NBER WORKING PAPER SERIES

MONETARY-FISCAL COORDINATION WITH INTERNATIONAL HEGEMON

Xuning Ding
Zhengyang Jiang

Working Paper 33123
http://www.nber.org/papers/w33123

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
November 2024

Zhengyang Jiang gratefully acknowledges the research support from the William and Mary Breen Fund at the Kellogg School of Management, Northwestern University. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

NBER working papers are circulated for discussion and comment purposes. They have not been peer-reviewed or been subject to the review by the NBER Board of Directors that accompanies official NBER publications.

© 2024 by Xuning Ding and Zhengyang Jiang. All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including © notice, is given to the source.

Monetary-Fiscal Coordination with International Hegemon
Xuning Ding and Zhengyang Jiang
NBER Working Paper No. 33123
November 2024
JEL No. E52, E63, F33

ABSTRACT

Monetary and fiscal policies require coordination to achieve desired macroeconomic outcomes. The literature since Leeper (1991) has focused on two regimes: monetary dominance and fiscal dominance. In both cases, one policy is active while the other is passive and accommodates the former. We study this coordination problem in an international economy, and find a third regime—hegemon dominance. In this case, one country (the hegemon)'s monetary and fiscal authorities can pursue separate policy goals, while the other country's monetary and fiscal policies are both accommodative. For example, the hegemon can pursue a monetary policy unbacked by its fiscal policy. When this happens, the foreign monetary authority has to take the same stance as the hegemon, the foreign fiscal authority has to provide fiscal backing for the monetary stance undertaken by both countries, and the exchange rate adjusts to equilibrate the economy. Our result suggests that the U.S. fiscal policy's independence from its own monetary policy can be made possible by accommodative foreign policies, and that the Fed's effort to fight inflation can succeed despite the high level of public debt which would have required enormous fiscal backing in a closed economy.

Xuning Ding
Kellogg School of Management
Northwestern University
Evanston, IL 60201
xuning.ding@kellogg.northwestern.edu

Zhengyang Jiang
Kellogg School of Management
Northwestern University
2211 Campus Drive
Evanston, IL 60208
and NBER
zhengyang.jiang@kellogg.northwestern.edu

1 Introduction

A common lesson in monetary economics is that coordination between monetary and fiscal policies is required to achieve desired outcomes. Leeper (1991) classifies two possible modes of coordination: monetary dominance and fiscal dominance. In the former case, the monetary authority uses the nominal interest rate policy to address inflation, while the fiscal authority accommodates the monetary stance by adjusting the primary surplus and keeping a stable debt level. In the latter case, the fiscal authority can pursue its own policy goals without committing to a stable debt path, while the monetary authority accommodates the fiscal stance by adjusting the nominal interest rate to stabilize the debt level.

In this paper, we study the monetary-fiscal coordination in an international economy. We manage to recover the standard monetary and fiscal dominance regimes, which are similar to closed-economy cases in isolated countries. Moreover, a third possibility arises: the monetary and fiscal authorities in one country pursue active policy goals, whereas the monetary and fiscal authorities in the other country accommodate the policies in the first country. We refer to this regime as *hegemon dominance*, and refer to the country taking the active role as the hegemon. We study this novel regime in a tractable international New Keynesian model, and report three key results.

First, under hegemon dominance, the hegemon country can pursue an active monetary policy without providing fiscal backing. When this happens, the other country's monetary policy has to take the same stance as the hegemon's monetary policy. To understand this result, first consider the standard monetary dominance regime in a closed economy. Suppose the monetary authority raises the nominal interest rate. As Leeper (2021) notes, a higher nominal rate increases households' interest income and, by reducing inflation, increases the real value of the households' nominal assets. Both effects increase the households' real spending power. In order to ensure that the households' expenditure remains feasible under the resource constraint, the fiscal authority needs to raise tax to "absorb" the excess demand. In this way, an active monetary policy stance requires coordination from the fiscal authority.

While tax adjustment is the only possible response to attain equilibrium in a closed economy, open economy allows for an alternative mechanism. When the home country's nominal interest rate increases without corresponding adjustment in the tax rate, the excess demand can also be absorbed by exchange rate movement. Specifically, the home currency has to depreciate to lower the home households' real spending power in the world numéraire, which offsets the positive effect of the home monetary policy on household budget. To engineer the required exchange rate response, the foreign country's monetary policy has to be in the same direction as the home country's, which generates comovements in home and foreign nominal interest rates under hegemon dominance.

Second, while the hegemon's fiscal authority can pursue its own policy goal without accommodating its own active monetary policy, the other country's fiscal authority has to accommodate the monetary stance undertaken by both countries. The foreign fiscal authority responds in the same way as a passive fiscal authority responds to an active monetary policy in the closed economy, which maintains a consolidated version of the intertemporal government budget condition. In this sense, the hegemon transfers the responsibility of providing fiscal backing to its monetary policy from the domestic fiscal authority to the foreign one.

Third, under hegemon dominance, the hegemon country's real exchange rate appreciates when the hegemon lowers its nominal and real rates. As we noted in the first result, this exchange rate adjustment is the key mechanism that equilibrates the hegemon's active monetary policy with its lack of fiscal response. Since expansionary monetary policy tends to occur in global downturns, this result implies that the hegemon country's currency strength is countercyclical, which is consistent with the notion that the hegemon tends to have a safe-haven currency.

This hegemon dominance regime offers a potentially useful perspective for understanding the global imbalances and the fiscal situation faced by the U.S. It has the following implications for the U.S. and the global economy.

Disconnect between U.S. Monetary and Fiscal Policies. There has been a disconnect between the U.S. monetary and fiscal policies, especially in recent years. The Federal Reserve's monetary policy is not strongly influenced by the fiscal condition, while the public debt grows regardless of the monetary stance. The hegemon dominance regime provides an explanation for prolonged coexistence of active monetary and fiscal policies in the U.S., whereas in the prior literature, such coexistence is either ruled out or perceived to be temporary and eventually resolved to either monetary or fiscal dominance regime (Davig and Leeper, 2007; Bianchi and Ilut, 2017).

Comovement between U.S. and Foreign Monetary Policies. Monetary policies tend to comove across countries. One obvious explanation is that business cycles are correlated across countries, and monetary authorities respond to the same global shock. Our result provides an additional mechanism based on policy coordination. Moreover, under the hegemon dominance regime, the dollar exchange rate tends to strengthen during global monetary easing, which is also consistent with the empirical evidence.

U.S. Fiscal Expansion and Global Savings Glut. On the fiscal side, while the U.S. government has been running large deficits and quickly accumulating public debt, China, Japan, and other countries have been saving and accumulating large reserves. These global imbalances are also consistent with the hegemon dominance regime, under which the fiscal backing for U.S. monetary policy lies beyond its national border. Under this regime, the U.S. monetary policy's independence from its own fiscal policy can be made possible by accommodative policies in foreign countries.

Can U.S. Control Its Inflation? The results in this paper also leave an optimistic message for the U.S. ability to control inflation. Under monetary dominance, the amount of fiscal backing in the form of tax increase required to accommodate monetary tightening and to successfully lower inflation is increasing in the level of public debt. A higher level of public debt requires more fiscal resources to offset the increase in its real value and interest expense in response to monetary tightening. As a result, Leeper (2021) worries that the U.S. today can no longer fight inflation as Volcker did in the 80s, because the debt level was 25% of GDP then and close to 100% of GDP now. Under hegemon dominance, the hegemon country's monetary authority can fight inflation while maintaining a low tax rate. The fiscal cost of fighting inflation is not borne domestically;

instead, the hegemon country can "export" inflation abroad by appreciating its currency, which is consistent with our experience post-Covid.

Model Details. We derive our results in a two-country New Keynesian model, which features standard consumption, production, and asset holding decisions. In Section 2, we first assume an extreme version of nominal rigidity: prices are fully sticky for one period. This allows us to derive algebraic solutions that clearly characterize monetary, fiscal, and hegemon dominance regimes. We obtain a very stark prediction: under monetary dominance, the monetary authorities in both countries set their nominal interest rates, while the fiscal authorities have no discretion at all; similarly, under fiscal dominance, the fiscal authorities set their primary surpluses which determine the nominal interest rates that the monetary authorities have to follow. In both cases, the active policies in home and foreign countries are independent of each other.

In contrast, under hegemon dominance, the monetary and fiscal authorities in the hegemon country can set their policies independently, and they dictate the monetary and fiscal policies that the other country has to follow. This third mode of monetary-fiscal coordination gives rise to the three results we discussed above, which characterize the foreign monetary, fiscal, and exchange rate responses to the hegemon's policy actions.

In Section 3, we relax the assumption of one-period full stickiness and consider the more realistic Calvo price setting. This allows us to introduce more flexible *policy rules* considered by Leeper (1991). For example, an active monetary policy rule means that the monetary authority can raise the nominal interest rate sufficiently to fight inflation; under monetary dominance, this requires the fiscal authority to systematically adjust government surpluses to stabilize the debt level. However, the fiscal authority can still have some discretion, which we model as exogenous policy shocks to the primary surplus. Similarly, an active fiscal policy rule means that the fiscal authority can set its primary surplus independently of the debt level; under fiscal dominance, this limits the monetary authority's ability to adjust the nominal interest rate in response to inflation, even though the monetary authority can still have some discretion, which we model as exogenous policy shocks to the nominal interest rate.

We again recover the monetary and fiscal dominance regimes, in which policies are set independently in the two countries. We also uncover the hegemon dominance regime, in which the hegemon's monetary and fiscal authorities are both active, while the other country's monetary and fiscal authorities are both passive. The impulse response patterns to the hegemon's policy shocks are consistent with the simpler setting with one-period full stickiness, while the Calvo price setting and policy rules generate more realistic and persistent dynamics.

In summary, our paper expands the modes of monetary-fiscal policy coordination by studying the international dimension and uncovering a new hegemon dominance regime. This result can potentially explain the disconnect between monetary and fiscal policies in the hegemon country, and provide a new perspective on the global imbalances and the fiscal situation faced by the U.S.

Literature. Our paper builds on the vast theoretical literature studying the interaction and coordination between monetary and fiscal policies (Sargent et al., 1981; Aiyagari and Gertler, 1985; Sims, 1994; Woodford, 1995; Cochrane, 1998; Loyo, 1999; Schmitt-Grohé and Uribe, 2000; Woodford, 2001; Benhabib et al., 2002; Bassetto, 2002; Uribe, 2006; Cochrane, 2011; Leeper and Zhou, 2013; Bassetto and Sargent, 2020; Gabaix, 2020; Alberola et al., 2021; Caramp and Silva, 2023). Leeper (1991) identifies two possible modes of monetary-fiscal coordination: monetary dominance and fiscal dominance. The policy regime may switch between these two patterns, and the time-varying policy mix has important macroeconomic consequences (Davig et al., 2006; Sims, 2011; Bianchi and Ilut, 2017; Bianchi and Melosi, 2017; Leeper et al., 2017; Bianchi and Melosi, 2019; Eusepi and Preston, 2018; Cochrane, 2018). Our paper contributes to this literature by studying the international dimension of policy coordination. We identify a new regime of hegemon dominance, in which monetary and fiscal policies can be both active in the hegemon country, and we study the implications of this regime for the global economy and policies.

A more recent literature studies mechanisms that equilibrate the intertemporal government budget condition (Reis, 2021; Mian, Straub, and Sufi, 2022; Brunnermeier, Merkel, and Sannikov, 2022; Angeletos, Lian, and Wolf, 2023; Gomez Cram, Kung, and Lustig, 2023; Cieslak, Li, and Pflueger, 2023; Diamond, Landvoigt, and Sánchez, 2024). Our paper proposes a new mechanism based on external adjustment and foreign policy coordination. Relatedly, Jiang et al. (2020, 2024) find evidence that the U.S. government issues more debt than backed by future primary surpluses, suggesting a potential violation of the intertemporal government budget condition. Chen et al. (2022) find this violation in past international hegemons. Our paper shows that the hegemon can appear to have no fiscal backing for its monetary policy in the presence of international policy coordination.

Another recent literature studies international policy coordination and the role of a global hegemon in the international monetary system (Farhi and Maggiori, 2018; He, Krishnamurthy, and Milbradt, 2019; Egorov, Mukhin, et al., 2019; Fontanier, 2023; Auray, Devereux, and Eyquem, 2024; Acharya, Jiang, Richmond, and Von Thadden, 2024; Jiang and Richmond, 2023; Clayton, Maggiori, and Schreger, 2023, 2024; De Leo, Gopinath, and Kalemli-Ozcan, 2024; Pflueger and Yared, 2024; Broner, Martin, Meyer, and Trebesch, 2024). Aizenman, Eldén, Jinjarak, Uddin, and Widholm (2024) find that concerns about the U.S. fiscal condition lower the policy rates in foreign countries, which is consistent with our notion of passive foreign policies. Our paper contributes to this literature by studying how a global hegemon affects the policy coordination in monetary and fiscal domains.

2 Baseline Model

We consider a two-country New Keynesian model. Most ingredients follow Corsetti and Pesenti (2007), and the fiscal part follows Jiang (2022, 2023). This model simplifies the standard New Keynesian framework by assuming that prices are fully sticky for one period, which allows us to derive algebraic solutions that characterize monetary, fiscal, and hegemon dominance regimes in a tractable way. We relax this assumption in Section 3 when we consider the more standard Calvo (1983) price setting.

Throughout this paper, we use uppercase letters to denote prices and nominal quantities, and lowercase letters to denote real quantities. For example, Q and q denote the nominal and real values of government debt, respectively. The only exceptions are π and i, which are reserved for inflation and nominal interest rate in log, and $p_t(h)$, which denotes the nominal price of a specific goods variety.

2.1 Households

There are two countries, home and foreign. Each country contains a unit mass of households, a unit mass of firms, and a government. Home households are indexed by $j \in [0,1]$, and home firms are indexed by $h \in [0,1]$. Foreign households are indexed by $j^* \in [0,1]$, and foreign firms are indexed by $f \in [0,1]$. Each firm produces a unique variety of good, which is an imperfect substitute for other varieties.

The lifetime expected utility of home household j is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \delta^t \left(\log c_t(j) - \chi \ell_t(j)\right),$$

where $\ell_t(j)$ is the labor effort, $c_t(j)$ is the consumption composed of a Cobb-Douglas basket of home and foreign bundles:

$$c_t(j) = c_{H,t}(j)^{\alpha} c_{F,t}(j)^{1-\alpha},$$

and $c_{H,t}(j)$ and $c_{F,t}(j)$ are constant-elasticity-of-substitution (CES) bundles of home and foreign varieties:

$$c_{H,t}(j) = \left(\int_0^1 c_t(h,j)^{1-1/\rho} dh\right)^{1/(1-1/\rho)}, \qquad c_{F,t}(j) = \left(\int_0^1 c_t(f,j)^{1-1/\rho} df\right)^{1/(1-1/\rho)}.$$

The parameter $\alpha > 1/2$ measures home bias in consumption, and the parameter ρ is the elasticity of substitution across varieties.

Let P_t denote the price of the home consumption basket in the unit of the home currency, and let W_t denote the nominal wage. In period t, the home household j receives wage $W_t\ell_t(j)$ and dividend $D_t(j)$ from home firms, and pays tax $P_t\tau_t(j)$ and consumption $P_tc_t(j)$. The financial markets are complete, so that the household can trade Arrow-Debreu securities denominated in home and foreign numéraires.

2.2 Firms

Each home firm produces a variety h using labor supplied by home households. The production technology is linear in labor input:

$$y_t(h) = z_t\ell_t(h),$$

where $y_t(h)$ is the output of firm h, $\ell_t(h)$ is the labor input, and z_t is a productivity process common to all home firms. Aggregating across home and foreign households, we obtain the following demand function for variety h:

$$y_t(h) = \int_0^1 c_t(h,j)dj + \int_0^1 c_t^*(h,j^*)dj^*.$$

Let $p_t(h)$ and $p_t^*(h)$ denote the home and foreign prices of variety h in local currencies, and let $\mathcal{E}_t$ denote the log nominal exchange rate which is increasing in the strength of the home currency. To produce $y_t(h)$ units of goods, firm h faces a wage cost of $W_t\ell_t(h)$. The firm's nominal revenue is

$$D_t(h) = p_t(h)\int_0^1 c_t(h,j)dj + \exp(-\mathcal{E}_t)p_t^*(h)\int_0^1 c_t^*(h,j^*)dj^* - W_t\ell_t(h).$$

We assume that the firm is entirely owned by the domestic households. The firm's objective function is to maximize the present value of this dividend stream.

We consider a simple form of nominal rigidities: firms have to set prices one period in advance.

Under this assumption, the firms' profit maximization problem only concerns one period. Take the home firm h as an example,

$$\max_{p_t(h), \exp(-\mathcal{E}_t)p_t^*(h)} \mathbb{E}_{t-1}[M_{t-1,t} D_t(h)],$$

where the firm uses the home households' stochastic discount factor $M_{t-1,t}$ to discount future dividends. International trade is conducted under *Producer Currency Pricing*: exports are priced and invoiced in the producer's currency, which is also rigid for one period.

