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ABSTRACT

The optimal income taxation problem has been extensively studied in one-period models. This paper analyzes optimal income taxation when consumers work for many periods. We also analyze what information, if any, that the government learns about abilities in one period can be used in later periods to attain more redistribution than in a one-period world. When the government must commit itself to future tax schedules, intertemporal nonstationarity of tax schedules could relax the self-selection constraints and lead to Pareto improvements. The effect of nonstationarity is analogous to that of randomization in one-period models. The use of information is limited since only a single lifetime self-selection constraint for each type of consumer exists. These results hold when individuals and the government have the same discount rates. The planner can make additional use of the information when individual and social rates of time discounting differ. In this case, the limiting tax schedule is a nondistorting one if the government has a lower discount rate than individuals.

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I. INTRODUCTION

Governments make some, but only limited, use of age-dependent income tax schedules. In the U.S., the elderly benefit from a larger standard deduction in the Federal income tax, and the subsidies provided through the Social Security system (which depend on individual earnings histories) can be viewed as an age-dependent transfer.

At first glance, this seems anomalous. Surely by the time an individual is, say, age 50, the government has accumulated an enormous amount of information about him. The government could make use of an individual's lifetime work history in designing tax schedules. This argument seems particularly forceful under the now widely accepted argument that limited information about individual abilities (combined with distributional objectives) provides the reason for distortionary taxation.

Upon further reflection, this failure may not be quite so anomalous. For if individuals knew that future tax rates would depend on current income, it would affect their current behavior. While the government might be able to use the information accumulated during an individual's lifetime to reduce the distortion in later years, doing so would increase the distortion in earlier years.

Indeed what appears more anomalous is that individuals act as if they assume that the government will not base future taxes on their current income, even though there is no explicit commitment to this. But this is also less of a paradox than it first appears: individuals save, even though the government has not committed itself not to confiscate their wealth. The government's desire to maintain a reputation can lead it to act as if it could make binding commitments not to use information in one period to engage in first best lump sum taxation in later periods. We analyze here the intertemporal structure of income taxation when the government can commit itself to lifetime tax schedules. It is easy to show that, with the possibility of commitment, optimal tax schedules will immediately lead individuals to reveal their ability levels. This information is of no value to the government since it has, in effect, committed itself not to use that information.

With preferences and productivity unchanged over time and the usual assumptions of quasi-concave indifference curves, we might expect the mathematics simply to confirm the optimality of what is commonly observed: tax schedules that do not vary with age. But in many cases, this does not hold. Upon further reflection, this result should not have been unexpected. Even with concave utility functions, the self-selection constraints introduce a nonconvexity. Just as earlier one-period analyzes had suggested that the optimum entailed random taxation (see, for example, Stiglitz [1982]), the optimum may entail nonstationary tax schedules. By varying tax rates over time, the government can simulate the outcome that would have occurred with randomization.

When the government can commit itself, we show:

(1) If first best is not optimal in a one period model, it is not optimal in later periods of a multi-period model.

(2) The self-selection constraint creates a potential nonconvexity which implies that sometimes it may be desirable to have different tax functions in different periods.

(3) When the tax structure differs across periods, the information from the first period incorporated in later periods' schedules is only an individual's ability class. The planner uses this to incorporate a large

penalty in later periods for an agent who acts in the second period as if his ability differs from that revealed in the first, thus forcing agents to be consistent in their behavior across periods.

These results follow from noting that a multiperiod optimal tax model and a one period model with random taxation as discussed in Stiglitz [1982] are essentially the same. The analogy between random taxation and nonstationary taxation is suggestive, but there are differences between the two. The random tax model has no restrictions on the relative frequencies with which different schedules are offered. With an infinite horizon, it is possible to duplicate exactly the random solution. However, in a finite period model, restrictions exist on the frequency with which bundles can be offered over time. In addition, the multiperiod model may be more restrictive if it has period-by-period government budget constraints instead of one intertemporal constraint. Despite these extra restrictions, similar arguments show that different tax schedules in different periods may be desirable. Intertemporal nonstationarity is an alternative method of implementing random taxes that does not require either ex ante or ex post violations of horizontal equity.

The discussion above applies when the government discounts the future at the same rate as individuals. When the government values the future more than individuals, more systematic use of age-dependent tax schedules than just mimicking randomization is possible. Over time, the government can approach the first-best allocation with nondistortionary redistribution.

To some readers, randomization may seem a mathematical curiosity, of limited interest to policy design.[1] However, some of the arguments against implementing randomization are less serious criticisms against nonstationarity of tax schedules. With the latter, as we have noted, it is still possible not to violate ex post horizontal equity. Intertemporal changes in the tax schedule only require keeping track of individuals' ability types, which may be far simpler than implementing a randomization procedure which needs to be perceived as fair and appropriate. Furthermore, the gains from using random taxes may well be large; Brito et al. [1986b] give an example in which these gains may be equivalent to as much as 12% of aggregate resources. With this in mind, one does not wish lightly to dismiss the use of age-dependent tax schedules.

Section II presents the basic model and summarizes bothsimple characterization results for efficient tax structures and necessary and sufficient conditions for randomization to be desirable. Section III presents our results for the model with equal discount rates. Section IV considers the problem when the government has a different discount rate from individuals and Section V contains our conclusions.

II. THE INTERTEMPORAL MODEL

Consider a society composed of two different classes of individuals denoted A and B. The individuals within each class are identical but the two classes differ either in tastes or abilities. The government is assumed initially not to know to which class any individual belongs but to know the numbers of individuals in each class, denoted N_i, i = a, b. Individuals discount future utility at the rate ρ. In each period individuals consume a single good, C, and earn income, Y. Within any period, all members of each class have the same utility function over these bundles $V^i(C, Y)$, i = a, b, with $\partial V^i/\partial C \equiv V^i_c > 0$ and $\partial V^i/\partial Y \equiv V^i_y < 0$.[2] Individuals live for M periods and have a lifetime utility function $\hat{V}^i = \sum_{t=1}^{M} \rho^{t-1} V^i$, i=a,b.

