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INFORMATION DISCOVERY FOR INDUSTRIAL POLICY

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ABSTRACT

Amid growing interest in industrial policy, we develop a model exploring the tension between market-driven information discovery and policymakers' career incentives. While market-based information discovery can help address informational barriers faced by policymakers, career incentives may lead them to aggressively pursue their agendas to signal political capability, shifting dynamics toward a government-centric equilibrium. In this equilibrium, market participants focus on policy-related information over industry fundamentals, weakening the market's role in information discovery and reducing policy efficiency. Our analysis highlights the importance of bureaucratic frictions and market-based information discovery in jointly shaping the effectiveness of industrial policy implementation.

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In the aftermath of the 2008 financial crisis and the COVID-19 pandemic, there has been a global resurgence of interest in industrial policy. Asian countries such as Japan, South Korea, Taiwan, and Singapore have long used state-led strategies to nurture emerging industries. China, in particular, has expanded its industrial policy efforts, becoming the world's largest exporter and second-largest economy. Inspired by such successes, other developing nations, like India, are now implementing large-scale state-led strategies to boost their manufacturing capabilities. Developed economies, such as the U.S. and the EU—historically reluctant to adopt industrial policies—are now launching initiatives like the U.S. CHIPS and Inflation Reduction Acts to promote technological innovation and new industries.[1] Furthermore, Gruber and Johnson (2019) advocate for a revival of public-private partnerships in the U.S. to enhance basic scientific research and technological progress. These shifts have reignited academic debate on the effectiveness of industrial policies, as underscored by a recent review from Juhász, Lane, and Rodrik (2023).

A central issue in this debate is whether policymakers have the necessary information to implement industrial policies effectively. This concern echoes the historic contest between central planning and free markets that dominated the twentieth century. Central planning, as seen in the former Soviet Union, sought to achieve economic efficiency through centralized decision-making aimed at optimizing social welfare. Critics like von Mises (1922) and Hayek (1945), however, highlighted a key flaw: the lack of crucial information needed for planners to make optimal economic decisions. The Soviet Union's collapse appeared to settle this debate in favor of free markets.

By integrating industrial policies within a market economy, policymakers can leverage the market's information discovery capabilities, potentially overcoming the informational challenges faced by command economies like the Soviet Union. However, as Juhász and Lane (2024) highlight, the effectiveness of industrial policy is closely linked to the political incentives of policymakers. These incentives not only influence policy choices but also affect how firms engage in information discovery. These interactions ultimately shape the effectiveness of industrial policies, warranting a systematic analysis.

In this paper, we develop a model to assess the informational efficiency of industrial policy in an economy where firms process information, while a policymaker—motivated by career incentives rather than purely maximizing household welfare—sets the industrial policy. We examine an emerging industry with a continuum of firms, each strategically investing in industry-specific capital to maximize shareholder value. Influenced by a common, unobservable factor that drives

[1] Other recent examples of industrial policy include the E.U. CHIPS Act, the European Battery Alliance, the European Green Deal, China's Belt and Road Initiative, and the Japan-U.S. Leading-Edge Semiconductor Technology Center.

productivity, firms invest effort to acquire information about the industry's fundamentals. This information is aggregated through their capital demand, making the equilibrium capital price an informative indicator of the industry's fundamental, following Grossman and Stiglitz (1980) and Hellwig (1980).

The government supports the industry through investments in infrastructure or basic research that enhance productivity across firms. Lacking direct knowledge of the industry's fundamentals, the policymaker extracts information from the capital price. However, government investment is also shaped by a policy agenda that reflects not only the industry fundamentals but also the policymaker's ability to advocate and coordinate across government branches. The policymaker can use government investment as a signal to demonstrate her political capability, making the policy agenda an additional factor influencing industry development and the capital price.

Consequently, firms allocate their limited capacity for information processing to learn about both the industry fundamental and the policy agenda. The equilibrium capital price, which balances aggregated firm demand with capital supply, transforms private information into a public signal, guiding investment decisions for both the government and firms. If the policymaker aggressively advances her policy agenda, firms may concentrate on gathering information about this agenda rather than the industry fundamentals, leading to a government-centric equilibrium where market-driven information discovery fails to capture industry fundamentals.

This raises a critical question: What drives the policymaker's responses to both the capital price and policy agenda? To explore this, we analyze a political economy equilibrium, where the policymaker aims to maximize her career prospects, influenced by the public's perception and precision of her political capability, while being constrained by the necessity to maintain household welfare above a certain threshold – a public outcry constraint.

The policymaker's career incentives encourage a strong response to the policy agenda, which diverts firms' information acquisition away from the industry fundamental. However, the stringency of the public outcry constraint tempers the policymaker's actions, preventing excessively aggressive behavior that might prioritize her career over household welfare. Our analysis thus underscores bureaucratic efficiency as a key factor in ensuring the effectiveness of industrial policy, even when market-based information discovery is present.

This implication is especially relevant for countries that heavily pursue industrial policies, specially China, which is a leader in this area. After four decades of successful economic reforms, China has entered a new phase where it can no longer depend solely on labor-intensive manufacturing and exports for growth. Instead, it must prioritize innovation in technology and services. As it approaches the technological frontier, rising uncertainty demands tough choices among competing technologies and products, making the market's role in identifying promising options critical. Bureaucrats alone cannot navigate these complexities effectively, highlighting the need for market-driven information discovery. Our model emphasizes the importance of managing internal agency frictions within the state to enhance the effectiveness of this market-based approach.

Our paper offers a novel contribution to the literature on dispersed information in economies influenced by government interventions, building on works like Angeletos and Pavan (2004, 2007), Bond and Goldstein (2015), and Brunnermeier, Sockin, and Xiong (2017, 2022). Although consistent with the idea that government interventions can distort private agents' information acquisition, our analysis uniquely connects these distortions to internal bureaucratic frictions. Specifically, Brunnermeier, Sockin, and Xiong (2022) show how interventions in financial markets, intended to counter noise trading, may unintentionally shift focus away from fundamentals. Our simpler model also highlights this crowding-out effect but ties it directly to firm investment, tracing its roots to agency frictions within the government system.

Our work also relates to the literature examining public service through the lens of principal-agent problems (e.g., Barro (1973), Ferejohn (1986), Besley and Case (2003), Chari and Kehoe (1990)). More recently, these issues have been explored in the context of career concerns following Holmstrom (1999), as seen in the work of Alesina and Tabellini (2007, 2008) and Bonfiglioli and Gancia (2013), which explain why politicians driven by career advancement often pursue myopic and socially suboptimal policies. Our paper offers a novel perspective on how policymakers' career incentives can distort the information choices of economic agents, thereby not only intensifying the principal-agent problem but also leading to capital misallocation.

1 The Model

We analyze a closed economy where both the government and firms make investment decisions, conceptualized as a region specializing in a specific industry. The model spans three dates $t \in \{0, 1, 2\}$. At date 2, a firm's output is determined by the production function:

$$Y_i = F G^{\alpha_G} K_i^{\alpha_K},$$

where K_i is the firm's capital, G is the government's investment, and F is a common productivity factor representing the industry fundamental. The parameters satisfy $\alpha_G \in (0, 1)$ and $\alpha_K = 1 - \alpha_G$.

The industry fundamental F is unobservable to both the government and firms. At $t = 0$, their prior belief about the log of F, denoted by f, follows a normal distribution:

$$f \equiv \log F \backsim \mathcal{N}\left(\bar{f}, \tau_f^{-1}\right).$$

Firms can acquire information about f and make investment decisions at $t = 1$.

Government investment G boosts firm productivity by providing infrastructure or funding for foundational research in emerging industries, such as broadband for IT or power grids for renewable energy. To justify government involvement, we present a microfoundation in Online Appendix B, based on coordination failure among firms in supplying a public good.

A policymaker oversees government investment G, but they often face information barriers and may lack the expertise to evaluate emerging industries effectively. Collaborating with private enterprises can help bridge these gaps. In our model, the policymaker sets G based on the capital price q, which reflects firms' investment decisions, forming the core mechanism of the model.

Additionally, the policymaker must also navigate bureaucratic frictions, as highlighted by Juhász and Lane (2024). Mobilizing public resources requires political skill and reputation, and policy outcomes become a measure of the policymaker's competence. This creates career incentives to use policy choices strategically to enhance political standing. For example, in China's bureaucracy, under a national policy of rapid economic growth, a city mayor must demonstrate her ability to grow the local economy beyond its fundamentals (Song and Xiong, 2023). Similarly, in American politics, when economic growth is a key voter concern, a mayor may aim to boost a local industry beyond its fundamental potential.

At date 1, the policymaker enacts a policy agenda:

$$\pi_g = f + \theta, \tag{1}$$

by advocating and coordinating within the government bureaucracy. This agenda combines the industry fundamental f with a component $\theta \backsim \mathcal{N}\left(0, \tau_\theta^{-1}\right)$, representing the policymaker's capability. A more ambitious agenda is easier to implement if the industry is more promising or the policymaker is more capable. However, the policymaker is unaware of the individual components, f and θ.

At date 0, the policymaker selects an investment policy for G, balancing household welfare with career incentives. For simplicity, we assume that she adheres to a log-linear rule relative to π_g and the capital price q, which reveals information about f.

At date 0, each firm decides how much private information to acquire about f and/or π_g. This is crucial, as firms' date 1 capital investments reflect this information, allowing the capital price q to aggregate their private information, as in Grossman and Stiglitz (1980) and Hellwig (1980). This feedback loop between market signals and policymaking is central to our model, with its efficiency shaped by firms' information acquisition strategies, which are influenced by the policymaker's investment decisions.

In summary, at date 0, the government sets its industry policy and firms choose their information acquisition strategy. At date 1, the government invests in infrastructure and firms invest in capital. By date 2, firms produce output and households consume.

1.1 Firms

There is a continuum of households, each owning a corresponding firm i, so the terms are used interchangeably. At date 2, household i receives its firm's profit as a dividend:

$$\Pi_i = e^f G^{\alpha_G} K_i^{\alpha_K} - qK_i,$$

where qK_i is the cost of purchasing capital from capital providers. Since all firms demand the same type of capital, the price q reflects the aggregate demand, thereby incorporating and aggregating firms' information.

The household has constant relative risk aversion preferences for consumption:

$$u(C_i) = \frac{C_i^{1-\gamma}}{1-\gamma}, \text{ for } \gamma \in [0, 1/\alpha_K),$$

where consumption $C_i = \Pi_i + \tau_i$. Households collectively own capital providers, and for simplicity, each household receives back the cost of capital paid by its firm. Thus, household i's consumption is

$$C_i = e^f G^{\alpha_G} K_i^{\alpha_K}. \tag{2}$$

The household holds undiversified risk in the firm. At date 1, it values the firm's profit as $\mathbb{E}[\Lambda_i \Pi_i | \mathcal{I}_i]$, where $\Lambda_i = \lambda_i \frac{u'(C_i)}{\mathbb{E}[u'(C_i)]}$ is the household's stochastic discount factor, with λ_i being a constant and $u'(C_i)$ representing marginal utility. The firm chooses its investment K_i to maximize

$$\max_{K_i} \mathbb{E}[\Lambda_i \Pi_i | \mathcal{I}_i] = \max_{K_i} \mathbb{E}\left[\Lambda_i \left(e^f G^{\alpha_G} K_i^{\alpha_K} - qK_i\right) | \mathcal{I}_i\right], \tag{3}$$

where $\mathcal{I}_i$ is the firm's information set.

At date 1, firm i observes two private signals about the industry fundamental f and the government's policy agenda:

$$s_i = f + \varepsilon_{si},$$

and

$$v_i = \pi_g + \varepsilon_{vi},$$

where $\varepsilon_{si} \backsim \mathcal{N}\left(0, \tau_s^{-1}\right)$ and $\varepsilon_{vi} \backsim \mathcal{N}\left(0, \tau_v^{-1}\right)$ are independent noise specific to firm i.

The total investment $K = \int K_i di$ aggregates firms' information about f and π_g, determining the capital price q. While q is publicly observable, total investment is not, making aggregate investment a noisy channel for firm information. Thus, firm i's information set is $\mathcal{I}_i = \{s_i, v_i, q\}$.

From the firm's problem in (3), the optimal investment K_i is

$$K_i = \left(\frac{\alpha_K}{q} \frac{\mathbb{E}\left[\left(e^f G^{\alpha_G}\right)^{1-\gamma} \mid \mathcal{I}_i \right]}{\mathbb{E}\left[\left(e^f G^{\alpha_G}\right)^{-\gamma} \mid \mathcal{I}_i \right]} \right)^{\frac{1}{1-\alpha_K}}. \tag{4}$$

This choice is similar to that of a risk-neutral firm but adjusted for the risk faced by the household owner.

At date 0, firm i chooses the precision of its private signals, s_i and v_i. Following the rational inattention framework of Sims (2003), the firm faces an information acquisition constraint in reducing the Shannon entropy of f and π_g through these noisy signals. Given firms have access to public information (the capital price q) at date 1, the entropy reduction occurs from the public information set $\mathcal{I}_P = \{q\}$ to the private information set $\mathcal{I}_i = \sigma\left(\{q, s_i, v_i\}\right)$.

The posteriors conditional on both public and private information are Gaussian:

$$\begin{bmatrix} f \\ \pi_g \end{bmatrix} | \mathcal{I}_P \backsim \mathcal{N}\left(\begin{bmatrix} \hat{f} \\ \hat{\pi}_g \end{bmatrix}, \Sigma_P \right), \quad \begin{bmatrix} f \\ \pi_g \end{bmatrix} — \mathcal{I}_i \backsim \mathcal{N}\left(\begin{bmatrix} \hat{f}_i \\ \hat{\pi}_{gi} \end{bmatrix}, \Sigma_i \right).$$

The entropy reduction from the two private signals with precision τ_s and τ_v, denoted $I\left(\tau_s, \tau_v\right)$, is

$$I\left(\tau_s, \tau_v\right) = \frac{1}{2} \log \left|\Sigma_P\right| - \frac{1}{2} \log \left|\Sigma_i\right|. \tag{5}$$

At date 0, the firm chooses signal precision to maximize the household's expected utility:

$$U_i = \sup_{\tau_s, \tau_v} \mathbb{E}\left[\frac{C_i^{1-\gamma}}{1-\gamma} \right] \tag{6}$$

subject to the entropy constraint

$$I\left(\tau_s, \tau_v\right) \leq \frac{\kappa}{2}, \tag{7}$$

where $\kappa/2$ is the firm's total information-processing capacity.

1.2 Capital Suppliers

There is a continuum of capital suppliers. Each supplier j produces capital k_j at a convex effort cost of $\frac{1}{1+1/\psi}\varepsilon^{\varphi_j}k_j^{1+1/\psi}$ for $\psi < 1$, where φ_j is supplier j's operating cost, known only to itself. We assume $\varphi_j = \varphi + \varepsilon_{\varphi j}$, where $\varphi \backsim \mathcal{N}\left(0, \tau_\varphi^{-1}\right)$ is the common operating cost, and $\varepsilon_{\varphi j} \backsim \mathcal{N}\left(0, \tau_{\varphi\epsilon}^{-1}\right)$ is the idiosyncratic cost.

Supplier j chooses k_j to maximize its profit at date 1:

$$\sup_{k_j} qk_j - \frac{1}{1+1/\psi}e^{\varphi_j}k_j^{1+1/\psi}, \tag{8}$$

given its information set $\mathcal{I}_j = \{q, \varphi_j\}$. The optimal choice is

$$k_j = \left(qe^{-\varphi_j}\right)^{\psi}. \tag{9}$$

Aggregating across capital suppliers, total capital supplied is

$$K_S = \int k_j dj = q^{\psi}e^{-\psi\varphi + \frac{1}{2}\psi^2\tau_{\varphi\epsilon}^{-1}}. \tag{10}$$

Since capital suppliers are owned by households, each supplier's revenue $\tau_j^S = qk_j$ is transferred to the corresponding household. This disutility from supplying capital is considered by the government when maximizing household welfare.

1.3 Government

At date 1, the policymaker chooses G, based on her information set, which includes the policy agenda π_g and the capital price q. We assume the policymaker's choice follows a log-linear form:

$$\log G = b_\pi \pi_g + b_q \log q + b_0. \tag{11}$$

At date 0, the policymaker selects the coefficients $\{b_\pi, b_q, b_0\}$, before π_g and q are realized at date 1. In analyzing market equilibrium, we treat the government's policy $\{b_\pi, b_q, b_0\}$ as given, and will address the policymaker's objectives and optimal policy in Section 3.

1.4 Equilibrium Definition

We analyze a Ramsey Noisy Rational Expectations Equilibrium, consisting of policy functions $\{K_i, k_j\}$ and capital price q such that: given the government policy $\{b_\pi, b_q, b_0\}$, firm i optimizes K_i in (3), and capital supplier j optimizes k_j in (8). The capital market clears with $K = K_S$, and the output market clears with $C_i = Y_i$. Firms and capital suppliers update their beliefs using Bayes' Law based on their information sets.

2 Market Equilibrium

In this section, we analyze the market equilibrium among firms and capital suppliers, taking the government's policy as given. Specifically, we establish a necessary and sufficient condition for the existence of a government-centric equilibrium, where all firms focus exclusively on learning about the government's policy agenda.

As derived in equation (4), firm i's optimal investment depends on its expectation of $\left(e^f G^{\alpha_G}\right)^{1-\gamma}$. Given the government's investment policy in (11), this expectation follows a log-linear expression of f and π_g. Since the firm's information set $\mathcal{I}_i$ includes public information $\mathcal{I}_P$ and its private signals s_i and v_i, the optimal investment can be expressed as:

$$\log K_i = a_f \hat{f} + a_\pi \hat{\pi}_g + a_s s_i + a_v v_i + a_q \log q + a_0, \tag{12}$$

where $\hat{f}$ and $\hat{\pi}_g$ are the public expectations of f and π_g:

$$\begin{bmatrix} f \\ \pi_g \end{bmatrix} |\mathcal{I}_P \backsim \mathcal{N}\left(\begin{bmatrix} \hat{f} \\ \hat{\pi}_g \end{bmatrix}, \begin{bmatrix} \hat{\tau}_f^{-1} & \hat{\tau}_{f\pi}^{-1} \\ \hat{\tau}_{f\pi}^{-1} & \hat{\tau}_\pi^{-1} \end{bmatrix}\right). \tag{13}$$

The terms $a_f \hat{f} + a_\pi \hat{\pi}_g$ capture the contribution of public information, $a_s s_i + a_v v_i$ represent the firm's private information, and $a_q \log q$ reflects the effect of the capital price, through the firm's capital cost and the policymaker's expectation.

