $S(x)$ by the expected discounted value of a unit flow over the match's duration. When discounting is suspended ($r = 0$), and when the arrival of an outside offer induces a full loss of value ($\beta = 1$), this thought experiment recovers expected remaining tenure. Figure 1A superimposes the implied shape of $\bar{\tau}(x)$, which commences at zero at $x_l$, and then increases with $x$. A kink emerges at $x_0$ as contacted workers cease to quit, and $\bar{\tau}(x)$ then converges to $1/\delta$.

**Instructive special cases**. It is possible to refine and extend Proposition 1 to solve additionally for the distribution of tenure in instructive special cases. These capture some of the key economics of the more general model, and are ingredients to our characterization of wage adjustment in these special cases later in section 1.4. Lemma 1 summarizes.

**Lemma 1** *The following results for special cases are available.*

*(i) If $s = 0$, or $\beta = 0$, then*

$$S(x) = \frac{x}{r+\delta-\mu} - \frac{rU}{r+\delta}\left[1 - \frac{\mu}{r+\delta+\mu}\left(\frac{x}{x_l}\right)^{-\sqrt{\frac{r+\delta}{\mu}}}\right], \text{ and } \; x_l = \frac{r+\delta-\mu}{r+\delta+\mu} rU. \tag{6}$$

*(ii) If $s = 0$, the distribution function of completed tenure spells $\tau$ is given by*

$$H(\tau) = 1 - \exp(-\delta\tau)\,[1 - H_0(\tau)]. \tag{7}$$

*where $H_0(\tau)$ is Inverse Gaussian with associated density*

$$h_0(\tau) = \frac{\ln(x_0/x_l)}{\sigma\tau^{3/2}} \phi\left(\frac{\ln(x_0/x_l)}{\sigma\tau^{1/2}}\right), \tag{8}$$

*and where* $\phi(\cdot)$ *is the standard normal density.*

*(iii) If* $rU \leq \beta s\lambda S(x_0)$*, then* $x_l \leq 0$*, and the distribution function of completed tenure spells* $\tau$ *is given by*

$$H(\tau) = 1 - \exp\left[-\left(\delta + \frac{s\lambda}{2}\right)\tau\right] I_0\left(\frac{s\lambda\tau}{2}\right), \tag{9}$$

*where* $I_.(\cdot)$ *is the modified Bessel function of the first kind.*

Result (i) underscores the link between the solution for match surplus in our environment, and standard results on optimal stopping problems for the special case in which surplus is unaffected by job-to-job turnover ($\beta s = 0$). Here, the solution for $S(x)$ takes the simpler form in (6), and a closed-form solution for $x_l$ is available, reiterating results reported in Dixit (1993), Moscarini (2005), Buhai and Teulings (2014), *inter alia.*

Results (ii) and (iii) then reveal that one additionally can infer the *distribution* of match tenure in two polar cases for the sources of endogenous separation.

Result (ii) suspends job-to-job separations ($s = 0$), and adapts classic results on first passage times originally applied to labor turnover by Whitmore (1979). Given initial productivity $x_0 > x_l$, an endogenous separation occurs in this case if, and only if, $x$ first hits the lower boundary $x_l$, where the total surplus is zero. The match survival distribution is then the product of the exponential survival distribution of exogenous separations, and the Inverse Gaussian survival distribution of first passage times.

More novel, result (iii) suspends endogenous separations into unemployment ($x_l \leq 0$). Matches terminate in this case exogenously at rate $\delta$, and endogenously at rate $s\lambda$ whenever $x < x_0$. The distribution of tenure thus depends on the distribution of the time spent by $x$ below $x_0$. The latter is provided by a classic result on occupation times due to Lévy (1939): The share of time spent by $x$ below $x_0$ has an arcsine distribution, which is U-shaped, and symmetric about its mean (and, thereby, median) of one half. The match survival distribution that emerges is the product of the exponential survival distribution that would arise if $x$ spent half the time below $x_0$ with certainty, and an uncertainty adjustment that is formalized by a modified Bessel function of the first kind. The latter is increasing and convex in $s\lambda\tau/2 \geq 0$, and satisfies $I_0(0) = 1$.

Away from these special cases, it is straightforward to infer the tenure density $h(\tau)$ numerically. Separation rates by tenure can then be recovered from the hazard $\varsigma(\tau) = h(\tau)/[1 - H(\tau)]$. Figure 1B illustrates $h(\tau)$ and $\varsigma(\tau)$. Of particular note are the latter separation rates, which are initially hump-shaped, and thereafter declining. These features are consistent with empirical evidence on hazard rates of job ending documented by Farber (1999). The hump-shape has its origins in result (ii) of Lemma 1: New matches are created at $x_0 \gg x_l$, and it takes time for shocks to $x$ to accumulate sufficiently to induce endogenous separations to unemployment. By contrast, the declining hazard emerges from both results (ii) and (iii) of Lemma 1. Both capture a form of dynamic selection whereby low productivity matches—which are more likely to separate both to unemployment (result (ii)), and from job to job (result (iii))—are weeded out at lower tenures.

## 1.3 Wage adjustment

A key contribution of our analysis is to show that the wage protocol further admits a characterization of the surpluses of both firm and worker and, thereby, the path of wages.

To see how, consider the firm surplus, which we denote by $J(w,x)$. As the notation anticipates, this will depend on both the current wage $w$, and the productivity of the match $x$. The presence of two state variables raises a potential analytical challenge: Since the firm faces expected capital gains not only from future changes in productivity $x$, but also from future changes in the wage $w$, the firm surplus $J(w,x)$ in general will satisfy a *partial* differential equation, analytical solution of which is complicated. A symmetric argument applies to the worker surplus, which we denote $V(w,x)$.

The wage protocol provides a considerable simplification, however. We show that it yields a solution for the wage policy characterized by three further thresholds for productivity for any current wage, which we shall denote by $x_e(w)$, $x_f(w)$, and $x_n(w)$. The boundaries $x_e(w)$ and $x_f(w)$ trace out loci along which a renegotiation is initiated by the *employee* and *firm* respectively to raise and cut the wage. The boundary $x_n(w)$ traces out the locus along which an outside offer is rendered *noncompetitive*.

Importantly, wages are adjusted if, and only if, the boundaries $x_e(w)$ and $x_f(w)$ are attained, or a competitive outside offer arrives. Otherwise, wages remain unchanged and, for any current wage $w$, firms face capital gains solely from future changes in productivity $x$. In this case, the firm surplus $J(w,x)$ satisfies the *ordinary* differential equation

$$\begin{aligned}(r+\delta)J(w,x) = {} & x - w - s\lambda \mathbf{1}_{\{x<x_0\}} J(w,x) \\ & + s\lambda \mathbf{1}_{\{x\geq x_0\}} \min\{[1-\beta(1-\Delta_e)][S(x)-S(x_0)] - J(w,x), 0\} \\ & + \mu x J_x + \frac{1}{2}\sigma^2 x^2 J_{xx},\end{aligned} \tag{10}$$

and the worker surplus $V(w,x)$ satisfies the analogous ordinary differential equation

$$(r+\delta)V(w,x) = w - rU + s\lambda \mathbf{1}_{\{x<x_0\}}\{S(x) + \beta[S(x_0) - S(x)] - V(w,x)\}$$
$$+ s\lambda \mathbf{1}_{\{x\geq x_0\}} \max\{S(x_0) + \beta(1-\Delta_e)[S(x) - S(x_0)] - V(w,x), 0\} \quad (11)$$
$$+ \mu x V_x + \frac{1}{2}\sigma^2 x^2 V_{xx}.$$

Absent wage adjustment, the firm receives a flow surplus equal to output $x$ less the current wage $w$, and the worker receives a flow surplus of $w$ less the annuitized value of unemployment $rU$. The match then faces capital gains from two sources.

First, at rate $s\lambda$ the worker receives an offer from an outside firm of productivity $x_0$. If $x < x_0$, the worker quits, the firm realizes a capital loss equal to its value of the match $J(w,x)$, and the worker realizes a capital gain $S(x) + \beta[S(x_0) - S(x)] - V(w,x)$. If $x \geq x_0$, the worker is retained, and receives a capital gain of $S(x_0) + \beta(1-\Delta_e)[S(x) - S(x_0)] - V(w,x)$, or zero, whichever is greater. The firm receives the remainder of the surplus.

The second source of capital gains arises from idiosyncratic shocks to match productivity, which evolve according to the stochastic law of motion (1). Application of Ito's lemma yields the latter two terms in (10) and (11). Importantly, these capture not only the direct value to the firm of future changes in output $x$, but also the indirect value of future adjustments in wages induced by changes in $x$, or the arrival of outside offers. Formally, the latter are encoded in the boundary conditions for this problem, which follow from the bounds to firm and worker surpluses implied by the wage adjustment protocol.

Specifically, along the $x_f(w)$ boundary, wages are cut such that the firm is indifferent between continuing under the adjusted wage and unilaterally initiating a renegotiation. Optimal exercise of such threats implies that the firm's surplus must satisfy the value-matching and smooth-pasting conditions,

$$J(w, x_f(w)) \equiv (1-\beta)(1-\Delta_f)S(x_f(w)), \text{ and}$$
$$J_x(w, x_f(w)) = (1-\beta)(1-\Delta_f)S'(x_f(w)), \text{ for all } w. \quad (12)$$

Symmetrically, along the $x_e(w)$ boundary, wages are increased such that the employee is indifferent between continuing under the adjusted wage and unilaterally initiating a renegotiation. The worker's surplus must then satisfy

$$V(w, x_e(w)) \equiv \beta(1-\Delta_e)S(x_e(w)), \text{ and}$$
$$V_x(w, x_e(w)) = \beta(1-\Delta_e)S'(x_e(w)), \text{ for all } w. \quad (13)$$

Finally, along the $x_n(w)$ boundary, the worker receives a surplus that renders her indifferent to unilateral initiation of a bargain subject to breakdown risk $\Delta_e$. Optimality further requires that $V(w,x)$ be continuously differentiable through the boundary,

$$
\begin{aligned}
&V\big(w, x_n(w)\big) \equiv S(x_0) + \beta(1-\Delta_e)\big[S\big(x_n(w)\big) - S(x_0)\big], \text{ and} \\
&V_x\big(w, x_n^-(w)\big) = V_x\big(w, x_n^+(w)\big), \text{ for all } w.
\end{aligned} \tag{14}
$$

The boundary conditions reiterate the analytical tractability afforded by the decoupling of match surplus, and the surplus of each party, implied by bilateral efficiency. Given the solution for match surplus $S(x)$ in Proposition 1, the boundary conditions in (12), (13) and (14) can be evaluated, and the wage adjustment problem can be solved.

To do so, we define the *inaction sets* $\mathrm{I}_e(w)$ and $\mathrm{I}_f(w)$ that describe, for each current wage $w$, the set of productivities $x$ such that employee and firm respectively cannot credibly issue a unilateral threat to renegotiate. It follows that $x_e(w) = \partial \mathrm{I}_e(w)$, and $x_f(w) = \partial \mathrm{I}_f(w)$, the boundaries of the respective inaction sets. Similarly, we define the *noncompetitive set* $\mathrm{N}(w)$ (in $\mathrm{I}_e(w) \cap \mathrm{I}_f(w)$) that describes, for each $w$, the set of $x$ such that an outside offer is rendered noncompetitive. Accordingly, $x_n(w) = \partial \mathrm{N}(w)$. Finally, to accommodate the possibility that $\mathrm{N}(w)$, or its complement $\bar{\mathrm{N}}(w)$ (in $\mathrm{I}_e(w) \cap \mathrm{I}_f(w)$), is nonconvex, we partition each into subintervals indexed by $i$, denoted respectively by $\mathrm{N}_i(w)$ and $\bar{\mathrm{N}}_i(w)$. With this notation in hand, Proposition 2 summarizes the solutions for the firm and worker surpluses, and their implications for the allocative effects of wages.

