NBER WORKING PAPER SERIES

THERE IS NO EXCESS VOLATILITY PUZZLE

Andrew Atkeson
Jonathan Heathcote
Fabrizio Perri

Working Paper 32481
http://www.nber.org/papers/w32481

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
May 2024

The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis, the Federal Reserve System, or the National Bureau of Economic Research. We would like to thank Ellen McGrattan, Moto Yogo, Stavros Panageas, Yueran Ma, Andrea Eisfeldt, Stijn Van Nieuwerburgh, Zhengyang Jiang, Dan Greenwald, Sydney Ludvigson, and Martin Lettau for many helpful conversations on this topic.

There is No Excess Volatility Puzzle
Andrew Atkeson, Jonathan Heathcote, and Fabrizio Perri
NBER Working Paper No. 32481
May 2024
JEL No. G0,G12

ABSTRACT

We present two valuation models that we use to account for the annual data on price per share and dividends per share for the CRSP Value-Weighted Index from 1929-2023. We show that it is a simple matter to account for these data based purely on a model of variation in the expected ratio of dividends per share to aggregate consumption over time under two conditions. First, investors must receive news shocks regarding the expected ratio of dividends per share to aggregate consumption in the long run. Second, the discount rate used to evaluate the impact of this news on the current price per share must be low. We argue that both of these conditions are likely satisfied in the data. Because our valuation model reproduces the data on price per share and dividends per share exactly over this long time period, it also reproduces realized values of returns, dividend growth, the dividend-price ratio, and all Campbell-Shiller-style regression results involving these variables. Thus, we conclude that the answer to Shiller (1981)'s question "Do stock prices move too much to be justified by subsequent movements in dividends?" is No.

Andrew Atkeson
Bunche Hall 9381
Department of Economics
UCLA
Box 951477
Los Angeles, CA 90095-1477
and NBER
andy@atkeson.net

Jonathan Heathcote
Federal Reserve Bank of Minneapolis
Research Department
90 Hennepin Ave.
Minneapolis, MN 55401
heathcote@minneapolisfed.org

Fabrizio Perri
Research Department
Federal Reserve Bank of Minneapolis
90 Hennepin Avenue
Minneapolis MN 55401
and CEPR
fperri@umn.edu

1 Introduction

Shiller (1981) famously posed the question "Do stock prices move too much to be justified by subsequent movements in dividends?" An important body of work in finance argues that the answer to this question is yes. See, for example, Leroy and Porter (1981), Campbell and Shiller (1987), Campbell and Shiller (1988), Cochrane (2011), and Shiller (2014).

In this paper, we call into question the conclusions drawn from this prior work. In particular, we present two valuation models that we use to account for the annual data on price per share and dividends per share for the CRSP Value-Weighted Index from 1929-2023 based on a simple model of the dynamics of agents' expectations of future dividends per share valued at constant discount rates. We use these valuation models to show that it is a straightforward exercise to account for these aggregate stock market data based purely on a model of variation in the expected ratio of dividends per share to aggregate consumption over time under two conditions:

1. First, investors must receive news shocks regarding the expected ratio of dividends per share to aggregate consumption in the long run.

2. Second, the discount rate that determines the impact of this news on the current price per share must be low.

We argue that both of these conditions are likely satisfied in the data.

We offer the results of these two valuation exercises as evidence that the fluctuations that we have observed in the value of the aggregate stock market from 1929-2023 can be accounted for by a reasonable model of fluctuations in investors' expectations of future dividends without reliance on changes in discount rates, bubbles, or behavioral explanations of stock market volatility. We do not see our accounting models as the final word on the question of what drives stock market volatility. Instead, we see our paper as breathing new life into the old hypothesis that changing expectations of future dividends play an important, or even dominant role, in driving aggregate stock market volatility.

Our results are related to the large literature following Campbell and Shiller (1987) and Campbell and Shiller (1988) looking for evidence of time-varying expected returns on the aggregate stock market as a rationalization of stock market volatility. Because our valuation model reproduces the data on price per share and dividends per share exactly over this long time period, it also reproduces realized values of returns, dividend growth, and the dividend-price ratio. It follows that all Campbell-Shiller style regression results involving these variables replicate exactly in simulated model output, including those on return predictability.

In our two valuation exercises, we wish to account for the dynamics of annual data on the ratio of price per share for the CRSP Value-Weighted Index to Personal Consumption Expenditures (PCE) over the period 1929-2023 as shown in the left panel of Figure 1. As is evident in this figure, the ratio of this measure of stock prices to consumption is quite volatile over time. We aim to account for the realized values of this series for stock prices for each year on the basis of a simple time series model of the dynamics of the ratio of dividends per share for the CRSP Value-Weighted Index to PCE, where the realized data for dividends per share relative to consumption are shown in the right panel of Figure 1.

One striking feature of these data on the ratio of dividends per share to consumption is that it is unclear what long-run value the ratio is converging to. Motivated by this uncertainty, our first valuation model assumes that the dynamics of the ratio of dividends per share to consumption, D_t/C_t, follows a first-order autoregressive process with a shifting endpoint. This endpoint, which we denote by X_t, represents agents' expectations at time t of the value of the ratio of dividends per share to consumption to which this series will converge in the long run. In particular,

$$\frac{D_{t+1}}{C_{t+1}} - X_t = \rho\left(\frac{D_t}{C_t} - X_t\right) + \sigma_D \epsilon_{D,t+1}, \tag{1}$$

where ρ is the AR(1) persistence parameter, and $\epsilon_{D,t+1}$ is a normally-distributed innovation.

This endpoint X_t itself follows a random walk, with innovations given by $\epsilon_{X,t+1}$:

$$X_{t+1} = X_t + \sigma_X \epsilon_{X,t+1}. \tag{2}$$

Given this process for dividends per share relative to consumption, we price equity in a standard way. We assume that the representative investor's pricing kernel has the property that the price of a perpetual claim to aggregate future consumption relative to current consumption, which we denote by γ^X, is time-invariant. It is in this sense that the discount rate in our model is constant. Estimates of this price-dividend ratio for a consumption claim in the literature are typically high, or even infinite (see, for example, Alvarez and Jermann, 2004, Lustig, Van Nieuwerburgh, and Verdelhan, 2013, or Gârleanu and Panageas, 2023).

Given our dividend process and this assumptions of a constant discount rate, we show that the level of the ratio of the price per share to consumption is linear in two state variables: (i) the transitory deviation of the ratio of dividends per share to consumption from its long-run value, and (ii) the long-run value as represented by X_t:

$$\frac{P_{Dt}}{C_t} = \gamma^D\left(\frac{D_t}{C_t} - X_t\right) + \gamma^X X_t + \phi. \tag{3}$$

The term ϕ captures the price impact of risk associated with fluctuations in the ratio of dividends to consumption. In our analysis we should that with a standard conditionally-lognormal pricing kernel this risk term ϕ does not vary over time. Given the affine structure of our valuation equation 3, we refer to this first valuation model as our affine valuation model.

Note that γ^X, the price of a claim to future consumption relative to current consumption, captures the impact at the margin of a news shock at t regarding expected long-run dividends, $\epsilon_{X,t+1}$ on the current price per share to consumption ratio. Thus, when we use high estimates of the value for γ^X, we find that the series $\{X_t\}$ for long-run expected dividends per share to consumption required to account for the annual data on the ratio of CRSP price per share to consumption shows only modest variation over time: see this result in Figure 4. That is, small news shocks about the long run are all that is required to account for observed stock market volatility.

In contrast, we find that if γ^X is small, meaning that claims to future consumption are discounted at a high rate, then news about the long run does not have a large impact on current stock prices, and implausibly large fluctuations in expected dividends X_t are required to account for the observed dynamics of stock prices.

Our affine valuation model has the property that a component of dividend growth is forecastable (deviations of dividends from their long run expected value should gradually disappear) and this forecastable component is reflected in the equilibrium price to dividend ratio. In contrast, given our constant discount rate assumption, equity returns are largely unpredictable, and are driven by unanticipated shocks to current and expected dividends.[1]

An important test of the model is to check whether the innovations that our measurement exercise uncovers to both the long-run expected value of the ratio of dividends per share to consumption ($\epsilon_{X,t+1}$) and to the autoregressive deviations of this ratio from its long-run value ($\epsilon_{D,t+1}$) are in fact unforecastable. If, contrary to the model assumptions, these innovations are in fact forecastable, then our model agents should be forecasting them, and they should show up in valuations. We find that these innovations are close to zero on average and do not show significant autocorrelation over time. Moreover, they do not appear to be forecast by the current price dividend ratio or by its logarithm.

To the best of our knowledge, the affine equity valuation model that we present is new to the literature. We derive it using standard techniques together with an application of Stein's Lemma to compute products of the exponent of a normal random variable with the level of that variable. This affine equity valuation model has the unfamiliar property that

[1]If expected consumption growth is constant and the risk term in the pricing equation ϕ is equal to zero, then expected equity returns are constant. We will show that when $\phi \neq 0$, the model generates modest time variation in expected returns.

adjustments for the risk of a claim to dividends over and above the risks of a claim to consumption are additive (the ϕ term in eq. 3). That property is what generates the result that, at the margin, news about the long run ratio of dividends per share to consumption is discounted at a rate associated with the pricing of a claim to aggregate consumption rather than at a rate associated with the pricing of a claim to equity itself.

In contrast to this affine valuation model for equity, most of the prior literature derives valuation formulas for equity claims in which the log (rather than the level) of a claim to dividends per share relative to consumption is linear the state variables. We develop such a model as our second valuation model. In this exponentially affine model, we assume that it is the log (rather than the level) of the ratio of the ratio of dividends per share to consumption that follows a first-order autoregressive process with a shifting endpoint, x_t, denoting the log of the value of this ratio that investors expect in the long run. We now assume that x_t follows a random walk and news about the long run corresponds to shocks to $x_{t+1} - x_t$.

In this second valuation model, the equity price impact of news about changes in x_t depends on the value of the price-dividend ratio for a claim to equity (rather than a claim to consumption) at the point at which the ratio of dividends per share to aggregate consumption is equal to its long-run value. We argue based on realized data for the CRSP price-dividend ratio shown in the right panel of Figure 3 that there is considerable uncertainty about this value. Early on in the past century, the price-dividend ratio was close to 20 or 25. More recently, it has been close to 45 or 50.

We use our second valuation model to show that if one assumes a price-dividend ratio of 45 or above, as in recent years, then our second valuation model delivers similar results to our affine model and thus no excess volatility puzzle. In contrast, if one assumes a price-dividend ratio of 20 or 25, as is often done in the literature (see, for example, Cochrane, 2011 or page 134 of Campbell, 2018), then there is an excess volatility puzzle.

Which of these long-run price dividend ratios is the right one for valuing equity? The answer to that question depends on one's view of what has happened in the stock market over the last century? Do we imagine that the ratio of dividends per share to consumption was close to its long run value early on in that time period and that this ratio of dividends per share to consumption has drifted away from its long run value due to unexpected negative shocks driving it down continuously over the following decades? Or do we imagine that the ratio of dividends per share to consumption was far away from its long run value early on in the sample and slowly converged towards its long run value as would be expected if this ratio followed an autoregressive process? With our exponentially affine valuation model, if one takes the first view, then concludes that stock prices are excessively volatile. If one takes the second view, one does not.

A reader who is not familiar with the construction of measures of price per share and dividends per share for equity indices such as the CRSP Value-Weighted Index might wonder why one would have significant uncertainty about the ratio of dividends per share to aggregate consumption in the long run and, likewise, about the long-run ratio of price per share to dividends per share. We argue that much of the uncertainty about the long-run values of these two ratios is not driven by economic fundamentals but instead is driven by what are called *corporate actions* that impact the number of shares outstanding for firms included in the index. These corporate actions include entry of new firms into the index, exit of firms from the index, mergers and acquisitions, new equity issuances and repurchases of shares by incumbent firms in the index.

As noted by Dichev (2006), Boudoukh et al. (2007), Larraine and Yogo (2008), Gârleanu and Panageas (2023), and Davydiuk et al. (2023) among others, these corporate actions generate large differences in the dynamics of dividends per share relative to aggregate dividends, large differences in aggregate dividends relative to total cash flows to owners of equity, and large differences in the dynamics of price per share relative to aggregate equity market capitalization (see Figure 2). In particular, it appears in that figure that a large share of the movements in the ratio of dividends per share to aggregate consumption that we see in the data over the past century have been driven by the dynamics of corporate actions. Thus, a large portion of the uncertainty about the long-run ratio of dividends per share to aggregate consumption that is so important for the volatility of the ratio of price per share to aggregate consumption likely reflects uncertainty about future corporate actions.

The impact of corporate actions on measures of stock market value and dividends extends to the observed ratio of price per share to dividends per share, which is typically used as a measure of the price-dividend ratio in empirical asset pricing. As argued by Miller and Modigliani (1961), what is likely fundamental in valuing equity are the total cash flows to equity holders. They note that firms can use corporate actions to alter the dynamics of their dividends while holding fixed these total cash flows to equity holders. Thus, they argue that firms can alter the dynamics of their price-dividend ratio simply through changing their policy for paying dividends. We suspect that such corporate actions have also played a large role in driving the observed dynamics of the ratio of price per share to dividends per share over the past century based on the findings of Larraine and Yogo (2008) and Atkeson, Heathcote, and Perri (2024) who argue that ratios of total payouts to value do not show the same trends in valuation ratios using price per share and dividends per share.

In terms of the literature, we see Barsky and De Long (1993) as the closest precursor to our work. That paper emphasizes the role of shocks to the dividend growth rate in the long run in accounting for the volatility of stock prices. Bansal and Lundblad (2002) and Bansal and Yaron (2004) also point to low-frequency movements in expected growth in dividends as an important source of changes in the price-dividend ratio for the aggregate stock market.

Greenwald, Lettau, and Ludvigson (2023) is an important precursor to our work in that it uses a model in which shocks to the ratio of earnings to output play a major role in accounting for the data on the evolution of the value of the stock market over time. We also follow them in using a valuation model to uncover the innovations needed to account for the data. In contrast to their work, however, we use the standard data on price per share and dividends per share that are used in many asset pricing studies. Their work, and work by Larraine and Yogo (2008) and our companion paper Atkeson, Heathcote, and Perri (2024), develop valuation models using alternative data on cash flows to owners of U.S. corporations.

We note that it is standard in the asset pricing literature to build models with separate dynamics for dividends and aggregate consumption and thus these models implicitly incorporate shocks to expectations of the ratio of dividends to consumption in the long-run. See, for example, Campbell and Cochrane (1999) and Bansal and Yaron (2004). But these alternative models do not appear to put these shocks to long-run expectations of the ratio of dividends per share to consumption at the center of their analysis. Given our results, it is unclear whether the other features of these models are needed to account for stock market data once they allow for news about the ratio of dividends to consumption in the long-run. In that vein, we do not attempt a full general equilibrium asset pricing model in this paper. As a result, we have nothing to say at this time about the economic sources of the equity premium as measured by the high average returns to equity.

The remainder of this paper is organized as follows. In Section 2 we review the data on the stock market that we use in our study. In Section 3 we sketch our first valuation model that delivers an affine model of equity prices. We describe how we use the model to reproduce the realized data on stock prices and dividends in Section 3.1 and we describe our model's implications for the sources of the realized equity premium over the last century in Section 3.5. We offer a more complete foundation for this affine model using Stein's Lemma in Section 4. In Section 5, we present our second valuation model with exponentially affine prices for claims to equity and repeat our excess volatility exercise with this second model. We also use this exponentially affine model in a Monte Carlo exercise to explore its implications for estimates of stock return predictability with log dividend-price ratios and show that our model is consistent with estimates of return predictability even if, in population with a log-linear approximation to returns, this predictability is not there. In Section 6, we conclude.

2 Stock Market Data

In this section we review key properties of the data we study.

We use data on the CRSP Value-Weighted Total Market Index 1929-2023. We are somewhat pedantic in our presentation of this data as some readers may not be familiar with its construction, and several elements of its construction are important in understanding the motivation for the key assumptions in our valuation model.

The original data we use are CRSP indices of annual returns without dividends (denoted by R^{nd}_{t+1}), returns with dividends (denoted by R^{d}_{t+1}), and total market capitalization (denoted by TMC_t) on the CRSP Value-Weighted Index combining stocks listed in the NYSE, AMEX, and NASDAQ exchanges for the years 1929-2023. We focus on this time period as this is the time period for which we also have NIPA data on Personal Consumption Expenditures.

As is well known, CRSP annual value-weighted returns on the total stock market are high on average and quite volatile. In our sample, the arithmetic averages of nominal and real returns with dividends (R^{d}_{t+1}) are 11.6% and 8.6% respectively (deflating with the PCE deflator), and these nominal and real returns have a standard deviations of 19.8% and 19.5% respectively.