Given each firm's investment in (12), aggregating across firms yields total investment:

$$K = \int K_i di.$$

Applying the Strong Law of Large Numbers gives

$$\log K = A_s f + A_v \pi_g + A_q \log q + A_f \hat{f} + A_\pi \hat{\pi}_g + A_0.$$

with

$$A_s = a_s, A_v = a_v, A_q = a_q, A_f = a_f, A_\pi = a_\pi, A_0 = a_0 + \frac{1}{2}\left(a_s^2 \tau_s^{-1} + a_v^2 \tau_v^{-1}\right). \tag{14}$$

Imposing market-clearing using equation (10) give the capital price:

$$\log q = \frac{1}{\psi - A_q}\left(A_s f + A_v \pi_g + A_f \hat{f} + A_\pi \hat{\pi}_g + A_0 + \psi\varphi + \frac{1}{2}\left(A_s^2 \tau_s^{-1} + A_v^2 \tau_v^{-1} - \psi^2 \tau_{\varphi\epsilon}^{-1}\right)\right). \tag{15}$$

After removing public information, the information content of $\log q$ is

$$\begin{aligned} z_q &= \frac{1}{A_s}\left((\psi - A_q)\log q - A_f \hat{f} - A_\pi \hat{\pi}_g - A_0 - \frac{1}{2}\left(A_s^2 \tau_s^{-1} + A_v^2 \tau_v^{-1} - \psi^2 \tau_{\varphi\epsilon}^{-1}\right)\right) \\ &= f + \frac{A_v}{A_s}\pi_g + \frac{\psi}{A_s}\varphi. \end{aligned} \tag{16}$$

In solving each firm's optimization in (3), it takes the coefficients A_s, A_v, A_q, $A_{f,}$ and A_π as given and determines its optimal coefficients a_s, a_v, a_q, a_f and a_π. Using the expression (16), we can derive $\hat{f}$ and $\hat{\pi}_g$ and their conditional variance. Further, conditional on each firm's private signals, we can drive its conditional expectations and optimal investment choice. Ultimately, (14) provides the fixed-point conditions for A_s, A_v, A_q, $A_{f,}$ and A_π, allowing us to solve the market equilibrium, summarized in the following proposition.

Proposition 1. *At date* 1*, firm i's optimal investment choice is*

$$\begin{aligned} \log K_i &= \frac{1}{1-\alpha_K}\hat{f} + \frac{\alpha_G b_\pi}{1-\alpha_K}\hat{\pi}_g + a_s\left(s_i - \hat{f}\right) + a_v\left(v_i - \hat{\pi}_g\right) + \frac{\alpha_G b_q - 1}{1-\alpha_K}\log q \\ &\quad + \frac{\log\alpha_K + \alpha_G b_0}{1-\alpha_K} + \frac{1-2\gamma}{2}\left(a_s\tau_s^{-1} + \alpha_G b_\pi a_v \tau_v^{-1}\right), \end{aligned} \tag{17}$$

where

$$a_s = \frac{1}{1-\alpha_K} + \frac{1}{1-\alpha_K}\frac{\alpha_G b_\pi \hat{\tau}_{f\pi}^{-1}\tau_v^{-1} - \tau_s^{-1}\left(\hat{\tau}_\pi^{-1} + \tau_v^{-1}\right)}{\left(\hat{\tau}_f^{-1} + \tau_s^{-1}\right)\left(\hat{\tau}_\pi^{-1} + \tau_v^{-1}\right) - \hat{\tau}_{f\pi}^{-1}\hat{\tau}_{f\pi}^{-1}}, \tag{18}$$

$$a_v = \frac{\alpha_G b_\pi}{1-\alpha_K} + \frac{1}{1-\alpha_K}\frac{\hat{\tau}_{f\pi}^{-1}\tau_s^{-1} - \alpha_G b_\pi\left(\hat{\tau}_f^{-1} + \tau_s^{-1}\right)\tau_v^{-1}}{\left(\hat{\tau}_f^{-1} + \tau_s^{-1}\right)\left(\hat{\tau}_\pi^{-1} + \tau_v^{-1}\right) - \hat{\tau}_{f\pi}^{-1}\hat{\tau}_{f\pi}^{-1}}, \tag{19}$$

and $\log q$ *is given by* (15)*.*

The market equilibrium in Proposition 1 is log-linear, resembling the linear noisy rational expectations equilibrium of Grossman and Stiglitz (1980) and Hellwig (1980). Differences in firm investments stem from variations in private signals s_i and v_i, with $a_s s_i + a_v v_i$ acting as a sufficient statistic for the idiosyncratic component of investment.

A firm's information choices minimize the conditional variance of its household's utility, subject to the rational inattention constraint. This is equivalent to minimizing the conditional variance of the sum of the industry fundamental f and the effect of the government's policy agenda, $\alpha_G b_\pi \pi_g$. The next proposition establishes a necessary and sufficient condition for a government-centric equilibrium, where all firms acquire private information solely about the government's policy agenda.

Proposition 2. *At date* 0*, each firm's optimal information choices are*

$$\tau_v = \min\left\{\max\left\{(\alpha_G b_\pi)^2\,\tau_s + \frac{(\alpha_G b_\pi)^2\,\hat{\tau}_f - \hat{\tau}_\pi}{1 - \frac{\hat{\tau}_f}{\hat{\tau}_{f\pi}}\frac{\hat{\tau}_\pi}{\hat{\tau}_{f\pi}}}, 0\right\}, (e^\kappa - 1)\,\hat{\tau}_g\right\}, \tag{20}$$

$$\tau_s = \min\left\{\begin{array}{c}\max\left\{\sqrt{\left(\tau_f + \left(\frac{a_s}{\psi}\right)^2\tau_\varphi\right)^2 - \tau_f^2 + \frac{\tau_f\hat{\tau}_\pi - (1-e^\kappa)\left(\frac{a_v}{\psi}\right)^2\tau_\varphi\tau_f}{(\alpha_G b_\pi)^2}} - \left(\tau_f + \left(\frac{a_s}{\psi}\right)^2\tau_\varphi\right), 0\right\}, \\ (e^\kappa - 1)\,\hat{\tau}_f\end{array}\right\} \tag{21}$$

Here, τ_s is (weakly) decreasing in $\hat{\tau}_f$ and $\alpha_G b_\pi$, while τ_v is (weakly) decreasing in $\hat{\tau}_\pi$ and increasing in $\alpha_G b_\pi$.

A government-centric equilibrium exists if and only if $b_\pi \in (-\infty, -\tilde{b}^*_\pi] \cup [b^*_\pi, \infty)$. The values $\tilde{b}^*_\pi$ and b^*_π exist if $\tau_f \geq \tau_\varphi e^\kappa \left(\frac{1}{\psi}\frac{1-e^{-\kappa}}{1-\alpha_K}\right)^2$, and are decreasing in α_G, τ_f, and ψ, increasing in τ_θ, κ, and τ_φ.

Conversely, a fundamental-centric equilibrium, where firms acquire signals only about the fundamental f, exists if and only if $b_\pi \in [-\underline{\tilde{b}}_\pi, \underline{b}_\pi]$. Here $\underline{\tilde{b}}_\pi, \underline{b}_\pi > 0$ are decreasing in α_G, τ_f, κ, and τ_φ, and increasing in τ_θ and ψ. For a given b_π, at most one extreme equilibrium (either fundamental- or government-centric) exits.

Proposition 2 outlines the potential equilibria based on the government's response elasticity b_π. A fundamental-centric equilibrium exists if $|b_\pi|$ is sufficiently low ($b_\pi \in [-\underline{\tilde{b}}_\pi, \underline{b}_\pi]$). If $|b_\pi|$ is high enough ($b_\pi < -\underline{\tilde{b}}_\pi$ or $b_\pi > \underline{b}_\pi$), a government-centric equilibrium emerges. Additionally, a mixed equilibrium may occur, where firms acquire signals about both the industry fundamental and the policy agenda.[2]

A government-centric equilibrium requires that ex-ante uncertainty about the policy agenda (τ_θ^{-1}) is sufficiently high relative to that of the fundamental (τ_f^{-1}). A high $|b_\pi|$ indicates a large impact of the policy agenda π_g on the industry. Since π_g also contains information about f, the government must act aggressively on its agenda, even if it provides minimal information about f.

Two additional points about b_π. First, its effect is monotonic—greater reliance on the policy agenda (larger b_π) increases the likelihood of a government-centric equilibrium, as firms become more focused on the agenda. Second, the thresholds $\tilde{b}^*_\pi$ and b^*_π decrease with τ_f, meaning lower uncertainty about f makes a government-centric equilibrium more likely.

[2] We allow b_π to take negative values. As discussed in Section 3, the government aims to maximize household welfare. When households are highly risk-averse, the government may choose a negative b_π as a hedge against firms' investment risks.

Figure 1: Firm information acquisition policies τ_s and τ_v across different b_π values, with $\tau_f = 20$, $\tau_\theta = 1$, $\tau_\phi = 1$, $\kappa = 2$, $\alpha_K = 0.33$, $\psi = 1$.

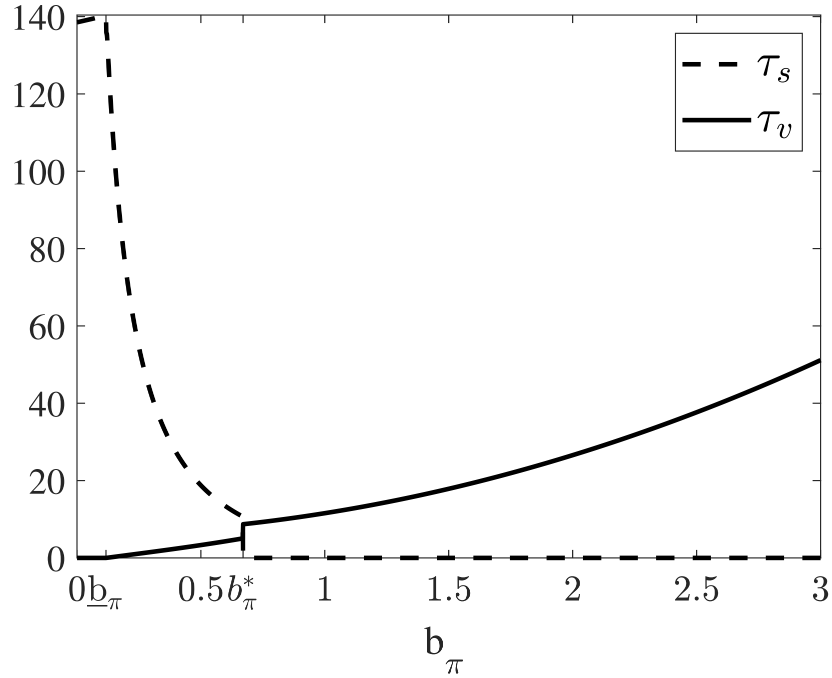

Figure 1 depicts firms' information acquisition strategies across different b_π.[3] If b_π is below $\underline{b}_\pi$, firms focus entirely on the fundamental, leading to a fundamental-centric equilibrium. As b_π rises above $\underline{b}_\pi$, a mixed equilibrium emerges, with firms shifting attention toward the government's policy agenda π_g and away from the fundamental f. When b_π exceeds b^*_π, a government-centric equilibrium forms, where firms focus solely on π_g.

When firms focus exclusively on π_g, the capital price no longer provides information to the government, impairing market-driven information discovery. This inefficiency implies that the government should regulate b_π to avoid a government-centric equilibrium. In the next section, we examine how the government sets its investment policy.

3 The Political Economy Equilibrium

Governments often act not just as social planners but as complex organizations with agency issues. As Juhász and Lane (2024) emphasize, the success of industrial policies depends significantly on bureaucratic capacity. Regardless of political systems, politicians and officials must demonstrate

[3]When multiple equilibria exist, we select the extreme equilibrium, leading to jumps in information acquisition policies.

policy implementation skills—either to gain voter support in democracies or to prove competence to superiors in authoritarian regimes. In China, local governments manage regional economic development under central oversight, with career-driven incentives fueling growth, as shown by Maskin, Qian, and Xu (2000), Li and Zhou (2005), and Song and Xiong (2023). In contrast, elected officials in the U.S. and other Western countries focus on demonstrating effectiveness to voters, a dynamic described by Besley and Case (2003), which can lead to shortsighted, volatile policies.

In this section, we analyze the political economy equilibrium where the policymaker, motivated by career incentives rather than solely maximizing household welfare, sets industrial policy.[4]

3.1 Career Incentives

We assume the policymaker's career trajectory depends on public perception of her political capability, θ. In line with the career concerns framework, θ is unobservable to both the policymaker and the public (including households, firms, voters, and superiors). At date 0, all parties share a common prior belief that $\theta \sim \mathcal{N}\left(\bar{\theta}, \tau_\theta^{-1}\right)$, where τ_θ is the prior precision. The public updates their beliefs about θ after observing log output and capital price, using Bayes' rule, as outlined in the following proposition.

Proposition 3. *The public's posterior distribution of θ after observing $\log C$ and $\log q$ is Gaussian, with conditional mean and precision given by:*

$$\hat{\theta} = \tau_\theta^{-1} \begin{pmatrix} a_v \\ 1 \end{pmatrix}' \Sigma^{-1} \begin{pmatrix} z_Q \\ z_C \end{pmatrix}, \tag{22}$$

$$\hat{\tau}_\theta = \tau_\theta + \frac{\left(\alpha_G b_\pi a_s - a_v\right)^2 \frac{\tau_\varphi}{\psi^2} + \left(\alpha_G b_\pi + \alpha_K a_v\right)^2 \tau_f}{\left(1 + \alpha_G b_\pi + \alpha_K \left(a_s + a_v\right)\right)^2}, \tag{23}$$

where

$$z_C = \theta + \left(1 + \frac{1}{\alpha_G b_\pi}\right)\left(f - \bar{f}\right) - \frac{\alpha_K \psi}{\alpha_G b_\pi}\varphi, \tag{24}$$

$$z_Q = a_v\theta + \left(a_s + a_v\right)\left(f - \bar{f}\right) + \psi\varphi, \tag{25}$$

are sufficient statistics recovered from log output and the log capital price, respectively, and

$$\Sigma = \begin{bmatrix} \left(a_s + a_v\right)^2 \tau_f^{-1} + a_v^2 \tau_\theta^{-1} + \psi^2 \tau_\varphi^{-1} & a_v \tau_\theta^{-1} + \left(a_s + a_v\right)\left(1 + \frac{1}{\alpha_G b_\pi}\right)\tau_f^{-1} - \frac{\alpha_K \psi^2}{\alpha_G b_\pi}\tau_\varphi^{-1} \\ a_v \tau_\theta^{-1} + \left(a_s + a_v\right)\left(1 + \frac{1}{\alpha_G b_\pi}\right)\tau_f^{-1} - \frac{\alpha_K \psi^2}{\alpha_G b_\pi}\tau_\varphi^{-1} & \tau_\theta^{-1} + \left(1 + \frac{1}{\alpha_G b_\pi}\right)^2 \tau_f^{-1} + \left(\frac{\alpha_K \psi}{\alpha_G b_\pi}\right)^2 \tau_\varphi^{-1} \end{bmatrix}. \tag{26}$$

[4]One may wonder whether the policymaker has incentive to disclose her agenda if she could at date 1. If her goal is to maximize social welfare, then she would to avoid distorting firms' information acquisition decisions. However, she aims to maximize her career incentives, then she would not because she would want firms to devote all their attention to learning about her agenda so that it is reflected in prices.

Intuitively, the policymaker's career prospects improve with higher $\hat{\theta}$ and $\hat{\tau}_\theta$. A larger $\hat{\theta}$ indicates greater capability, while a higher $\hat{\tau}_\theta$ reflects increased confidence in public perception. These factors are captured in an ex post scoring function $t\left(\hat{\theta}, \hat{\tau}_\theta\right)$, which increases with both variables, similar to a scoring auction, as discussed by Asker and Cantillon (2008).

We assume the policymaker's career prospects rise linearly with her score. Thus, she aims to maximize its ex ante expected value when setting policy at date 0:

$$T\left(\hat{\tau}_\theta, \tau_\theta\right) = \mathbb{E}\left[t\left(\hat{\theta}, \hat{\tau}_\theta\right)\right].$$

Since $t\left(\hat{\theta}, \hat{\tau}_\theta\right)$ depends on a Gaussian random variable, its expectation is driven by the first two moments, and θ has a zero unconditional expectation. As $t\left(\hat{\theta}, \hat{\tau}_\theta\right)$ increases with $\hat{\tau}_\theta$, on a realization-by-realization basis, so does $T\left(\hat{\tau}_\theta, \tau_\theta\right)$. For simplicity, we assume

$$T\left(\hat{\tau}_\theta, \tau_\theta\right) = \bar{\theta} + \frac{1}{2}\log\left(\frac{\hat{\tau}_\theta}{\tau_\theta}\right).$$

The first term, $\bar{\theta}$, represents the policymaker's ex ante expected capability, while the second term, $\frac{1}{2}\log\left(\frac{\hat{\tau}_\theta}{\tau_\theta}\right)$, reflects the reduction in the entropy of public beliefs after observing public signals, $\log C$ and $\log q$, motivating the policymaker to ensure precise signals of her capability.

The policymaker's career incentives prevent her from credibly announcing her agenda, as she has an incentive to exaggerate. Consequently, firms may seek private information about the policy agenda.

3.2 Career-Driven Policy Choice

We start by considering an extreme case where the policymaker aims solely to maximize her career prospects. Her problem at date at date 0, in choosing the investment policy $\left\{b_0, b_\pi, b_q\right\}$, can be summarized as:

$$V = \sup_{\left\{b_0, b_\pi, b_q\right\}} T\left(\hat{\tau}_\theta, \tau_\theta\right) = \sup_{\left\{b_\pi\right\}} \frac{1}{2}\log\left(\frac{\hat{\tau}_\theta}{\tau_\theta}\right). \tag{27}$$

Since the public fully accounts for the policymaker's choices, attempting to influence the ex-ante expected evaluation $\bar{\theta}$ through policy is ineffective. However, the policymaker can directly influence the public's posterior precision $\hat{\tau}_\theta$ through her choice of b_π. Thus, the optimal b_π is the one that maximizes the posterior precision:

$$b_\pi = \arg\sup_{b'_\pi} \hat{\tau}_\theta.$$

to maximize $\hat{\tau}_\theta$, the policymaker prefers a higher b_π even within a fundamental-centric equilibrium. This is because, even if firms' information choices remain unchanged, a higher b_π increases the impact of the policy agenda π_g on both $\log C$ and $\log q$, making these signals more informative about the policymaker's capability. However, b_π is constrained by an upper bound $\underline{b}_\pi$. Exceeding this threshold would shift the market towards a mixed or government-centric equilibrium.

