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ABSTRACT

We study international trade and macroeconomic dynamics triggered by the imposition of sanctions. We begin with a tractable two-country model where Home and Foreign countries have comparative advantages in production of differentiated consumption goods and a commodity (e.g., gas), respectively. Home imposes sanctions on Foreign. Financial sanctions exclude a fraction of Foreign agents from the international bond market. Gas sanctions take the form of a ban on gas trade, equivalent to an appropriate price cap in our model. Differentiated goods trade sanctions exclude a fraction of Foreign and Home exporters from international trade. All sanctions lead to resource reallocation in both economies. Exchange rate movements reflect the direction of reallocation and the type of sanctions imposed rather than the success of the sanctions. Welfare analysis shows that gas sanctions are more costly for Home, while differentiated consumption goods trade sanctions are more costly for Foreign. A third country that refrains from joining the sanctions mitigates welfare losses in Foreign, but refraining from joining the sanctions is beneficial for the third country. These findings highlight the importance and the difficulty of international coordination when imposing sanctions.

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1. Introduction

Understanding the mechanisms of international economic interdependence is crucial, especially during periods of geopolitical tensions. The large-scale invasion of Ukraine by Russia on February 24, 2022, prompted the governments of 38 countries to impose sanctions on Russia and Belarus. This response aimed to punish the aggression, cut off resources supporting it, and engage in economic warfare to support Ukraine.[1] Unlike countries that faced sanctions since the end of World War II, Russia is a major economy, ranking 11th and 13th by nominal GDP and goods exports in 2021. Not since the 1930s had an economy of a similar size been subjected to such restrictions. Furthermore, today's global economy is more integrated, and Russia, as one of the leading energy suppliers globally, plays a crucial role in global value chains. Its significant energy exports make Russia a major net creditor in the global financial market, boasting the fourth-largest foreign exchange reserves in the world in 2021. These factors complicate and amplify the consequences of sanctions on both the targeted and sanctioning economies. When both sanctioning and sanctioned economies are sizable, understanding the resulting intended and unintended outcomes becomes particularly challenging. This study aims to contribute to this understanding by employing a micro-founded model of international trade and macroeconomic dynamics.

We begin with a suitable extension of the two-country model proposed by Ghironi and Melitz (2005)—henceforth referred to as GM. Our modification involves assuming that both countries, Home and Foreign, possess a raw source of energy—for convenience, natural gas. In each country, an upstream, perfectly competitive production sector combines sector-specific labor and natural gas to produce usable gas. A downstream, monopolistically competitive sector uses gas and sector-specific labor to manufacture differentiated consumption goods. Firm entry into this sector is endogenous and subject to an initial sunk cost, with firms producing at heterogeneous productivities drawn upon entry. Fixed trade costs dictate that only the relatively more productive firms engage in export. In the absence of sanctions, Home and Foreign gas are perfect substitutes, and their price is determined by the equalization of world demand and supply. We assume that Foreign has a larger endowment of natural gas but faces higher sunk costs of firm entry. Consequently, in the absence of sanctions, Home imports gas from Foreign. Additionally, there is a larger mass of producers of differentiated goods in Home compared to Foreign. Households in the two countries

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[1] For historical context, see Blackwill and Harris (2016) and Mulder (2022). See also Caldara and Iacoviello (2022) on the economic effects of adverse geopolitical events.

hold non-contingent bonds and shares. As in GM, only bonds are traded internationally. Each household comprises gas-sector and consumption-sector workers who pool their incomes, leading to the presence of a representative household in each country in the absence of sanctions.[2]

We consider sanctions imposed by Home, which come in two forms: financial sanctions and trade sanctions. Both types of sanctions can result in exclusion from the international market. Financial market sanctions specifically involve excluding a fraction of Foreign households from participating in international bond trading. Consequently, two distinct types of Foreign households emerge: the representative sanctioned household and the representative non-sanctioned household. While the non-sanctioned households can continue trading bonds with Home households, the sanctioned households are restricted to trading bonds solely with the non-sanctioned Foreign households. In the extreme case, the exclusion of all Foreign households result in financial autarky.

Trade sanctions can be applied to both gas trade and trade in differentiated consumption goods. In the context of gas trade, we explore a scenario where the quantity of traded gas is constrained. Imposing a ban on gas imports is equivalent to a case where Home regulates the price of imported gas below the marginal cost of Foreign gas production.

In the market for consumption goods, Home enforces sanctions by prohibiting the export of products from Home firms with productivity exceeding a specific threshold, and/or prohibits import of goods from Foreign firms with productivity surpassing a potentially different threshold. This approach implies that sanctions primarily target larger, more productive firms, typically producers of high-tech products, and sectors requiring advanced technologies.[3] In the absence of sanctions, all Home (Foreign) firms, with productivity above a cutoff determined by the fixed cost of trade, export to Foreign (Home). However, with the introduction of sanctions, a second, higher productivity cutoff is established. Only firms with productivity falling between these two cutoffs are permitted to engage in international trade. In the extreme case, if the sanction-determined cutoff is equal to or lower than the trade-cost determined one, there is no international trade in

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[2] We intentionally keep our setup relatively simple compared to quantitative extensions of the GM framework in subsequent literature and analyses of sanctions in quantitative, but static, trade models, such as Bachmann et al. (2022). Our goal is to provide a framework and a set of benchmark results that can serve as a foundation for future research addressing issues on macroeconomics of geoeconomic conflicts."

[3] The primary reason for not modeling trade sanctions as blanket bans or imposing higher tariff costs is rooted in the nature of the sanctions imposed by Western countries on Russia. These sanctions explicitly prohibit the trade of specific goods and/or transactions with designated entities. Guidance notes issued by these Western countries provide a list of sanctioned goods, emphasizing that the sanctions are not intended as blanket bans. For instance, see https://www.consilium.europa.eu/en/policies/sanctions/ and https://crsreports.congress.gov/product/ pdf/R/R45415 . It is noteworthy that the industries targeted by these sanctions, when compared to non-sanctioned sectors like non-durable goods, demonstrate higher productivity levels.

differentiated goods.[4]

We study the effects of sanctions in the short, medium, and long term, focusing on international relative prices, balances, standard macroeconomic aggregates, and Home and Foreign welfare. Similar to most conventional open macroeconomic models, our model cannot be fully solved analytically. Nevertheless, following the approach in GM, we derive analytical results that are crucial for understanding the effects of sanctions. We specifically analyze two variables of particular interest: the price of gas and the real exchange rate.

We demonstrate that, as long as there is some gas trade, the price of gas depends on two key variables: Home consumption and the extent to which relative consumption dynamics deviate from the outcome under internationally complete asset markets. If Home demand for consumption goods rises, consumption good producers demand more inputs to satisfy the demand, leading to increased input prices, including Home gas prices. In response to a rise in Foreign demand for consumption goods, the supply of Foreign gas to Home firms decreases due to its higher domestic use. Consequently, Home consumption good producers substitute toward more expensive Home gas, resulting in an increase in Home gas prices.