**Proposition 2** *(i) For any $w$, $i$, and $x \in N_i(w)$, the firm surplus has general solution*

$$
J(w,x) = \frac{x}{r+\delta-\mu} - \frac{w}{r+\delta} + J_{i1}(w) x^{-\sqrt{\frac{r+\delta}{\mu}}} + J_{i2}(w) x^{\sqrt{\frac{r+\delta}{\mu}}}. \tag{15}
$$

*For any $w$, $i$, and $x \in \bar{N}_i(w)$, the firm surplus has general solution*

$$
J(w,x) = \frac{x}{r+\delta-\mu+s\lambda} - \frac{w}{r+\delta+s\lambda} + s\lambda\mathcal{P}(x) + \mathcal{J}_{i1}(w) x^{-\sqrt{\frac{r+\delta+s\lambda}{\mu}}} + \mathcal{J}_{i2}(w) x^{\sqrt{\frac{r+\delta+s\lambda}{\mu}}}, \tag{16}
$$

*where*

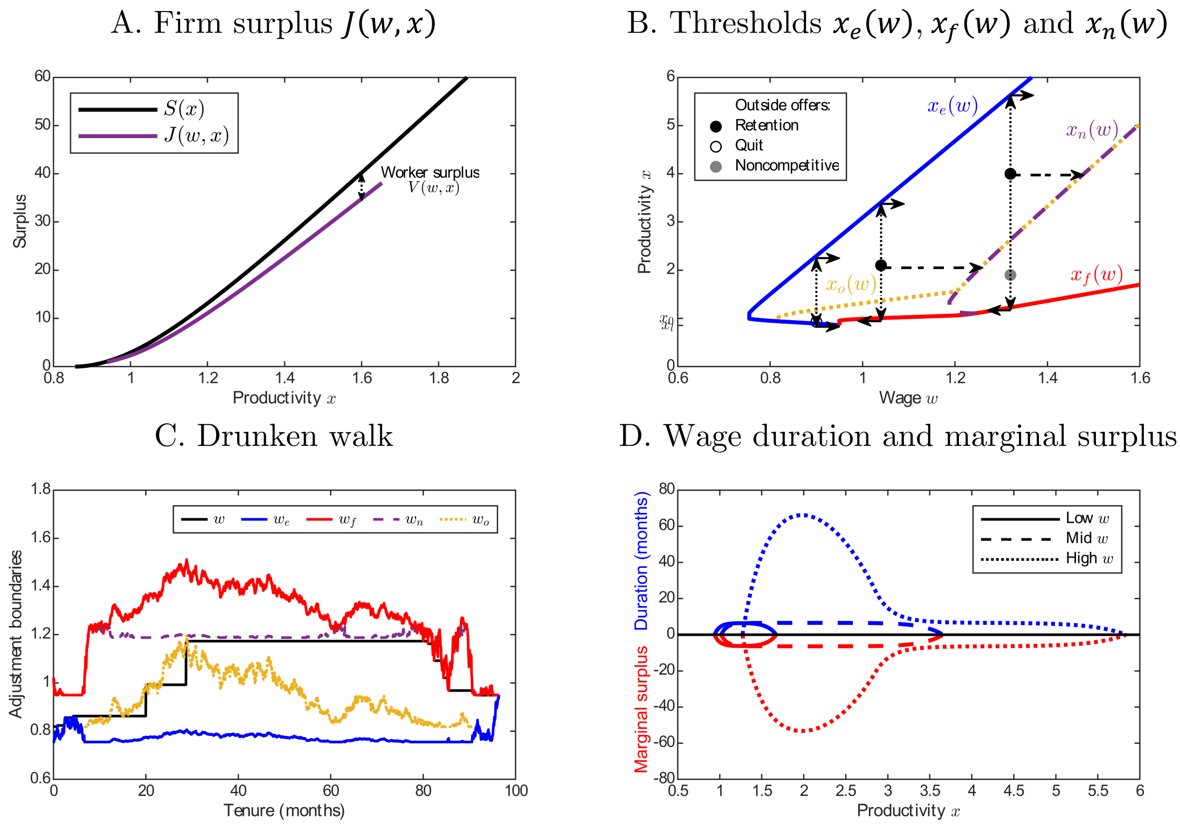
$$
\mathcal{P}(x) = -\frac{1-\beta(1-\Delta_e)}{2\sqrt{(r+\delta+s\lambda)\mu}} \int_{x_0}^{\max\{x_0, x\}} \left[\left(\frac{x}{\tilde{x}}\right)^{\sqrt{\frac{r+\delta+s\lambda}{\mu}}} - \left(\frac{x}{\tilde{x}}\right)^{-\sqrt{\frac{r+\delta+s\lambda}{\mu}}}\right] \frac{S(\tilde{x}) - S(x_0)}{\tilde{x}} \mathrm{d}\tilde{x}. \tag{17}
$$

*The coefficients $J_{i1}(w)$, $J_{i2}(w)$, $\mathcal{J}_{i1}(w)$, $\mathcal{J}_{i2}(w)$, and boundaries $x_e(w)$, $x_f(w)$, $x_n(w)$, are implied by the boundary conditions (12), (13), and (14). The worker surplus $V(w,x) = S(x) - J(w,x)$ follows from (4).*

*(ii) Holding fixed the wage adjustment boundaries, $x_e(w)$, $x_f(w)$, and $x_n(w)$, the expected duration until next wage adjustment can be recovered from*

$$
\bar{\tau}^w(w,x) = -J_w(w,x)|_{r=0} = V_w(w,x)|_{r=0}. \tag{18}
$$

Figure 2. Baseline model outcomes




Notes. Parameter values are based on the model calibrated as described in Table 1.

Proposition 2 has several useful properties. First, it emphasizes important parallels between the economics of our model of long-lived employment relationships and the economics of long-lived capital investments. It uses the fact that the firm and worker values in (10) and (11), and their boundary conditions in (12) to (14), comprise a sequence of instantaneous control problems, indexed by the current wage $w$. As a result, the solution in Proposition 2 echoes that for irreversible investment in Abel and Eberly (1996). As there, the approach yields an analytical general solution for the firm (and, thereby, worker) value in (15) and (16) up to a given set of constants. The boundary conditions can then be used to solve for the constants and the adjustment boundaries numerically. Proposition 2 extends this logic to sporadic adjustment of the factor price $w$, and the presence of on-the-job search. This parallel with the investment literature will be shown to be especially useful in section 1.6, where we exploit an analogy between the user cost of labor and the user cost of capital in these models.

Second, the interpretation of the general solution in (15) and (16) also echoes its analogue in Abel and Eberly (1996). The affine terms in (15) and (16) capture the firm's surplus absent the option to adjust wages, and absent the possibility of switching between states in which outside offers are competitive ($x \in \overline{\mathrm{N}}(w)$), or noncompetitive ($x \in \mathrm{N}(w)$). The power functions in (15) and (16) respectively value the prospects of wage cuts in adverse future states, wage increases in favorable future states, and of switching in and out of $\mathrm{N}(w)$, for each current wage $w$.

Third, a key analytical challenge resolved by Proposition 2 is the presence of additional capital gains associated with the prospect of an outside offer in (10) and (11). The insight is twofold: The solution for the total match surplus $S(x)$ in Proposition 1 delivers a solution for these additional capital gains; and, the value of these capital gains to firm and worker can be inferred analytically in the form of the particular solution $\mathcal{P}(x)$ given in (17). This, in turn, implies a solution for the option value of on-the-job search.

Fourth, we find that a solution approach based on Proposition 2 is faster and more accurate relative to brute-force numerical simulation. Intuitively, for any given current wage $w$, Proposition 2 reduces a problem of iterating over a *function* to one of iterating over a *constant*. Appendix C provides further detail.

Finally, result (ii) of Proposition 2 establishes that the expected duration of wage spells is intimately related to the allocative effects of intermittent wage adjustment, as captured by the marginal values of the wage to firm and worker, $J_w(w,x)$ and $V_w(w,x)$. Intuitively, each party knows that any future adjustment of the wage will be made solely on the basis of contemporaneous productivity, and thus will otherwise be independent of the history of wages and productivity up to that point. Firm and worker therefore care simply about the duration of the current wage, appropriately discounted. We will see that this aspect of Proposition 2 has important implications for the allocative effects of intermittent wage adjustment on unemployment.

Figure 2A illustrates the firm and worker values that emerge, reiterating the intuition that the intermittent ability of firm and worker to issue credible unilateral threats to renegotiate the wage places bounds on their valuations of the match.

By providing an analytical characterization of the firm and worker valuations of the match, Proposition 2 aids solution for the boundaries $x_e(w)$, $x_f(w)$, and $x_n(w)$. These are illustrated in Figure 2B, which reveals that, in general, the wage adjustment boundaries take the form of correspondences. Most notably, the boundary $x_e(w)$ acts as both an upper and a lower bound for productivity at low current wages $w$. The upper bound reflects the fact that increases in productivity $x$ raise the possible surplus that the employee can capture by issuing a unilateral threat to renegotiate. The lower bound reflects the fact that, as $x$ declines, the surplus that the worker can extract in the event of an outside offer also declines. Both act as a potential stimulus to wage increases in the model. An analogous logic applies to the boundary along which outside offers become noncompetitive—note that the value-matching conditions in (13) and (14) differ only by a constant—and so an echo of these same forces can be seen in the shape of $x_n(w)$.

The boundaries in turn determine the path of wages for any given initial wage and realization of the path of match productivity $x$. As indicated by the arrows in Figure 2B, wages remain constant for periods of time, punctuated by adjustments from two sources.

First, productivity shocks trigger incremental[10] wage adjustments at the upper and lower boundaries induced by unilateral bargaining threats. These adjustments regulate the joint path of $(w, x)$ to remain within the boundaries $x_e(w)$ and $x_f(w)$.

Second, the arrival of outside offers also may trigger an adjustment to compensation. In matches with productivity $x < x_0$, contacted workers quit to the outside match. In matches of productivity $x \geq x_0$, however, contacted workers are retained. If, in addition, the outside offer is competitive, $x \in \overline{\mathrm{N}}(w)$, workers realize a retention package that raises their compensation just enough to render the outside offer noncompetitive.

Observe that these two stimuli to adjustment are fundamentally different. Adjustments at $x_e(w)$ and $x_f(w)$ are *incremental*, triggered by shocks to *flow* productivity $x$ that are *persistent*. By contrast, adjustments induced by outside offers are *discrete*, triggered by shocks to the outside *value* available to the worker *temporarily*.

This distinction has natural implications for the structure of compensation between flow wages and lump-sum bonuses in each case. Changes to flow productivity $x$ at the boundaries $x_e(w)$ and $x_f(w)$ naturally are resolved by adjustments to the flow wage $w$: Given the persistence of shocks to $x$, lump-sum bonuses would instead resolve the impetus to renegotiation only temporarily, requiring a period of *continual* bonuses.[11]

By contrast, temporary realizations of outside offers naturally can be resolved either by flow wage increases, or by instantaneous bonus pay. Interestingly, a novel implication of the model is that it places an upper bound on the former: Firms can credibly commit to flow wage increases only up to $x_f^{-1}(x)$; a further increase would trigger a subsequent renegotiation, and be reversed, almost immediately. Any remaining value must then be delivered as a lump-sum bonus—lump-sum because the firm has no other means to pledge

---

[10] To see that such adjustments are incremental, recall that if, say, the firm's surplus under the existing agreement falls below $(1-\beta)(1-\Delta_f)S$, the worker will accede to a wage cut *by mutual consent* that restores the firm's surplus to $(1-\beta)(1-\Delta_f)S$, rather than countenance the risk of breakdown $\Delta_f$.

[11] Furthermore, at the wage cut boundary $x_f(w)$, this alternative additionally would require a period of continual lump-sum payments *from worker to firm*.

value credibly at the point of counteroffer. Thus, the model provides a novel theory of non-base pay in the form of recruitment and retention bonuses.