According to Proposition 2, when $\tau_f \geq \tau_\varphi e^\kappa \left(\frac{1}{\psi}\frac{1-e^{-\kappa}}{1-\alpha_K}\right)^2$, a government-centric equilibrium is possible. In this equilibrium, firms focus on acquiring information about π_g, making $\log C$ and $\log q$ more informative about π_g and, consequently, θ. To maximize $\hat{\tau}_\theta$, the policymaker is incentivized to select the highest feasible b_π, inducing a government-centric equilibrium. Proposition 4 formalizes this:

Proposition 4. *Under the condition $\tau_f \geq \tau_\varphi e^\kappa \left(\frac{1}{\psi}\frac{1-e^{-\kappa}}{1-\alpha_K}\right)^2$, the policymaker optimally chooses the largest possible b_π, inducing a government-centric equilibrium to maximize the precision of the public's posterior beliefs, $\hat{\tau}_\theta$.*

This proposition shows that the policymaker's career incentives drive the implementation of industrial policy, allowing her to strategically influence public perceptions of her capability. This results in more assertive policy interventions, potentially distorting the information acquisition process of market participants.

This focus on uncertainty reduction in career assessment differs from the typical emphasis on first-moment effects in the literature. For instance, Maskin, Qian, and Xu (2000) explore how performance evaluations affect officials' effort choices through the conditional mean of their abilities, while Song and Xiong (2023) emphasize the short-term behaviors of local officials driven by career incentives. By introducing a second-moment channel, our analysis broadens the understanding of the complex relationship between governance and economic performance.

3.3 Welfare-Maximizing Choice

As a benchmark, we now examine the policymaker's optimal policy when her goal is to maximize household welfare. We define household welfare, considering households own firms and capital suppliers, as

$$W = \mathbb{E}\left[\int_0^1 C_i^{1-\gamma} di\right]^{\frac{1}{1-\gamma}} - \mathbb{E}\left[\frac{qK}{1+1/\psi}\right] - R_G \mathbb{E}\left[G\right], \tag{28}$$

representing the certainty-equivalent of households' aggregate utility from consumption, minus the effort costs of capital suppliers and the government's investment cost, $R_G\mathbb{E}[G]$.[5] Given that all terms are log-linear in the market equilibrium and recognizing $\mathbb{E}[qK] = \alpha_K \mathbb{E}[C_i]\, e^{-\gamma Var\left[f+\alpha_G b_\pi \pi_g |\, \mathcal{I}_i\right]}$,

[5] From equations (9), (10), and $K = K_S$, we have $\int e^{\varphi_j} k_j^{1+1/\psi} dj = qK$.

we can rewrite the policymaker's optimal program as

$$\bar{W} = \sup_{\{b_0, b_\pi, b_q\}} \left(e^{-\frac{\gamma}{2} Var[\log C_i]} - \frac{\psi \alpha_K}{1+\psi} e^{-\gamma Var\left[f + \alpha_G b_\pi \pi_g | \mathcal{I}_i\right]} \right) \mathbb{E}\left[C_i\right] - R_G \mathbb{E}\left[G\right], \tag{29}$$

where C_i is the consumption of a representative household. Detailed expressions for C_i and other variables in the market equilibrium are provided in Online Appendix C.

Proposition 5 characterizes this optimal program. Using a log-linear approximation, we show that if the policymaker seeks to maximize household welfare, the optimal policy will not induce a government-centric equilibrium when households are sufficiently risk-averse.

Proposition 5. *In a log-linear approximation of social welfare around $\gamma = 0$, there exists a γ^* such that if $\gamma \geq \gamma^*$, the optimal policy will not induce a government-centric equilibrium.*

As shown in the proof, the optimal b_π and b_q balance the costs and benefits of log consumption volatility and government expenditure. In a government-centric equilibrium, log consumption volatility rises with $|b_\pi|$, prompting the policymaker to reduce scale when households have higher risk aversion (γ). For sufficiently high γ, the policymaker will select a scale small enough to avoid a government-centric equilibrium, favoring a fundamental-centric or mixed equilibrium instead.

3.4 Constrained Policy Choice

More generally, the policymaker must balance career incentives with household welfare. We define her objective as maximizing her career reward, as in equation (27), subject to a constraint ensuring household welfare W, from equation (28), does not fall below a specified reservation level, $\underline{W}$. This constraint resembles the participation constraint in Acemoglu, Golosov, and Tsyvinksi (2008), designed to prevent public dissatisfaction that could lead to her removal by the central authority or voters.

This leads to a modified optimization problem for the policymaker:

$$\mathcal{V} = \sup_{\{b_0, b_\pi, b_q\}} \frac{1}{2} \log\left(\frac{\hat{\tau}_\theta}{\tau_\theta}\right), \tag{30}$$

subject

$$\log W \geq \log \underline{W}. \tag{31}$$

Now, the policymaker's choice of $b_\pi$must account for how increased responsiveness to the policy agenda affects household welfare. Since b_0 and b_q do not influence her evaluation (as shown in Proposition 5), she sets these to maximize household welfare, meaning her career incentives primarily distort the choice of b_π.

Assume households are sufficiently risk-averse (high γ), ensuring that the welfare-maximizing policy avoids a government-centric equilibrium, as established in Proposition 5. Under this condition, we derive the following proposition:

Proposition 6. *If $\gamma \geq \gamma^*$, the policymaker's optimal b_π decreases as $\underline{W}$ increases, reaching the level that maximizes household welfare when $\underline{W} = \bar{W}$. However, if $\underline{W}$ is sufficiently low, the policymaker's choice of b_π will induce a government-centric equilibrium.*

Proposition 6 shows that the public outcry constraint serves as a regulatory mechanism, limiting the policymaker's ability to favor her career over household welfare. The constraint strengthens as the minimum acceptable welfare level, $\underline{W}$, increases. As $\underline{W}$ approaches the maximum household welfare, $\bar{W}$, the policymaker is compelled to select b_π that aligns with maximizing household welfare. Conversely, if $\underline{W}$ is low, the constraint becomes lax, allowing the policymaker to prioritize her career, potentially leading to a government-centric equilibrium.

Since more established politicians typically face a more lenient public outcry constraint (lower $\underline{W}$), Proposition 6 predicts that more secure policymakers are likely to adopt more aggressive policy agendas, resulting in weaker information discovery by the market.

This model underscores the tension between the policymaker's career incentives and the market's efficiency in information discovery. In signaling her political capability, the policymaker may aggressively pursue her agenda, diverting market participants from acquiring information about fundamental economic factors. Ensuring efficient information discovery requires a binding constraint that disciplines the policymaker's actions, aligning with Juhász and Lane's (2024) emphasis on bureaucratic capacity as crucial for the effective implementation of industrial policy.

4 Conclusion

This paper develops a model to highlight the tension between two key aspects of implementing industrial policy—the policymaker's career incentives and the market's role in information discovery. Although market-based information discovery aids policymakers in overcoming informational barriers when conducting industrial policy, an overly aggressive pursuit of the policymaker's agenda can shift market dynamics toward a government-centric equilibrium. In such a scenario, market participants focus more on information related to the government's policy agenda than on fundamental economic factors, which undermines the market's role in information discovery and reduces the efficiency of industrial policy. Our analysis, therefore, emphasizes the need to balance the policymaker's career incentives with household welfare considerations when designing industrial policy.

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Appendix A: Proof of Propositions

Proof of Proposition 1

We follow the standard approach for solving noisy rational expectations models. Below are the key steps, with detailed formulas provided in the Online Appendix.

Step 1: Firm i's Beliefs

Applying Bayes' Rule, we derive the posterior beliefs about f and π_g conditional on $\mathcal{I}_P$: $\begin{bmatrix} f \\ \pi_g \end{bmatrix} | \mathcal{I}_P \backsim \mathcal{N}\left(\begin{bmatrix} \hat{f} \\ \hat{\pi}_g \end{bmatrix}, \begin{bmatrix} \hat{\tau}_f^{-1} & \hat{\tau}_{f\pi}^{-1} \\ \hat{\tau}_{f\pi}^{-1} & \hat{\tau}_{\pi}^{-1} \end{bmatrix} \right)$, with conditional expectation

$$\begin{bmatrix} \hat{f} \\ \hat{\pi}_g \end{bmatrix} = \begin{bmatrix} \bar{f} \\ \bar{f} \end{bmatrix} + \frac{\begin{bmatrix} (A_s + A_v)\, \tau_f^{-1} \\ (A_s + A_v)\, \tau_f^{-1} + A_v \tau_\theta^{-1} \end{bmatrix} \left(A_s z_q - (A_s + A_v)\, \bar{f} \right)}{(A_s + A_v)^2\, \tau_f^{-1} + A_v^2 \tau_\theta^{-1} + \psi^2 \tau_\varphi^{-1}}, \tag{A.1}$$

and conditional covariance matrix

$$\begin{bmatrix} \hat{\tau}_f^{-1} & \hat{\tau}_{f\pi}^{-1} \\ \hat{\tau}_{f\pi}^{-1} & \hat{\tau}_{\pi}^{-1} \end{bmatrix} = \frac{\psi^2 \tau_\varphi^{-1} \tau_f^{-1} \iota_2 \iota_2' + \begin{bmatrix} A_v^2 \tau_f^{-1} \tau_\theta^{-1} & -A_s A_v \tau_f^{-1} \tau_\theta^{-1} \\ -A_s A_v \tau_f^{-1} \tau_\theta^{-1} & A_s^2 \tau_f^{-1} \tau_\theta^{-1} + \psi^2 \tau_\varphi^{-1} \tau_\theta^{-1} \end{bmatrix}}{(A_s + A_v)^2\, \tau_f^{-1} + A_v^2 \tau_\theta^{-1} + \psi^2 \tau_\varphi^{-1}}. \tag{A.2}$$

We then update from the public beliefs to the private beliefs of firm i, which is also Gaussian with conditional expectation

$$\begin{bmatrix} \hat{f}_i \\ \hat{\pi}_{gi} \end{bmatrix} = \frac{\left\{ \begin{array}{l} \begin{bmatrix} \tau_s^{-1} \left(\hat{\tau}_\pi^{-1} + \tau_v^{-1} \right) & -\hat{\tau}_{f\pi}^{-1} \tau_s^{-1} \\ -\hat{\tau}_{f\pi}^{-1} \tau_v^{-1} & \tau_v^{-1} \left(\hat{\tau}_f^{-1} + \tau_s^{-1} \right) \end{bmatrix} \begin{bmatrix} \hat{f} \\ \hat{\pi}_g \end{bmatrix} \\ + \begin{bmatrix} \hat{\tau}_f^{-1} \left(\hat{\tau}_\pi^{-1} + \tau_v^{-1} \right) - \hat{\tau}_{f\pi}^{-1} \hat{\tau}_{f\pi}^{-1} & \hat{\tau}_{f\pi}^{-1} \tau_s^{-1} \\ \hat{\tau}_{f\pi}^{-1} \tau_v^{-1} & \hat{\tau}_\pi^{-1} \left(\hat{\tau}_f^{-1} + \tau_s^{-1} \right) - \hat{\tau}_{f\pi}^{-1} \hat{\tau}_{f\pi}^{-1} \end{bmatrix} \begin{bmatrix} s_i \\ v_i \end{bmatrix} \end{array} \right\}}{\left(\hat{\tau}_\pi^{-1} + \tau_v^{-1} \right) \left(\hat{\tau}_f^{-1} + \tau_s^{-1} \right) - \hat{\tau}_{f\pi}^{-1} \hat{\tau}_{f\pi}^{-1}}, \tag{A.3}$$

and conditional covariance matrix

$$\begin{bmatrix} \hat{\tau}_{f,i}^{-1} & \hat{\tau}_{f\pi,i}^{-1} \\ \hat{\tau}_{f\pi,i}^{-1} & \hat{\tau}_{\pi,i}^{-1} \end{bmatrix} = \frac{\begin{bmatrix} \tau_s^{-1} \hat{\tau}_f^{-1} \left(\hat{\tau}_\pi^{-1} + \tau_v^{-1} \right) - \tau_s^{-1} \hat{\tau}_{f\pi}^{-1} \hat{\tau}_{f\pi}^{-1} & \tau_s^{-1} \tau_v^{-1} \hat{\tau}_{f\pi}^{-1} \\ \tau_s^{-1} \tau_v^{-1} \hat{\tau}_{f\pi}^{-1} & \tau_v^{-1} \hat{\tau}_\pi^{-1} \left(\hat{\tau}_f^{-1} + \tau_s^{-1} \right) - \tau_v^{-1} \hat{\tau}_{f\pi}^{-1} \hat{\tau}_{f\pi}^{-1} \end{bmatrix}}{\left(\hat{\tau}_\pi^{-1} + \tau_v^{-1} \right) \left(\hat{\tau}_f^{-1} + \tau_s^{-1} \right) - \hat{\tau}_{f\pi}^{-1} \hat{\tau}_{f\pi}^{-1}}. \tag{A.4}$$

Step 2: Firm i's Optimal Investment Policy

By substituting the government's policy function (11) into equation (4), incorporating the learning expressions, and matching coefficients, we derive the optimal investment policy in (17).

Step 3: Price of Capital

Substituting equation (17) into (15), we obtain the capital price where the equilibrium fixed-point conditions in (14) hold.

Proof of Proposition 2

Step 1: Firm *i*'s Optimal Information Acquisition

Applying the results from Steps 1-3 to (6), household i's information acquisition problem when $\gamma < \frac{1}{\alpha_K}$ reduces to

$$\begin{aligned} u_i &= \sup_{\tau_v, \tau_s} -Var\left[f + \alpha_G b_\pi \pi_g | \, \mathcal{I}_i\right], \\ s.t. &: \log\left|\frac{\hat{\tau}_f + \tau_s}{\hat{\tau}_f}\frac{\hat{\tau}_\pi + \tau_v}{\hat{\tau}_\pi} - \frac{\tau_s}{\hat{\tau}_{f\pi}}\frac{\tau_v}{\hat{\tau}_{f\pi}}\right| \leq \kappa. \end{aligned} \tag{A.5}$$

Taking the first-order conditions when the capacity constraint binds, we derive the optimal precision and comparative statics stated in the proposition.

Step 2: Existence of a Government-centric Equilibrium

From Proposition 1, in a government-centric equilibrium, $a_s = 0$ and

$$a_v = \alpha_G b_\pi \frac{1 - e^{-\kappa}}{1 - \alpha_K} + e^{-\kappa}\frac{\hat{\tau}_{f\pi}^{-1}\tau_v}{1 - \alpha_K} = \left(\frac{\tau_\theta}{\tau_\theta + \tau_f} + \alpha_G b_\pi\right)\frac{1 - e^{-\kappa}}{1 - \alpha_K}. \tag{A.6}$$

For a government-centric equilibrium to exist, $\tau_s = 0$, which requires the first argument in the max of equation (21) to be less than or equal to 0. After some manipulation, this condition becomes

$$\tau_f \left(\alpha_G b_\pi\right)^2 - e^\kappa \left(\frac{\tau_\theta}{\tau_\theta + \tau_f} + \alpha_G b_\pi\right)^2 \left(\frac{1}{\psi}\frac{1 - e^{-\kappa}}{1 - \alpha_K}\right)^2 \tau_\varphi \geq \left(\tau_\theta^{-1} + \tau_f^{-1}\right)^{-1}. \tag{A.7}$$

There are critical values of b_π, $\tilde{b}_\pi^*$ and b_π^*, such that a government-centric equilibrium exists if and only if $b_\pi \in (-\infty, -\tilde{b}_\pi^*] \cup [b_\pi^*, \infty)$ and $\tau_f \geq \tau_\varphi e^\kappa \left(\frac{1}{\psi}\frac{1-e^{-\kappa}}{1-\alpha_K}\right)^2$. Since α_G and b_π appear together as $\alpha_G b_\pi$, both $\tilde{b}_\pi^*$ and b_π^* decrease with α_G. These cutoffs also decrease with τ_f/τ_θ and increase with $\frac{\tau_\varphi}{\psi^2}$ and κ.

Step 3: Existence of a Fundamental-centric Equilibrium

From Proposition 1, in a fundamental-centric equilibrium, $a_v = 0$ and

$$a_s = (1 + \alpha_G b_\pi)\frac{1 - e^{-\kappa}}{1 - \alpha_K}. \tag{A.8}$$

For a fundamental-centric equilibrium to exist, $\tau_v = 0$, which requires the first argument in the max of equation (20) to be less than or equal to 0, or

$$(\alpha_G b_\pi)^2 \left(1 + \frac{e^\kappa}{\tau_\theta}\left(\frac{\tau_\varphi}{\psi^2} a_s^2 + \tau_f\right)\right) \leq 1. \tag{A.9}$$

There are two critical b_π, $\underline{b}_\pi$ and $-\tilde{\underline{b}}_\pi$, such that a fundamental-centric equilibrium exists if and only if $b_\pi \in [-\tilde{\underline{b}}_\pi, \underline{b}_\pi]$. Because α_G and b_π appear together as $\alpha_G b_\pi$, $\underline{b}_\pi$ and $\tilde{\underline{b}}_\pi$ decrease with α_G. These cutoffs also decreases with τ_f/τ_θ, κ, and $\frac{\tau_\varphi}{\psi^2}$.

Step 4: Ranking Cutoffs

Consider the threshold $\underline{b}_\pi$, the upper bound for a fundamental-centric equilibrium. Manipulating equation (A.9) when it holds with equality, we can bound $\underline{b}_\pi$ according to:

$$\tau_f \left(\alpha_G \underline{b}_\pi\right)^2 - e^\kappa \left(\frac{\tau_\theta}{\tau_\theta + \tau_f} + \alpha_G \underline{b}_\pi\right)^2 \left(\frac{1}{\psi}\frac{1 - e^{-\kappa}}{1 - \alpha_K}\right)^2 \tau_\varphi < \left(\tau_\theta^{-1} + \tau_f^{-1}\right)^{-1}. \tag{A.10}$$

This implies that $\underline{b}_\pi$ does not meet the condition for a government-centric equilibrium, ensuring $\underline{b}_\pi < b_\pi^*$.

Similarly, when equation (A.9) holds with equality, we can bound $\tilde{\underline{b}}_\pi$ according to:

$$\tau_f \left(\alpha_G \tilde{\underline{b}}_\pi\right)^2 - e^\kappa \left(\frac{\tau_\theta}{\tau_\theta + \tau_f} - \alpha_G \tilde{\underline{b}}_\pi\right)^2 \left(\frac{1}{\psi}\frac{1 - e^{-\kappa}}{1 - \alpha_K}\right)^2 \tau_\varphi < \tau_f \left(\alpha_G \tilde{\underline{b}}_\pi\right)^2 < \left(\tau_\theta^{-1} + \tau_f^{-1}\right)^{-1}. \tag{A.11}$$

Again, $-\tilde{\underline{b}}_\pi$ fails to meet the condition for a government-centric equilibrium, ensuring $\tilde{\underline{b}}_\pi > \tilde{b}_\pi^*$.