The relative cost of effective labor across countries (in GM terminology, the terms of labor) is a key driver of real exchange rate fluctuations. Similar to GM, an appreciation of the terms of labor results in real exchange rate appreciation. Furthermore, the real exchange rate responds to changes in average exporter productivity and/or alterations in the composition of consumption baskets. For example, a decrease in the average productivity of Foreign exporters leads to Home real exchange rate appreciation because less productive Foreign exporters charge higher prices. If the share of imported goods in the total product variety available to Home consumers rises relative to Foreign, the real exchange rate depreciates because, on average, exporters charge lower prices than non-exporters.

These analytical results guide our understanding of the effects of sanctions by shedding light on the roles of sanction-induced changes in the determinants of gas price and the real exchange rate. Numerical exercises are employed to illustrate the analytical results and explore the implications of our model for how other variables respond to sanctions.

We find that the impact of financial sanctions on Foreign consumption and welfare is substantial

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[4] It is worth noting that for our results, the distinction between firms ceasing exports due to government-imposed sanctions or voluntary decisions on their part is inconsequential. In practice, some companies independently ceased trade with specific other countries due to their own concerns (ranging from moral principles to public relations) in response to events like Russia's invasion of Ukraine. We do not explicitly model these concerns, and as such, we do not differentiate between government-imposed and self-imposed termination of trade.

only when a considerable proportion of Foreign households face sanctions. The rationale behind this is intuitive: if the share of Foreign households excluded from international asset trade is not significant enough, international borrowing and lending by non-sanctioned households on behalf of sanctioned ones tend to mitigate the consumption and welfare effects of sanctions. In scenarios where a substantial share of Foreign households faces sanctions, the availability of resources to finance domestic producer entry into the Foreign economy diminishes, resulting in adverse effects on entry and the number of Foreign firms.[5] Notwithstanding the decrease in the overall number of Foreign producers, the number of Foreign exporters must increase for Home and Foreign to reach a steady-state in which trade is balanced. This surge in exporter numbers results in a decline in the average productivity of Foreign exporters, consequently leading to a higher average price of Home imports. As a consequence, the Home real exchange rate appreciates.

Imposing sanctions on Foreign gas compels the Foreign economy to shift resources toward the production of differentiated goods, compensating for lost labor income and export revenue. This shift results in a surge in the number of entrants and producers in Foreign. Simultaneously, in the Home economy, resources are redirected to gas production to offset the shortfall in gas imports. Sanctions that curtail exports of Home consumption goods prompt a shift in Foreign consumption demand towards domestic products. Analogous to sanctions on Foreign gas, this measure also stimulates a higher influx of entrants and producers in the consumption goods sector of the Foreign economy. The reallocation in Foreign reduces gas production, subsequently diminishing Home imports of gas from Foreign. Consequently, Home rebalances its economic focus towards gas production.

The real exchange rate, however, exhibits distinct behaviors in response to various types of sanctions. Gas sanctions prompt an appreciation of the Home real exchange rate, whereas consumption good sanctions lead to depreciation. This disparity in real exchange rate responses stems from the divergent effects on the number of Foreign exporters. Following a gas sanction, Foreign consumption good exports expand, prompting an increase in the number of Foreign exporters to compensate for the loss of gas exports. Conversely, after a consumption good sanction, the consumption demand in Foreign shifts to domestically produced goods, causing a reduction in the supply of Foreign consumption goods to export markets. The higher number of Foreign exporters

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[5] As in GM, we assume a one-to-one identification between a producer, a product, and a firm for convenience. However, continuity makes it possible to interpret our model as one of multi-product firms whose boundaries are left unspecified, and where product-line managers within each firm act independently of each other. See Ghironi and Melitz (2005) and Bilbiie, Ghironi, and Melitz (2012) for more discussion.

(and their lower average productivity) contributes to Home real exchange rate appreciation following gas sanctions, while the real exchange rate depreciates after consumption good sanctions due to the fall in the number of Foreign exporters (and the rise in their average productivity).

The combined effects of price and variety dynamics and inefficient resource allocations result in significant welfare losses for both Foreign and Home households. While economic sanctions may harm adversaries, they inevitably come at a cost when the sanctioned economy has non-negligible size. The effectiveness and costs of sanctions are contingent on the type of sanction and comparative advantage. Specifically, the impact on an economy's welfare is more pronounced when sanctions target sectors with comparative disadvantages. For example, gas sanctions compel the Home economy to redirect resources toward gas production, the less efficient Home sector. This implies that gas sanctions impose greater hardships on the Home economy, while consumption goods trade sanctions prove to be more costly for the Foreign economy.

After completing the analysis of our two-country model, we extend our study to a three-country version to address questions that cannot be tackled in a two-country model. The three-country model allows us to investigate whether the effects of sanctions are dampened when the sanctioned economy can substitute trade with a third country. Our findings reveal that while the mechanisms from our analytical results and simulations of the two-country model are preserved, introducing a third country can dampen the effect of sanctions if the third country does not join in sanctioning. Coordinating sanctions with the third country results in the most pronounced welfare losses in the sanctioned economy, while the third country shares the burden with the Home country.

Our findings hinge on two crucial model features: heterogeneous producers and extensive margin dynamics. Firstly, the differentiation in productivity between exporters and non-exporters enables us to analyze how changes in producer composition within the downstream sector impact real exchange rate dynamics. For instance, under both consumption-good export sanctions and gas sanctions, more labor is used in the consumption-good sector. However, the exchange rate responds to movements in the producer composition in the consumption-good sector, depreciating under export sanctions and appreciating under gas sanctions. Additionally, our model suggests imperfect substitution toward the third country in response to sanctions. Only more productive producers can cover the fixed export costs and, therefore, enter export markets. This trade friction attenuates entry into export markets in the third country, so that additional demand from the sanctioned country cannot be perfectly fulfilled. Although sanctions that are imposed on Foreign hurt the third country, third country is still better off than joining the sanctions, highlighting the difficulty of coordinating sanctions.

Our paper is related to at least two literatures. The invasion of Ukraine by Russia has spurred a series of papers examining the impacts of sanctions. Notable works in this context include Albrizio et al. (2022), Bachmann et al. (2022), Bianchi and Sosa-Padilla (2022), Chupilkin et al. (2023), Eichengreen et al. (2022), Itskhoki and Mukhin (2022), Lorenzoni and Werning (2022), and Sturm (2023, 2024). Work that pre-dates Russia's attack on Ukraine includes Korhonen (2019), van Bergeijk (2021), and references therein. These papers present quantitative, multi-country, static analyses of trade effects (for instance, Bachmann et al. 2022), analyses that abstract from extensive margin effects (for instance, Lorenzoni and Werning 2022), or small open economy, New Keynesian models that cannot address the full range of consequences of sanctioning a large economy (for instance, Itskhoki and Mukhin 2022). Our approach differs in that we present a dynamic analysis within a canonical trade and macroeconomic framework. Although our model is not explicitly quantitative or multi-country, it allows us to explore the dynamic effects of sanctions, incorporating extensive margin effects that we deem crucial to understanding the functioning of sanctions. Moreover, our model makes it possible to study repercussions for both the sanctioning economy and the rest of the world. In line with the findings of Eichengreen et al. (2022), our results confirm their conclusions that exchange rate movements reflect the type and scale of sanctions rather than measuring their success or failure. By expanding on these insights, we highlight the role played by the producer composition in the downstream sector in determining the exchange rate fluctuations in response to sanctions.