Concretely, we specify how recruitment and retention compensation is apportioned between base and bonus pay as follows. Per (12), the maximum share of the match surplus that the firm credibly can deliver in the form of base pay is $1-(1-\beta)\left(1-\Delta_f\right)$. As we have noted, however, were the base wage raised to satisfy this bound, it subsequently would be adjusted almost surely. We adopt a simple middle ground whereby, in the event of an outside offer, the recruitment or retention package delivers a value up to a share $\beta$ of its surplus by raising the worker's base wage, and delivers any remaining value as a lump-sum bonus. This rule is both simple and intuitive, implying a form of equal treatment whereby base wages in hiring firms deliver the same surplus to new hires, regardless of whether they are hired from unemployment, or poached from another firm.[12]

Returning to Figure 2B, the dotted line traces out a locus of match productivities, $x \geq x_0$, and wages, $w$, that determines the adjustment of the base wage in response to an outside offer, denoted $x_o(w)$. To the left of this locus, outside offers induce a base wage increase such that the match moves horizontally to lie on the locus; if, in turn, the latter lies to the left of the noncompetitive boundary, $x_n(w)$, the difference in value is delivered to the worker in the form of a retention bonus. Alternatively, to the right of the dotted line, outside offers induce no change in the base wage, but may induce a retention bonus if the match lies to the left of the noncompetitive boundary, $x_n(w)$.[13]

Finally, Figure 2B anticipates a useful simplification of the form of the adjustment boundaries—namely that they appear to approach linear functions as the wage $w$ rises.

**Lemma 2** *For $\beta \in (0,1]$, and $\Delta_e$, $\Delta_f \in [0,1)$, the adjustment boundaries $x_e(w)$, $x_f(w)$ and $x_n(w)$ become linear and increasing as $w \to \infty$.*

The seemingly complex wage dynamics implied by Proposition 2 are thus quite simple for sufficiently productive matches. The model therefore does not demand excessive sophistication of firms and workers who adhere to near-linear adjustment rules.

Figure 2C provides a parallel view of the implied wage dynamics. The adjustment thresholds can be inverted to define three wage thresholds: A lower wage threshold, $w_e(x) \equiv x_e^{-1}(x)$, an upper threshold $w_f(x) \equiv x_f^{-1}(x)$, and a threshold wage at which

[12] Clearly, one could generalize this simple rule to accommodate further moments of wage adjustment. In the interest of parsimony, though, we start here.

[13] Note also that, once a match enters the right-hand region of Figure 2B in which the wage cut boundary $x_f(w)$ is operative, it will remain in that region until its termination.

outside offers cease to be competitive, $w_n(x) \equiv x_n^{-1}(x)$. Given an initial wage $w_0$, and sample path of productivity $x$, wages evolve according to the *drunken walk* depicted in Figure 2C, remaining constant on the interior of the thresholds, adjusting minimally when a shock induces them to bind, and adjusting discretely upon arrival of a competitive outside offer. Figure 2C also reiterates the nature of bonus compensation in the model. Echoing Figure 2B, it depicts the threshold that determines the adjustment of the base wage in response to an outside offer, $w_o(x) \equiv x_o^{-1}(x)$. Where this lies below the noncompetitive wage $w_n(x)$, the worker receives any additional value as a lump-sum retention bonus. Observe that, although bonus compensation appears naturally asymmetric, delivering only raises in compensation, it also can contribute to the realization of *reductions* in compensation among workers who have received bonuses in the past, but not in the present, a theme we will return to later.

Figure 2D illustrates the link between the marginal firm surplus and the expected duration of wage spells. Because wage adjustment is two-sided—wage spells terminate when match productivity $x$ first hits *either* the relevant lower *or* upper boundary—the expected duration until next wage adjustment is zero at these boundaries (and, by extension, at the separation boundary $x_l$). Away from these extremes, the expected time until adjustment is hump-shaped for productivities outside the noncompetitive set $\mathsf{N}(w)$. If productivity further traverses through $\mathsf{N}(w)$, a larger second hump emerges as outside offers cease to be competitive, and wages no longer need adjust to their arrival.

## 1.4 Instructive special cases

To build further intuition, we now consider two special cases which correspond to the two stimuli to wage adjustment in the general model, and are contributions in themselves.

**Costly renegotiation**. First, we consider the case in which there is no on-the-job search ($s = 0$), and wage adjustment is prompted solely by intermittent unilateral threats to initiate a renegotiation. This case has the tractable implication that simple analytical solutions are available for match surplus $S(x)$, the separation boundary $x_l$, and the distribution of completed tenure spells $h(\tau)$, as summarized in results (i) and (ii) of Lemma 1. The solution for $S(x)$ can in turn be used directly to evaluate the boundary conditions in (12) and (13), permitting solution to the wage adjustment problem. A tractable feature of this case is that the wage adjustment boundaries $x_e(w)$ and $x_f(w)$ can be distilled into a pair of nonlinear equations which we provide in the appendix.

**Proposition 3** *In the special case $s = 0$, the general solution for the firm surplus is*

$$J(w,x) = \frac{x}{r+\delta-\mu} - \frac{w}{r+\delta} + J_1(w)x^{-\sqrt{\frac{r+\delta}{\mu}}} + J_2(w)x^{\sqrt{\frac{r+\delta}{\mu}}}, \tag{19}$$

*for all* $w \geq \beta x_l + (1-\beta)rU$, *and* $x \in (x_f(w), x_e(w))$. *The coefficients* $J_1(w)$, $J_2(w)$, *can be recovered from the boundaries* $x_e(w)$, *and* $x_f(w)$, *which in turn solve a known pair of equations (provided in the appendix).*

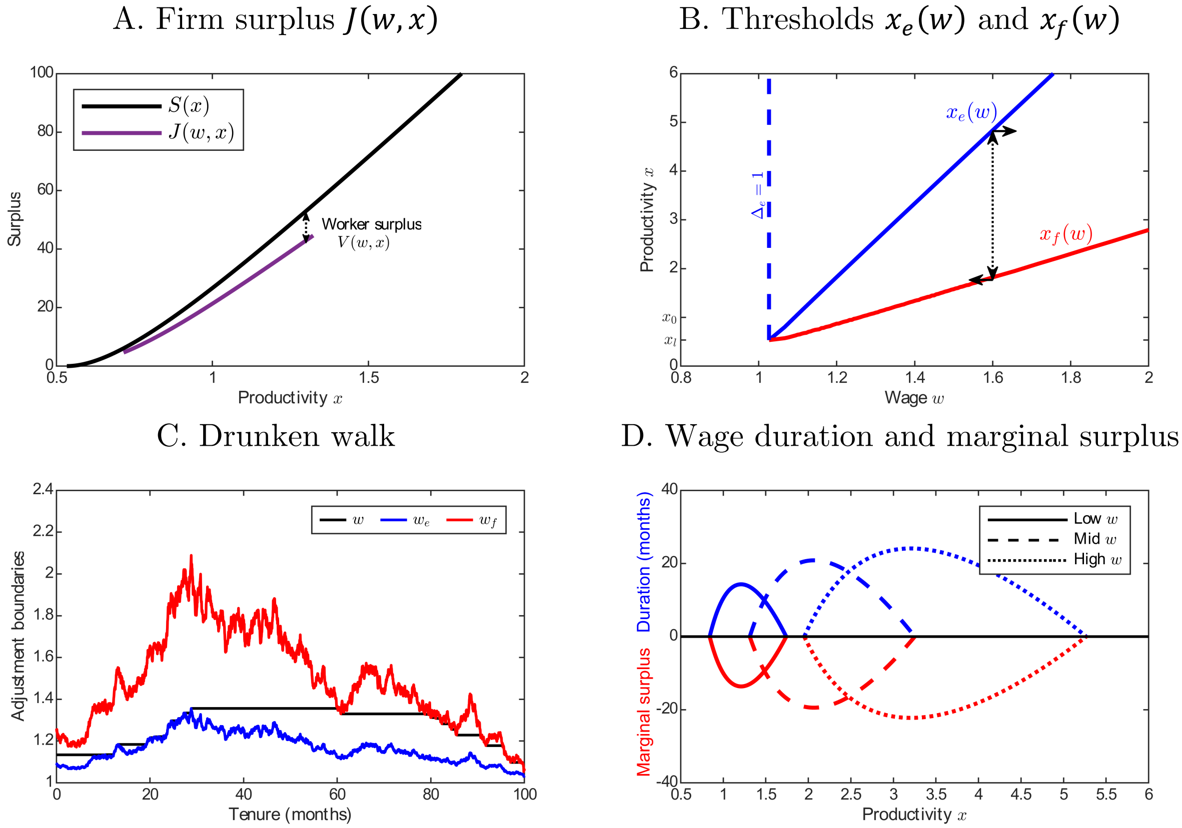
Figure 3 illustrates model outcomes in this case as a point of comparison to Figure 2. The boundaries simplify to an upper bound provided by the employee's unilateral renegotiation threshold $x_e(w)$, and a lower bound provided by the firm threshold $x_f(w)$, as in Figure 3B. This reinforces the role of the credibility of unilateral threats to initiate a renegotiation, indexed by the $\Delta$s, in the incidence of equilibrium wage adjustment. Smaller $\Delta$s raise the credibility of such threats, narrowing the adjustment thresholds, and raising the frequency of wage adjustment. Larger $\Delta$s have the opposite effect. When both $\Delta$s are zero, threats to initiate a unilateral renegotiation are always credible, the adjustment boundaries degenerate, and the model recovers the continual Nash bargaining applied extensively in the search and matching literature (Mortensen and Pissarides 1994).

Notice, however, a fundamental asymmetry in this special case: Firms receive shocks to their flow payoff $x$; workers *do not*. Even in the face of complete breakdown risk ($\Delta_f = 1$), there remain states in which the firm can credibly threaten to walk away from the match at the current wage, and thereby enforce a wage cut. By contrast, the worker can *never* credibly enforce a wage increase in the face of complete breakdown risk ($\Delta_e = 1$), as illustrated in Figure 3B. This asymmetry is resolved in the general model by the presence of shocks to the worker's payoff, driven by the arrival of outside job offers.

**Sequential auctions**. The second special case we explore turns to the opposite polar case in which wage adjustment is prompted solely by the availability of outside offers generated by on-the-job search ($s > 0$). To isolate its signature implications for wage dynamics, we suspend the forces of the previous subsection: Unilateral bargaining threats are no longer credible, $\Delta_f = 1$, and workers no longer have bargaining power, $\beta = 0$.

This special case corresponds to the canonical sequential auctions model of Postel-Vinay and Robin (2002), extended to incorporate persistent idiosyncratic shocks to match productivity $x$, and drunken walk wage dynamics. Postel-Vinay and Turon (2010) study a similar environment, but with i.i.d. productivity shocks. As they note, the present case with persistent shocks poses an analytical challenge that has necessitated the use of numerical methods in prior work (Yamaguchi 2010; Lise et al. 2016). We show how analytical progress can be made for this canonical special case.

Figure 3. Model outcomes: Costly renegotiation case ($s = 0$)




Notes. Parameter values based on a recalibration that omits the target moment for $s$ in Table 1.

To begin, note that total match surplus takes the same form as in the preceding case: Although the presence of on-the-job search induces additional turnover, match surplus remains unimpaired since, upon job-to-job transition, the worker receives the entire surplus of her previous match. A direct implication is that the solutions in Lemma 1 for the match surplus $S(x)$ and the threshold for separation into unemployment $x_l$ in (6) apply equally to this environment with sequential auctions. This again facilitates evaluation of the boundary conditions in (12), (13), and (14), which again reduces the solution for the boundaries $x_e(w)$, $x_f(w)$, and $x_n(w)$ to a set of nonlinear equations.

**Proposition 4** *In the special case in which $\beta = 0$ and $\Delta_f = 1$, the general solution for the firm surplus is*

$$J(w,x) = \begin{cases} \dfrac{x}{r+\delta-\mu+s\lambda} - \dfrac{w}{r+\delta+s\lambda} + s\lambda\mathcal{P}(x) + \mathcal{J}_1(w)x^{-\sqrt{\frac{r+\delta+s\lambda}{\mu}}} + \mathcal{J}_2(w)x^{\sqrt{\frac{r+\delta+s\lambda}{\mu}}} & \text{if} \quad x < x_n(w), \\ \dfrac{x}{r+\delta-\mu} - \dfrac{w}{r+\delta} + J_1(w)x^{-\sqrt{\frac{r+\delta}{\mu}}} & \text{if} \quad x \geq x_n(w), \end{cases} \quad (20)$$

*for all* $w$*, and* $x \in \left(max\left\{x_e(w), x_f(w)\right\}, \infty\right)$*, where* $\mathcal{P}(x)$ *is given in (17). The coefficients* $\mathcal{J}_1(w)$*,* $\mathcal{J}_2(w)$*, and* $J_1(w)$*, and boundaries* $x_e(w)$*,* $x_f(w)$*, and* $x_n(w)$*, are known (implicit) functions of the parameters of the problem (provided in the appendix).*

Figure 4 illustrates model outcomes for this case, revealing several points of contrast relative to Figures 2 and 3 above. Figure 4A reveals that the worker's share of the surplus is highest (and the firm's share lowest) in the neighborhood of $x_0 = 1$, since the worker's outside option is bounded above by $S(x_0)$. And, because the latter is small compared to $S(x)$ for $x \gg x_0$, the worker's surplus share decays as $x$ rises.