Thus, the cutoff ranking is $-\tilde{b}_\pi^* < -\tilde{\underline{b}}_\pi < \underline{b}_\pi < b_\pi^*$. This implies that for a given b_π, at most one pure equilibrium (either fundamental- or government-centric) exists.

Proof of Proposition 3

The proof follows directly from Bayes' rule, with detailed expressions given in the Online Appendix.

Proof of Proposition 4

The proof strategy follows the outline in the text, with details available in the Online Appendix.

Proof of Proposition 5

Step 1: Optimal b_0

Manipulating the social welfare objective (29), the first-order condition for the optimal b_0 is

$$e^{\mathbb{E}[\log C_i] + \frac{1}{2}Var[\log C_i]} \left(e^{-\frac{\gamma}{2}Var[\log C_i]} - \frac{\psi \alpha_K}{1+\psi} e^{-\gamma Var\left[f + \alpha_G b_\pi \pi_g \mid \mathcal{I}_i\right]}\right) = \frac{1 + \psi(1 - \alpha_K)}{(1+\psi)\alpha_G} R_G e^{\mathbb{E}[\log G] + \frac{1}{2}Var[\log G]}. \tag{A.12}$$

Define

$$A = \frac{\alpha_G \mathbb{E}\left[C_i\right]}{R_G \mathbb{E}\left[G\right]} e^{-\frac{\gamma}{2} Var[\log C_i]} \geq \frac{1 + \psi\left(1 - \alpha_K\right)}{1 + \psi}. \tag{A.13}$$

When $\gamma = 0$ (risk-neutral households), $A = 1$.

Step 2: Optimal b_π and b_q

Using the first-order conditions for b_π and b_q along with equation (A.12) and expressions from Online Appendix C, we have:

- For b_π:

$$\begin{aligned} 0 = & \left(\frac{\psi \alpha_K}{1+\psi} + 2\gamma\left(A - 1\right)\right) \partial_{b_\pi} Var\left[f + \alpha_G b_\pi \pi_g \middle| \mathcal{I}_i\right] - \frac{\alpha_K}{1+\psi} \partial_{b_\pi}\left(a_v^2 \tau_v^{-1}\right) \\ & + \left(\frac{1 + \psi\left(1 - \alpha_K\right)}{1 + \psi} - \gamma A\right) \partial_{b_\pi} Var\left[\log C_i\right] - \alpha_G \partial_{b_\pi} Var\left[\log G\right]. \end{aligned} \tag{A.14}$$

- For π_q:

$$\left(\frac{1 + \psi\left(1 - \alpha_K\right)}{1 + \psi} - \gamma A\right) \partial_{b_q} Var\left[\log C_i\right] - \alpha_G \partial_{b_q} Var\left[\log G\right] = 0. \tag{A.15}$$

Step 3: Log-linear Approximating a Government-centric Equilibrium

Using a log-linear approximation of the welfare objective around $\gamma = 0$ (where $A = 1$), applying the Implicit Function Theorem to equations (A.14) and (A.15) gives

$$\partial_\gamma b_\pi \propto -\partial_{b_\pi} Var\left[\log C_i\right]. \tag{A.16}$$

We therefore focus on $Var\left[\log C_i\right]$ and make use of the following Lemma.

Lemma 7. *In a government-centric equilibrium, if $b_\pi < \hat{b}_\pi^*$, then $\partial_{b_\pi} Var\left[\log C_i\right] < 0$, while if $b_\pi > b_\pi^*$, then $\partial_{b_\pi} Var\left[\log C_i\right] > 0$.*

Consequently, if $b_\pi < \hat{b}_\pi^*$, then $\partial_\gamma b_\pi > 0$; if $b_\pi > b_\pi^*$, then $\partial_\gamma b_\pi < 0$.

For sufficiently large γ, the optimal b_π falls between $\hat{b}_\pi^*$ and b_π^*, preventing a government-centric equilibrium.

While this analysis uses a log-linear approximation, numerical analyses suggest its reasonableness, as A typically remains close to unity in the fully nonlinear model when τ_f and τ_φ are modestly large.

Proof of Proposition 6

Assume households are sufficiently risk-averse such that, from Proposition 5, the optimal b_π avoids a government-centric equilibrium. The maximum posterior precision about the policymaker's ability $\bar{\tau}_\theta$ from Proposition 4 is achieved in a government-centric equilibrium as $b_\pi \to \infty$, and the maximum evaluation payoff:

$$\sup_{b_\pi} \frac{1}{2} \log \frac{\bar{\tau}_\theta}{\tau_\theta} = \frac{1}{2} \log \left(1 + \frac{\tau_f}{\tau_\theta} + \frac{1}{\tau_\theta} \left(\frac{1 - e^{-\kappa}}{1 - \alpha_K e^{-\kappa}} \right)^2 \frac{\tau_\varphi}{\psi^2} \right) < \infty,$$

which is bounded.

Define $\beta\left(\underline{W}\right) = \frac{\lambda(\underline{W})}{1+\lambda(\underline{W})}$, where $\lambda\left(\underline{W}\right)$ is the Lagrange multiplier on the public outcry constraint (31) when the reservation welfare level is $\underline{W}$. The policymaker's optimization problem (30) becomes:

$$\hat{\mathcal{V}} = \sup_{\{b_0, b_s, b_q\}} \left(1 - \beta\left(\underline{W}\right)\right) \frac{1}{2} \log \left(\frac{\hat{\tau}_\theta}{\tau_\theta} \right) + \beta\left(\underline{W}\right) \log W. \tag{A.17}$$

Since b_0 and b_q impact household welfare W but not $\frac{1}{2} \log \left(\frac{\hat{\tau}_\theta}{\tau_\theta} \right)$, they are set to maximize W, as shown in Proposition 5. Consequently, the log of household welfare, $\log W$, is always well-defined, as

$$W = \frac{\alpha_K R_G}{(1 + \psi)\, \alpha_G} \mathbb{E}\left[G\right] \geq 0.$$

If $\underline{W} = \bar{W}$ (the maximum welfare), then $\beta\left(\bar{W}\right) = 1$, and the policymaker must maximize household welfare. In this case, b_π follows its optimal value from Proposition 5 and is small enough to avoid a government-centric equilibrium.

If $\underline{W}$ is sufficiently low ($\underline{W} = 0$), then $\beta\left(\underline{W}\right) \to 0$, and the policymaker maximizes her evaluation. From Proposition 4, this leads to b_π being chosen arbitrarily large, pushing the economy into a government-centric equilibrium. By continuity of problem (A.17) in $\underline{W}$, this occurs when $\underline{W}$ is sufficiently small as $\underline{W} \to 0$.

As the public outcry constraint (31) tightens with higher $\underline{W}$, both $\lambda\left(\underline{W}\right)$ and $\beta\left(\underline{W}\right)$ increase, causing the optimal b_π to decrease with $\underline{W}$.

Online Appendix for Information Discovery for Industrial Policy

In this Online Appendix, we provide the complete proofs for Propositions 1, 2, 3, 4, and 5 and the proof of Lemma 7.

Proof of Proposition 1

We follow the standard approach to solving noisy rational expectations models.

Step 1: Solve for Firm i's Beliefs

We begin with the beliefs conditional on $\mathcal{I}_P$: $\begin{bmatrix} f \\ \pi_g \end{bmatrix} | \mathcal{I}_P \backsim \mathcal{N}\left(\begin{bmatrix} \hat{f} \\ \hat{\pi}_g \end{bmatrix}, \begin{bmatrix} \hat{\tau}_f^{-1} & \hat{\tau}_{f\pi}^{-1} \\ \hat{\tau}_{f\pi}^{-1} & \hat{\tau}_\pi^{-1} \end{bmatrix}\right)$. Define the Kalman Gain H as

$$H = \frac{A_s}{\left(A_s + A_v\right)^2 \tau_f^{-1} + A_s^2 \tau_\theta^{-1} + \psi^2 \tau_\varphi^{-1}} \begin{bmatrix} \left(A_s + A_v\right) \tau_f^{-1} \\ \left(A_s + A_v\right) \tau_f^{-1} + A_v \tau_\theta^{-1} \end{bmatrix}, \tag{OA.1}$$

Then, the conditional expectation is given by

$$\begin{bmatrix} \hat{f} \\ \hat{\pi}_g \end{bmatrix} = \begin{bmatrix} \bar{f} \\ \bar{f} \end{bmatrix} + \frac{\begin{bmatrix} \left(A_s + A_v\right) \tau_f^{-1} \\ \left(A_s + A_v\right) \tau_f^{-1} + A_v \tau_\theta^{-1} \end{bmatrix} \left(A_s z_q - \left(A_s + A_v\right) \bar{f}\right)}{\left(A_s + A_v\right)^2 \tau_f^{-1} + A_v^2 \tau_\theta^{-1} + \psi^2 \tau_\varphi^{-1}}, \tag{OA.2}$$

and the conditional covariance matrix by

$$\begin{aligned} \begin{bmatrix} \hat{\tau}_f^{-1} & \hat{\tau}_{f\pi}^{-1} \\ \hat{\tau}_{f\pi}^{-1} & \hat{\tau}_\pi^{-1} \end{bmatrix} &= \begin{bmatrix} \tau_f^{-1} & \tau_f^{-1} \\ \tau_f^{-1} & \tau_f^{-1} + \tau_\theta^{-1} \end{bmatrix} - H \begin{bmatrix} \left(A_s + A_v\right) \tau_f^{-1} \\ \left(A_s + A_v\right) \tau_f^{-1} + A_v \tau_\theta^{-1} \end{bmatrix}' \\ &= \frac{\psi^2 \tau_\varphi^{-1} \tau_f^{-1} \iota_2 \iota_2' + \begin{bmatrix} A_v^2 \tau_f^{-1} \tau_\theta^{-1} & -A_s A_v \tau_f^{-1} \tau_\theta^{-1} \\ -A_s A_v \tau_f^{-1} \tau_\theta^{-1} & A_s^2 \tau_f^{-1} \tau_\theta^{-1} + \psi^2 \tau_\varphi^{-1} \tau_\theta^{-1} \end{bmatrix}}{\left(A_s + A_v\right)^2 \tau_f^{-1} + A_v^2 \tau_\theta^{-1} + \psi^2 \tau_\varphi^{-1}} \end{aligned} \tag{OA.3}$$

Because firms are Bayesian, we can update from the public beliefs to the private beliefs of firm i. Conditional on observing its private signals s_i and v_i, the posterior beliefs of firm i are also jointly normally distributed $\begin{bmatrix} f \\ \pi_g \end{bmatrix} | \mathcal{I}_i \backsim \mathcal{N}\left(\begin{bmatrix} \hat{f}_i \\ \hat{s}_{Gi} \end{bmatrix}, \begin{bmatrix} \hat{\tau}_{f,i}^{-1} & \hat{\tau}_{fG,i}^{-1} \\ \hat{\tau}_{fG,i}^{-1} & \hat{\tau}_{G,i}^{-1} \end{bmatrix}\right)$. Define the Kalman Gain H_i as

$$\begin{aligned} H_i &= \begin{bmatrix} \hat{\tau}_f^{-1} & \hat{\tau}_{f\pi}^{-1} \\ \hat{\tau}_{f\pi}^{-1} & \hat{\tau}_\pi^{-1} \end{bmatrix} \begin{bmatrix} \hat{\tau}_f^{-1} + \tau_s^{-1} & \hat{\tau}_{f\pi}^{-1} \\ \hat{\tau}_{f\pi}^{-1} & \hat{\tau}_\pi^{-1} + \tau_v^{-1} \end{bmatrix}^{-1} \\ &= \frac{\begin{bmatrix} \hat{\tau}_f^{-1}\left(\hat{\tau}_\pi^{-1} + \tau_v^{-1}\right) - \hat{\tau}_{f\pi}^{-1} \hat{\tau}_{f\pi}^{-1} & \hat{\tau}_{f\pi}^{-1} \tau_s^{-1} \\ \hat{\tau}_{f\pi}^{-1} \tau_v^{-1} & \hat{\tau}_\pi^{-1}\left(\hat{\tau}_f^{-1} + \tau_s^{-1}\right) - \hat{\tau}_{f\pi}^{-1} \hat{\tau}_{f\pi}^{-1} \end{bmatrix}}{\left(\hat{\tau}_\pi^{-1} + \tau_v^{-1}\right)\left(\hat{\tau}_f^{-1} + \tau_s^{-1}\right) - \hat{\tau}_{f\pi}^{-1} \hat{\tau}_{f\pi}^{-1}}. \end{aligned}$$

Then, the conditional expectation of beliefs of firm i are given by

$$\begin{bmatrix} \hat{f}_i \\ \hat{\pi}_{gi} \end{bmatrix} = \frac{\begin{bmatrix} \tau_s^{-1}\left(\hat{\tau}_\pi^{-1}+\tau_v^{-1}\right) & -\hat{\tau}_{f\pi}^{-1}\tau_s^{-1} \\ -\hat{\tau}_{f\pi}^{-1}\tau_v^{-1} & \tau_v^{-1}\left(\hat{\tau}_f^{-1}+\tau_s^{-1}\right) \end{bmatrix}}{\left(\hat{\tau}_\pi^{-1}+\tau_v^{-1}\right)\left(\hat{\tau}_f^{-1}+\tau_s^{-1}\right)-\hat{\tau}_{f\pi}^{-1}\hat{\tau}_{f\pi}^{-1}} \begin{bmatrix} \hat{f} \\ \hat{\pi}_g \end{bmatrix} + H_i \begin{bmatrix} s_i \\ v_i \end{bmatrix}, \tag{OA.4}$$

and the conditional covariance matrix by

$$\begin{aligned} \begin{bmatrix} \hat{\tau}_{f,i}^{-1} & \hat{\tau}_{f\pi,i}^{-1} \\ \hat{\tau}_{f\pi,i}^{-1} & \hat{\tau}_{\pi,i}^{-1} \end{bmatrix} &= \begin{bmatrix} \hat{\tau}_f^{-1} & \hat{\tau}_{f\pi}^{-1} \\ \hat{\tau}_{f\pi}^{-1} & \hat{\tau}_\pi^{-1} \end{bmatrix} - H_i \begin{bmatrix} \hat{\tau}_f^{-1} & \hat{\tau}_{f\pi}^{-1} \\ \hat{\tau}_{f\pi}^{-1} & \hat{\tau}_\pi^{-1} \end{bmatrix}' \\ &= \frac{\begin{bmatrix} \tau_s^{-1}\hat{\tau}_f^{-1}\left(\hat{\tau}_\pi^{-1}+\tau_v^{-1}\right)-\tau_s^{-1}\hat{\tau}_{f\pi}^{-1}\hat{\tau}_{f\pi}^{-1} & \tau_s^{-1}\tau_v^{-1}\hat{\tau}_{f\pi}^{-1} \\ \tau_s^{-1}\tau_v^{-1}\hat{\tau}_{f\pi}^{-1} & \tau_v^{-1}\hat{\tau}_\pi^{-1}\left(\hat{\tau}_f^{-1}+\tau_s^{-1}\right)-\tau_v^{-1}\hat{\tau}_{f\pi}^{-1}\hat{\tau}_{f\pi}^{-1} \end{bmatrix}}{\left(\hat{\tau}_\pi^{-1}+\tau_v^{-1}\right)\left(\hat{\tau}_f^{-1}+\tau_s^{-1}\right)-\hat{\tau}_{f\pi}^{-1}\hat{\tau}_{f\pi}^{-1}} \end{aligned} \tag{OA.5}$$

Step 2: Solve for Firm i's Optimal Investment Policy

By substituting the government's policy function (11) into equation (4) and substituting our learning expressions, we have

$$\begin{aligned} \log K_i &= \frac{1}{1-\alpha_K}\log \mathbb{E}\left[\frac{e^{(1-\gamma)f+(1-\gamma)\alpha_G b_\pi \pi_g+(1-\gamma)a_G b_q \log q}}{\mathbb{E}\left[e^{-\gamma f-\gamma\alpha_G b_\pi \pi_g-\gamma a_G b_q \log q} \mid \mathcal{I}_i\right]} \mid \mathcal{I}_i\right] + \frac{\log\alpha_K+\alpha_G b_0-\log q}{1-\alpha_K}, \\ &= \frac{1}{1-\alpha_K}\frac{\tau_s^{-1}\left(\hat{\tau}_\pi^{-1}+\tau_v^{-1}\right)-\alpha_G b_\pi \hat{\tau}_{f\pi}^{-1}\tau_v^{-1}}{\left(\hat{\tau}_\pi^{-1}+\tau_v^{-1}\right)\left(\hat{\tau}_f^{-1}+\tau_s^{-1}\right)-\hat{\tau}_{f\pi}^{-1}\hat{\tau}_{f\pi}^{-1}}\hat{f} \\ &\quad + \frac{1}{1-\alpha_K}\frac{\alpha_G b_\pi \tau_v^{-1}\left(\hat{\tau}_f^{-1}+\tau_s^{-1}\right)-\hat{\tau}_{f\pi}^{-1}\tau_s^{-1}}{\left(\hat{\tau}_\pi^{-1}+\tau_v^{-1}\right)\left(\hat{\tau}_f^{-1}+\tau_s^{-1}\right)-\hat{\tau}_{f\pi}^{-1}\hat{\tau}_{f\pi}^{-1}}\hat{\pi}_g \\ &\quad + \frac{\alpha_G b_q-1}{1-\alpha_K}\log q + \frac{\log\alpha_K+\alpha_G b_0}{1-\alpha_K} \\ &\quad + \frac{1}{1-\alpha_K}\frac{\hat{\tau}_f^{-1}\left(\hat{\tau}_\pi^{-1}+\tau_v^{-1}\right)-\hat{\tau}_{f\pi}^{-1}\hat{\tau}_{f\pi}^{-1}+\alpha_G b_\pi \hat{\tau}_{f\pi}^{-1}\tau_v^{-1}}{\left(\hat{\tau}_\pi^{-1}+\tau_v^{-1}\right)\left(\hat{\tau}_f^{-1}+\tau_s^{-1}\right)-\hat{\tau}_{f\pi}^{-1}\hat{\tau}_{f\pi}^{-1}}s_i \\ &\quad + \frac{1}{1-\alpha_K}\frac{\hat{\tau}_{f\pi}^{-1}\tau_s^{-1}+\alpha_G b_\pi\left(\hat{\tau}_\pi^{-1}\left(\hat{\tau}_f^{-1}+\tau_s^{-1}\right)-\hat{\tau}_{f\pi}^{-1}\hat{\tau}_{f\pi}^{-1}\right)}{\left(\hat{\tau}_\pi^{-1}+\tau_v^{-1}\right)\left(\hat{\tau}_f^{-1}+\tau_s^{-1}\right)-\hat{\tau}_{f\pi}^{-1}\hat{\tau}_{f\pi}^{-1}}v_i \\ &\quad + \frac{1-2\gamma}{2}\frac{\hat{\tau}_{f,i}^{-1}+\left(\alpha_G b_\pi\right)^2\hat{\tau}_{g,i}^{-1}+2\alpha_G b_\pi \hat{\tau}_{fG,i}^{-1}}{1-\alpha_K}, \end{aligned} \tag{OA.6}$$