2.3 Monetary and Fiscal Policies

In each country, the monetary authority controls the nominal interest rate i_t, and the fiscal authority controls government tax and spending processes. We assume that governments only issue one-period debt in local currency units. The government debt does not default on its notional value, but its real value can vary due to inflation. Let Q_{t+1} denote the quantity of outstanding home government debt that is issued in period t and due in period $t+1$. The government budget condition in nominal terms is

$$Q_t + P_t g_t = P_t \tau_t + Q_{t+1} \exp(-i_t). \tag{1}$$

Let $s_t = \tau_t - g_t$ denote the home real government surplus, and let $m_{t,t+k}$ denote the home households' real stochastic discount factor. Iterate forward the home government's intertemporal budget condition and impose the transversality condition. We obtain the intertemporal government budget condition:

$$\frac{Q_t}{P_t} = \mathbb{E}_t \left[\sum_{k=0}^{\infty} m_{t,t+k} s_{t+k} \right], \tag{2}$$

which states that the real market value of government debt is equal to the real present value of government surpluses.

In this section, we assume all shocks are i.i.d., which implies that

$$A = \mathbb{E}_t \left[\sum_{k=1}^{\infty} \delta^k \frac{s_{t+k}}{c_{t+k}} \right]$$

is a constant. This condition implies that the present value of future government surpluses, i.e., $\mathbb{E}_t \left[\sum_{k=1}^{\infty} m_{t,t+k} s_{t+k} \right]$, is equal to the current consumption c_t times the constant A, because future surpluses are unpredictable and the only variation in the present value is driven by the real discount rate, which is related to consumption growth. We can write the intertemporal budget condition (2) as[1]

$$\frac{Q_t}{P_t} = s_t + c_t A.$$

2.4 Equilibrium Conditions

This model can be characterized by three pairs of equilibrium conditions. The details of model derivation are presented in Appendix A.1. First, we have the standard bond Euler equations:

$$\begin{aligned} i_t &= \log \left(\frac{1}{P_t c_t} \right) - \log \mathbb{E}_t \left[\delta \frac{1}{P_{t+1} c_{t+1}} \right], \\ i_t^* &= \log \left(\frac{1}{P_t^* c_t^*} \right) - \log \mathbb{E}_t \left[\delta \frac{1}{P_{t+1}^* c_{t+1}^*} \right]; \end{aligned}$$

second, we have the intertemporal government budget conditions:

$$\begin{aligned} \frac{Q_t}{P_t} &= s_t + c_t A, \\ \frac{Q_t^*}{P_t^*} &= s_t^* + c_t^* A^*; \end{aligned}$$

[1] By assuming i.i.d. shocks and constant A, we implicitly rule out the possibility that the expectation of future surpluses can adjust in response to policy shocks today. This assumption imposes a strong restriction between the price level P_t and the surplus s_t today in this equation, which is useful for illustrating the key results in the model. We will relax this assumption in Section 3.

third, we also obtain the following equilibrium conditions based on firm optimization and sticky prices:

$$\log c_t = \kappa_{t-1}^{c} + \alpha \log \left(P_t c_t\right) + (1-\alpha) \log \left(P_t^* c_t^*\right), \tag{3}$$

$$\log c_t^* = \kappa_{t-1}^{c^*} + (1-\alpha) \log \left(P_t c_t\right) + \alpha \log \left(P_t^* c_t^*\right). \tag{4}$$

Corsetti and Pesenti (2007) interpret the nominal expenditures $P_t c_t$ and $P_t^* c_t^*$ as the home and foreign countries' aggregate demand in this model. When prices are sticky, monetary and fiscal policies affect the aggregate demand, and hence real consumption through Eq. (3) and (4).

We can further substitute out the price levels and obtain the following equation system:

$$\begin{aligned}
i_t - \Delta \log Q_{t+1} &= \log \left(\frac{s_t + c_t A}{c_t}\right) - \log \mathbb{E}_t \left[\delta \frac{s_{t+1} + c_{t+1} A}{c_{t+1}}\right], \\
i_t^* - \Delta \log Q_{t+1}^* &= \log \left(\frac{s_t^* + c_t^* A^*}{c_t^*}\right) - \log \mathbb{E}_t \left[\delta \frac{s_{t+1}^* + c_{t+1}^* A^*}{c_{t+1}^*}\right], \\
\log c_t &= \kappa_{t-1}^{c} + \alpha \log \left(\frac{c_t}{s_t + c_t A}\right) + (1-\alpha) \log \left(\frac{c_t^*}{s_t^* + c_t^* A^*}\right), \\
\log c_t^* &= \kappa_{t-1}^{c^*} + (1-\alpha) \log \left(\frac{c_t}{s_t + c_t A}\right) + \alpha \log \left(\frac{c_t^*}{s_t^* + c_t^* A^*}\right),
\end{aligned}$$

which has 4 equations with 6 unknowns: the nominal interest rates i_t and i_t^*, the government surpluses s_t and s_t^*, and the equilibrium consumption c_t and c_t^*. We assume that the debt growth rates are $\Delta \log Q_{t+1} = \Delta \log Q_{t+1}^* = 0$. This is without loss of generality, because what matters for surprise inflation and real outcomes is the difference between i_t and $\Delta \log Q_{t+1}$. Given the difference, varying $\Delta \log Q_{t+1}$ only creates expected inflation and has no real effects.

From this equation system, we can already see that nominal interest rates and government surpluses cannot both be exogenous policy variables. When the monetary authorities set the nominal interest rates, consumption and government surpluses need to endogenously adjust to satisfy this equation system. Conversely, when the fiscal authorities set the government surpluses, consumption and nominal interest rates need to endogenously adjust. As we will show in the next subsection, these two cases correspond to the monetary dominance and fiscal dominance regimes, respectively.

2.5 Model Characterization

We linearize the equation system around a symmetric steady state with $\bar{i} = \bar{i}^*$, $\bar{s} = \bar{s}^*$, $\bar{c} = \bar{c}^*$, and $A = A^*$. Using the notation $\hat{s}_t = s_t - \bar{s}$ and $\hat{i}_t = i_t - \bar{i}$, we can substitute out consumption and obtain the following equilibrium relationships.

Proposition 1. *(a) The nominal interest rates and government surpluses are related by*

$$\begin{aligned} \hat{s}_t &= A\bar{c}\hat{i}_t + (1-\alpha)\bar{s}(\hat{i}_t - \hat{i}_t^*), \\ \hat{s}_t^* &= A\bar{c}\hat{i}_t^* + (1-\alpha)\bar{s}^*(\hat{i}_t^* - \hat{i}_t). \end{aligned} \tag{5}$$

(b) Consumption, real exchange rate, and price levels are given by

$$\begin{aligned} \log \hat{c}_t &= -\alpha\hat{i}_t - (1-\alpha)\hat{i}_t^*, \qquad \log \hat{c}_t^* = -(1-\alpha)\hat{i}_t - \alpha\hat{i}_t^*, \\ \hat{e}_t &= (2\alpha - 1)(\hat{i}_t - \hat{i}_t^*), \\ \log \hat{P}_t &= -(1-\alpha)(\hat{i}_t - \hat{i}_t^*), \qquad \log \hat{P}_t^* = (1-\alpha)(\hat{i}_t - \hat{i}_t^*), \end{aligned}$$

where $\log \hat{c}_t = \log c_t - \log \bar{c}$*,* $\hat{e}_t = e_t - \bar{e}$*, and* $\log \hat{P}_t = \log P_t - \log \bar{P}$ *denote deviations from the steady state.*

The proof is presented in Appendix A.2. Part (a) of the proposition shows that monetary and fiscal policies are tightly connected. Part (b) of the proposition is consistent with the notion that a higher nominal interest rate i_t has tightening effects which lower local consumption and price level, increase the local real interest rate, and appreciate the local currency in real terms. Because of international risk-sharing, local consumption and price level also depend on the foreign nominal interest rate i_t^*.

Monetary and Fiscal Dominance. Let us start with the closed-economy limit: as the home bias parameter α goes to 1, Eq. (5) becomes

$$\hat{s}_t = A\bar{c}\hat{i}_t, \qquad \hat{s}_t^* = A\bar{c}\hat{i}_t^*. \tag{6}$$

This result recovers the insight in Leeper (1991): under monetary dominance, the monetary authorities in both countries set the nominal interest rates i_t and i_t^*, and the fiscal authorities accommodate the monetary stance by adjusting the primary surpluses s_t and s_t^* according to this equation. Conversely, under fiscal dominance, the fiscal authorities in both countries set the primary surpluses s_t and s_t^*, and the monetary authorities accommodate the fiscal stance by adjusting the nominal interest rates i_t and i_t^*.

Eq. (6) imposes a positive relationship between the monetary policy rate and the fiscal surplus. In the language of Leeper (2021), a higher monetary policy rate increases households' interest income and, by reducing inflation, increases the real value of the households' nominal assets. To ensure that the households' expenditure remains feasible, the government needs to raise tax.[2]

Even when we consider the more general case with $\alpha < 1$, Eq. (5) still imposes tight constraints on monetary and fiscal policies. If the foreign monetary rate i_t^* increases in response to an increase in the home monetary rate i_t, it alleviates the need for the home fiscal policy to respond. Next, we consider this case explicitly.

Hegemon Dominance. When the home bias parameter $\alpha < 1$, there is a third, overlooked possibility: Eq. (5) can also be balanced by taking the home policies i_t and s_t as exogenous shocks and the foreign policies i_t^* and s_t^* as endogenous variables. This corresponds to the case in which the monetary and fiscal authorities in the home country dictate the policies in the foreign country. In this case, the home country's monetary and fiscal authorities can pursue separate policy goals, while the foreign country's monetary and fiscal policies are both accommodative. We refer

[2]This discussion takes the perspective of the households. We can also interpret this result from the government side. Because prices are sticky, a higher nominal interest rate raises the government's real interest expense, which requires the government to increase tax to balance the budget.

to this situation as the *hegemon dominance* regime. Formally, rearranging the system of linearized equations, we obtain the following result:

Proposition 2. *When the foreign country's monetary and fiscal policies follow the home country's policies, we can express them as*

$$\hat{i}_t^* = \frac{A\bar{c} + (1-\alpha)\bar{s}}{(1-\alpha)\bar{s}}\hat{i}_t - \frac{1}{(1-\alpha)\bar{s}}\hat{s}_t,$$
$$\hat{s}_t^* = \frac{2A\bar{c}(1-\alpha)\bar{s} + A^2\bar{c}^2}{(1-\alpha)\bar{s}}\hat{i}_t - \frac{(1-\alpha)\bar{s} + A\bar{c}}{(1-\alpha)\bar{s}}\hat{s}_t.$$

Suppose the home country pursues a tightening monetary policy (i.e., $\hat{i}_t > 0$) without providing fiscal backing (i.e., $\hat{s}_t = 0$).

(a) The foreign monetary policy has to tighten as well: $\hat{i}_t^* > 0$.

(b) The foreign fiscal policy has to accommodate the monetary stance: $\hat{s}_t^* > 0$.

(c) The home currency depreciates in real terms: $\hat{e}_t < 0$.

The proof is presented in Appendix A.3. In this hegemon dominance regime, foreign monetary and fiscal policies are fully determined by the hegemon's policies. Parts (a)-(c) of this proposition highlight three properties of the foreign policies.

First, $\hat{i}_t^* > 0$ implies that, when the home country pursues a tightening monetary policy without fiscal accommodation, the foreign monetary policy has to take the same tightening stance as well. This response gives rise to a positive correlation between the home and foreign monetary stances. Under hegemon dominance, the foreign country gives up its monetary independence and accommodates the home monetary policy, just like the case of monetary dominance in which the fiscal authority gives up its independence and accommodates the monetary policy in the same country.

To understand why i_t^* has to increase, recall that a higher nominal interest rate i_t in the home country increases home households' real spending power. While monetary dominance restores equilibrium by raising tax in the home country, hegemon dominance does so by adjusting the exchange rate. Specifically, as the foreign nominal interest rate i_t^* increases, the home currency depreciates and lowers the home households' real spending power in the world numéraire, which offsets the effect of the home monetary policy on the home households' budget. In this way, the exchange rate adjustment accommodates the home monetary stance, while allowing the home country's fiscal authority to pursue a separate policy goal.

Second, $\hat{s}_t^* > 0$ implies that the foreign fiscal policy also responds to the hegemon's monetary shock. Again, it is useful to compare this result with the case of monetary dominance, in which case the home fiscal authority has to increase surplus to accommodate monetary tightening. Under hegemon dominance, the hegemon's fiscal authority is not responding, whereas the foreign fiscal authority responds by increasing its own surplus. In this sense, the hegemon transfers the responsibility of providing fiscal backing to its monetary policy from the domestic fiscal authority to the foreign one, which accommodates the monetary stance undertaken by both countries.

Third, $\hat{e}_t < 0$ implies that the home currency depreciates in real terms, which follows from the fact that the foreign interest rate $\hat{i}_t^*$ increases more than the home interest rate $\hat{i}_t$ in response to the home monetary shock. This home currency depreciation is the key equilibrium force that enforces the home households' budget constraint in the absence of adjustment in the home fiscal policy. More precisely, recall the intertemporal government budget condition for the home country:

$$\frac{Q_t}{P_t} = \mathbb{E}_t \left[\sum_{k=0}^{\infty} m_{t,t+k} s_{t+k} \right] = s_t + c_t A.$$

When the home country pursues a tightening monetary policy (i.e., $\hat{i}_t > 0$) without providing fiscal backing (i.e., $\hat{s}_t = 0$), home consumption declines and real rate rises. As a result, the discounted value of future surpluses on the right-hand side declines, which requires the home country's price level on the left-hand side to increase. This increase in price level is achieved by a foreign currency appreciation which weakens the home country's terms of trade. As a result, the hegemon's currency tends to be weaker when both countries raise nominal interest rates, and stronger when both countries lower nominal interest rates. Empirically, the U.S. dollar tends to appreciate during global downturns, when central banks around the world lower interest rates.

In sum, under hegemon dominance, implementing home policies $(\hat{i}_t > 0, \hat{s}_t = 0)$ requires accommodation by foreign policies ($\hat{i}^*_t > 0, \hat{s}^*_t > 0$). These policies jointly determine other outcome variables according to Proposition 1(b): home and foreign real rates both increase (i.e., $\hat{r}_t > 0$ and $\hat{r}^*_t > 0$), and home and foreign consumption both decrease (i.e., $\hat{c}_t < 0$ and $\hat{c}^*_t < 0$), with foreign responses stronger than home responses. As the home currency depreciates and erodes the home households' real spending power, home inflation π_t increases and foreign inflation π^*_t decreases.

Our discussion so far assumes no response in the hegemon's fiscal policy. The formula in Proposition 2 shows that the required responses in the foreign nominal interest rate and government surplus also load on the home government surplus $\hat{s}_t$ with negative signs. If the home country's fiscal authority is accommodative to the domestic monetary policy to some extent, which means $\hat{s}_t > 0$ as in the case of monetary dominance, then, the required responses in the foreign country's monetary and fiscal policies become smaller: they do not have to be as accommodative. In this sense, there is a spectrum of policy choices which trade off between accommodative domestic fiscal policy and accommodative foreign monetary and fiscal policies.

3 Active and Passive Policy Rules

So far, we have considered a stylized setting in which prices are fully sticky for one period. This setting allows us to derive algebraic solutions and obtain a stark result: the active policy can pursue its own goal, whereas the passive policy is entirely pinned down and has no room for any discretion.

Leeper (1991) considers more realistic monetary and fiscal policy rules that respond systematically to inflation and the real debt level, respectively. For example, the monetary policy can follow a version of the Taylor rule:

$$i_t = \phi_0 + \phi\pi_t + \theta_t, \tag{7}$$

where ϕ describes how it responds to inflation. Similarly, letting $b_{t-1} = Q_t \exp(-i_{t-1})/P_{t-1}$ denote the real market value of government debt outstanding at the end of period $t-1$, the fiscal policy follows something similar:

$$s_t = \gamma_0 + \gamma b_{t-1} + \psi_t, \tag{8}$$

where γ describes how it responds to debt level. θ_t and ψ_t are exogenous policy shocks.

In this setting, an active monetary policy means a large ϕ coefficient: the nominal interest rate can respond to inflation more than one-for-one, while a passive monetary policy means that the nominal interest rate cannot respond to inflation as much. An active fiscal policy means a small γ coefficient: the fiscal authority can pursue its own goal without paying attention to the real debt level, while a passive fiscal policy means that the government surplus needs to increase when the debt level is high. In these cases, passive policies can still have exogenous innovations.

This section generalizes our analysis by modifying two assumptions in the baseline setting: we adopt these more general policy rules, and we replace the assumption of one-period full price stickiness by Calvo (1983) price setting. We develop this generalization in three steps. First, we present a closed-economy case as similar to Leeper (1991) as possible, which introduces the policy rules, and replicate the standard monetary and fiscal dominance regimes found in that paper. Second, we introduce Calvo (1983) price setting which allows for more gradual price adjustments and gives rise to the standard New-Keynesian Phillips curve. Third, we extend the model to an international setting. Doing so helps clarify which assumptions are crucial for the existence of hegemon dominance.

3.1 Leeper (1991) Set-up

We consider a closed economy similar to Leeper (1991). We abstract away from non-interest-bearing money in the set-up because it is not central to our analysis. Leeper (1991) has the same bond Euler equation and government budget condition, reproduced below,

$$i_t = \log\left(\frac{1}{P_t c_t}\right) - \log \mathbb{E}_t\left[\delta \frac{1}{P_{t+1} c_{t+1}}\right], \tag{9}$$

$$q_t = s_t + q_{t+1}\exp(\pi_{t+1} - i_t). \tag{10}$$

However, instead of deriving the equilibrium condition (3) based on production and sticky prices, consumption is assumed to be constant:

$$c_t = \bar{c}. \tag{11}$$

After linearization, we obtain the following equation system.

