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A DEFENSE OF THE EFFICIENCIES OFFENSE

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ABSTRACT

This paper uses an aggregative games framework to predict consumer welfare when market structure is endogenously determined. Our main results characterize mergers whose synergies reduce consumer welfare by inducing rivals to exit. The conditions under which such mergers arise are broad, regardless of whether we consider quantity competition among homogeneous products or price competition among multi-product firms facing multinomial logit demand. Calibrated models based on commonly used parameter values indicate that the synergies required to avoid consumer harm can be much higher than those implied by traditional merger analysis. Neither subsequent entry nor follow-on mergers necessarily mitigate the problem.

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1 Introduction

Horizontal mergers create efficiencies through economies of scale and scope, improved coordination, and the exchange of "best practices," which reduce costs. All else equal, as savings are passed through, they put downward pressure on prices. For these reasons, merger review traditionally treats efficiency gains as an unambiguously pro-competitive factor, pitted against upward pressure applied by increased market power. However, this treatment implicitly assumes that market structure is unaffected by the merger. This paper relaxes the restriction by allowing market structure to endogenously adapt. As we show, doing so can dramatically change how certain mergers are evaluated.

The change occurs because merger synergies can cause rivals to exit the market, further weakening the competitive process. In such cases, efficiencies could constitute an "offense" rather than "defense" under antitrust laws. Federal courts and agencies have sharply disagreed about the validity of this claim over time. In the US, the argument was almost uniformly accepted under the Warren Court of the 1960s only to be rejected out-of-hand by the Chicago School, which has dominated most judicial thinking and agency analysis since the 1980s. For example, in 1967, the Federal Trade Commission successfully argued against Procter & Gamble's acquisition of Clorox on the grounds that the scale of the merged entity would deter competitors from challenging it.[1] Yet, in 2000, the same fundamental force that worked against Procter & Gamble worked in favor of General Electric when it brought its proposed acquisition of Honeywell to the Department of Justice (Majoras, 2001). Despite these facts, to the best of our knowledge, the general case has never been formally studied.

We use an aggregative games framework to predict merger-related changes in consumer welfare when market structure is endogenously determined.[2] Leveraging recent technical advances by Nocke and Schutz (2018, 2024), we analyze quantity competition in homogeneous product markets as well as price competition between multi-product firms facing multinomial-logit demand. We find that relaxing the usual assumption, which holds market structure fixed, can lead to very different decisions about which mergers should be allowed under a consumer welfare (i.e., consumer surplus or simply "CS") standard. Moreover, we find that neither subsequent entry nor follow-on mergers necessarily mitigate the problem. Our findings support claims by Nocke and Whinston (2022) that *US Horizontal Merger Guidelines* published between 1982 and 2010 were likely too lax.

Our main focus is merger-induced exit by competitors, which we model using a two-stage perfect information game that can be summarized as follows. In the first stage, each firm decides whether to stay in the market or exit. If the firm stays, then it pays a fixed cost; otherwise, it receives a payoff normalized to zero. In the second stage, firms set prices or quantities and earn gross profits, while consumers demand the products. Equilibrium strategies reflect profit maximization. Our approach parallels ones commonly used in modern empirical work (e.g., Mazzeo (2002); Ciliberto and Tamer (2009); Fan (2013); Eizenberg (2014); Wollmann (2018); Fan and Yang (2020)).[3] It also draws from, complements, and, in one instance, extends recent work by Caradonna, Miller, and Sheu (2024), which cleverly characterizes the limits of postmerger entry (but abstracts away from the efficiencies offense).

Keeping with convention, we evaluate hypothetical mergers by solving each game twice. First, we

[1] *FTC v. Procter & Gamble Co.*, 386 US 568 (1968).

[2] We restrict attention to changes in consumer welfare, which is the substantive legal standard employed in nearly all US merger analysis (Farrell and Shapiro, 2010). See, e.g., *FTC v. Elders Grain*, 868 F.2d 901, 904 (1989), which affirms the standard.

[3] It relates to analogous games with incomplete information, which typically mitigate complications from multiplicity but increase computational demands (Seim, 2006). It also relates to dynamic games with entry, exit, repositioning, and/or investment (see, e.g., (Bajari et al., 2007; Ryan, 2012; Sweeting, 2013)). All have their roots in seminal work by Bresnahan and Reiss (1990) and Berry (1992).

consider a "premerger" environment in which each firm is independently owned. Second, we consider a "postmerger" environment in which two firms are combined just prior to the start of the game.[4] As we alluded, most theoretical work spotlights the second stage, thereby illuminating the key static tradeoff for consumers: mergers not only reduce cost and raise quality but also cause firms to internalize a larger share of business-stealing externalities (Williamson, 1968). Based on this tradeoff, influential work by Farrell and Shapiro (1990) and more recently Nocke and Whinston (2022) concisely characterize which mergers should be allowed. We diverge from their approach by incorporating first stage equilibrium behavior and then analyzing how the set of allowable mergers changes.

We begin with a special case in which the least profitable nonmerging firm in the market breaks even prior to the transaction. We find that if a merger is CS-positive when market structure is held fixed, then it will cause the least profitable nonmerging firm to exit and, as a consequence, reduce CS below its premerger level.[5] In short, exactly when traditional merger analysis indicates that synergies are large enough to raise CS, we find that exit occurs, "ratcheting" CS down. By extension, but more broadly, CS is non-monotone in synergies.

We then solve for the synergies required for a CS-positive merger. The closed-form expression that we obtain often far exceeds what one derives when market structure is assumed fixed. For instance, if we assume symmetric firms compete à la Cournot, then required synergies are *twice those reported in the existing literature*. To illustrate, in their leading example, Nocke and Whinston (2022) choose primitives such that a CS-positive merger requires synergies greater than 5%. Using identical parameter values, our figure is 10%.

We then relax the assumption that the least profitable firm breaks even prior to the merger and instead impose only that fixed costs rationalize the premerger market structures we analyze. The bounds we construct are intuitive. Suppose, for example, that exactly N firms stay in the market prior to the merger. Were fixed costs "too high," not all of the N firms could profitably stay. Conversely, were fixed costs "too low," at least $N+1$ firms could profitably stay. We find that, given any vector of fixed costs that satisfy the necessary conditions for equilibrium, there always exist synergies that create an efficiencies offense (i.e., that reduce CS by inducing exit among nonmerging firms).

Finally, we address other market responses that might mitigate the problems we identified. As one example, we consider subsequent entry. All else equal, exit makes the market more attractive to a potential entrant. If entry occurs and the incoming firm is more efficient than the departing one, then the combination of the merger, exit, and subsequent entry might raise CS even when the combination of the merger and exit alone would reduce it. However, we find that only a narrow range of parameter values permit this to occur. The limiting factor is the entrant, which must be efficient enough to profitably enter and offset harm but not so efficient that it would have already entered prior to the merger.

As another example, we consider follow-on mergers. A firm that becomes unprofitable because its rivals have combined to become more efficient may itself merge (and generate its own synergies). Our analysis underscores an earlier, underappreciated argument made by Nocke and Whinston (2010): two mergers that occur together can be CS-positive even when each of them, in isolation, would be CS-negative. Yet, practical reliance on follow-on mergers requires caution. The likelihood that a subsequent, synergistic transaction emerges depends critically on what "merger technologies" firms possess. At the present time, such capabilities are not well understood (Demirer and Karaduman, 2022). Moreover, to the best of our knowledge, neither agencies nor courts jointly evaluate mergers, so formally relying on follow-on mergers

[4] We agree with Farrell and Shapiro (2000) that "absent-merger" and "with-merger" are more accurate terms, but we opted, as they have, for standard descriptors.

[5] As we show, there is one uninteresting exception, which involves synergies so large that all nonmerging firms exit after the merger.

will require different forms of analysis.

To be clear, this paper does not advocate against crediting synergies—or against any particular policy, for that matter. Instead, it relaxes a standard assumption, explores the consequences, and, in turn, questions the rationale of the decades-long blanket dismissal of the efficiencies offense. If one juxtaposes our propositions with the common refrain that merger review must be tailored to each transaction's diverse circumstances (i.e., "every merger is different" (Benkard, 2010)), then, in our view, it seems hard to claim that "an allegation of an efficiency offense has no actual basis in [antitrust] law" (Schmitz, 2002).[6] Furthermore, even one who worries about the unintended consequences of scrutinizing synergies would still want to carefully understand the intended consequences of doing so.

We should also clarify our framework's objective function. In many cases, we find that whether to allow a merger hinges critically on whether it will drive nonmerging firms from the market. In these instances, enforcement decisions are highly correlated with the *protection of competitors*. In turn, they are closely aligned with the "New Brandeis" movement.[7] Interestingly, however, we only consider *consumer welfare*. That is, we place zero welfare weight on either firm profits or spillovers that may accrue through non-market mechanisms (e.g., the possibility that concentration facilitates lobbying and capture, thereby raising entry barriers in unmodeled future periods or undermining democratic processes, as in Teachout and Khan (2014) or Teachout (2020)). In other words, the relationship that we discover between the current substantive legal standard and Neo-Brandeisian objectives arises endogenously from our model (even though questions about which standard is correct are beyond its reach).

Finally, while measuring the frequency at which these situations arise is also outside the scope of our paper, anecdotes abound in trade press and business media that market structure endogenously responds to mergers among rivals. For example, when the FCC relaxed ownership restrictions on radio broadcasting, the industry rapidly consolidated. Following the initial wave of mergers, firms wishing to remain independent allegedly struggled to survive. An industry analyst quoted in the *New York Times* summarized the concern: "Mom and pop broadcasters will be forced to sell" (Adelson, 1993). As another example, similar pressures have been faced by wholesalers of products ranging from building materials and industrial gases to electronics and food. As "distribution consolidators . . . rapidly acquired former competitors, . . . smaller, regional wholesalers . . . were forced to sell out or dissolve" (Fein and Jap, 1999).

This is not to say that merger-induced exit only arises in fragmented markets with niche producers. Concentrated markets populated by firms with billions of dollars in revenue may pose risks. For example, when Canadian Pacific proposed to buy Norfolk Southern in 2015, industry sources indicated that the merger could "trigger further industry consolidation," with one former railroad executive indicating, "companies like CSX and [Canadian National] would also explore a merger to remain competitive" (Dummett and Mickle, 2015). Nearly a decade earlier, the chief executive of Continental Airlines used similar language following a bid by US Airways to takeover Delta, telling employees that their company "wants to remain independent" but "would consider a merger to remain competitive if the industry continues

[6]The quote references European Commission law in particular (the omitted word is "EC"). However, the surrounding text states that the EC is a comparatively favorable jurisdiction for this type of argument, implying that the statement applies broadly.