Matching coefficients in equation (12) with (OA.6), we find

$$\begin{aligned}
a_q &= \frac{\alpha_G b_q - 1}{1-\alpha_K}, && \text{(OA.7)}\\
a_f &= \frac{1}{1-\alpha_K} - a_s, && \text{(OA.8)}\\
a_\pi &= \frac{\alpha_G b_\pi}{1-\alpha_K} - a_v, && \text{(OA.9)}\\
a_s &= \frac{1}{1-\alpha_K} + \frac{1}{1-\alpha_K}\frac{\alpha_G b_\pi \hat{\tau}_{f\pi}^{-1}\tau_v^{-1} - \tau_s^{-1}\left(\hat{\tau}_\pi^{-1}+\tau_v^{-1}\right)}{\left(\hat{\tau}_f^{-1}+\tau_s^{-1}\right)\left(\hat{\tau}_\pi^{-1}+\tau_v^{-1}\right) - \hat{\tau}_{f\pi}^{-1}\hat{\tau}_{f\pi}^{-1}}, && \text{(OA.10)}\\
a_v &= \frac{\alpha_G b_\pi}{1-\alpha_K} + \frac{1}{1-\alpha_K}\frac{\hat{\tau}_{f\pi}^{-1}\tau_s^{-1} - \alpha_G b_\pi\left(\hat{\tau}_f^{-1}+\tau_s^{-1}\right)\tau_v^{-1}}{\left(\hat{\tau}_f^{-1}+\tau_s^{-1}\right)\left(\hat{\tau}_\pi^{-1}+\tau_v^{-1}\right) - \hat{\tau}_{f\pi}^{-1}\hat{\tau}_{f\pi}^{-1}}, && \text{(OA.11)}\\
a_0 &= \frac{1-2\gamma}{2}\frac{\hat{\tau}_{f,i}^{-1} + (\alpha_G b_\pi)^2\hat{\tau}_{g,i}^{-1} + 2\alpha_G b_\pi \hat{\tau}_{fG,i}^{-1}}{1-\alpha_K}\\
&\quad + \frac{\log\alpha_K + \alpha_G b_0}{1-\alpha_K}\\
&= \frac{1-2\gamma}{2}\left(a_s\tau_s^{-1} + \alpha_G b_\pi a_v \tau_v^{-1}\right) + \frac{\log\alpha_K + \alpha_G b_0}{1-\alpha_K}. && \text{(OA.12)}
\end{aligned}$$

Thus, we obtain the expression for $\log K_i$ in equation (17). This confirms that if other firms and the government follow log-linear policies, it is optimal for firm i to follow a log-linear investment policy.

Step 3: Solve for the Price of Capital

By substituting equations (OA.7), (OA.8), (OA.9) and (OA.12) into equation (15), we have

$$\begin{aligned}
\log q &= \frac{1-\alpha_K}{1-a_G b_q + \psi(1-\alpha_K)}\left(\frac{1}{1-\alpha_K}\hat{f} + \frac{\alpha_G b_\pi}{1-\alpha_K}\hat{\pi}_g + A_s\left(f-\hat{f}\right) + A_v\left(\pi_g - \hat{\pi}_g\right) + \psi\varphi\right)\\
&\quad + \frac{1-\alpha_K}{1-a_G b_q + \psi(1-\alpha_K)}\left(A_0 + \frac{1}{2}\left(A_s^2\tau_s^{-1} + A_v^2\tau_v^{-1} - \psi^2\tau_{\varphi\epsilon}^{-1}\right)\right), && \text{(OA.13)}
\end{aligned}$$

where, in equilibrium, the fixed-point conditions in (14) hold.

Proof of Proposition 2

Step 1: Solve for Firm *i*'s Optimal Information Acquisition Decision

Recall that the household maximizes (6). Substituting with equation (4) into (6), the household's optimal information acquisition policy solves the time 0 problem

$$\begin{aligned}
U_i &= \sup_{\tau_v,\tau_s}\frac{1}{1-\gamma}\mathbb{E}\left[\left(e^f G^{\alpha_G}K_i^{\alpha_K}\right)^{1-\gamma}\right] && \text{(OA.14)}\\
s.t. &: I(\tau_s,\tau_v) \le \frac{\kappa}{2}.
\end{aligned}$$

Define

$$h = f + \alpha_G b_\pi \pi_g.$$

Then, recognizing for a constant a and log-normal random variable h

$$\mathbb{E}\left[e^{ah} | \mathcal{I}_i\right] = e^{a\mathbb{E}[h|\mathcal{I}_i] + \frac{a^2}{2} Var[h|\mathcal{I}_i]}, \tag{OA.15}$$

and substituting equations (17) and (OA.13) into (OA.14), the objective in equation (OA.14) reduces to

$$\begin{aligned} U_i &= \frac{e^{(1-\gamma)\alpha_K \frac{\log \alpha_K + \alpha_G b_0}{1-\alpha_K}}}{1-\gamma} \mathbb{E}\left[e^{\frac{1-\gamma}{1-\alpha_K}\left(\alpha_G b_q - \alpha_K\right)\log q} e^{(1-\gamma)h} \left(\frac{\mathbb{E}\left[e^{(1-\gamma)h} | \mathcal{I}_i\right]}{\mathbb{E}\left[e^{-\gamma h} | \mathcal{I}_i\right]} \right)^{\frac{(1-\gamma)\alpha_K}{1-\alpha_K}} \right] \\ &= \frac{e^{(1-\gamma)\alpha_K \frac{\log \alpha_K + \alpha_G b_0}{1-\alpha_K}}}{1-\gamma} \mathbb{E}\left[e^{\frac{1-\gamma}{1-\alpha_K}\left(\alpha_G b_q - \alpha_K\right)\log q} e^{(1-\gamma)h} e^{\frac{(1-\gamma)\alpha_K}{1-\alpha_K}\left(\mathbb{E}[h|\mathcal{I}_i] + \frac{1-2\gamma}{2} Var[h|\mathcal{I}_i]\right)} \right]. \end{aligned} \tag{OA.16}$$

Applying the Law of Iterated Expectations by conditioning first on firm i's information set $\mathcal{I}_i$, and invoking equation (OA.15), equation (OA.16) simplifies to

$$\begin{aligned} U_i &= \frac{e^{(1-\gamma)\alpha_K \frac{\log \alpha_K + \alpha_G b_0}{1-\alpha_K}}}{1-\gamma} \mathbb{E}\left[e^{\frac{1-\gamma}{1-\alpha_K}\left(\alpha_G b_q - \alpha_K\right)\log q} e^{\frac{(1-\gamma)\alpha_K}{1-\alpha_K}\left(\mathbb{E}[h|\mathcal{I}_i] + \frac{1-2\gamma}{2} Var[h|\mathcal{I}_i]\right)} \mathbb{E}\left[e^{(1-\gamma)h} | \mathcal{I}_i\right] \right] \\ &= \frac{e^{(1-\gamma)\alpha_K \frac{\log \alpha_K + \alpha_G b_0}{1-\alpha_K}}}{1-\gamma} \mathbb{E}\left[e^{\frac{1-\gamma}{1-\alpha_K}\left(\alpha_G b_q - \alpha_K\right)\log q} e^{\frac{1-\gamma}{1-\alpha_K}\mathbb{E}[h|\mathcal{I}_i] + \frac{1}{2}\frac{1-\gamma}{1-\alpha_K}(1-\gamma-\gamma\alpha_K))Var[h|\mathcal{I}_i]} \right] \\ &= \frac{e^{(1-\gamma)\alpha_K \frac{\log \alpha_K + \alpha_G b_0}{1-\alpha_K}}}{1-\gamma} \mathbb{E}\left[e^{\frac{1-\gamma}{1-\alpha_K}\left(\alpha_G b_q - \alpha_K\right)\log q + \frac{1-\gamma}{1-\alpha_K}\mathbb{E}[h|\mathcal{I}_P] + \frac{1}{2}\left(\frac{1-\gamma}{1-\alpha_K}\right)^2 Var[\mathbb{E}[h|\mathcal{I}_i]|\mathcal{I}_P] + \frac{1}{2}\frac{1-\gamma}{1-\alpha_K}(1-\gamma-\gamma\alpha_K))\frac{Var[h|\mathcal{I}_i]}{1-\alpha_K}} \right]. \end{aligned} \tag{OA.17}$$

We recognize that

$$\begin{aligned} \mathbb{E}\left[h | \mathcal{I}_P\right] &= \hat{f} + \alpha_G b_\pi \hat{\pi}_g, \\ Var\left[\mathbb{E}\left[h | \mathcal{I}_i\right] | \mathcal{I}_P\right] &= Var\left[h | \mathcal{I}_P\right] - \mathbb{E}\left[Var\left[h | \mathcal{I}_i\right] | \mathcal{I}_P\right] = Var\left[h | \mathcal{I}_P\right] - Var\left[h | \mathcal{I}_i\right], \\ Var\left[h | \mathcal{I}_P\right] &= \hat{\tau}_f^{-1} + \left(\alpha_G b_\pi\right)^2 \hat{\tau}_\pi^{-1} + 2\alpha_G b_\pi \hat{\tau}_{f\pi}^{-1} \end{aligned}$$

and consequently equation (OA.17) can be expressed as

$$\begin{aligned} U_i = & \frac{e^{-\frac{1-\gamma}{2(1-\alpha_K)}\frac{1-\gamma\alpha_K}{1-\alpha_K}\alpha_K Var[h|\mathcal{I}_i]}}{1-\gamma} \mathbb{E}\left[e^{\frac{1-\gamma}{1-\alpha_K}\left(\hat{f} + \alpha_G b_\pi \hat{\pi}_g + \left(\alpha_G b_q - \alpha_K\right)\log q\right)} \right] \\ & \times e^{\frac{1}{2}\left(\frac{1-\gamma}{1-\alpha_K}\right)^2 Var[h|\mathcal{I}_P] + (1-\gamma)\alpha_K \frac{\log \alpha_K + \alpha_G b_0}{1-\alpha_K}}. \end{aligned} \tag{OA.18}$$

It is clear from (OA.18) that $Var\left[h | \mathcal{I}_i\right]$ is the only term in U_i that varies with τ_s and τ_v.

Let $\tilde{\theta}_i$ be the Lagrange multiplier on the information acquisition constraint. Simplifying equation (OA.18), we arrive at the Lagrangian

$$U_i = \sup_{\tau_v, \tau_s} \frac{\Xi}{1-\gamma} e^{-\frac{1-\gamma}{2(1-\alpha_K)} \frac{1-\gamma\alpha_K}{1-\alpha_K} \alpha_K Var[h|\ \mathcal{I}_i]} - \frac{\tilde{\theta}_i}{2} \left(I\left(\tau_s, \tau_v\right) - \kappa \right), \tag{OA.19}$$

where $\Xi \geq 0$ given by

$$\begin{aligned} \Xi &= \mathbb{E}\left[e^{\frac{1-\gamma}{1-\alpha_K}\left(\hat{f}+\alpha_G b_\pi \hat{\pi}_g + \left(\alpha_G b_q - \alpha_K\right)\log q\right)} \right] \\ &\quad \times e^{\frac{(1-\gamma)^2}{2(1-\alpha_K)^2}\left(\hat{\tau}_f^{-1} + (\alpha_G b_\pi)^2 \hat{\tau}_\pi^{-1} + 2\alpha_G b_\pi \hat{\tau}_{f\pi}^{-1}\right) + (1-\gamma)\frac{\alpha_K}{1-\alpha_K}(\log \alpha_K + \alpha_G b_0)}. \end{aligned} \tag{OA.20}$$

Because the firm behaves competitively, it takes Ξ, $\hat{\tau}_f^{-1}$, $\hat{\tau}_\theta^{-1}$, and $\hat{\tau}_{f\theta}^{-1}$ as given.

If we define

$$\theta_i = \frac{2}{1-\gamma\alpha_K} \frac{1-\alpha_K}{\alpha_K} e^{-\frac{1-\gamma}{2(1-\alpha_K)} \frac{1-\gamma\alpha_K}{1-\alpha_K} \alpha_K Var[h|\ \mathcal{I}_i]} \Xi^{-1} \tilde{\theta}_i,$$

to be the normalized Lagrange multiplier, we can write the first-order necessary conditions of the Lagrangian for τ_s and τ_v as

$$\tau_s \quad : \quad -\frac{\partial Var\left[h|\ \mathcal{I}_i\right]}{\partial \tau_s} - \theta_i \frac{\partial I\left(\tau_s, \tau_v\right)}{\partial \tau_s} \leq 0 \ \left(= \text{binds if } \tau_s > 0\right), \tag{OA.21}$$

$$\tau_v \quad : \quad -\frac{\partial Var\left[h|\ \mathcal{I}_i\right]}{\partial \tau_v} - \theta_i \frac{\partial I\left(\tau_s, \tau_v\right)}{\partial \tau_v} \leq 0 \ \left(= \text{binds if } \tau_v > 0\right), \tag{OA.22}$$

If $\gamma < \frac{1}{\alpha_K}$ and $\tilde{\theta}_i \geq 0$, then $\theta_i > 0$.

Notice, however, that these first-order necessary conditions are equivalent to the simpler information acquisition program

$$\begin{aligned} u_i &= \sup_{\tau_v, \tau_s} -Var\left[h|\ \mathcal{I}_i\right], \\ s.t. &: \quad I\left(\tau_s, \tau_v\right) \leq \kappa/2, \end{aligned} \tag{OA.23}$$

because $h = f + \alpha_G b_\pi \pi_g$ by definition, taking as given $\hat{\tau}_f^{-1}$, $\hat{\tau}_\pi^{-1}$, and $\hat{\tau}_{f\pi}^{-1}$.

To take the first-order conditions more formally, we recognize substituting equation (OA.5) into equations (OA.10) and (OA.11) that

$$a_s = \frac{\hat{\tau}_{f,i}^{-1} + a_G b_\pi \hat{\tau}_{fG,i}^{-1}}{1-\alpha_K} \tau_s, \tag{OA.24}$$

$$a_v = \frac{\hat{\tau}_{fG,i}^{-1} + a_G b_\pi \hat{\tau}_{G,i}^{-1}}{1-\alpha_K} \tau_v, \tag{OA.25}$$

so that

$$\frac{Var\left[h|\ \mathcal{I}_i\right]}{1-\alpha_K} = a_s \tau_s^{-1} + a_G b_\pi a_v \tau_v^{-1}. \tag{OA.26}$$

Finally, we recognize the entropy reduction from the firm's information acquisition $I\left(\tau_s, \tau_v\right)$ can be expressed as

$$I(\tau_s, \tau_v) = \frac{1}{2}\log|\Sigma_P| - \frac{1}{2}\log|\Sigma_P - H_i\Sigma_P'| = -\frac{1}{2}\log\left| I_2 - \Sigma_P \begin{bmatrix} \hat{\tau}_f^{-1} + \tau_s^{-1} & \hat{\tau}_{f\pi}^{-1} \\ \hat{\tau}_{f\pi}^{-1} & \hat{\tau}_\pi^{-1} + \tau_v^{-1} \end{bmatrix}^{-1} \right|,$$

where I_2 is the 2×2 identity matrix. With some manipulation, this reduces to

$$I(\tau_s, \tau_v) = \frac{1}{2}\log\left|\frac{\hat{\tau}_f + \tau_s}{\hat{\tau}_f}\frac{\hat{\tau}_\pi + \tau_v}{\hat{\tau}_\pi} - \frac{\tau_s}{\hat{\tau}_{f\pi}}\frac{\tau_v}{\hat{\tau}_{f\pi}}\right|.$$

Substituting this expression for $I(\tau_s, \tau_v)$ yields that in the statement in the proposition. Notice that the capacity constraint will bind in equilibrium, which implies

$$\frac{\hat{\tau}_f + \tau_s}{\hat{\tau}_f}\frac{\hat{\tau}_\pi + \tau_v}{\hat{\tau}_\pi} - \frac{\tau_s}{\hat{\tau}_{f\pi}}\frac{\tau_v}{\hat{\tau}_{f\pi}} = e^\kappa. \tag{OA.27}$$

Substituting with equation (OA.27), we can express $\frac{\partial Var[h|\ \mathcal{I}_i]}{\partial \tau_s}$ and $\frac{\partial Var[h|\ \mathcal{I}_i]}{\partial \tau_v}$ as

$$\begin{aligned} -\frac{\partial Var[h|\ \mathcal{I}_i]}{\partial \tau_s} &= (a_G b_\pi)^2 e^{-\kappa}\tau_v^{-1}\hat{\tau}_f^{-1} \\ &\quad + \frac{\tau_s^{-1}\left(e^\kappa - 1 - \frac{\tau_v}{\hat{\tau}_\pi}\right) + 2\alpha_G b_\pi \hat{\tau}_{f\pi}^{-1} - (\alpha_G b_\pi)^2 \tau_v^{-1}\left(1 + \frac{\tau_s}{\hat{\tau}_f}\right)}{e^{2\kappa}}\tau_s^{-1}\left(e^\kappa - 1 - \frac{\tau_v}{\hat{\tau}_\pi}\right), \end{aligned}$$

and

$$-\frac{\partial Var[h|\ \mathcal{I}_i]}{\partial \tau_v} = e^{-\kappa}\tau_s^{-1}\hat{\tau}_\pi^{-1} + \frac{-\tau_s^{-1}\left(1 + \frac{\tau_v}{\hat{\tau}_g}\right) + 2\alpha_G b_\pi \hat{\tau}_{f\pi}^{-1} + (\alpha_G b_\pi)^2 \tau_v^{-1}\left(e^\kappa - 1 - \frac{\tau_s}{\hat{\tau}_f}\right)}{e^{2\kappa}}\tau_v^{-1}\left(e^\kappa - 1 - \frac{\tau_s}{\hat{\tau}_f}\right),$$

from which follows from the first-order conditions for τ_s and τ_v that we can identify τ_s and τ_v from equation (OA.27) and

$$\frac{(a_G b_\pi)^2 \tau_v^{-1}}{e^\kappa - 1 - \frac{\tau_v}{\hat{\tau}_\pi}} = \frac{\tau_s^{-1}}{e^\kappa - 1 - \frac{\tau_s}{\hat{\tau}_f}}. \tag{OA.28}$$

Notice the left-hand side of equation (OA.28) that the left-hand side is monotonically decreasing in τ_v while the right-hand side is monotonically decreasing in τ_s. Consequently, as $a_G b_\pi$ increases, τ_v increases while τ_s (weakly) decreases. Similarly, τ_s is decreasing in $\hat{\tau}_f$ while τ_v is decreasing in $\hat{\tau}_\pi$.