Proposition 3. *In the one-country economy summarized by the bond Euler equation (9), government budget condition (10), constant consumption (11), and monetary and fiscal rules (7) and (8), the equilibrium dynamics is characterized by*

$$\begin{bmatrix} \hat{b}_t \\ \hat{\pi}_t \\ \mathbb{E}_t[\hat{\pi}_{t+1}] \end{bmatrix} = \begin{bmatrix} (\delta^{-1}-\gamma) & \phi\delta^{-1}\bar{q} & -\delta^{-1}\bar{q} \\ 0 & 0 & 1 \\ 0 & 0 & \phi \end{bmatrix} \begin{bmatrix} \hat{b}_{t-1} \\ \hat{\pi}_{t-1} \\ \hat{\pi}_t \end{bmatrix} + \begin{bmatrix} \delta^{-1}\bar{q} & 0 & -\delta^{-1} \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \theta_{t-1} \\ \theta_t \\ \psi_t \end{bmatrix}.$$

The transition matrix has 3 eigenvalues: $\delta^{-1} - \gamma$, 0, ϕ. Since only $\hat{\pi}_{t+1}$ is not predetermined, the system has a unique saddle-path equilibrium if and only if one of the eigenvalues is outside the unit circle (Blanchard and Kahn, 1980).

The proof is presented in Appendix A.4. Based on this result, Figure (1) plots the number of eigenvalues outside the unit circle as we vary ϕ and γ. We find two parametric regions that correspond to stable solutions (Leeper, 1991).

First, if $|\phi| > 1$ and $|\delta^{-1} - \gamma| < 1$, we are in the monetary dominance regime, which corresponds to the bottom-left corner of the figure. In this regime, the monetary policy can respond to inflation with a greater-than-one coefficient ϕ, whereas the fiscal policy has to stabilize the debt level with a large enough coefficient γ.

Second, if $|\phi| < 1$ and $|\delta^{-1} - \gamma| > 1$, we are in the fiscal dominance regime, which corresponds to the top-right corner of the figure. In this regime, the fiscal authority can pursue its own goal without having to worry about the debt level, while the monetary authority has to give up its inflation-targeting policy.

In this way, Leeper (1991) identifies two separate parametric regions representing different modes of policy coordination. While we can trivially generalize this setting to two countries, it does not allow for the possibility of hegemon dominance. In comparison, our baseline model in the previous section replaces the constant consumption assumption (11) by the aggregate demand equation (3), which links consumption in one country with monetary and fiscal policies in the other country. This is the key ingredient that allows foreign policies to play a role in equilibrium dynamics.

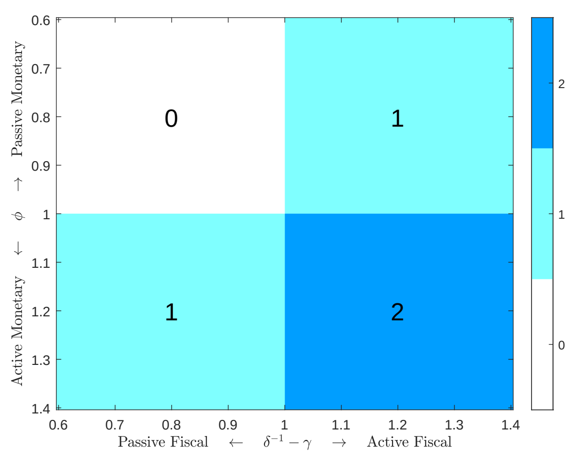


Figure 1: Number of Eigenvalues Outside the Unit Circle in Closed Economy

Note: We report the number of eigenvalues outside the unit circle as we vary the policy rule coefficients ϕ and γ.

3.2 Introducing New-Keynesian Phillips Curve

Next, we consider a more realistic case in which we replace the constant consumption assumption (11) by the standard New-Keynesian Phillips curve:

$$\hat{\pi}_t = \delta \mathbb{E}_t[\hat{\pi}_{t+1}] + \kappa \log \hat{c}_t, \tag{12}$$

which is derived from a Calvo pricing setting in which only a random fraction $(1 - \xi)$ of firms can reset their prices in each period. We still consider a closed economy, but generalize the setting also by assuming that the agents have CRRA utility with risk aversion parameter σ^{-1}. The previous log utility setting can be obtained as a special case by setting $\sigma = 1$. We obtain a very similar result.

Proposition 4. *In the one-country economy summarized by the bond Euler equation* (9)*, government budget condition* (10)*, the Phillips curve* (12)*, and monetary and fiscal rules* (7) *and* (8)*, the equilibrium dynamics is characterized by*

$$\begin{bmatrix} \hat{b}_t \\ \hat{\pi}_t \\ \mathbb{E}_t[\hat{\pi}_{t+1}] \\ \mathbb{E}_t[\log \hat{c}_{t+1}] \end{bmatrix} = \begin{bmatrix} (\delta^{-1} - \gamma) & \phi\delta^{-1}\bar{q} & -\delta^{-1}\bar{q} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \delta^{-1} & -\delta^{-1}\kappa \\ 0 & 0 & \sigma(\phi - \delta^{-1}) & 1 + \sigma\delta^{-1}\kappa \end{bmatrix} \begin{bmatrix} \hat{b}_{t-1} \\ \hat{\pi}_{t-1} \\ \hat{\pi}_t \\ \log \hat{c}_t \end{bmatrix} + \begin{bmatrix} \delta^{-1}\bar{q} & 0 & -\delta^{-1} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \sigma & 0 \end{bmatrix} \begin{bmatrix} \theta_{t-1} \\ \theta_t \\ \psi_t \end{bmatrix}.$$

Since there are 2 non-predetermined endogenous variables, we need exactly two eigenvalues outside the unit circle to guarantee a unique saddle-path equilibrium, which implies either $|\phi| > 1$ and $|\delta^{-1} - \gamma| < 1$, or $|\phi| < 1$ and $|\delta^{-1} - \gamma| > 1$.

The proof is presented in Appendix A.5. In this closed-economy setting, we obtain the same monetary and fiscal dominance regions. This result confirms the generality of Leeper (1991)'s insight, and sets the stage for the international setting we next consider.

3.3 Calvo Pricing in Open Economy

Finally, we extend our analysis to a two-country setting. The set-up is very similar to the baseline model presented in Section 2, except we introduce Calvo pricing and policy rules. This model is also similar to Chari et al. (2002), except we additionally consider the fiscal side of the economy. The details of the model set-up are presented in Appendix A.6.

Like the closed-economy set-up, we obtain microfounded New Keynesian Phillips curves based on firms' optimal pricing decisions. At the level of home or foreign goods, the Phillips curves are very similar to the closed-economy case. For home households, the changes in their price levels of home and foreign goods satisfy

$$\hat{\pi}_{H,t} = \delta \mathbb{E}_t[\hat{\pi}_{H,t+1}] + \frac{(1-\xi)(1-\delta\xi)}{\xi}(\log \widehat{MC}_{H,t} - \log \hat{P}_{H,t}), \tag{13}$$

$$\hat{\pi}^*_{F,t} = \delta \mathbb{E}_t[\hat{\pi}^*_{F,t+1}] + \frac{(1-\xi)(1-\delta\xi)}{\xi}(\log \widehat{MC}^*_{F,t} - \log \hat{P}^*_{F,t}), \tag{14}$$

where MC denotes the nominal marginal cost of production, and π_H and P_H denote the inflation and price index of the home goods, respectively.

At the aggregate level of home and foreign consumption bundles, the price levels and the Phillips curves additionally depend on the exchange rate. We obtain the following results.

Proposition 5. *In open economy, the home and foreign countries' Phillips curves are given by*

$$\hat{\pi}_t = \delta \mathbb{E}_t[\hat{\pi}_{t+1}] + \delta \frac{1-\alpha}{2\alpha-1}\mathbb{E}_t[\hat{e}_{t+1}] - \eta \frac{1-\alpha}{2\alpha-1}\hat{e}_t + \frac{1-\alpha}{2\alpha-1}\hat{e}_{t-1} + \kappa \log \hat{c}_t - \kappa\sigma \log \hat{z}_t, \tag{15}$$

$$\hat{\pi}^*_t = \delta \mathbb{E}_t[\hat{\pi}^*_{t+1}] - \delta \frac{1-\alpha}{2\alpha-1}\mathbb{E}_t[\hat{e}_{t+1}] + \eta \frac{1-\alpha}{2\alpha-1}\hat{e}_t - \frac{1-\alpha}{2\alpha-1}\hat{e}_{t-1} + \kappa \log \hat{c}^*_t - \kappa\sigma \log \hat{z}^*_t. \tag{16}$$

The proof is presented in Appendix A.7. Similar to the equilibrium conditions (3) and (4) obtained in the baseline model, this proposition shows that the exchange rate adjustment plays a crucial role in transmitting the effects of monetary and fiscal policies across countries.

Moreover, we consider the monetary and fiscal policy rules, which are Eq. (7) and (8) for the home country, and

$$i_t^* = \phi_0^* + \phi^* \hat{\pi}_t^* + \theta_t^*, \tag{17}$$

$$s_t^* = \gamma_0^* + \gamma^* b_{t-1}^* + \psi_t^*, \tag{18}$$

for the foreign country. Active monetary policies correspond to $|\phi| > 1$ and $|\phi^*| > 1$, and active fiscal policies correspond to $|\delta^{-1} - \gamma| > 1$ and $|\delta^{-1} - \gamma^*| > 1$. In this set-up, we obtain the following forward-looking linear equation system.

Proposition 6. *In the two-country economy summarized by the domestic bond Euler equation* (9) *and its foreign counterpart, the domestic government budget condition* (10) *and its foreign counterpart, the Phillips curves* (15) *and* (16)*, and monetary and fiscal rules* (7)*,* (17)*,* (8)*, and* (18)*, the equilibrium dynamics is characterized by*

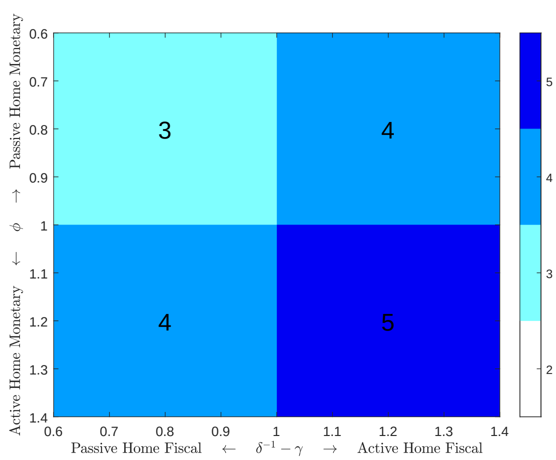
$$\begin{bmatrix} \hat{b}_t \\ \hat{b}_t^* \\ \hat{\pi}_t \\ \hat{\pi}_t^* \\ \log \hat{c}_t \\ \log \hat{c}_t^* \\ \mathbb{E}_t[\hat{\pi}_{t+1}] \\ \mathbb{E}_t[\hat{\pi}_{t+1}^*] \\ \mathbb{E}_t[\log \hat{c}_{t+1}] \\ \mathbb{E}_t[\log \hat{c}_{t+1}^*] \end{bmatrix} = \Psi \begin{bmatrix} \hat{b}_{t-1} \\ \hat{b}_{t-1}^* \\ \hat{\pi}_{t-1} \\ \hat{\pi}_{t-1}^* \\ \log \hat{c}_{t-1} \\ \log \hat{c}_{t-1}^* \\ \hat{\pi}_t \\ \hat{\pi}_t^* \\ \log \hat{c}_t \\ \log \hat{c}_t^* \end{bmatrix} + \Phi \begin{bmatrix} \theta_{t-1} \\ \theta_{t-1}^* \\ \theta_t \\ \theta_t^* \\ \psi_t \\ \psi_t^* \\ \log \hat{z}_t \\ \log \hat{z}_t^* \end{bmatrix}.$$

Since there are 4 *non-predetermined endogenous variables, we need exactly four eigenvalues outside the unit circle to guarantee a unique saddle-path equilibrium.*

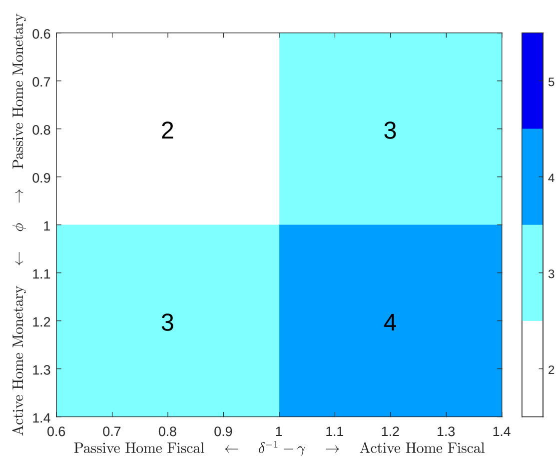
The proof and the definitions of the matrices Ψ and Φ are presented in Appendix A.8. By studying the eigenvalues of the transition matrix Ψ, we can likewise identify the regions of monetary and fiscal dominance.

Active Foreign Monetary Policy and Passive Foreign Fiscal Policy. To begin with, we consider the conventional case in which the foreign country has an active monetary policy and a passive fiscal policy. We study the parameter space of ϕ and γ which characterize the home country's monetary and fiscal policies. Appendix A.10 provides the details of the calibration.

Figure (2)(a) plots the number of eigenvalues outside the unit circle as we vary ϕ and γ. We find two separate regions in which this number is 4, which implies a unique equilibrium. Similar to the closed-economy case depicted in Figure (1), the first region requires an active home monetary policy and a passive home fiscal policy (i.e., $|\phi| > 1$ and $|\delta^{-1} - \gamma| < 1$), which corresponds to the bottom-left corner of the figure. The second region requires a passive home monetary policy and an active home fiscal policy (i.e., $|\phi| < 1$ and $|\delta^{-1} - \gamma| > 1$), which corresponds to the top-right corner of the figure.



(a) Active Foreign Monetary and Passive Fiscal Policies



(b) Passive Foreign Monetary and Fiscal Policies

Figure 2: Number of Eigenvalues Outside the Unit Circle in Open Economy

Note: We fix the foreign policy rules and report the number of eigenvalues outside the unit circle as we vary the home policy rule coefficients ϕ and γ. In Panel (a), the foreign country has an active monetary policy and a passive fiscal policy. In Panel (b), the foreign country has a passive monetary policy and a passive fiscal policy.

Passive Foreign Monetary and Fiscal Policies. We next consider an alternative case in which the foreign country has passive monetary and fiscal policies. We pick $\phi^* = 0.8$ and $\gamma^* = 0.2$. In the closed-economy case, this pair of policy rules does not allow for a unique equilibrium.

Figure (2)(b) plots the number of eigenvalues outside the unit circle as we vary ϕ and γ in the open-economy model. When both monetary and fiscal policies are passive in the foreign country, the number of eigenvalues outside the unit circle declines by 1 across the (ϕ, γ) parameter space. As a result, there is only one region consistent with a unique equilibrium, which corresponds to the bottom-right corner of the figure. This region requires $|\phi| > 1$ and $|\delta^{-1} - \gamma| > 1$, which implies that the home country's monetary and fiscal policies are both active: the home monetary policy responds sufficiently to local inflation, whereas the home fiscal policy is not committed to stabilizing the debt level.

In this way, we identify a new hegemon dominance regime in which the home monetary and fiscal policies are both active, while the foreign monetary and fiscal policies are both passive. This result generalizes the insight in the baseline setting by allowing for more realistic policy rules.

3.4 Inspecting the Mechanism

In this final section, we examine the impulse responses in this model, and confirm that they are consistent with the algebraic characterization in Proposition 2(a)-(c) obtained in the more stylized baseline setting.

Monetary Dominance. To set the stage, let us begin with the monetary dominance regime in closed economy as we specified in Section 3.2. We consider a negative monetary shock θ_t to the nominal interest rate i_t, but it is useful to think of this shock as a surprise decrease in the real rate r_t. Figure (3) reports the impulse responses. Let us consider the pairwise relationships between the real rate r_t, the real consumption c_t, and the inflation π_t, which are characterized by the Euler equation, the Phillips curve, and the intertemporal government budget condition.

First, the Euler equation (9) relates the real rate r_t to consumption c_t. A lower real rate requires a lower expected consumption growth, which increases the current consumption.

Second, the Phillips curve (12) relates consumption c_t to inflation π_t. If the slope coefficient κ is positive as in our calibration, a higher consumption c_t requires a higher inflation π_t, which is consistent with the notion that an expansionary monetary shock stimulates the aggregate demand and raises both consumption and inflation.

Third, the intertemporal government budget condition (2) relates inflation π_t to the real rate r_t. In this equation, if government surpluses remain unaffected, a lower real rate increases the present value of government surpluses, which requires the price level to decrease. Therefore, to ensure that a lower real rate is consistent with a higher inflation, current and future government surpluses s_{t+k} have to decrease to lower the present value of government surpluses, which also lowers the debt level b_t in real terms.