The maximum income that individuals in each class can earn per period is bounded from above by K^i so that $Y^i \leq K^i$, $i = a, b$. The marginal rate of substitution for individual i is denoted $MRS^i(C, Y) \equiv -V^i_y/V^i_c > 0$, $i = a, b$.

The following assumptions are made about $V^i(C, Y)$:

(A1) $V^i(C, Y)$, $i = a, b$, is twice continuously differentiable in C and Y;

(A2) $V^i(C, Y)$, $i = a, b$, is strictly concave in C and Y;

(A3) $MRS^a(C, Y)$ and $MRS^b(C, Y)$ differ at almost every (C, Y) bundle and, in particular, the indifference curves are not tangent to the no-tax budget line at the same bundle.

Assumption (A1) is made for convenience in exposition but can be relaxed without difficulty. Assumption (A2) of concavity (instead of quasiconcavity) insures that lifetime discounted utility describes convex preferences. Assumption (A3) is crucial to guarantee that the groups actually have different preferences since, if their indifference curves coincide, redistribution between groups is impossible. Assumption (A3) allows multiple crossings of the indifference curves. With multiple crossings, there will be bundles at which the indifference curves of the two groups are tangent, having equal marginal rates of substitution. Such tangencies are not ruled out as long as they form discrete curves in the (C, Y) plane.[3] A special case satisfying these assumptions is that considered in Stiglitz [1982] in which the utility functions $V^i(C, Y)$ arise from common underlying preferences over consumption and labor with the classes A and B having different abilities (and wages). Indifference curves of the classes cross only once.[4] As shown in Brito, et al. [1988a], such a "single crossing" assumption is unnecessary for much of the characterization of Pareto efficient tax schedules.

A major theme of this paper is that nonstationary tax schedules may be used by the government to mimic the effects of randomization in the single period problem. Baron and Besanko [1984] and Laffont and Tirole [1988] report that, with commitment by the principal, nonstationary reward schedules are of no value. However, they use particular functional forms for which randomization is not desirable in one-shot settings. We consider general utility functions, including those for which randomization may be useful in single-period applications.

Brito et al. [1988b] present necessary and sufficient conditions for randomization and characterize the optimal randomization in the single period version of this model. Our results there include the following:

1) if consumers are sufficiently risk-averse, there exists a randomization scheme which preserves horizontal equity;
2) with ordinal preferences for the two groups which are nearly identical, no randomization is desirable unless one group is much more risk averse than the other;
3) the optimal randomization scheme requires at most three distinct consumption bundles to be included in the lottery offered one type of consumers; and
4) if a local randomization is desirable with some probability vector over three bundles, then for any other probability triple, there exists three bundles such that randomization over those bundles at the given probabilities is feasible and improves on the deterministic solution.[5]

Individuals are unable to save or to borrow across periods and thus face M separate budget constraints. This assumption is made to focus purely on the role of information transfer across periods in affecting taxation without complicating the analysis with possibility of wealth or interest taxation. In the first period, every individual faces the same tax function $T^1(Y_1)$ since the government has no basis upon which to distinguish individuals. Thereafter, the government can recall the incomes reported in previous periods by that individual and can condition the tax functions on previous periods' income. Thus, the tax function in period t > 1, is written as $T^t(Y_t|Y_1,\ldots,Y_{t-1})$. The government cannot condition current taxes on current or past behavior of others.[6] Taking the sequence of tax functions as given, each individual chooses lifetime consumption and income to solve the following maximization:

$$\underset{(C_i,Y_i)}{\text{Max}} \quad \sum_{t=1}^{M} \rho^{t-1} V^i(C_t^i, Y_t^i)$$

$$\text{s.t.} \quad C_1^i \leq Y_1^i - T^1(Y_1^i)$$

$$C_t^i \leq Y_t^i - T^t(Y_t^i|Y_1^i,\ldots,Y_{t-1}^i),\ t = 2,\ldots,M$$

The solution gives lifetime consumption and income vectors as functions of the vector of tax functions, $\hat{C}^i(T^1,\ldots,T^M)$, $\hat{Y}^i(T^1,\ldots,T^M)$.

Given the choices by individuals in each class and subject to budget balance requirements, the government chooses the set of tax functions to maximize social welfare (to be defined below). We can transform the decision on tax functions into a choice of lifetime consumption-income vectors for each class with $(\hat{C}^a, \hat{Y}^a)$ and $(\hat{C}^b, \hat{Y}^b)$ sustainable by a system of tax functions if and only if lifetime self-selection constraints are satisfied for each class. There is only one lifetime constraint for each class and not period by period self-selection constraints. In the first period, individuals decide whether or not to reveal their type when choosing their current income based on the entire lifetime consequences that follow. If individuals reveal their type in the first period, the government knows thereafter who they are and can prevent them from acting as if they belonged to a different class. Any attempted deviation could be punished by imposition of a large penalty. After revealing through first period choices, individuals in later periods can no longer choose any bundle other than the one the government desires them to consume. Hence, after the first period, the government's choices are constrained only because of its initial commitment. It is well known that in the single period problem, optimality requires separation, that is, the government chooses tax schedules such that individuals reveal their types. Because the government can commit itself, and separation is desirable in a single period problem, separation will occur in every period.