The measure of price per share for the CRSP Value-Weighted Index that we use as the measure of the value of the stock market in our study is constructed from the cumulation of annual returns without dividends R^{nd}_{t+1}. Specifically, if we let P_{Dt} denote the level of price per share on the last day of year t, we construct $P_{D,t+1} = R^{nd}_{t+1} P_{Dt}$. Note that P_{Dt} is an index number in that the initial value must be normalized.

We plot the ratio of this index of price per share to Personal Consumption Expenditures (PCE) in the left panel of Figure 1. We have normalized the index of price per share so that the initial value of this ratio is equal to one. As is clearly evident in this figure, this ratio is quite volatile.

The measure of dividends per share for the CRSP Value-Weighted Index that we denote by D_t and use as our measure of cash flows to someone holding the CRSP Value-Weighted Index is constructed to solve the following equation

$$\frac{D_{t+1} + P_{D,t+1}}{P_{Dt}} = R^{d}_{t+1}$$

That is, given the index for price per share, the index for annual dividends per share is chosen so that returns match value-weighted returns with dividends. This equation pins down the ratio of dividends per share to price per share. The scale of dividends per share is set by the normalization of the level of price per share.

We plot the ratio of this index of dividends per share to PCE in the right panel of Figure

1. Henceforth, we denote this ratio of dividends per share to PCE by $\frac{D_t}{C_t}$.

Our goal with our valuation models is to account for the data on the ratio of price per share to PCE shown in the left panel of Figure 1 in terms of a plausible model of the expected discounted present value of the data on the ratio of dividends per share to PCE shown in the right panel of this figure.

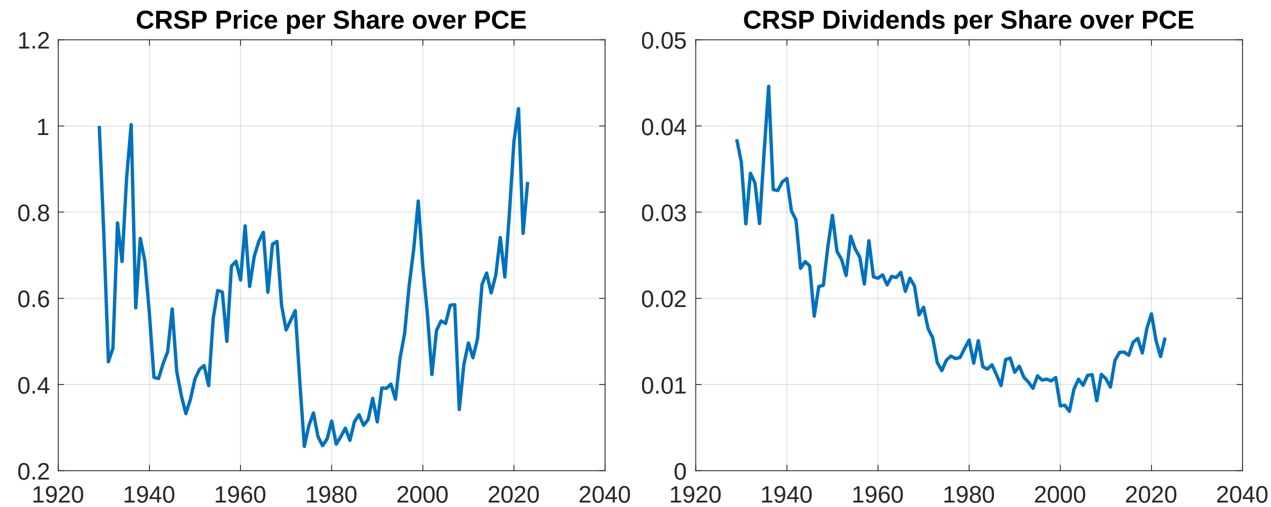

Figure 1: Left Panel: The ratio of price per share for the CRSP Value-Weighted Total Market Index to Personal Consumption Expenditures 1929-2023. The value of this ratio in 1929 is normalized to one. Right Panel: The ratio of dividends per share for the CRSP Value-Weighted Total Market Index to Personal Consumption Expenditures 1929-2023. This series for dividends per share is normalized so that the ratio of the two series equals the ratio of dividends per share to price per share at every date.

One striking feature of the data on the ratio of dividends per share to PCE shown in the right panel of Figure 1 is that it does not appear to be stationary. Instead, it shows a marked downward trend since 1929. The first key assumption in our valuation models is that investors do not have a fixed expectation of the value of this ratio in the long run. Instead, they receive news each period that leads them to revise their expectation of the long-run value of this ratio. We argue that uncertainty about the long-run value of this ratio is plausible as a matter of econometrics given its historical path as shown in the figure – it is not at all clear what long-run value the series is converging to.

There are also multiple economic reasons why the ratio of dividends per share to PCE might vary in the long run.

First, it may be the case that the total cash flows paid to owners of U.S. corporations relative to consumption might vary over time. Greenwald, Lettau, and Ludvigson (2023) and Atkeson, Heathcote, and Perri (2024) argue that this is indeed the case.

Second, the fraction of economic activity carried out in publicly-traded corporations relative to all corporations might vary over time. This is likely the case as well over the course of the past century.

Third, the methodology used in the construction of the indices of price per share and dividends per share implies that these measures do not track the total value of the stock market as measured by total market capitalization of the stocks in the index nor the total value of cash payouts to owners of these equities. Instead, the ratio of the index of price per share to total market capitalization varies over time as a result of a large number of actions that result in changes in the number of shares outstanding for the firms in the index. These corporate actions include entry and exit of firms in public markets, mergers of firms, issuance of new shares or repurchases of shares by incumbent firms, etc. What these corporate actions imply is that an investor who maintained a portfolio to track the CRSP Value-Weighted Index would hold a share of the total market that varies over time. We give further details on these points in Appendix B.

We show the variation of the ratio of the index of price per share to total market capitalization for the CRSP Value-Weighted Index over the period 1929-2023 in Figure 2. This ratio represents the fraction of the total stock market held by an investor tracking the CRSP Value-Weighted Index. In this figure, we normalized 1929 price per share so that the fraction is equal to one in 1929.

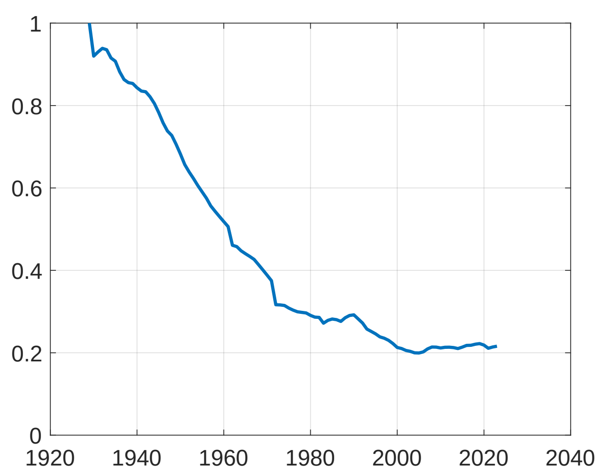

Figure 2: The fraction of the total market capitalization of the stocks in the CRSP Value-Weighted Total Market Index held by an investor tracking that index, 1929-2023. The initial value of this fraction has been normalized to one.

We see in this figure a sharp downward trend in the share of the total market held by an index investor over this time period. An investor managing his or her portfolio to track the CRSP Value-Weighted Index would end up holding a shrinking share of the total stock market because he or she would not be purchasing the new shares being issued on net from corporate actions.[2] Thus, it is natural that the ratio of dividends per share to consumption would also fall over time, as an investor tracking the index of price per share would have claims to a shrinking share of total dividends. This figure also makes clear that part of investors' uncertainty regarding the long-run value of the ratio of dividends per share to consumption is also driven by uncertainty regarding the future course of corporate actions.

A great deal of research focuses on modeling the dynamics of the ratio of price per share to dividends per share. See, for example, Campbell and Shiller (1987) and Campbell and Shiller (1988) and the large body of work following these papers. Our valuation model reproduces the data on price per share and dividends per share exactly, so it also reproduces data on the ratio of these two series.

We plot the logarithm of the ratio of dividends per share to price per share in the left panel of Figure 3. This is the measure of the dividend-price ratio frequently used in Campbell-Shiller style regressions aiming to forecast returns and/or growth in dividends per share. We see a sharp downward trend in this series over time, which raises the question of whether this series is stationary and thus suitable for use in regression analysis. We discuss in Appendix B how this measure of the dividend-price ratio is impacted in arbitrary ways by the dynamics of corporate actions. The logic of this impact follows from the arguments of Miller and Modigliani (1961) that the choice of whether firms return money to shareholders via dividends or stock repurchases does not impact total market capitalization or stock returns, but does impact dividend payouts, price per share, and the observed dividend-price ratio. This disconnect between total cash flows to equity owners and dividends per share is compounded by changes in the composition of firms in the index. See, for example, Dichev (2006) and Davydiuk et al. (2023). The lesson of these arguments is that the ratio of price per share to dividends per share should not be interpreted as a meaningful economic fundamental.

We plot the level of the ratio of price per share to dividends per share in the right panel of Figure 3. Naturally, we see a sharp upward trend in this series. This ratio rises from a value close to 20 at the beginning of the sample to a value above 45 at the end of the sample. It is unclear what long-run value this ratio is converging to. This uncertainty regarding the long-run value of this ratio plays an important role in our valuation models.

[2] Gârleanu and Panageas (2023) document that most of this decline in the share of the total market held by an index investor is driven by the entry of new firms into public markets through initial public offerings.

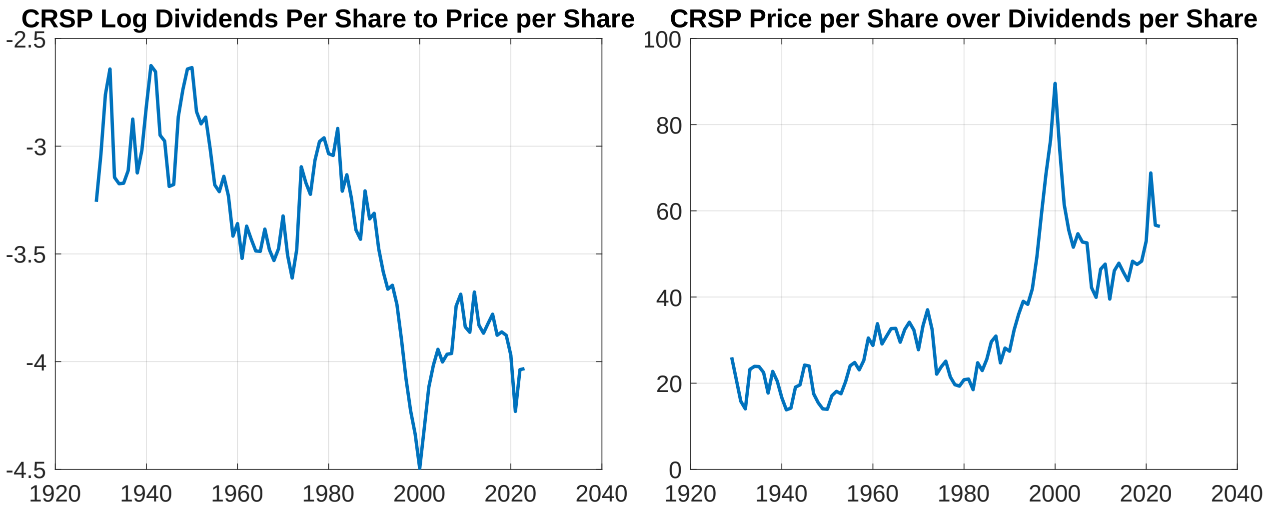

Figure 3: Left Panel: The logarithm of the ratio of dividends per share to price per share for the CRSP Value-Weighted Total Market Index 1929-2023. Right Panel: The level of the ratio of price per share to dividends per share.

3 A First Valuation Model

We now present a simple valuation framework to assess whether data on price per share for the CRSP Value-Weighted Index from 1929 to 2023 are too volatile to be accounted for by a plausible model of the dynamics of dividends per share for that index under the "straw-man" assumption that discount rates are constant over time. We make this assumption regarding discount rates not because we believe that it is strictly true but instead to directly address the question posed in Shiller (1981).

In this section, we first present the model with a minimum of formal structure to make the valuation equation accessible to a reader with only a basic knowledge of finance. In Section 4 we present a more complete affine pricing model that provides a formal basis for our key valuation equation. In Section 5, we present a second valuation model based on an exponentially affine pricing model that is closer to the asset pricing models used more widely in the literature.

Let P_{Dt} be the level of the CRSP Value-Weighted Index at the end of year t and D_t dividends per share for that index in year t. Let C_t be aggregate PCE in year t. We use nominal data. Let $\frac{D_t}{C_t}$ denote the ratio of dividends per share to consumption.

We begin with the standard valuation equation for the level of the index

$$P_{Dt} = \sum_{k=1}^{\infty} \mathbb{E}_t \left[M_{t,t+k} D_{t+k} \right] \tag{4}$$

where $M_{t,t+k}$ is the pricing kernel between periods t and $t+k$.

We find it convenient to work with ratios of price per share and dividends per share to PCE. We therefore rewrite this pricing equation (4) as

$$\frac{P_{Dt}}{C_t} = \sum_{k=1}^{\infty} \mathbb{E}_t \left[M_{t,t+k} \frac{C_{t+k}}{C_t} \frac{D_{t+k}}{C_{t+k}} \right].$$

Using the result that the expectation of a product of two random variables is the product of the expectations plus the covariance between these variables, we have

$$\frac{P_{Dt}}{C_t} = \sum_{k=1}^{\infty} \mathbb{E}_t \left[M_{t,t+k} \frac{C_{t+k}}{C_t} \right] \mathbb{E}_t \left[\frac{D_{t+k}}{C_{t+k}} \right] + \sum_{k=1}^{\infty} \mathbb{C}ov_t \left(M_{t,t+k} \frac{C_{t+k}}{C_t}, \frac{D_{t+k}}{C_{t+k}} \right). \tag{5}$$

Note that the term

$$\frac{P_{Ct}^{(k)}}{C_t} \equiv \mathbb{E}_t \left[M_{t,t+k} \frac{C_{t+k}}{C_t} \right]$$

is the price at t of a claim to aggregate consumption at delivered at $t+k$ relative to aggregate consumption at t. We define the price at t of a claim to aggregate consumption in perpetuity relative to the current level of aggregate consumption as

$$\frac{P_{Ct}}{C_t} \equiv \sum_{k=1}^{\infty} \frac{P_{Ct}^{(k)}}{C_t}$$

The terms

$$H_t^{(k)} \equiv \mathbb{C}ov_t \left(M_{t,t+k} \frac{C_{t+k}}{C_t}, \frac{D_{t+k}}{C_{t+k}} \right)$$

constitute a risk adjustment to the price of claims to dividends due to risk associated with fluctuations in the ratio $\frac{D_{t+k}}{C_{t+k}}$.

With this notation, we obtain the following valuation equation for the ratio of price per share to consumption

$$\frac{P_{Dt}}{C_t} = \sum_{k=1}^{\infty} \frac{P_{Ct}^{(k)}}{C_t} \mathbb{E}_t \left[\frac{D_{t+k}}{C_{t+k}} \right] + \phi_t \tag{6}$$

where

$$\phi_t \equiv \sum_{k=1}^{\infty} H_t^{(k)}.$$

Note that equation (6) has not required any assumptions beyond the original valuation equation (4) and thus should hold for all of the main models used in the asset pricing literature for the aggregate stock market.[3]

[3]We do assume that the infinite sums in this valuation equation each converge separately. This need not

3.1 An Excess Volatility Exercise

We now specialize the valuation equation (6) to conduct a Shiller (1981)-style excess volatility calculation by imposing three assumptions.

Assumption 1: Assume that the ratio of the price of aggregate consumption at horizon $k = 1$ relative to aggregate consumption today is constant over time, i.e.,

$$\frac{P_{Ct}^{(1)}}{C_t} = \beta \equiv \frac{P_C^{(1)}}{C}$$

for all t.[4]

Given this assumption, we have

$$\frac{P_{Ct}^{(k)}}{C_t} = \beta^k$$

for all k and t.[5]

Assumption 2: Assume that the risk adjustment terms $H_t^{(k)}$ and thus ϕ_t are constant over time. We present a full affine pricing model under which this assumption holds in Section 4. This assumption does not hold in the more commonly used exponentially affine pricing model. We conduct an alternative valuation exercise using such an alternative model in Section 5.