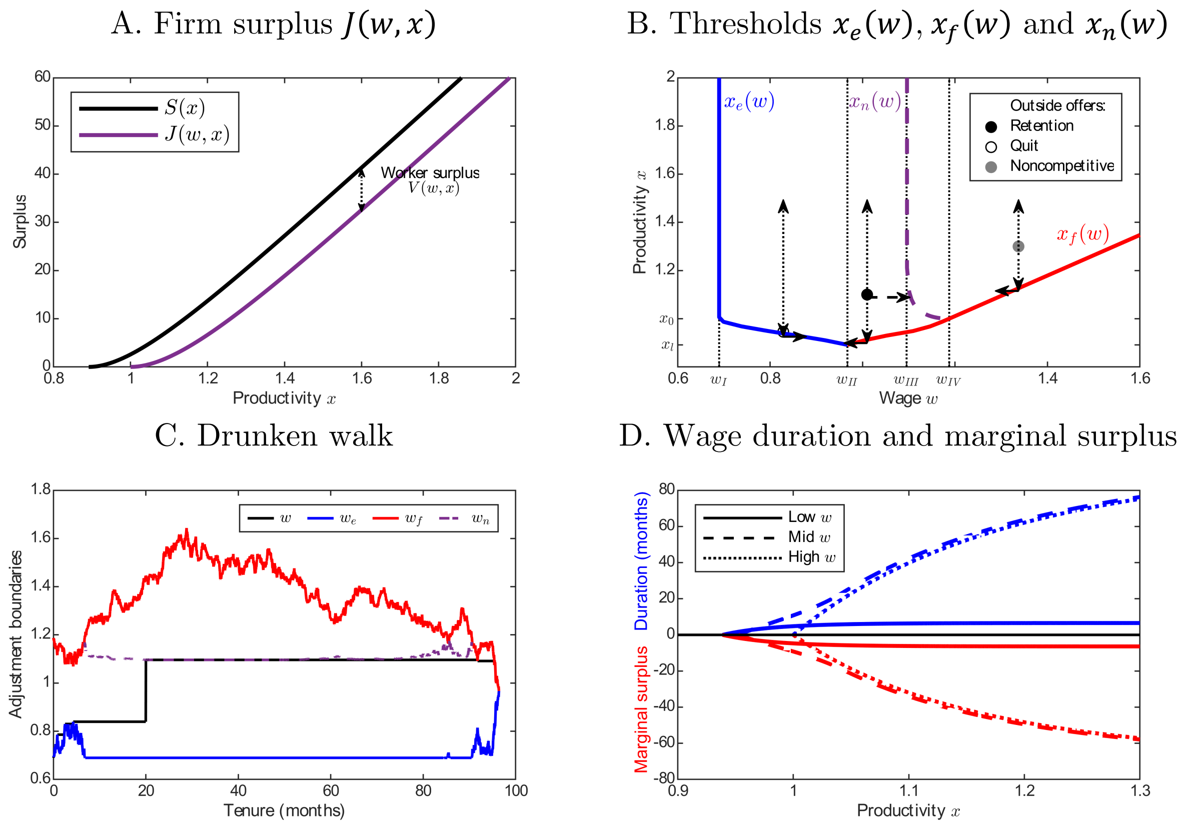
Figure 4B then illustrates the implied wage adjustment thresholds. This reveals that the downward-sloping sections in Figure 2B have their origins in the presence of on-the-job search: Both $x_e(w)$ and $x_n(w)$ slope downward in the sequential auctions environment. Intuitively, since the worker has no bargaining power, these trace out indifference loci over wages and productivity for the worker, $V\left(w, x_e(w)\right) \equiv 0$ and $V\left(w, x_n(w)\right) \equiv S(x_0)$. Since increases in $x$ raise recruitment compensation paid in the event of an outside offer, $S(x)$, workers in matches with productivity $x < x_0$ value both higher wages and higher productivity, and $x_e(w)$ slopes downward for such matches. And since increases in $x$ reduce the likelihood of future wage cuts, $x_n(w)$ also slopes downward.

Relatedly, Figure 4B reveals that wage adjustment at the thresholds is *one-sided* in this case. Fixing the current wage $w$, and successively raising match productivity $x$ no longer induces wage increases, due to the absence of worker bargaining power. A perhaps-surprising corollary is that wage increases induced at the $x_e(w)$ threshold must arise from *reductions* in match productivity. Intuitively, reductions in $x$ lower the worker's recruitment compensation in the event of an outside job offer, lowering her value of the match, and necessitating a raise to obviate a quit to unemployment.

The remaining panels of Figure 4 highlight two corollaries of the one-sided nature of wage adjustment. First, the drunken walk in Figure 4C reveals that the bounds for wages are *inversely* correlated in the sequential auctions case. Second, in Figure 4D the expected duration of the wage rises monotonically with productivity $x$, converging to $1/(\delta + s\lambda)$ when outside offers are competitive, and to $1/\delta$ when they are not. Reiterating the link between the duration of wages and their allocative effects, the marginal value of the wage behaves (near-)symmetrically, converging to $1/(r + \delta + s\lambda)$, and $1/(r + \delta)$, respectively.

Like the previous costly renegotiation example, the sequential auctions case also has extreme implications: The absence of worker bargaining power implies that workers can capture surplus only through receipt of outside offers, no matter how productive the match. Juxtaposing Figures 2, 3, and 4 suggests that the two forces of wage adjustment

Figure 4. Model outcomes: Sequential auctions case ($\Delta_f = 1$, $\beta = 0$)




Notes. Parameter values as described in Table 1, but with $\Delta_f = 1$ and $\beta = 0$.

encapsulated in these two special cases complement one another, capturing adjustments prompted by rent sharing on one hand, and external competition on the other. This, in turn, further underscores the value of the general model characterized in Proposition 2.

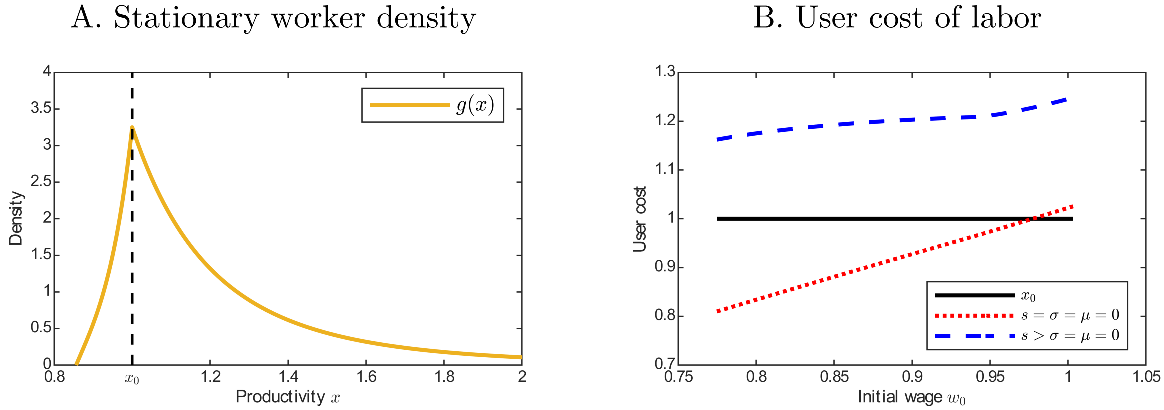
## 1.5 Labor market equilibrium

We now show how this model of wage determination can be embedded into aggregate labor market equilibrium, facilitating an analysis of its implications for unemployment.

To underscore the novel features of the theory, we consider its implications in the context of standard models in the Diamond-Mortensen-Pissarides search and matching tradition, extended to accommodate on-the-job search. The flow of new contacts arising from $u$ unemployed searchers, $1-u$ employed searchers, and $v$ vacancies, is determined by a constant-returns-to-scale matching function $m(u + s(1-u), v)$. Labor market tightness, $\theta \equiv v/[u + s(1-u)]$, is thus a sufficient statistic for contact rates: Vacancies contact a searcher at rate $q(\theta) = m/v = m(1/\theta, 1)$; unemployed and employed searchers respectively contact a vacant job at rates $\lambda(\theta) = m/[u + s(1-u)] = m(1, \theta)$, and $s\lambda(\theta)$.

Four further conditions then complete aggregate labor market equilibrium.

Figure 5. Stationary worker density and user cost of labor




Notes. Parameter values are based on the model calibrated as described in Table 1.

First, the value of unemployment to a worker $U$ satisfies the Bellman equation,

$$rU(w_0, x_0; \theta) = b + \lambda(\theta)V(w_0, x_0; \theta). \tag{21}$$

While unemployed, a worker receives a flow payoff $b$. At rate $\lambda(\theta)$ she finds a new job with initial wage $w_0$ and initial productivity $x_0$ yielding a surplus to the worker of $V(w_0, x_0; \theta)$. Observe that the solution for $V(w_0, x_0; \theta)$ is provided by Proposition 2 above.

Second, the model is amenable to different protocols for the determination of the entry wage of a new hire from unemployment $w_0$. Consistent with our assumption that bargaining is costless when both parties wish to initiate a negotiation, our benchmark will be that $w_0$ is determined by Nash bargaining. A newly hired worker from unemployment thus receives a value $\beta S(x_0)$.[14] But the model also is amenable to alternatives, such as Hall's (2005) proposed fixity of $w_0$, and variations thereof, a point to which we will return.

Third, equilibrium labor market tightness is determined by free entry into vacancy creation. This requires the expected firm surplus from meeting a searcher to be equal to the expected cost of meeting a searcher. Denoting the distribution function of employees over productivity by $G(x; \theta)$, and the flow cost of a vacancy by $c_v$, this implies

$$\frac{u}{u + s(1-u)} J(w_0, x_0; \theta) + \frac{s(1-u)}{u + s(1-u)} (1-\beta) \int_{x_l(\theta)}^{x_0} [S(x_0; \theta) - S(x; \theta)] \mathrm{d}G(x; \theta) = \frac{c_v}{q(\theta)}. \tag{22}$$

[14] An implication is that all new hires from unemployment receive the same wage. This should not be taken literally, however. For example, suppose there are (time-invariant) worker types that differ in their efficiency units. If $x$, $b$ (and $c_v$, if search markets are segmented) scale with these efficiency units, then so will the boundaries in Figure 2B and the initial wage $w_0$, whereas the implied log wage change distribution will be identical for each worker type.

In the event of hiring an unemployed worker, the firm realizes a surplus of $J(w_0, x_0; \theta)$. Recruitment of a worker previously employed in a match of productivity $x < x_0$ realizes a surplus of $(1-\beta)[S(x_0;\theta) - S(x;\theta)]$. Weighting the latter by their respective shares of the population of searchers delivers the expected firm surplus in (22). Observe that solutions for the total surplus $S(x;\theta)$, and the firm's valuation of a new job $J(w_0, x_0; \theta)$, are provided respectively by Propositions 1 and 2.

Finally, denoting the aggregate separation rate into unemployment by $\varsigma(\theta)$, the unemployment rate evolves according to the law of motion,

$$\frac{\mathrm{d}u}{\mathrm{d}t} = \varsigma(\theta)(1-u) - \lambda(\theta)u. \tag{23}$$

To close the model, it remains to solve for the stationary density of workers over productivity, $g(x;\theta)$. This takes a standard "double-Pareto" form illustrated in Figure 5. Job creation "pours" workers into jobs at $x_0$, yielding a peak at that point. Endogenous job destruction "pours" workers out of jobs at $x_l(\theta)$ such that there is no density at this point. Derivations of $g(x;\theta)$ and $\varsigma(\theta)$ are provided in the online appendix.