We consider two separate budget balance requirements for the government. One possibility is that the government has a single multiperiod budget constraint $\sum_{t=1}^{M} \delta^{t-1}[N_a T_a^t + N_b T_b^t] \geq 0$, where T_i^t is the tax revenue raised from group i in period t and δ is the discount factor faced by the government. Alternatively, the government could be required to balance its budget separately in each period with $N_a T_a^t + N_b T_b^t \geq 0$, $t = 1,\ldots,M$. Clearly, the second is a tighter constraint on the government's choices. It reflects more closely the goal of considering pure information transfer between periods and is consistent with individuals' inability to save. On the other hand, the single multiperiod constraint is justifiable if the government has possibilities not available to individuals, such as a storage technology feasible only on a large scale or access to a world market closed to trade by individuals.[7]

The government maximizes the present discounted value of a weighted sum of lifetime utilities where the weights α and $(1 - \alpha)$ are arbitrary and can vary to change the distribution between the groups. The government's discount factor δ need not equal that of individuals. When they are equal $(\delta = \rho)$, the government's maximization corresponds to finding the multiperiod Pareto frontier as α varies from 0 to 1. When they differ $(\delta \neq \rho)$, the problem is no longer a Pareto problem since the government does not respect individuals' intertemporal preferences. While much literature analyzes why private and social discount rates could differ, these do not constitute our major reason for considering $\delta \neq \rho$. The major focus is on the case of equal discount rates. Allowing them to differ gives rise to a case which serves as a useful benchmark for comparison when discussing the uses of information in the optimal tax structure. The use of information across periods when $\delta = \rho$ is much less systematic than when $\delta \neq \rho$.

We consider two maximization problems, (P_1) and (P_2), depending upon which budget constraint is used. For the single multiperiod constraint the problem is:[8]

$$(P_1) \quad \text{Max} \sum_{t=1}^{M} \delta^{t-1}[\alpha N_a V^a(C_t^a, Y_t^a) + (1 - \alpha)N_b V^b(C_t^b, Y_t^b)]$$

$$\text{s.t.} \sum_{t=1}^{M} \rho^{t-1}[V^i(C_t^i, Y_t^i) - V^i(C_t^j, Y_t^j)] \geq 0, \; i = a, b, \quad j \neq i: \lambda_i$$

$$\sum_{t=1}^{M} \delta^{t-1}[N_a(C_t^a - Y_t^a) + N_b(C_t^b - Y_t^b)] \leq 0 \qquad : \mu$$

$$0 \leq Y_t^i \leq K^i, \quad i = a, b, \quad t = 1,\ldots,M$$

$$C_t^i \geq 0, \quad i = a, b, \quad t = 1,\ldots,M$$

For the separate constraints on each period, the problem is identical except that the constraint $\sum_{t=1}^{M} \delta^{t-1}[N_a(C_t^a - Y_t^a) + N_b(C_t^b - Y_t^b)] \le 0$ is replaced by:

$$(P_2) \qquad N_a(C_t^a - Y_t^a) + N_b(C_t^b - Y_t^b) \le 0, \quad t = 1,\ldots,M \qquad : \mu_t$$

The Lagrange multipliers on the self-selection constraints in both problems are denoted λ_a and λ_b, but their values will differ between the two problems. The Lagrange multiplier on the single budget balance constraint in (P_1) is denoted by μ while μ_t, $t = 1,\ldots,M$, denote the multipliers on each period's budget balance constraint in (P_2).

III. OPTIMAL TAXATION WHEN $\delta = \rho$

Let $C_t^i(\alpha)$ and $Y_t^i(\alpha)$, $i = a, b$, and $t = 1,\ldots,M$, be the solutions to (P_1) as functions of α. Let $V^{it}(\alpha)$ denote the optimal utility in period t of individuals in class i for (P_1). V^{io}, $i = a, b$, is the one period utility when no taxes are imposed. All proofs are deferred to the Appendix.

Theorem I: The optimal solution to (P_1) satisfies the following properties:

(i) If $\sum_{t=1}^{M} \rho^{t-1}V^i(\alpha) \ge V^{io} \sum_{t=1}^{M} \rho^{t-1}$ then $\sum_{t=1}^{M} \rho^{t-1}V^i(C_t^j(\alpha), Y_t^j(\alpha)) < \sum_{t=1}^{M} \rho^{t-1}V^{it}(\alpha)$, $i = a, b$ and $j \ne i$.

(ii) If $\sum_{t=1}^{M} \rho^{t-1}V^{it}(\alpha) = \sum_{t=1}^{M} \rho^{t-1}V^i(C_t^j(\alpha), Y_t^j(\alpha))$ then $\sum_{t=1}^{M} \rho^{t-1}V^{jt}(\alpha) > \sum_{t=1}^{M} \rho^{t-1}V^j(C_t^i(\alpha), Y_t^i(\alpha))$, $i = a, b$ and $j \ne i$.

(iii) $\mu > 0$

(iv) If $\lambda_i = 0$ then $C_t^j(\alpha) = C_1^j(\alpha)$, $Y_t^j(\alpha) = Y_1^j(\alpha)$ and $MRS^j(C_t^j(\alpha), Y_t^j(\alpha)) = 1$,

$i = a, b, j \neq i, t = 1,\ldots,M$

(v) For $i = a$, b and $j \neq i$, if $\lambda_i > 0$ then at each $t = 1,\ldots,M$ either:

$$1 < MRS^j(C_t^j(\alpha), Y_t^j(\alpha)) < MRS^i(C_t^j(\alpha), Y_t^j(\alpha))$$

$$MRS^i(C_t^j(\alpha), Y_t^j(\alpha)) < MRS^j(C_t^j(\alpha), Y_t^j(\alpha)) < 1$$

$$MRS^i(C_t^j(\alpha), Y_t^j(\alpha)) = MRS^j(C_t^j(\alpha), Y_t^j(\alpha)) = 1$$

Part (i) asserts that, at the optimum, a group which has higher utility than in the no tax situation cannot have a binding self-selection constraint, while part (ii) rules out both groups having binding self-selection constraints. Part (iii) guarantees that the optimum is production efficient. Part (iv) considers the optimal allocation for a group whose bundle is not desired by the other group. Such a group's allocation is stationary over time at a nondistorted bundle, that is, at a bundle where the MRS equals 1. Finally, part (v) considers the allocation to a group whose bundle is acceptable to the other group. In each period, the MRS of that group at its bundle must lie between the marginal rate of transformation (which equals unity) and the MRS of the other group at that bundle. However, the MRT and the MRS of the other group could have any relation to each other and this relation could differ in different periods. Distortions exist in all periods, except for the possibility that in some periods a bundle is assigned at which the MRS's of both groups equal 1. Generically, such a bundle will not exist. Hence, information learned in the first period is not used to move to the first best allocation in later periods. If the government uses the information it learns in specifying tax functions after the first, it is commited to making only a limited use of this information.