With assumptions one and two, our valuation equation (6) can be written as

$$\frac{P_{Dt}}{C_t} = \sum_{k=1}^{\infty} \beta^k \mathbb{E}_t \left[\frac{D_{t+k}}{C_{t+k}} \right] + \phi. \tag{7}$$

We take this valuation equation as corresponding to a case with constant discounting as we have assumed that both $\frac{P_C^{(1)}}{C}$ and ϕ are constant over time. In this case, fluctuations

be the case.

[4]A simple example economy in which this assumption is satisfied is a model in which investors consume C_t at each date t and value consumption streams using logarithmic utility and a constant discount factor β, in which case $\frac{P_C^{(1)}}{C} = \beta$. In such an economy $\phi_t = 0$ since the realized values of $M_{t,t+1}\frac{C_{t+1}}{C_t}$ are constant at β.

[5]To prove this statement, observe that these prices for consumption at horizon k satisfy the recursion

$$\frac{P_{Ct}^{(k)}}{C_t} = \mathbb{E}_t \left[M_{t,t+1} \frac{C_{t+1}}{C_t} \frac{P_{C,t+1}^{(k-1)}}{C_{t+1}} \right]$$

We then prove our result by induction. Starting with $k = 2$ and using the assumption that $\frac{P_{Ct}^{(1)}}{C_t}$ is constant and equal to β at each date t, we have

$$\frac{P_{Ct}^{(2)}}{C_t} = \beta^2$$

which is also constant over time. Repeating this argument then delivers the result for all k.

in the model-implied value of the ratio of price per share to consumption are accounted for entirely by fluctuations in the future expected values of the ratio of dividends per share to consumption.

Assumption 3: Assume that the ratio of dividends per share to consumption $\frac{D_t}{C_t}$ follows a first-order autoregressive process with a shifting endpoint X_t as described in equations (1) and (2) which we reproduce here:

$$
\begin{aligned}
\frac{D_{t+1}}{C_{t+1}} - X_t &= \rho \left(\frac{D_t}{C_t} - X_t \right) + \sigma_D \epsilon_{D,t+1}, \\
X_{t+1} &= X_t + \sigma_X \epsilon_{X,t+1}
\end{aligned}
$$

where ρ is the AR(1) persistence parameter.

These dynamics imply

$$
\frac{D_{t+1}}{C_{t+1}} - X_{t+1} = \rho \left(\frac{D_t}{C_t} - X_t \right) + \sigma_D \epsilon_{D,t+1} - \sigma_X \epsilon_{X,t+1} \tag{8}
$$

with $\epsilon_{X,t+1}$ being the innovation to the endpoint X_{t+1} and $\epsilon_{D,t+1}$ being the innovation to $\frac{D_{t+1}}{C_{t+1}}$ relative to its expected value at t. We assume that these are independent and have standard normal distributions.[6] We use the notation

$$
\sigma_A \epsilon_{A,t+1} \equiv \sigma_D \epsilon_{D,t+1} - \sigma_X \epsilon_{X,t+1}
$$

to refer to the combined innovation to the gap between the current dividend-consumption ratio and its long-run expected value, $\frac{D_t}{C_t} - X_t$.

Note that our third assumption implies that

$$
\mathbb{E}_t \left[\frac{D_{t+k}}{C_{t+k}} \right] = \rho^k \left(\frac{D_t}{C_t} - X_t \right) + X_t
$$

and

$$
\lim_{k \to \infty} \mathbb{E}_t \left[\frac{D_{t+k}}{C_{t+k}} \right] = X_t.
$$

Thus, with Assumption 3, we obtain from equation (7) the valuation equation (3) that we

[6] It is straightforward to allow these innovations to be contemporaneously correlated.

apply to account for the observed data on price per share:

$$\frac{P_{Dt}}{C_t} = \gamma^D \left(\frac{D_t}{C_t} - X_t \right) + \gamma^X X_t + \phi$$

where

$$\gamma^D \equiv \frac{\beta\rho}{1-\beta\rho}$$

and

$$\gamma^X \equiv \frac{\beta}{1-\beta} = \frac{P_C}{C}.$$

Note that this valuation model has only three parameters: (i) the persistence of the autoregressive component of the ratio of dividends to consumption ρ, (ii) the price-dividend ratio for a one period consumption claim β (or equivalently $\frac{P_C}{C} = \beta/(1-\beta)$), and (iii) the risk adjustment parameter ϕ. The coefficients γ^D and γ^X are derived from the first two of these parameters. These coefficients define, respectively, (i) the marginal response of the equity price-to-consumption ratio to innovations to the gap between the current dividend-to-consumption ratio and its expected long-run value (ii) the marginal response of the equity price-to-consumption ratio to innovations to the expected long-run dividend-to-consumption ratio.

3.2 Accounting for the Value of the Stock Market, 1929-2023

We use the valuation model in equation (3) to account for the data on equity price per share using the data on dividends per share as follows.

First, we need values for the three model parameters β, ρ, and ϕ.

We cite Lustig, Van Nieuwerburgh, and Verdelhan (2013) for an estimate of the value of the price dividend ratio for a claim to consumption of $\frac{P_C}{C} = \gamma^X = 80$. That is, we set $\beta = \frac{80}{81}$. Alvarez and Jermann (2004) also offer estimates of the value of a claim to aggregate consumption that are consistent with a high price-dividend ratio for such a claim. This price-dividend ratio for a claim to consumption is quite high compared to price-dividend ratios for equity (except recently), but one would reasonably expect that a claim to consumption is safer than a claim to equity and hence should have a higher price-dividend ratio. Note that the average growth rate of real PCE over the 1929-2023 time period is 3.14%. Many estimates of the real interest lie below this number. As a result, some might argue that in fact the present value of future relative to current consumption might be much higher or even infinite. See, for example, Blanchard (2019) and Gârleanu and Panageas (2023).

We estimate ρ and ϕ as follows. Equation (3) can be rearranged to give

$$\frac{D_t}{C_t} - X_t = \frac{\gamma^X \frac{D_t}{C_t} - \frac{P_{Dt}}{C_t}}{(\gamma^X - \gamma^D)} + \frac{\phi}{(\gamma^X - \gamma^D)}.$$

We substitute this expression into equation (8) and estimate ρ and ϕ by least squares by regressing $\gamma^X \frac{D_{t+1}}{C_{t+1}} - \frac{P_{D,t+1}}{C_{t+1}}$ on a constant and the same variable at date t.[7] The slope coefficient provides a direct estimate of ρ, while the constant corresponds to $\phi(\rho - 1)$. This procedure gives $\rho = 0.9447$ and $\phi = -0.6203$.

With these values for β and ρ, the values for the coefficients in equation (3) are $\gamma^D = 15.2$ and $\gamma^X = 80$.

Next, we construct the values of X_t implied by the data and these parameters by rearranging equation (3) to give

$$X_t = \frac{1}{\gamma^X - \gamma^D} \left(\frac{P_{Dt}}{C_t} - \gamma^D \frac{D_t}{C_t} - \phi \right) \tag{9}$$

Observe from this equation (9) that the terms representing the ratio of price per share to consumption $\frac{P_{Dt}}{C_t}$ and dividends per share to consumption $\frac{D_t}{C_t}$ are taken straight from the data. Note that the value of the parameter ϕ affects the *level* of X_t, but not the time series for the implied innovations $\sigma_X \epsilon_{X,t+1} = X_{t+1} - X_t$.

We show what our valuation exercise implies for $\{X_t\}$ in Figure 4. The blue line reproduces the data on $\frac{D_t}{C_t}$. The red line shows the sequence of values for X_t, given our parameters, such that the predicted ratio of price per share to consumption from equation (3) matches the data on the ratio of price per share to consumption.

[7]Note that the coefficient γ^X that appears in this term is a function of β but not of ρ.

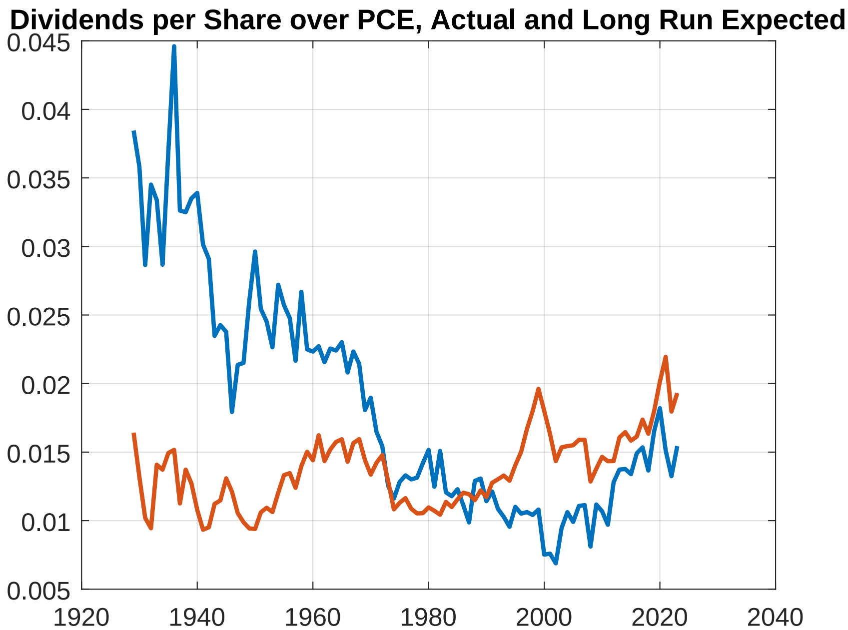

Figure 4: Blue Line: The ratio of dividends per share for the CRSP Value-Weighted Total Market Index to PCE ($\frac{D_t}{C_t}$), 1929-2023. Red Line: The expected long-run ratio of dividends per share for the CRSP Value-Weighted Total Market Index to PCE, X_t, that rationalizes the observed price per share of this index using equation (3), 1929-2023.

We use this figure to ask the following question: does the red line for X_t in Figure 4 represent a reasonable model of agents expectations of where the ratio of dividends per share to consumption will converge in the long run? Or is that red line somehow unreasonably volatile? We argue that this variation in the red line is reasonable and that, as a result, there is no excess volatility puzzle. The data on price per share of the stock market can be accounted for by a reasonable model of expected fluctuations in future dividends per share with no fluctuations in discount rates and no bubbles.

To substantiate this point, we examine properties of the innovations $\sigma_X \epsilon_{X,t+1} = X_{t+1} - X_t$ and $\sigma_A \epsilon_{A,t+1} = (\frac{D_{t+1}}{C_{t+1}} - X_{t+1}) - \rho(\frac{D_t}{C_t} - X_t)$ implied by our accounting procedure. We find a sample mean value of $\sigma_X \epsilon_{Xt+1}$ equal to 3.1×10^{-5} and a sample standard deviation of 0.0014. Since we have only 94 annual observations, this sample mean is well within one standard error of zero. The sample mean of $\sigma_A \epsilon_{A,t+1}$ is -4.5×10^{-7}. The sample standard deviation of these innovations is 0.0027.

In the left panel of Figure 5, we show the estimated autocorrelation function of the implied series $\frac{D_t}{C_t} - X_t$ in blue crosses and the model-implied autocorrelation function for this series given $\rho = 0.9447$ as red dots (the values of the red dots are simply ρ^j, where j is the autocorrelation lag on the x axis). We see that this choice of $\rho = 0.9447$ gives a reasonable fit to the persistence of $\frac{D_t}{C_t} - X_t$.

In the right panel of Figure 5, we show the autocorrelation function for the implied innovations $\sigma_x \epsilon_{Xt+1}$ as blue crosses and the autocorrelation function for the implied innovations $\sigma_A \epsilon_{At+1}$ as red dots. This figure shows little autocorrelation of the model-implied innovations to agents' expectations.

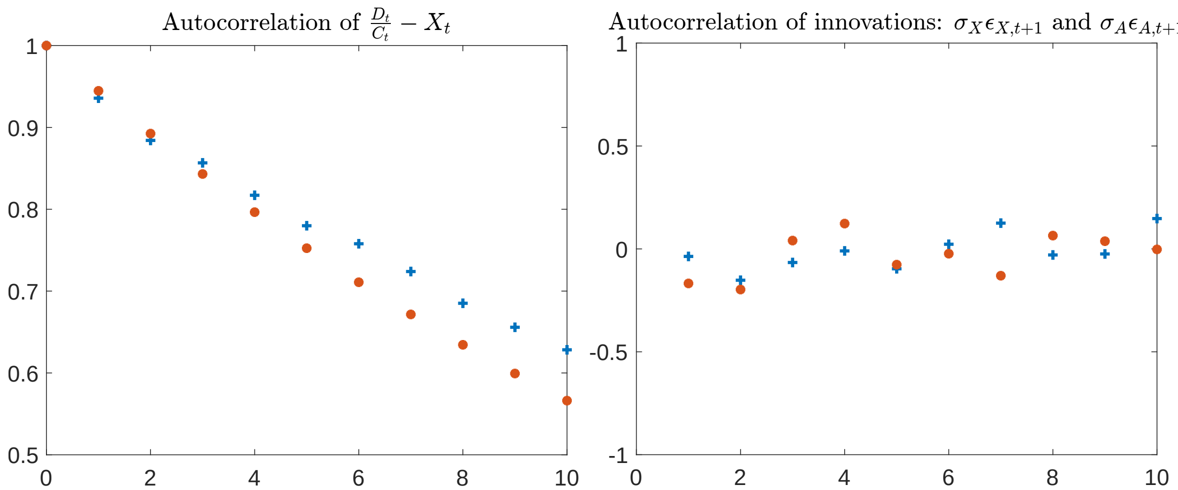

Figure 5: Left Panel: The autocorrelation function for the estimated series $\frac{D_t}{C_t} - X_t$ in blue and the model-implied autocorrelation function for this series in red given $\rho = 0.95$. Right Panel: The autocorrelation functions for the innovations $\sigma_X \epsilon_{X,t+1}$ in blue and for $\sigma_A \epsilon_{A,t+1}$ in red.

For completeness, in Figure 6, we show, in the left panel, the sequence of implied innovations to X_{t+1} given by $\sigma_X \epsilon_{X,t+1}$ and in the right panel the sequence of implied innovations to the AR(1) part given by $\sigma_A \epsilon_{A,t+1}$. We do not see any patterns in these model-implied innovations that would suggest a deviation from rational expectations.

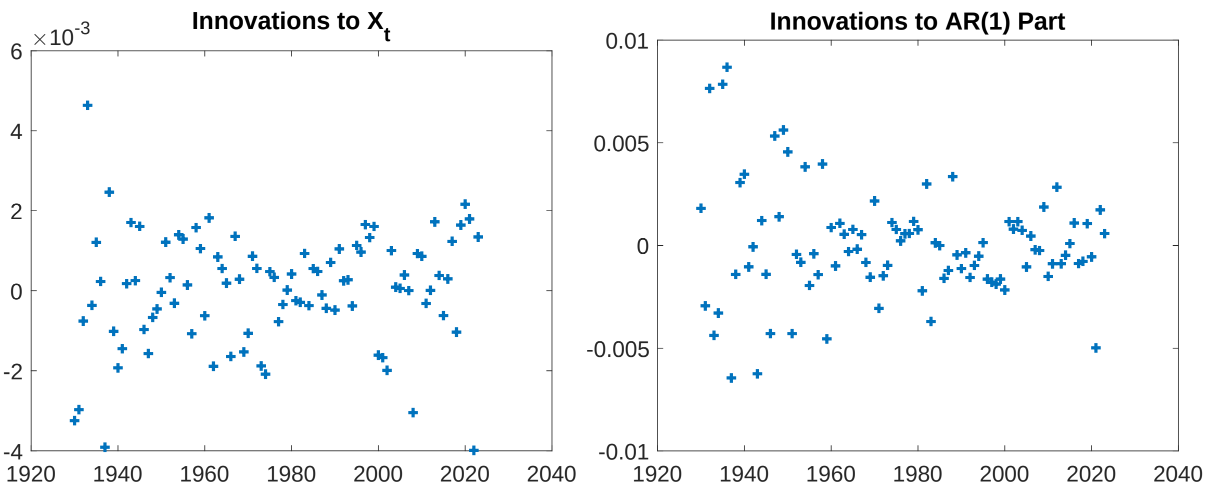

Figure 6: Left Panel: The innovations $\sigma_X \epsilon_{X,t+1} = X_{t+1} - X_t$ uncovered from our valuation exercise. Right Panel: The innovations $\sigma_A \epsilon_{A,t+1} = (\frac{D_{t+1}}{C_{t+1}} - X_{t+1}) - \rho(\frac{D_t}{C_t} - X_t)$ uncovered from our valuation exercise.