## 1.6 User cost of labor

The preceding analysis characterizes outcomes in terms of *values*—most notably the value of a prospective match to a firm, $J(w_0, x_0; \theta)$. We now explore a parallel interpretation in terms of *flows*. This is aided by an approach identified by Kudlyak (2014), who notes that the shadow flow price of labor in long-term employment relationships has a *user cost* interpretation, mirroring the analogous concept in capital theory (Jorgenson 1963).

Kudlyak's user cost of labor concept can be extended to our environment using the out-of-steady-state analogue of the Bellman equation for the firm surplus in (10). Specifically, evaluating the latter at initial productivity $x_0$, a straightforward extension of the approach to user cost devised by Abel and Eberly (1996) in a model of investment under uncertainty implies that the user cost of labor for a new match takes the form

$$\begin{aligned} \omega(w_0, x_0; \theta) = w_0 + [r + \delta + s\lambda(\theta)]J(w_0, x_0; \theta) - \mu x_0 J_x(w_0, x_0; \theta) \\ - \frac{1}{2}\sigma^2 x_0^2 J_{xx}(w_0, x_0; \theta) - \frac{\partial}{\partial t} J(w_0, x_0; \theta). \end{aligned} \tag{24}$$

A useful point of contrast is provided by the analogous expression for the canonical Diamond-Mortensen-Pissarides model considered by Kudlyak, in which $s = \mu = \sigma = 0$:

$$\omega(w_0, x_0; \theta)|_{s=\mu=\sigma=0} = w_0 + (r+\delta)J(w_0, x_0; \theta)|_{s=\mu=\sigma=0} - \frac{\partial}{\partial t} J(w_0, x_0; \theta)\Big|_{s=\mu=\sigma=0}. \tag{25}$$

Comparing the latter expressions provides a useful perspective on the economic forces in the model. Absent idiosyncratic shocks that induce *ex post* adjustments of the wage—arising either from on-the-job search ($s = 0$), or variations in productivity ($\mu = \sigma = 0$)—user cost in (25) is comprised simply by the flow wage $w_0$, and the flow-equivalent costs associated with discounting and exogenous job destruction $(r + \delta)J(w_0, x_0; \theta)$, set against the flow value of any future changes in entry wages $w_0$, productivity $x_0$, and tightness $\theta$.

The presence of on-the-job search ($s > 0$), productivity shocks ($\mu \neq 0 \neq \sigma$), and the *ex post* wage adjustments that they induce, further gives rise to two additional components of user cost in (24), with important economic implications.

First, on-the-job search generates additional costs to the firm associated with job-to-job quits, and the retention compensation for contacted workers. Among new matches of productivity $x_0$, the latter coincide, with a user cost contribution of $s\lambda(\theta)J(w_0, x_0; \theta)$. Although reminiscent of the analogous contribution of exogenous job destruction $\delta$, a fundamental economic difference is the *endogeneity* of the contact rate $s\lambda(\theta)$, which will naturally vary, for example, with the economic cycle.

Second, drift and variance in productivity give rise to the user cost contributions $-\mu x_0 J_x(w_0, x_0; \theta) - (\sigma^2/2)x_0^2 J_{xx}(w_0, x_0; \theta)$, which capture the flow value of future changes in productivity. Notice, however, that these terms encompass a variety of economic forces: Both the direct effect of changes in productivity, as well as indirect effects associated with endogenous wage adjustments, and endogenous job destruction at $x_l$, induced by them.

To illustrate, note first from the Bellman equation for firm surplus (10) that the sum of all components of the user cost in (24) by definition must equal the productivity of a new match $x_0$. Figure 5 then traces out the remaining subcomponents of the steady-state user cost of labor as a function of the initial wage $w_0$.

The first captures the user cost contribution $w_0 + (r + \delta)J(w_0, x_0; \theta)$. Dominated by the initial flow wage, this subcomponent is increasing in $w_0$ and near-linear. Adding in the turnover costs, $s\lambda(\theta)J(w_0, x_0; \theta)$, then has two effects. First, it substantially raises the user cost contribution, indicating that turnover costs are nontrivial to the firm. Second, the slope of the user cost contribution in the initial wage $w_0$ is reduced. Intuitively, the prospect that the worker may quit, or require retention compensation, in the future blunts the salience of the initial wage in the firm's effective flow cost of labor.

What remains to satisfy the identity that user cost equals initial productivity $x_0$ is the effect of shocks to idiosyncratic productivity. The message of Figure 5 is that these act as a source of moderation to the user cost of labor, and one that further diminishes the importance of the initial wage $w_0$. Intuitively, mirroring an earlier lesson of the model—that the allocative effects of wages are tied to their expected duration—the prospect that future changes in productivity will induce future changes in wages limits the effects of the initial wage $w_0$ on the user cost of labor.

# 2. Quantitative exploration

We now provide a quantitative illustration of the model's ability to capture relevant empirical evidence on the prevalence of long-term jobs and the sporadic nature of wage adjustment, and their associated implications for the allocative effects of wages.

Table 1 summarizes an illustrative calibration of the model. We focus on a parsimonious case in which $\mu = \sigma^2/2$, so that log match productivity is driftless, and $\Delta_e = \Delta_f$, so that the credibility of unilateral threats is symmetric across employees and firms.

Given this, we begin by normalizing the initial productivity of a match, $x_0 \equiv 1$, and setting the monthly discount rate $r$ to replicate an annual real interest rate of 5 percent. The remaining parameters of the model are then calibrated as follows.

First, since the exogenous separation rate $\delta$ is the limit of the total separation rate as tenure rises in our model (Whitmore 1979; Buhai and Teulings 2014), we set $\delta$ equal to the limiting monthly separation rate among high-tenure jobs of about 1 percent reported by Farber (1999, Figure 5). We then choose the annuitized value of unemployment $rU$ and the intensity of on-the-job search $s$ to match a steady-state unemployment rate of 6 percent and a monthly job-to-job (E-to-E) transition rate of 2.5 percent (Fujita, Moscarini and Postel-Vinay 2021). Intuitively, $rU$ shapes the separation rate into unemployment via the reservation productivity $x_l$, and the intensity of on-the-job search $s$ determines the arrival rate of outside offers and, thereby, the rate of job-to-job transitions.

Second, we choose the volatility of idiosyncratic productivity shocks $\sigma$ and the breakdown probabilities $\Delta_e = \Delta_f$ to target, respectively, estimates of the size and incidence of base wage and compensation changes among job stayers documented by Grigsby et al. (2021, section 8). Recalling our initial focus on a zero-inflation environment, a convenient aspect of Grigsby et al.'s data on nominal pay changes is that they pertain to a period of very low inflation. Accordingly, we choose $\sigma$ to target a standard deviation of annual log base wage changes among job stayers of 0.053, and $\Delta_e = \Delta_f$ to replicate an incidence of compensation (base plus bonus pay) freezes of 17 percent. Intuitively, the volatility of shocks $\sigma$ naturally passes through into dispersion in wage changes, while the credibility of unilateral threats, indexed by $\Delta_e = \Delta_f$, shapes the incidence of pay changes. Appendix C confirms the latter numerically. We return to the model's implications for nominal wage adjustments in an inflationary environment in section 3.

Table 1. Parameters and targeted moments (monthly frequency)

| Parameter | | Value | Reason / Moment | Target |
|---|---|---|---|---|
| *A. Initial parameters* | | | | |
| $x_0$ | Initial productivity | 1 | Normalization | — |
| $r$ | Discount rate | 0.004 | Annual real interest rate | 0.05 |
| *B. Long-term employment relationships* | | | | |
| $\delta$ | Exogenous separation rate | 0.010 | Limiting high-tenure sep. rate | 0.010 |
| $rU$ | Annuitized value of unemployment | 1.119 | Unemployment rate | 0.060 |
| $s$ | Employed search intensity | 0.569 | E-to-E rate | 0.025 |
| *C. Wage adjustment* | | | | |
| $\sigma$ | Standard deviation of $x$ | 0.036 | Std. dev. annual log wage change | 0.053 |
| $\Delta_{e,f}$ | Breakdown probability | 0.330 | Incidence of compensation freezes | 0.170 |
| $\beta$ | Worker bargaining power | 0.200 | See text | — |
| *D. Aggregate parameterization* | | | | |
| $\alpha$ | Matching elasticity | 0.5 | Standard | 0.5 |
| $m_0$ | Matching efficiency | 0.25 | Tightness $\theta$ | 1 |
| $c_v$ | Flow vacancy cost | 0.097 | Job-finding rate $\lambda$ | 0.25 |
| $b$ | Flow unemployment payoff | 0.974 | Annuitized value $rU$ | 1.119 |

Notes. The rationale and source for each targeted moment are explained in detail in the main text.

It remains to specify worker bargaining power $\beta$. We find that setting $\beta$ equal to 0.2 gives rise to several reasonable outcomes in the calibrated model.[15] First, it yields a wage passthrough elasticity—the log change in wages induced by a one log point rise in match productivity—equal to 0.22, somewhat greater than the recent estimates of Lamadon et al. (2022, Table 2) and somewhat less than the range of estimates reported by Kline et al. (2019, Table VIII). Second, the magnitude of hiring costs (net of recruitment bonuses) in the calibrated model corresponds to 1.4 months of hiring wages. This is consistent with the early results of Oi (1962), which have been borne out in more-recent work (Manning 2011; Gavazza et al. 2018). Third, the calibrated model implies that the base pay share of compensation is 97 percent at the median, which coincides with the analogous statistic reported by Grigsby et al. (2021, Table 1). Fourth, the flow payoff to unemployment $b$ corresponds to approximately two-thirds of average match productivity. This implies an

[15] Mirroring our later analysis of the implied distributions of wage and compensation changes, the outcomes that follow are based on a two-year panel of job stayers simulated from the calibrated model.

average flow surplus to employment relationships that is in the range of values considered plausible in related literature (Hall and Milgrom 2007; Mortensen and Nagypal 2007).[16]

The final ingredients relate to aggregate labor market equilibrium. Since our goal is to draw out the novel implications of the theory, we adopt a conventional approach. The matching function is assumed to be Cobb-Douglas,

$$m(u+s(1-u),v)=m_0\cdot[u+s(1-u)]^{\alpha}v^{1-\alpha}. \tag{26}$$

We set the matching elasticity $\alpha$ equal to 0.5. Matching efficiency $m_0$, and the flow vacancy cost $c_v$, are then chosen to deliver a (normalized) equilibrium labor market tightness $\theta$ of 1, and a monthly job-finding rate $\lambda(\theta)$ of 0.25, consistent with the unemployment-to-employment transition rate in the Current Population Survey "gross flows" data. Finally, the flow payoff from unemployment $b$ is set to generate the equilibrium annuitized value of unemployment $rU$ pinned down earlier.

In the subsections that follow, we explore the model's implications for the allocative effects of wages and bonus pay. Before we do so, we first confront the model with further moments of the stylized facts that informed it: the prevalence of long-term employment relationships, and the intermittent adjustment of wages, and compensation more broadly.

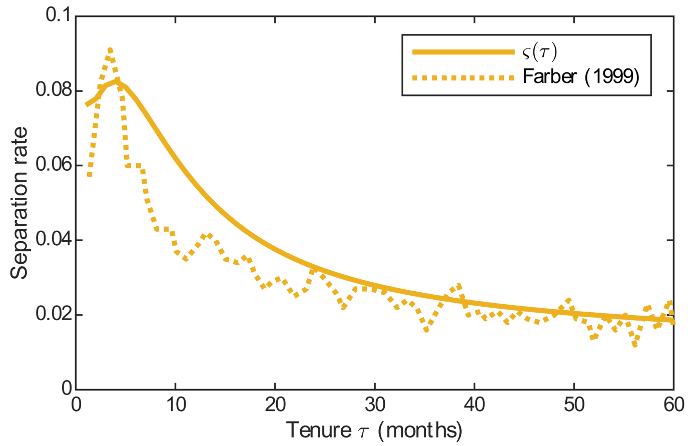
## 2.1 Durability of employment relationships

Figure 6 reiterates the model-generated pattern of separation rates by tenure from Figure 1 above, and juxtaposes it against analogous estimates reported by Farber (1999). In addition to replicating the (targeted) limiting separation rate, the calibrated model also generates a (nontargeted) pattern of separation rates by tenure that resembles Farber's estimates: Similar to the data, separation rates in the model are hump-shaped among low-tenure matches, with a peak of just over 0.08 at around 3 to 5 months of tenure, and declining thereafter. The model understates the hump, and overstates separation rates at middling levels of tenure; but there is a clear broad similarity between model and data.