[7]Justice Brandeis was an early, influential critic of not just market power but also "bigness." His ideas shaped competition policy through much of the 20th century. Many continue to influence legal scholarship (Khan, 2016; Wu, 2018; Asil, 2024). In the absence of formal models, it is often hard to know precisely where he and his ideological opponents disagree. In any event, we find it most interesting that our analysis microfounds one of his most controversial and counterintuitive claims. Brandeis criticized "destructive competition," especially when it is "cutthroat" in nature, as it paves the "road to monopoly" (see, e.g., his dissent in *New State Ice Co. v. Liebmann*, 285 US 262, 311 (1932) or Brandeis (1913)). Nearly all modern analysis, such as Greve (2001), ridicules this view, ruling out the mere possibility. However, as we show, it is possible, and in some cases even probable, for cost savings to raise prices by eliminating competition.

to consolidate" (Carpenter, 2006). By the same token, when Comcast proposed to buy Time Warner Cable, analysts "speculat[ed] . . . that DirecTV and Dish Network Corp., the second and third biggest pay TV providers, respectively, could seek to merge to gain scale and remain competitive" (Ramachandran, 2014).

The remainder of the paper is organized as follows. Section 2 studies the efficiencies offense under Cournot competition. Section 3 conducts similar analysis under Nash-Bertrand competition. That is, whereas firms set quantities for homogenous products in Section 2, they set prices for differentiated products in Section 3. Section 4 models and evaluates potential defenses such as subsequent entry and follow-on mergers. Section 5 concludes.

2 Quantity competition with homogeneous products

2.1 Primitives and equilibrium

Consider a market with a set $\mathscr{F}$ of firms. In the first stage, each firm $f \in \mathscr{F}$ decides to stay or leave. If f stays, then it pays a fixed cost ϕ_f and moves to the second stage. If not, then it receives a payoff normalized to zero and exits. In the second stage, each firm that stays in the market produces a homogeneous product with constant returns to scale. f sets a quantity q_f and receives gross profit equal to $(P(Q) - c_f)q_f$, where $P(Q)$, Q, and c_f denote inverse demand, aggregate quantity, and per-unit variable cost, respectively.

For simplicity, we assume that $\phi_f > 0$, $c_f > 0$, and $\phi_f < \phi_{f'}$ if and only if $c_f < c_{f'}$. For expositional ease, firms are sorted in decreasing order of their costs from 1 to N.[8] Also, to ensure a unique equilibrium exists, we assume that firms make first stage decisions in reverse order—the lowest cost firm decides first. For the same reason, we make the standard assumptions on inverse demand. $P'(Q) < 0$ and $P'(Q) + QP''(Q) < 0$ for all Q satisfying $P(Q) > 0$, and $\lim_{Q\to\infty} P(Q) = 0$.

Firms maximize profits. Equilibrium strategies may be obtained by backwards induction. In the second stage, given all $f < J$ exit, each $f \geq J$ sets the first order condition of its gross profit equal to zero. It chooses

$$q_f = \max\left\{\frac{P(Q) - c_f}{-P'(Q)}, 0\right\}, \tag{1}$$

where Q is understood to equal $\sum_{f' \geq J} q_{f'}$. Assuming $q_f > 0$, it earns

$$\pi_f(J) = \frac{(P(Q) - c_f)^2}{-P'(Q)}. \tag{2}$$

In the first stage, given the order in which firms make decisions,[9] each firm f stays if and only if

$$\pi_f(f) \geq \phi_f. \tag{3}$$

We study the merger of two firms, m and n. Under Cournot competition, it is natural to assume that the transaction consolidates production into a single entity and gives rise to synergies, meaning it reduces variable costs. We denote the merger by $\mathscr{M}$ and the merged firm by M. To rule out uninteresting cases, we

[8] If there exist f and f' such that $c_f = c_{f'}$ and $\phi_f = \phi_{f'}$, then the firms can be arbitrarily ordered without loss of generality.

[9] If f exits, then its fixed cost exceeds its variable profit. Given the indexing of firms, $f' < f$ has weakly smaller variable profit and weakly larger fixed cost, so f' also exits. By analogous logic, if f stays, then any $f' > f$ stays.

restrict attention to $c_M \leq \min\{c_m, c_n\}$, $\phi_M \leq \min\{\phi_m, \phi_n\}$, and $\min\{m, n\} > 1$ (i.e., neither m nor n is the least profitable firm prior to the merger),[10] and we impose that all firms produce positive quantity after the merger.[11] Finally, we denote premerger equilibrium objects using an asterisk and, unless otherwise stated, make the standard assumption that all firms $f \in \mathscr{F}$ earn positive profit prior to the merger (i.e., $\pi_1^\star(1) \geq \phi_1$).

2.2 Zero-rival-profit and consumer-welfare-neutrality curves

Borrowing nomenclature from Caradonna et al. (2024), who analyze entry induced by increased postmerger market power, we introduce two sets of equilibrium objects. First, we define zero-rival-profit (ZRP) curves. ZRP_i plots the synergies required for firm i to break even, assuming all $i' < i$ exit. Since i breaks even,

$$\phi_i = \left(P(Q^i) - c_i\right) q_i = \frac{\left(P(Q^i) - c_i\right)^2}{-P'(Q^i)}, \tag{4}$$

where Q^i is the aggregate quantity at which i earns exactly zero profit. Equation 4 shows that Q^i can be written as a function of ϕ_i. If we write aggregate quantity as a sum of individual quantities and rearrange terms, then we have

$$\tilde{c}_M^i = (N-i)P(Q^i(\phi_i)) + P'(Q^i(\phi_i))Q^i(\phi_i) - \sum_{h \in \mathscr{H}_i} c_h, \tag{5}$$

where $\mathscr{H}_i = \{\{i, i+1, ..., N\} \setminus \{m, n\}\}$ and $\tilde{c}_M^i$ denotes the value of c_M associated with ZRP_i. Equation 5 shows that $\tilde{c}_M^i$ can be written as a function of ϕ_i. To facilitate comparisons, we normalize fixed costs and variable cost improvements. Specifically, we replace ϕ_i with $\Phi_i \equiv \phi_i / \pi_i^\star$ and $\tilde{c}_M^i$ with $(c - \tilde{c}_M^i)/c$ so that each lies within the closed unit interval. ZRP_i is given by

$$\frac{c - \tilde{c}_M^i}{c} = \frac{1}{c}\left(c - (N-i)P(Q^i(\Phi_i)) - P'(Q^i(\Phi_i))Q^i(\Phi_i) + \sum_{h \in \mathscr{H}_i} c_h\right). \tag{6}$$

Second, we define CS-neutrality (CSN) curves. CSN_i plots the synergies required to make the merger CS-neutral, given all $i' \leq i$ exit. Since the second stage is an aggregative game, a CS-neutral merger must leave aggregate quantity unchanged from its premerger level Nocke and Whinston (2022). This implies

$$Q^\star = \sum_{f \in 0\mathscr{F}} \frac{P(Q^\star) - c_f}{-P'(Q^\star)} = \sum_{g \in \mathscr{G}_i} \frac{P(Q^\star) - c_g}{-P'(Q^\star)} + \frac{P(Q^\star) - \tilde{c}_M^i}{-P'(Q^\star)}, \tag{7}$$

[10] We impose the first condition because a court that uses a consumer welfare standard will block any merger that raises variable cost. We impose the second condition for two reasons. One is that, in this framework, fixed costs are unlikely to increase due to the merger. The other is to guarantee that firms do not merge and immediately exit. Such cases complicate exposition but are unlikely to ever occur as they are never profitable. As for the third condition, we make frequent references to the "least profitable firm prior to the merger" (i.e., $f = 1$) as it often exits following the merger of m and n. Without this restriction, such references would be erroneously directed were the merging firm to be the least profitable. Rather than repeatedly restate this, we rule it out here.

[11] In terms of the model's primitives, this requires $c_1 < P(Q^{\star\star})$, where $Q^{\star\star} = \sum_{h \in \mathscr{H}} -(P(Q^{\star\star}) - c_h)/P'(Q^{\star\star})$ and $\mathscr{H} \equiv \{\{\mathscr{F} \setminus \{m, n\}\} \cup M\}$. Doing so rules out cases where a nonmerging firm exits because its variable cost exceeds the equilibrium price charged when the firm stays in the market. An efficiencies offense can still arise in such cases, although they are much less interesting, and if we do not rule them out, then they complicate exposition.

which in turn implies

$$\bar{c}_M^i = (N - i - 1)P(Q^\star) + P'(Q^\star)Q^\star - \sum_{g \in \mathcal{G}_i} c_g, \tag{8}$$

where $\mathcal{G}_i = \{\{i+1, ..., N\} \backslash \{m, n\}\}$ and $\bar{c}_M^i$ denotes the value of c_M associated with CSN_i. Applying the same normalizations as we did above, CSN_i is given by

$$\frac{c - \bar{c}_M^i}{c} = \frac{1}{c}\left(c - (N - i - 1)P(Q^\star) - P'(Q^\star)Q^\star + \sum_{g \in \mathcal{G}_i} c_g\right). \tag{9}$$

We can extend the indexing of the CSN curves to construct CSN_0, which measures the synergies required for a CS-neutral merger when no firm exits. As this particular case is isomorphic to a model in which market structure is held fixed, it coincides with traditional merger analysis. Specifically, if we replace c with the share-weighted average of premerger variable costs of the merging firms, then CSN_0 corresponds to what is typically called the "compensating marginal cost reduction" (or simply "CMCR," as in Miller and Sheu (2020)). To avoid confusion, we refer to it for the remainder of this paper as the "fixed-market-structure CMCR."

We can also now formally define the efficiencies offense.

Definition 1. *If $\bar{c}_M^i < c_M < \tilde{c}_M^i$ for any $i \in \{\mathcal{F} \backslash \{m, n\}\}$, then an efficiencies offense arises under Cournot competition.*

Three criteria are met. First, $c_M < \tilde{c}_M^i$ ensures that synergies induce nonmerging firms to exit. Second, given the aforementioned exit, $c_M > \bar{c}_M^i$ ensures that the merger reduces CS. Third, were market structure held fixed, $c_M < \bar{c}_M^0$ ensures that the merger would have raised CS.[12]

By extension, we can now depict the contours of the efficiency offense. To do so, we compare ZRP and CSN curves. For each, we plot normalized variable cost reductions on the vertical axis against normalized fixed costs on the horizontal axis. We then shade regions corresponding to mergers that reduce CS. For comparability, our parameter choices are the same as the ones Nocke and Whinston (2022) use in their leading example. Fourteen firms whose variable costs are symmetric prior to the merger face linear demand and a price elasticity equal to 1.5.

Figure I reports the result. First, turn attention to CSN_0. As we mentioned above, its height corresponds to the synergies that would be required for a CS-neutral merger under the assumption that market structure is held fixed. As such, it reflects one of the key takeaways in the paper from which our parameters are drawn: even in a market like this one where several firms compete for price-sensitive consumers, synergies of 5% are required to prevent consumer harm.[13] However, as the shaded regions above CSN_0 indicate, when we allow market structure to equilibrate, 5% no longer suffices. Many mergers whose synergies far exceed this figure reduce CS.