3.3 Is Dividend Growth Forecastable?

Are fluctuations in the latent long-run expectations factor X_t that our model relies on to account for equity price dynamics plausibly consistent with investors having rational expectations? Or should these fluctuations instead be interpreted as capturing waves or irrational exuberance or pessimism?

Figure 4 suggests that X_t does help forecast growth in future dividends per share relative to consumption. The figure indicates that from the start of our sample until around 1990, investors were expecting dividends per share relative to consumption to decline over time (the red line is below the blue line). Were these expectations reasonable? Well, dividends per share relative to consumption did in fact decline steadily over this period! As described in Gârleanu and Panageas (2023), this is what one would have expected in an economy in which new firms continuously displace old ones.

From 1990 onward, the model indicates that investors were expecting future ratios of dividends per share to consumption to exceed the current level (the red line is above the blue), with this gap being especially pronounced around the dot com stock boom in 2000. And in fact, from around 2000 onward, dividends per share relative to consumption have been generally rising, suggesting this optimism about future dividends per share was more than irrational exuberance. Future research can determine the extent to which this more recent increasing trend in the ratio of dividends per share to consumption is due to increased firm profitability versus changes in corporate actions that have had a more mechanical impact on dividends per share.

To further explore the extent to which X_t helps forecast future dividends relative to consumption, we now run some simple forecasting regressions. In particular, for different forecasting horizons s, we regress growth between t and $t+s$ in dividends per share relative to consumption on $X_t - \frac{D_t}{C_t}$. Thus, we estimate coefficients α_s for the model

$$\frac{D_{t+s}}{C_{t+s}} - \frac{D_t}{C_t} = \alpha_s \left(X_t - \frac{D_t}{C_t} \right) + \epsilon_{t+s}. \tag{10}$$

We compare these coefficients to those that are rational given our shock process. In particular, given equations (1) and (2),

$$\mathbb{E}\left[\frac{D_{t+s}}{C_{t+s}}\right] - \frac{D_t}{C_t} = (1-\rho^s)\frac{D_t}{C_t}. \tag{11}$$

Figure 7 plots the results. It is clear that our latent expectation variable X_t is strongly predictive of future dividend growth. Average realized growth in dividends per share relative to consumption is close to the model-implied rational expectations value at every forecasting horizon s. We conclude that dividend growth does have a large predictable component, and that fluctuations in equilibrium equity prices can reasonably be interpreted as reflecting rational changes in expectations about that growth.

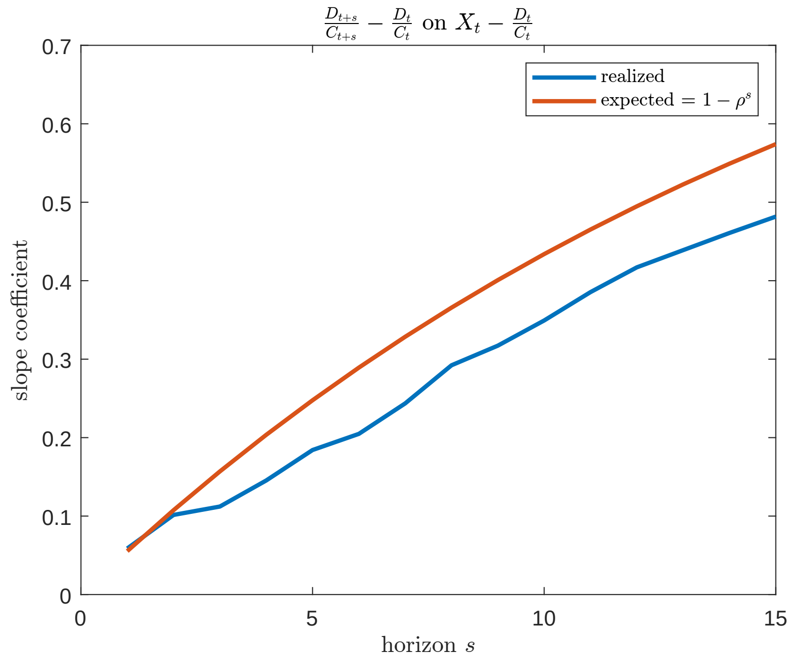

Figure 7: Blue Line: Estimates of the coefficient α_s as described in equation (10) for different forecasting horizons s. The right-hand side variable $\left(X_t - \frac{D_t}{C_t}\right)$ runs from 1929 to 2008 in each regression. Red Line: The corresponding rational expectations consistent expected value $1 - \rho^s$ (see equation 11).

3.4 Sensitivity to Our Model of News Shocks

Our ability to account for the dynamics of the ratio of price per share to consumption for the CRSP Value-Weighted Index is dependent on two key features of our valuation model. One is that there are shocks to agents' expectations of the long-run value of the ratio of dividends per share to consumption as indexed by innovations to X_{t+1}. The other is that the impact of that long-run news on the model-implied price per share as represented by the price-dividend ratio of a claim to consumption ($\gamma^X = P_C/C$) is large.

To demonstrate the quantitative importance of these two assumptions, we first compute the portion of the value of the ratio of price per share to consumption accounted for by X_t in our baseline specification of the model, where X_t denotes agents' expectations of the long-run value of the ratio of dividends per share to consumption.

The blue line in Figure 8 shows the data for the ratio of price per share to consumption P_{Dt}/C_t. The red line shows the quantity $\gamma^X X_t + \phi$, which corresponds to the model-predicted movements in P_{Dt}/C_t driven by movements in X_t. We see in this figure that these model-implied movements in agents' long-run expectations for dividends account for most of the fluctuations in stock prices seen in the data. The transitory dynamics of the ratio of dividends per share to consumption are much less important in our accounting. We conclude from this figure that innovations to agents' long-run expectations of the ratio of dividends per share to consumption are the key driver of stock market volatility in our valuation framework.

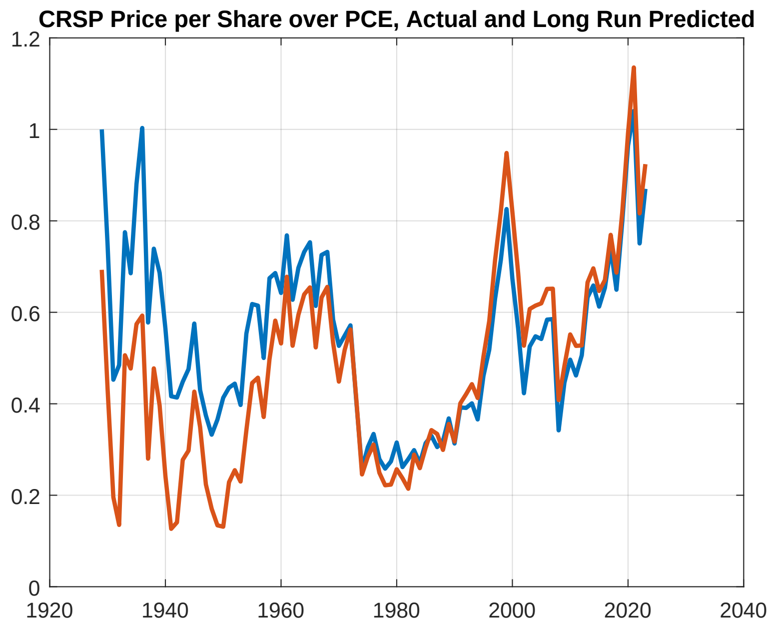

Figure 8: Blue Line: The ratio of price per share for the CRSP Value-Weighted Total Market Index to PCE (P_{Dt}/C_t), 1929-2023. Red Line: The portion of this measure of stock market valuation accounted for by long-run expectations as measured by $\gamma^X X_t + \phi$, 1929-2023.

Next, we recompute the implied series for X_t under the assumption that the price-dividend ratio for a perpetual claim to consumption $\gamma^X = P_C/C = 25$ rather than our baseline value of 80. We do this twice: once leaving the parameters ρ and ϕ unchanged, and once re-estimating them.

We show the results of this experiment in Figure 9. As in Figure 4, the blue line in Figure 9 reproduces the data on $\frac{D_t}{C_t}$. The red line shows the sequence of values for X_t required so that the predicted ratio of price per share to consumption from equation 3 matches the data on the ratio of price per share to consumption when $\gamma^X = P_C/C = 25$. We regard these red series as implausibly volatile. Hence, we regard our assumption of a high price-dividend ratio for a claim to consumption as key to our model's ability to account for observed stock market volatility.

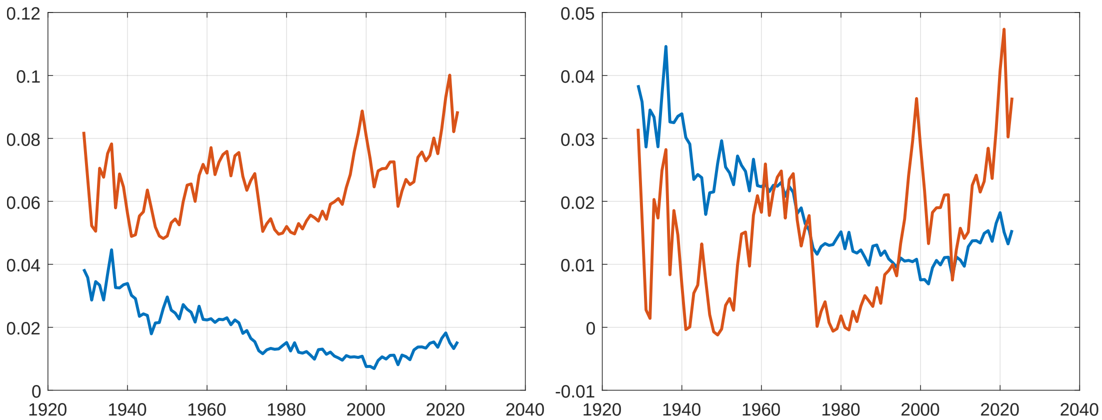

Figure 9: Left Panel: Results when $\gamma^X = P_C/C = 25$ and ρ and ϕ take their baseline values ($\rho = 0.9447$ and $\phi = -0.620$). Right Panel: Results when $\gamma^X = P_C/C = 25$ and ρ and ϕ are recalibrated to give $\rho = 0.9367$ and $\phi = 0.149$. Blue Lines: $\frac{D_t}{C_t}$, the ratio of dividends per share for the CRSP Value-Weighted Total Market Index to PCE, 1929-2023. Red Lines: X_t, the long-run expected ratio of dividends per share for the CRSP Index to PCE needed to rationalize the observed price per share of this index using equation (3).

3.5 Sources of the Equity Premium

The realized return on equity from 1929-2023 has been quite high. As we noted above, the sample average nominal and real returns to the CRSP Value-Weighted Index including dividends have been 11.6% and 8.6% respectively. Here we examine our model's implications for the sources of these high returns.

We focus on our model's implications for the decomposition of realized returns to equity in excess of consumption growth as given by

$$R^x_{D,t+1} \equiv \frac{\frac{D_{t+1}}{C_{t+1}} + \frac{P_{D,t+1}}{C_{t+1}}}{\frac{P_{Dt}}{C_t}}.$$

The sample average for this return in excess of consumption growth is 5.3%. Note that the sample average growth rate of real consumption over this time period is 3.1%.

Given our model for the dynamics of the ratio of dividends per share to consumption and our affine pricing model in equation (3), we can decompose these realized returns into a component that was expected one period ahead, and components due to the innovations to current dividends per share relative to consumption $\sigma_D \epsilon_{D,t+1}$ and the innovations to long-run expected ratio of dividends per share to consumption $\sigma_X \epsilon_{X,t+1}$ as follows.

Returns expected one period ahead are given by

$$\mathbb{E}_t\left[R^x_{D,t+1}\right] \equiv \frac{1}{\frac{P_{Dt}}{C_t}}\left[(\gamma^D+1)\rho\left(\frac{D_t}{C_t}-X_t\right)+(\gamma^X+1)X_t+\phi\right].$$

With our baseline choice of parameters, the sample average of these expected returns in excess of consumption growth is 2.88%, which is 2.42 percentage points below the sample average of *realized* returns in excess of consumption growth. Thus, the model interprets a significant portion of high realized returns to equity as reflecting unanticipated shocks that boosted expected cash flow and thus equity value.

Which shocks drive these high realized returns? The portions of realized returns due to innovations to current dividends per share relative to consumption $\sigma_D\epsilon_{D,t+1}$ and innovations to long-run expected ratio of dividends per share to consumption $\sigma_X\epsilon_{X,t+1}$ are given by

$$R^x_{D,t+1}-\mathbb{E}_t\left[R^x_{D,t+1}\right] = \frac{1}{\frac{P_{Dt}}{C_t}}\left[(\gamma^D+1)\sigma_D\epsilon_{D,t+1}+(\gamma^X-\gamma^D)\sigma_X\epsilon_{X,t+1}\right] \tag{12}$$

The sample averages of these innovations to returns in excess of consumption growth are 0.31% for the innovations to D_{t+1} and 2.11% for the innovations to X_{t+1}.[8]

We see from these calculations that, in our model, a substantial portion of realized returns to equity are due to in-sample positive innovations to the ratio of dividends per share to consumption expected in the long run. Note, however, that this positive surprise to returns is within one standard error of zero given the high volatility of realized returns (19%) and our sample size of 94 years, so it does not seem inconsistent with rational expectations.

One feature of our affine pricing model for equity is that the risk adjustment to the ratio of price per share to consumption is additive. This means that model-implied expected returns to equity one period ahead $\mathbb{E}_t\left[R^x_{D,t+1}\right]$ vary over time as the values of the state variables $\frac{D_t}{C_t}-X_t$ and X_t move up and down relative to the constant level risk adjustment ϕ. We show the path for the model-implied expected return to equity in Figure 10.

[8]Of course, if these two innovations are correlated, then this decomposition is not into two distinct components.

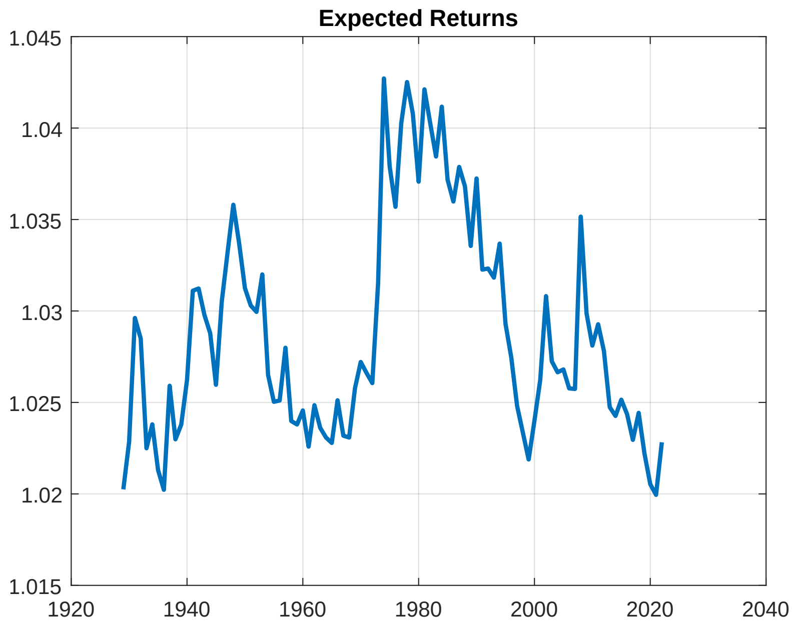

Figure 10: Model-implied one-year ahead expected return to equity $\mathbb{E}_t\left[R^x_{D,t+1}\right]$, 1929-2023

3.6 Relationship to the Literature on Forecasting Aggregate Stock Returns

In Figure 10, we show that our affine model with its baseline calibration produces modest variation over time in the expected (arithmetic) return to equity due to the additive risk adjustment for the price of equity in this affine structure. But we see from our pricing equation (3) that our model attributes all variation over time in the ratio of price per share to consumption to movements over time in the discounted present value of expected dividends per share relative to consumption.

How then do our results regarding the role of news about the long-run ratio of dividends per share to consumption in driving the bulk of the volatility in the ratio of price per share to consumption relate to the empirical literature on the forecastability or lack thereof of aggregate stock returns?

The connection between our work and this literature comes in our assumption that the innovations $\epsilon_{D,t+1}$ and $\epsilon_{X,t+1}$ are forecast errors relative to all information at time t. Thus no variable known at t should forecast these innovations. We see from equation (12) that this statement regarding these innovations to the ratio of dividends per share relative to consumption is equivalent to the statement that no variable known at t should forecast realized equity returns in excess of the conditional expectation of these returns implied by the model at time t.