Relatedly, we also have confirmed that, despite its parsimony, the model has sensible implications for the relationship between wages and tenure. Although we have confined ourselves in the main text to the case in which there is no general tendency to accumulate specific human capital on the job ($\mu=\sigma^2/2$), the presence of dynamic selection—the process of weeding out low productivity matches—implies that average wages rise with

[16] One could imagine calibrating $\beta$ more formally to minimize the distance between model outcomes and these targets, with some specified weights. The simple motivation we provide in the main text of course corresponds to one such set of weights that we think is reasonable. We think this motivation is more straightforward and direct.

Figure 6. Separation rates by tenure: Model vs. data




Notes. Parameter values are based on the model calibrated as described in Table 1. Data are transcribed from the lower-left panel of Figure 5 in Farber (1999).

tenure. Moreover, these wage gains would be lost if the job were to end exogenously. The implied return to tenure turns out to be quantitatively sensible: Cumulative growth in average wages after 10 years of tenure is approximately 15 percent in the model. The latter is less than Topel's (1991) estimate of over 25 percent, and a little more than Altonji and Williams' (2005) estimate of 11 percent.

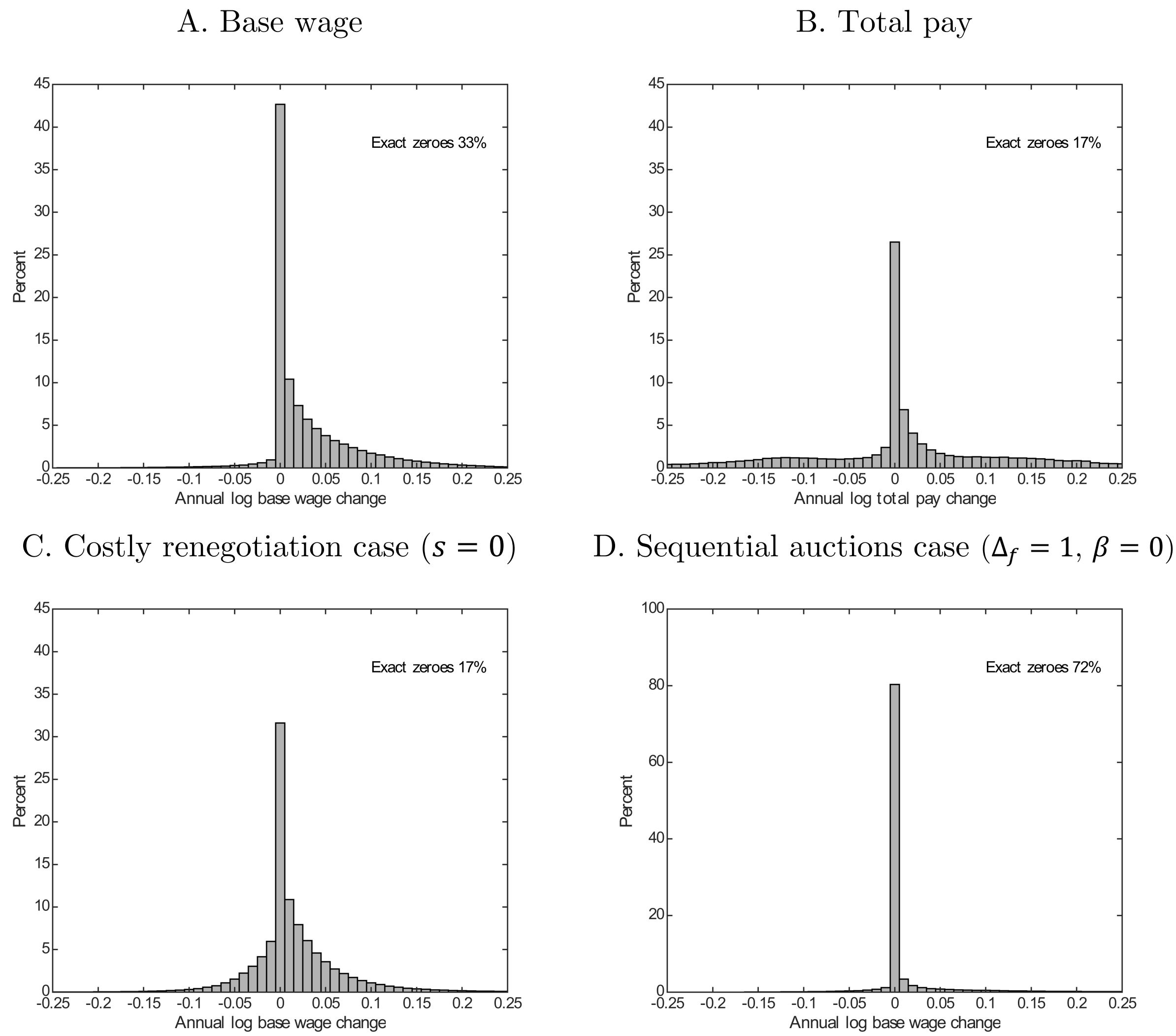
## 2.2 Wage and compensation adjustment

A natural implication of Figure 2C is that the distribution of wage and compensation changes takes a form reminiscent of their analogues in microdata (Malcomson 1997). Intermittent adjustment yields a mass point at zero change; idiosyncratic shocks and outside offers generate tails of wage cuts and increases. The calibration targets a subset of these outcomes—the dispersion of base wage changes, and frequency of compensation changes. We now assess model implications for these outcomes more generally.

Figure 7 depicts the distributions of annual base wage (panel A) and compensation (i.e. base wage plus bonus pay, panel B) changes among job stayers implied by the model. These display several key properties that resemble their empirical analogues.

Consider first Figure 7A. In addition to the paucity of base wage cuts, the model-implied distribution features a large incidence of base wage freezes: Over 30 percent of job stayers in the model realize no change in their base pay from year to year. Furthermore, the distribution is prominently asymmetric, with many more realizations of base pay increases than cuts. Both are key features of empirical distributions of base pay changes, as documented by Altonji and Devereux (2000) using data for a large corporation, and by Grigsby et al. (2021) using microdata from the ADP payroll processing company.

Figure 7. Model-implied distributions of wage changes among job stayers




Notes. Panels A and B: Parameter values for the baseline model are calibrated as described in Table 1. Panel C: Parameter values based on a recalibration that sets the E-to-E rate target to zero in Table 1. Panel D: Parameter values based on a recalibration that omits the incidence of compensation freezes target in Table 1 and sets $\Delta_f = 1$ and $\beta = 0$.

An interesting corollary is that the model is able to capture the asymmetry of base wage changes *despite* the presence of a symmetric friction—recall that the parameters underlying Figure 7 are such that $\Delta_e = \Delta_f$. This observation, anticipated qualitatively by Malcomson (1997), emerges in the model from three natural forces. First, and most directly, positive drift in match productivity, $\mu > 0$, generates more wage increases than wage cuts. Second, separations are concentrated in matches that otherwise would have cut the wage. A third reason for the asymmetry of base wage changes relates to the structure of compensation. Figure 7B depicts the analogous distribution of compensation (i.e., base wage plus bonus pay) changes in the model. In addition to the much lower (targeted) incidence of pay freezes, this further reveals that compensation changes are also much more symmetric than base wage changes. Intuitively, when a portion of pay is delivered as a bonus—in this case, for recruitment and retention purposes—two additional forces come into play. First, there is a greater mass of increases in overall compensation driven by the realization of these bonuses. Second, the flipside is that there is a greater mass of cuts in overall compensation driven by workers who have received bonuses in the past, but not in the present.

This implication of the model in turn dovetails with two dimensions of related empirical evidence. First, recent studies of annual changes in overall hourly earnings report substantially greater incidence of cuts than the picture for base wages in Figure 7A (Kurmann and McEntarfer 2018; Jardim et al. 2019). Second, studies that differentiate components of pay find that non-base pay is responsible for much of the flexibility in overall compensation. Shin and Solon (2007) note that a large part of the procyclicality of real wages can be traced to variation in non-base pay. And Grigsby et al. (2021) report that the distribution of annual changes in base plus bonus pay exhibits considerably greater flexibility, and symmetry, compared to their analogous results for base wages.[17]

The remaining panels of Figure 7 depict analogous results for special cases, recalibrated to match the relevant targets in Table 1. Outcomes are very similar in the costly renegotiation case (with on-the-job search suspended, $s = 0$) in Figure 7C. However, the sequential auctions case ($\Delta_f = 1$, $\beta = 0$) in Figure 7D is considerably different, greatly overstating the incidence of pay freezes.[18] Echoing Postel-Vinay and Turon (2010), the arrival rate of job offers $s\lambda$ consistent with the empirical job-to-job transition rate is too low to account for the empirical incidence of wage change. This, in turn, underscores the important role of renegotiation costs in our model in accommodating intermediate, empirically realistic degrees of wage adjustment.

The theory of wage determination developed in the preceding sections is thus able to capture many of the salient features of both the durability of employment relationships, and the evolution of wages over their course. That the model can faithfully replicate these stylized facts lends credence to its use in studying the allocative effects of wages and bonus pay, topics that we now take up in the subsections that follow.

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[17] Lebow et al. (2003) report similar results using job-level microdata from the Employment Cost Index.

[18] We have also confirmed that a related recalibration in which $\Delta_e = \Delta_f = 1$ and $\beta > 0$ also delivers a similarly counterfactual distribution of wage changes—indeed, the share of pay freezes in this case is very similar to that in Figure 7D, at 75 percent.

## 2.3 The allocative effects of entry wages

We begin by revisiting an influential theory of the allocative effects of rigidity in entry wages developed by Hall (2005). Hall's model can be viewed as a simple special case of our environment in which there is no on-the-job search ($s = 0$), there are no idiosyncratic shocks to match productivity ($\mu = \sigma = 0$), and neither firm nor worker credibly can issue unilateral threats to renegotiate the wage ($\Delta_e = \Delta_f = 1$).

The drunken walk illustrated in Figure 2C is particularly simple in this case, as any initial wage $w_0$ need *never* be adjusted *ex post*. This feature of the model yields a theory of unemployment fluctuations whereby rigidity in entry wages squeezes firm surplus in recession, retarding job creation, and raising unemployment. A celebrated feature is that the theory generates unemployment fluctuations without violating bilateral efficiency.

In this subsection, we show that the presence of on-the-job search and idiosyncratic shocks to match productivity has an important bearing on the role of entry wage rigidity in unemployment fluctuations. To do so, we modify the match product to equal $px$, and consider a perturbation to aggregate labor productivity $p$. We allow the entry wage of a new hire from unemployment $w_0$ to vary flexibly with this perturbation, with elasticity $\epsilon_{w_0,p}$. Thus, as in Hall (2005), the response of $w_0$ to $p$ in general will deviate from that implied by Nash surplus sharing. And, as in Hall (2005), this will imply no violation of the wage adjustment bounds, given a small perturbation to $p$.