This is especially true when we restrict attention to fixed costs that rationalize premerger market structure, which coincide with market conditions that firms are likely to face. Since exactly N firms enter prior to the merger, Φ must lie on the half-open interval $(0.88, 1]$.[14] In this region of the graph, which lies to

[12] The definition impose that $c_M < \tilde{c}_M^i$. By comparing the expressions for $\tilde{c}_M^i$ and $\bar{c}_M^0$, it is easy to see that $\tilde{c}_M^i < \bar{c}_M^0$ for all $i \geq 1$. Hence, $c_M < \bar{c}_M^0$. Thus, the merger would have been CS positive in the absence of exit.

[13] For the first instance, see the abstract of Nocke and Whinston (2022).

[14] When firms have symmetric constant marginal costs and face linear inverse demand, individual gross profit takes form

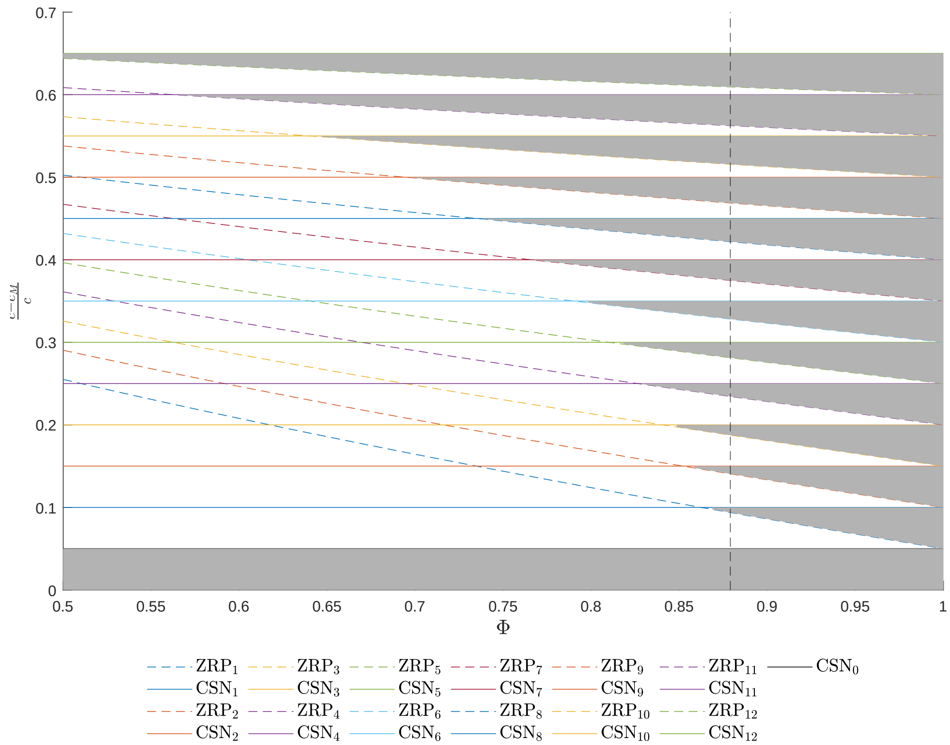

Figure I: *Contours of the efficiency offense in the Cournot model*

This figure plots normalized fixed costs (Φ_f) on the horizontal axis against normalized merger synergies ($(c - c_M)/c$) on the vertical axis. Dashed lines represent ZRP curves, while solid lines represented CSN curves. The height of CSN$_0$ corresponds to the fixed-market-structure CMCR, which is the benchmark against which synergies are compared in traditional merger analysis. To improve legibility, horizontal and vertical axis values range from 0.5 to 1 and from 0 to 0.7, respectively.

the right of the vertical dashed line, most of the area is shaded—*the majority of mergers here reduce consumer welfare by inducing exit.*

It is equally striking that each CSN curve intersects its subsequently indexed ZRP curve. To see the importance, assume that the least profitable nonmerging firm breaks even prior to the merger, i.e., $\Phi = 1$. Under this assumption, comparative statics on synergies equate to movements along the right-hand side vertical axis. Now, starting at zero, gradually increase synergies so that they cross CSN$_0$ and ZRP$_1$. By definition of CSN$_0$, this is exactly the point at which, were market structure held fixed, a merger would turn CS-positive. Yet, by definition of ZRP$_1$, this is also exactly the point at which the least profitable nonmerging firm exits, resulting in a discontinuous decline in CS. In other words, precisely when traditional merger analysis predicts synergies are large enough to reduce price *below* its premerger level, we predict that price rises sharply *above* its premerger level—CS is non-monotone in synergies, exhibiting a saw-tooth pattern as they increase.

$(P(0) - c)^2/((N+1)^2 P')$, where $P(0)$, c, and P' are constants (wherever P' is well-defined). Since exactly 14 firms stay in the market, fixed cost must exceed the gross profit that a 15^{th} firm would earn, so $\phi > (P(0) - c)^2/(16^2 P')$. We normalize fixed cost using premerger equilibrium gross profit. In this numerical example, this means $\Phi = \phi/[(P(0) - c)^2/(15^2 P')]$. Putting this all together, we have $\Phi > [(P(0) - c)^2/(16^2 P')]/[(P(0) - c)^2/(15^2 P')] = 15^2/16^2 \approx 0.88$. By similar logic, fixed cost must not exceed the gross profits that 14 firms earn, so $\phi \leq (P(0) - c)^2/(15^2 P')$, which implies $\Phi \leq 15^2/15^2 = 1$.

2.3 General characterizations

The lessons learned from Figure I apply more broadly. We formalize them here. Our first proposition considers a market in which, prior to the merger, the least profitable firm breaks even. The proposition states that any merger which raises CS when market structure is held fixed will cause the least profitable firm to exit. This implies that our earlier observation—CSN_0 intersects ZRP_1 at $\Phi_1 = 1$—always holds. Neither symmetry, a specific elasticity, nor a particular number of firms are required.

Proposition 1. *In the Cournot model, assume that the least profitable firm breaks even prior to the merger. If $\mathscr{M}$ would be CS-positive were market structure held fixed, then $\mathscr{M}$ causes the least profitable firm to exit.*

Proof. Define $\pi_1(Q)$ equal to the gross profit that the least profitable firm earns given that Q is produced. We have

$$\begin{aligned}
\frac{\partial \pi_1(\cdot)}{\partial Q} &= \frac{-2(P(Q)-c_1)(P'(Q))^2 + (P(Q)-c_1)^2 P''(Q)}{P'(Q)^2} \\
&= 2\frac{P(Q)-c_1}{-P'(Q)}P'(Q) + \left(\frac{P(Q)-c_1}{-P'(Q)}\right)^2 P''(Q) \\
&\propto 2P'(Q) + q_1 P''(Q) \\
&< 0.
\end{aligned}$$

To arrive at the first equality, differentiate, and to arrive at the second, one, simplify the resulting expression. To obtain the proportionality relationship, substitute q_1 for $-[P(Q)-c_1]/P'(Q)$, and factor q_1 out. To arrive at the inequality, recall that $P'(Q) < 0$ and $P'(Q) + QP''(Q) \leq 0$. If $P''(Q) \leq 0$, then $2P'(Q) + q_1 P''(Q) \leq 2P'(Q) < 0$. Alternatively, if $P''(Q) > 0$, then $2P'(Q) + q_1 P''(Q) < 2P'(Q) + QP''(Q) < P'(Q) + QP''(Q) < 0$. In either case, $2P'(Q) + q_1 P''(Q) < 0$.

Let $\tilde{Q}$ denote postmerger aggregate quantity. Since CS is strictly increasing in Q, and given that $\mathscr{M}$ would be CS-positive were market structure held fixed, $\tilde{Q} > Q^\star$. Given that $\partial\pi_1(\cdot)/\partial Q < 0$ and $\tilde{Q} > Q^\star$, $\pi_1(\tilde{Q}) < \pi_1(Q^\star)$. Since $\pi_1(Q^\star) = \phi_1$, $\pi_1(\tilde{Q}) < \phi_1$, so the least profitable firm exits after the merger. ■

Under the same conditions, we can produce a closed-form solution to the synergies required for a merger to increase CS.

Corollary 1. *In the Cournot model, assume that the least profitable firm breaks even prior to the merger. Denote the price elasticity of demand by $\epsilon = -\partial log(Q)/\partial log(P(Q))$, and denote the sum of the premerger shares of the merging firms and the least profitable firm by $\check{s} = s_1^\star + s_m^\star + s_n^\star$. Also, denote their average share-weighted premerger variable cost by $\check{c} = (s_1^\star c_1 + s_m^\star c_m + s_n^\star c_n)/\check{s}$. Finally, denote their within-merger concentration by $HHI_M = ({s_1^\star}^2 + {s_m^\star}^2 + {s_n^\star}^2)/\check{s}^2$ and their within-merger concentration change by $\Delta HHI_M = 2(s_m^\star s_n^\star + s_m^\star s_1^\star + s_n^\star s_1^\star)$. If $\mathscr{M}$ is CS-positive, then*

$$\frac{\check{c} - c_M}{\check{c}} > \frac{\Delta HHI_M}{\check{s}\,(\epsilon - \check{s}HHI_M)}. \tag{10}$$

Proof. We have

$$
\begin{aligned}
\frac{\check{c} - c_M}{\check{c}} &> \frac{\check{c} - \bar{c}_M^1}{\check{c}} \\
&= \frac{s_m^\star c_m + s_n^\star c_n + s_1^\star c_1 - \check{s}\bar{c}_M^1}{s_m^\star c_m + s_n^\star c_n + s_1^\star c_1} \\
&= \frac{s_m^\star c_m + s_n^\star c_n + s_1^\star c_1 - \check{s}\left[c_m + c_n + c_1 - 2P\left(Q^*\right)\right]}{s_m^\star c_m + s_n^\star c_n + s_1^\star c_1} \\
&= \frac{s_m^\star \left[P\left(Q^*\right) - c_n\right] + s_m^\star \left[P\left(Q^*\right) - c_1\right] + s_n^\star \left[P\left(Q^*\right) - c_m\right]}{s_m^\star c_m + s_n^\star c_n + s_1^\star c_1} \\
&\quad + \frac{s_n^\star \left[P\left(Q^*\right) - c_1\right] + s_1^\star \left[P\left(Q^*\right) - c_m\right] + s_1^\star \left[P\left(Q^*\right) - c_n\right]}{s_m^\star c_m + s_n^\star c_n + s_1^\star c_1} \\
&= \frac{s_m^\star P\left(Q^*\right) \frac{s_n^\star}{\epsilon} + s_m^\star P\left(Q^*\right) \frac{s_1^\star}{\epsilon} + s_n^\star P\left(Q^*\right) \frac{s_m^\star}{\epsilon}}{s_m^\star P\left(Q^*\right)\left[1 - \frac{s_m^\star}{\epsilon}\right] + s_n^\star P\left(Q^*\right)\left[1 - \frac{s_n^\star}{\epsilon}\right] + s_1^\star P\left(Q^*\right)\left[1 - \frac{s_1^\star}{\epsilon}\right]} \\
&\quad + \frac{s_n^\star P\left(Q^*\right) \frac{s_1^\star}{\epsilon} + s_1^\star P\left(Q^*\right) \frac{s_m^\star}{\epsilon} + s_1 P\left(Q^*\right) \frac{s_n^\star}{\epsilon}}{s_m^\star P\left(Q^*\right)\left[1 - \frac{s_m^\star}{\epsilon}\right] + s_n^\star P\left(Q^*\right)\left[1 - \frac{s_n^\star}{\epsilon}\right] + s_1^\star P\left(Q^*\right)\left[1 - \frac{s_1^\star}{\epsilon}\right]} \\
&= \frac{\frac{2s_m^\star s_n^\star + 2s_m^\star s_1^\star + 2s_n^\star s_1^\star}{\epsilon}}{\check{s}\left[1 - \frac{s_m^{\star 2} + s_n^{\star 2} + s_1^{\star 2}}{\check{s}\epsilon}\right]} \\
&= \frac{\Delta HHI_M}{\check{s}\left[\epsilon - \check{s}HHI_M\right]}.
\end{aligned}
$$