Hence, to test our model, we run OLS regressions aimed at forecasting the innovations $\epsilon_{X,t+1}$ and $\epsilon_{D,t+1}$ shown in the two panels of Figure 6. We run separate univariate OLS regressions to forecast these innovations with their lagged values and with the ratio of price per share to dividends per share, its inverse, and the log of this ratio. We find that none of these variables are significant in forecasting these innovations. As a result, we conclude that one does not need to assume that stock returns are forecastable beyond that shown in Figure 10 to account for the observed volatility of the ratio of price per share to consumption.

We examine our model's implications for estimates of returns forecastability using log dividend price ratios further in section 5.2 below.

4 An Explicit Pricing Kernel

In equation (3) in the previous section we presented a simple affine valuation model for equity. To derive that model, we made assumptions regarding the dynamics of the price of claims to consumption and of the risk adjustment terms represented by the conditional covariances that we denoted by $H_t^{(k)}$. We now present a more complete pricing model to justify these assumptions.

We begin with standard assumptions regarding the dynamics of consumption growth and of the pricing kernel used to value assets. Let the log of consumption growth between t and $t+1$ be given by

$$g_{C,t+1} = \bar{g}_C + \sigma_{g_C}\epsilon_{C,t+1},$$

where $\bar{g}_C$ measures trend growth, and shocks to the log growth rate $\sigma_{g_C}\epsilon_{C,t+1}$ are drawn from a Normal distribution with mean zero variance $\sigma^2_{g_C}$.

Let the log of the pricing kernel be given by

$$m_{t+1} = \bar{m} + \lambda_C\epsilon_{C,t+1} + \lambda_D\epsilon_{D,t+1} + \lambda_X\epsilon_{X,t+1},$$

where the parameters λ_C, λ_D and λ_X capture, respectively, the pricing kernel loadings on the three shocks in the model: innovations to consumption growth $\epsilon_{C,t+1}$, and the transitory and permanent innovations to the ratio of dividends per share to consumption, $\epsilon_{D,t+1}$ and $\epsilon_{X,t+1}$, that we introduced in the previous section.

These assumptions for consumption growth and the pricing kernel jointly imply that the following three variables are all constant over time: (i) the price of a claim to consumption one period ahead relative to current consumption, (ii) the expected growth of consumption, and (iii) the riskless interest rate.

In particular, the price of a claim to consumption one period ahead relative to consumption today is given by

$$\frac{P_{Ct}^{(1)}}{C_t} = \beta_t = \mathbb{E}_t\left[\exp(m_{t+1} + g_{C,t+1})\right] = \exp\left(\bar{m} + \bar{g}_C + \frac{1}{2}\left((\lambda_C + \sigma_{gC})^2 + \lambda_D^2 + \lambda_X^2\right)\right).$$

Because this price is constant over time, Assumption 1 of our simple valuation model is satisfied.

The gross one-period risk-free interest rate implied by this pricing kernel is also constant and given by

$$R^{RF} = \frac{1}{\mathbb{E}_t\left[\exp(m_{t+1})\right]} = \exp\left(-\bar{m} - \frac{1}{2}\left(\lambda_C^2 + \lambda_D^2 + \lambda_X^2\right)\right).$$

The expected growth rate of the level of consumption is

$$\mathbb{E}_t\left[\exp(g_{C,t+1})\right] = \exp\left(\bar{g}_C + \frac{1}{2}\sigma_{gC}^2\right).$$

Observe that the expected return on a one-period consumption bond is

$$R^C = \frac{\mathbb{E}_t\left[\exp(g_{C,t+1}\right] C_t}{P_{Ct}^{(1)}} = \frac{\exp\left(\bar{g}_C + \frac{1}{2}\sigma_{gC}^2\right)}{\beta}$$

Thus the expected return on a consumption bond in excess of the risk free rate is

$$R^C - R^{RF} = \exp(-\lambda_C \sigma_{gC})$$

Thus,

$$\beta = \frac{\mathbb{E}_t\left[\exp(g_{C,t+1})\right]}{R^C} = \frac{\mathbb{E}_t\left[\exp(g_{C,t+1})\right]}{R^{RF} + \exp(-\lambda_c \sigma_{gC})}$$

As we have noted above, in the data, the risk free interest rate appears to be below the expected growth rate of consumption. For us to have a finite value for the coefficient $\gamma_X = \beta/(1-\beta)$, as is standard, we need to have a sufficiently large risk premium on a claim to consumption as determined by $\exp(-\lambda_C \sigma_{gC})$.

We do not want to argue that these moments are all constant over time in the data. But the fact that they are constant in our model allows us to transparently make the point that it is possible to account for the observed volatility of stock prices based entirely on volatility of expected cash flows. We leave to future work the project of extending our valuation framework to richer models for consumption growth or for the pricing kernel under which these data moments vary over time.

4.1 Pricing Dividends

We now turn to pricing claims to dividends. We assume that the dynamics of the ratio of dividends per share to consumption are given by equations (2) and (8). Note that this model departs from standard asset pricing formulations in that innovations to the ratio of dividends to consumption are normal rather than log-normal. We now show how to compute prices of claims to dividends and equity given the dynamics of the pricing kernel and consumption growth using Stein's Lemma.

We guess and verify that the price of a claim to dividends k periods ahead has the following form:

$$\frac{P_{Dt}^{(k)}}{C_t} = A_k\left(\frac{D_t}{C_t} - X_t\right) + B_k X_t + H_k \tag{13}$$

We solve for the coefficients A_k, B_k, and H_k recursively using the method of undetermined coefficients as follows.

The price for a claim to dividends in the current period is given by

$$\frac{P_{Dt}^{(0)}}{C_t} = \frac{D_t}{C_t}$$

so $H_0 = 0$, $A_0 = B_0 = 1$.

We have the following recursion for all other horizons k:

$$\frac{P_{Dt}^{(k)}}{C_t} = \mathbb{E}_t\left[\exp(m_{t+1} + g_{C,t+1})\frac{P_{D,t+1}^{(k-1)}}{C_{t+1}}\right]. \tag{14}$$

We use this to solve for A_k, B_k and H_k as follows.

This recursive equation implies that[9]

$$A_k\left(\frac{D_t}{C_t} - X_t\right)+B_kX_t+H_k = \mathbb{E}_t\left[\exp(m_{t+1} + g_{C,t+1})\right]\left[\rho A_{k-1}\left(\frac{D_t}{C_t} - X_t\right) + B_{k-1}X_t + H_{k-1}\right] +$$

$$\exp(\bar{m}+\bar{g}_C)\mathbb{E}_t\left[\exp((\lambda_C + \sigma_{g_C})\epsilon_{C,t+1} + \lambda_D\epsilon_{D,t+1} + \lambda_X\epsilon_{X,t+1})\left[A_{k-1}\sigma_D\epsilon_{D,t+1} + (B_{k-1} - A_{k-1})\sigma_X\epsilon_{X,t+1}\right]\right]$$

Matching coefficients on $(\frac{D_t}{C_t} - X_t)$ and X_t gives us that the coefficients A_k, B_k satisfy the recursion

$$A_k = \beta\rho A_{k-1} = (\beta\rho)^k$$

$$B_k = \beta B_{k-1} = \beta^k$$

[9] This equation is derived by substituting eq. (13) into the left-hand side of eq. (14) and again into the right-hand side (this time evaluated at $t+1$ and $k-1$). Next we used eqs. (8) and (2) to express $(\frac{D_{t+1}}{C_{t+1}} - X_{t+1})$ and X_{t+1} in terms of $(\frac{D_t}{C_t} - X_t)$ and X_t plus innovation terms.

To solve for the recursion for the constant term H_k, we need to solve for the term

$$\exp(\bar{m}+\bar{g}_C)\mathbb{E}_t\left[\exp((\lambda_C+\sigma_{g_C})\epsilon_{C,t+1}+\lambda_D\epsilon_{D,t+1}+\lambda_X\epsilon_{X,t+1})\left[A_{k-1}\sigma_D\epsilon_{D,t+1}+(B_{k-1}-A_{k-1})\sigma_X\epsilon_{X,t+1}\right]\right]$$

To do so, we use the result that if x and y and z are independent standard normal random variables and a, b, c, d are scalar constants, then

$$\mathbb{E}\left[\exp(ax+by)(cx+dz)\right]=ca\exp((a^2+b^2)/2) \tag{15}$$

This formula is an application of Stein's Lemma. We prove it in Appendix C.

This gives us that

$$\exp(\bar{m}+\bar{g}_C)\mathbb{E}_t\left[\exp((\lambda_C+\sigma_{g_C})\epsilon_{C,t+1}+\lambda_D\epsilon_{D,t+1}+\lambda_X\epsilon_{X,t+1})\left[A_{k-1}\sigma_D\epsilon_{D,t+1}+(B_{k-1}-A_{k-1})\sigma_X\epsilon_{X,t+1}\right]\right]=$$

$$\exp(\bar{m}+\bar{g}_C)\exp\left(\frac{1}{2}(\lambda_C+\sigma_{g_C})^2+\frac{1}{2}(\lambda_D^2+\lambda_X^2)\right)(\lambda_D A_{k-1}\sigma_D+\lambda_X(B_{k-1}-A_{k-1})\sigma_X)=$$
$$\beta\left(\lambda_D A_{k-1}\sigma_D+\lambda_X(B_{k-1}-A_{k-1})\sigma_X\right).$$

This result implies that we can solve for the coefficients H_k recursively from

$$H_k=\beta\left(H_{k-1}+\lambda_D A_{k-1}\sigma_D+\lambda_X(B_{k-1}-A_{k-1})\sigma_X\right).$$

Note that these values are independent of date t. We can then construct the value of a claim to equity as in equation (3) from

$$\frac{P_{Dt}}{C_t}=\sum_{k=1}^{\infty}\frac{P_{Dt}^{(k)}}{C_t}=\gamma^D\left(\frac{D_t}{C_t}-X_t\right)+\gamma^X X_t+\phi,$$

where $\gamma^D=\frac{\beta\rho}{1-\beta\rho}$, $\gamma^X=\frac{\beta}{1-\beta}$, and $\phi=\sum_{k=1}^{\infty}H_k$. Note that H_k and thus ϕ are constant over time, so this affine model satisfies Assumption 2 of our valuation framework.

5 An Exponentially Affine Model

In Sections 3 and 4, we derived an affine model for the value of equity relative to consumption based on the assumption that the level of the ratio of dividends per share to consumption followed an AR(1) with a shifting endpoint. The more commonly made assumption in the asset pricing literature is that the log of the ratio of dividends per share to consumption follows a process with normal innovations so that innovations to the level of the ratio of dividends per share to consumption are log-normal rather than normal. With such a model, the conditional covariances that we denote by the terms $H_t^{(k)}$ are not constant over time and thus our simple valuation equation (3) does not apply. In this section, we consider an exponentially affine model and develop an alternative argument that there is no excess volatility puzzle.

In this alternative model, we assume the same processes for the logarithm of the pricing kernel and consumption growth as in Section 4. This results in the same prices for claims to consumption and risk free interest rates as in that section.

In this section, we make the alternative assumption that the log of the ratio of dividends per share to consumption, denoted by $dc_t = \log(\frac{D_t}{C_t})$ is given by an AR(1) with a drifting endpoint.

Let the endpoint of the log ratio of dividends per share to consumption be denoted by x_t and evolve according to

$$x_{t+1} = x_t + \sigma_x \epsilon_{x,t+1}$$

Then let the log of the ratio of Dividends per Share to consumption evolve according to

$$dc_{t+1} - x_{t+1} = \rho(dc_t - x_t) + \sigma_d \epsilon_{d,t+1} - \sigma_x \epsilon_{x,t+1}$$

Assume that $\sigma_{d,t+1}$ and $\sigma_{x,t+1}$ are standard normal random variables that are independent over time and have contemporaneous correlation ρ_{dx}.

With these assumptions, the price of a claim to consumption one period ahead relative to consumption today is given by

$$\frac{P_{Ct}^{(1)}}{C_t} = \mathbb{E}_t \left[\exp(m_{t+1} + g_{Ct+1})\right] = \exp\left(\bar{m} + \bar{g}_C + \frac{1}{2}\left((\lambda_c + \sigma_{gC})^2 + \lambda_d^2 + \lambda_x^2 + 2\lambda_d\lambda_x\rho_{dx}\right)\right)$$

Note that this price is constant over time, so Assumption 1 of our first affine model is satisfied.

The prices of dividends relative to consumption satisfy the recursive formula that

$$\frac{P_{D,t}^{(k)}}{C_t} = \mathbb{E}_t \left[\exp(m_{t+1} + g_{Ct+1}) \frac{P_{D,t+1}^{(k-1)}}{C_{t+1}}\right] \tag{16}$$

We guess and verify that these prices are given by

$$\frac{P_{D,t}^{(k)}}{C_t} = \left(\frac{P_C^{(1)}}{C}\right)^k \exp(G_k) \mathbb{E}_t \left[\frac{D_{t+k}}{C_{t+k}}\right] \tag{17}$$

We prove that the prices of dividends relative to consumption have this form below.

This pricing formula gives us two main differences from our first affine pricing model.

First, in this exponentially affine case, our general pricing formula (5) gives us that

$$\mathbb{C}ov_t\left(M_{t,t+k}\frac{C_{t+k}}{C_t}, \frac{D_{t+k}}{C_{t+k}}\right) = \left(\frac{P_C^{(1)}}{C}\right)^k \mathbb{E}_t\left[\left(\exp(G_k) - 1\right)\frac{D_{t+k}}{C_{t+k}}\right]$$

so that this covariance term is no longer constant over time. Thus, our Assumption 2 is not satisfied unless $G_k = 0$. Thus, at the margin, an innovation to $\mathbb{E}_t\left[\frac{D_{t+k}}{C_{t+k}}\right]$ of size 1 moves the price of this dividend strip relative to current consumption by

$$\left(\frac{P_C^{(1)}}{C}\right)^k \exp(G_k)$$

where $\exp(G_k)$ corrects for the fact that the covariance term that was constant in the affine model moves with shocks to expected dividends in the exponentially affine model.

Second, with our log-normal model of innovations to dividends relative to consumption, we have

$$\mathbb{E}_t\left[\frac{D_{t+k}}{C_{t+k}}\right] = \exp\left(\rho^k(dc_t - x_t) + x_t\right)\exp(J_k) \tag{18}$$

where

$$J_k = \frac{1}{2}\mathbb{V}ar\left(dc_{t+k} - \mathbb{E}_t dc_{t+k}\right)$$

(which we will prove below is constant over time t).

With these two modifications of our first affine model, in the exponentially affine case, we can write equation (17) as

$$\frac{P_{D,t}^{(k)}}{C_t} = \tilde{\beta}_k \exp\left(\rho^k(dc_t - x_t) + x_t\right) \tag{19}$$

where

$$\tilde{\beta}_k \equiv \left(\frac{P_C^{(1)}}{C}\right)^k \exp(G_k + J_k).$$

In Appendix D we show that the terms G_k satisfy the recursion $G_0 = 0$ and

$$G_{k+1} = G_k + (\lambda_d + \lambda_x\rho_{dx})\rho^k\sigma_d + (\lambda_x + \lambda_d\rho_{dx})(1-\rho^k)\sigma_x$$

while the terms J_k satisfy the recursion $J_0 = 0$ and

$$J_{k+1} = J_k + \frac{1}{2}\left(\rho^k\right)^2\sigma_d^2 + \frac{1}{2}\left(1-\rho^k\right)^2\sigma_x^2 + \rho^k(1-\rho^k)\rho_{dx}\sigma_d\sigma_x.$$

Now consider the terms $\tilde{\beta}_k$ in equation (19). What are they in the data? The answer is that these are the price-current dividend ratios for dividend strips of maturity k when the current ratio of dividends to consumption is at its long run value (i.e. $dc_t = x_t$). To see this point, observe that when $dc_t = x_t$, the current ratio of dividends to consumption is given by $\exp(x_t)$ and the current ratio of the price of a dividend strip of maturity k to consumption is given from equation (17) and equation (18) by

$$\left.\frac{P_{D,t}^{(k)}}{C_t}\right|_{dc_t=x_t} = \tilde{\beta}_k \exp(x_t)$$

Dividing both sides of this equation by $D_t/C_t = \exp(dc_t) = \exp(x_t)$ gives us our result.