Our paper also contributes to the literature on international macroeconomic models with microfoundations, which evolved following work by Melitz (2003) and Ghironi and Melitz (2005). Several studies have extended the GM framework to address several questions in international macroeconomics. These include works by Auray and Eyquem (2011), Bergin and Corsetti (2019), Cacciatore and Ghironi (2021), Corsetti, Martin, and Pesenti (2013), Hamano and Zanetti (2017), Imura and Shukayev (2019), Kim (2021), and Zlate (2016) among others. In our contribution, we enhance this literature by incorporating a straightforward model of energy production into the GM framework and by utilizing the extended model, along with a three-country version, to examine the effects of sanctions.

The rest of the paper is organized as follows: Section 2 presents the two-country model. Section 3 presents analytical findings on gas price and real exchange rate determination. Section 4 presents the calibration and model dynamics. Section 5 studies the effects of sanctions in the two-country

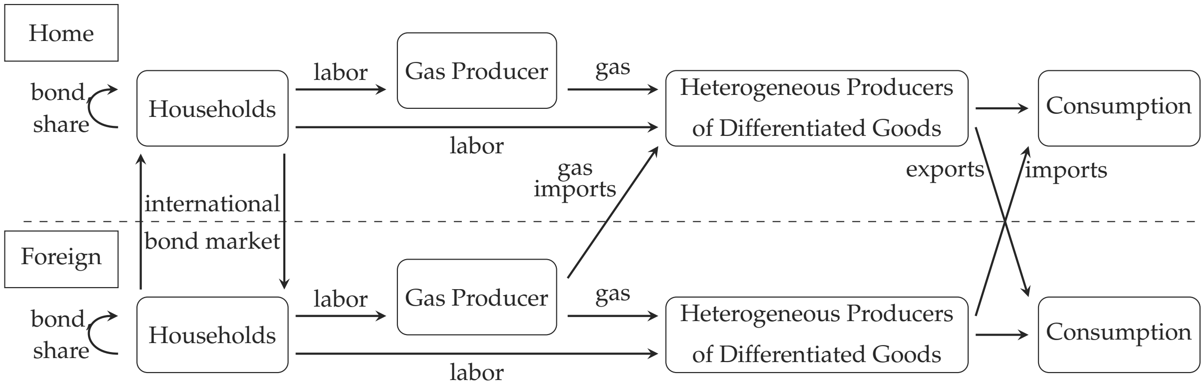

Figure 1: Model Architecture

model. Section 6 introduces a third country to study sanctions and international coordination. Section 7 concludes. The Appendix contains details on analytical derivations and additional figures and tables.

2. The Model

The world is composed of two asymmetric regions, Home and Foreign. Both Home and Foreign are populated by a unit mass of atomistic households. The representative household in each country consists of two groups of workers who supply labor to the two sectors of the economy, consumption goods producers and gas producers. Labor is assumed immobile across the two sectors in each country and across countries. Home is an importer of gas, whereas Foreign is an exporter of gas. We use Melitz (2003)'s monopolistic competition and heterogenous producers framework for the microeconomic underpinning of the consumption good producing sector as in GM. Prices are flexible. Figure 1 exhibits the model architecture.

2.1. Household Preferences

The representative household obtains utility from consumption of a basket of goods, C_t, and disutility from supplying labor, L_t, to the sector that produces consumption goods and $L_{G,t}$ to the sector that produces gas. The expected intertemporal utility function that the household maximizes is:

$$\mathbb{E}_t \left[\sum_{s=t}^{\infty} \beta^{s-t} \left(\log C_s - \frac{\kappa}{2} L_s^2 - \frac{\kappa_G}{2} L_{G,s}^2 \right) \right]$$

with $\beta \in (0,1)$ and $\kappa, \kappa_G > 0$. The consumption basket is defined over a continuum of goods Ω: $C_t = [\int_{\omega\in\Omega} c_t(\omega)^{\frac{\theta-1}{\theta}} d\omega]^{\frac{\theta}{\theta-1}}$ where $\theta > 1$ is the symmetric elasticity of substitution across goods. At any time t, only a subset of goods $\Omega_t \subset \Omega$ is available. Demand for individual goods is $c_t(\omega) = [p_t(\omega)/P_t]^{-\theta} C_t$ where $p_t(\omega)$ is the home currency price of a good $\omega \in \Omega_t$ and $P_t = [\int_{\omega\in\Omega_t} p_t(\omega)^{1-\theta} d\omega]^{\frac{1}{1-\theta}}$. Letting $\rho_t(\omega)$ be the price of good ω relative to the price of the basket, demand for good ω is $c_t(\omega) = [\rho_t(\omega)]^{-\theta} C_t$. Everything is similar in Foreign unless otherwise noted. Foreign variables are denoted with a star, and the location of gas use or good consumption below is denoted with a subscript H or F.

2.2. Gas Production

Home and Foreign are endowed with amounts of natural gas G_N and G_N^*, respectively, and we assume that Foreign has a larger endowment ($G_N^* > G_N$) and is an exporter. In each country, a perfectly competitive upstream sector produces usable gas by combining labor and natural gas. The production of usable gas by Foreign is given by:

$$G_t^* = G_N^* L_{G,t}^*, \tag{1}$$

This gas can be used domestically ($G_{H,t}^*$) or exported ($G_{F,t}^*$). Hence, in equilibrium, we have $G_N^* L_{G,t}^* = G_{H,t}^* + G_{F,t}^*$. Similarly, Home production of usable gas is

$$G_t = G_N L_{G,t}, \tag{2}$$

where we assume that this gas is used only domestically ($G_{H,t}$). Equilibrium implies $G_N L_{G,t} = G_{H,t}$.

First-order conditions for optimal labor demand in gas production in Home and Foreign imply, respectively, $w_{G,t} = \rho_{G,t} G_N$ and $w_{G,t}^* = \rho_{G,t}^* G_N^*$, where $w_{G,t}$ and $w_{G,t}^*$ are the real wages paid to workers in this sector in Home and Foreign, and $\rho_{G,t}$ and $\rho_{G,t}^*$ are the real prices of usable gas in the two countries (both wages and prices are in units of the relevant country's consumption basket). Foreign exports gas to Home. Home and foreign produced gas is perfectly substitutable, and thus home gas market price determination ensures $\rho_{G,t} = \tau_{G,t} Q_t \rho_{G,t}^*$, where $\tau_{G,t}$ is iceberg gas trade costs, and Q_t is the consumption-based real exchange rate (units of Home consumption per unit of Foreign).

2.3. Consumption Good Production

Consumption Goods Producer. Differentiated consumption goods are produced by monopolistically competitive firms using gas and labor as inputs. Home and Foreign gas are perfect substitutes in production of consumption goods. Home firm ω produces output $y_t(\omega)$ of good ω with production function:

$$y_t(\omega) = zZ_t \left[g_{H,t}(\omega) + \frac{g^*_{H,t}(\omega)}{\tau_{G,t}} \right]^{\alpha} [l_t(\omega)]^{1-\alpha}, \tag{3}$$

where z is exogenous, heterogeneous productivity determined upon firm entry, Z_t is an exogenous sector-wide productivity shock, $g_{H,t}(\omega) + g^*_{H,t}(\omega)/\tau_{G,t}$ is the firm's total use of gas (domestic and imported, with gas import subject to an iceberg trade cost $\tau_{G,t} \geq 1$), $l_t(\omega)$ is the firm's use of labor in production, and $0 \leq \alpha < 1$. We set Foreign not to import gas from Home. Foreign firms use only domestic gas, $g_{F,t}(\omega) = 0$.