This discussion shows that fiscal responses in the form of government surplus adjustments are crucial for equilibrating the economy under monetary dominance. Under our calibration, the

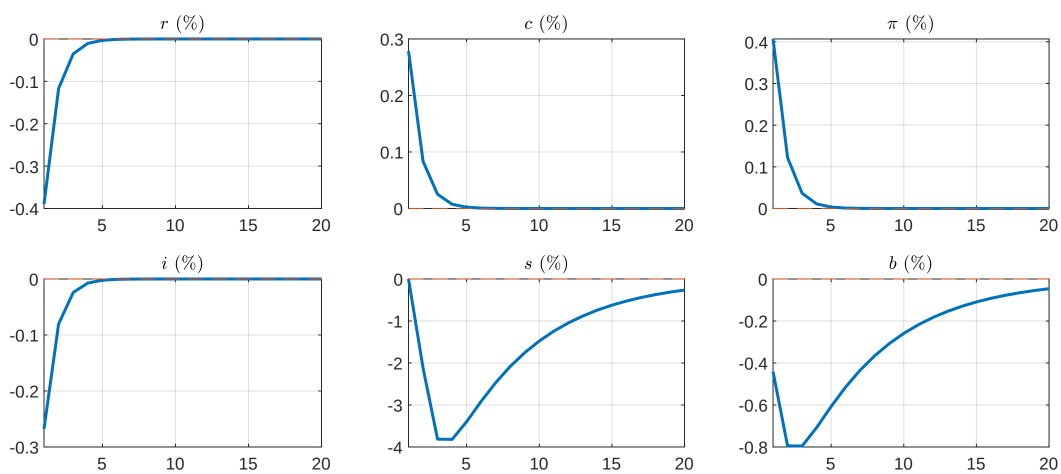


Figure 3: Impulse Responses to a Monetary Shock in the Closed Economy under Monetary Dominance

Note: We plot the impulse responses to a monetary shock θ_t in the closed economy under monetary dominance. The outcome variables include the nominal interest rate i_t, surplus s_t, debt level b_t, consumption c_t, inflation π_t, and real rate r_t.

nominal interest rate i_t decreases with the real rate in response to the monetary shock.[3]

Hegemon Dominance. Next, we study the impulse response under the hegemon dominance regime in open economy. In this regime, the home monetary and fiscal policies are both active ($\phi = 1.8$ and $\gamma = 0$), while the foreign monetary and fiscal policies are both passive ($\phi^* = 0.2$ and $\gamma^* = 0.2$). Appendix A.10 provides calibration details.

We consider a monetary shock to θ_t in the home country which similarly lowers the home real rate r_t. Figure (4) reports the impulse responses, which are different from the closed-economy case in Figure (3) in significant ways. The best way to understand this result is again to consider the pairwise relationships between the real rate r_t, consumption c_t, and inflation π_t, which include the Euler equation, the Phillips curve, and the intertemporal government budget condition.

First, given the negative real rate shock, the Euler equation (9) requires home consumption c_t to increase, which is similar to the closed-economy case.

Next, consider the relationship between the real rate r_t and inflation π_t as imposed by the intertemporal government budget condition (2), which implies that a lower real rate boosts the present value of government surpluses and puts a downward pressure on the price level. In the closed-economy case above, to accommodate a higher inflation predicted by the Phillips curve, government surpluses s_t have to adjust. In the open-economy case, since the home fiscal policy is assumed to be active, government surpluses do not adjust. As a result, inflation has to go down when the real rate declines, which explains a lower inflation π_t observed in Figure (4).

Finally, consider the relationship between inflation π_t and consumption c_t as imposed by the Phillips curve (15). Compared to the closed-economy Phillips curve (12), the open-economy version additionally includes the exchange rate e_t. As a result, while a higher consumption implies a higher inflation in the closed-economy case, the open-economy case allows for a lower inflation as the exchange rate adjusts. More precisely, home currency appreciation increases the home

[3] We derive the algebraic solution of the nominal rate i_t, real rate r_t, consumption c_t, and inflation π_t in Appendix A.9. We find that a negative monetary shock θ_t unambiguously lowers the real rate r_t and raises consumption c_t and inflation π_t, while the nominal rate i_t can increase or decrease depending on the persistence of the monetary shock and the shape of the Phillips curve, as well as risk aversion and discount rate.

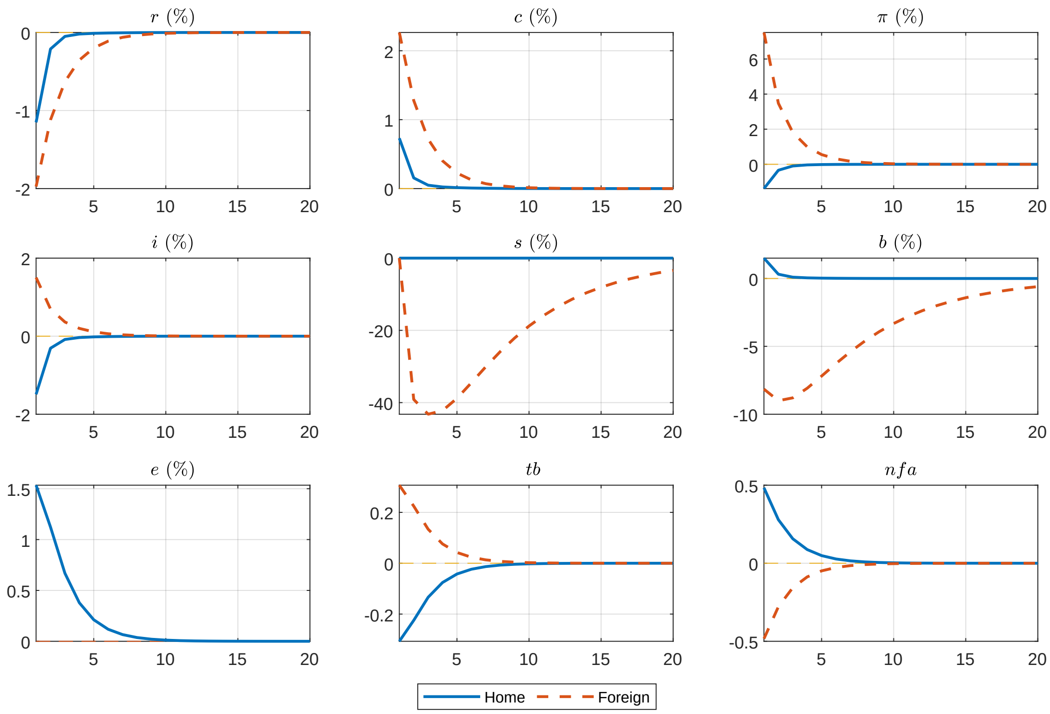


Figure 4: Impulse Responses to a Monetary Shock in the Open Economy under Hegemon Dominance

Note: We plot the impulse responses to a monetary shock θ_t in the open economy under hegemon dominance. The solid blue curve represents the home country, and the dashed red curve represents the foreign country. The outcome variables include the nominal interest rate i_t, surplus s_t, debt level b_t, consumption c_t, inflation π_t, and real rate r_t in both countries, and the real exchange rate e_t.

households' spending power and lowers home inflation by making foreign goods cheaper. In this way, while government surplus adjusts to equilibrate these three conditions under the monetary dominance regime, the exchange rate adjusts under the hegemon dominance regime.

Appreciating the home currency in real terms requires policy coordination from the foreign country. The foreign monetary authority needs to lower the foreign real rate r_t^* below the home real rate r_t. The foreign fiscal authority needs to accommodate the real rate declines in both countries by lowering its fiscal surplus s_t^*. This fiscal response is similar to the response of a passive fiscal policy in the closed economy, which lowers surplus whenever the monetary policy lowers the real rate, except that this fiscal response is done by the foreign, not the home, fiscal authority under hegemon dominance.

Moreover, through the foreign Euler equation, a lower foreign real rate also leads to a higher foreign consumption level c_t^* upon impact. As the open-economy New Keynesian Phillips curves (15) and (16) depend on the exchange rate level e_t, home inflation loads negatively on the real exchange rate, because a stronger home currency leads to more favorable terms of trade which make imported goods cheaper, whereas foreign inflation loads positively on the real exchange rate for the opposite reason. This terms of trade channel is the dominating force in the determination of inflation. As a result, the foreign country experiences a higher inflation π_t^* due to the external depreciation of its currency, which also leads to a higher nominal interest rate i_t^* even though its monetary response coefficient ϕ^* is much lower.[4]

Finally, we also report the trade balance tb_t and net foreign assets nfa_t in local currency units in the figure. The trade balance is defined in the usual way as the value of imports minus that of exports, and the net foreign assets are defined as the present value of consumption minus that of production. The coordinated monetary and fiscal policies transfer resources from the foreign country to the home country during monetary expansions, because a stronger home currency increases the purchasing power of the home households and allows them to run persistent trade deficits. This international wealth transfer can be thought of as the home country sharing the stimulating effects of the foreign monetary and fiscal policies, which lower the foreign real rate and surplus more than the home country.

In summary, the hegemon dominance regime is different from the closed-economy case in two important ways. First, due to an active home fiscal policy, the equilibrium conditions governing inflation, consumption, and the real rate in the home country are equilibrated not by the response in the government surplus s_t, but instead by adjusting the exchange rate e_t. Second, inflation and the Phillips curves in open economy load on the real exchange rate movement, which explains the opposite responses in inflation and nominal interest rates between the home and foreign countries. These features allow the home monetary authority to lower domestic inflation by transferring it to

[4]It is interesting to note that, in the closed-economy monetary-dominance case depicted by Figure (3), lower inflation is always accompanied by lower consumption in response to the monetary shock θ_t. In the open-economy case, low inflation and high consumption are possible because of the exchange rate adjustment, which makes foreign goods cheaper (hence lower inflation) and home spending power higher (hence higher consumption).

the foreign country.

These patterns confirm the main results in the baseline setting: when the hegemon country lowers its real rate without fiscal adjustment, the foreign policy responds in such way that (a) home and foreign real rates r and r^* both decline, while home and foreign consumption levels c and c^* both increase. The foreign real rate and consumption respond more than the home real rate and consumption; (b) the foreign fiscal authority accommodates the monetary stance by lowering surplus s^*; (c) the home currency e experiences real appreciation; and (d) home inflation π declines while foreign inflation π^* increases. These outcomes arise from the hegemon dominance regime in both the baseline setting and in the more realistic policy rule setting.

The only difference between the baseline setting and the policy rule setting is the response of the foreign nominal interest rate i^*, which moves in the same direction as the home nominal interest rate i in the baseline setting, but in the opposite direction in the policy rule setting. This is because the foreign country's policy rule is specified such that its nominal interest rate systematically responds to the inflation rate. In our calibration, the response coefficient is $\phi^* = 0.2$, which is still positive. If we set $\phi^* = 0$, i.e., the foreign monetary policy is so passive that it does not respond to inflation at all, the foreign nominal interest rate i_t^* would not increase. Alternatively, if the foreign monetary authority systematically lowers its nominal interest rate when the dollar appreciates, which helps mitigate the tightening effect of the dollar cycle, then, the foreign nominal interest rate i_t^* also declines along with the home nominal interest rate i_t. See Appendix A.11 for details of this alternative calibration. In this case, the responses in real rates, consumption, inflation, and the exchange rate are still consistent with our characterization above.

4 Conclusion

In this paper, we study the coordination problem between monetary and fiscal policies in an international setting. We recover the standard monetary and fiscal dominance regimes, in which one policy is active and the other is passive in each country. We also identify a third regime, in which the hegemon country's monetary and fiscal policies are both active, while the other country's policies are both passive. This new mode of policy coordination has novel implications for the transmission of monetary and fiscal policies, and provides a new perspective for understanding the global hegemon in a connected world.

A natural follow-up question is how and which equilibrium regime is selected, and which country becomes the hegemon. To answer this question, we need to specify the off-equilibrium strategies just like the earlier works on equilibrium selection in closed economy (Kocherlakota and Phelan, 1999; Buiter, 2002; Bassetto, 2002; Niepelt, 2004; Atkeson et al., 2010; Cochrane, 2011; Angeletos and Lian, 2023). For example, following Cochrane (2011)'s argument that the monetary authority selects the desired equilibrium path for inflation in a closed economy by threatening to blow up inflation in off-equilibrium paths, perhaps a similar argument can select the hegemon country based on which country's monetary and fiscal authorities have more credible threats.

Another interesting direction is to consider how the exchange rate adjustment also affects policy coordination through valuation effects in the financial market. Specifically, our main mechanism relies on adjustments in the terms of trade in the goods market to enforce resource constraints. When the monetary policy increases domestic households' real spending power and hence their aggregate demand for goods, we need either higher tax to absorb the excess demand as in the standard monetary dominance regime (Leeper, 2021), or a real depreciation of the local currency to make the imported goods more expensive. The exchange rate adjustment also affects the valuation of home and foreign financial assets, which shifts the wealth distribution between the two countries and potentially complements the international trade channel in the hegemon dominance regime. This is particularly relevant when we consider the U.S. as the hegemon country, as the U.S. monetary policy also has large spill-over effects on global asset prices by driving the global financial cycle (Rey, 2015).

References

Viral V Acharya, Zhengyang Jiang, Robert J Richmond, and Ernst-Ludwig Von Thadden. International policy coordination in a multisectoral model of trade and health policy. Technical report, National Bureau of Economic Research, 2024.

S Rao Aiyagari and Mark Gertler. The backing of government bonds and monetarism. Journal of Monetary Economics, 16(1):19–44, 1985.

Joshua Aizenman, William Eldén, Yothin Jinjarak, Gazi Salah Uddin, and Frida Widholm. International spillovers of u.s. fiscal challenges. Working Paper 32868, National Bureau of Economic Research, August 2024. URL http://www.nber.org/papers/w32868.

Enrique Alberola, Carlos Cantú, Paolo Cavallino, and Nikola Mirkov. Fiscal regimes and the exchange rate. Bank for International Settlements, Monetary and Economic Department, 2021.

George-Marios Angeletos and Chen Lian. Determinacy without the taylor principle. Journal of Political Economy, 131(8):2125–2164, 2023.

George-Marios Angeletos, Chen Lian, and Christian K Wolf. Can deficits finance themselves? Technical report, National Bureau of Economic Research, 2023.

Andrew Atkeson, Varadarajan V Chari, and Patrick J Kehoe. Sophisticated monetary policies. The Quarterly Journal of Economics, 125(1):47–89, 2010.

Stéphane Auray, Michael B Devereux, and Aurélien Eyquem. Trade wars and the optimal design of monetary rules. Technical report, National Bureau of Economic Research, 2024.

Marco Bassetto. A game–theoretic view of the fiscal theory of the price level. Econometrica, 70 (6):2167–2195, 2002.

Marco Bassetto and Thomas J Sargent. Shotgun wedding: Fiscal and monetary policy. Annual Review of Economics, 12(1):659–690, 2020.

Jess Benhabib, Stephanie Schmitt-Grohé, and Martin Uribe. Avoiding liquidity traps. Journal of Political Economy, 110(3):535–563, 2002.

Francesco Bianchi and Cosmin Ilut. Monetary/fiscal policy mix and agents' beliefs. Review of economic Dynamics, 26:113–139, 2017.

Francesco Bianchi and Leonardo Melosi. Escaping the great recession. American Economic Review, 107(4):1030–1058, 2017.

Francesco Bianchi and Leonardo Melosi. The dire effects of the lack of monetary and fiscal coordination. Journal of Monetary Economics, 104:1–22, 2019.

Olivier Jean Blanchard and Charles M Kahn. The solution of linear difference models under rational expectations. Econometrica: Journal of the Econometric Society, pages 1305–1311, 1980.

Fernando Broner, Alberto Martin, Josephin Meyer, and Christoph Trebesch. Hegemonic globalization. Technical report, Working Paper, 2024.

Markus K Brunnermeier, Sebastian A Merkel, and Yuliy Sannikov. Debt as safe asset. Technical report, National Bureau of Economic Research, 2022.

Willem H Buiter. The fiscal theory of the price level: A critique. The Economic Journal, 112(481): 459–480, 2002.

Guillermo A Calvo. Staggered prices in a utility-maximizing framework. Journal of monetary Economics, 12(3):383–398, 1983.

Nicolas Caramp and Dejanir H Silva. Fiscal policy and the monetary transmission mechanism. Review of Economic Dynamics, 51:716–746, 2023.

Varadarajan V Chari, Patrick J Kehoe, and Ellen R McGrattan. Can sticky price models generate volatile and persistent real exchange rates? The review of economic studies, 69(3):533–563, 2002.

Zefeng Chen, Zhengyang Jiang, Hanno Lustig, Stijn Van Nieuwerburgh, and Mindy Z Xiaolan. Exorbitant privilege gained and lost: Fiscal implications. Technical report, National Bureau of Economic Research, 2022.

Anna Cieslak, Wenhao Li, and Carolin E Pflueger. Inflation and treasury convenience. USC Marshall School of Business Research Paper Sponsored by iORB, 2023.

Christopher Clayton, Matteo Maggiori, and Jesse Schreger. A framework for geoeconomics. Technical report, National Bureau of Economic Research, 2023.

Christopher Clayton, Matteo Maggiori, and Jesse Schreger. A theory of economic coercion and fragmentation. Available at SSRN 4767131, 2024.

John H Cochrane. A frictionless view of us inflation. NBER macroeconomics annual, 13:323–384, 1998.