If $\mathscr{M}$ is CS positive, then $c_M < \bar{c}_M^1$, which implies the inequality. To obtain the first equality, substitute $(s_1^\star c_1 + s_m^\star c_m + s_n^\star c_n)/\check{s}$ for $\check{c}$. By definition, $\bar{c}_M^1$ leaves CS unchanged, so it leaves aggregate quantity unchanged. Since $\mathscr{M}$ does not affect the variable costs of nonmerging firms, and since each firm's equilibrium quantity depends only on its variable cost and aggregate quantity, the equilibrium quantities produced by the nonmerging firms do not change as a result of the merger. Thus, the postmerger quantity produced by M must equal the sum of the premerger quantities produced by m, n, and the least profitable firm. Since each firm f produces $-(P(Q^\star) - c_f)/P'(Q^\star)$, $P(Q^\star) - \bar{c}_M^1 = 3P(Q^\star) - c_m - c_n - c_1$, so $\bar{c}_M^1 = c_m + c_n + c_1 - 2P(Q^\star)$. Make this substitution to obtain the second equality. Rearrange terms to obtain the third equality, and simplify the expression to obtain the fourth equality. Substitute in for ϵ to obtain the fifth equality, and substitute in with ΔHHI_M, HHI_M, and $\check{s}$ to obtain the sixth equality. ■

To put Corollary 1 in perspective, consider the case where all firms are symmetric. If market structure is held fixed, then the synergies required for a CS-positive merger must exceed $s^\star/(\epsilon - s^\star)$, as shown by Nocke and Whinston (2022). However, if market structure equilibrates, then synergies must exceed $\Delta HHI_M/3s^\star(\epsilon - 3s^\star HHI_M)$, which is exactly twice the previous figure.[15]

Proposition 1 and Corollary 1 are special in that they require a firm to break even prior to the merger. We now relax that restriction. Instead, we make the natural assumption that fixed costs rationalize premerger market structure. Doing so requires hypothetically expanding the market by one firm. For simplicity, we

[15] If we substitute in for ΔHHI_M and HHI_M, then the numerator equals $6s^{\star 2}$ and denominator equals $3s^\star(\epsilon - 3s^\star[3s^{\star 2}/9s^{\star 2}]) = 3s^\star(\epsilon - s^\star)$. The fraction simplifies to $2 \times s^\star/(\epsilon - s^\star)$.

impose that the added firm's variable and fixed costs are equal to those of the least profitable firm.[16] As our next proposition shows, these conditions ensure that synergies exist which reduce CS by inducing exit.[17]

Proposition 2. *In the Cournot model, assume the market has one additional firm whose variable and fixed costs equal those of the least profitable firm. Then, given any fixed costs that rationalize N firms stay prior to the merger, there exist synergies that create an efficiencies offense.*

Proof. See the appendix.

3 Price competition with differentiated products

3.1 Primitives and equilibrium

Now, consider a game that is similar to the one described above with one exception. In the prior section, firms set quantities for homogeneous products. In this one, firms set prices for differentiated products that face multinomial logit demand.

Just prior to the start of the game, each firm $f \in \mathscr{F}$ is assigned a set $\mathscr{K}_f$ of products. This set determines the firm's type, T_f, which is defined as $\sum_{k \in \mathscr{K}_j}(v_k - \alpha c_k)$. Here, k indexes products, v_k denotes the mean utility of consumption, c_k denotes the variable cost of production, and α measures sensitivity to price. Firms are sorted in increasing order of their types from 1 to N. The first stage proceeds as it did earlier—firms decide to stay or exit. In the second stage, each firm j that stays in the market sets a price p_k for each $k \in \mathscr{K}_j$ and earns gross profit equal to $\sum_{j \in \mathscr{K}_j}(p_k - c_k)s_k(p))$, where s_k denotes market share. Given multinomial logit demand, k's market share takes the form

$$s_k(p) = \frac{e^{v_k - \alpha p_k}}{1 + \sum_{k' \in \mathscr{K}} e^{v_{k'} - \alpha p_{k'}}}, \tag{11}$$

where $\mathscr{K} = \cup_j \mathscr{K}_j$.

Recall that firms maximize profits and that the production technology provides constant returns to scale. In the second stage, given all firms $f < J$ exit, the price that firm $f \geq J$ charges for k satisfies

$$(p_k - c_k)\,\alpha s_k(p)\,(1 - s_k(p)) - s_k(p) + \sum_{k' \in \mathscr{K}_f} \left[(p_{k'} - c_{k'})\,\alpha s_{k'}(p)\,(1 - s_{k'}(p)) - s_{k'}(p) \right] = 0, \tag{12}$$

which implies

$$\alpha(p_k - c_k) = 1 + \alpha \sum_{k' \in \mathscr{K}_f} (p_{k'} - c_{k'})\,s_{k'}(p). \tag{13}$$

Remarkably, Nocke and Schutz (2018) show that not only is the pricing game described here aggregative but also each firm's strategy can be neatly summarized as a scalar. Borrowing their notation, we denote the

[16]This model nests one with symmetric fixed costs and one with asymmetric fixed costs in which the disturbances are expectational errors (i.e., ν_2 in the notation of Pakes et al. (2015), which has been widely adopted in empirical literature).

[17]The added firm did not enter prior to the merger, so we can bound its fixed cost, which are also the least profitable firm's fixed costs. For this reason, the added firm provides information about when the least profitable firm will exit.

aggregator by H. It equals $1 + \sum_{k \in \mathscr{K}} e^{v_k - \alpha p_k}$. We also denote the aforementioned scalars by μ_f and refer to them as ι-markups. Each is given by the left-hand side of equation 13.

Equilibrium is characterized by three conditions that derive from profit maximization, demand, and an adding-up constraint. Given all $f < J$ exit, the conditions provide

$$\mu_f = \frac{1}{1 - s_f}, \tag{14}$$

$$s_f = \frac{T_f e^{-\mu_f}}{H}, \tag{15}$$

and

$$\frac{1}{H} + \sum_{f \geq J} s_f = 1, \tag{16}$$

respectively. Each firm f earns gross profit equal to

$$\pi_f(J) = \frac{1}{\alpha}(\mu_f - 1). \tag{17}$$

In the first stage, each firm f stays in if and only if $\pi_f(f) \geq \phi_f$.

Merger $\mathscr{M}$ combines m and n into M and creates synergies by increasing types such that $T_M \geq T_m + T_n$. As in Section 2, we denote premerger equilibrium objects using an asterisk and, unless otherwise stated, make the standard assumption that all firms earn positive profit prior to the merger.

3.2 Zero-rival-profit and consumer-welfare-neutrality curves

The type of merged firm required for firm i to break even, provided all $i' < i$ exit, is given by

$$\tilde{T}_M^i = \left(1 - \frac{1}{\mu_M^i(\phi_i)}\right) e^{-\mu_M^i(\phi_i)} H^i(\phi^i), \tag{18}$$

where

$$H^i(\phi_i) = \frac{T_i(1 + \alpha\phi_i)}{\alpha\phi_i} e^{-(1+\alpha\phi_i)},$$

$$\mu_M^i(\phi_i) = \frac{H(\phi^i)}{2 + \alpha\phi_i + \sum_{f \in \mathscr{G}_i} T_f e^{-\mu_f^i(\phi_i)}},$$

and $\mathscr{G}_i = \{\{i+1, ..., N\} \backslash \{m, n\}\}$, and where ι-markups are implicitly defined by

$$\mu_f^i(\phi_i)\left(1 - \frac{T_f}{H^i(\phi_i)} e^{-\mu_f^i(\phi_i)}\right) = 1$$

for all $f \in \mathscr{G}$. In the Nash-Bertrand model, it is most natural to normalize synergies by dividing through by the sum of the types of the merging firms, as the resulting ratio has a straightforward interpretation: since M can obtain $T_m + T_n$ through mere joint ownership, it scales $\mathscr{M}$'s contribution to CS when all products are priced at variable cost. It also directly maps to variable cost reductions. As Nocke and Whinston (2022) show, if $\mathscr{M}$ reduces the variable costs of products offered by m and n by the same amount Δc, then the log of the type ratio is directly proportional to the variable cost reduction:

$$\begin{aligned}\log\left(\frac{T^M}{T^m+T^n}\right) &= \log\left(\frac{\sum_{j\in\{m,n\}}\exp(v_j-\alpha c_j-\alpha\Delta c)}{\exp(v_m-\alpha c_m)+\exp(v_n-\alpha c_n)}\right)\\ &= \log\left(\frac{\exp\left(-\alpha\Delta c\right)\sum_{j\in\{m,n\}}\exp(v_j-\alpha c_j)}{\exp(v_m-\alpha c_m)+\exp(v_n-\alpha c_n)}\right)\\ &= -\alpha\Delta c.\end{aligned}$$

We substitute $\Phi_i \equiv \phi_i/\pi_i^\star$ for ϕ_i and $\tilde{T}_M^i/(T_m+T_n)$ for $\tilde{T}_M^i$, which provides the curve ZRP$_i$:

$$\frac{\tilde{T}_M^i}{T_m+T_n} = \frac{1}{T_m+T_n}\left(1-\frac{1}{\mu_M^i(\Phi_i)}\right)e^{-\mu_M^i(\Phi_i)}H^i(\Phi^i). \tag{19}$$

The type of merged firm required to make $\mathscr{M}$ CS-neutral, provided all $i'<i$ exit, is given by

$$\bar{T}_M^i = \left(s_m^\star+s_n^\star+\sum_{f<i}s_f^\star\right)e^{-\mu_M(H^\star)}H^\star. \tag{20}$$

Apply the normalization, we have the curve CSN$_i$:

$$\frac{\bar{T}_M^i}{T_m+T_n} = \frac{1}{T_m+T_n}\left(s_m^\star+s_n^\star+\sum_{f<i}s_f^\star\right)e^{-\mu_M(H^\star)}H^\star. \tag{21}$$

As in the previous section, we can extend the indexing of the CSN curves to construct CSN$_0$ by computing $\bar{c}_M^i$ when $i=0$.

We can now formally define the efficiencies offense when firms choose prices rather than quantities.