Theorem I does not show that the government uses information gained in the first period to affect later period taxes. In fact, a simple repetition of the one period nonrandom solution satisfies all the first order conditions for (P_1). However, despite the apparent symmetry of the first order conditions, multiple asymmetric solutions may arise in the form of nonstationarity of the optimal consumption-income vectors. Such nonstationary solutions arise from the same nonconvexity of the self-selection constraints that make random solutions in the one period problem optimal. In fact, the following theorems show that there is an exact analogy between existence of a nondegenerate solution to the random tax problem and a nonstationary solution to (P_1).

Let $\hat{V}^i(\alpha) = \sum_{t=1}^{M} \rho^{t-1} V^i(C_t^i(\alpha), Y_t^i(\alpha)) / \sum_{t=1}^{M} \rho^{t-1}$, $i = a, b$, be the average utility achieved by each group over its lifetime. Let the normalized utility possibility frontier be the utility possibility frontier in average utilities.

<u>Theorem II</u>: Assume $\rho = \delta \geq 2/3$ and $M = \infty$. Then, for every α, there exists a solution to (P_1). This solution involves nonstationarity if and only if randomization is desirable in the one period problem. In addition, the normalized Pareto frontier arising in (P_1) is identical to that in the one period case.

With an infinite horizon and a large enough discount factor, any one period random solution can be exactly duplicated by a deterministic nonstationary solution. The circumstances from Brito, <u>et al</u>. [1988b] listed above under which randomization arises in the one period optimal solution are thus sufficient for nonstationary solutions in the multiperiod context. When

M is finite or δ is small, nonstationarity can arise even if the optimal randomization cannot be duplicated.

Theorem III: If $\delta = \rho$ and if M is finite or $\delta < 2/3$, then, for any α, the solution to (P_1) is nonstationary only if the one-period solution involves randomization. The solution to (P_1) is nonstationary if a local randomization would improve on the one-period deterministic solution or if the one-period solution involves nonlocal randomization, $\delta \geq 2/3$, and M is sufficiently large. When the solution is nonstationary, the normalized Pareto frontier for (P_1) may be interior to that in the one period random problem.

(P_2) is the case of pure information transfer across periods since neither the government nor individuals can borrow or save. It is not possible to duplicate the one period random solution by nonstationarity unless the randomization is over bundles such that $C^{ih} - Y^{ih} = (N_j/N_i)(Y^j - C^j)$, h = 1, 2, 3, that is, in the random solution, the government raises the same revenue in each contract. This condition will not generally hold. Consider the case when all groups have the same additively separable utility function over consumption and leisure but differ in ability. Then only income is random in the optimal random solution. This solution cannot be duplicated by a nonstationary solution in (P_2). Nevertheless, nonstationarity may still arise in (P_2) as long as randomization arises in the one-period problem.

The only significant difference between the results for (P_2) and those for (P_1) is that it cannot be shown that the same bundle is given in every period to group j if group i's self-selection constraint is not binding. However, any nonstationarity for group j is over bundles with no distortion.

Theorem IV: If $\rho = \delta$, nonstationarity is possible in the solution to (P_2) and arises only if randomization is used in the one period problem. The normalized Pareto frontier found in (P_2) is generally interior to that in (P_1) whenever the solution to (P_1) involves nonstationarity.

IV. OPTIMAL TAXATION WHEN $\delta \neq \rho$

When the government and individuals have different discount rates, systematic nonstationarity arises in the optimal solution. To contrast with the nonsystematic nonstationarity when $\delta = \rho$, only results for (P_1) are given. Similar results hold if period-by-period budget balance is required or if additional randomization is allowed. Note that similar results arise if the two classes had different private discount rates instead of identical private rates different from the government's.

The first order conditions in (P_1) are:

$$[\delta^{t-1}\alpha N_a + \lambda_a\rho^{t-1}](\partial V^a/\partial C_t^a) - \lambda_b\rho^{t-1}(\partial V^b/\partial C_t^a) - \mu\delta^{t-1}N_a = 0, \tag{1a}$$

$t = 1,\ldots,M$

$$[\delta^{t-1}\alpha N_a + \lambda_a\rho^{t-1}](\partial V^a/\partial Y_t^a) - \lambda_b\rho^{t-1}(\partial V^b/\partial Y_t^a) - \mu\delta^{t-1}N_a = 0, \tag{1b}$$

$t = 1,\ldots,M$

$$[\delta^{t-1}(1-\alpha)N_b + \lambda_b\rho^{t-1}](\partial V^b/\partial C_t^b) - \lambda_a\rho^{t-1}(\partial V^a/\partial C_t^b) - \mu\delta^{t-1}N_b = 0, \tag{1c}$$

$t = 1,\ldots,M$

$$[\delta^{t-1}(1-\alpha)N_b + \lambda_b\rho^{t-1}](\partial V^b/\partial Y_t^b) - \lambda_a\rho^{t-1}(\partial V^a/\partial Y_t^b) + \mu\delta^{t-1}N_b = 0, \tag{1d}$$

$t = 1,\ldots,M$

When $\delta > \rho$, it follows from these conditions that as t increases, the economy approaches the single period first best UPF for any values of λ_a and λ_b. Because of the distortions in early periods, the normalized UPF based on average utilities is interior to the first best UPF.