John H Cochrane. Determinacy and identification with taylor rules. Journal of Political economy, 119(3):565–615, 2011.

John H Cochrane. Stepping on a rake: The fiscal theory of monetary policy. European Economic Review, 101:354–375, 2018.

Giancarlo Corsetti and Paolo Pesenti. The simple geometry of transmission and stabilization in closed and open economies [with comments]. In NBER international seminar on macroeconomics, volume 2007, pages 65–129. The University of Chicago Press Chicago, IL, 2007.

Troy Davig and Eric M Leeper. Generalizing the taylor principle. American Economic Review, 97(3):607–635, 2007.

Troy Davig, Eric M Leeper, Jordi Galí, and Christopher Sims. Fluctuating macro policies and the fiscal theory [with comments and discussion]. NBER macroeconomics annual, 21:247–315, 2006.

Pierre De Leo, Gita Gopinath, and Sebnem Kalemli-Ozcan. Monetary policy and the short-rate disconnect in emerging economies. National Bureau of Economic Research, 2024.

William F Diamond, Tim Landvoigt, and Germán Sánchez Sánchez. Printing away the mortgages: Fiscal inflation and the post-covid boom. Technical report, National Bureau of Economic Research, 2024.

Konstantin Egorov, Dmitry Mukhin, et al. Optimal monetary policy under dollar pricing. In Meeting Papers, volume 1510, 2019.

Stefano Eusepi and Bruce Preston. Fiscal foundations of inflation: imperfect knowledge. American Economic Review, 108(9):2551–2589, 2018.

Emmanuel Farhi and Matteo Maggiori. A model of the international monetary system. The Quarterly Journal of Economics, 133(1):295–355, 2018.

Paul Fontanier. Dollar debt and the global financial cycle. 2023.

Xavier Gabaix. A behavioral new keynesian model. American Economic Review, 110(8):2271–2327, 2020.

Roberto Gomez Cram, Howard Kung, and Hanno N Lustig. Can us treasury markets add and subtract? Available at SSRN, 2023.

Zhiguo He, Arvind Krishnamurthy, and Konstantin Milbradt. A model of safe asset determination. American Economic Review, 109(4):1230–1262, 2019.

Zhengyang Jiang. Fiscal cyclicality and currency risk premia. The Review of Financial Studies, 35(3):1527–1552, 2022.

Zhengyang Jiang. Lecture notes on international finance. Lecture Notes on International Finance (December 1, 2023), 2023.

Zhengyang Jiang and Robert Richmond. Reserve asset competition and the global fiscal cycle. Available at SSRN, 2023.

Zhengyang Jiang, Hanno Lustig, Stijn Van Nieuwerburgh, and Mindy Z Xiaolan. Manufacturing risk-free government debt. Technical report, National Bureau of Economic Research, 2020.

Zhengyang Jiang, Hanno Lustig, Stijn Van Nieuwerburgh, and Mindy Z Xiaolan. The us public debt valuation puzzle. Econometrica, 92(4):1309–1347, 2024.

Narayana Kocherlakota and Christopher Phelan. Explaining the fiscal theory of the price level. Federal Reserve Bank of Minneapolis Quarterly Review, 23(4):14–23, 1999.

Eric M Leeper. Equilibria under 'active'and 'passive'monetary and fiscal policies. Journal of monetary Economics, 27(1):129–147, 1991.

Eric M Leeper. Panel on the interaction of fiscal and monetary policy: Shifting policy norms and policy interactions. 2021.

Eric M Leeper and Xuan Zhou. Inflation's role in optimal monetary-fiscal policy. Technical report, National Bureau of Economic Research, 2013.

Eric M Leeper, Nora Traum, and Todd B Walker. Clearing up the fiscal multiplier morass. American Economic Review, 107(8):2409–2454, 2017.

Eduardo Loyo. Tight money paradox on the loose: a fiscalist hyperinflation. Manuscript, Harvard University, 4, 1999.

Atif R Mian, Ludwig Straub, and Amir Sufi. A goldilocks theory of fiscal deficits. Technical report, National Bureau of Economic Research, 2022.

Dirk Niepelt. The fiscal myth of the price level. The Quarterly Journal of Economics, 119(1): 277–300, 2004.

Carolin Pflueger and Pierre Yared. Global hegemony and exorbitant privilege. Technical report, National Bureau of Economic Research, 2024.

Ricardo Reis. The constraint on public debt when r< g but g< m. CEPR Discussion Papers, 15950, 2021.

Hélène Rey. Dilemma not trilemma: the global financial cycle and monetary policy independence. Technical report, National Bureau of Economic Research, 2015.

Thomas J Sargent, Neil Wallace, et al. Some unpleasant monetarist arithmetic. Federal reserve bank of minneapolis quarterly review, 5(3):1–17, 1981.

Stephanie Schmitt-Grohé and Martın Uribe. Price level determinacy and monetary policy under a balanced-budget requirement. Journal of Monetary Economics, 45(1):211–246, 2000.

Christopher A Sims. A simple model for study of the determination of the price level and the interaction of monetary and fiscal policy. Economic theory, 4:381–399, 1994.

Christopher A Sims. Stepping on a rake: The role of fiscal policy in the inflation of the 1970s. European Economic Review, 55(1):48–56, 2011.

Martin Uribe. A fiscal theory of sovereign risk. Journal of Monetary Economics, 53(8):1857–1875, 2006.

Michael Woodford. Price-level determinacy without control of a monetary aggregate. In Carnegie-Rochester conference series on public policy, volume 43, pages 1–46. Elsevier, 1995.

Michael Woodford. Fiscal requirements for price stability, 2001.

Appendix

A Theory Appendix

A.1 Derivation of the Baseline Setting

Household's Within-period Solution Let $p_t(h)$ and $p_t(f)$ denote the home-currency prices of varieties h and f. Let $P_{H,t}$ and $P_{F,t}$ denote the home-currency prices of the home and foreign bundles $c_{H,t}(j)$ and $c_{F,t}(j)$, which can be shown to be the CES indices with elasticity $1/\rho$:

$$P_{H,t} = \left(\int_0^1 p_t(h)^{1-\rho} dh\right)^{1/(1-\rho)}, \qquad P_{F,t} = \left(\int_0^1 p_t(f)^{1-\rho} df\right)^{1/(1-\rho)}.$$

Similarly, let P_t denote the home-currency price of the aggregate consumption basket in the home country. This utility-based CPI can be expressed as

$$P_t = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} P_{H,t}^\alpha P_{F,t}^{1-\alpha}.$$

We show that the home household makes the following choices for different goods:

$$c_t(h,j) = \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\rho} c_{H,t}, \qquad c_t(f,j) = \left(\frac{p_t(f)}{P_{F,t}}\right)^{-\rho} c_{F,t},$$

and

$$c_{H,t} = \alpha \frac{P_t}{P_{H,t}} c_t, \qquad c_{F,t} = (1-\alpha) \frac{P_t}{P_{F,t}} c_t.$$

Indeed, home household's cost minimization problem is

$$\min_{c_t(h,j)} \quad \int_0^1 p_t(h) c_t(h,j) dh + \mu_{H,t} \left(c_{H,t}(j) - \left(\int_0^1 c_t(h,j)^{1-1/\rho} dh\right)^{\rho/(\rho-1)} \right).$$

Let $P_{H,t} := \mu_{H,t}$. The first order condition is

$$c_t(h,j) = \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\rho} c_{H,t}.$$

Raise to the power $1 - 1/\rho$ and integrate by h to obtain

$$c_{H,t}^{1-1/\rho} = \left(\int_0^1 \left(\frac{p_t(h)}{P_{H,t}}\right)^{-(\rho-1)} dh\right) c_{H,t}^{1-1/\rho},$$

i.e.,

$$P_{H,t} = \left(\int_0^1 p_t(h)^{1-\rho} dh\right)^{1/(1-\rho)}.$$

Similarly, we obtain $c_t(f,j) = \left(\frac{p_t(f)}{P_{F,t}}\right)^{-\rho} c_{F,t}$ and $P_{F,t} = \left(\int_0^1 p_t(f)^{1-\rho} df\right)^{1/(1-\rho)}$. Now consider the following cost minimization problem

$$\min_{c_{H,t}(j), c_{F,t}(j)} \left(P_{H,t} c_{H,t}(j) + P_{F,t} c_{F,t}(j)\right) + \mu_t \left(c_t(j) - c_{H,t}(j)^{\alpha} c_{F,t}(j)^{1-\alpha}\right)$$

The first-order conditions are

$$P_{H,t} = \alpha \mu_t c_{H,t}(j)^{\alpha-1} c_{F,t}(j)^{1-\alpha},$$
$$P_{F,t} = (1-\alpha) \mu_t c_{H,t}(j)^{\alpha} c_{F,t}(j)^{-\alpha},$$

which imply

$$\mu_t = \frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} P_{H,t}^{\alpha} P_{F,t}^{1-\alpha} =: P_t.$$

The first-order conditions also imply

$$
\begin{aligned}
c_{H,t}(j) &= \alpha \frac{P_t}{P_{H,t}} c_t(j) \\
c_{F,t}(j) &= (1-\alpha) \frac{P_t}{P_{F,t}} c_t(j)
\end{aligned}
$$

and hence

$$
\frac{P_{H,t} c_{H,t}(j) + P_{F,t} c_{F,t}(j)}{c_t(j)} = P_t,
$$

which verifies that the shadow price of the home consumption bundle is the CPI itself.

Intertemporal Solution In period t and state σ_t, home household maximize the lifetime utility subject to the budget constraint

$$
\begin{aligned}
&\sum_{\sigma_{t+1}} \Omega_t(\sigma_{t+1}, j)\Theta(\sigma_{t+1}|\sigma_t) + \exp(-\mathcal{E}_t) \sum_{\sigma_{t+1}} \Omega_t^*(\sigma_{t+1}, j)\Theta^*(\sigma_{t+1}|\sigma_t) \\
\leq &W_t \ell_t(j) + D_t(j) - P_t \tau_t(j) - P_t c_t(j) + \Omega_{t-1}(\sigma_t, j) + \exp(-\mathcal{E}_t)\Omega_{t-1}^*(\sigma_t, j),
\end{aligned}
$$

where $\Theta(\sigma_{t+1}|\sigma_t)$ denote the time-t home-currency price for one unit of home currency delivered in period $t+1$ contingent on the state being σ_{t+1}. At time t, home household j holds $\Omega_t(\sigma_{t+1}, j)$ unit of the Arrow-Debreu security that pays off in state σ_{t+1}. $\Theta^*(\sigma_{t+1}|\sigma_t)$ is similarly defined as the time-t foreign-currency price for one unit of foreign currency delivered in period $t+1$ contingent on the state being σ_{t+1}, and $\Omega_t^*(\sigma_{t+1}, j)$ is the quantity of this security held by home household j.

The home household's Lagrangian is

$$
\begin{aligned}
\mathcal{L}(j) = \max \mathbb{E}_0 \sum_{t=0}^{\infty} \delta^t \left(\log c_t(j) - \chi \ell_t(j)\right) \\
&- \mathbb{E}_0 \sum_{t=0}^{\infty} \delta^t \zeta_t(j) \{ \sum_{\sigma_{t+1}} \Omega_t(\sigma_{t+1}, j)\Theta(\sigma_{t+1}|\sigma_t) + \exp(-\mathcal{E}_t) \sum_{\sigma_{t+1}} \Omega_t^*(\sigma_{t+1}, j)\Theta^*(\sigma_{t+1}|\sigma_t) \\
&- W_t \ell_t(j) - D_t(j) + P_t \tau_t(j) + P_t c_t(j) - \Omega_{t-1}(\sigma_t, j) - \exp(-\mathcal{E}_t)\Omega_{t-1}^*(\sigma_t, j) \}
\end{aligned}
$$

The first-order conditions are

$$
\begin{aligned}
\zeta_t(j) &= \frac{1}{P_t c_t(j)}, \\
\Theta(\sigma_{t+1}|\sigma_t) &= \delta \mathbb{P}(\sigma_{t+1}|\sigma_t) \frac{P_t c_t(j)}{P_{t+1} c_{t+1}(j)}, \\
\chi P_t c_t(j) &= W_t,
\end{aligned}
$$

We define the home country's nominal SDF as

$$
M_{t,t+1} := \delta \frac{P_t c_t(j)}{P_{t+1} c_{t+1}(j)},
$$

which implies the Euler equation for nominal risk-free rate

$$
\exp(-i_t) = \mathbb{E}_t \left[\delta \frac{P_t c_t}{P_{t+1} c_{t+1}} \right],
$$

i.e.,

$$
i_t = \log \left(\frac{1}{P_t c_t} \right) - \log \mathbb{E}_t \left[\delta \frac{1}{P_{t+1} c_{t+1}} \right].
$$

Similarly,

$$
i_t^* = \log \left(\frac{1}{P_t^* c_t^*} \right) - \log \mathbb{E}_t \left[\delta \frac{1}{P_{t+1}^* c_{t+1}^*} \right].
$$

Optimal Price Setting Based on information available in period $t-1$, the price at home $p_t(h)$ is set to maximize the expected profit from the home market:

$$\max_{p_t(h)} \mathbb{E}_{t-1}\left[M_{t-1,t}\left(p_t(h)-\frac{W_t}{z_t}\right)\left(\frac{p_t(h)}{P_{H,t}}\right)^{-\rho} c_{H,t}\right].$$

The future profit is discounted by the domestic households' SDF $M_{t-1,t}$, as they are the shareholders whose interests the firms maximize.

The first-order condition is

$$0=\mathbb{E}_{t-1}\left[\delta\frac{P_{t-1}c_{t-1}}{P_t c_t}\left((1-\rho)p_t(h)^{-\rho}+\rho\frac{W_t}{z_t}p_t(h)^{-\rho-1}\right)\left(\frac{1}{P_{H,t}}\right)^{-\rho}\alpha\frac{P_t}{P_{H,t}}c_t\right],$$

which, under symmetry $p_t(h)=P_{H,t}$, implies

$$P_{H,t}=\frac{\rho}{\rho-1}\mathbb{E}_{t-1}\left[\frac{W_t}{z_t}\right].$$

The home firms also need to set their sale price in the foreign market. Under *Producer Currency Pricing*, the price of the home firms' production sold in the foreign country is set according to

$$\max_{\exp(-\mathcal{E}_t)p_t^*(h)} \mathbb{E}_{t-1}\left[M_{t-1,t}\left(\exp(-\mathcal{E}_t)p_t^*(h)-\frac{W_t}{z_t}\right)\left(\frac{p_t^*(h)}{P_{H,t}^*}\right)^{-\rho} c_{H,t}^*\right],$$

which implies

$$\exp(-\mathcal{E}_t)P_{H,t}^*=\frac{\rho}{\rho-1}\mathbb{E}_{t-1}\left[\frac{W_t}{z_t}\right].$$

Similarly, entire price block can be described as

$$P_{H,t} = \exp(-\mathcal{E}_t) P^*_{H,t} = \frac{\rho}{\rho-1} \mathbb{E}_{t-1}\left[\frac{W_t}{z_t}\right], \tag{A.1}$$

$$P^*_{F,t} = \exp(\mathcal{E}_t) P_{F,t} = \frac{\rho}{\rho-1} \mathbb{E}_{t-1}\left[\frac{W^*_t}{z^*_t}\right]. \tag{A.2}$$

Equilibrium The market clearing condition for the home consumption good is

$$z\ell_t = \alpha \frac{P_t}{P_{H,t}}(c_t + g_t) + (1-\alpha)\frac{P^*_t}{P^*_{H,t}}(c^*_t + g^*_t).$$

i.e.,

$$z_t \ell_t = \frac{P_t c_t}{P_{H,t}} \left(1 + \alpha \frac{g_t}{c_t} + (1-\alpha)\frac{g^*_t}{c^*_t}\right) \tag{A.3}$$

Note that households' first order condition w.r.t. labor implies

$$W_t = \chi P_t c_t \tag{A.4}$$

Plug Eq. (A.4) and (A.1) into the market clearing condition to obtain

$$\ell_t = \frac{P_t c_t}{P_{H,t} z_t} \left(1 + \alpha \frac{g_t}{c_t} + (1-\alpha)\frac{g^*_t}{c^*_t}\right) = \frac{P_t c_t / z_t}{\mathbb{E}_{t-1}\left[P_t c_t / z_t\right]} \bar{\ell} \left(1 + \alpha \frac{g_t}{c_t} + (1-\alpha)\frac{g^*_t}{c^*_t}\right),$$

where $\bar{\ell} = \frac{\rho-1}{\rho\chi}$. Use Eq. (A.3) again,

$$c_t = z_t \ell_t \cdot \frac{P_{H,t}}{P_t} \left(1 + \alpha \frac{g_t}{c_t} + (1-\alpha)\frac{g^*_t}{c^*_t}\right)^{-1} = \frac{P_{H,t}}{\frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} P^\alpha_{H,t} P^{1-\alpha}_{F,t}} \frac{P_t c_t}{\mathbb{E}_{t-1}\left[P_t c_t / z_t\right]} \bar{\ell}$$
$$= \frac{P_t c_t}{\mathbb{E}_{t-1}\left[P_t c_t / z_t\right]} \bar{\ell} (\alpha^\alpha (1-\alpha)^{1-\alpha}) \frac{P^{1-\alpha}_{H,t}}{P^{1-\alpha}_{F,t}}$$