Manipulating equations (OA.28) and (OA.27), we can solve for τ_s and τ_v explicitly according to

$$\tau_v = \min\left\{\max\left\{(\alpha_G b_\pi)^2 \tau_s + \frac{(\alpha_G b_\pi)^2 \hat{\tau}_f - \hat{\tau}_\pi}{1 - \frac{\hat{\tau}_f}{\hat{\tau}_{f\pi}}\frac{\hat{\tau}_\pi}{\hat{\tau}_{f\pi}}}, 0\right\}, (e^\kappa - 1)\hat{\tau}_g\right\}, \tag{OA.29}$$

and

$$\tau_s = \min \left\{ \begin{array}{c} \max \left\{ \sqrt{\left(\tau_f + \left(\frac{a_s}{\psi} \right)^2 \tau_\varphi \right)^2 - \tau_f^2 + \frac{\tau_f \hat{\tau}_\pi - (1-e^\kappa) \left(\frac{a_v}{\psi} \right)^2 \tau_\varphi \tau_f}{(\alpha_G b_\pi)^2}} - \left(\tau_f + \left(\frac{a_s}{\psi} \right)^2 \tau_\varphi \right), 0 \right\}, \\ \\ (e^\kappa - 1) \hat{\tau}_f \end{array} \right\}. \tag{OA.30}$$

Finally, substituting with equation (OA.27), from equations (OA.10) and (OA.11), a_s and a_v become

$$a_s = \frac{1-e^{-\kappa}}{1-\alpha_K} + e^{-\kappa} \frac{\alpha_G b_\pi \tau_s \hat{\tau}_{f\pi}^{-1} - \tau_v \hat{\tau}_\pi^{-1}}{1-\alpha_K}, \tag{OA.31}$$

$$a_v = \alpha_G b_\pi \frac{1-e^{-\kappa}}{1-\alpha_K} + e^{-\kappa} \frac{\tau_v \hat{\tau}_{f\pi}^{-1} - \alpha_G b_\pi \tau_s \hat{\tau}_f^{-1}}{1-\alpha_K}. \tag{OA.32}$$

Step 2: Existence of a Government-centric Equilibrium

Suppose the equilibrium is a government-centric equilibrium in which all households choose to learn only about the government political agenda (i.e., $\tau_v > 0$ and $\tau_s = 0$). In this case, the entropy constraint (7), substituting with equation (5), reduces to

$$I(\tau_s, \tau_v) = \frac{1}{2} \log \frac{\hat{\tau}_\pi + \tau_v}{\hat{\tau}_\pi} \le \kappa/2. \tag{OA.33}$$

By the entropy constraint (OA.33)

$$\tau_v = (e^\kappa - 1) \hat{\tau}_\pi, \tag{OA.34}$$

where $\hat{\tau}_\pi$ depends on τ_v. From equations (OA.31) and (OA.32), substituting with (OA.34), a_s and a_v reduce to

$$a_s = 0, \tag{OA.35}$$

$$a_v = \alpha_G b_\pi \frac{1-e^{-\kappa}}{1-\alpha_K} + e^{-\kappa} \frac{\hat{\tau}_{f\pi}^{-1} \tau_v}{1-\alpha_K} = \left(\frac{\tau_\theta}{\tau_\theta + \tau_f} + \alpha_G b_\pi \right) \frac{1-e^{-\kappa}}{1-\alpha_K}. \tag{OA.36}$$

Further, from equation (OA.3)

$$\hat{\tau}_f^{-1} = \frac{A_v^2 \tau_\theta^{-1} + \psi^2 \tau_\varphi^{-1}}{A_v^2 \left(\tau_f^{-1} + \tau_\theta^{-1} \right) + \psi^2 \tau_\varphi^{-1}} \tau_f^{-1}, \tag{OA.37}$$

and

$$\hat{\tau}_\pi^{-1} = \frac{\psi^2 \tau_\varphi^{-1} \left(\tau_f^{-1} + \tau_\theta^{-1} \right)}{A_v^2 \left(\tau_f^{-1} + \tau_\theta^{-1} \right) + \psi^2 \tau_\varphi^{-1}}. \tag{OA.38}$$

In equilibrium, $A_v = a_v$ from equation (OA.36), and from equations (OA.34) and (OA.38) $\hat{\tau}_g$ satisfies

$$\hat{\tau}_\pi = \left(\tau_\theta^{-1} + \tau_f^{-1} \right)^{-1} + \left(\frac{\tau_\theta}{\tau_\theta + \tau_f} + \alpha_G b_\pi \right)^2 \left(\frac{1}{\psi} \frac{1-e^{-\kappa}}{1-\alpha_K} \right)^2 \tau_\varphi. \tag{OA.39}$$

from which we can recover τ_v from equation (OA.34).

For the equilibrium to be a government-centric equilibrium $\tau_s = 0$, which requires in the optimal choice of τ_s from equation (21) that the first argument in the max be less than or equal to 0, or

$$\tau_f \left(\alpha_G b_\pi\right)^2 - e^\kappa \left(\frac{\tau_\theta}{\tau_\theta + \tau_f} + \alpha_G b_\pi\right)^2 \left(\frac{1}{\psi}\frac{1-e^{-\kappa}}{1-\alpha_K}\right)^2 \tau_\varphi \geq \left(\tau_\theta^{-1} + \tau_f^{-1}\right)^{-1}, \tag{OA.40}$$

from which follows either

$$\alpha_G b_\pi \leq \frac{\frac{\tau_\theta}{\tau_\theta+\tau_f} e^\kappa \left(\frac{1}{\psi}\frac{1-e^{-\kappa}}{1-\alpha_K}\right)^2 \tau_\varphi}{\tau_f - e^\kappa \left(\frac{1}{\psi}\frac{1-e^{-\kappa}}{1-\alpha_K}\right)^2 \tau_\varphi} - \sqrt{\left(\frac{\frac{\tau_\theta}{\tau_\theta+\tau_f} e^\kappa \left(\frac{1}{\psi}\frac{1-e^{-\kappa}}{1-\alpha_K}\right)^2 \tau_\varphi}{\tau_f - e^\kappa \left(\frac{1}{\psi}\frac{1-e^{-\kappa}}{1-\alpha_K}\right)^2 \tau_\varphi}\right)^2 + \frac{\left(\frac{\tau_\theta}{\tau_\theta+\tau_f}\right)^2 e^\kappa \left(\frac{1}{\psi}\frac{1-e^{-\kappa}}{1-\alpha_K}\right)^2 \tau_\varphi + \left(\tau_\theta^{-1} + \tau_f^{-1}\right)^{-1}}{\tau_f - e^\kappa \left(\frac{1}{\psi}\frac{1-e^{-\kappa}}{1-\alpha_K}\right)^2 \tau_\varphi}}$$

which would imply a government-centric equilibrium exists if $\alpha_G b_\pi \leq -\alpha_G \tilde{b}_s^* < -\frac{\tau_\theta}{\tau_\theta+\tau_f}$, or

$$\alpha_G b_\pi \geq \frac{\frac{\tau_\theta}{\tau_\theta+\tau_f} e^\kappa \left(\frac{1}{\psi}\frac{1-e^{-\kappa}}{1-\alpha_K}\right)^2 \tau_\varphi}{\tau_f - e^\kappa \left(\frac{1}{\psi}\frac{1-e^{-\kappa}}{1-\alpha_K}\right)^2 \tau_\varphi} + \sqrt{\left(\frac{\frac{\tau_\theta}{\tau_\theta+\tau_f} e^\kappa \left(\frac{1}{\psi}\frac{1-e^{-\kappa}}{1-\alpha_K}\right)^2 \tau_\varphi}{\tau_f - e^\kappa \left(\frac{1}{\psi}\frac{1-e^{-\kappa}}{1-\alpha_K}\right)^2 \tau_\varphi}\right)^2 + \frac{\left(\frac{\tau_\theta}{\tau_\theta+\tau_f}\right)^2 e^\kappa \left(\frac{1}{\psi}\frac{1-e^{-\kappa}}{1-\alpha_K}\right)^2 \tau_\varphi + \left(\tau_\theta^{-1} + \tau_f^{-1}\right)^{-1}}{\tau_f - e^\kappa \left(\frac{1}{\psi}\frac{1-e^{-\kappa}}{1-\alpha_K}\right)^2 \tau_\varphi}} \tag{OA.41}$$

from which follows a government-centric equilibrium exists if $\alpha_G b_\pi \geq \sqrt{\frac{\tau_\theta}{\tau_\theta+\tau_f}} \geq \frac{\tau_\theta}{\tau_\theta+\tau_f}$ because $\frac{\tau_\theta}{\tau_\theta+\tau_f} \leq 1$. A necessary condition for solutions to exist is $\tau_f \geq \tau_\varphi e^\kappa \left(\frac{1}{\psi}\frac{1-e^{-\kappa}}{1-\alpha_K}\right)^2$; otherwise, both roots are imaginary.

It follows there exists critical values of b_π, $\tilde{b}_\pi^*$ and b_π^*, such that there exists a government-centric equilibrium if and only if $b_\pi \in (-\infty, -\tilde{b}_\pi^*] \cup [b_\pi^*, \infty)$, and there does not exist one otherwise. Because α_G and b_π enter the inequality together as $\alpha_G b_\pi$, it follows $\tilde{b}_\pi^*$ and b_π^* are decreasing in α_G. It is further immediate that they are decreasing in τ_f/τ_θ and increasing in $\frac{\tau_\varphi}{\psi^2}$ and κ.

Step 3: Existence of a Fundamental-centric Equilibrium

Suppose instead the equilibrium is a fundamental-centric equilibrium in which all households choose to learn only about the fundamental (i.e., $\tau_s > 0$ and $\tau_v = 0$). In this case, $\hat{\tau}_\pi^{-1} = \hat{\tau}_f^{-1} + \tau_\theta^{-1}$, $\hat{\tau}_{f\pi} = \hat{\tau}_f$ and by similar arguments to Step 1, $a_v = 0$,

Suppose instead the equilibrium is a fundamental-centric equilibrium in which all households choose to learn only about the fundamental (i.e., $\tau_s > 0$ and $\tau_v = 0$). In this case, $\hat{\tau}_\pi^{-1} = \hat{\tau}_f^{-1} + \tau_\theta^{-1}$, $\hat{\tau}_{f\pi} = \hat{\tau}_f$ and by similar arguments to Step 1, $a_v = 0$,

$$\tau_s = \left(e^\kappa - 1\right) \hat{\tau}_f, \tag{OA.42}$$

and we have

$$a_s = (1 + \alpha_G b_\pi) \frac{1 - e^{-\kappa}}{1 - \alpha_K}, \tag{OA.43}$$

and

$$\hat{\tau}_f = \tau_f + (1 + \alpha_G b_\pi)^2 \left(\frac{1}{\psi} \frac{1 - e^{-\kappa}}{1 - \alpha_K} \right)^2 \tau_\varphi. \tag{OA.44}$$

For the equilibrium to be a fundamental-centric equilibrium $\tau_v = 0$, which requires in the optimal choice of τ_v from equation (20) that the first argument in the max be less than or equal to 0, or

$$(\alpha_G b_\pi)^2 \left(1 + \frac{e^\kappa}{\tau_\theta} \left(\frac{\tau_\varphi}{\psi^2} a_s^2 + \tau_f \right) \right) \le 1. \tag{OA.45}$$

This can be expanded into the quartic polynomial

$$\begin{aligned} \frac{e^\kappa}{\tau_\theta} \tau_\varphi \left(\frac{1}{\psi} \frac{1 - e^{-\kappa}}{1 - \alpha_K} \right)^2 (\alpha_G b_\pi)^4 + 2 \frac{e^\kappa}{\tau_\theta} \tau_\varphi \left(\frac{1}{\psi} \frac{1 - e^{-\kappa}}{1 - \alpha_K} \right)^2 (\alpha_G b_\pi)^3 \\ + \left(1 + \frac{e^\kappa}{\tau_\theta} \tau_\varphi \left(\frac{1}{\psi} \frac{1 - e^{-\kappa}}{1 - \alpha_K} \right)^2 + \frac{e^\kappa}{\tau_\theta} \tau_f \right) (\alpha_G b_\pi)^2 - 1 \quad \le \quad 0, \end{aligned}$$

which has one positive and one negative root.

Notice a_v is monotonically increasing in $\alpha_G b_\pi$ from equation (OA.43). When $b_\pi = 0$, the left-hand side reduces to 0, and consequently a fundamental-centric equilibrium exists. It is immediate that the left-hand side is monotonically increasing in $\alpha_G b_\pi$ for $b_\pi > 0$. There therefore exists a critical b_π, $\underline{b}_\pi$, such that a fundamental-centric equilibrium exists if $b_\pi \le \underline{b}_\pi$, and does not exist otherwise. Similarly, there exists a second critical b_π, $-\tilde{\underline{b}}_\pi$,such that a fundamental equilibrium exists if $b_\pi \ge -\tilde{\underline{b}}_\pi$. Consequently, a fundamental-centric equilibrium exists if and only if $b_\pi \in [-\tilde{\underline{b}}_\pi, \underline{b}_\pi]$.

From equation (OA.43), it is immediate that $\alpha_G \tilde{\underline{b}}_\pi, \alpha_G \underline{b}_\pi < \sqrt{\frac{\tau_\theta}{\tau_f + \tau_\theta}}$ because

$$\frac{\tau_\theta}{\tau_f + \tau_\theta} \left(1 + \frac{e^\kappa}{\tau_\theta} \left(\frac{\tau_\varphi}{\psi^2} a_s^2 + \tau_f \right) \right) = 1 + \frac{(e^\kappa \tau_f - 1)\, \tau_f}{\tau_f + \tau_\theta} + \frac{e^\kappa}{\tau_f + \tau_\theta} \frac{\tau_\varphi}{\psi^2} a_s^2 > 1.$$

Because α_G and b_π enter the inequality together as $\alpha_G b_\pi$, it follows $\underline{b}_\pi$ and $\tilde{\underline{b}}_\pi$ are decreasing in α_G. By the Implicit Function Theorem applied to equation (OA.45) when it holds with equality, the critical $\underline{b}_\pi$ and $\tilde{\underline{b}}_\pi$ are decreasing in τ_f / τ_θ, κ, and $\frac{\tau_\varphi}{\psi^2}$.

Step 4: Ranking the Cutoffs

Consider the critical $b_\pi > 0$, $\underline{b}_\pi$, that is the upper bound for a fundamental-centric equilibrium. From equation (A.9) when it holds with equality, we can bound this critical $\underline{b}_\pi$

$$\begin{aligned} 1 &= (\alpha_G \underline{b}_\pi)^2 \left(1 + \frac{e^\kappa}{\tau_\theta} \left((1 + \alpha_G \underline{b}_\pi)^2 \left(\frac{1}{\psi} \frac{1 - e^{-\kappa}}{1 - \alpha_K} \right)^2 \tau_\varphi + \tau_f \right) \right) \\ &> (\alpha_G \underline{b}_\pi)^2 \left(1 + \frac{e^\kappa}{\tau_\theta} \left(\left(\frac{1}{\psi} \frac{1 - e^{-\kappa}}{1 - \alpha_K} \right)^2 \tau_\varphi + \tau_f \right) \right), \end{aligned}$$

which implies

$$(\alpha_G \underline{b}_\pi)^2 < \frac{1}{1+\frac{e^\kappa}{\tau_\theta}\left(\left(\frac{1}{\psi}\frac{1-e^{-\kappa}}{1-\alpha_K}\right)^2\tau_\varphi+\tau_f\right)}. \tag{OA.46}$$

It is then immediate from this bound (OA.46) that

$$\begin{aligned}\tau_f(\alpha_G\underline{b}_\pi)^2 - e^\kappa\left(\frac{\tau_\theta}{\tau_\theta+\tau_f}+\alpha_G\underline{b}_\pi\right)^2\left(\frac{1}{\psi}\frac{1-e^{-\kappa}}{1-\alpha_K}\right)^2\tau_\varphi &< \tau_f(\alpha_G\underline{b}_\pi)^2 \\ &< \frac{\tau_f\tau_\theta}{e^\kappa\tau_f+\tau_\theta+e^\kappa\left(\frac{1}{\psi}\frac{1-e^{-\kappa}}{1-\alpha_K}\right)^2\tau_\varphi}\end{aligned} \tag{OA.47}$$

Note, however, because $\kappa \geq 0$ that

$$\frac{\tau_f\tau_\theta}{e^\kappa\tau_f+\tau_\theta+e^\kappa\left(\frac{1}{\psi}\frac{1-e^{-\kappa}}{1-\alpha_K}\right)^2\tau_\varphi} < \frac{\tau_f\tau_\theta}{\tau_f+\tau_\theta} = \left(\tau_\theta^{-1}+\tau_f^{-1}\right)^{-1}, \tag{OA.48}$$

which consequently implies from inequality (OA.47) that

$$\tau_f(\alpha_G\underline{b}_\pi)^2 - e^\kappa\left(\frac{\tau_\theta}{\tau_\theta+\tau_f}+\alpha_G\underline{b}_\pi\right)^2\left(\frac{1}{\psi}\frac{1-e^{-\kappa}}{1-\alpha_K}\right)^2\tau_\varphi < \left(\tau_\theta^{-1}+\tau_f^{-1}\right)^{-1}. \tag{OA.49}$$

Comparing (OA.49) to (OA.40), it is immediate that $\underline{b}_\pi$ does not satisfy the condition for the existence of a government-centric equilibrium. As such, $\underline{b}_\pi < b_\pi^*$.