Using w_t to denote the real wage paid to consumption-sector workers (in units of consumption), the firm's marginal cost is $\rho^{\alpha}_{G,t} w^{1-\alpha}_t/(zZ_t)$. Given Dixit-Siglitz preferences, the real price charged by the firm for sales in the Home market is

$$\rho_{H,t}(z) = \left(\frac{\theta}{\theta - 1} \right) \frac{\rho^{\alpha}_{G,t} w^{1-\alpha}_t}{zZ_t}, \tag{4}$$

where we dropped the identifier ω and replaced it with the heterogeneous productivity z. Exporting is costly, and producers are subject to an iceberg trade cost, $\tau_t \geq 1$, and a per-period fixed export cost, f_X. The fixed export cost requires use of consumption-sector labor with effectiveness determined by the aggregate shock Z_t. We assume that f_X is in units of effective labor. Hence, the fixed export cost in units of consumption is $w_t f_X/Z_t$. The fixed export cost implies that only firms with sufficiently high productivity z will export. The iceberg cost implies that, if a firm exports, the price it charges in the Foreign market (in units of the Foreign consumption basket) is

$$\rho_{X,t}(z) = \left(\frac{\theta}{\theta - 1} \right) \frac{\tau_t \rho^{\alpha}_{G,t} w^{1-\alpha}_t}{Q_t z Z_t}. \tag{5}$$

Number of Firms, Exporters, and Their Averages. Following Melitz (2003), define the market-share weighted productivity average $\tilde{z}_D$ for all producing firms in each country as:

$$\tilde{z}_D \equiv \left[\int_{z_{\min}}^{\infty} z^{\theta-1} \mathrm{d}\Phi(z) \right]^{\frac{1}{\theta-1}}, \tag{6}$$

and the market-share weighted productivity averages for Home and Foreign exporters as, respectively:

$$\tilde{z}_{X,t} \equiv \left[\frac{1}{1-\Phi(z_{X,t})} \int_{z_{X,t}}^{\infty} z^{\theta-1} \mathrm{d}\Phi(z) \right]^{\frac{1}{\theta-1}} \quad \text{and} \quad \tilde{z}^*_{X,t} \equiv \left[\frac{1}{1-\Phi(z^*_{X,t})} \int_{z^*_{X,t}}^{\infty} z^{\theta-1} \mathrm{d}\Phi(z) \right]^{\frac{1}{\theta-1}}. \tag{7}$$

As shown by Melitz (2003), the model is isomorphic to one in which $N_{D,t}$ ($N^*_{D,t}$) firms with productivity $\tilde{z}_D$ produce in the Home (Foreign) country and $N_{X,t}$ ($N^*_{X,t}$) firms with productivity $\tilde{z}_{X,t}$ ($\tilde{z}^*_{X,t}$) export to Foreign (Home). The expression of the Home price index P_t then implies $N_{D,t}(\tilde{\rho}_{D,t})^{1-\theta} + N^*_{X,t}(\tilde{\rho}^*_{X,t})^{1-\theta} = 1$, where $\tilde{\rho}_{D,t} \equiv \rho_{D,t}(\tilde{z}_D)$ and $\tilde{\rho}^*_{X,t} \equiv \rho^*_{X,t}(\tilde{z}^*_{X,t})$ are the average relative prices of Home producers and Foreign exporters in the Home market. Moreover, given average profits from domestic and export sales$\tilde{d}_{D,t} \equiv d_{D,t}(\tilde{z}_D)$ and $\tilde{d}_{X,t} \equiv d_{X,t}(\tilde{z}_{X,t})$, average total profits of Home firms are $\tilde{d}_t \equiv \tilde{d}_{D,t} + [1-\Phi(z_{X,t})]\,\tilde{d}_{X,t}$, where $1-\Phi(z_{X,t})$ is the proportion of Home firms that export, i.e., $1-\Phi(z_{X,t}) = N_{X,t}/N_{D,t}$.

Firm Entry and Exit. There is an unbounded mass of potential entrants in each country. Entry requires use of consumption-sector labor with effectiveness determined by the aggregate shock Z_t. Prior to entry, all firms are identical and face a sunk entry cost f_E in units of effective labor. Hence, the sunk entry cost in units of consumption is $w_t f_E / Z_t$. Upon entry, firms draw the firm-specific productivity level z from a cumulative distribution function $\Phi(z) = 1 - (z/z_{\min})^{-k}$ with support $[z_{\min}, \infty)$. This productivity level remains fixed thereafter. We assume that $f^*_E \geq f_E$, allowing for the possibility that the gas-rich country features less consumption-sector firms as a consequence of inefficiencies of various type that can characterize the firm creation process.

We also assume a one-period time-to-build requirement: It takes one period between the time of entry and the time when firms start producing and generating profits. All firms in the economy, incumbent and new entrants, are subject to an exogenous shock that causes them to exit with probability $\delta \in (0,1)$ at the end of each period. Therefore, the mass $N_{D,t}$ of producing Home firms in period t is determined by $N_{D,t} = (1-\delta)(N_{D,t-1} + N_{E,t-1})$, where $N_{E,t-1}$is the number of firms that entered in period $t-1$.

Given these definition, firm entry decisions are determined as follows. Prospective entrants are forward looking and compute the rational expectation of the stream of average total profits that they will generate post entry. This determines the average value of an entrant, $\tilde{v}_t$, as:

$$\tilde{v}_t \equiv \mathbb{E}_t\left[\sum_{s=t+1}^{\infty}[\beta(1-\delta)]^{s-t}\left(\frac{C_s}{C_t}\right)^{-1}\tilde{d}_s\right], \tag{8}$$

when share holdings are $x_{t+1}=x_t=1$. Entry occurs until this value is equated to the sunk entry cost, implying the free-entry condition $\tilde{v}_t = w_t f_E/Z_t$. We assume that macroeconomic shocks are never large enough to cause zero entry in any period (or $\tilde{v}_t < w_t f_E/Z_t$) so that the entry condition always holds with equality (in other words, there is always a positive number of entrants). Since both new entrants and incumbent firms face the same probability δ of exit at the end of each period regardless of their firm-specific productivity, $\tilde{v}_t$ is also the average value of incumbent firms after production has occurred.