Given that the price of a claim to equity in perpetuity relative to consumption is given by

$$\frac{P_{Dt}}{C_t} = \sum_{k=1}^{\infty} \frac{P_{D,t}^{(k)}}{C_t}$$

we then have that the aggregate price dividend ratio for equity when the ratio of dividends to consumption is equal to its long run value is given by

$$\left.\frac{P_{Dt}}{D_t}\right|_{dc_t=x_t} = \sum_{k=1}^{\infty} \tilde{\beta}_k$$

We solve for the model's implications for the aggregate price-dividend ratio away from this point by dividing both sides of equation (19) by $D_t/C_t = \exp((dc_t - x_t) + x_t)$ and then summing across k to get

$$\frac{P_{Dt}}{D_t} = \sum_{k=1}^{\infty} \tilde{\beta}_k \exp\left((\rho^k - 1)(dc_t - x_t)\right). \tag{20}$$

We use this equation to solve for the sequence of values of $\{x_t\}$ that exactly replicate the ratio of the price per share of the CRSP Value-Weighted index to consumption expenditure.

5.1 A Second Excess Volatility Exercise

To use equation (20) to conduct our second excess volatility exercise, we must choose the parameters ρ and the sequence of parameters $\tilde{\beta}_k$. These parameters $\tilde{\beta}_k$ are implied by the full set of model parameters $\beta, \rho, \sigma_d, \sigma_x, \rho_{dx}, \lambda_d, \lambda_x$. They do not necessarily decay geometrically as would be consistent with a constant discount rate. We present a full calibration of this model in Appendix E.

Here we consider an approximation to our model in which we assume that the terms $\tilde{\beta}_k$ decay at a geometric rate so that we have $\tilde{\beta}_k = \tilde{\beta}^k$. In this case, to use equation (20) to solve for the sequence of $\{x_t\}$ that reconciles the data on the ratio of price per share to dividends per share on the left side of that equation with the data on the log of the ratio of dividends per share to consumption on the right side of that equation, we need choose only two parameters: ρ and $\tilde{\beta}$ where $\tilde{\beta}/(1-\tilde{\beta})$ corresponds to the price dividend ratio predicted by our model when $dc_t = x_t$.

We first consider model results when $\rho = 0.93$ and $\tilde{\beta}/(1-\tilde{\beta}) = 45$. The model-implied series for the value of the ratio of dividends per share to consumption expected in the long run shown in Figure 11. As in Figure 4, the blue line in this figure shows the data for the level of the ratio of dividends per share to consumption while the red line shows the long run expected value of this ratio $\exp(x_t)$ implied by our exponentially affine model.

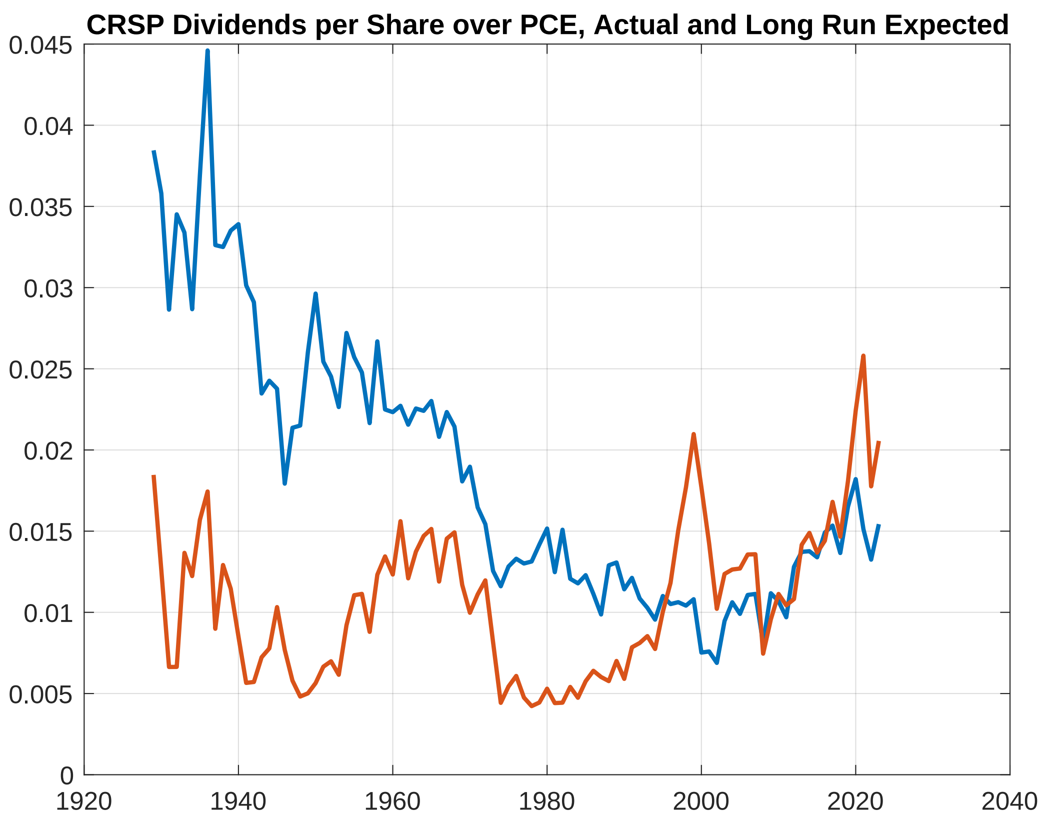

Figure 11: Blue Line: The ratio of dividends per share for the CRSP Value-Weighted Total Market Index to PCE ($\frac{D_t}{C_t}$), 1929-2023. Red Line: The expected long-run ratio of dividends per share for the CRSP Value-Weighted Total Market Index to PCE, X_t, that rationalizes the observed price per share of this index using equation (20) with $\tilde{\beta}_k = \tilde{\beta}^k$, $\tilde{\beta}/(1-\tilde{\beta}) = 45$, and $\rho = 0.93$. 1929-2023.

We find that with these parameters, the valuation model also matches the sample autocorrelation function of the model-implied sequence for $dc_t - x_t$ quite well and that the innovations to $dc_t - x_t$ and x_t do not appear to be forecastable by their lagged values or by the log price dividend ratio.

The results from this valuation exercise with our exponentially affine pricing model appear quite similar to those for the affine model. We do not see evidence of an excess volatility puzzle here.

In contrast, if we use our model with parameters $\rho = 0.93$ and long-run price dividend ratio $\tilde{\beta}/(1-\tilde{\beta}) = 25$, we get very different results. We show these results in Figure 12. Here we find that the model implies an implausibly volatile series for $\exp(x_t)$ (shown in red).

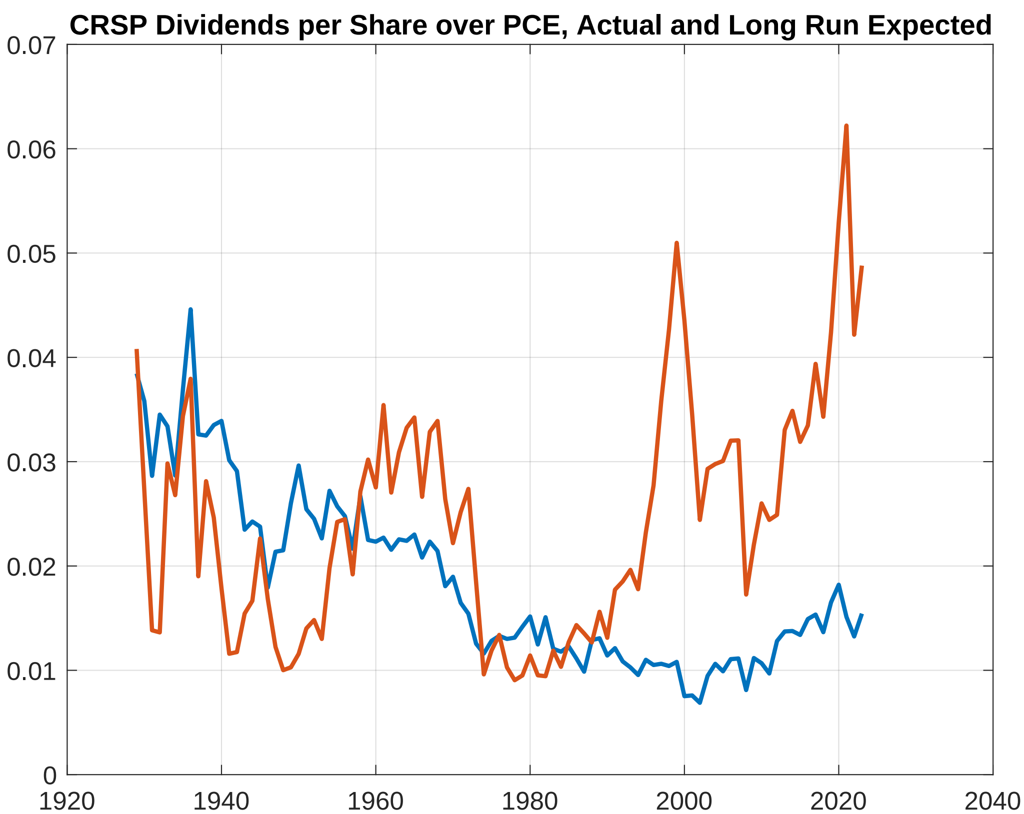

Figure 12: Blue Line: The ratio of dividends per share for the CRSP Value-Weighted Total Market Index to PCE ($\frac{D_t}{C_t}$), 1929-2023. Red Line: The expected long-run ratio of dividends per share for the CRSP Value-Weighted Total Market Index to PCE X_t that rationalizes the observed price per share of this index using equation (20) with $\tilde{\beta}_k = \tilde{\beta}^k$, $\tilde{\beta}/(1-\tilde{\beta}) = 25$, and $\rho = 0.93$. 1929-2023.

We see this exercise as confirming our main premise that it is straightforward to account for observed movements in stock prices based on a simple model of expectations of future dividends under the conditions that agents receive news about the ratio of dividends per share to consumption in the long run and that this news is not discounted at a high rate.

5.2 Monte Carlo Estimates of Return Forecastability

We now consider the question of how we might reconcile our model with findings in the literature that stock returns are forecastable using the log dividend price ratio as a forecasting variable? We note that in the data from 1929-2023, a regression of the form

$$r_{t+1}^{wd} = \alpha + \beta_r dp_t + \epsilon_{t+1} \tag{21}$$

where r_{t+1}^{wd} is the log of realized returns on the CRSP Value Weighted Index in excess of realized growth in PCE expenditures and dp_t is the log of the ratio of dividends per share to price per share yields an estimated slope coefficient of $\hat{\beta}_r = 0.07343$. We conduct the following Monte Carlo exercise to assess whether this estimated coefficient is unusual from the perspective of our exponentially affine valuation model.

Specifically, we take 100,000 random draws of sequences of 94 innovations $\{\epsilon_{xt+1}, \epsilon_{dt+1}\}$ of length 94 from a joint normal distribution with covariance matrix equal to that of the sample innovations implied by our affine pricing model above. We then construct alternative paths for realized returns and dividend price ratios using these simulated innovations, an initial value for the log of the ratio of dividends per share to consumption equal to its initial value in the data, and an initial value for x_0 drawn from the ergodic distribution of $dc_t - x_t$ given dc_0. The sampling distribution of these estimates $\hat{\beta}_r$ from equation 21 is shown in Figure 13. We obtain similar results using the initial value of x_0 implied by our model.

We see in Figure 13 that the data estimate of this slope coefficient (0.07343) is fairly likely even from our model with no time variation in discount rates. We thus conclude that there is no contradiction between our valuation model and estimates of aggregate stock market return forecastability using the log dividend price ratio as a forecasting variable.

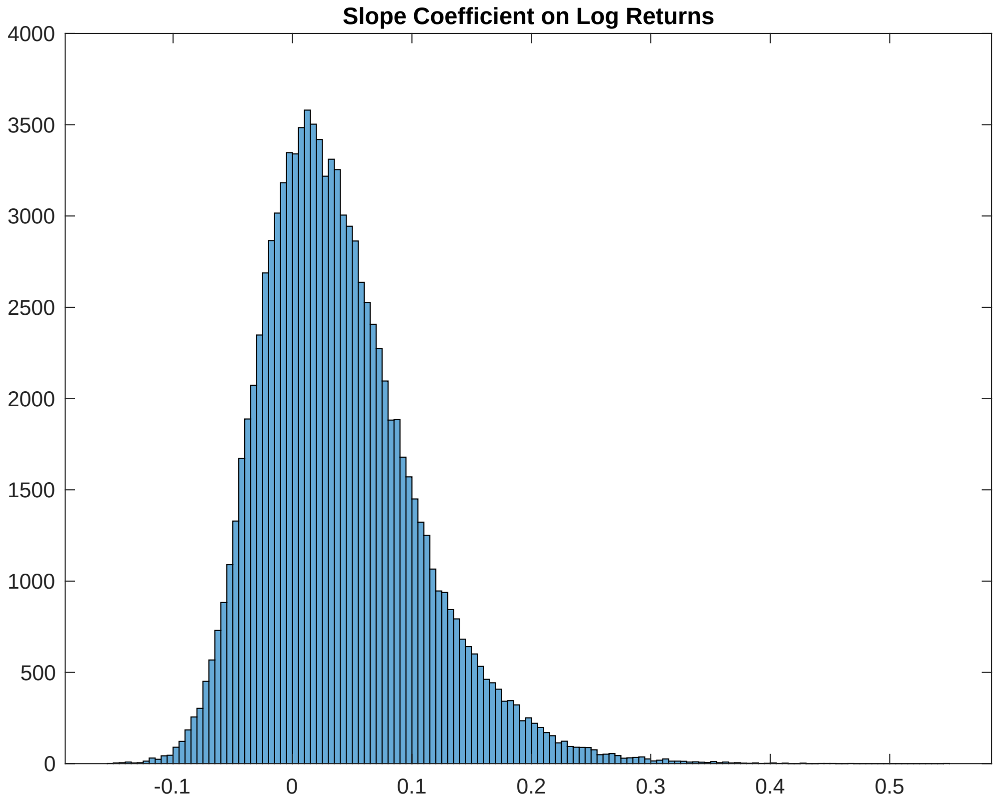

Figure 13: Histogram of estimates of $\hat{\beta}_r$ from return forecasting regression 21 from 100,000 simulations of 94 observations each from our exponentially affine model

6 Conclusion

The claim by Shiller (1981) that data on price per share for the stock market is too volatile to be accounted for by subsequent fluctuations in dividends has had a large impact on subsequent work in asset pricing.

Our aim in this paper is to call into question this basic claim that the stock market is excessively volatile. We present a simple and tractable model of the dynamics of dividends per share that accounts for realized values of price per share for the CRSP Value-Weighted Total Market Index over the period 1929-2023. In a companion paper, Atkeson, Heathcote, and Perri (2024), we conduct a closely related valuation exercise using data from the Integrated Macroeconomic Accounts on the market value and free cash flow of the U.S. corporate sector as a whole and again find that aggregate cash flows can account for these more aggregated data on market value over the period 1929-2023.

Because we do not have direct evidence on investors' expectations of future dividends per share, we cannot definitively say whether stock prices are driven by news about future cash flows or news about future discount rates. We simply see our study as a basis for arguing that simple asset pricing models based largely or even entirely on news about future cash flows are useful frameworks for understanding the market value of U.S. corporations.

Appendices

A The Argument for Excess Volatility in Shiller (1981)

In this appendix, we review the criticisms of the interpretation of Figure 1 in Shiller (2014) updating Shiller (1981) as evidence for excess volatility levied in Kleidon (1986) and Marsh and Merton (1986) on the grounds that both the levels of dividends per share and price per share are non stationary. We focus on the criticisms of Kleidon (1986).

Let the realized data on price per share be given by $\{P_t\}$. Let the realized data on dividends per share be given by $\{D_t\}$.

Consider the following simple valuation model. In this valuation model, assume that the logarithm of dividends per share, denoted by d_t, evolves according to

$$d_{t+1} = \tilde{g} + d_t + \sigma\epsilon_{t+1}$$

where $\epsilon_{t+1} \sim N(0,1)$ and $\tilde{g}$ is a constant. With this assumption, we have

$$\mathbb{E}_t D_{t+1} = (1+g)D_t$$

and, more generally

$$\mathbb{E}_t D_{t+k} = (1+g)^k D_t$$

where

$$g = \exp(\tilde{g} + \frac{1}{2}\sigma^2) - 1$$

With this model of expected dividends, create the model's implications for price per share based under constant discounting as

$$P_t = \sum_{k=1}^{\infty}(1+r)^{-k}\mathbb{E}_t D_{t+k} = \frac{1+g}{r-g}D_t \tag{22}$$

The prediction for the price constructed in Shiller (1981) and Shiller (2014) under the assumption that have an infinite realized sequence of dividends is

$$P_t^\star = \sum_{k=1}^{\infty}(1+r)^{-k}D_{t+k}$$

That is, we use realized dividends without the expectation.