Theorem V: Consider (P_1) when $\delta > \rho$ and $M = \infty$. Assume that the utility functions satisfy the conditions that $\lim_{C \to 0}(\partial V^a/\partial C)/(\partial V^b/\partial C)$ and $\lim_{Y \to K^b}(\partial V^a/\partial Y)/(\partial V^b/\partial Y)$ are finite. Then, when $\lambda_b = 0$ and $\lambda_a > 0$, $\lim_{t \to \infty} MRS^b(C_t^b, Y_t^b) = 1$. Since $MRS^a(C_t^a, Y_t^a) = 1$, for all t, as t grows, the optimum approaches the period-by-period Pareto frontier.

As the following example shows, the assumptions of this theorem can be satisfied. With identical, additively separable utility functions ($V^i(C^i,Y^i) = \Psi(C^i)-\gamma(Y^i/w_i)$) and $w_a > w_b$, $(\partial V^a/\partial C)/(\partial V^b/\partial C) = 1$ at all C. The maximum income earned by the able exceeds that earned by the unable ($K^a > K^b$). Hence, even if $\partial\gamma/\partial L$ goes to infinity as L approaches its maximum value of K^i/w_i, since $K^b/w_a < K^a/w_a$, then $\partial\gamma(K^b/w_a)/\partial Y$ and $\lim_{Y \to K^b}(\partial V^a/\partial Y)/(\partial V^b/\partial Y)$ are finite.

Different discount rates cause the government and individuals of type A to have different intertemporal preferences for income. The government places a higher value on the future than do individuals. Therefore, trade between them is possible. The government can offer type A individuals higher current utility and lower future utility while doing the reverse for type B (as compared to the solution when $\delta = \rho$), while maintaining the self-selection constraints. Doing this increases social welfare. To see this, note that from equation (1a) and (1b), $\partial V^a/\partial C_t^a = -\partial V^a/\partial Y_t^a = \mu N_a/[\alpha N_a + \lambda_a(\rho/\delta)^{t-1}]$. It follows that the marginal utility of consumption rises over time indicating that consumption declines. As t goes to infinity, $\alpha\partial V^a/\partial C_t^a$ goes to μ. For group B consumption, $(1 - \alpha)(\partial V^b/\partial C_t^b) = \mu + (\lambda_a/N_b)(\rho/\delta)^{t-1}(\partial V^a/\partial C_t^b)$. Since $\lim\inf C_t^b > 0$, at least eventually $\partial V^b/\partial C_t^b$ declines with C_t^b rising. As t goes to infinity, $(1 - \alpha)(\partial V^b/\partial C_t^b)$ goes to μ. Thus, in the limit, the solution is not only Pareto optimal, but it is first best in the sense of being the same as the solution to the one period problem without self-selection constraints.[9]

V. CONCLUSIONS

Our results indicate that, when the government respects individual discount rates, only in a weak sense does the optimal tax system incorporate any information about individuals learned from their responses over time. First, if the government is able to randomize in each period, then no benefit is gained by keeping track of what individuals have earned in past periods. A lottery can be offered in each period, independent of other periods, satisfying self-selection constraints and yielding the best possible outcome. Second, if the government cannot randomize directly, then it can duplicate randomization by intertemporal nonstationarity. Such nonstationarity requires that the government keep track of individuals' past behavior since, after the first period, some individuals would like to choose different bundles than those assigned to them. Third, the information on past earning experience does not yield systematic increases in the value of the government's objective function. The time pattern of tax schedules is not motivated by attempts to gather information about abilities, but is designed to provide lifetime utilities consistent with the self-selection constraints. Only the first period choice by individuals is relevant information; after that, individuals are assigned a single bundle in each period.

By contrast, if the government discounts the future at a different rate from individuals, then there is systematic change in the bundles given to individuals over their lifetimes. In the limit, the distortions may be eliminated. This arises because the different intertemporal preferences of the government and individuals leaves room for "trade" between them. Over time the differences between the utilities of the groups grows larger.

These results show that nonstationarity over time and randomization within each period can substitute for each other in the optimal intertemporal income tax. It is not clear which approach is preferable since each has some advantages.

First, they are not perfect substitutes. Even if the government has a single intertemporal budget constraint, nonstationarity is guaranteed to do as well as randomization only with an infinite horizon and a sufficiently large discount factor. If the government has a separate budget constraint in each period, the optimal randomization cannot be completely duplicated by intertemporal variability, so that randomization along with nonstationarity would be needed to reach this Pareto frontier.

Second, political and administrative difficulties could prevent implementation of either method. On one hand, the government may be reluctant to incorporate randomization explicitly in the tax code. This is especially true since the optimal randomization requires individuals to declare their type and then receive at random a tax schedule before choosing their labor supplies. The optimal randomization can generally not be implemented by random collection or enforcement after labor supply decisions. On the other hand, intertemporal nonstationarity requires keeping track of past incomes to determine individuals' current tax payments. However, this is simplified since the government needs only to recall each individual's type as revealed by past decisions.

Third, it is desirable that the system be fair, and to increase acceptance of the tax system by society, it is desirable that the system be perceived as fair. A standard notion of fairness is horizontal equity, that individuals in the same circumstances be treated the same. Randomization satisfies horizontal equity ex ante but not ex post. Before the random selection, all individuals of the same type face the same lottery. After receiving a random draw of tax functions, individuals of the same type will be induced to choose bundles which need not yield the same utility. Intertemporal nonstationarity achieves horizontal equity both ex ante and ex post in each period. Individuals of the same type are induced to choose the same bundles as each other in every period even though the choice varies over time.