Use the price setting Eq. (A.1) and (A.2),

$$\frac{P_{H,t}}{P_{F,t}} = \frac{1}{\exp(-\mathcal{E}_t)} \frac{\mathbb{E}_{t-1}\left[W_t/z_t\right]}{\mathbb{E}_{t-1}\left[W_t^*/z_t^*\right]} = \frac{P_t^* c_t^*}{P_t c_t} \frac{\mathbb{E}_{t-1}\left[P_t c_t/z_t\right]}{\mathbb{E}_{t-1}\left[P_t^* c_t^*/z_t^*\right]}.$$

Then

$$c_t = \bar{\ell}\alpha^\alpha(1-\alpha)^{1-\alpha}\frac{(P_t c_t)^\alpha (P_t^* c_t^*)^{1-\alpha}}{(\mathbb{E}_{t-1}[P_t c_t/z_t])^\alpha (\mathbb{E}_{t-1}[P_t^* c_t^*/z_t^*])^{1-\alpha}}.$$

So the equilibrium price levels and consumption can be solved by

$$c_t = \bar{\ell}\alpha^\alpha(1-\alpha)^{1-\alpha}\frac{(P_t c_t)^\alpha (P_t^* c_t^*)^{1-\alpha}}{(\mathbb{E}_{t-1}[P_t c_t/z_t])^\alpha (\mathbb{E}_{t-1}[P_t^* c_t^*/z_t^*])^{1-\alpha}}, \tag{A.5}$$

$$c_t^* = \bar{\ell}\alpha^\alpha(1-\alpha)^{1-\alpha}\frac{(P_t c_t)^{1-\alpha} (P_t^* c_t^*)^{\alpha}}{(\mathbb{E}_{t-1}[P_t c_t/z_t])^{1-\alpha} (\mathbb{E}_{t-1}[P_t^* c_t^*/z_t^*])^{\alpha}}, \tag{A.6}$$

$$P_t = \frac{Q_t}{s_t + c_t \mathbb{E}_t\left[\sum_{k=1}^\infty \delta^k \frac{s_{t+k}}{c_{t+k}}\right]}, \tag{A.7}$$

$$P_t^* = \frac{Q_t^*}{s_t^* + c_t^* \mathbb{E}_t\left[\sum_{k=1}^\infty \delta^k \frac{s_{t+k}^*}{c_{t+k}^*}\right]}, \tag{A.8}$$

which implies

$$\begin{aligned}
\log c_t &= \kappa_{t-1}^c + \alpha\log(P_t c_t) + (1-\alpha)\log(P_t^* c_t^*),\\
\log c_t^* &= \kappa_{t-1}^{c^*} + \alpha\log(P_t^* c_t^*) + (1-\alpha)\log(P_t c_t),\\
-e_t = \log c_t - \log c_t^* &= \kappa_{t-1}^e + (2\alpha - 1)(\log(P_t c_t) - \log(P_t^* c_t^*)),
\end{aligned}$$

where

$$\begin{aligned}
\kappa_{t-1}^c &= \log\left(\frac{\bar{\ell}\alpha^\alpha(1-\alpha)^{1-\alpha}}{(\mathbb{E}_{t-1}[P_t c_t/z_t])^\alpha (\mathbb{E}_{t-1}[P_t^* c_t^*/z_t^*])^{1-\alpha}}\right),\\
\kappa_{t-1}^c &= \log\left(\frac{\bar{\ell}\alpha^\alpha(1-\alpha)^{1-\alpha}}{(\mathbb{E}_{t-1}[P_t c_t/z_t])^{1-\alpha} (\mathbb{E}_{t-1}[P_t^* c_t^*/z_t^*])^{\alpha}}\right),\\
\kappa_{t-1}^e &= \kappa_{t-1}^c - \kappa_{t-1}^{c^*}.
\end{aligned}$$

Denote

$$A_t = \mathbb{E}_t\left[\sum_{k=1}^{\infty}\delta^k \frac{s_{t+k}}{c_{t+k}}\right], \qquad A_t^* = \mathbb{E}_t\left[\sum_{k=1}^{\infty}\delta^k \frac{s_{t+k}^*}{c_{t+k}^*}\right].$$

We show that A_t is a constant if all shocks are i.i.d, i.e., $A_t = A$. Conjecture that c_t and c_t^* are functions of s_t and s_t^* only, hence A_t and A_t^* are constant if surplus shocks are i.i.d. Substitute Eq. (A.7) and (A.8) into (A.5) and (A.6) to obtain

$$c_t = \bar{\ell}\alpha^\alpha(1-\alpha)^{1-\alpha}\frac{(\frac{1}{s_t+c_tA}c_t)^\alpha(\frac{1}{s_t^*+c_t^*A^*}c_t^*)^{1-\alpha}}{(\mathbb{E}_{t-1}[\frac{1}{s_t+c_tA}c_t/z_t])^\alpha(\mathbb{E}_{t-1}[\frac{1}{s_t^*+c_t^*A^*}c_t^*/z_t^*])^{1-\alpha}}$$
$$c_t^* = \bar{\ell}\alpha^\alpha(1-\alpha)^{1-\alpha}\frac{(\frac{1}{s_t+c_tA}c_t)^{1-\alpha}(\frac{1}{s_t^*+c_t^*A^*}c_t^*)^\alpha}{(\mathbb{E}_{t-1}[\frac{1}{s_t+c_tA}c_t/z_t])^{1-\alpha}(\mathbb{E}_{t-1}[\frac{1}{s_t^*+c_t^*A^*}c_t^*/z_t^*])^\alpha}.$$

By the i.i.d. assumption of s_t, s_t^*, z_t, and z_t^*, the expectation terms $\mathbb{E}_{t-1}[\frac{1}{s_t+c_tA}c_t/z_t]$ and $\mathbb{E}_{t-1}[\frac{1}{s_t^*+c_t^*A^*}c_t^*/z_t^*]$ do not vary across periods. The two equations above can be used to solve c_t and c_t^* as functions of s_t and s_t^*, which confirms the conjecture.

A.2 Proof of Proposition 1

Note that under i.i.d. shocks, assuming $\Delta \log Q_{t+1} = \Delta \log Q_{t+1}^* = 0$, i.e., $Q_{t+1} = Q_{t+1}^* = \bar{Q}$, we obtain

$$\mathbb{E}_t\left[\frac{1}{P_{t+1}c_{t+1}}\right] = \frac{1}{\bar{Q}}\mathbb{E}_t\left[\frac{Q_{t+1}}{P_{t+1}c_{t+1}}\right] = \frac{1}{\bar{Q}}\left(A + \mathbb{E}_t\left[\frac{s_{t+1}}{c_{t+1}}\right]\right),$$

which is a constant. Plug in Euler equation into Eq. (3) to obtain

$$\log c_t = \kappa_{t-1}^c - \alpha\left(i_t + \log\mathbb{E}_t\left[\delta\frac{1}{P_{t+1}c_{t+1}}\right]\right) - (1-\alpha)\left(i_t^* + \log\mathbb{E}_t\left[\delta\frac{1}{P_{t+1}^*c_{t+1}^*}\right]\right),$$

which implies

$$\log \hat{c}_t = -\alpha \hat{i}_t - (1-\alpha)\hat{i}_t^*.$$

Similarly,

$$\log \hat{c}_t^* = -\alpha \hat{i}_t^* - (1-\alpha)\hat{i}_t,$$

and

$$\hat{e}_t = \log \hat{c}_t^* - \log \hat{c}_t = (2\alpha - 1)(\hat{i}_t - \hat{i}_t^*).$$

By Euler equation,

$$\log P_t = -\log c_t - i_t - \log \mathbb{E}_t \left[\delta \frac{1}{P_{t+1} c_{t+1}} \right],$$

hence

$$\log \hat{P}_t = -(1-\alpha)(\hat{i}_t - \hat{i}_t^*),$$

and similarly

$$\log \hat{P}_t^* = (1-\alpha)(\hat{i}_t - \hat{i}_t^*).$$

Recall that

$$\begin{aligned}
\log c_t &= \kappa_{t-1}^c + \alpha \log\left(\frac{c_t}{s_t + c_t A}\right) + (1-\alpha)\log\left(\frac{c_t^*}{s_t^* + c_t^* A^*}\right), \\
\log c_t^* &= \kappa_{t-1}^{c^*} + (1-\alpha) \log\left(\frac{c_t}{s_t + c_t A}\right) + \alpha\log\left(\frac{c_t^*}{s_t^* + c_t^* A^*}\right).
\end{aligned}$$

Linearizing both sides around the symmetric steady state yields

$$\log \hat{c}_t = \alpha \log \hat{c}_t + (1-\alpha)\log \hat{c}_t^* - \alpha \frac{1}{\bar{s}+\bar{c}A}(\hat{s}_t + \bar{c}A \log \hat{c}_t) - (1-\alpha)\frac{1}{\bar{s}+\bar{c}A}(\hat{s}_t^* + \bar{c}A\log \hat{c}_t^*),$$
$$\log \hat{c}_t^* = \alpha \log \hat{c}_t^* + (1-\alpha)\log \hat{c}_t - \alpha \frac{1}{\bar{s}+\bar{c}A}(\hat{s}_t^* + \bar{c}A \log \hat{c}_t^*) - (1-\alpha)\frac{1}{\bar{s}+\bar{c}A}(\hat{s}_t + \bar{c}A\log \hat{c}_t),$$

plug in $\log \hat{c}_t$ and $\log \hat{c}_t^*$ as functions of $\hat{i}_t$ and $\hat{i}_t^*$ to solve for $\hat{s}_t$ and $\hat{s}_t^*$

$$\begin{aligned}\hat{s}_t &= \frac{(1-\alpha)\bar{s} + \alpha A\bar{c}}{1-2\alpha}\log \hat{c}_t + \frac{-(1-\alpha)\bar{s} + (\alpha-1)A\bar{c}}{1-2\alpha}\log \hat{c}_t^* \\ &= A\bar{c}\hat{i}_t + (1-\alpha)\bar{s}(\hat{i}_t - \hat{i}_t^*), \\ \hat{s}_t^* &= A\bar{c}\hat{i}_t^* + (1-\alpha)\bar{s}(\hat{i}_t^* - \hat{i}_t).\end{aligned}$$

A.3 Proof of Proposition 2

Rearrange Eq. (5) to obtain

$$\begin{aligned}\hat{i}_t^* &= \frac{A\bar{c} + (1-\alpha)\bar{s}}{(1-\alpha)\bar{s}}\hat{i}_t - \frac{1}{(1-\alpha)\bar{s}}\hat{s}_t, \\ \hat{s}_t^* &= \left[\frac{(A\bar{c} + (1-\alpha)\bar{s})^2}{(1-\alpha)\bar{s}} - (1-\alpha)\bar{s}\right]\hat{i}_t - \frac{A\bar{c}+(1-\alpha)\bar{s}}{(1-\alpha)\bar{s}}\hat{s}_t \\ &= \frac{A^2\bar{c}^2 + 2A\bar{c}(1-\alpha)\bar{s}}{(1-\alpha)\bar{s}}\hat{i}_t - \frac{A\bar{c}+(1-\alpha)\bar{s}}{(1-\alpha)\bar{s}}\hat{s}_t.\end{aligned}$$

Let $\hat{i}_t > 0$ and $\hat{s}_t = 0$, we get

$$\begin{aligned}\hat{i}_t^* &= \frac{A\bar{c} + (1-\alpha)\bar{s}}{(1-\alpha)\bar{s}}\hat{i}_t > 0, \\ \hat{s}_t^* &= \frac{A^2\bar{c}^2 + 2A\bar{c}(1-\alpha)\bar{s}}{(1-\alpha)\bar{s}}\hat{i}_t > 0, \\ \hat{e}_t &= (2\alpha-1)(\hat{i} - \hat{i}_t^*) \\ &= -(2\alpha-1)\frac{A\bar{c}}{(1-\alpha)\bar{s}}\hat{i}_t < 0.\end{aligned}$$

A.4 Proof of Proposition 3

Since consumption is constant, the Euler equation can be restated as

$$\exp(-i_t) = \delta \mathbb{E}_t \left[\frac{1}{\exp(\pi_{t+1})} \right]$$

Linearize the system to obtain

$$\begin{aligned} \hat{i}_t &= \mathbb{E}_t[\hat{\pi}_{t+1}], \\ \hat{q}_t &= \hat{s}_t + \delta \hat{q}_{t+1} + \delta \bar{q}(\hat{\pi}_{t+1} - \hat{i}_t), \end{aligned}$$

where we plugged in $\exp(\bar{\pi} - \bar{i}) = \delta$. Plug in the monetary rule to obtain

$$\mathbb{E}_t[\hat{\pi}_{t+1}] = \phi \hat{\pi}_t + \theta_t. \tag{A.9}$$

Plug the fiscal rule into the intertemporal government budget constraint to obtain

$$\hat{q}_t = \gamma \delta \hat{q}_t + \gamma \delta \bar{q}(\hat{\pi}_t - \hat{i}_{t-1}) + \psi_t + \delta \hat{q}_{t+1} + \delta \bar{q}(\hat{\pi}_{t+1} - \hat{i}_t),$$

i.e.,

$$\hat{q}_{t+1} + \bar{q}(\hat{\pi}_{t+1} - \hat{i}_t) = (\delta^{-1} - \gamma)[\hat{q}_t + \bar{q}(\hat{\pi}_t - \hat{i}_{t-1})] - \delta^{-1}\bar{q}(\hat{\pi}_t - \phi\hat{\pi}_{t-1}) + \delta^{-1}\bar{q}\theta_{t-1} - \delta^{-1}\psi_t,$$

where $\hat{q}_{t+1} + \bar{q}(\hat{\pi}_{t+1} - \hat{i}_t)$ is predetermined, which we denote as $\hat{b}_t$. The system can be summarized by

$$\begin{bmatrix} \hat{b}_t \\ \hat{\pi}_t \\ \mathbb{E}_t[\hat{\pi}_{t+1}] \end{bmatrix} = \begin{bmatrix} (\delta^{-1} - \gamma) & \phi\delta^{-1}\bar{q} & -\delta^{-1}\bar{q} \\ 0 & 0 & 1 \\ 0 & 0 & \phi \end{bmatrix} \begin{bmatrix} \hat{b}_{t-1} \\ \hat{\pi}_{t-1} \\ \hat{\pi}_t \end{bmatrix} + \begin{bmatrix} \delta^{-1}\bar{q} & 0 & -\delta^{-1} \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \theta_{t-1} \\ \theta_t \\ \psi_t \end{bmatrix},$$

which has 3 eigenvalues: $\delta^{-1}-\gamma$, 0, ϕ. Since only $\hat{\pi}_{t+1}$ is non-predetermined, the system has a unique solution if and only if one of the eigenvalues is outside the unit circle (Blanchard and Kahn, 1980).

A.5 Proof of Proposition 4

The Euler equation implies

$$\exp(-i_t) = \mathbb{E}_t\left[\delta \exp(\sigma^{-1}\log c_t - \sigma^{-1}\log c_{t+1} - \pi_{t+1})\right],$$

i.e.,

$$\log \hat{c}_t = \mathbb{E}_t[\log \hat{c}_{t+1}] - \sigma(\hat{i}_t - \mathbb{E}_t[\hat{\pi}_{t+1}]), \tag{A.10}$$

which can be restated as, by plugging in monetary rule and the Phillips curve

$$\log \hat{c}_t = \mathbb{E}_t[\log \hat{c}_{t+1}] - \sigma(\phi\hat{\pi}_t + \theta_t - \delta^{-1}\hat{\pi}_t + \delta^{-1}\kappa \log \hat{c}_t).$$

The system can be summarized as

$$\begin{bmatrix} \hat{b}_t \\ \hat{\pi}_t \\ \mathbb{E}_t[\hat{\pi}_{t+1}] \\ \mathbb{E}_t[\log \hat{c}_{t+1}] \end{bmatrix} = \begin{bmatrix} (\delta^{-1}-\gamma) & \phi\delta^{-1}\bar{q} & -\delta^{-1}\bar{q} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \delta^{-1} & -\delta^{-1}\kappa \\ 0 & 0 & \sigma(\phi-\delta^{-1}) & 1+\sigma\delta^{-1}\kappa \end{bmatrix} \begin{bmatrix} \hat{b}_{t-1} \\ \hat{\pi}_{t-1} \\ \hat{\pi}_t \\ \log \hat{c}_t \end{bmatrix} + \begin{bmatrix} \delta^{-1}\bar{q} & 0 & -\delta^{-1} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \sigma & 0 \end{bmatrix} \begin{bmatrix} \theta_{t-1} \\ \theta_t \\ \psi_t \end{bmatrix}.$$

Since there are only 2 non-predetermined endogenous variables, the number of eigenvalues outside the unit circle must be 2 to guarantee unique solution. The eigenvalues are

$$
\begin{aligned}
\lambda_1 &= \delta^{-1} - \gamma, \\
\lambda_2 &= 0, \\
\lambda_3 &= \frac{1 + \delta^{-1} + \sigma\delta^{-1}\kappa}{2} + \frac{\sqrt{(1 - \delta^{-1} - \sigma\delta^{-1}\kappa)^2 + 4\sigma\delta^{-1}\kappa(1-\phi)}}{2}, \\
\lambda_4 &= \frac{1 + \delta^{-1} + \sigma\delta^{-1}\kappa}{2} - \frac{\sqrt{(1 - \delta^{-1} - \sigma\delta^{-1}\kappa)^2 + 4\sigma\delta^{-1}\kappa(1-\phi)}}{2}.
\end{aligned}
$$

We focus on the usual case where $\delta < 1$, $\phi \geq 0$ and $\kappa \geq 0$, and that the parameters must satisfy that $\lambda_3, \lambda_4 \in \mathbb{R}$ exist. In this case, $\phi > 1$ indicates active monetary policy. Note that the last two eigenvalues satisfy

$$
P(\lambda) = \lambda^2 - (1 + \delta^{-1} + \sigma\delta^{-1}\kappa)\lambda + \delta^{-1} + \sigma\delta^{-1}\kappa\phi = 0,
$$

where

$$
P(0) = \delta^{-1} + \sigma\delta^{-1}\kappa\phi > 0, \quad P(1) = \sigma\delta^{-1}\kappa(\phi - 1).
$$

The axis of symmetry is $(1 + \delta^{-1} + \sigma\delta^{-1}\kappa)/2 > 1$, which implies polynomial $P(\lambda)$ has at most one root inside the unit circle. Hence, if $\phi > 1$, i.e., the monetary policy is active, then λ_3 and λ_4 are both outside the unit circle, which requires $|\delta^{-1} - \gamma| < 1$, i.e., passive fiscal policy. If $\phi < 1$, i.e., the monetary policy is passive, then $P(1) < 0$, which implies $\lambda_3 > 1$, $\lambda_4 < 1$. To guarantee the uniqueness of the solution, $|\lambda_1| > 0$ must hold, i.e., $|\delta^{-1} - \gamma| > 1$, i.e., active fiscal policy.