2.4. Household Budget Constraint, Asset Holding, and Labor Supply Decisions

International financial markets are incomplete as only non-contingent, riskless real bonds are traded internationally. The representative Home household's holdings of Home bonds entering period t are denoted with $B_{H,t}$. The household receives the risk-free real interest rate r_t on these bonds during period t. The household's holdings of Foreign real bonds entering period t are denoted with $B^*_{H,t}$, and they pay the risk-free real interest rate r^*_t (Foreign bonds and interest rate are in units of Foreign consumption). We assume that firms are fully owned domestically. Specifically, the representative household enters the period with share holdings x_t in a mutual fund of $N_{D,t}$ Home producing firms. During period t, the household receives dividends from its share holdings, $\tilde{d}_t$ per share, and the value of selling its share portfolio at the price $\tilde{v}_t$ per share. Besides its financial assets and the income they generate, the representative household's resources in period t also include the income from labor supplied in the gas production sector ($w_{G,t}L_{G,t}$) and in the consumption sector ($w_t L_t$). Finally, the household also receives a lump-sum rebate of fees that it pays to financial intermediaries in order to enter period $t+1$ (these fees serve the purpose of pinning down holdings of Home and Foreign bonds at their steady-state values in the deterministic steady-state of the model). During period t, the household uses its resources to buy consumption, to buy bonds with which it will enter period $t+1$ ($B_{H,t+1}$ and $B^*_{H,t+1}$), to pay fees

$0.5\eta(B_{H,t+1} - B_H)^2$ and $0.5\eta Q_t(B^*_{H,t+1} - B^*_H)^2$, with $\eta > 0$, and to buy share holding x_{t+1} in a mutual fund of $N_t \equiv N_{D,t} + N_{E,t}$ firms. Only $1 - \delta$ of these N_t firms will be around to produce and generate profits in period $t + 1$. The household does not know which firms will be hit by the exit-inducing shock and, therefore, it finances continued operations by all currently producing firms and entry by all producers who choose to enter the market, with the risk of firm exit at the end of period t reflected in the share price that will be determined by the Euler equation for optimal share holdings. The budget constraint of the representative Home household is thus:

$$\begin{aligned} &C_t + B_{H,t+1} + Q_t B^*_{H,t+1} + \tilde{v}_t N_t x_{t+1} + \frac{\eta}{2}(B_{H,t+1} - B_H)^2 + \frac{\eta}{2} Q_t (B^*_{H,t+1} - B^*_H)^2 + \frac{\eta}{2}\tilde{v}_t N_t (x_{t+1} - 1)^2 \\ &\quad = (1 + r_t) B_{H,t} + Q_t (1 + r^*_t) B^*_{H,t} + w_{G,t} L_{G,t} + w_t L_t + (\tilde{d}_t + \tilde{v}_t) N_{D,t} x_t + T_t. \end{aligned} \tag{9}$$

where $T_t = 0.5\eta(B_{H,t+1} - B_H)^2 + 0.5\eta Q_t(B^*_{H,t+1} - B^*_H)^2 + 0.5\eta\tilde{v}_t N_t(x_{t+1} - 1)^2$.

The Euler equations for optimal holdings of Home and Foreign bonds are, respectively:

$$1 + \eta(B_{H,t+1} - B_H) = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-1} \right] (1 + r_{t+1}), \tag{10}$$

$$1 + \eta(B^*_{H,t+1} - B^*_H) = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-1} \frac{Q_{t+1}}{Q_t} \right] (1 + r^*_{t+1}). \tag{11}$$

The Euler equation for optimal share holdings implies:

$$\tilde{v}_t [1 + \eta(x_{t+1} - 1)] = \beta(1 - \delta) \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-1} (\tilde{v}_{t+1} + \tilde{d}_{t+1}) \right]. \tag{12}$$

Forward iteration of this equation and the relevant transversality condition imply the expression for $\tilde{v}_t$ in the free-entry condition above, thus establishing the general equilibrium link between firm entry decisions and household decisions regarding the financing of entry.

Finally, the first-order conditions for optimal supply of labor to the gas and consumption sectors are $\kappa_G L_{G,t} = w_{G,t}/C_t$ and $\kappa L_t = w_t/C_t$.

2.5. Market Clearing and Aggregate Accounting

The price of usable gas, $\rho_{G,t}$, is determined by gas market clearing conditions:

$$G_N L_{G,t} = G_{H,t}, \tag{13}$$

$$G_N^* L_{G,t}^* = G_{H,t}^* + G_{F,t}^*, \tag{14}$$

where $G_{H,t} = N_{D,t} g_{H,t}(\tilde{z}_D) + N_{X,t} g_{H,t}(\tilde{z}_{X,t})$, $G_{H,t}^* = N_{D,t} g_{H,t}^*(\tilde{z}_D) + N_{X,t} g_{H,t}^*(\tilde{z}_{X,t})$, and $G_{F,t}^* = N_{D,t}^* g_{F,t}^*(\tilde{z}_D) + N_{X,t}^* g_{F,t}^*(\tilde{z}_{X,t}^*)$.

Market clearing for individual goods requires $y_t(z) = c_{H,t}(z) + c_{F,t}(z)$ for the product of a Home firm with specific productivity z and $y_t^*(z) = c_{H,t}^*(z) + c_{F,t}^*(z)$ for the product of a Foreign firm with the same productivity.

Labor market clearing in gas production in Home and Foreign requires $L_{G,t} = w_{G,t}/(\kappa_G C_t)$ and $L_{G,t}^* = w_{G,t}^*/(\kappa_G C_t^*)$, respectively. Since $w_{G,t} = \rho_{G,t} G_N$ and $w_{G,t}^* = \rho_{G,t}^* G_N^*$, it follows that $L_{G,t} = \rho_{G,t} G_N/(\kappa_G C_t)$ and $L_{G,t}^* = \rho_{G,t}^* G_N^*/(\kappa_G C_t^*) = \rho_{G,t} G_N^*/(\kappa_G Q_t C_t^*)$, where the last equality uses the fact that $\rho_{G,t} = \tau_{G,t} Q_t \rho_{G,t}^*$. Ceteris paribus, the amount of labor employed in gas production in each country is larger the larger the country's endowment of natural gas and the higher the price of gas; instead, labor in the gas sector is smaller the higher the country's consumption and, intuitively, the higher the weight of the disutility of labor. Since a real depreciation of the Home currency (an increase in Q_t) causes a higher real price of usable gas in Home, it causes a decrease in gas-sector employment in Foreign, as there is an incentive to shift production to Home.

Labor market clearing conditions in the consumption sectors of the two countries require

$$L_t = N_{D,t} l_t(\tilde{z}_D) + N_{X,t} l_t(\tilde{z}_{X,t}) + N_{E,t} \frac{f_E}{Z_t} + N_{X,t} \frac{f_X}{Z_t}, \tag{15}$$

$$L_t^* = N_{D,t}^* l_t^*(\tilde{z}_D) + N_{X,t}^* l_t^*(\tilde{z}_{X,t}^*) + N_{E,t}^* \frac{f_E^*}{Z_t^*} + N_{X,t}^* \frac{f_X^*}{Z_t^*}, \tag{16}$$

which equal to $w_t/(\kappa C_t)$ and $w_t^*/(\kappa C_t^*)$, respectively.