Definition 2. *If $\tilde{T}_M^i < T_M < \bar{T}_M^i$ for any $i \in \{\mathscr{F}\backslash\{m,n\}\}$, then an efficiencies offense arises under Nash-Bertrand competition.*

Recall that we require the following: synergies induce a nonmerging firm to exit; accounting for the ensuing exit, the merger reduces CS; and ignoring the ensuing exit, the merger raises CS. First, $T_M > \tilde{T}_M^i$ ensures all $i' \le i$ exit. Second, given all $i' \le i$ exit, $T_M < \bar{T}_M^i$ ensures that the merger reduces CS. Third, were no firms to exit, $T_M > T_M^0$ ensures that the merger would have raised CS.

We can also now visualize the contours of the efficiencies offense under price competition. We plot the curves defined by the equations 19 and 21. Borrowing parameters from the leading example in Nocke and Whinston (2022), we assume symmetric firms, each of which have a 10% market share prior to the merger and face an own-price elasticity of 5.

Figure II reports the result. Its basic features closely resemble those of Figure I. Evidenced by the shaded portions of the graph above CSN$_0$, many mergers that would be CS-positive were market structure held fixed are CS-negative when market structure is allowed to endogenously adjust. This is especially true when we restrict attention to fixed costs that rationalize premerger market structure—the area to the right of the vertical, dashed line—suggesting that efficiencies offense may commonly arise in "real world" situations. Third, each CSN curve intersects its subsequently indexed ZRP curve on the unit interval. Thus, in markets where one or more firms are only marginally profitable, the efficiencies offense may be the rule rather than exception.

Equally salient, each CSN curve intersects its subsequently indexed ZRP curve on the unit interval. Thus, in markets where one or more firms are only marginally profitable, the efficiencies offense may be the rule rather than exception.

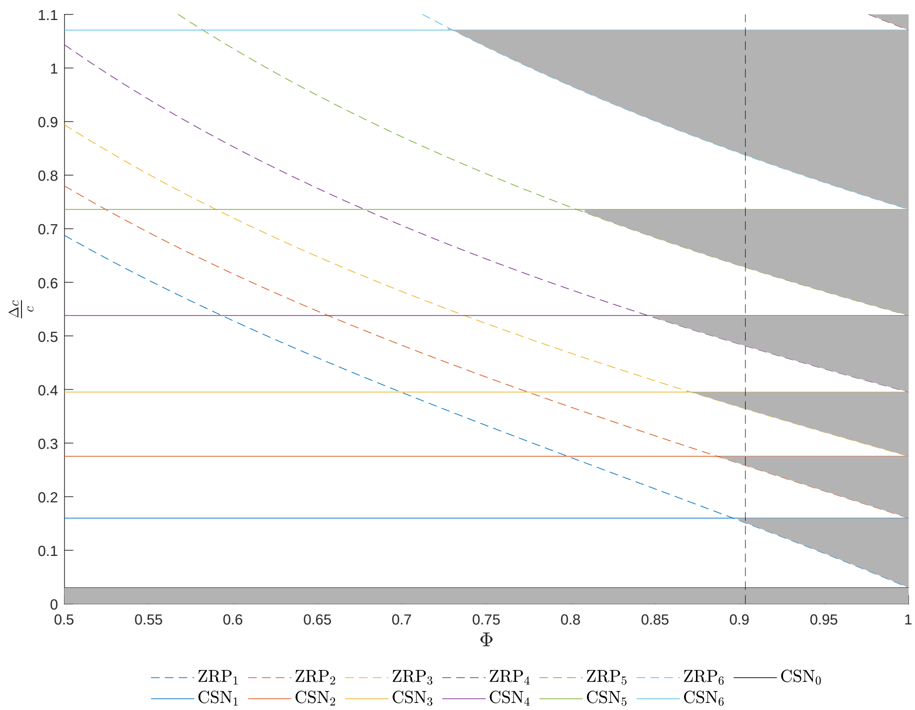

Figure II: *Contours of the efficiency offense in the Nash-Bertrand model*

This figure plots normalized fixed costs (Φ_f) on the horizontal axis against marginal cost changes (as function of normalized type changes $T_M/(T_m + T_n)$, as shown above) on the vertical axis. Dashed lines represent ZRP curves, while solid lines represented CSN curves. The height of CSN_0 corresponds to the fixed-market-structure CMCR, which is the benchmark against which synergies are compared in traditional merger analysis. To improve legibility, horizontal and vertical axis values range from 0.5 to 1 and from 0 to 1.1, respectively.

3.3 General characterizations

The Nash-Bertrand framework we analyze bears important similarities to the one in which firms compete à la Cournot. For instance, their first stages are qualitatively the same, while their second stages are both aggregative. As a consequence, the broad lessons we took away from Section 2.3 apply here. We begin with Proposition 3, which is analogous to Proposition 1.

Proposition 3. *In the Nash-Bertrand model, assume that the least profitable firm breaks even prior to the merger. If $\mathscr{M}$ would be CS-positive were market structure held fixed, then $\mathscr{M}$ causes the least profitable firm to exit.*

Proof. See the appendix.

We can also produce a closed-form solution for the type increase required for a CS-positive merger. It is identical to what one obtains when market structure is held fixed but for one difference, which is that $s_1 \exp(1/(1-s_1))$ also appears in the denominator.

Corollary 2. *In the Nash-Bertrand model, assume that the least profitable firm breaks even prior to the merger. Let $\check{s} = s_1^\star + s_m^\star + s_n^\star$ denote the sum of the premerger shares of the merging firms and the least profitable firm. If $\mathscr{M}$ is CS-positive, then*

$$\frac{T_M}{T_m + T_n} > \frac{\check{s}\exp(\frac{1}{1-\check{s}})}{s_m^\star \exp(\frac{1}{1-s_m^\star}) + s_n^\star \exp(\frac{1}{1-s_n^\star})}.$$

Proof. See the appendix.

Finally, we can relax the restriction that at least one firm breaks even prior to the merger and instead consider fixed costs that rationalize premerger market structure. As in the prior section, under these conditions, synergies always exist that create an efficiencies offense.

Proposition 4. *In the Nash-Bertrand model, assume the market has one additional firm whose type and fixed cost equal those of the least profitable firm. Then, given any fixed costs that rationalize N firms stay prior to the merger, there exist synergies that create an efficiencies offense.*

Proof. See the appendix.

4 Potential defenses

In this section, we address other market responses that might mitigate the efficiency offense.

4.1 Subsequent entry

All else equal, exit makes markets more attractive to potential entrants, so a merger that causes exit is more likely to induce subsequent entry. If this occurs and the entrant has an especially low variable cost (or high type), then the merger, exit, and subsequent entry together might raise CS, even though the merger and exit alone would reduce CS. That is, in theory, entry can remedy the harm that consumers would otherwise endure. In practice, however, this may be hard to achieve, as it requires a very special type of potential entrant. Conceptually, the new arrival must be efficient enough to offset harm caused by the loss of two competitors, efficient enough to profitably enter after this has occurred, but not so efficient as to enter before this has occurred. Technically, such cases permit only a narrow set of parameters.

We formalize this argument under Cournot competition.[18] Let e index the new entrant, and denote its variable and fixed costs by c_e and ϕ_e, respectively. Define $\mathscr{H} = \{\mathscr{F} \backslash \{m, n, 1\}\}$, $\mathscr{I} = \{\{\mathscr{F} \backslash \{m, n, 1\}\} \cup \{M, e\}\}$, and $\mathscr{J} = \{\mathscr{F} \cup e\}$, and let $\hat{Q}$ and $\hat{\hat{Q}}$ denote the aggregate quantities produced when $\mathscr{I}$ and $\mathscr{J}$ stay in the market, respectively. Before the merger occurs, e does not enter, so we can bound ϕ_e from below in relation to c_e. Similarly, after the merger occurs and the least profitable firm exits, e enters, so we can bound ϕ_e from above in relation to c_e. That is, we can write

$$\underline{\phi}_e \equiv \frac{(P(\hat{\hat{Q}}) - c_e)^2}{-P'(\hat{\hat{Q}})} < \phi_e \leq \frac{(P(\hat{Q}) - c_e)^2}{-P'(\hat{Q})} \equiv \overline{\phi}_e. \tag{22}$$

The merger, exit, and subsequent entry are together CS-positive, we can also bound c_e in a way that does not depend on ϕ_e. We compute the sum of the first order conditions, solve for $Q^\star$ and $\hat{Q}$, compare their values, and rearrange terms to arrive at

$$c_e < P(\hat{Q}) + \sum_{h \in \mathscr{H}} (P(\hat{Q}) - c_h) + P(\hat{Q}) - c_M - \frac{\hat{P}'(\hat{Q})}{P'(Q^\star)} \sum_{f \in \mathscr{F}} (P(Q^\star) - c_f). \tag{23}$$

To visualize the set of mergers that satisfy these inequalities, we plot the set of permissible fixed and variable costs. To preserve comparability to earlier results, we impose the same assumptions used to construct Figure I. Likewise, we normalize the entrant's fixed costs by dividing them through by premerger gross profit, and we express the entrant's variable costs as a percentage off those of the incumbents.[19] In this example, we assume that $\mathscr{M}$ reduces variable cost by 7%. This is 2% points greater than the fixed-market-structure CMCR (see, e.g., Figure I), so $\mathscr{M}$ reduces CS by causing the least profitable firm to exit.

Figure III reports the result. Two features are especially salient. One is the narrowness of the shaded area. Only a thin "slice" of the parameter space is permitted by the model, which is consistent with our claim that entrants cannot be widely relied upon to remedy the efficiencies offense. The other is that the shaded area extends to the right of zero. Even inefficient entrants—ones whose variable costs exceed those of incumbents—can restore CS. Of course, such firms must have very low fixed costs to profitably enter. For example, an entrant with 1% higher variable costs requires 37% lower fixed costs.

4.2 Follow-on mergers

Managers that prefer to run their firms independently (for any number of business or personal reasons) will be forced to at least reconsider when faced with exit. As a result, firms that turn unprofitable because their rivals have merged will themselves be more likely to merge. If this occurs and the follow-on merger generates synergies, then the two mergers together might be CS-neutral, even though the first merger (and ensuing exit) would reduce it.

Again, we formally assess this scenario under Cournot competition. Consider a follow-on merger

[18] Caradonna et al. (2024) characterize entry induced by postmerger profit increases, concisely enumerating its limits (e.g., if a merger attracts entrants whose products strongly appeal to consumers, then the merger will never be proposed in the first place, as it will be unprofitable). As we consider the joint effect of entry *and* postmerger exit on postmerger entry, we view this section as extending part of their results to reflect the efficiencies offense.

[19] Specifically, we set $\underline{\Phi}_e = \underline{\phi}_e / \pi_1^\star$, $\overline{\Phi}_e = \overline{\phi}_e / \pi_1^\star$, and replace c_e with $(c_e - c)/c$.