Given the conventional search and matching structure, a summary statistic for the implied volatility of job creation is the elasticity of labor market tightness $\theta$ with respect to $p$, denoted $\epsilon_{\theta,p}$. Rewrite the job creation condition (22) as $J(w_0, p; \theta) \cdot R(p, \theta) = c_v/q(\theta)$, where $R$ is the scaling of the firm's effective surplus share due to on-the-job search. Totally differentiating allows $\epsilon_{\theta,p}$ to be decomposed into four components,[19]

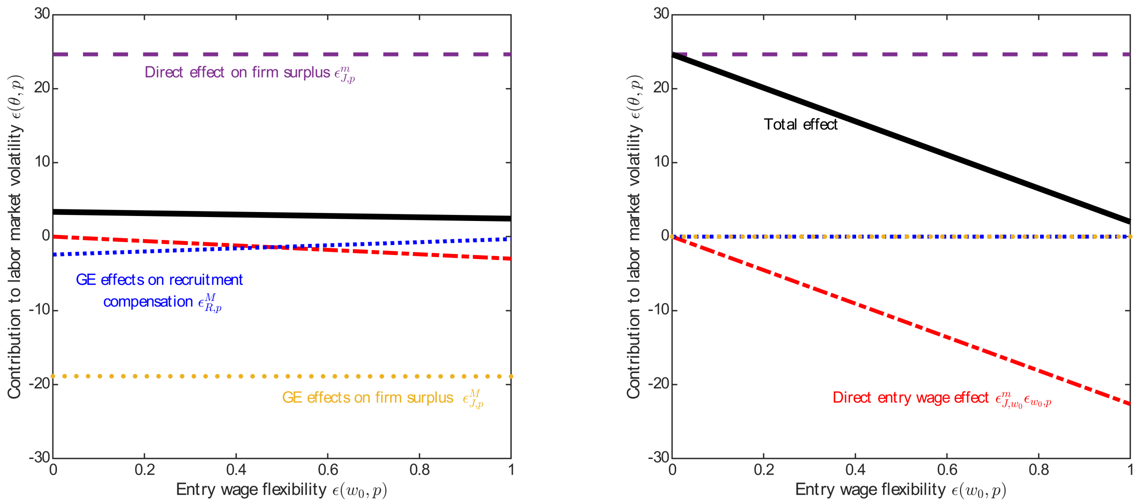
$$\epsilon_{\theta,p} = \left(\epsilon^m_{J,p} + \epsilon^m_{J,w_0} \cdot \epsilon_{w_0,p} + \epsilon^M_{J,p} + \epsilon^M_{R,p}\right)/\alpha, \tag{27}$$

where a superscript $m$ denotes a *micro* (or partial equilibrium) elasticity, and a superscript $M$ denotes a *macro* (or general equilibrium) elasticity.

[19] This decomposition applies to *steady-state* changes in labor market tightness. Prior work has shown that the latter provides a good approximation to transition dynamics in standard search-and-matching models (Shimer 2005). The same argument does not apply immediately to our environment with on-the-job search and idiosyncratic shocks, since the job creation condition (22) depends on both the unemployment rate $u$ and the worker distribution $G$ for $x < x_0$. It is well-known that, given the magnitude of U.S. worker flows, the dynamics of $u$ are short-lived (Shimer 2005). In addition, under the calibration, the movements in $G$ in response to $p$ are quantitatively negligible for the job creation condition (22), relative to the movements in either $p$ or $u$. The decomposition in (27) is thus likely a good approximation to the dynamics of our model.

Figure 8. The allocative effects of entry wages

A. Baseline model ($s, \sigma > 0$) B. No *ex post* adjustments ($s = \sigma = 0$)




Notes. Panel A: Parameter values are based on the model calibrated as described in Table 1. Panel B: Idiosyncratic shocks and on-the-job search are eliminated $s = \mu = \sigma = 0$; unilateral threats to renegotiate are not credible, $\Delta_e = \Delta_f = 1$, so bargaining power $\beta$ is redundant; $w_0$ is set to replicate the direct effect on firm surplus $\epsilon^m_{J,p}$. The latter, in turn, is similar to the empirical magnitudes emphasized by Shimer (2005).

The micro elasticities capture changes in a firm's job creation incentives for a fixed labor market equilibrium. $\epsilon^m_{J,p} > 0$ summarizes the direct effect of an increase in aggregate productivity on a firm's surplus. $\epsilon^m_{J,w_0} \cdot \epsilon_{w_0,p} \leq 0$ is the partial equilibrium effect of entry wages. To the extent that the entry wage is flexible, $\epsilon_{w_0,p} \geq 0$, and the entry wage affects a firm's surplus, $\epsilon^m_{J,w_0} \leq 0$, this will moderate the rise in firm's incentives to create jobs.

The macro elasticities capture changes in job creation incentives due to changes in labor market equilibrium. $\epsilon^M_{J,p} \leq 0$ summarizes the total effect of equilibrium changes on firm surplus (combining the effects of increases in the offer arrival rate for employed workers $s\lambda$, and rises in the annuitized value of search for unemployed workers $rU$). Similarly, $\epsilon^M_{R,p} \leq 0$ is the total effect of equilibrium changes on recruitment compensation. Intuitively, firms anticipate that their workers will quit—or require retention compensation—with greater frequency, and that wages will have to be raised more in the future when outside options improve. Both moderate job creation incentives.

Finally, the implications of all four effects for labor market volatility are shaped by the denominator in (27), the elasticity of the matching function, $\alpha = 0.5$.

This decomposition provides an important point of contrast with Hall (2005). There, the absence of on-the-job search and idiosyncratic productivity shocks implies that wages need never be adjusted. The legacy of any initial entry wage is thus indefinite, and the initial wage has profound effects on the valuation of a prospective match to a firm, and thereby on job creation. Formally, result (ii) of Proposition 2 implies that the marginal valuation of the wage to the firm, and thereby the allocative effects of entry wages, are *maximal* in this special case, $J_w(w,x)|_{s=\mu=\sigma=0} = -1/(r+\delta)$.

By contrast, in the presence of on-the-job search, and idiosyncratic productivity shocks, the duration of the hiring wage is truncated by *ex post* wage adjustments, and the allocative implications of entry wages are moderated. Note that these *ex post* wage adjustments are not forced in the model, but arise endogenously through agents' decisions, which in turn preserve bilaterally efficient matches. Intuitively, wages must change *ex post* to preserve profitable matches. In turn, adjusted wages are naturally independent of the initial wage. This imparts on wages a form of *memorylessness*: Return to the drunken walk in Figure 2C, and consider a perturbation of the initial wage. Conditional on adjustment, the subsequent sample path of wages is unaltered by the initial perturbation. Upon adjustment, the entry wage is "forgotten," and its allocative effects end.

This implication of the model dovetails well with the substantial empirical literature initiated by Beaudry and DiNardo (1991). Most of that literature finds that, once one conditions on the history of economic conditions since initiation of the employment relationship, economic conditions at the outset of the relationship have no discernible explanatory power for the current wage.

Returning to the special case studied by Hall (2005), note that the absence of both on-the-job search and idiosyncratic shocks to match productivity implies that no *ex post* wage adjustments are required by bilateral efficiency in this case. The magnitude of the allocative effects of entry wages rests on this.

Figure 8 provides a quantitative illustration of this point. It applies the decomposition of labor market volatility in (27) to the baseline model parameterized as in Table 1, and to a comparable model without on-the-job search ($s = 0$), or idiosyncratic shocks ($\mu = \sigma = 0$). To underscore the role of entry wage flexibility, the decomposition is presented as a function of the flexibility of wages for hires from unemployment, summarized by $\epsilon_{w_0,p}$.

This highlights two implications of *ex post* wage adjustments. First, as foreshadowed above, the entry wage effect, $\epsilon^m_{J,w_0} \cdot \epsilon_{w_0,p}$, is much less consequential. Second, general equilibrium effects on firm surplus, $\epsilon^M_{J,p}$, provide a considerable source of moderation of labor market volatility. Put simply, firms anticipate that wages will have to be raised more frequently in the future following expansionary aggregate shocks, limiting the increased incentive to create jobs.

Table 2. The allocative effects of recruitment and retention bonuses

| **Outcome** | **Baseline** | **Without bonuses** | |
|---|---|---|---|
| | | Retention | Retention and recruitment |
| Incidence of base pay cuts | 0.050 | 0.095 | 0.091 |
| Loss of surplus in new match | 0% | 15.1% | 16.0% |
| Rise in E-to-U rate | 0% | 9.5% | 10.3% |

Notes. Parameter values are based on the model calibrated as described in Table 1.

The upshot is that *ex post* wage adjustments required by the combination of on-the-job search, idiosyncratic shocks and bilateral efficiency dampen labor market volatility, and render it much less responsive to wages at the point of hiring.

Figure 8 thus provides a novel answer to a quantitative question: How much of Hall's insight on the allocative effects of hiring wages survives in a model of efficient long-term employment relationships with empirically realistic wage adjustments? Figure 8 suggests not much. This simple point has not been articulated formally, to our knowledge, and is quantitatively significant.

## 2.4 The allocative effects of recruitment and retention bonuses

A further key implication of the model is that it provides a novel interpretation of base and non-base pay. Specifically, the flexibility afforded by non-base pay has important *allocative* consequences, that take a novel form. Absent an ability to respond fully to outside offers via recruitment and retention bonuses, job-to-job separations will be bilaterally *inefficient*, and matches will face additional costs associated with turnover. Anticipating this *ex ante*, valuations of a match will be retarded.

We now explore these novel implications of the model. Since bonuses arise from firms responding to workers' outside offers, we explore their allocative effects using an extension of the approach to incomplete offer matching developed in Elsby and Gottfries (2022). Specifically, we consider a simple extension of the bargaining model of Cahuc et al. (2006) invoked thus far. There, upon realization of an outside offer, the two firms bid for the worker in an initial stage, and the worker then uses the less-productive firm as an outside option for bargaining with the more-productive firm in a second stage. Our approach here is to introduce a probability $\zeta$ that the bid of the less-productive firm remains available after breakdown in negotiation with the more-productive match. (With complementary probability $1 - \zeta$, the worker becomes unemployed following such a breakdown.) This limits the role of outside offers in the determination of overall compensation and, thereby, the necessity of recruitment and retention bonuses.

Accordingly, the recruitment surplus delivered to a contacted worker currently in a match of productivity $x \leq x_0$ will be $\zeta S(x) + \beta[S(x_0) - \zeta S(x)]$. And the retention surplus delivered to a contacted worker currently in a match of productivity $x > x_0$ will be $\zeta S(x_0) + \beta(1 - \Delta_e)[S(x) - \zeta S(x_0)]$. These in turn have some convenient implications. First, note that the special case in which $\zeta = \Delta_e = \Delta_f = 0$ gives rise to continual linear surplus sharing, as in models of continual *ex post* renegotiation without offer matching (see Pissarides 1994, and the microfoundation provided by Gottfries 2021). Second, coupled with the maximum value that credibly can be delivered to a worker via increases in the flow wage, $[1 - (1 - \beta)(1 - \Delta_f)]S(x)$, one can find values of $\zeta$ that eliminate retention and recruitment bonuses, facilitating a study of their allocative consequences.[20]

Table 2 reports the implications of these two counterfactuals in the model, and compares them with the baseline model. A first message is that eliminating firms' ability to respond to outside offers with bonus pay considerably alters the incidence of cuts in base pay, which approximately doubles. Thus, the patterns of base and overall compensation changes in the baseline model studied earlier are intrinsically related.

A second message of Table 2 is that removing firms' ability to deliver bonuses has detrimental allocative consequences. First, Table 2 reports the percentage reduction in total match surplus among new matches, $S(x_0)$, implied by the elimination of bonus pay. This is substantial, around 15 percent of total surplus in the baseline model. Second, Table 2 reports the rise in the separation rate into unemployment. This too is nontrivial, rising by around 10 percent relative to the baseline model.

Our model's account of non-base pay is surely incomplete insofar as it does not address types of non-base pay aside from recruitment and retention bonuses. But our results regarding bonus pay vividly illustrate the important point that, even if non-base pay is transitory and a small percentage of overall compensation, the marginal adjustments it enables can matter considerably for the preservation of efficient matches.[21]

# 3. Nominal wage adjustment and inflation

Recall that the foregoing analyses can be interpreted equally as a characterization of *real*

---

[20] Specifically, setting $\zeta$ equal to $[\beta\Delta_e + (1 - \beta)\Delta_f]/[1 - \beta(1 - \Delta_e)]$ eliminates the payment of retention bonuses. Further lowering $\zeta$ to equal $\Delta_f$ additionally eliminates the payment of recruitment bonuses.

[21] Relatedly, our results also underscore the importance of sourcing high-quality microdata on the components of non-base pay.

wage adjustments, or of *nominal* wage adjustments in a zero-inflation environment. In this section, we address this distinction by extending the model to accommodate inflation. We suppose instead that, absent adjustment, it is the *nominal* wage that is held fixed; equivalently, the real wage $w$ drifts downward at the rate of inflation $\pi$, $\mathrm{d}w/\mathrm{d}t = -\pi w$.