Market clearing for bonds issued by Home requires $B_{H,t+1} + B_{F,t+1} = B_{H,t} + B_{F,t} = 0$ in every period, and for bonds issued by Foreign: $B_{H,t+1}^* + B_{F,t+1}^* = B_{H,t}^* + B_{F,t}^* = 0$ in every period. Stock market clearing in each country requires $x_{t+1} = x_t = 1$ and $x_{t+1}^* = x_t^* = 1$ in every period. Since costs of adjusting bond holdings away from zero are rebated back to households in equilibrium, imposing equilibrium conditions on the household budget constraint yields:

$$C_t + \tilde{v}_t N_{E,t} + B_{H,t+1} + Q_t B^*_{H,t+1} = (1+r_t)B_{H,t} + Q_t(1+r^*_t)B^*_{H,t} + w_{G,t}L_{G,t} + w_t L_t + N_{D,t}\tilde{d}_t, \tag{17}$$

in Home and:

$$C^*_t + \tilde{v}^*_t N^*_{E,t} + \frac{B_{F,t+1}}{Q_t} + B^*_{F,t+1} = \frac{(1+r_t)B_{F,t}}{Q_t} + (1+r^*_t)B^*_{F,t} + w^*_{G,t}L^*_{G,t} + w^*_t L^*_t + N^*_{D,t}\tilde{d}^*_t. \tag{18}$$

These two equations together, and bond market equilibrium, imply that Home net foreign assets obey the law of motion:

$$\begin{aligned} &B_{H,t+1} + Q_t B^*_{H,t+1} \\ &= (1+r_t)B_{H,t} + Q_t(1+r^*_t)B^*_{H,t} + \frac{1}{2}(w_{G,t}L_{G,t} - Q_t w^*_{G,t}L^*_{G,t}) + \frac{1}{2}(w_t L_t - Q_t w^*_t L^*_t) \\ &\quad + \frac{1}{2}(N_{D,t}\tilde{d}_t - Q_t N^*_{D,t}\tilde{d}^*_t) - \frac{1}{2}(C_t - Q_t C^*_t) - \frac{1}{2}(\tilde{v}_t N_{E,t} - Q_t \tilde{v}^*_t N^*_{E,t}), \end{aligned} \tag{19}$$

or that Home's current account is determined by:

$$CA_t \equiv B_{H,t+1} + Q_t B^*_{H,t+1} - (B_{H,t} + Q_t B^*_{H,t}) = r_t B_{H,t} + Q_t r^*_t B^*_{H,t} + TB_t, \tag{20}$$

where TB_t is the trade balance:

$$\begin{aligned} TB_t \equiv &\frac{1}{2}(w_{G,t}L_{G,t} - Q_t w^*_{G,t}L^*_{G,t}) + \frac{1}{2}(w_t L_t - Q_t w^*_t L^*_t) \\ &+ \frac{1}{2}(N_{D,t}\tilde{d}_t - Q_t N^*_{D,t}\tilde{d}^*_t) - \frac{1}{2}(C_t - Q_t C^*_t) - \frac{1}{2}(\tilde{v}_t N_{E,t} - Q_t \tilde{v}^*_t N^*_{E,t}). \end{aligned} \tag{21}$$

Finally, the trade balance can be rewritten as:

$$TB_t = \frac{1}{2}(Y_t - Q_t Y^*_t) - \frac{1}{2}(C_t - Q_t C^*_t) - \frac{1}{2}(\tilde{v}_t N_{E,t} - Q_t \tilde{v}^*_t N^*_{E,t}). \tag{22}$$

Once we recognize that $w_{G,t}L_{G,t} + w_t L_t + N_{D,t}\tilde{d}_t$ is total Home income from labor and dividends (or Home GDP, Y_t) and $w^*_{G,t}L^*_{G,t} + w^*_t L^*_t + N^*_{D,t}\tilde{d}^*_t$ is total Foreign income from labor and dividends (or Foreign GDP, Y^*_t). Home and Foreign current accounts and trade balances are such that $CA_t + Q_t CA^*_t = TB_t + Q_t TB^*_t = 0$.

3. Analytical Insights

Like the GM model we build on, our model cannot be fully solved analytically. It, however, is possible to obtain intermediate analytical results on key variables of interest. We present some of these results below, focusing on two prices: the price of gas and the real exchange rate.

3.1. Gas Price

Using gas market clearing conditions, production functions, optimal prices, and marginal cost expressions, it is possible to express the price of gas, $\rho_{G,t}$, as:

$$\rho_{G,t} = \frac{(1-\alpha)^{(1-\alpha)(\theta-1)}\kappa_G C_t}{G_N^2} \left\{ \frac{1+\xi_t+\tau_{G,t}^{-1}\left[1-(1-\alpha)^{(1-\alpha)(\theta-1)}\right]\xi_t}{\left[1+\tau_{G,t}^{-1}\left[1-(1-\alpha)^{(1-\alpha)(\theta-1)}\right]\xi_t\right](1+\xi_t)} \right\}, \tag{23}$$

where $\xi_t \equiv (G_N^*/G_N)^2[\kappa_G/(\kappa_G^*\tau_{G,t})][C_t/(Q_tC_t^*)]$.[6] For given level of gas trade cost, $\tau_{G,t}$, fluctuations in the price of gas paid by Home consumption-sector firms are driven by fluctuations in Home consumption and in the extent to which the relation between Home and Foreign consumptions deviates from the complete markets outcome (under complete markets, the ratio $C_t/(Q_tC_t^*)$ would be constant, and changes in $\tau_{G,t}$ would be the only reason for ξ_t to move).

The Home gas price $\rho_{G,t}$ is measured in units of consumption, i.e., in welfare-consistent units. It can fluctuate because of pure variety effects on the price index P_t that are not accounted for in available data. This implies that, while understanding the dynamics of $\rho_{G,t}$ is important to understand the welfare-effects of sanctions through their impact on the price of gas, if we want to have a model-implied measure of real gas price that can be compared to data, we must deflate the nominal price of gas $p_{G,t}$ using a measure of the Home price index that has been purged of pure variety effects. As in Feenstra (1994) and GM, this measure of the Home price level is given by $\tilde{P}_t \equiv N_t^{\frac{1}{\theta-1}}P_t$, where $N_t \equiv N_{D,t}+N_{X,t}^*$ is the total number of products available to Home consumers. Deflating $p_{G,t}$ with $\tilde{P}_t$ yields the data-consistent gas price $\tilde{\rho}_{G,t} \equiv p_{G,t}/\tilde{P}_t$. Notice that this gas price is such that $\tilde{\rho}_{G,t} = N_t^{\frac{1}{1-\theta}}\rho_{G,t}$.

Figure 2 shows the effect of a change in Home consumption, Foreign consumption (in Home units) and the total number products available to Home consumers on the gas price, using equation (23). In this figure, we set the initial state as $C = QC^* = 1$ and $N = 1$, with $\alpha = 0.1$,

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[6] See Appendix 1 for details.