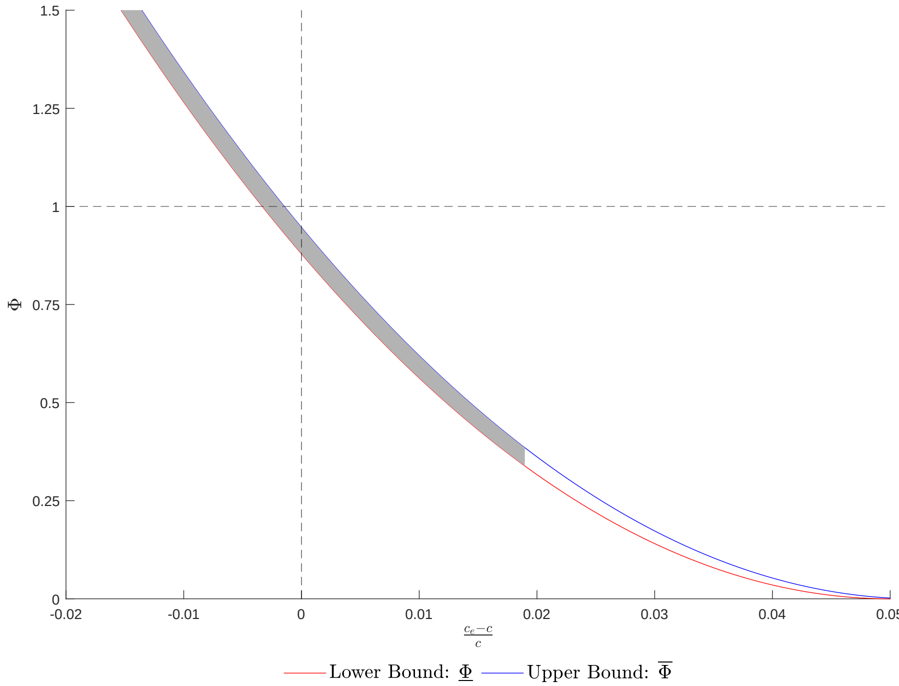

Figure III: *Types of entrants that remedy an efficiency offense*

This figure plots normalized merger synergies ($(c_e - c)/c$) on the horizontal axis against normalized fixed costs (Φ_e) on the vertical axis. The red and blue lines correspond to lower and upper bounds provided by inequalities 22, respectively. Synergies are also bounded from above in absolute terms by inequality 23. The shaded area corresponds to parameter values that remedy the efficiencies offense.

$\mathscr{M}'$ that combines the least profitable firm with another firm in $\mathscr{F}$, indexed by m', to form M'. To ease exposition, assume the follow-on merger is "disjoint" in the sense that $m' \neq M'$, and to rule out uninteresting cases, assume synergies satisfy $c_{M'} < \min\{c_1, c_{m'}\}$. Define $\mathscr{G} = \{\mathscr{F} \backslash \{1, m, m', n\}\}$ and $\mathscr{H} = \{\mathscr{F} \backslash \{1, m'\}\}$. We compute the sum of the first order conditions and then solve for aggregate quantity, which remains unchanged at $Q^\star$, as $\mathscr{M}$ and $\mathscr{M}'$ are together CS-neutral:

$$Q^\star = \sum_{g \in \mathscr{G}} \frac{P(Q^\star) - c_g}{-P'(Q^\star)} + \frac{P(Q^\star) - c_M}{-P'(Q^\star)} + \frac{P(Q^\star) - c_{M'}}{-P'(Q^\star)}. \tag{24}$$

We then multiply through by $P'(Q^\star)$, add and subtract $2P - c_m - c_n$, and solve for $c_{M'}$. We arrive at

$$c_{M'} = \left[P(Q^\star) + P'(Q^\star)Q^\star + \sum_{h \in \mathscr{H}} (P(Q^\star) - c_h) + P(Q^\star) \right] + \left[\Big(c_m + c_n - P(Q^\star) \Big) - c_M \right]. \tag{25}$$

The first bracketed term represents the variable cost required of M' to make $\mathscr{M}'$ CS-neutral were it to occur in isolation. The second bracketed term represents the difference between the variable cost required of M to make $\mathscr{M}$ CS-neutral were it to occur in isolation and the actual variable cost of M.

Since $\mathscr{M}$ reduces c_M beyond the fixed-market-structure CMCR (see, e.g., Figure I or Proposition 1),

$c_M < c_m + c_n - P(Q^\star)$. This implies that the second bracketed term is greater than zero and, in turn, that the first bracketed term is less than $c_{M'}$. In short, the synergies required to make $\mathscr{M}'$ CS-neutral are lower when the merger occurs alongside $\mathscr{M}$. That is, as Nocke and Whinston (2010) have shown, two disjoint, CS non-decreasing mergers are complementary in the sense that once one occurs, it is harder for the next to harm consumers. By extension, if $\mathscr{M}'$ creates the same synergies as $\mathscr{M}$, then the two mergers together are guaranteed to increase CS. This is precisely the assumption made by Motta and Vasconcelos (2005), who use it to show that forward-looking antitrust authority facing a series of mergers could make different decisions than a myopic one.

Whether follow-on mergers can be practically relied upon is an open question. In a broad sense, the answer depends critically on what *merger technology* firms possess. At the present moment, little is known about these capabilities. Few papers cleanly identify merger synergies; even fewer use firm or market characteristics to broadly predict how mergers will reduce cost or improve quality. In a narrow sense, it depends how "special" m and n are. On the one hand, business conditions might permit any pair of firms m' and n' to consolidate their operations as adroitly as m and n, mitigating the concerns we raise. On the other hand, m and n might represent a rare opportunity—one that has been carefully selected by highly experienced, skilled, and motivated bankers, consultants, and managers for its match value—in which case any follow-on merger $\mathscr{M}'$ will yield few improvements in terms of quality or cost.

We expect that the latter rather than former view conforms with reality. One reason is that, as Farrell and Shapiro (1990) state, "mergers differ enormously in the extent to which productive assets can usefully be recombined." Another reason is that firms commonly express skepticism about their own prospects to generate synergies. As just one example, consider remarks made by American Vanguard, a fertilizing producer, in its recent report to investors. It first states that "industry consolidation may threaten the Company's position in various markets," that "many of the Company's competitors have grown or are expected to grow through mergers and acquisitions," and that "consequently, the Company may find it more difficult to compete in various markets." Most interestingly, it then states that "while such merger activity may generate acquisition opportunities for the Company, there is no guarantee that the Company will benefit from such opportunities."[20]

4.3 Other considerations

At least three other issues may arise in practice. Katz (2002) crisply summarizes them as follows.[21] First, he argues that authorities must "balance post-exit harms against the consumer gains that would arise during any period in which the merged firm enjoyed the efficiencies and its rivals still were viable competitors." This is undoubtedly an important consideration. Even though a comprehensive analysis of dynamic situations in which viability depreciates over time is beyond the scope of this paper, we can capture its essential features by modifying our framework.

For the purposes of this illustration, we assume Cournot competition calibrated to the parameters used to produce Figure I. Given the point at issue, we also assume the least profitable firm breaks even prior to

[20] See the firm's December 31, 2022 10-K filing, which is available at https://shorturl.at/Lr2gc.

[21] Katz (2002) also questions "how likely it is that merged firm would have incentives to price so low that it would deter existing or new rivals from making investments needed to remain or become viable competitors." Our analysis reveals that only static profit maximization is necessary to produce an efficiencies offense. In short, such incentives may be very likely. To be clear, though, we are making generalizations, whereas the author's statements should be interpreted in the context of a particular case—General Electric's proposed acquisition of Honeywell—and its individual circumstances. Moreover, given the author's knowledge of the transaction and cogent analysis of its expected effects, we defer to his assessment.

the merger, which would be CS-positive were market structure held fixed. We denote the discount factor δ and the number of years it would take an unprofitable firm to exit by y. We define $\mathscr{H} = \{\mathscr{F} \backslash \{m, n\} \cup M\}$ and $\mathscr{I} = \{\mathscr{F} \backslash \{1, m, n\} \cup M\}$, define $\hat{Q}$ and $\hat{\hat{Q}}$ as the aggregate quantities produced when $\mathscr{H}$ and $\mathscr{I}$ stay in the market.

The present value of the consumer welfare change caused by $\mathscr{M}$ is given by

$$\text{CS Present Value} = \frac{\delta(1-\delta^y)}{1-\delta}\left(\int_0^{Q'}(P(x)-P(Q'))dx - \int_0^{Q^\star}(P(x)-P(Q^\star))dx\right) \tag{26}$$

$$+\frac{\delta^{y+2}}{1-\delta}\left(\int_0^{Q''}(P(x)-P(Q''))dx - \int_0^{Q^\star}(P(x)-P(Q^\star))dx\right). \tag{27}$$

The first term is an annuity comprised of consumer gains, which accrue while the least profitable firm remains in the market. The second term is a perpetuity comprised of CS losses, which begin accruing when the least profitable firm exits and must be discounted to the time of the merger. To put these tradeoffs in perspective, if $\delta = 0.9$, then approval turns on whether the least profitable firm would exit in under 6.1 years. If consumers are more or less patient, then the period is longer or shorter. For example, if $\delta = 0.925$, then approval turns on whether the least profitable firm will sustain losses for 8.4 years. Our subjective view is that most firms would exit long before this time has elapsed.

Second, one may question "the abilities of enforcement authorities to reliably predict this type of harm even after a detailed investigation of market conditions" (Katz, 2002). Our analysis highlights that, at least under the stated assumptions, the only inputs that are required to study the efficiencies offense but absent from traditional merger analysis are fixed costs; demand, marginal costs, and synergies are required irrespective of whether the agencies and courts wish to consider the postmerger viability of rivals. Further, at the risk of oversimplifying their measurement, fixed costs can often be read directly off the firms' income statements, making them much cheaper to obtain than any of the data required for second stage estimation.

Moreover, were a preliminary investigation to suggest that one or more firms is only marginally profitable prior to the merger, our analysis suggests that uncertainty about synergies works *against* the merger rather than *for* it. (Recall, for example, the right-hand sides of Figures I or II, where the majorities of the graphs are shaded.) Alternatively, of course, were a preliminary investigation to reveal operationally and financial sound competitors, then one can presumably set these concerns aside. As an example, in the case referenced by Katz (2002), which involved General Electric's proposed acquisition of Honeywell, the competitive set comprised Rolls-Royce and Pratt & Whitney. At the time, both rivals boasted "growing revenues and profits" as well as plans to "heavily [invest] in the development of their next-generation [products]." Facts such these mitigate or even eliminate the risk of merger-induced exit.

Third, one should "consider the broader or longer-term implications of a policy that attacks mergers suspected of having these effects." In markets comprised mainly of firms that would survive against a more efficient rival, exhaustively investigating synergistic mergers amounts to a tax on transactions that help rather than harm consumers. However, in markets where relevant rivals are clearly at risk, one could incorporate an efficiency offense without deterring CS-positive mergers. Other unintended consequences are harder to sign. For instance, foreseeing an efficiency offense, merging parties may delay CS-positive investments until their transaction is complete but may also opt to internally develop capabilities that they would otherwise have obtained through acquisition. In the latter case, even if the least profitable firm exits, two rather than one efficient firms may remain.