In this case, both P_t and $P_t^\star$ are non-stationary. But, theoretically, since the model's implication for the price P_t is directly proportional to the currently realized dividend D_t, we have that the standard deviation of log changes in price is given as

$$\mathbb{S}td\left(\log(P_{t+1}) - \log(P_t)\right) = \sigma$$

In contrast, it is straightforward to verify via a Monte Carlo simulation that the standard deviation of log changes in the predicted price constructed using the method above is

$$\mathbb{S}td\left(\log(P^\star_{t+1}) - \log(P^\star_t)\right)$$

is typically at least an order of magnitude smaller than σ. Kleidon (1986) shows several results from such Monte Carlo simulations that lead to figures with these simulated data very similar in appearance to those in Shiller (1981).

The issue of why this approach to assessing stock market volatility goes wrong can be seen clearly from equation 22. If dividends are a random walk, then news that arrives between t and $t+1$ in the form of the shock ϵ_{t+1} moves agents' expectations of future dividends out into the infinite future since

$$\mathbb{E}_t D_{t+k} = D_t$$

and

$$\mathbb{E}_{t+1} D_{t+k} = D_{t+1}$$

In contrast, if we follow the procedure in Shiller (1981) to construct $P^\star_t$, the we are effectively assuming that agents' expectations of future dividends never move at all. That is

$$\mathbb{E}^\star_t D_{t+k} = D_{t+k}$$

and

$$\mathbb{E}^\star_{t+1} D_{t+k} = D_{t+k}$$

The only updating to P^*_t that occurs is that the first dividend is dropped and the discounting of future dividends is update by $(1+r)$. That is P^*_t satisfies

$$P^\star_t = \frac{1}{1+r}\left[D_{t+1} + P^\star_{t+1}\right]$$

This equation implies that

$$\log(P^\star_{t+1}) - \log(P^\star_t) = \log\left(1 + \frac{D_{t+1}}{P^\star_{t+1}}\right) \approx \frac{D_{t+1}}{P^\star_{t+1}}$$

Given that $\{\log(D_t)\}$ is assumed to be a random walk with a constant drift and we have assumed a constant discount rate r, one should not expect $\frac{D_{t+1}}{P^{\star}_{t+1}}$ to be variable. It varies only because of random runs of positive or negative values of ϵ_t leading to positive or negative runs of realized dividend growth above or below the mean. Monte Carlo simulation reveals this variance to be very small.

B How a value-weighted stock index is constructed

The concepts of price per share and dividends per share for a broad stock market index are constructed to meet specific needs that are not the same as those of an academic researcher seeking to understand fluctuations in the value of the stock market. In particular, the measure of price per share represents the dynamics of the value of and payouts to the portfolio of an investor who follows a very specific trading strategy that does not correspond to equilibrium notions of "holding the market" as in Sharpe (1964) and Lucas (1978). An investor who invested to track the CRSP Value-Weighted Total Market Index, would end up holding a constantly changing share of the total market capitalization of that index, with the changes in that share of the market held engineered specifically to reduce the volatility of the cash flows to that investor, leaving that investor only with payouts from dividends. We argue, then, that it is no surprise that empirical work using these data would arrive at the conclusion that stock prices move too much to be justified by subsequent changes in payouts. This finding is hard-wired into the construction of the data.[10]

An alternative approach to assessing whether the volatility of the stock market is too high relative to the volatility of the cash flows going to someone invested in the market is to examine the cash flows that would flow to an investor who followed an "equilibrium" strategy of holding a constant fraction of the total market capitalization of the stocks in a broad stock index at every moment in time. This is the portfolio strategy that we take as the equilibrium strategy of "holding the market". As we describe next, it is a simple exercise to construct these cash flows using data on the index returns including dividends, index returns excluding dividends, the level of the index in question, and the total market capitalization of the stocks in the index. This methodology is presented in Dichev (2006) who notes that it is commonly used to in the mutual fund industry to reconcile fund returns, fund flows, and fund market values.

When we do so, using the CRSP Value-Weighted Total Market Index as an illustration, we

[10]This concern is heightened by the recognition that in the decades following World War II, firms smoothed their dividend payouts. See Marsh and Merton (1986) and Chen, Da, and Priestly (2012) and the papers cited therein for a discussion of the impact of dividend smoothing on variance bounds tests and predictive regressions.

find that the cash flows associated with this "equilibrium" investment strategy are massively volatile, calling into question the conclusion that stock prices move too much to be justified by subsequent movements in dividends. It is straightforward to illustrate the same findings with other broad value-weighted stock indices.

To begin, it is helpful to review the basics of the construction of a broad value-weighted stock market index. We do that now.

At any point in time, t, a value-weighted stock index, denoted here by $X(t)$ is given as a time-varying fraction of the total market capitalization of the stocks in the index. That is, if we let $\Omega(t)$ be the set of stocks in the index, and $p_i(t)$ and $s_i(t)$ be the prices and shares outstanding for those stocks, then the total market capitalization of the stocks in the index, denoted here by $TMC(t)$, is given by

$$TMC(t) = \sum_{i\in\Omega(t)} p_i(t)s_i(t) \tag{23}$$

The level of the index at t, which we denote by $X(t)$, is given by

$$X(t) = \frac{1}{\theta(t)} TMC(t) \tag{24}$$

where $\theta(t)$ is called the "divisor" for the index at t. The argument t in $\theta(t)$ is there to denote that this divisor changes over time. Note here that $1/\theta(t)$ represents the fraction of the total market capitalization of the stocks in the index held at t by an investor tracking the level of index rather than the total market capitalization of the stocks in the index.

The gross value-weighted return on this index between periods t and $t+1$ not including dividends is given by

$$R_{t,t+1}^{no\ dividends} = \sum_{i\in\Omega(t)} \left(\frac{p_i(t)s_i(t)}{\sum_{j\in\Omega(t)} p_j(t)s_j(t)} \right) \frac{p_i(t+1)}{p_i(t)} \tag{25}$$

If we denote by $d_i(t+1)$ the dividend paid by firm i at time $t+1$ to someone who owned the share at time t, then aggregate dividends paid in $t+1$ are given by

$$D(t+1) = \sum_{i\in\Omega(t)} d_i(t+1)s_i(t) \tag{26}$$

and dividends per share are given by

$$DPS(t+1) = \frac{1}{\theta(t)} D(t+1) \tag{27}$$

The gross value-weighted return on this index between periods t and $t+1$ including dividends is given by

$$R_{t,t+1}^{w\ dividends} = \sum_{i\in\Omega(t)} \left(\frac{p_i(t)s_i(t)}{\sum_{j\in\Omega(t)} p_j(t)s_j(t)} \right) \left(\frac{p_i(t+1)+d_i(t+1)}{p_i(t)} \right) \tag{28}$$

The divisor at $t+1$, denoted by $\theta(t+1)$ is chosen so that the change in the index level from t to $t+1$ corresponds to the gross value-weighted return without dividends, i.e.

$$\frac{X(t+1)}{X(t)} = R_{t,t+1}^{no\ dividends} \tag{29}$$

From equation 25, this implies that

$$X(t+1) = \frac{1}{\theta(t)} \sum_{i\in\Omega(t)} p_i(t+1)s_i(t)$$

With this construction, it is also the case that the gross value-weighted return including dividends corresponds in the natural manner to the returns defined in terms of price per share and dividends per share. That is

$$\frac{X(t+1)+DPS(t+1)}{X(t)} = R_{t,t+1}^{w\ dividends} \tag{30}$$

What we have in equations 29 and 30 is that data on the *price per share* and *dividends per share* for the index can be used to reproduce the value-weighted returns on the stocks in the index without and with dividends between periods t and $t+1$ in a natural manner consistent in notation as if the entire index were a single firm.

But how is this construction achieved? In reality, the stocks in the index are not a single firm since some stocks are added and some a removed and since the incumbent firms in the index often take actions to change the number of their shares outstanding. To deal with these issues, the divisor of the index is adjusted so that equation 24 is also satisfied in period $t+1$. This approach to index construction implies that the divisor changes from period t to period $t+1$ according to

$$\theta(t+1) = \frac{\sum_{i\in\Omega(t+1)} p_i(t+1)s_i(t+1)}{\sum_{i\in\Omega(t)} p_i(t+1)s_i(t)} \theta(t) \tag{31}$$

It is here in equation 31 that we see that the construction of the index implies a certain trading strategy that does not correspond to holding a constant share of the market capitalization of the stocks in the index. An investor who aims to hold a portfolio that tracks this index would be required to adjust the fraction of the total market capitalization of the stocks in the index that he or she held from $1/\theta(t)$ to $1/\theta(t+1)$ as indicated in equation 31. That is, if, at $t+1$, the shares outstanding for the firms in the index at $t+1$ have increased when evaluated at $t+1$ prices, either due to incumbent firms issuing more shares on net (raising capital), or due to firms being added to the index at $t+1$ being more valuable than firms leaving the index between t and $t+1$, the divisor rises and the implied share of the total market capitalization of the stocks in the index held by an investor tracking the index falls. Likewise, if incumbent firms buy back their shares (returning capital), of if firms being added to the index at $t+1$ are less valuable than firms leaving the index between t and $t+1$, the divisor falls and the implied share of the total market capitalization of the stocks in the index held by an investor tracking the index rises.

More generally, there is a long list of circumstances that lead to changes in the number of shares outstanding for the firms in the index between t and $t+1$ that are referred to as *Corporate Actions*. These include Initial Public Offerings, Delistings, Mergers and Acquisitions, Reverse Mergers/Takeovers, Tendered Shares, Spin-Offs, Rights Offerings, and certain transactions connected with warrants, options, partly paid shares, convertible bonds, contingent value rights, etc. The staff at CRSP (and S&P Dow Jones Indices for their indices) invest considerable resources tracking all of these events and adjusting the index divisor accordingly.

What this index construction methodology implies is that an investor who aims to hold a portfolio that tracks the level of the index over time will not participate in any of these corporate actions. As a result, this investor receives only the cash flows associated with dividends paid at $t+1$ by incumbent firms in period t. This investor will not receive the cash flows associated with new share issuance or share buybacks by these incumbent firms nor the cash flows associated with the entry and exit of firms from the index (or any of the other possible corporate actions). Instead of participating in these cash flows, an investor who aims to track the level of the index simply adjusts the fraction held of the total market capitalization of the stocks in the index rather than contribute or remove cash as indicated by these corporate actions.

How then can we use the data from the index to recover the cash flows received by an investor following the equilibrium trading strategy of "holding the market" at all times. To do this, we invoke the theorem of Miller and Modigliani (1961) that asserts that changes in a firm's dividend policy to return cash to share holders in the form of net buybacks do not change either the returns or the market capitalization of the firm. Using this principle, following Dichev (2006), we construct the additional cash flows to accruing to an investor received by an investor following the equilibrium trading strategy of "holding the market" at all times, denoted here by $CAFC_{t+1}$ for *corporate action cash flows* using the equation

$$CACF(t+1) = R_{t,t+1}^{no\ dividends} TMC(t) - TMC(t+1) \tag{32}$$

We then have the total cash flows to an equilibrium investor holding the market at $t+1$ are $D(t+1) + CACF(t+1)$.

This equation 32 is an accounting identity that follows from a reconciliation of returns on the market from t to $t+1$ and the change in market capitalization of the market as a whole. This accounting identity implies that these cash flows from corporate actions can be stated equivalently as

$$CACF(t+1) = \sum_{i \in \Omega(t)} p_i(t+1)s_i(t) - \sum_{i \in \Omega(t+1)} p_i(t+1)s_i(t+1)$$

That is, these are the cash flows that arise from all changes in the number of shares outstanding from time t to time $t+1$ when valued at prices at time $t+1$.

Now, what impact do these calculations have on the ratio of dividends per share to price per share as measured by this index? We have by definition that the ratio of dividends per share to price per share is equal to the ratio of total dividends to total market capitalization of the stocks in the index, i.e.

$$\frac{DPS(t)}{X(t)} = \frac{D(t)}{TMC(t)}$$

This then implies that

$$\frac{DPS(t)}{X(t)} = \frac{D(t) + CACF(t)}{TMC(t)} - \frac{CACF(t)}{TMC(t)} \tag{33}$$

Consider the implications of Miller and Modigliani (1961) for the terms in this equation. In their analysis, they take the total cash flows to equity investors $D(t) + CACF(t)$ as given. With this assumption, they show that total market capitalization $TMC(t)$ is independent of the payout policy as determined by the split of total payouts into dividends $D(t)$ and cash flows arising from corporate actions $CACF(t)$. Thus, the first ratio on the right side of equation 33, given by $\frac{D(t)+CACF(t)}{TMC(t)}$ is fundamental. It is not impacted by changes in corporate actions. Of course, the other two ratios, $\frac{DPS(t)}{X(t)}$ and $\frac{CACF(t)}{TMC(t)}$ are impacted by corporate actions. To the extent that the ratio $\frac{CACF(t)}{TMC(t)}$ is volatile, the relative volatility of the fundamental ratio $\frac{D(t)+CACF(t)}{TMC(t)}$ and the ratio of dividends per share and price per share $\frac{DPS(t)}{X(t)}$ will be different. To the extent that there is are low frequency movements in the ratio $\frac{CACF(t)}{TMC(t)}$ not present in the fundamental valuation ratio $\frac{D(t)+CACF(t)}{TMC(t)}$, there will be low frequency movements in the ratio of dividends per share to price per share $\frac{DPS(t)}{X(t)}$ not driven by fundamentals but instead driven by corporate actions.

In this appendix, we compare our measure of total payouts to equilibrium investors in equity as represented by the CRSP Value-Weighted Total Market Index to that constructed in Davydiuk et al. (2023) which builds on the work of Boudoukh et al. (2007) but also uses the CRSP Stock file as we do. In Figure 6, we show in blue the ratio of total payouts on the CRSP Value-Weighted Total Market Index to total market capitalization of the stocks in that index $((D(t)+CACF(t))/TMC(t))$ from 1926-2023. In red, we show the ratio of equity cash payouts less net equity issuance to total market capitalization as measured in Davydiuk et al. (2023) for the time period 1975-2017 obtained from the *Journal of Finance* website for this article. Note that the measure constructed in Davydiuk et al. (2023) accounts for share buybacks but also accounts for changes in entity structure due to initial public offerings (IPOs), mergers, acquisitions, and exchanges.

As is evident in this figure, these two measures are quite similar where they overlap.

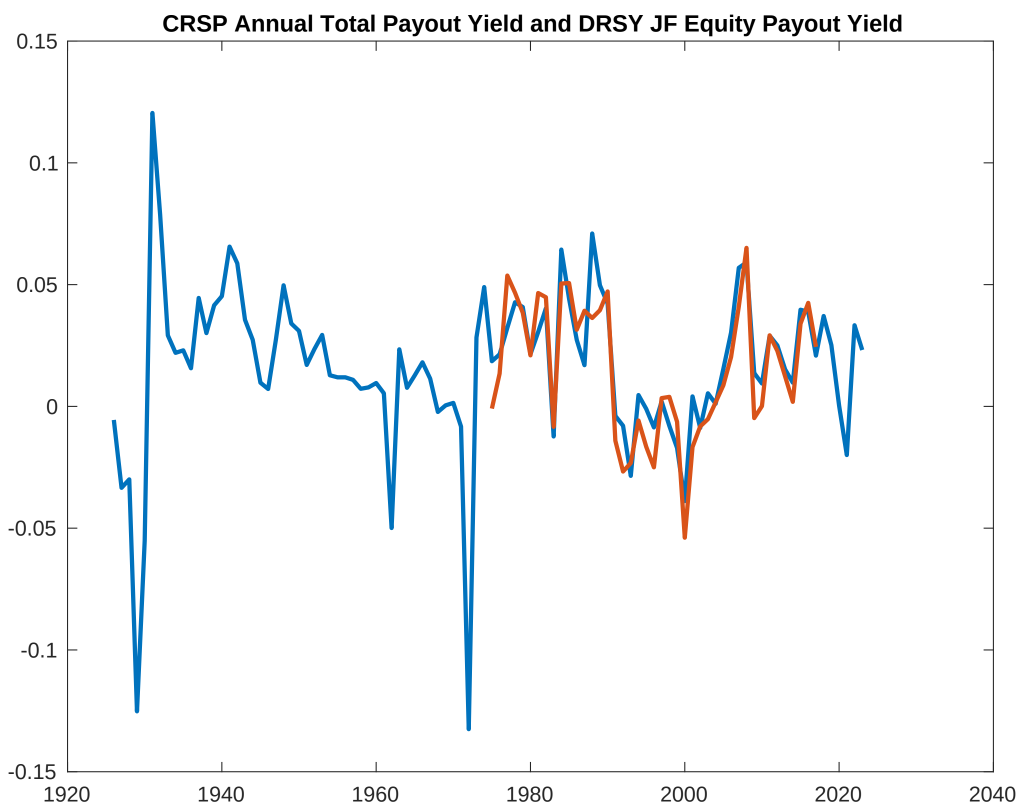

Figure B.1: In blue: the ratio of payouts to an equilibrium investor to total market capitalization of the stocks in the CRSP Value-Weighted Total Market Index $((D(t) + CACF(t))/TMC(t))$, where payouts are summed over the calendar year. In red: the ratio of total payouts to equity to total market capitalization of equity from Davydiuk et al. (2023).