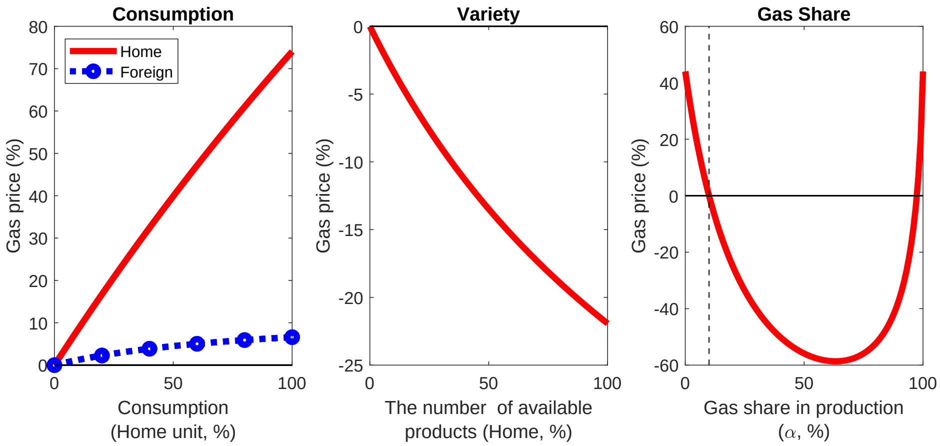

Figure 2: Decomposition of the Gas Price, $\tilde{\rho}_G$

Notes: The figures illustrate the response of the gas price (Home, $\tilde{\rho}_G$), as expressed in equation (2), to changes in Home or Foreign consumption (C or QC^*), the total number of available products in Home (variety, N), or the share of gas in consumption good production (α), ceteris paribus. On the y-axis of all figures, the gas price is represented as a percentage deviation from the initial state. The x-axis in the first and second figures indicates the percentage increase in consumption and variety from their initial values. Here, 0% corresponds to the initial values, with $C = QC^* = 1$ and $N = 1$. In the third figure, the x-axis represents the share, with the vertical dashed line denoting the initial share value ($\alpha = 0.1$). For all figures, we set the initial state as $C = QC^* = 1$ and $N = 1$, with $\alpha = 0.1$, $G_N = G_N^* = 1$, $\tau_G = 1$, $\theta = 3.8$, and $\kappa_G = \kappa_G^* = 0.75$.

$G_N = G_N^* = 1$, $\tau_G = 1$, $\theta = 3.8$, and $\kappa_G = \kappa_G^* = 0.75$.

The figure shows that, ceteris paribus, an increase in Home consumption or Foreign consumption generates an increase in the Home gas price. The total number of products available to Home consumers is negatively related with the data-consistent gas price, all else equal.

To build intuition for the implications of equation (23), suppose that markets are indeed complete, so that, up to a constant, $C_t = Q_t C_t^*$. Suppose also that $\tau_{G,t} = 1$, $G_N = G_N^*$, and $\kappa_G = \kappa_G^*$. then, equation (23) becomes:

$$\rho_{G,t} = \frac{(1-\alpha)^{(1-\alpha)(\theta-1)}\kappa_G C_t}{G_N^2} \left\{ \frac{3-(1-\alpha)^{(1-\alpha)(\theta-1)}}{2\left[2-(1-\alpha)^{(1-\alpha)(\theta-1)}\right]} \right\}. \tag{24}$$

The expression in curly brackets is smaller than 1. It tends to 1 if the share of gas in consumption production, α, tends to 0 or 1. Interestingly, both the cases in which there is no international trade in gas ($\alpha \to 0$) or there is the highest need for Home to import gas ($\alpha \to 1$) imply that the price of gas tends to $\kappa_G C_t / G_N^2$. We show in Appendix 2 that there is a non-monotonicity (U-shape) in gas price behavior as the share of gas in consumption production varies as shown in the last panel of Figure 2. For given Home consumption, if α is sufficiently high, further increases in α cause a higher gas price. If instead α is sufficiently low, increases in α have the opposite effect on $\rho_{G,t}$.

When α is high, the effect of rising α on gas demand prevails, resulting in a higher price. If α is low, demand does not increase enough to offset the effect of substitution toward labor, and the price of gas falls.

The effects of κ_G, C_t, and G_N on $\rho_{G,t}$ in equation (24) are also consistent with intuition: If the weight of the disutility of supplying labor to gas production increases, the price of gas increases as agents reduce gas labor supply. If consumption increases, the price of gas increases, because there is more demand for consumption goods. If efficiency in gas production (or the endowment of natural gas) increases, the price of gas decreases as its supply rises.

In the general case in which $\rho_{G,t}$ is determined by equation (23), we can build intuition by considering the version of equation (23) that is obtained by log-linearizing it around the steady-state. We show in Appendix 3 that it is:

$$\hat{\rho}_{G,t} = \mathsf{C}_t + (\Gamma_1 - \Gamma_2)(\mathsf{C}_t - \mathsf{Q}_t - \mathsf{C}_t^*) \tag{25}$$

where Sans Serif fonts denote percentage deviations from the steady-state, and the coefficients Γ_1 and Γ_2 are given by, respectively.

$$\Gamma_1 \equiv \frac{(1 + \bar{\tau}_G^{-1} A)(1 + \bar{\tau}_G^{-1} A\bar{\xi})(1 + \bar{\xi})\bar{\xi}}{(1 + \xi + \tau_G^{-1} A\bar{\xi})(1 + \bar{\xi})(1 + \tau_G^{-1} A\bar{\xi})}, \tag{26}$$

$$\Gamma_2 \equiv \frac{\left[\bar{\tau}_G^{-1} A(1 + \bar{\xi}) + (1 + \bar{\tau}_G^{-1} A\bar{\xi})\right]\bar{\xi}}{(1 + \bar{\tau}_G^{-1} A\bar{\xi})(1 + \bar{\xi})}. \tag{27}$$

In these expressions, $A \equiv 1 - (1 - \alpha)^{(1-\alpha)(\theta-1)}$. We denote steady-state levels of variables by dropping the time subscript and using an overbar, and we assumed that $\tau_{G,t}$ does not change. If $-1 < \Gamma_1 - \Gamma_2 < 0$, the effects of Home consumption, the real exchange rate, and foreign consumption on the gas price paid by Home firms are intuitive: Higher C_t causes higher demand of gas for production by Home firms, hence a higher price of gas. The effect of Q_t in equation (25) is tied to the role of the real exchange rate in international risk sharing and is best understood in conjunction with that of C_t^*.[7] Higher $\mathsf{Q}_t + \mathsf{C}_t^*$ implies an increase in gas demand by Foreign firms relative to Home (given a share on non-traded consumption goods larger than $1/2$). Supply of Foreign gas to Home firms decrease because of higher domestic use. The latter implies that Home consumption good producers substitute towards Home gas, generating an increase in the

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[7] With complete markets, we would have $\mathsf{C}_t - \mathsf{C}_t^* = \mathsf{Q}_t$, which would imply that the ceteris paribus scenario of a change in Q_t in equation (25) without at least one between C_t and C_t^* also moving would be impossible.

Home gas price. Any policy action (including sanctions) that causes Home consumption, the real exchange rate, and Foreign consumption to change will have an effect on the price of gas facing Home consumption-sector firms that can be understood based on these results.[8]

A final observation on the gas price concerns its measurement and variety channel. The log-linear equation for $\tilde{\rho}_{G,t}$ follows immediately from this relation and equation (25) as:

$$\hat{\tilde{\rho}}_{G,t} = \mathsf{C}_t + (\Gamma_1 - \Gamma_2)(\mathsf{C}_t - \mathsf{Q}_t - \mathsf{C}^*_t) - \frac{1}{\theta - 1}\mathsf{N}_\mathsf{t}. \tag{28}$$

In addition to the effects through $\rho_{G,t}$, policy actions affect the data-consistent gas price by changing the number of products available to Home consumers. Actions that reduce product variety in the Home country cause $\tilde{\rho}_{G,t}$ to rise. The reason follows from the effect of product variety on welfare via the price index P_t. Holding product prices constant, this price index decreases if product variety expands, implying that consumers can buy more consumption (and hence obtain more welfare) by spending a given nominal amount. The data-consistent price index $\tilde{P}_t$ removes this pure variety effect. Since $\tilde{\rho}_{G,t}$ is obtained by deflating $p_{G,t}$ with $\tilde{P}_t$, it follows that higher N_t causes $\tilde{\rho}_{G,t}$ to decrease, and lower N_t causes it to increase, consistent with the non-linear scenario of Figure 2.