Fourth, both procedures induce differences in the bundles chosen by individuals of a type either within a period in an expected sense under randomization or over time under nonstationarity. With strictly concave utility functions, individuals desire to reduce these differences. Under randomization, individuals might gain by purchasing insurance counteracting the randomness in the tax system. If such policies were forbidden, then similar effects could be achieved by trades with other individuals of the same type. For the same reason, under nonstationarity, individuals desire to smooth consumption and leisure over time by saving or borrowing. Saving or insurance serves to counteract the weakening of self-selection constraints which motivated the asymmetry of bundles in the first place. The ability to save or buy insurance will be a factor in the decision to reveal one's type truthfully. The choice between nonstationarity or randomness may depend upon whether it is easier to prevent saving or insurance. If these are desirable for other reasons or cannot be prevented, then the simple repetition of the solution to the nonrandom one-period problem may be the best feasible solution. However, the opposite problem arises if only symmetric solutions are allowed when individuals have nonconvex opportunity sets. Individuals might randomize consumption bundles to convexify budget sets, and to attain self-selection, more distortionary taxation would have to be imposed. Prohibiting gambling would, under these circumstances, be welfare increasing.

Footnotes

1. Examples of papers exploring the role of randomization in adverse selection and other problem include Weiss [1976], Stiglitz [1982], Fellingham, Kwon and Newman [1984], Maskin and Riley [1984], and Arnott and Stiglitz [forthcoming].

2. The utility functions defined over {C, Y} can be derived from the more fundamental utility functions defined over goods and leisure.

3. For the one-period problem, Guesnerie and Seade [1982] derived some results without global single crossing, but they assumed that MRS's were not equal at the optimal bundles. We only assume that tangencies do not lie on the no-tax budget line and show that the MRS's are not equal at an optimum. They also imposed Inada-type conditions on preferences, which we do not.

4. Let L^i be hours worked and w_i the wage rate of group i. Then $L^i = Y/w_i$ and $V^i(C, Y) = U(C, Y/w_i)$ where U is the common utility function over C and L. If A is the more able group ($w_a > w_b$), then $MRS^a(C, Y) < MRS^b(C, Y)$ at each (C, Y), provided an additional assumption on U(C,L) is satisfied. Note that $MRS^i(C,Y) = -[U_L(C,L^i)/w_iU_C(C,L^i)]$. If $w_a > w_b$ then $L^a < L^b$. The result holds if the direct effect of the higher wage is not countered by the effects of a lower L on the MRS. Differentiating $-[U_L(C,Y/w)/wU_C(C,Y/w)]$ with respect to w yields $dMRS(C,Y)/dw = (U_L/wU_C)-(L/w^2)d(-U_L/U_C)/dL$. A sufficient condition for $dMRS(C,Y)/dw < 0$ is $d(-U_L/U_C)/dL \geq 0$ which holds if C is not inferior. See Sadka [1976].

5. A local randomization scheme is one in which, even for only arbitrarily small deviations from the deterministic solution, the value of social welfare is increased.

6. This constrasts with Harris [1987]. If the government knows that half the population are type A and half type B, it can "force" truthful revelation in a Nash equilibrium by imposing heavy penalties on <u>all</u> individuals if

more than half claim to be of a particular type. We find these Nash equilibria unpersuasive.

7. Alternatively, the government faces a new cohort each period and is constrained to use the same tax structure for all cohorts. Such a framework gives rises to a single within period budget constraint when redistribution across cohorts is possible. The independent problems in each period are identical to the problem with a single intertemporal constraint.

8. A self-selection constraint may hold with equality so that one of the groups may be indifferent between the two bundles offered. The solution requires that all individuals in the group choose the bundle aimed at that group. This can be achieved by assuming that the government can assign indifferent individuals to whichever group it desires. Given that the government does not know to which group a particular individual belongs, this is not a reasonable assumption. An alternative view is that the solution is really an ε-equilibrium. Although it cannot itself be achieved, a bundle arbitrarily close to that solution can be found which satisfies resource balance and which has the self-selection constraint hold with strict inequality. If the self-selection constraints must hold with strict inequality, then there may exist no solution to the maximization problem.

9. If $\delta < \rho$, trade is still possible but tends to go in the opposite direction. A characterization of this solution is more difficult because the interior first order conditions may violate transversality conditions.

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APPENDIX

PROOFS OF THEOREMS

PROOF OF THEOREM I: (i) Replace the lifetime consumption and income vectors or lotteries by their mean values. Given concavity, the utilities of these mean values repeated in each period is greater than the utility in the no-tax situation. The violation of budget balance then follows as in the proofs in the random one period problem. See Brito, et al. [1988b].

(ii)-(v) See Brito, et al [1988b]; Theorem II]. Q.E.D.

PROOF OF THEOREM II: The proof requires the following lemma.

Lemma I: Consider an infinite sequence defined by $(1 - x)x^{t-1}$, $t = 1,\dots,\infty$. Consider three numbers Π_1, Π_2, Π_3 with $\Pi_i > 0$ and $\Pi_1 + \Pi_2 + \Pi_3 = 1$. If $2/3 \le x < 1$, then there exists a partition, I_i, of the positive integers indexing the terms of the sequence such that

$$(1 - x) \sum_{t\epsilon I_i} x^{t-1} = \Pi_i,\ i = 1, 2, 3.$$

Proof: Construct the desired partition by the following procedure. Define Ω^r_1, Ω^r_2 and Ω^r_3 as the sums of the terms in each partition using the first r terms of the sequence. For r = 1, one of $\Omega^1_i = 1 - x$ and the rest are zero. Let $S_r \equiv \Omega^r_1 + \Omega^r_2 + \Omega^r_3 = \sum_{t=1}^{r} (1 - x)x^{t-1}$. By summing, it follows that $1 - S_r = x^r$. If $x \ge 2/3$ then for all r, $(1 - x)x^{r-1} \le \frac{1}{3} x^{r-1} = 1/3(1 - S_{r-1})$. That is, the rth term is less than one-third the sum of the remaining terms including itself. Thus, the rth term can always be put into at least one of the partial sums without exceeding that Π_i. Then for all r, the partitions can be formed with $\Pi_i - \Omega^{r-1}_i \ge 0$, i = 1, 2, 3. As r goes to infinity, $\sum_{t=1}^{r} (1 - x)x^{t-1}$ goes to one so that $\Omega^r_1 + \Omega^r_2 + \Omega^r_3$ goes to 1. It then follows that Ω^r_i goes to Π_i as r goes to infinity. Q.E.D.