A.6 Derivation of the Two-Country Calvo Model

We preserve the assumptions of Calvo pricing and producer currency pricing: in each period, only a fraction $(1 - \xi)$ of firms can reset their prices. Firms set the price in their own currency and the law of one price holds (PCP). Denote the log nominal exchange rate as $\mathcal{E}_t$. Following A.1, we obtain the price aggregation rule

$$\begin{aligned}
P_{H,t} &= \left(\int_0^1 P_{H,t}(j)^{1-\rho}dj\right)^{1/(1-\rho)}, \\
P^*_{F,t} &= \left(\int_0^1 P^*_{F,t}(j)^{1-\rho}dj\right)^{1/(1-\rho)}, \\
P_t &= \frac{1}{\alpha^\alpha(1-\alpha)^{1-\alpha}} P^\alpha_{H,t}(P^*_{F,t})^{1-\alpha}\exp(-(1-\alpha)\mathcal{E}_t), \\
P^*_t &= \frac{1}{\alpha^\alpha(1-\alpha)^{1-\alpha}} (P^*_{F,t})^\alpha P^{1-\alpha}_{H,t}\exp((1-\alpha)\mathcal{E}_t).
\end{aligned}$$

Law of one price implies $P^*_{H,t}(j) = \exp(\mathcal{E}_t)P_{H,t}(j)$, $P^*_{F,t}(j) = \exp(\mathcal{E}_t)P_{F,t}(j)$, i.e.,

$$\frac{P_{H,t}(j)}{P_{H,t}} = \frac{P^*_{H,t}(j)}{P^*_{H,t}}, \quad \frac{P_{F,t}(j)}{P_{F,t}} = \frac{P^*_{F,t}(j)}{P^*_{F,t}}.$$

Denote the home aggregate purchase (i.e., households' consumption plus government purchase) of home and foreign intermediate good as $y_{H,t}$ and $y_{F,t}$. The demand curves for home intermediate goods are given by

$$\begin{aligned}
y_{H,t}(j) &= y_{H,t}\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\rho}, \\
y^*_{H,t}(j) &= y^*_{H,t}\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\rho}.
\end{aligned}$$

Firms take the aggregate demand for home goods $y_{H,t}$ and $y^*_{H,t}$ as given. Home firms that get the change to reset their price at time t maximize the discounted profit over the course of this price contract, i.e.,

$$\max_{P^\#_{H,t}} \quad \mathbb{E}_t\left[\sum_{k=0}^\infty (\delta\xi)^k \frac{P_t}{P_{t+k}}\left(\frac{c_{t+k}}{c_t}\right)^{-\sigma^{-1}}\left((y_{H,t+k}+y^*_{H,t+k})P^\rho_{H,t+k}(P^\#_{H,t})^{1-\rho} - C_H((y_{H,t+k}+y^*_{H,t+k})P^\rho_{H,t+k}(P^\#_{H,t})^{-\rho})\right)\right],$$

where $C_H(.)$ is the nominal cost function and $\delta^{t+k}\frac{P_t}{P_{t+k}}\left(\frac{c_{t+k}}{c_t}\right)^{-\sigma^{-1}}$ is the nominal stochastic discount factor. The first order condition is

$$\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\delta\xi)^k\frac{P_t}{P_{t+k}}\left(\frac{c_{t+k}}{c_t}\right)^{-\sigma^{-1}}\left((1-\rho)(y_{H,t+k}+y^*_{H,t+k})P^{\rho}_{H,t+k}(P^{\#}_{H,t})^{-\rho}+\rho MC_{H,t+k}(y_{H,t+k}+y^*_{H,t+k})P^{\rho}_{H,t+k}(P^{\#}_{H,t})^{-\rho-1}\right)\right]=0,$$

i.e.,

$$P^{\#}_{H,t}=\frac{\rho}{\rho-1}\frac{\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\delta\xi)^k\frac{P_t}{P_{t+k}}\left(\frac{c_{t+k}}{c_t}\right)^{-\sigma^{-1}}(y_{H,t+k}+y^*_{H,t+k})P^{\rho}_{H,t+k}MC_{H,t+k}\right]}{\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\delta\xi)^k\frac{P_t}{P_{t+k}}\left(\frac{c_{t+k}}{c_t}\right)^{-\sigma^{-1}}(y_{H,t+k}+y^*_{H,t+k})P^{\rho}_{H,t+k}\right]}.$$

Log-linearize the first order condition to obtain

$$\hat{P}^{\#}_{H,t}=(1-\delta\xi)\sum_{k=0}^{\infty}(\delta\xi)^k\mathbb{E}_t[\log\widehat{MC}_{H,t+k}],$$

which can be restated as

$$\hat{P}^{\#}_{H,t}=(1-\delta\xi)\log\widehat{MC}_{H,t}+\delta\xi\mathbb{E}_t[\hat{P}^{\#}_{H,t+1}].$$

Similarly,

$$\hat{P}^{*\#}_{F,t}=(1-\delta\xi)\log\widehat{MC}^{*}_{F,t}+\delta\xi\mathbb{E}_t[\hat{P}^{*\#}_{F,t+1}].$$

The aggregate price of intermediate goods follow

$$\hat{P}_{H,t}=(1-\xi)\hat{P}^{\#}_{H,t}+\xi\hat{P}_{H,t-1},$$
$$\hat{P}^{*}_{F,t}=(1-\xi)\hat{P}^{*\#}_{F,t}+\xi\hat{P}^{*}_{F,t-1},$$

which jointly imply

$$\hat{\pi}_{H,t} = \delta\mathbb{E}_t[\hat{\pi}_{H,t+1}] + \frac{(1-\xi)(1-\delta\xi)}{\xi}(\log \widehat{MC}_{H,t} - \log \hat{P}_{H,t}),$$
$$\hat{\pi}^*_{F,t} = \delta\mathbb{E}_t[\hat{\pi}^*_{F,t+1}] + \frac{(1-\xi)(1-\delta\xi)}{\xi}(\log \widehat{MC}^*_{F,t} - \log \hat{P}^*_{F,t}),$$

where $\hat{\pi}_{H,t} := \log \hat{P}_{H,t} - \log \hat{P}_{H,t-1}$, $\hat{\pi}^*_{F,t} := \log \hat{P}^*_{F,t} - \log \hat{P}^*_{F,t-1}$.

A.7 Proof of Proposition 5

Recall that the price aggregation rule implies

$$\log \hat{P}_t = \alpha \log \hat{P}_{H,t} + (1-\alpha)\log \hat{P}^*_{F,t} - (1-\alpha)\hat{\mathcal{E}}_t,$$
$$\log \hat{P}^*_t = (1-\alpha)\log \hat{P}_{H,t} + \alpha \log \hat{P}^*_{F,t} + (1-\alpha)\hat{\mathcal{E}}_t,$$

which solve for the price level of intermediate goods

$$\log \hat{P}_{H,t} = \log \hat{P}_t + \frac{1-\alpha}{2\alpha-1}(\log \hat{P}_t - \log \hat{P}^*_t + \hat{\mathcal{E}}_t)$$
$$\log \hat{P}_{H,t} = \log \hat{P}_t + \frac{1-\alpha}{2\alpha-1}\hat{e}_t,$$
$$\log \hat{P}^*_{F,t} = \log \hat{P}^*_t - \frac{1-\alpha}{2\alpha-1}\hat{e}_t.$$

Plug into Eq. (13) and (14) to obtain

$$\begin{aligned}\hat{\pi}_t + \frac{1-\alpha}{2\alpha-1}\Delta\hat{e}_t &= \delta\mathbb{E}_t[\hat{\pi}_{t+1} + \frac{1-\alpha}{2\alpha-1}\Delta\hat{e}_{t+1}] + \frac{(1-\xi)(1-\delta\xi)}{\xi}(\log \widehat{mc}_{H,t} + \log \hat{P}_t - \log \hat{P}_{H,t})\\ &= \delta\mathbb{E}_t[\hat{\pi}_{t+1} + \frac{1-\alpha}{2\alpha-1}\Delta\hat{e}_{t+1}] + \frac{(1-\xi)(1-\delta\xi)}{\xi}(\log \widehat{mc}_{H,t} - \frac{1-\alpha}{2\alpha-1}\hat{e}_t),\end{aligned}$$

i.e.,

$$
\begin{aligned}
\hat{\pi}_t &= \delta \mathbb{E}_t[\hat{\pi}_{t+1}] + \delta \frac{1-\alpha}{2\alpha-1} \mathbb{E}_t[\Delta \hat{e}_{t+1}] - \frac{1-\alpha}{2\alpha-1} \Delta \hat{e}_t \\
&\quad + \frac{(1-\xi)(1-\delta\xi)}{\xi} (\log \widehat{MC}_{H,t} - \log \hat{P}_t) - \frac{(1-\xi)(1-\delta\xi)}{\xi} \frac{1-\alpha}{2\alpha-1} \hat{e}_t \\
&= \delta \mathbb{E}_t[\hat{\pi}_{t+1}] + \delta \frac{1-\alpha}{2\alpha-1} \mathbb{E}_t[\hat{e}_{t+1}] - \frac{1+\delta\xi^2}{\xi} \frac{1-\alpha}{2\alpha-1} \hat{e}_t + \frac{1-\alpha}{2\alpha-1} \hat{e}_{t-1} + \frac{(1-\xi)(1-\delta\xi)}{\xi} \log \widehat{mc}_{H,t},
\end{aligned}
$$

where $mc_{H,t}$ is real marginal cost of home firms. Under linear technology $y_t(h) = z_t \ell_t(h)$, the real marginal cost gap can be related to output gap, i.e.,

$$
mc_{H,t} = \frac{1}{z_t} w_t.
$$

Households' first order condition w.r.t. ℓ_t yields the labor supply curve

$$
c_t^{-\sigma^{-1}} w_t = \chi.
$$

Hence

$$
\log \widehat{mc}_{H,t} = \log \hat{w}_t - \log \hat{z}_t = \sigma^{-1} \log \hat{c}_t - \log \hat{z}_t,
$$

which implies open-economy NKPCs:

$$
\begin{aligned}
\hat{\pi}_t &= \delta \mathbb{E}_t[\hat{\pi}_{t+1}] + \delta \frac{1-\alpha}{2\alpha-1} \mathbb{E}_t[\hat{e}_{t+1}] - \eta \frac{1-\alpha}{2\alpha-1} \hat{e}_t + \frac{1-\alpha}{2\alpha-1} \hat{e}_{t-1} + \kappa \log \hat{c}_t - \kappa\sigma \log \hat{z}_t, \\
\hat{\pi}_t^* &= \delta \mathbb{E}_t[\hat{\pi}_{t+1}^*] - \delta \frac{1-\alpha}{2\alpha-1} \mathbb{E}_t[\hat{e}_{t+1}] + \eta \frac{1-\alpha}{2\alpha-1} \hat{e}_t - \frac{1-\alpha}{2\alpha-1} \hat{e}_{t-1} + \kappa \log \hat{c}_t^* - \kappa\sigma \log \hat{z}_t^*,
\end{aligned}
$$

where $\kappa \overset{\text{def}}{=} \frac{(1-\xi)(1-\delta\xi)}{\xi} \sigma^{-1}$, $\eta \overset{\text{def}}{=} (1+\delta\xi^2)/\xi$.

A.8 Proof of Proposition 6

The home and foreign government implement the following monetary and fiscal policies

$$i_t = \phi_0 + \phi\pi_t + \theta_t,$$

$$i_t^* = \phi_0^* + \phi^*\pi_t^* + \theta_t^*,$$

$$s_t = \gamma_0 + \gamma b_{t-1} + \psi_t,$$

$$s_t^* = \gamma_0^* + \gamma^* b_{t-1}^* + \psi_t^*,$$

subject to the intertemporal government budget constraints

$$\hat{b}_t = (\delta^{-1} - \gamma)\hat{b}_{t-1} - \delta^{-1}\bar{q}(\hat{\pi}_t - \phi\hat{\pi}_{t-1}) + \delta^{-1}\bar{q}\theta_{t-1} - \delta^{-1}\psi_t,$$

$$\hat{b}_t^* = (\delta^{-1} - \gamma^*)\hat{b}_{t-1}^* - \delta^{-1}\bar{q}(\hat{\pi}_t^* - \phi^*\hat{\pi}_{t-1}^*) + \delta^{-1}\bar{q}\theta_{t-1}^* - \delta^{-1}\psi_t^*.$$

Recall that σ is the inverse of CRRA coefficient. The Euler equations are

$$1 = \mathbb{E}_t\left[\delta\exp(\sigma^{-1}\log c_t - \sigma^{-1}\log c_{t+1} - \pi_{t+1} + i_t)\right],$$

$$1 = \mathbb{E}_t\left[\delta\exp(\sigma^{-1}\log c_t^* - \sigma^{-1}\log c_{t+1}^* - \pi_{t+1}^* + i_t^*)\right],$$

$$1 = \mathbb{E}_t\left[\delta\exp(\sigma^{-1}\log c_t - \sigma^{-1}\log c_{t+1} - \pi_{t+1} + i_t^* - \Delta\mathcal{E}_{t+1})\right],$$

$$1 = \mathbb{E}_t\left[\delta\exp(\sigma^{-1}\log c_t^* - \sigma^{-1}\log c_{t+1}^* - \pi_{t+1}^* + i_t + \Delta\mathcal{E}_{t+1})\right].$$

Log-linearize the Euler equations to obtain

$$\log\hat{c}_t = \mathbb{E}_t[\log\hat{c}_{t+1}] - \sigma(\hat{i}_t - \mathbb{E}_t[\hat{\pi}_{t+1}]), \tag{A.11}$$

$$\log\hat{c}_t^* = \mathbb{E}_t[\log\hat{c}_{t+1}^*] - \sigma(\hat{i}_t^* - \mathbb{E}_t[\hat{\pi}_{t+1}^*]), \tag{A.12}$$

$$\log\hat{c}_t = \mathbb{E}_t[\log\hat{c}_{t+1}] - \sigma(\hat{i}_t^* - \mathbb{E}_t[\hat{\pi}_{t+1}^*] - \mathbb{E}_t[\Delta\hat{e}_{t+1}]). \tag{A.13}$$