5 Conclusions

This paper formalizes the efficiencies offense, which arises when a merger's synergies *reduce* consumer welfare by inducing rivals to exit. Mid-twentieth century jurisprudence broadly accepted this theory of harm, although the argument was especially easy to endorse in an era when judges valued competition per se, thereby placing positive weight on the surplus of rival producers. However, as the Chicago School ushered in the consumer welfare standard, it also rejected the efficiencies offense—not just unconditionally but also often derisively.

We find that the efficiencies offense deserves more nuanced treatment and may arise much more frequently than previously thought. This is especially true when one or more nonmerging firms are only marginally profitable, and it is even more so the case when high fixed costs are the underlying cause. In these instances, the synergies required for a CS-positive merger often far exceed the figures suggested by traditional merger analysis, which holds market structure fixed (i.e., does not allow nonmerging firms to endogenously exit if they turn unprofitable). By extension, our results strengthen recent, influential claims by Nocke and Whinston (2022), who argue that the *Guidelines* published by the DOJ and FTC over the last four decades are too lax, resulting in too few deals being screened in, challenged, and even blocked.

New questions arise that are beyond this paper's scope. How often does synergy-induced exit by nonmerging firms occur? Is it rare, or, like many other economic phenomena, simply hard to identify? If it is rare, then which of our assumptions is most commonly violated? If it is not, then are even large firms in economically important industries at risk, as anecdotes presented at the outset of the paper indicate. Is this mechanism obscured because firms are slow to exit, reflecting underlying transaction costs, contracting problems, option value in the face of uncertainty, or even mistakes by boundedly rational managers who learn about the viability of their enterprises over time? Alternatively, does the mechanism simply manifest as follow-on mergers, precipitated by investors who are "forced" to sell?

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Appendix

Proof of Proposition 2.

First, recall that gross profit takes the form $-(P(Q) - c_f)^2/P'(Q)$. Since exactly N firms stay prior to $\mathscr{M}$, $-(P(Q^\star) - c_1)^2/P'(Q^\star) \geq \phi_1 \equiv \overline{\phi}$ and $-(P(\hat{\hat{Q}}) - c_1)^2/P'(\hat{\hat{Q}}) < \phi_1 = \underline{\phi}$, where $\hat{\hat{Q}}$ denotes the quantity produced if all $N+1$ firms stay in the market. Hence, fixed costs lie weakly above $\underline{\phi}$ and strictly below $\overline{\phi}$.

Next, recall that $\tilde{c}^1_M$ is given by $(N-1)P(Q^1(\phi_1)) + P'(Q^1(\phi_1))Q^1(\phi_1) - \sum_{h \in \mathscr{H}_1} c_h$, where $\mathscr{H}_i = \{\mathscr{F} \backslash \{m, n\}\}$, and recall that $\bar{c}^1_M$ is given by $(N-2)P(Q^\star) + P'(Q^\star)Q^\star - \sum_{g \in \mathscr{G}_1} c_g$, where $\mathscr{G}_1 = \{\{2, 3, ..., N\} \backslash \{m, n\}\}$. Let $\phi^\star$ denote the value of ϕ_1 that sets these expressions equal. Since $\tilde{c}^1_M$ is increasing in ϕ_1 while $\bar{c}^1_M$ does not depend on it, $\tilde{c}^1_M > \bar{c}^1_M$ for all $\phi_1 > \phi^\star$. Hence, so long as $\phi_1 > \phi^\star$, there exists a c_M that satisfies $\bar{c}^1_M < c_M < \tilde{c}^1_M$. Thus, it suffices to prove that $\underline{\phi} > \phi^\star$.

Let $\mathscr{A}$ denote a hypothetical market with a set $\mathscr{I}$ of firms, where $\mathscr{I} = \{\mathscr{F} \backslash \{1, m, n\} \cup M\}$ and $c_M = \tilde{c}^1_M = \bar{c}^1_M$. Let $\mathscr{A}'$ denote $\mathscr{A}$ with one additional firm that has variable cost c_1. Let $\mathscr{B}$ denote a hypothetical market with the set $\mathscr{F}$ of firms, and let $\mathscr{B}'$ denote $\mathscr{B}$ with one additional firm that has variable cost c_1. Additionally, let $\hat{Q}$ denote the aggregate quantity produced in $\mathscr{A}'$. Notice that $Q^\star$ is produced in $\mathscr{A}$ and $\mathscr{B}$ and that $\hat{\hat{Q}}$ is produced in $\mathscr{B}'$.

By definition, when the variable cost M is $\tilde{c}^1_M$ and the set of firms that stay in the market is given by $\mathscr{H}_1$, the least profitable firm breaks even. $\mathscr{A}'$ corresponds exactly to these conditions, so the least profitable firm breaks even. Also, by definition, when $\tilde{c}^1_M = \bar{c}^1_M$, $\phi_1 = \phi^\star$. Hence, the gross profit of the least profitable firm in $\mathscr{A}'$ equals $\phi^\star$. Separately, notice that the gross profit of the least profitable firm in $\mathscr{B}'$ equals $\underline{\phi}$. Thus, it suffices to prove that the gross profit of the least profitable firm in $\mathscr{A}'$ is less than the gross profit of either of the least profitable firms in $\mathscr{B}'$.

Take the first order condition of gross profit with respect to individual quantity and sum over firms. In $\mathscr{A}$, arrive at

$$Q^\star P'(Q^\star) + (N-2)P(Q^\star) - \sum_{i \in \mathscr{I}} c_i - c_M = 0, \tag{28}$$

and in $\mathscr{B}$, arrive at

$$Q^\star P'(Q^\star) + NP(Q^\star) - \sum_{f \in \mathscr{F}} c_f = 0. \tag{29}$$

In both $\mathscr{A}$ and $\mathscr{B}$, aggregate quantity equals $Q^\star$. Solve the system of equations and obtain $c_M = c_1 + c_m + c_n - 2P(Q^\star)$.

In $\mathscr{A}'$, sum the first order conditions and arrive at

$$\hat{Q}P'(\hat{Q}) + (N-1)P(\hat{Q}) - \sum_{i \in \mathscr{I}} c_i - c_1 - c_M = 0, \tag{30}$$

substitute $c_1 + c_m + c_n - 2P(Q^\star)$ for c_M, and rearrange terms to obtain

$$\hat{Q}P'(\hat{Q}) + (N+1)P(\hat{Q}) - \sum_{f \in \mathscr{F}} c_f - c_1 + 2(P(Q^\star) - P(\hat{Q})) = 0. \tag{31}$$

Finally, in $\mathscr{B}'$, obtain the same sum and arrive at

$$\hat{\hat{Q}}P'(\hat{\hat{Q}}) + (N+1)P(\hat{\hat{Q}}) - \sum_{f \in \mathscr{F}} c_f - c_1 = 0. \tag{32}$$

Set the left-hand sides of equations 31 and 32 equal and rearrange terms so that

$$[\hat{Q}P'(\hat{Q}) - \hat{\hat{Q}}P'(\hat{\hat{Q}})] + N[P(\hat{Q}) - P(\hat{\hat{Q}})] - 2[P(\hat{Q}) - P(Q^\star)] = 0. \tag{33}$$

All else equal, adding a firm increases aggregate quantity, so $\hat{Q} > Q^\star$. Since $P'(Q) < 0$, $P(\hat{Q}) < P(Q^\star)$, which implies $P(\hat{Q}) - P(Q^\star) < 0$, which in turn implies $-2(P(\hat{Q}) - P(Q^\star)) > 0$. To balance the equation, $\hat{\hat{Q}}P'(\hat{\hat{Q}}) + NP(\hat{\hat{Q}})$ must exceed $\hat{Q}P'(\hat{Q}) + NP(\hat{Q})$. Since $P'(Q) + QP''(Q) < 0$, the first set of terms are decreasing in Q, and since $P'(Q) < 0$ and $N > 0$, the second set of terms are decreasing in Q, so $\hat{Q}$ must exceed $\hat{\hat{Q}}$. Hence, aggregate quantity is greater in $\mathscr{A}'$ than $\mathscr{B}'$. Since the gross profit of the least profitable firm is declining in aggregate quantity, the gross profit of the least profitable firm in $\mathscr{A}'$ is less than the gross profit of one of the least profitable firms in $\mathscr{B}'$. ■

Proof of Proposition 3.
Let $H^{\star\star}$, $\mu_1^{\star\star}$, $\pi_1^{\star\star}$ denote postmerger equilibrium values of H, μ_1, and π_1, respectively. The least profitable firm breaks even prior to the merger, meaning that $\pi_1^\star = \phi_1$. If $\pi_1^{\star\star} < \pi_1^\star$, then $\pi_1^{\star\star} < \phi_1$, which means that the least profitable firm exits following $\mathscr{M}$. Hence, it suffices to prove that $\pi_1^{\star\star} < \pi_1^\star$. $\mathscr{M}$ would be CS-positive were market structure held fixed, and CS is strictly increasing in H, so $H^{\star\star} > H^\star$. Thus, it suffices to prove that variable profit is strictly decreasing in H.

Combine equations 14 and 15 and then arrange terms to arrive at $\mu_f(1 - (T_f/H)\exp(-\mu_f)) = 1$. Based on this relationship, let $m(T_f/H)$ denote the implicit function that maps T_f/H to μ_f. Proposition 6 of Nocke and Schutz (2018) states that $m' > 0$, implying that μ_f is strictly decreasing in H. Per equation 17, variable profit is given by $(\mu_f - 1)/\alpha$. α is positive, so this expression is strictly increasing in μ_f. Since variable profit is strictly increasing in μ_f, which is strictly decreasing in H, variable profit is strictly decreasing in H. ■

Proof of Corollary 2.
We have

$$\begin{aligned}
\frac{T_M}{T_m + T_n} &> \frac{\bar{T}_M^1}{T_m + T_n} \\
&= \frac{\bar{T}_M^1/H^\star}{T_m/H^\star + T_n/H^\star} \\
&= \frac{\bar{T}_M^1/H^\star}{s_m^\star \exp(\frac{1}{1-s_m^\star}) + s_n^\star \exp(\frac{1}{1-s_n^\star})} \\
&= \frac{S(\bar{T}_M^1/H^\star)\exp\left(\frac{1}{1-S(\bar{T}_M^1/H^\star)}\right)}{s_m^\star \exp(\frac{1}{1-s_m^\star}) + s_n^\star \exp(\frac{1}{1-s_n^\star})} \\
&= \frac{(s_m^\star + s_n^\star + s_1^\star)\exp(\frac{1}{1-s_m^\star-s_n^\star-s_1^\star})}{s_m^\star \exp(\frac{1}{1-s_m^\star}) + s_n^\star \exp(\frac{1}{1-s_n^\star})} \\
&= \frac{\check{s}\exp(\frac{1}{1-\check{s}})}{s_m^\star \exp(\frac{1}{1-s_m^\star}) + s_n^\star \exp(\frac{1}{1-s_n^\star})}.
\end{aligned}$$