Suppose now $b_\pi < 0$. By similar arguments, when equation (A.9) holds with equality, we can bound this critical $\underline{\tilde{b}}_\pi$

$$1 = \left(\alpha_G\underline{\tilde{b}}_\pi\right)^2\left(1+\frac{e^\kappa}{\tau_\theta}\left(\left(1-\alpha_G\underline{\tilde{b}}_\pi\right)^2\left(\frac{1}{\psi}\frac{1-e^{-\kappa}}{1-\alpha_K}\right)^2\tau_\varphi+\tau_f\right)\right) > \left(\alpha_G\underline{\tilde{b}}_\pi\right)^2\left(1+\frac{e^\kappa}{\tau_\theta}\tau_f\right),$$

which implies

$$\left(\alpha_G\underline{\tilde{b}}_\pi\right)^2 < \frac{\tau_\theta}{e^\kappa\tau_f+\tau_\theta}. \tag{OA.50}$$

It is then immediate from this bound (OA.50) that

$$\tau_f\left(\alpha_G\underline{\tilde{b}}_\pi\right)^2 - e^\kappa\left(\frac{\tau_\theta}{\tau_\theta+\tau_f}-\alpha_G\underline{\tilde{b}}_\pi\right)^2\left(\frac{1}{\psi}\frac{1-e^{-\kappa}}{1-\alpha_K}\right)^2\tau_\varphi < \tau_f\left(\alpha_G\underline{\tilde{b}}_\pi\right)^2 < \frac{\tau_f\tau_\theta}{e^\kappa\tau_f+\tau_\theta}. \tag{OA.51}$$

It is then again immediate that

$$\frac{\tau_f\tau_\theta}{e^\kappa\tau_f+\tau_\theta} < \frac{\tau_f\tau_\theta}{\tau_f+\tau_\theta} = \left(\tau_\theta^{-1}+\tau_f^{-1}\right)^{-1},$$

and again $-\underline{\tilde{b}}_\pi$ is does not satisfy the condition (OA.40) necessary for a government-centric equilibrium to exist. As such, $\underline{\tilde{b}}_\pi > \tilde{b}_\pi^*$.

We consequently have the cutoff ranking $-\tilde{b}_\pi^* < -\underline{\tilde{b}}_\pi < \underline{b}_\pi < b_\pi^*$. It then follows that for a given b_π, at most one pure equilibrium (i.e., fundamental- or government-centric) exists.

Proof of Proposition 3

We derive the posterior beliefs of the public regarding θ. From the observations of $\log C$ and $\log q$, the public can construct two de-trended linear sufficient statistics:[6]

$$\begin{aligned} z_C &= \frac{1}{\alpha_G b_\pi}\left(\log C_s - \mathbb{E}\left[\log C\right] - \frac{\alpha_G b_q - \alpha_K}{1-\alpha_K}\log q - \frac{\alpha_K}{1-\alpha_K}\sigma_z z_Q\right) \\ &= \theta + \left(1+\frac{1}{\alpha_G b_\pi}\right)\left(f-\bar{f}\right) - \frac{\alpha_K \psi}{\alpha_G b_\pi}\varphi, \qquad \text{(OA.52)} \\ z_Q &= \frac{1-a_G b_q + \psi\left(1-\alpha_K\right)}{\sigma_z}\left(\log q - \mathbb{E}\left[\log q\right]\right) \\ &= a_v \theta + \left(a_s + a_v\right)\left(f-\bar{f}\right) + \psi\varphi. \qquad \text{(OA.53)} \end{aligned}$$

Note that z_Q is equivalent to z_q used before but with π_g now expressed as a linear combination of f and θ. These two statistics, z_C and z_Q, capture all relevant information contained in $\log C$ and $\log q$.

Given that the public have a normal prior about θ, $\theta \sim \mathcal{N}\left(0, \tau_\theta^{-1}\right)$, and observe Gaussian signals z_C and z_Q, given by equations (24) and (25), respectively, their posterior is Gaussian $\theta | z_C, z_Q \sim \mathcal{N}\left(\hat{\theta}, \hat{\tau}_\theta^{-1}\right)$, where

$$\hat{\theta} = \tau_\theta^{-1}\begin{pmatrix} a_v \\ 1 \end{pmatrix}' \Sigma^{-1} \begin{pmatrix} z_Q \\ z_C \end{pmatrix} = \tau_\theta^{-1}\begin{pmatrix} a_v \\ 1 \end{pmatrix}' \Sigma^{-1} \begin{pmatrix} z_Q \\ z_C \end{pmatrix}, \qquad \text{(OA.54)}$$

$$\hat{\tau}_\theta^{-1} = \tau_\theta^{-1} - \tau_\theta^{-2}\begin{pmatrix} a_v \\ 1 \end{pmatrix}' \Sigma^{-1} \begin{pmatrix} a_v \\ 1 \end{pmatrix}, \qquad \text{(OA.55)}$$

and

$$\Sigma = \begin{bmatrix} \left(a_s + a_v\right)^2 \tau_f^{-1} + a_v^2 \tau_\theta^{-1} + \psi^2 \tau_\varphi^{-1} & a_v \tau_\theta^{-1} + \left(a_s + a_v\right)\left(1 + \frac{1}{\alpha_G b_\pi}\right)\tau_f^{-1} - \frac{\alpha_K \psi^2}{\alpha_G b_\pi}\tau_\varphi^{-1} \\ a_v \tau_\theta^{-1} + \left(a_s + a_v\right)\left(1 + \frac{1}{\alpha_G b_\pi}\right)\tau_f^{-1} - \frac{\alpha_K \psi^2}{\alpha_G b_\pi}\tau_\varphi^{-1} & \tau_\theta^{-1} + \left(1+\frac{1}{\alpha_G b_\pi}\right)^2 \tau_f^{-1} + \left(\frac{\alpha_K \psi}{\alpha_G b_\pi}\right)^2 \tau_\varphi^{-1} \end{bmatrix}. \qquad \text{(OA.56)}$$

It is immediate from equation (OA.56) that

$$\tau_\theta^{-2}\begin{pmatrix} a_v \\ 1 \end{pmatrix}' \Sigma^{-1} \begin{pmatrix} a_v \\ 1 \end{pmatrix} = \tau_\theta^{-2}\frac{\left(a_s - \frac{a_v}{\alpha_G b_\pi}\right)^2 \tau_f^{-1} + \left(1 + \frac{\alpha_K a_v}{\alpha_G b_\pi}\right)^2 \psi^2 \tau_\varphi^{-1}}{|\Sigma|}, \qquad \text{(OA.57)}$$

and

$$|\Sigma| = \tau_\theta^{-1}\left(a_s - \frac{a_v}{\alpha_G b_\pi}\right)^2 \tau_f^{-1} + \tau_\theta^{-1}\left(1 + \frac{\alpha_K a_v}{\alpha_G b_\pi}\right)^2 \psi^2 \tau_\varphi^{-1} + \psi^2 \tau_\varphi^{-1}\left(1 + \frac{1}{\alpha_G b_\pi} + \frac{\alpha_K}{\alpha_G b_\pi}\left(a_s + a_v\right)\right)^2 \tau_f^{-1}. \qquad \text{(OA.58)}$$

[6]Because both signals would be flat with respect to π_g when $b_\pi = 0$, the policymaker will never choose $b_\pi = 0$ in equilibrium.

It then follows from equation (OA.57) and (OA.58) that

$$\hat{\tau}_\theta^{-1} = \frac{\psi^2\tau_\varphi^{-1}\left(1+\frac{1}{\alpha_G b_\pi}+\frac{\alpha_K}{\alpha_G b_\pi}\left(a_s+a_v\right)\right)^2\tau_f^{-1}}{\tau_\theta^{-1}\left(a_s-\frac{a_v}{\alpha_G b_\pi}\right)^2\tau_f^{-1}+\tau_\theta^{-1}\left(1+\frac{\alpha_K a_v}{\alpha_G b_\pi}\right)^2\psi^2\tau_\varphi^{-1}+\psi^2\tau_\varphi^{-1}\left(1+\frac{1}{\alpha_G b_\pi}+\frac{\alpha_K}{\alpha_G b_\pi}\left(a_s+a_v\right)\right)^2\tau_f^{-1}}\tau_\theta^{-1},$$

and therefore

$$\hat{\tau}_\theta = \tau_\theta + \frac{\left(\alpha_G b_\pi a_s - a_v\right)^2\frac{\tau_\varphi}{\psi^2}+\left(\alpha_G b_\pi+\alpha_K a_v\right)^2\tau_f}{\left(1+\alpha_G b_\pi+\alpha_K\left(a_s+a_v\right)\right)^2}. \tag{OA.59}$$

Proof of Proposition 4

We follow the following steps: first to examine the policymaker's choice of b_π within a fundamental-centric equilibrium, then her choice within a government-centric equilibrium, and finally examine whether she prefers a fundamental- or government-centric equilibrium.

Step 1: Fundamental-centric Equilibrium

In a fundamental-centric, $\hat{\tau}_\theta$ from equation (OA.59) based on Proposition 2 simplifies to

$$\hat{\tau}_\theta = \tau_\theta + \left(\alpha_G b_\pi\right)^2\left(\frac{1-e^{-\kappa}}{1-\alpha_K e^{-\kappa}}\right)^2\frac{\tau_\varphi}{\psi^2}+\left(\frac{1-\alpha_K}{1-\alpha_K e^{-\kappa}}\right)^2\left(\frac{\alpha_G b_\pi}{1+\alpha_G b_\pi}\right)^2\tau_f. \tag{OA.60}$$

It is immediate that to maximize $\hat{\tau}_\theta$, the policymaker chooses $b_\pi > 0$ as large as possible. Thus, the optimal choice is $\underline{b}_\pi$, the maximum value of b_π that supports a fundamental-centric equilibrium.

Step 2: Government-centric Equilibrium

In a government-centric, $\hat{\tau}_\theta$ from equation (OA.59) based on Proposition 2 simplifies to

$$\hat{\tau}_\theta = \tau_\theta + \left(\frac{\frac{\tau_\theta}{\tau_\theta+\tau_f}+\alpha_G b_\pi}{1+\alpha_G b_\pi-\alpha_K\frac{\tau_f}{\tau_\theta+\tau_f}\frac{1-e^{-\kappa}}{1-\alpha_K e^{-\kappa}}}\right)^2\left(\frac{1-e^{-\kappa}}{1-\alpha_K e^{-\kappa}}\right)^2\frac{\tau_\varphi}{\psi^2}+\left(\frac{\alpha_G b_\pi+\alpha_K\frac{\tau_\theta}{\tau_\theta+\tau_f}\frac{1-e^{-\kappa}}{1-\alpha_K e^{-\kappa}}}{1+\alpha_G b_\pi-\alpha_K\frac{\tau_f}{\tau_\theta+\tau_f}\frac{1-e^{-\kappa}}{1-\alpha_K e^{-\kappa}}}\right)^2\tau_f. \tag{OA.61}$$

It is immediate that to maximize $\hat{\tau}_\theta$, the policymaker chooses $b_\pi > 0$ as large as possible. Because a government-centric equilibrium exists if $b_\pi \geq b_\pi^*$, maximizing b_π is consistent with a government-centric equilibrium.

Step 3: Comparing Fundamental- and Government-centric Equilibria

Notice from comparing equations (OA.60) and (OA.61) that

$$\left(\frac{\alpha_G b_\pi+\alpha_K\frac{\tau_\theta}{\tau_\theta+\tau_f}\frac{1-e^{-\kappa}}{1-\alpha_K e^{-\kappa}}}{1+\alpha_G b_\pi-\alpha_K\frac{\tau_f}{\tau_\theta+\tau_f}\frac{1-e^{-\kappa}}{1-\alpha_K e^{-\kappa}}}\right)^2\tau_f \geq \left(\frac{1-\alpha_K}{1-\alpha_K e^{-\kappa}}\right)^2\left(\frac{\alpha_G b_\pi}{1+\alpha_G b_\pi}\right)^2\tau_f,$$

recognizing that $\frac{1-\alpha_K}{1-\alpha_K e^{-\kappa}} \leq 1$. Consequently, it is sufficient to focus only on the second terms in $\hat{\tau}_\theta$. Note that the policymaker would choose $b_\pi \to \infty$ in government-centric equilibrium, consequently causing the coefficient of the second term to $\left(\frac{1-e^{-\kappa}}{1-\alpha_K e^{-\kappa}}\right)^2 \frac{\tau_\varphi}{\psi^2}$. In the fundamental-centric equilibrium, the policymaker would choose $b_\pi = \underline{b}_\pi$, causing the second term to be $(\alpha_G \underline{b}_\pi)^2 \left(\frac{1-e^{-\kappa}}{1-\alpha_K e^{-\kappa}}\right)^2 \frac{\tau_\varphi}{\psi^2}$. From the proof of Proposition 2, $\alpha_G \underline{b}_\pi \leq \sqrt{\frac{\tau_\theta}{\tau_\theta+\tau_f}} < 1$ in a fundamental-centric equilibrium. Thus, $\hat{\tau}_\theta$ is higher in the government-centric equilibrium.

Consequently, the policymaker maximizes $\hat{\tau}_\theta$ by choosing a government-centric over a fundamental-centric equilibrium. It is immediate by continuity that such arguments also exclude a mixed equilibrium as being optimal.

Proof of Proposition 5

Step 1: Optimal Choice of b_0

With some manipulation of the social welfare objective (29), the first-order condition for the optimal choice of b_0 is

$$e^{\mathbb{E}[\log C_i]+\frac{1}{2}Var[\log C_i]}\left(e^{-\frac{\gamma}{2}Var[\log C_i]} - \frac{\psi\alpha_K}{1+\psi}e^{-\gamma Var\left[f+\alpha_G b_\pi \pi_g \mid \mathcal{I}_i\right]}\right) = \frac{1+\psi\left(1-\alpha_K\right)}{\left(1+\psi\right)\alpha_G} R_G e^{\mathbb{E}[\log G]+\frac{1}{2}Var[\log G]}. \tag{OA.62}$$

from which we can derive b_0 explicitly. Let $\mathbb{E}\left[\widehat{\log C_i}\right]$ be $\mathbb{E}\left[\log C_i\right]$ from equation (OC.7) without its b_0 term, and similarly for $\mathbb{E}\left[\widehat{\log qK}\right]$ and $\mathbb{E}\left[\widehat{\log G}\right]$. Then, the optimal b_0 is

$$\begin{aligned} b_0 \quad = \quad & \frac{1-a_G b_q + \psi\left(1-\alpha_K\right)}{1-\alpha_G}\left[\mathbb{E}\left[\widehat{\log C_i}\right] - \mathbb{E}\left[\widehat{\log G}\right] + \log\left(\frac{1+\psi}{1+\psi\left(1-\alpha_K\right)}\frac{\alpha_G}{R_G}\right)\right. . \\ & \left. + \log\left(\frac{e^{-\frac{\gamma}{2}Var[\log C_i]} - \frac{\psi\alpha_K}{1+\psi}e^{-\gamma Var\left[f+\alpha_G b_\pi\pi_g \mid \mathcal{I}_i\right]}}{e^{\frac{1}{2}Var[\log G]-\frac{1}{2}Var[\log C]}}\right)\right] \end{aligned} \tag{OA.63}$$

In what follows, we define

$$A = \frac{\alpha_G \mathbb{E}\left[C_i\right]}{R_G \mathbb{E}\left[G\right]} e^{-\frac{\gamma}{2}Var[\log C_i]}.$$

In the special case $\gamma = 0$ (i.e., households are risk-neutral), equation (OA.63) implies $\alpha_G \mathbb{E}\left[C_i\right] = R_G \mathbb{E}\left[G\right]$ and $A = 1$. Otherwise, by definition from equation (OA.62), we recognize

$$A \geq \frac{1+\psi\left(1-\alpha_K\right)}{1+\psi}. \tag{OA.64}$$

Step 2: Optimal Choices of b_π and b_q

With respect to b_π and b_q, we can manipulate their first-order necessary conditions with equations (OA.63), (OC.4), and (OC.7) to express them as

$$0 = 2\partial_{b_\pi}\mathbb{E}\left[\frac{1+\psi\left(1-\alpha_K\right)}{1+\psi}\log C_i - \alpha_G \log G\right] + \left(\frac{1+\psi\left(1-\alpha_K\right)}{1+\psi} - \gamma A\right)\partial_{b_\pi} Var\left[\log C_i\right], \tag{OA.65}$$

$$- \alpha_G \partial_{b_\pi} Var\left[\log G\right] + 2\gamma B \partial_{b_\pi} Var\left[f + \alpha_G b_\pi \pi_g \mid \mathcal{I}_i\right],$$

and

$$\left(\frac{1+\psi\left(1-\alpha_K\right)}{1+\psi}-\gamma A\right)\partial_{b_q} Var\left[\log C_i\right]-\alpha_G \partial_{b_q} Var\left[\log G\right]=0, \tag{OA.66}$$

where

$$\begin{aligned}\mathbb{E}\left[\frac{1+\psi\left(1-\alpha_K\right)}{1+\psi}\log C_i-\alpha_G\log G\right]=\bar{f}+\frac{\psi\alpha_K}{1+\psi}\left(\log\alpha_K+\frac{1-2\gamma}{2}Var\left[f+\alpha_G b_\pi\pi_g\middle|\,\mathcal{I}_i\right]\right)\\-\frac{\alpha_K}{1+\psi}\frac{1}{2}\left(a_s^2\tau_s^{-1}+a_v^2\tau_v^{-1}-\psi^2\tau_{\varphi\epsilon}^{-1}\right),\end{aligned} \tag{OA.67}$$

and

$$B=A-\frac{1+\psi\left(1-\alpha_K\right)}{1+\psi}\geq 0, \tag{OA.68}$$

because $A\geq\frac{1+\psi(1-\alpha_K)}{1+\psi}$.