Note that after linearization, one Euler equation becomes redundant. Combining Eq. (A.11) and (A.13) yields the uncovered interest parity (UIP), i.e.,

$$\hat{i}^*_t - \mathbb{E}_t[\hat{\pi}^*_{t+1}] - \mathbb{E}_t[\Delta \hat{e}_{t+1}] = \hat{i}_t - \mathbb{E}_t[\hat{\pi}_{t+1}], \tag{A.14}$$

which is satisfied under complete market

$$\hat{e}_t = \sigma^{-1}(\log \hat{c}^*_t - \log \hat{c}_t).$$

The equation system can be summarized by

$$\begin{bmatrix} \hat{b}_t \\ \hat{b}^*_t \\ \hat{\pi}_t \\ \hat{\pi}^*_t \\ \log \hat{c}_t \\ \log \hat{c}^*_t \\ \mathbb{E}_t[\hat{\pi}_{t+1}] \\ \mathbb{E}_t[\hat{\pi}^*_{t+1}] \\ \mathbb{E}_t[\log \hat{c}_{t+1}] \\ \mathbb{E}_t[\log \hat{c}^*_{t+1}] \end{bmatrix} = \Psi \begin{bmatrix} \hat{b}_{t-1} \\ \hat{b}^*_{t-1} \\ \hat{\pi}_{t-1} \\ \hat{\pi}^*_{t-1} \\ \log \hat{c}_{t-1} \\ \log \hat{c}^*_{t-1} \\ \hat{\pi}_t \\ \hat{\pi}^*_t \\ \log \hat{c}_t \\ \log \hat{c}^*_t \end{bmatrix} + \Phi \begin{bmatrix} \theta_{t-1} \\ \theta^*_{t-1} \\ \theta_t \\ \theta^*_t \\ \psi_t \\ \psi^*_t \\ \log \hat{z}_t \\ \log \hat{z}^*_t \end{bmatrix},$$

where

$$\Psi = \begin{bmatrix} \delta^{-1}-\gamma & 0 & \delta^{-1}\phi\bar{q} & 0 & 0 & 0 & -\delta^{-1}\bar{q} & 0 & 0 & 0 \\ 0 & \delta^{-1}-\gamma^* & 0 & \delta^{-1}\phi^*\bar{q} & 0 & 0 & 0 & -\delta^{-1}\bar{q} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & \aleph_{11} & \aleph_{12} & \aleph_{13} & \aleph_{14} & \aleph_{15} & \aleph_{16} \\ 0 & 0 & 0 & 0 & \aleph_{21} & \aleph_{22} & \aleph_{23} & \aleph_{24} & \aleph_{25} & \aleph_{26} \\ 0 & 0 & 0 & 0 & \aleph_{31} & \aleph_{32} & \aleph_{33} & \aleph_{34} & \aleph_{35} & \aleph_{36} \\ 0 & 0 & 0 & 0 & \aleph_{41} & \aleph_{42} & \aleph_{43} & \aleph_{44} & \aleph_{45} & \aleph_{46} \end{bmatrix},$$

$$\aleph = \delta^{-1}$$
$$\begin{bmatrix} (1-\alpha)\sigma^{-1} & -(1-\alpha)\sigma^{-1} & \alpha+(1-\alpha)\delta\phi & (1-\alpha)(1-\delta\phi^*) & (1-\alpha)(\delta-\eta)\sigma^{-1}-\alpha\kappa & -(1-\alpha)(\delta-\eta)\sigma^{-1}-(1-\alpha)\kappa \\ -(1-\alpha)\sigma^{-1} & (1-\alpha)\sigma^{-1} & (1-\alpha)(1-\delta\phi) & \alpha+(1-\alpha)\delta\phi^* & -(1-\alpha)(\delta-\eta)\sigma^{-1}-(1-\alpha)\kappa & (1-\alpha)(\delta-\eta)\sigma^{-1}-\alpha\kappa \\ -(1-\alpha) & 1-\alpha & -\alpha(1-\delta\phi)\sigma & -(1-\alpha)(1-\delta\phi^*)\sigma & \alpha(\delta-\eta+\kappa\sigma)+\eta & (1-\alpha)(\delta-\eta+\kappa\sigma) \\ 1-\alpha & -(1-\alpha) & -(1-\alpha)(1-\delta\phi)\sigma & -\alpha(1-\delta\phi^*)\sigma & (1-\alpha)(\delta-\eta+\kappa\sigma) & \alpha(\delta-\eta+\kappa\sigma)+\eta \end{bmatrix},$$

$$\Phi = \begin{bmatrix} \delta^{-1}\bar{q} & 0 & 0 & 0 & -\delta^{-1} & 0 & 0 & 0 \\ 0 & \delta^{-1}\bar{q} & 0 & 0 & 0 & -\delta^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1-\alpha & -(1-\alpha) & 0 & 0 & \delta^{-1}\alpha\kappa\sigma & \delta^{-1}(1-\alpha)\kappa\sigma \\ 0 & 0 & -(1-\alpha) & 1-\alpha & 0 & 0 & \delta^{-1}(1-\alpha)\kappa\sigma & \delta^{-1}\alpha\kappa\sigma \\ 0 & 0 & \alpha\sigma & (1-\alpha)\sigma & 0 & 0 & -\delta^{-1}\alpha\kappa\sigma^2 & -\delta^{-1}(1-\alpha)\kappa\sigma^2 \\ 0 & 0 & (1-\alpha)\sigma & \alpha\sigma & 0 & 0 & -\delta^{-1}(1-\alpha)\kappa\sigma^2 & -\delta^{-1}\alpha\kappa\sigma^2 \end{bmatrix}.$$

A.9 Algebraic Solution of Monetary Dominance in Closed Economy

From Proposition 4, the expected inflation and consumption satisfy

$$\begin{bmatrix} \mathbb{E}_t[\hat{\pi}_{t+1}] \\ \mathbb{E}_t[\log \hat{c}_{t+1}] \end{bmatrix} = \begin{bmatrix} \delta^{-1} & -\delta^{-1}\kappa \\ \sigma(\phi-\delta^{-1}) & 1+\sigma\delta^{-1}\kappa \end{bmatrix} \begin{bmatrix} \hat{\pi}_t \\ \log \hat{c}_t \end{bmatrix} + \begin{bmatrix} 0 \\ \sigma\theta_t \end{bmatrix}.$$

Diagnolize the transition matrix to obtain

$$\begin{bmatrix} \mathbb{E}_t[\hat{\pi}_{t+1}] \\ \mathbb{E}_t[\log \hat{c}_{t+1}] \end{bmatrix} = V \begin{bmatrix} \lambda_4 & 0 \\ 0 & \lambda_3 \end{bmatrix} V^{-1} \begin{bmatrix} \hat{\pi}_t \\ \log \hat{c}_t \end{bmatrix} + \begin{bmatrix} 0 \\ \sigma\theta_t \end{bmatrix},$$

i.e.,

$$V^{-1} \begin{bmatrix} \mathbb{E}_t[\hat{\pi}_{t+1}] \\ \mathbb{E}_t[\log \hat{c}_{t+1}] \end{bmatrix} = \begin{bmatrix} \lambda_4 & 0 \\ 0 & \lambda_3 \end{bmatrix} V^{-1} \begin{bmatrix} \hat{\pi}_t \\ \log \hat{c}_t \end{bmatrix} + V^{-1} \begin{bmatrix} 0 \\ \sigma\theta_t \end{bmatrix}.$$

Let

$$\begin{bmatrix} \breve{\pi}_t \\ \log \breve{c}_t \end{bmatrix} := V^{-1} \begin{bmatrix} \hat{\pi}_t \\ \log \hat{c}_t \end{bmatrix}, \quad \begin{bmatrix} \breve{\theta}_t^{\pi} \\ \breve{\theta}_t^{c} \end{bmatrix} := V^{-1} \begin{bmatrix} 0 \\ \sigma\theta_t \end{bmatrix},$$

We obtain

$$\begin{bmatrix} \mathbb{E}_t[\breve{\pi}_{t+1}] \\ \mathbb{E}_t[\log \breve{c}_{t+1}] \end{bmatrix} = \begin{bmatrix} \lambda_4 & 0 \\ 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} \breve{\pi}_t \\ \log \breve{c}_t \end{bmatrix} + \begin{bmatrix} \breve{\theta}_t^{\pi} \\ \breve{\theta}_t^{c} \end{bmatrix},$$

which implies

$$\begin{bmatrix} \mathbb{E}_t[\breve{\pi}_{t+2}] \\ \mathbb{E}_t[\log \breve{c}_{t+2}] \end{bmatrix} = \begin{bmatrix} \lambda_4^2 & 0 \\ 0 & \lambda_3^2 \end{bmatrix} \begin{bmatrix} \breve{\pi}_t \\ \log \breve{c}_t \end{bmatrix} + \begin{bmatrix} \mathbb{E}_t[\breve{\theta}_{t+1}^{\pi}] \\ \mathbb{E}_t[\breve{\theta}_{t+1}^{c}] \end{bmatrix} + \begin{bmatrix} \lambda_4 & 0 \\ 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} \breve{\theta}_t^{\pi} \\ \breve{\theta}_t^{c} \end{bmatrix},$$

where

$$\begin{bmatrix} \mathbb{E}_t[\breve{\theta}_{t+1}^{\pi}] \\ \mathbb{E}_t[\breve{\theta}_{t+1}^{c}] \end{bmatrix} = \rho_1 \begin{bmatrix} \breve{\theta}_t^{\pi} \\ \breve{\theta}_t^{c} \end{bmatrix}$$

hence

$$
\begin{aligned}
\begin{bmatrix} \mathbb{E}_t[\check{\pi}_{t+k}] \\ \mathbb{E}_t[\log \check{c}_{t+k}] \end{bmatrix} &= \begin{bmatrix} \lambda_4^k & 0 \\ 0 & \lambda_3^k \end{bmatrix} \begin{bmatrix} \check{\pi}_t \\ \log \check{c}_t \end{bmatrix} + \rho_1^{k-1} \sum_{j=0}^{k-1} \left(\frac{1}{\rho_1} \right)^j \begin{bmatrix} \lambda_4^j & 0 \\ 0 & \lambda_3^j \end{bmatrix} \begin{bmatrix} \check{\theta}_t^\pi \\ \check{\theta}_t^c \end{bmatrix} \\
&= \begin{bmatrix} \lambda_4^k & 0 \\ 0 & \lambda_3^k \end{bmatrix} \begin{bmatrix} \check{\pi}_t \\ \log \check{c}_t \end{bmatrix} + \rho_1^{k-1} \begin{bmatrix} \frac{1-\left(\frac{\lambda_4}{\rho_1}\right)^k}{1-\frac{\lambda_4}{\rho_1}} & 0 \\ 0 & \frac{1-\left(\frac{\lambda_3}{\rho_1}\right)^k}{1-\frac{\lambda_3}{\rho_1}} \end{bmatrix} \begin{bmatrix} \check{\theta}_t^\pi \\ \check{\theta}_t^c \end{bmatrix} \\
&= \begin{bmatrix} \lambda_4^k & 0 \\ 0 & \lambda_3^k \end{bmatrix} \begin{bmatrix} \check{\pi}_t \\ \log \check{c}_t \end{bmatrix} + \begin{bmatrix} \frac{\rho_1^k - \lambda_4^k}{\rho_1 - \lambda_4} & 0 \\ 0 & \frac{\rho_1^k - \lambda_3^k}{\rho_1 - \lambda_3} \end{bmatrix} \begin{bmatrix} \check{\theta}_t^\pi \\ \check{\theta}_t^c \end{bmatrix}.
\end{aligned}
$$

We require that expected consumption and inflation do not explode exponentially, hence

$$
\begin{bmatrix} \check{\pi}_t \\ \log \check{c}_t \end{bmatrix} = \begin{bmatrix} \frac{1}{\rho_1 - \lambda_4} & 0 \\ 0 & \frac{1}{\rho_1 - \lambda_3} \end{bmatrix} \begin{bmatrix} \check{\theta}_t^\pi \\ \check{\theta}_t^c \end{bmatrix}.
$$

Multiply both sides by V to recover $\hat{\pi}_t$ and $\log \hat{c}_t$, i.e.,

$$
\begin{bmatrix} \hat{\pi}_t \\ \log \hat{c}_t \end{bmatrix} = V \begin{bmatrix} \frac{1}{\rho_1 - \lambda_4} & 0 \\ 0 & \frac{1}{\rho_1 - \lambda_3} \end{bmatrix} V^{-1} \begin{bmatrix} 0 \\ \sigma \theta_t \end{bmatrix},
$$

which simplifies to

$$
\begin{aligned}
\hat{\pi}_t &= -\frac{\kappa \sigma}{1 + \kappa \phi \sigma - \rho_1(1 + \delta - \delta \rho_1 + \kappa \sigma)} \theta_t, \\
\log \hat{c}_t &= -\frac{(1 - \delta \rho_1)\sigma}{1 + \kappa \phi \sigma - \rho_1(1 + \delta - \delta \rho_1 + \kappa \sigma)} \theta_t.
\end{aligned}
$$

The nominal rate is given by

$$
\begin{aligned}
\hat{i}_t &= \phi \hat{\pi}_t + \theta_t \\
&= \frac{1 - \rho_1(1 + \delta - \delta \rho_1 + \kappa \sigma)}{1 + \kappa \phi \sigma - \rho_1(1 + \delta - \delta \rho_1 + \kappa \sigma)} \theta_t.
\end{aligned}
$$

According to the NKPC, the expected inflation is given by

$$\begin{aligned}\mathbb{E}_t[\hat{\pi}_{t+1}] &= \delta^{-1}\hat{\pi}_t - \delta^{-1}\kappa \log \hat{c}_t \\ &= -\frac{\kappa\sigma\rho_1}{1+\kappa\phi\sigma-\rho_1(1+\delta-\delta\rho_1+\kappa\sigma)}\theta_t,\end{aligned}$$

and the real rate is

$$\begin{aligned}\hat{r}_t &= \hat{i}_t - \mathbb{E}_t[\hat{\pi}_{t+1}] \\ &= \frac{(1-\rho_1)(1-\rho_1\delta)}{1+\kappa\phi\sigma-\rho_1(1+\delta-\delta\rho_1+\kappa\sigma)}\theta_t.\end{aligned}$$

Real government can be solved "backwardly" from the stable difference equation

$$\hat{b}_t = (\delta^{-1}-\gamma)\hat{b}_{t-1} - \delta^{-1}\bar{q}(\hat{\pi}_t - \hat{i}_{t-1}) - \delta^{-1}\psi_t,$$

and the real surplus is pinned down by fiscal policy

$$\hat{s}_t = \gamma\hat{b}_{t-1} + \psi_t.$$

A.10 Model Calibration

The parameters of the open-economy model in Section 3 are shown in Table A.1. The closed-economy model shares the same parameters.

Table A.1: Parameters

	Notation	Value
Discount rate	δ	0.96
Inverse of constant relative risk aversion	σ	0.50
Home bias	α	0.80
Chance of resetting price	$1-\xi$	0.50
Steady state government bond oustanding	$\bar{q}$	1.00
Home monetary policy	ϕ	1.80
Home fiscal policy	γ	0.00
Foreign monetary policy	ϕ^*	0.20
Foreign fiscal policy	γ^*	0.20
Home monetary policy shock persistence	ρ_1	0.30

A.11 Alternative Specification

We consider the alternative foreign monetary policy rule with direct response to exchange rate, i.e.,

$$i_t^* = \phi_0^* + \phi^* \pi_t + \chi^e e_t + \theta_t^*,$$

where we calibrate $\chi^e = -2$. In this case, the foreign monetary authority lowers the nominal interest rate when the dollar appreciates. Other parameters are inherited from Table A.1. The impulse responses are shown in Figure A.1.

In this specification, we obtain qualitatively identical results as in the baseline setting: when the home country lowers the nominal interest rate i without changing its government surplus s, the

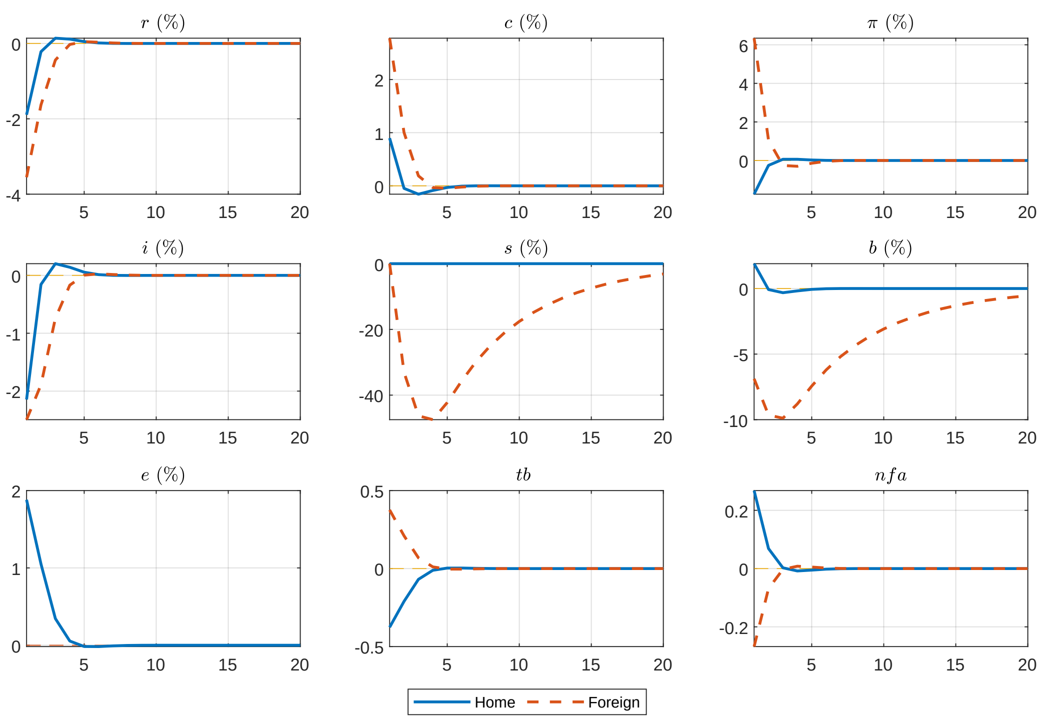


Figure A.1: Impulse Responses to a Monetary Shock in the Open Economy under Hegemon Dominance: Alternative Specification.

Note: We plot the impulse responses to a monetary shock θ_t in the open economy under hegemon dominance. The solid blue curve represents the home country, and the dashed red curve represents the foreign country. The outcome variables include the nominal interest rate i_t, surplus s_t, debt level b_t, consumption c_t, inflation π_t, and real rate r_t in both countries, and the real exchange rate e_t.

foreign nominal interest rate i^* also declines while the foreign government surplus s^* decreases. The real rates r and r^* decrease in both countries, the dollar strengthens in real terms, and the consumption c and c^* increase in both countries. Moreover, inflation π goes down in the home country and π^* up in the foreign country.