By Proposition 6 of Nocke and Schutz (2018), consumer surplus is increasing in T_f for every f. Hence, if $\mathscr{M}$ is CS-positive, then T_M must exceed $\bar{T}_M^1$, which implies the inequality. Multiply the numerator and denominator by $1/H^\star$ to obtain the first equality. Combine equations 14 and 15 to obtain

$$\frac{T_f}{H} = s_f \exp\left(\frac{1}{1 - s_f}\right) \tag{34}$$

for every f. Substitute in for $T_m/H^\star$ and $T_n/H^\star$ using equation 34 to obtain the second equality. Equation 34 shows that each firm's market share depends only on its type and the aggregator, so write

$$s_f = S(T_f/H). \tag{35}$$

Combine equations 34 and 35, and substitute in for $\bar{T}_M^1/H^\star$ to obtain the third equality. Rearrange terms in equation 16 to write

$$1 - \frac{1}{H^\star} = \sum_{f\in\mathscr{F}} S(T_f/H^\star) = S(\bar{T}_M^1/H^\star) + \sum_{h\in\mathscr{H}} S(T_h/H^\star), \tag{36}$$

where $\mathscr{H} = \{\{\mathscr{F}\backslash\{1, m, n\}\}$, which implies that $S(\bar{T}_M^1/H^\star) = S(T_m/H^\star) + S(T_n/H^\star) + S(T_1/H^\star) = s_1^\star + s_m^\star + s_n^\star$. Substitute in for $S(\bar{T}_M^1/H^\star)$ to obtain the fourth equality. Finally, substitute in with $\check{s}$ to obtain the fifth equality. ■

Proof of Proposition 4.
Since exactly N firms stay prior to $\mathscr{M}$, $(\hat{\hat{\mu}}_i - 1)/\alpha \leq \phi_1$ and $\leq (\mu^\star - 1)/\alpha \geq \phi_1$, where $\hat{\hat{\mu}}_i$ denote i's ι-markup if all of the $N+1$ firms stay. The inequalities imply that $\bar{\phi} = (\mu^\star - 1)/\alpha$ and $\underline{\phi} = (\hat{\hat{\mu}}_i - 1)/\alpha$, where $\overline{\phi}$ and $\underline{\phi}$ denote the lower and upper bounds, respectively, for feasible values of ϕ_1.

Let $\phi^\star$ be the value of ϕ_1 at which $\tilde{T}_M^1 = \bar{T}_M^1$. $\tilde{T}_M^1$ is strictly increasing in ϕ_1, while $\bar{T}_M^1$ does not depend on ϕ_1, so for any $\phi > \phi^\star$, there exists a T_M such that $\tilde{T}_M^1 < T_M < \bar{T}_M^1$. Therefore, it suffices to prove $\phi^\star < \underline{\phi}$.

Let $\mathscr{A}$ denote a hypothetical market with a set $\mathscr{H}$ of firms, where $\mathscr{H} = \{\mathscr{F} \backslash \{1, m, n\} \cup M\}$ and $T_M = \tilde{T}_M^i = \bar{T}_M^i$. Let $\mathscr{A}'$ denote $\mathscr{A}$ but with one additional firm that has type T_1. Let $\mathscr{B}'$ denote a hypothetical market with the set $\mathscr{F}$ of firms, and let $\mathscr{B}'$ denote $\mathscr{B}$ with one additional firm that has type T_1. Notice that $\phi^\star$ equals the variable profit that the least profitable firm earns in $\mathscr{A}'$. Also, notice that $\underline{\phi}$ equals the variable profit that either of the least profitable firms earn in in $\mathscr{B}'$. Hence, it suffices to prove that the variable profit of the least profitable firm in $\mathscr{A}'$ is less than the variable profit of either of the least profitable firms in $\mathscr{B}'$.

In both $\mathscr{A}$ and $\mathscr{B}$, the aggregator equals $H^\star$. Thus, we can use the adding-up constraints to obtain

$$\frac{1}{H^\star} + \sum_{f \in \mathscr{H}} s_f^\star + s_M^\star = 1. \tag{37}$$

and

$$\frac{1}{H^\star} + \sum_{f \in \mathscr{F}} s_f^\star = 1. \tag{38}$$

We can then solve the system of equations and obtain $s_M^\star = s_1^\star + s_m^\star + s_n^\star$. The adding-up constraint also provides

$$\frac{1}{\hat{H}} + \sum_{f \in \mathscr{H}} \hat{s}_f + \hat{s}_1 + \hat{s}_M = 1 \tag{39}$$

where $\hat{H}$ denotes the aggregator and $\hat{s}_i$ denote i's share in $\mathscr{A}'$, which can be rewritten as

$$\frac{1}{\hat{H}} + \sum_{f \in \mathscr{F}} \hat{s}_f + \hat{s}_1 + \hat{s}_M - (\hat{s}_1 + \hat{s}_m + \hat{s}_n) = 1. \tag{40}$$

Again, the adding-up constraint provides

$$\frac{1}{\hat{\hat{H}}} + \sum_{f \in \mathscr{F}} \hat{\hat{s}}_f + \hat{\hat{s}}_1 = 1, \tag{41}$$

where $\hat{\hat{H}}_i$ denotes the aggregator and $\hat{\hat{s}}_i$ denotes i's share in $\mathscr{B}'$. Set the left-hand sides of equations 40 and 41 equal and rearrange terms to obtain

$$\left[\frac{1}{\hat{H}} - \frac{1}{\hat{\hat{H}}_i}\right] + \sum_{f \in \mathscr{F}} \left[\hat{s}_f - \hat{\hat{s}}_f\right] + \left[\hat{s}_1 - \hat{\hat{s}}_1\right] + \left[\hat{s}_M - \hat{s}_1 - \hat{s}_m - \hat{s}_n\right] = 0. \tag{42}$$

We will now prove that $\hat{s}_M - \hat{s}_1 - \hat{s}_m - \hat{s}_n > 0$. We combine equations 14 and 15 to obtain

$$\frac{T_f}{H} = s_f \exp\left(\frac{1}{1 - s_f}\right) \tag{43}$$

for every f, which implies that each firm's market share depends only on its type and the aggregator, which, in turn, implies that we can write $s_f = S(T_f/H)$. We differentiate $S(T_M/H) - S(T_1/H) - S(T_m/H) - S(T_n/H)$ with respect to H and evaluate the resulting expression at $H^\star$ to obtain

$$\begin{aligned}
-\frac{T_M}{H^\star} S'\left(\frac{T_M}{H^\star}\right) + \frac{T_1}{H^\star} S'\left(\frac{T_1}{H^\star}\right) + \frac{T_m}{H^\star} S'\left(\frac{T_m}{H^\star}\right) + \frac{T_n}{H^\star} S'\left(\frac{T_n}{H^\star}\right)& \\
= -\epsilon\left(\frac{T_M}{H^\star}\right) S\left(\frac{T_M}{H^\star}\right) + \epsilon\left(\frac{T_1}{H^\star}\right) S\left(\frac{T_1}{H^\star}\right) + \epsilon\left(\frac{T_m}{H^\star}\right) S\left(\frac{T_m}{H^\star}\right) + \epsilon\left(\frac{T_n}{H^\star}\right) S\left(\frac{T_n}{H^\star}\right)& \\
= -\epsilon\left(\frac{T_M}{H^\star}\right) \left[S\left(\frac{T_1}{H^\star}\right) + S\left(\frac{T_m}{H^\star}\right) + S\left(\frac{T_n}{H^\star}\right)\right] + \epsilon\left(\frac{T_1}{H^\star}\right) S\left(\frac{T_1}{H^\star}\right) + \epsilon\left(\frac{T_m}{H^\star}\right) S\left(\frac{T_m}{H^\star}\right) + \epsilon\left(\frac{T_n}{H^\star}\right) S\left(\frac{T_n}{H^\star}\right)& \\
= \left[\epsilon\left(\frac{T_1}{H^\star}\right) - \epsilon\left(\frac{T_M}{H^\star}\right)\right] S\left(\frac{T_1}{H^\star}\right) + \left[\epsilon\left(\frac{T_m}{H^\star}\right) - \epsilon\left(\frac{T_M}{H^\star}\right)\right] S\left(\frac{T_m}{H^\star}\right) + \left[\epsilon\left(\frac{T_n}{H^\star}\right) - \epsilon\left(\frac{T_M}{H^\star}\right)\right] S\left(\frac{T_n}{H^\star}\right)& \\
> 0.&
\end{aligned}$$

To arrive at the second line, use Lemma 5 of Nocke and Schutz (2024), which states that the elasticity of S is given by $\epsilon(x) = xS'/S(x)$, and replace $xS'(x)$ with $\epsilon(x)S(x)$, where $x \in \{T_M/H^\star, T_m/H^\star, T_n/H^\star, T_1/H^\star\}$. (The derivation of Lemma 5 of Nocke and Schutz (2024) appears in Section XXXi.3 of the Online Appendix of Nocke and Schutz (2018).) To arrive at the third line, replace $S(T_M/H^\star)$ with $s^\star_m + s^\star_n + s^\star_1$, and to arrive at the fourth line, rearrange terms. To arrive at the fifth line, use Lemma XXV of Nocke and Schutz (2018), which states that $\epsilon(x)$ is strictly decreasing in x, and the fact that $T_M > \max\{T_1, T_m, T_n\}$. Together, they imply that $\epsilon(T_\ell/H^\star) - \epsilon(T_M/H^\star) > 0$. Since $S(T_M/H) - S(T_1/H) - S(T_m/H) - S(T_n/H)$ is increasing in H, and since$\tilde{H} > H^\star$, $\tilde{s}_M - \tilde{s}_m - \tilde{s}_n - \tilde{s}_1 > \hat{s}_M - \hat{s}_1 - \hat{s}_m - \hat{s}_n$. Moreover, since $\hat{s}_M - \hat{s}_1 - \hat{s}_m - \hat{s}_n = 0$, $\tilde{s}_M - \tilde{s}_m - \tilde{s}_n - \tilde{s}_1 > 0$.

To balance both sides of equation 42, given $\hat{s}_M - \hat{s}_1 - \hat{s}_m - \hat{s}_n > 0$, we must have

$$\left[\frac{1}{\hat{H}} - \frac{1}{\hat{\hat{H}}_i}\right] + \sum_{f \in \mathscr{F}} \left[\hat{s}_f - \hat{\hat{s}}_f\right] + \left[\hat{s}_1 - \hat{\hat{s}}_1\right] < 0. \tag{44}$$

$1/\hat{H} - 1/\hat{\hat{H}}_i < 0$ if and only if $\hat{H} > \hat{\hat{H}}$. Also, shares are strictly decreasing in the aggregator, so $\hat{s}_f < \hat{\hat{s}}_f$ if and only if $\hat{H} > \hat{\hat{H}}$. Thus, inequality 44 implies $\hat{H} > \hat{\hat{H}}$, which implies $\hat{\mu}_1 < \hat{\hat{\mu}}_1$, which implies that the variable profit of the least profitable firm in $\mathscr{A}'$ is less than the variable profit of either of the least profitable firms in $\mathscr{B}'$. ■