Note that if $x < 2/3$, then there exist triples Π_i, $i = 1, 2, 3$, which cannot be found as the sum of terms in a partition of the sequence $(1 - x)x^t$, $t = 1,\ldots,\infty$. For example, consider $\Pi_1 = \Pi_2 = \Pi_3 = 1/3$, and $x = 1/2$.

Proof of Theorem: First, for any α, the random one-period solution can be duplicated in (P_1). For some j, $k(j) = 1$ with (C^{j1}, Y^{j1}) received by j with certainty, while for $i \neq j$, $k(j) \leq 3$ can hold at an optimal solution. Hence, there exist (C^{ih}, Y^{ih}) and π_{ih}, $h = 1, 2, 3$ in an optimal solution. Consider (P_1) and multiply the objective function, the self-selection constraints, and the budget constraint by the constant $(1/\sum_{t=1}^{\infty} \rho^{t-1}) = 1 - \rho$. This leaves the solution unchanged. From Lemma I, there is a partition of the integers such that $\sum_{t\epsilon I_h} (1 - \rho)^{t-1} = \pi_{ih}$, $h = 1, 2, 3$. Then assign (C^{ih}, Y^{ih}) to i in all periods $t \epsilon I_h$, $h = 1, 2, 3$, and assign (C^{j1}, Y^{j1}) to j in all periods. By construction, this is feasible in (P_1) since it is feasible in the one period problem.

Second, this solution is optimal in (P_1). If it were not optimal, then there would exist a feasible lifetime bundle $(\hat{C}^i, \hat{Y}^i)$, $i = a, b$, such that

$$[\alpha N_a \sum_{t=1}^{\infty} \rho^{t-1} V^a(\hat{C}^a_t, \hat{Y}^a_t) + (1 - \alpha)N_b \sum_{t=1}^{\infty} \rho^{t-1} V^b(\hat{C}^b_t, \hat{Y}^b_t)](1 - \rho) >$$

$$\alpha_i N_i \sum_{h=1}^{3} \sum_{t\epsilon I_h} (\rho^{t-1})(1 - \rho)V^i(C^{ih}, Y^{ih}) + \alpha_j N_j V^j(C^{j1}, Y^{j1}) =$$

$$\alpha_i N_i \sum_{h=1}^{3} \pi_{ih} V^i(C^{ih}, Y^{ih}) + \alpha_j N_j V^j(C^{j1}, Y^{j1}),$$ $i = a, b$, and $j \neq i$. Since $\rho < 1$ and since $V^a(C^a, Y^a)$ and $V^b(C^b, Y^b)$ have upper bounds given the bounds on C^a and Y^a, there would exist a finite T such that

$$[\alpha N_a \sum_{t=1}^{T} \rho^{t-1} V^a(\hat{C}^a_t, \hat{Y}^a_t) + (1 - \alpha)N_b \sum_{t=1}^{T} \rho^{t-1} V^b(\hat{C}^b_t, \hat{Y}^b_t)](1 - \rho) >$$

$$\alpha_i N_i \sum_{h=1}^{3} \pi_{ih} V^i(C^{ih}, Y^{ih}) + \alpha_j N_j V^j(C^{j1}, Y^{j1}). \qquad \text{(A1)}$$

Since $\sum_{h=1}^{3} \sum_{t \in I_h} \rho^{t-1} < 1/(1 - \rho)$, the inequality (A1) will still hold if the left hand side is multiplied by $1/[(1 - \rho) \sum_{t=1}^{T} \rho^{t-1}]$. Similarly, for T large enough, a self-selection constraint which held with inequality will still hold with inequality while one that held with equality will be violated by, at most, an arbitrarily small ε. Similarly, resource balance will be violated by, at most, an arbitrarily small ε. If the self-selection constraint violated is that for group i, then there exists a $\delta > 0$, such that substituting $\bar{Y}_t^j = \hat{Y}_t^j + \delta$, for $\hat{Y}_t^j$, will have all constraints satisfied and still leave (A39) satisfied. Then the lotteries $((\hat{C}_t^i, \hat{Y}_t^i), \pi_{it})$ and $((\hat{C}_t^j, \bar{Y}_t^j), \pi_{jt})$ with $\pi_{it} = \pi_{jt} = \rho^{t-1}/ \sum_{t=1}^{T} \rho^{t-1}$ are feasible in the one period problem and yield a higher value than the optimum, which is a contradiction.

This shows existence by construction of an optimal solution. Since the solution duplicates that of the one period problem, the remainder of the theorem follows immediately. Q.E.D.

<u>PROOF OF THEOREM III</u>: If the optimal solution in the one period problem is nonrandom for both a and b, this can be duplicated in (P_1). If this were not the optimum, a better solution would have an analogous lottery which would be feasible in the one period problem and have a higher value than the nonrandom optimum, a contradiction. Hence, one period randomization is necessary for multiperiod nonstationarity. When M is finite or $\delta < 2/3$, only a subset of the probability simplex $\sum_{h=1}^{3} \pi_{ih} = 1$ can be achieved by intertemporal variation in the optimal bundles. If the optimal lotteries in the one period problem involve probabilities in the attainable subset, then the optimal solution can

be duplicated. If not, but a local randomization does improve the deterministic outcome, then, as shown in Brito, et al. [1988b], a local randomization with any probabilities improves on the deterministic solution. Thus, randomization with probabilities from the attainable part of the simplex will improve on the deterministic solution and these can be duplicated by nonstationary solutions in (P_1). For $\delta \geq 2/3$, if a random one period optimum exists, it will achieve a strictly higher value of the objective function than the nonrandom one period solution. For M sufficiently large but finite, a nonstationary solution can come sufficiently close to the nonrandom solution to improve on the best nonstationary solution. When some nonstationarity is desirable, but the optimal random solution can not be duplicated, the outcome on the normalized Pareto frontier will be interior to that in the one period problem. Q.E.D.