C Proof of formula (15)

One can prove this formula using the moment generating function for normal random variables. In particular, we start by computing for a normal random variable

$$\mathbb{E}\exp(atx) = \exp(at\mu + \frac{1}{2}a^2t^2\sigma^2)$$

We then have

$$\mathbb{E}ax\exp(atx) = \mathbb{E}\frac{d}{dt}\exp(atx) = \exp(at\mu + \frac{1}{2}a^2t^2\sigma^2)(a\mu + ta^2\sigma^2)$$

If we evaluate this expression at $t = 1$ with $\mu = 0$ and $\sigma = 1$ for a standard normal, we have

$$\mathbb{E}ax\exp(ax) = \exp(\frac{1}{2}a^2)a^2$$

we multiply by c/a to obtain

$$\mathbb{E}cx\exp(ax) = \exp(\frac{1}{2}a^2)ca$$

We then have

$$\mathbb{E}\exp(ax + by)(cx + dz) = \mathbb{E}\exp(by)\mathbb{E}cx\exp(ax) + \mathbb{E}\exp(by)\mathbb{E}\exp(ax)\mathbb{E}dz$$

by the independence of x, y and z. Finally, since $\mathbb{E}z = 0$ and $\mathbb{E}\exp(by) = \exp(\frac{1}{2}b^2)$ we get equation 15.

D Solving for G_k and J_k

The terms G_k satisfy the recursion $G_0 = 0$ and

$$G_{k+1} = G_k + (\lambda_d + \lambda_x\rho_{dx})\rho^k\sigma_d + (\lambda_x + \lambda_d\rho_{dx})(1 - \rho^k)\sigma_x$$

The terms J_k satisfy the recursion $J_0 = 0$ and

$$J_{k+1} = J_k + \frac{1}{2}\left(\rho^k\right)^2\sigma_d^2 + \frac{1}{2}\left(1 - \rho^k\right)^2\sigma_x^2 + \rho^k(1 - \rho^k)\rho_{dx}\sigma_d\sigma_x$$

We now derive equation 17 and our recursive formulas for G_k and J_k in two steps. First we use equation 18 to derive our formulas for J_k. Then we use equation 16 to derive equation

19 and the associated formulas for G_k.

We start with the observation that, for $k = 0$

$$\mathbb{E}_t \frac{D_{t+k}}{C_{t+k}} = \exp\left(\rho^k (dc_t - x_t) + x_t\right) \exp(J_k)$$

with $J_0 = 0$. This confirms equation 18 for $k = 0$.

We now prove our recursive formula for J_{k+1} by induction. Observe that conditional expectations of the ratio of dividends per share to consumption satisfy the following recursive formula by the Law of Iterated Expectations

$$\mathbb{E}_t \frac{D_{t+k}}{C_{t+k}} = \mathbb{E}_t \mathbb{E}_{t+1} \frac{D_{t+k}}{C_{t+k}}$$

Take as given formula 18 for $k \geq 0$. These two statements then imply that

$$\mathbb{E}_t \frac{D_{t+k+1}}{C_{t+k+1}} = \mathbb{E}_t \exp\left(\rho^k (dc_{t+1} - x_{t+1}) + x_{t+1} + J_k\right) =$$

$$\mathbb{E}_t \exp\left(\rho^{k+1} (dc_t - x_t) + x_t + \rho^k \sigma_d \epsilon_{d,t+1} + (1-\rho^k)\sigma_x \epsilon_{x,t+1} + J_k\right) =$$

$$\exp\left(\rho^{k+1}(dc_t - x_t) + x_t + \frac{1}{2}\left(\rho^k\right)^2 \sigma_d^2 + \frac{1}{2}\left(1-\rho^k\right)^2 \sigma_x^2 + \rho^k(1-\rho^k)\rho_{dx}\sigma_d\sigma_x + J_k\right)$$

This confirms equation 18 for k with our recursive formula for J_k.

Moving on to pricing dividends, we have for $k = 0$ that

$$\frac{P_{Dt}^{(0)}}{C_t} = \exp(dc_t)$$

and thus equation 19 is satisfied with $G_0 = 0$ (and $J_0 = 0$).

We now prove our recursive formula for G_{k+1} by induction. We have that the prices of dividends satisfy the recursion in equation 16. Assume that equation 19 holds for $k \geq 0$. We then have from these two equations that

$$\frac{P_{Dt}^{(k+1)}}{C_t} = \left[\left(\frac{P_C^{(1)}}{C}\right)^k \exp(G_k + J_k)\right] \mathbb{E}_t \exp(m_{t+1} + g_{Ct+1}) \exp\left(\rho^k (dc_{t+1} - x_{t+1}) + x_{t+1}\right) =$$

$$\left[\left(\frac{P_C^{(1)}}{C}\right)^k \exp(G_k + J_k) \exp\left(\rho^{k+1}(dc_t - x_t) + x_t\right)\right] \times$$

$$\mathbb{E}_t \exp(m_{t+1} + g_{Ct+1}) \exp\left(\rho^k \sigma_d \epsilon_{d,t+1} + (1-\rho^k)\sigma_x \epsilon_{x,t+1}\right)$$

The term

$$\mathbb{E}_t \exp\left(m_{t+1} + g_{Ct+1} + \rho^k \sigma_d \epsilon_{d,t+1} + (1-\rho^k)\sigma_x \epsilon_{x,t+1}\right) =$$

$$\exp\left(\bar{m} + \bar{g}_C\right) \mathbb{E}_t \exp\left((\lambda_c + \sigma_{g_C})\epsilon_{c,t+1} + (\lambda_d + \rho^k \sigma_d)\epsilon_{d,t+1} + (\lambda_x + (1-\rho^k)\sigma_x)\epsilon_{x,t+1}\right) =$$

$$\exp\left(\bar{m} + \bar{g}_C\right) \exp\left(\frac{1}{2}(\lambda_c + \sigma_{g_C})^2 + \frac{1}{2}(\lambda_d + \rho^k \sigma_d)^2 + \frac{1}{2}(\lambda_x + (1-\rho^k)\sigma_x)^2 + (\lambda_d + \rho^k \sigma_d)(\lambda_x + (1-\rho^k)\sigma_x)\rho_{dx}\right) =$$

$$\left(\frac{P_C^{(1)}}{C}\right) \exp\left((\lambda_d + \lambda_x \rho_{dx})\rho^k \sigma_d + (\lambda_x + \lambda_d \rho_{dx})(1-\rho^k)\sigma_x + \frac{1}{2}\rho^{2k}\sigma_d^2 + \frac{1}{2}(1-\rho^k)^2 \sigma_x^2 + \rho^k(1-\rho^k)\rho_{dx}\sigma_d \sigma_x\right)$$

This algebra and our prior results for J_k give us our inductive result and formula for G_k.

E Full Calibration of Exponentially Affine Model

In this section, we produce a full calibration of the Exponentially Affine valuation model and compare the exact values of the discount rates $\tilde{\beta}_k$ to the geometrically declining series $\tilde{\beta}^k$ that we used in section 5.1. We show that, at least with respect to this calibration, the geometrically declining series $\tilde{\beta}^k$ is a close approximation to the exact discount rates $\tilde{\beta}_k$.

To calibrate the model, we must choose parameters $\beta \equiv \frac{P_C^{(1)}}{C}$ and $\rho, \sigma_d, \sigma_x, \lambda_d, \lambda_x$ and ρ_{dx}. In choosing parameters, we can set β as before consistent with a price-dividend ratio for a claim to consumption of 80. Given parameter choices, we compute the implied terms G_k and J_k and the implied series for $\tilde{\beta}_k$.

We choose the parameters $\rho, \sigma_d, \sigma_x, \lambda_d, \lambda_x$ and $\rho_d x$ jointly to match the following data targets.

1. We set $\rho = 0.93$. We show that this well-approximates the autocorrelation function of $dc_t - x_t$ when we use the values of $\{x_t\}$ implied by this calibration.

2. Given the values of $\{x_t\}$ implied by our parameters and the data and the choice of ρ, we can construct the model-implied innovations $\sigma_d \epsilon_{d,t+1}$ and $\sigma_x \epsilon_{x,t+1}$. We check that $\sigma_d = 0.147$, $\sigma_x = 0.24$ and $\rho_{dx} = 0.50$ match the sample moments of these innovations.

3. The parameters λ_d and λ_x are not separately identified. We set $\lambda_d = \lambda_x/4$ and we set $\lambda_x = -0.1371$. These parameters together give us a ratio of price per share to dividends per share when the ratio of dividends per share to consumption is equal to its long run value of 45.

We compare the first 100 discount factors from this calibrated model to a geometrically declining series with the same value of 45 for the price dividend ratio when the ratio of dividends per share to consumption is equal to its long run value ($dc_t = x_t$) in Figure E.1. We show the exact solution for the first 100 of these discount factors $\tilde{\beta}_k$ in blue. We show the corresponding values of $\tilde{\beta}^k$ in red. It is clear from this figure that the two series are quite close.

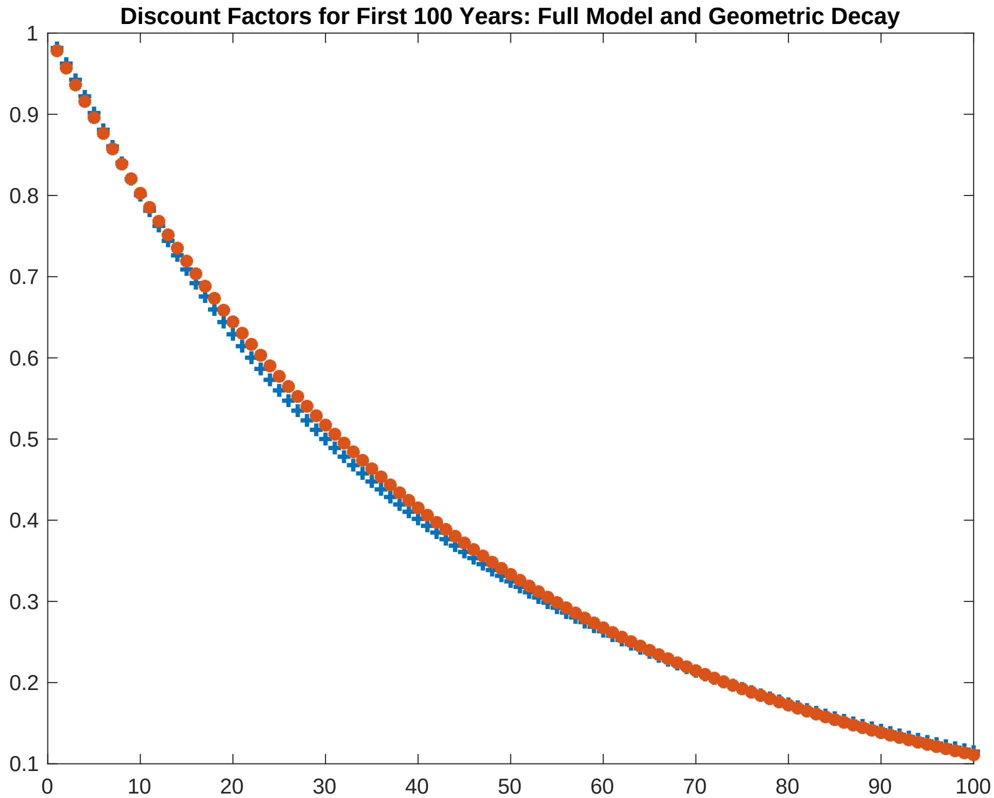

Figure E.1: In blue: the first 100 discount factors $\tilde{\beta}_k$ computed with the calibrated parameters of the model. In red: the first 100 of the geometrically declining discount factors $\tilde{\beta}^k$ when this $\tilde{\beta}$ is calibrated to match a price dividend ratio of 45 when $DC_t = x_t$.

References

Alvarez, Fernando and Urban J. Jermann. 2004. "Using Asset Prices to Measure the Cost of Business Cycles." *Journal of Political Economy* 112 (6):1223–1256.

Atkeson, Andrew, Jonathan Heathcote, and Fabrizio Perri. 2024. "Reconciling Macroeconomics and Finance for the US Corporate Sector: 1929 - Present."

Bansal, Ravi and Christian Lundblad. 2002. "Market efficiency, asset returns, and the size of the risk premium in global equity markets." *Journal of Econometrics* 109 (2):195–237.

Bansal, Ravi and Amir Yaron. 2004. "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles." *Journal of Finance* 59 (4):1481–1509.

Barsky, Robert B. and J. Bradford De Long. 1993. "Why Does the Stock Market Fluctuate?" *Quarterly Journal of Economics* 107 (2):291–311.

Blanchard, Olivier. 2019. "Public Debt and Low Interest Rates." *American Economic Review* 109 (4):1197–1299.

Boudoukh, Jacob, Roni Michaely, Matthew Richardson, and Michael R. Roberts. 2007. "On the Importance of Measuring Payout Yield: Implications for Empirical Asset Pricing." *Journal of Finance* 62 (2):877–915.

Campbell, John Y. 2018. *Financial Decisions and Markets: A Course in Asset Pricing.* Princeton University Press.

Campbell, John Y. and John Cochrane. 1999. "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior." *Journal of Political Economy* 107 (2):205–251.

Campbell, John Y. and Robert J. Shiller. 1987. "Cointegration and tests of present value models." *Journal of Political Economy* 95 (1062-1088).

———. 1988. "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors." *The Review of Financial Studies* 1 (3):195–228.

Chen, Long, Zhi Da, and Richard Priestly. 2012. "Dividend Smoothing and Predictability." *Management Science* 58 (10):1834–1853.

Cochrane, John. 2011. "Presidential Address: Discount Rates." *Journal of Finance* 66 (4):1047–1108.

Davydiuk, Tetiana, Scott Richard, Ivan Shaliastovich, and Amir Yaron. 2023. "How Risky Are U.S. Corporate Assets?" *Journal of Finance* 78 (1):141–208.

Dichev, Ilia D. 2006. "What Are Stock Investors' Actual Historical Returns? Evidence from Dollar-Weighted Returns." *American Economic Review* 97 (1):386–401.

Greenwald, Daniel, Martin Lettau, and Sydney Ludvigson. 2023. "How the Wealth Was Won: Factor Shares as Market Fundamentals." Working Paper 25769, NBER.

Gârleanu, Nicolae and Stavros Panageas. 2023. "Heterogeneity and Asset Prices: An Intergenerational Approach." *Journal of Political Economy* 121 (4):839–876.

Kleidon, Allan W. 1986. "Variance Bounds Tests and Stock Price Valuation Models." *Journal of Political Economy* 94 (5):953–1001.

Larraine, Borja and Motohiro Yogo. 2008. "Does firm value move too much to be justified by subsequent changes in cash flow?" *Journal of Financial Economics* 87 (1):200–226.

Leroy, Stephen and Richard Porter. 1981. "The Present-Value Relation: Tests Based on Implied Variance Bounds." *Econometrica* 49 (3):555–574.

Lucas, Jr., Robert E. 1978. "Asset Prices in an Exchange Economy." *Econometrica* 46 (6):1429–1445.

Lustig, Hanno N., Stijn Van Nieuwerburgh, and Adrien Verdelhan. 2013. "The Wealth-Consumption Ratio." *The Review of Asset Pricing Studies* 3 (1):38–94.

Marsh, Terry A. and Robert C. Merton. 1986. "Dividend Variability and Variance Bounds Tests for the Rationality of Stock Market Prices." *American Economic Review* 76 (3):483–498.

Miller, Merton H. and Franco Modigliani. 1961. "Dividend Policy, Growth, and the Valuation of Shares." *Journal of Business* 34 (4):411–433.

Sharpe, William F. 1964. "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk." *Journal of Finance* 19 (3):425–442.

Shiller, Robert J. 1981. "Do Stock Prices Move Too Much to be Justified by Subsequent Movements in Dividends?" *American Economic Review* 71 (3):421–436.

———. 2014. "Speculative Asset Prices." *American Economic Review* 104 (6):1486–1517.