We study this case for two reasons. First, it formalizes a view of nominal wage adjustments proposed by Malcomson (1997) whereby wage contracts set in nominal terms are renegotiated only by mutual consent. Second, as noted by Malcomson, it provides a simple interpretation of the wealth of microdata-based evidence on nominal wage changes.

In contrast to the baseline model, the firm and worker now must anticipate not only future capital gains associated with changes in match productivity $x$, but also in the real wage $w$. Formally, absent adjustment, the Bellman equation for the firm's surplus now satisfies a *partial* differential equation,

$$
\begin{aligned}
(r+\delta)J(w,x;\pi) = x - w - s\lambda \mathbf{1}_{\{x<x_0\}} J(w,x;\pi) \\
+ s\lambda \mathbf{1}_{\{x\geq x_0\}} \min\{[1-\beta(1-\Delta_e)][S(x)-S(x_0)] - J(w,x;\pi), 0\} \\
-\pi w J_w + \mu x J_x + \frac{1}{2}\sigma^2 x^2 J_{xx}
\end{aligned}
\tag{28}
$$

Relative to its analogue in the baseline model (10), downward drift in real wages is valued by the firm in proportion to the marginal value of the wage, yielding the capital gain $-\pi w J_w$. The presence of this additional capital gain complicates analytical solution.

It is possible to make progress, however, by seeking an approximate solution. Specifically, we derive a Taylor series expansion of $J(w,x;\pi)$ that decouples the partial differential equation (28) into two ordinary differential equations (a method due to Fleming 1971).[22] These in turn can be solved by an extension of our baseline approach.

**Proposition 5** *For any $w$, $i$, and $x \in N_i(w;\pi)$, the firm surplus has general solution*

$$
J(w,x;\pi) = J(w,x;0) + \pi \mathbb{P}_{\mathrm{N}}(w,x) + \pi j_{i1}(w) x^{-\sqrt{\frac{r+\delta}{\mu}}} + \pi j_{i2}(w) x^{\sqrt{\frac{r+\delta}{\mu}}} + O(\pi^2),
\tag{29}
$$

*where $J(w,x;0)$ is given in (15), and*

$$
\mathbb{P}_{\mathrm{N}}(w,x) = -\frac{w}{2\sqrt{(r+\delta)\mu}} \int^x \left[ \left(\frac{x}{\tilde{x}}\right)^{\sqrt{\frac{r+\delta}{\mu}}} - \left(\frac{x}{\tilde{x}}\right)^{-\sqrt{\frac{r+\delta}{\mu}}} \right] \frac{J_w(w,\tilde{x};0)}{\tilde{x}} \mathrm{d}\tilde{x}.
\tag{30}
$$

*For any $w$, $i$, and $x \in \bar{N}_i(w;\pi)$, the firm surplus has general solution*

[22] Thanks to Ben Moll for alerting us to Fleming's early work. The method has also recently been applied in consumption-savings problems by Kasa and Lei (2018) and Fagereng et al. (2021).

$$J(w,x;\pi) = J(w,x;0) + \pi\mathbb{P}_{\bar{\mathrm{N}}}(w,x) + \pi\jmath_{i1}(w)x^{-\sqrt{\frac{r+\delta+s\lambda}{\mu}}} + \pi\jmath_{i2}(w)x^{\sqrt{\frac{r+\delta+s\lambda}{\mu}}} + O(\pi^2), \tag{31}$$

*where* $J(w,x;0)$ *is given in (16), and*

$$\mathbb{P}_{\bar{\mathrm{N}}}(w,x) = -\frac{w}{2\sqrt{(r+\delta+s\lambda)\mu}}\int^{x}\left[\left(\frac{x}{\tilde{x}}\right)^{\sqrt{\frac{r+\delta+s\lambda}{\mu}}} - \left(\frac{x}{\tilde{x}}\right)^{-\sqrt{\frac{r+\delta+s\lambda}{\mu}}}\right]\frac{J_w(w,\tilde{x};0)}{\tilde{x}}\,\mathrm{d}\tilde{x}. \tag{32}$$

*The coefficients* $j_{i1}(w)$, $j_{i2}(w)$, $\jmath_{i1}(w)$, $\jmath_{i2}(w)$, *and boundaries* $x_e(w;\pi)$, $x_f(w;\pi)$, $x_n(w;\pi)$, *are implied by the boundary conditions (12), (13), and (14). The worker surplus* $V(w,x;\pi) = S(x) - J(w,x;\pi)$ *follows from (4).*

Two insights are key to the solution in Proposition 5. First, the marginal value of the wage in the absence of inflation, $J_w(w,x;0)$, is provided by the baseline solution, allowing approximation of the capital gain due to inflation, $-\pi w J_w$. Second, the value of this capital gain can be inferred analytically in the form of the particular solutions $\mathbb{P}_{\mathrm{N}}(w,x)$ and $\mathbb{P}_{\bar{\mathrm{N}}}(w,x)$ in (30) and (32), implying a solution for the value of inflation to firm and worker.

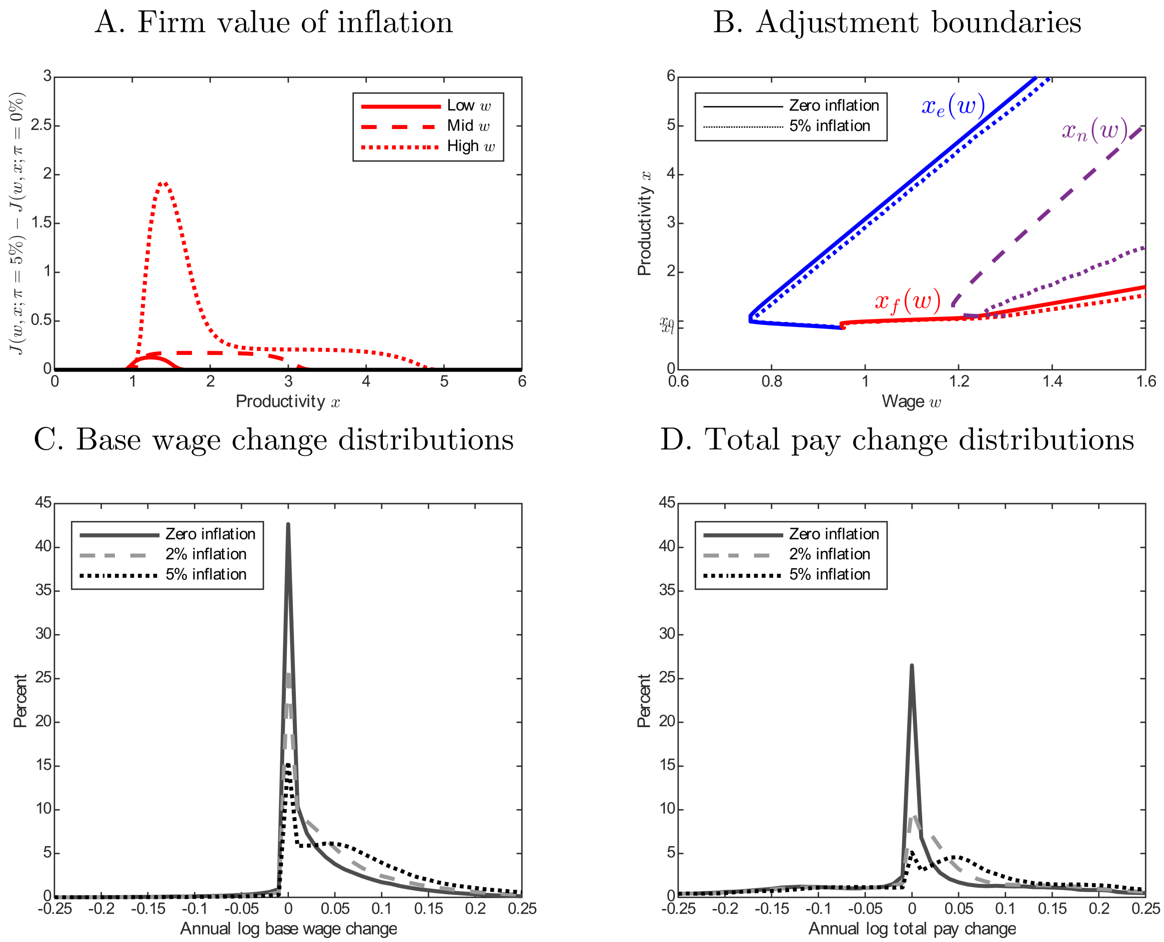
Inflation induces downward drift in real wages in the absence of renegotiation, providing an additional source of value to the firm (and loss to the worker). We illustrate this in Figure 9A. A consequence is that positive inflation lowers the adjustment boundaries. For a given productivity, firms are less likely to demand (nominal) wage cuts, and workers are more likely to demand (nominal) wage increases, as in Figure 9B.

The final panels of Figure 9 demonstrate that this simple extension of the baseline model has sensible implications for how nominal wage adjustments vary with inflation. They plot implied distributions of nominal base wage and compensation growth. These exhibit a spike at zero change that decays with inflation, a familiar feature of the empirical literature on nominal wage adjustment. Again, model outcomes are very sensible.

The upshot is that a simple interpretation of the baseline model extended to an inflationary environment can accommodate further salient empirical facts emphasized by the large empirical literature on nominal wage adjustments, and their relation to inflation. More generally, the solution in Proposition 5 accommodates additional interpretations. Partial indexation would manifest as downward drift in real wages, but at a slower rate. Contracts that link wages to expected productivity growth would instead manifest as *upward* drift in real wages. Proposition 5 can also be extended to *state-dependent* contracts that link wage drift to match productivity $x$.[23] We leave these extensions for future work.

[23] Furthermore, analogous methods to those used in Proposition 5 may facilitate solution of related models with (small or infrequent) aggregate shocks, by approximating the solution around the no-shock case.

Figure 9. Nominal wage adjustment and inflation




Notes. Parameter values are based on the model calibrated as described in Table 1.

# 4. Summary and discussion

In this paper, we have explored an interpretation of sporadic wage adjustment in which long-term employment relationships are held to be bilaterally efficient—a view that has its origins in the influential early work of Becker (1962), and which pervades important subsequent insights due to Barro (1977), Malcomson (1997), and Hall (2005).

We offer two contributions. First, we propose a model in which efficient long-term employment relationships are sustained by wage adjustments prompted by idiosyncratic shocks and outside job offers. A useful feature of the model is that it nests canonical approaches to wage determination (Mortensen and Pissarides 1994; Postel-Vinay and Robin 2002; Cahuc et al. 2006).

Our second contribution is to draw out insights on the durability of employment relationships, wage adjustments, the structure of pay, and unemployment dynamics.

Echoing the qualitative insights of Malcomson (1997), our quantitative exploration of the model reveals that it is able to account for a range of empirical facts. Separation hazards are hump-shaped in tenure, mirroring the empirical results of Farber (1999). Distributions of wage changes for job stayers exhibit spikes at zero, and a relative lack of wage cuts, mirroring the data (e.g., Elsby and Solon 2019).

The model naturally yields a novel theory of base and non-base pay. The implied dynamics of the components of pay further resemble recent empirical findings (Grigsby et al. 2021). Interestingly, the model also implies that this form of non-base pay is allocative: Absent the ability to use non-base pay, firm value is reduced, and separations are inefficiently elevated.

Despite these successes, an important failing of the model is that it is unable to account for unemployment fluctuations. This emerges from a novel perspective on Hall's (2005) influential theory of rigid hiring wages. The presence of idiosyncratic shocks and outside job offers mutes this source of unemployment fluctuations due to a *memorylessness* property: *Ex post* adjustments required to preserve bilaterally efficient employment relationships render future wages independent of the hiring wage.

This insight suggests fruitful directions for work on the sources of unemployment fluctuations. First, one could entertain possible violations of bilateral efficiency. Second, one could pursue theories of history dependence in wages, such that the legacy of any (hiring) wage is longer than the time it takes to be adjusted. Finally, one could appeal to sources of unemployment fluctuations that emphasize instead volatility in labor *demand*. We hope the present paper will stimulate further research along these lines.

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