PROOF OF THEOREM IV: A nonrandom one period solution is feasible to repeat in every period in (P_2). If there exists a different solution to (P_2) which is superior, then a feasible lottery exists which would improve on the one period optimal solution, a contradiction. Hence, randomization in the one period problem is necessary for nonstationary in (P_2).

The following example shows that nonstationarity is possible. Assume that $\lambda_a > 0$ in the one period problem and that $V^i(C,Y) = \psi(C) - \gamma(Y/w_i)$. If $L^i = Y/w_i$, there exists a local randomization for any probabilities $\Pi_1 + \Pi_2 + \Pi_3 = 1$ around the nonrandom one period solution if

$$\frac{\gamma''(Y^b/w_a)}{w_a^{\,2}(1-MRS^a(C^b,Y^b))} > \frac{\gamma''(Y^b/w_b)}{w_b^{\,2}(1-MRS^b(C^b, Y^b))}$$

(see Corollary II of Brito et al. [1988b]). These conditions do not depend upon $\psi(C)$. Hence, randomization will arise for all functions $\psi(C)$, including $\psi(C) = dC$, with individuals risk neutral over consumption. The random one period solution can be duplicated in (P_1) if $\delta \geq 2/3$ and M is infinite. In each period, budget balance may not be satisfied. Define $E_t \equiv N_a(C^a - Y^a) + N_b(C_t^b - Y_t^b)$ where A's bundles are constant over time. From overall budget balance, $\sum_{t=1}^{\infty} \delta^{t-1} E_t = 0$. Substituting $C_t^a = C^a - E_t/N_a$ for C^a in each period, ensures period by period budget balance. Then $\sum_{t=1}^{\infty} \delta^{t-1} C_t^a = C^a \sum_{t=1}^{\infty} \delta^{t-1} - \sum_{t=1}^{\infty} \delta^{t-1} E_t/N_a = C^a \sum_{t=1}^{\infty} \delta^{t-1}$. This change leaves the utility of both A and B from A's bundle unchanged since $\sum_{t=1}^{\infty} d\, \delta^{t-1} C_t^a = d\, C^a \sum_{t=1}^{\infty} \delta^{t-1}$ and the sequence of Y^a was unchanged. Thus, with linear utility for consumption, the individuals in group A are willing to engage in the borrowing or lending which had previously been done with the rest of world. Alternatively, since in this example, the B's were also risk neutral the consumption of the A's could have been unchanged at C^a but the consumption of the B's could have been adjusted to maintain budget balance. If the utility functions of the two groups differ but A is still risk neutral, it might be necessary to have A's consumption be nonstationary. In this case, the normalized Pareto frontier in (P_2) is the same as that in (P_1). In other cases, the need to have period by period balance can reduce the utility levels in (P_2) below those in (P_1). Q.E.D.

Note that the example in the above proof with utility linear in consumption can be generalized. If with linear utility the nonstationary solution is strictly better than the stationary one, then a nonstationary solution will still be better when $\psi(C)$ has some curvature.

PROOF OF THEOREM V : Given $\lambda_b = 0$, equations (1a) and (1b) yield $MRS^a(C_t^a, Y_t^a) = 1$, for all t. Divide equation (1c) by $\delta^{t-1}(\partial V^b/\partial C_t^b)$ and (1d) by $\delta^{t-1}(\partial V^b/\partial Y_t^b)$.

$$(1 - \alpha)N_b - \lambda_a(\rho/\delta)^{t-1}[(\partial V^a/\partial C_t^b)/(\partial V^b/\partial C_t^b)] - \mu N_b/(\partial V^b/\partial C_t^b) = 0, \quad t = 1,\ldots,M \tag{A2a}$$

$$(1 - \alpha)N_b - \lambda_a(\rho/\delta)^{t-1}[(\partial V^a/\partial Y_t^b)/(\partial V^b/\partial Y_t^b)] - \mu N_b/(\partial V^b/\partial Y_t^b) = 0, \quad t = 1,\ldots,M \tag{A2b}$$

Since $\partial V^i/\partial C$ is finite for $C > 0$ and $\lim_{C\to 0}(\partial V^a/\partial C)/(\partial V^b/C)$ is finite, $\lim\sup_{t\to\infty}(\rho/\delta)^{t-1}[(\partial V^a/\partial C_t^b)/(\partial V^a/C_t^b)] = 0$. Hence, should $\lim\sup_{t\to\infty}(\partial V^b/\partial C) = \infty$ then after some $\hat{t}$, the left hand side of (A2a) would be strictly positive violating the first order condition. Thus, no C_t^b sequence goes to zero, guaranteeing that $\lim\inf_{t\to\infty} C_t^b > 0$. Similarly, from (A2b), $\lim\sup_{t\to\infty} Y_t^b < K^b$. Therefore, $\lim\sup(\rho/\delta)^{t-1}(\partial V^b/\partial C_t^b) = \lim\sup_{t\to\infty} (\rho/\delta)^{t-1} (\partial V^b/\partial Y_t^b) = 0$.

Given this, divide (1c) and (1d) by δ^{t-1} and solve for $MRS^b(C_t^b, Y_t^b) = -(\partial V^b/\partial Y_t^b)/(\partial V^b/\partial C_t^b)$.

$$MRS^b(C_t^b, Y_t^b) = \frac{\mu N_b - \lambda_a(\rho/\delta)^{t-1}(\partial V^a/\partial Y_t^b)}{\mu N_b + \lambda_a(\rho/\delta)^{t-1}(\delta V^a/\partial C_t^b)} \tag{A3}$$

then $\lim_{t\to\infty} MRS^b(C_t^b, Y_t^b) = 1$. Q.E.D.