Substituting with equations (OA.67) and (OA.68) into equation (OA.65)

$$\begin{aligned}0=\left(\frac{\psi\alpha_K}{1+\psi}+2\gamma\left(A-1\right)\right)\partial_{b_\pi}Var\left[f+\alpha_G b_\pi\pi_g\middle|\,\mathcal{I}_i\right]-\frac{\alpha_K}{1+\psi}\partial_{b_\pi}\left(a_v^2\tau_v^{-1}\right)\\+\left(\frac{1+\psi\left(1-\alpha_K\right)}{1+\psi}-\gamma A\right)\partial_{b_\pi}Var\left[\log C_i\right]-\alpha_G\partial_{b_\pi}Var\left[\log G\right].\end{aligned} \tag{OA.69}$$

Step 3: Log-linear Approximating a Government-centric Equilibrium

Consider a log-linear approximation of the welfare objective around $\gamma=0$ in which case $A=1$. We focus on the first-order conditions for b_π and b_q. Let $X_s=0$ and $X_q=0$ be the left-hand side of equations (OA.69) and (OA.66) when $A=1$, respectively. Because welfare will be twice continuously differentiable, notice equations (OA.69) and (OA.66) imply when $A=1$

$$\partial_{b_q}X_s=0, \tag{OA.70}$$

and

$$\partial_\gamma X_s=-\partial_{b_\pi}Var\left[\log C\right]. \tag{OA.71}$$

Let the left-hand side of equation (OA.69) be X_q. Invoking the Implicit Function Theorem for b_π and b_q

$$\begin{bmatrix}\partial_\gamma b_\pi\\ \partial_\gamma b_q\end{bmatrix}=-\begin{bmatrix}\partial_{b_\pi}X_s & \partial_{b_q}X_s\\ \partial_{b_\pi}X_q & \partial_{b_q}X_q\end{bmatrix}^{-1}\begin{bmatrix}\partial_\gamma X_s\\ \partial_\gamma X_q\end{bmatrix}, \tag{OA.72}$$

from which follows, because $\partial_{b_q}X_s=0$, that

$$\partial_\gamma b_\pi=-\frac{\partial_\gamma X_s}{\partial_{b_\pi}X_s}, \tag{OA.73}$$

where Δ is the determinant of the matrix in equation (OA.72). If the government's problem has a unique local maximum, this matrix must be negative definite everywhere, and consequently its eigenvalues must all be negative. Because the eigenvalues of a triangular matrix are (proportional to) its diagonal entries, it follows that $\partial_{b_\pi}X_s<0$. Consequently, this and equation (OA.71) imply

$$\partial_\gamma b_\pi\propto-\partial_{b_\pi}Var\left[\log C_i\right].$$

We consequently focus on $Var\left[\log C_i\right]$ and make use of the following Lemma.

Lemma 8. *7 In a government-centric equilibrium, if $b_\pi < \hat{b}^*_\pi$, then $\partial_{b_\pi} Var\left[\log C_i\right] < 0$, while if $b_\pi > b^*_\pi$, then $\partial_{b_\pi} Var\left[\log C_i\right] > 0$.*

As a consequence of the lemma, if $b_\pi < \hat{b}^*_\pi$, then $\partial_\gamma b_\pi > 0$, while if $b_\pi > b^*_\pi$, then $\partial_\gamma b_\pi < 0$. It is then immediate that if γ is sufficiently large, then the optimal choice of b_π is below b^*_π and above $\hat{b}^*_\pi$. As such, it follows that if γ is sufficiently large, then the government's optimal policy does not induce a government-centric equilibrium.

Although we resort to a linear approximation, numerical analyses suggest this approximation is reasonable. The variable A is typically close to unity in the fully nonlinear model if τ_f and τ_φ are modestly large.

Proof of Lemma 7

In what follows, we focus on a government-centric equilibrium in which we can rewrite σ_z as

$$\sigma_z = \frac{1-\alpha_K}{1-e^{-\kappa}}\left(1 - \frac{e^{-\kappa}\psi^2\tau_\varphi^{-1}}{a_v^2\left(\tau_f^{-1}+\tau_\theta^{-1}\right)+\psi^2\tau_\varphi^{-1}}\right). \tag{OA.74}$$

It is immediate σ_z is increasing in $|b_\pi|$ and $\sigma_z \in \left[1-\alpha_K, \frac{1-\alpha_K}{1-e^{-\kappa}}\right]$.

In addition, in a government-centric equilibrium, $\partial_{b_\pi} Var\left[\log C\right]$ is given by

$$\begin{aligned}
&\frac{\partial_{b_\pi} Var\left[\log C\right]}{2\alpha_G\left(\tau_f^{-1}+\tau_\theta^{-1}\right)} \qquad\qquad \text{(OA.75)}\\
&= \left(\begin{array}{c}\left(1+\frac{a_G b_q+\psi\alpha_K}{1-\alpha_G b_q+\psi(1-\alpha_K)}\frac{a_v^2\left(\tau_f^{-1}+\tau_\theta^{-1}\right)+\left(\frac{1-\alpha_K e^{-\kappa}}{1-\alpha_K}\sigma_z-\alpha_K\right)\frac{1-e^{-\kappa}}{1-\alpha_K}\psi^2\tau_\varphi^{-1}}{a_v^2\left(\tau_f^{-1}+\tau_\theta^{-1}\right)+\psi^2\tau_\varphi^{-1}}\right)^2\\
+\frac{2e^{-\kappa}}{1-e^{-\kappa}}\left(\frac{\alpha_K\frac{1-e^{-\kappa}}{1-\alpha_K}\psi^2\tau_\varphi^{-1}}{\psi^2\tau_\varphi^{-1}+a_v^2\left(\tau_\theta^{-1}+\tau_f^{-1}\right)}\right)^2\end{array}\right)\frac{1-\alpha_K}{1-e^{-\kappa}}a_v\\
&+\left(\frac{a_G b_q+\psi\alpha_K}{1-\alpha_G b_q+\psi\left(1-\alpha_K\right)}\right)^2\frac{2e^{\kappa}\frac{1-\alpha_K e^{-\kappa}}{1-\alpha_K}-1-\left(\frac{1-\alpha_K e^{-\kappa}}{1-\alpha_K}\frac{\psi^2\tau_\varphi^{-1}}{a_v^2\left(\tau_f^{-1}+\tau_\theta^{-1}\right)+\psi^2\tau_\varphi^{-1}}\right)^2}{a_v^2\left(\tau_f^{-1}+\tau_\theta^{-1}\right)+\psi^2\tau_\varphi^{-1}}\left(e^{-\kappa}\psi^2\tau_\varphi^{-1}\right)^2\frac{1-\alpha_K}{1-e^{-\kappa}}a_v.
\end{aligned}$$

Because $e^\kappa > \frac{1-\alpha_K e^{-\kappa}}{1-\alpha_K}$, we have

$$\begin{aligned}
2e^{\kappa}\frac{1-\alpha_K e^{-\kappa}}{1-\alpha_K}-1-\left(\frac{1-\alpha_K e^{-\kappa}}{1-\alpha_K}\frac{\psi^2\tau_\varphi^{-1}}{a_v^2\left(\tau_f^{-1}+\tau_\theta^{-1}\right)+\psi^2\tau_\varphi^{-1}}\right)^2 &\geq 2e^{\kappa}\frac{1-\alpha_K e^{-\kappa}}{1-\alpha_K}-1-e^{\kappa}\frac{1-\alpha_K e^{-\kappa}}{1-\alpha_K}\\
&\geq e^{\kappa}\frac{1-\alpha_K e^{-\kappa}}{1-\alpha_K}-1>0.
\end{aligned}$$

It then follows that if $\alpha_G b_\pi \leq \alpha_G \hat{b}^*_\pi < -\frac{\tau_\theta}{\tau_f+\tau_\theta}$, and $a_v < 0$, then $\partial_{b_\pi} Var\left[\log C\right] < 0$, while if $\alpha_G b_\pi > \alpha_G b^*_\pi > \frac{\tau_\theta}{\tau_f+\tau_\theta}$ and $a_v > 0$, then $\partial_{b_\pi} Var\left[\log C\right] > 0$.

Online Appendix B: An Investment Game

For simplicity, the main paper assumes government provision of infrastructure. In this online appendix, we provide a microfoundation, showing how government intervention addresses potential coordination failures among firms.

Suppose each firm invests in two types of capital to produce output: private capital K_i, as in the baseline model, and public capital g_i, shared by all firms and priced at R_G. The production function is

$$Y_i = F\left(\int_0^1 g_j dj\right)^{\alpha_G} K_i^{\alpha_K},$$

and firm profits are

$$\Pi_i = e^f\left(\int_0^1 g_j dj\right)^{\alpha_G} K_i^{\alpha_K} - qK_i - R_G g_i,$$

where qK_i is the cost of purchasing capital from capital providers.

As in the main model, households have constant relative risk aversion preferences for consumption:

$$u\left(C_i\right) = \frac{C_i^{1-\gamma}}{1-\gamma}, \text{ for } \gamma \in [0, 1/\alpha_K),$$

where consumption $C_i = \Pi_i + \tau_i$. As owners of capital providers, each household receives the cost of capital back from its firm, making household consumption:

$$C_i = e^f\left(\int_0^1 g_j dj\right)^{\alpha_G} K_i^{\alpha_K}. \tag{OB.1}$$

The household holds undiversified risk in the firm. At date 1, it values firm profit as $\mathbb{E}\left[\Lambda_i \Pi_i | \mathcal{I}_i\right]$, where $\Lambda_i = \lambda_i \frac{u'(C_i)}{\mathbb{E}[u'(C_i)]}$ is the stochastic discount factor, with λ_i as a constant and $u'\left(C_i\right)$ as the marginal utility.

The firm maximizes:

$$\max_{K_i, g_i} \mathbb{E}\left[\Lambda_i \Pi_i | \mathcal{I}_i\right] = \max_{K_i, g_i} \mathbb{E}\left[\Lambda_i\left(e^f\left(\int_0^1 g_j dj\right)^{\alpha_G} K_i^{\alpha_K} - qK_i - R_G g_i\right) | \mathcal{I}_i\right], \tag{OB.2}$$

where $\mathcal{I}_i$ is the firm's information set.

The first-order necessary condition for the optimal choice of public capital, g_i, is

$$\alpha_G \mathbb{E}\left[\Lambda_i Y_i | \mathcal{I}_i\right]\left(\int_0^1 g_j dj\right)^{-1} dj - \mathbb{E}\left[\Lambda_i | \mathcal{I}_i\right] R_G \leq 0 \;\; (= \text{ if } g_i > 0).$$

Since each firm bears the full cost, R_G, of the public capital it supplies but receives only dj of the benefit, the privately optimal choice for g_i is zero. As public capital benefits all firms, no firm will supply it privately. However, if all firms collectively invested G in public capital, they would each pay $R_G G$ and receive G units of public capital in their production functions. This creates a coordination failure because the positive externalities of public capital benefit other firms.

This coordination failure justifies government provision of public capital G, or public infrastructure. To finance its cost, $R_G G$, the government levies taxes on households, which are incorporated into the social welfare function in Section 3.3.

Online Appendix C: Additional Expressions

In this online appendix, we provide explicit expressions for the first and second moments of log output, capital expenditure and government infrastructure.

Define

$$\sigma_z = \frac{(1+\alpha_G b_\pi)(a_s+a_v)\tau_f^{-1} + \alpha_G b_\pi a_v \tau_\theta^{-1} + (1-\alpha_K)\psi^2\tau_\varphi^{-1}}{(a_s+a_v)^2\tau_f^{-1} + a_v^2\tau_\theta^{-1} + \psi^2\tau_\varphi^{-1}},$$

and

$$\epsilon_q = a_s z_q - (a_s+a_v)\bar{f},$$

to be the innovation to the log capital price $\log q$ relative to its mean. Then,

$$\hat{f} + \alpha_G b_\pi \hat{\pi}_g - (1-\alpha_K)a_s\hat{f} - (1-\alpha_K)a_v\hat{\pi}_g + (1-\alpha_K)a_s z_q = (1+\alpha_G b_\pi)\bar{f} + \sigma_z\epsilon_q.$$

Then, we can rewrite the price of capital from equation (OA.13) as

$$\begin{aligned}\log q \quad = \quad & \frac{1}{1-a_G b_q + \psi(1-\alpha_K)}\left[(1+\alpha_G b_\pi)\bar{f} + \sigma_z\epsilon_q + \log\alpha_K + \alpha_G b_0\right. \\ & \left. + \frac{1-2\gamma}{2}(1-\alpha_K)\left(a_s\tau_s^{-1} + \alpha_G b_\pi a_v\tau_v^{-1}\right) + \frac{1-\alpha_K}{2}\left(a_s^2\tau_s^{-1} + a_v^2\tau_v^{-1} - \psi^2\tau_{\varphi\epsilon}^{-1}\right)\right],\end{aligned} \tag{OC.1}$$

firm capital from equation (17) as

$$\begin{aligned}\log K_i \quad = \quad & \frac{1}{1-\alpha_G b_q + \psi(1-\alpha_K)}\left\{\left(\alpha_G b_q - 1\right)\frac{1}{2}\left(a_s^2\tau_s^{-1} + a_v^2\tau_v^{-1} - \psi^2\tau_{\varphi\epsilon}^{-1}\right)\right. \\ & \left. +\psi\left[\left((1+\alpha_G b_\pi)\bar{f} + \sigma_z\epsilon_q\right) + \log\alpha_K + \alpha_G b_0 + \frac{1-2\gamma}{2}(1-\alpha_K)\left(a_s\tau_s^{-1} + \alpha_G b_\pi a_v\tau_v^{-1}\right)\right]\right\} \\ & -\psi\varphi + a_s\varepsilon_{si} + a_v\varepsilon_{vi},\end{aligned} \tag{OC.2}$$

and government infrastructure as

$$\log G = \frac{1+\psi\left(1-\alpha_K\right)}{1-\alpha_G b_q+\psi\left(1-\alpha_K\right)} b_0 + b_\pi \pi_g + \frac{b_q\left(\left(1+\alpha_G b_\pi\right)\bar{f}+\sigma_z \epsilon_q\right)}{1-\alpha_G b_q+\psi\left(1-\alpha_K\right)}.$$

$$+ b_q \frac{\log \alpha_K + \frac{1-2\gamma}{2}\left(1-\alpha_K\right)\left(a_s\tau_s^{-1}+\alpha_G b_\pi a_v \tau_v^{-1}\right)+\frac{1-\alpha_K}{2}\left(a_s^2\tau_s^{-1}+a_v^2\tau_v^{-1}-\psi^2\tau_{\varphi\epsilon}^{-1}\right)}{1-\alpha_G b_q+\left(1-\alpha_K\right)} \quad \text{(OC.3)}$$

These expressions imply

$$\mathbb{E}\left[\log G\right] = \frac{1+\psi\left(1-\alpha_K\right)}{1-\alpha_G b_q+\psi\left(1-\alpha_K\right)} b_0 + b_\pi \bar{f} + \frac{b_q\left(1+\alpha_G b_\pi\right)}{1-\alpha_G b_q+\psi\left(1-\alpha_K\right)}\bar{f}$$

$$+ b_q \frac{\log \alpha_K + \frac{1-2\gamma}{2}\left(1-\alpha_K\right)\left(a_s\tau_s^{-1}+\alpha_G b_\pi a_v \tau_v^{-1}\right)+\frac{1-\alpha_K}{2}\left(a_s^2\tau_s^{-1}+a_v^2\tau_v^{-1}-\psi^2\tau_{\varphi\epsilon}^{-1}\right)}{1-\alpha_G b_q+\psi\left(1-\alpha_K\right)}, \quad \text{(OC.4)}$$

and

$$Var\left[\log G\right] = \left(b_\pi + \frac{b_q}{1-\alpha_G b_q+\psi\left(1-\alpha_K\right)}\sigma_z\left(a_s+a_v\right)\right)^2 \tau_f^{-1} + \left(b_\pi + \frac{b_q}{1-\alpha_G b_q+\psi\left(1-\alpha_K\right)}\sigma_z a_v\right)^2 \tau_\theta^{-1}$$

$$+ \left(\frac{b_q}{1-\alpha_G b_q+\psi\left(1-\alpha_K\right)}\sigma_z\right)^2 \psi^2 \tau_\varphi^{-1}. \quad \text{(OC.5)}$$

Substituting these expressions into household consumption equation (2), which in aggregate is equal to output Y, we also have

$$\log C_i = \left(1+\alpha_K a_s\right)\left(f-\bar{f}\right) + \left(\alpha_G b_\pi + \alpha_K a_v\right)\left(\pi_g - \bar{f}\right) + \alpha_K\left(\frac{\sigma_z}{1-\alpha_K}-1\right)\epsilon_q + \alpha_K a_s \varepsilon_{si} + \alpha_K a_v \varepsilon_{vi}$$

$$+ \frac{\alpha_G b_q - \alpha_K}{1-\alpha_K}\log q + \frac{1+\alpha_G b_\pi}{1-\alpha_K}\bar{f} + \frac{\alpha_K \log \alpha_K + \alpha_G b_0}{1-\alpha_K} + \frac{1-2\gamma}{2}\alpha_K\left(a_s\tau_s^{-1}+\alpha_G b_\pi a_v \tau_v^{-1}\right), \quad \text{(OC.6)}$$

from which follows

$$\mathbb{E}\left[\log C_i\right] = \frac{\left(1+\psi\right)\alpha_G b_0 + \left(1+\psi\right)\left(1+\alpha_G b_\pi\right)\bar{f} + \left(\alpha_G b_q - \alpha_K\right)\frac{1}{2}\left(a_s^2\tau_s^{-1}+a_v^2\tau_v^{-1}-\psi^2\tau_{\varphi\epsilon}^{-1}\right)}{1-\alpha_G b_q+\psi\left(1-\alpha_K\right)}$$

$$+ \frac{\alpha_K\psi+\alpha_G b_q}{1-\alpha_G b_q+\psi\left(1-\alpha_K\right)}\left(\log \alpha_K + \frac{1-2\gamma}{2}\left(1-\alpha_K\right)\left(a_s\tau_s^{-1}+\alpha_G b_\pi a_v \tau_v^{-1}\right)\right), \quad \text{(OC.7)}$$

and

$$\begin{aligned}Var\left[\log C_i\right] = & \left(1+\alpha_G b_\pi + \frac{\alpha_G b_q + \psi\alpha_K}{1-\alpha_G b_q + \psi\left(1-\alpha_K\right)}\sigma_z\left(a_s+a_v\right)\right)^2 \tau_f^{-1} \\ & + \left(\alpha_G b_\pi + \frac{\alpha_G b_q + \psi\alpha_K}{1-\alpha_G b_q + \psi\left(1-\alpha_K\right)}\sigma_z a_v\right)^2 \tau_\theta^{-1} \\ & + \left(\frac{\alpha_G b_q + \psi\alpha_K}{1-\alpha_G b_q + \psi\left(1-\alpha_K\right)}\sigma_z - \alpha_K\right)^2 \psi^2\tau_\varphi^{-1} + \alpha_K^2\left(a_s^2\tau_s^{-1} + a_v^2\tau_v^{-1}\right). \end{aligned} \tag{OC.8}$$

Notice the first-order condition for optimal capital is

$$qK_i = \alpha_K \mathbb{E}\left[C_i e^{-\gamma Var\left[f+\alpha_G b_\pi \pi_g \mid \mathcal{I}_i\right]}\right],$$

from which it is immediate by the Law of Iterated Expectations and the linearity of the integral operator

$$\mathbb{E}\left[qK\right] = \alpha_K \mathbb{E}\left[\int_0^1 C_i e^{-\gamma Var\left[f+\alpha_G b_\pi \pi_g \mid \mathcal{I}_i\right]} di\right] = \alpha_K \mathbb{E}\left[C_i\right] e^{-\gamma Var\left[f+\alpha_G b_\pi \pi_g \mid \mathcal{I}_i\right]}. \tag{OC.9}$$