3.2. Real Exchange Rate

Similar to the gas price $\rho_{G,t}$, the real exchange rate Q_t is in welfare-consistent units that are not comparable to data because of unmeasured variety effects. As in GM, the data-consistent real exchange rate $\tilde{Q}_t$ is related to Q_t by the equation:

$$\tilde{Q}_t = \left(\frac{N^*_t}{N_t}\right)^{\frac{1}{\theta-1}} Q_t, \tag{29}$$

where $N^*_t \equiv N^*_{D,t} + N_{X,t}$ is the total number of products available to Foreign consumers.

Using price index equations and optimal price setting by Home and Foreign consumption-sector firms yields:

$$\tilde{Q}_t^{1-\theta} = \frac{\frac{N^*_{D,t}}{N^*_t}\left[TOL_t^{1-\alpha}\left(\frac{Z_t}{\tau_{G,t}Z^*_t}\right)^{\alpha}\frac{\tilde{z}_D}{\tilde{z}^*_D}\right]^{1-\theta} + \frac{N_{X,t}}{N^*_t}\left[\frac{\tau\tilde{z}_D}{\tilde{z}_{X,t}}\right]^{1-\theta}}{\frac{N_{D,t}}{N_t} + \frac{N^*_{X,t}}{N_t}\left[TOL_t^{1-\alpha}\left(\frac{Z_t}{\tau_{G,t}Z^*_t}\right)^{\alpha}\frac{\tau^*\tilde{z}_D}{\tilde{z}^*_{X,t}}\right]^{1-\theta}}, \tag{30}$$

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[8] If Home imposes a full embargo on Foreign gas, there no longer is any arbitrage force that ensures the condition $\rho_{G,t} = \tau_{G,t}Q_t\rho^*_{G,t}$, which is used in obtaining equation (23). In case of a full embargo, the price of gas in Home is determined solely by $\rho_{G,t} = w_{G,t}/G_N$.

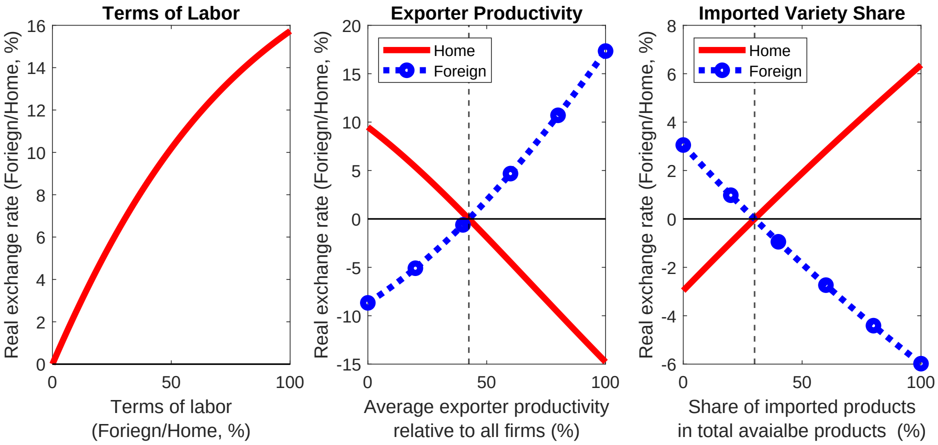

Figure 3: Decomposition of the Real Exchange Rate, $\tilde{Q}$

Notes: The figures illustrate the response of the (data-consistent) real exchange rate (Foreign/Home, $\tilde{Q}$), as expressed in Equation (2), to changes in terms of labor (Foreign/Home, $TOL = Q(w^*/Z^*)/(w/Z)$), the average Home or Foreign exporter productivity (relative to all firm average productivity, $\tilde{z}_X/\tilde{z}_D$ or $\tilde{z}^*_X/\tilde{z}_D$), or the share of imported varieties in the total available varieties in the Home or Foreign market (N^*_X/N or N_X/N^*), ceteris paribus. On the y-axis of all figures, the real exchange rate is represented as a percentage deviation from the initial state. In the first figure, the x-axis indicates the percentage increase in terms of labor. Here, 0% corresponds to identical labor costs in efficiency, with $TOL = 1$. The x-axis in the second figure represents how much exporters are more productive than all firms, with the vertical dashed lines denoting the initial value ($100 \times (0.3^{-1/3.4} - 1) \approx 42\%$). Here, 0% corresponds to no productivity difference between exporters and non-exporters. In the third figure, the x-axis represents the share, with the vertical dashed line denoting the initial share value (30%). For all figures, we set the initial state as $TOL = 1$, $\tilde{z}_X/\tilde{z}_D = \tilde{z}^*_X/\tilde{z}D = 0.3^{-1/k}$, and $N^*_X/N = N_X/N^* = 0.3$, with $\alpha = 0.1$, $Z = Z^* = 1.5$, $\tau = \tau^* = 1.3$, $\tau_G = 1$, $k = 3.4$, $\theta = 3.8$, and $\kappa_G = \kappa^*_G = 0.75$.

where $TOL_t \equiv Q_t(w^*_t/Z^*_t)/(w_t/Z_t)$. As in GM, this variable measures the relative cost of effective consumption-sector labor in the two countries. Interestingly, gas prices do not enter the real exchange rate expression directly. Factor prices enter the equation through cross-country ratios of variables. The ratio of Home to Foreign gas prices is such that $\rho_{G,t}/(Q_t\rho^*_{G,t}) = \tau_{G,t}$. Hence, only the iceberg cost paid by Home (the importer) appears in equation (30). In addition to the terms of labor and the iceberg cost of gas trade, the real exchange rate can change because of changes in the total number of products available to Home and Foreign consumers, in the numbers of producers serving the domestic or export market, and in average export productivities.

Consider a permanent decline in Home gas imports, a scenario that we study below as resulting from gas sanctions. In response to lower Home demand of Foreign gas, resources in the Foreign economy will be shifted toward production of consumption goods in order to sustain exports by increasing consumption-sector output. This translates into an increase in labor demand by Foreign consumption good producers, which puts upward pressure on consumption sector wages. In turn, this leads to a depreciation (an increase) in TOL_t. We will show below that, to a first order, terms of labor depreciation is associated with depreciation of $\tilde{Q}_t$.

Figure 3 plots the relationship between the real exchange rate and average Foreign exporter productivity, average Home exporter productivity, terms of labor, and the share of imported goods in total available products in Foreign and in Home, respectively. All of the lines in both figures show the relationship while all other variables stay constant. W e set the initial state as $TOL = 1$, $\tilde{z}_X/\tilde{z}_D = \tilde{z}^*_X/\tilde{z}D = 0.3^{-1/k}$, and $N^*_X/N = N_X/N^* = 0.3$, with $\alpha = 0.1$, $Z = Z^* = 1.5$, $\tau = \tau^* = 1.3$, $\tau_G = 1$, $k = 3.4$, $\theta = 3.8$, and $\kappa_G = \kappa^*_G = 0.75$.

We observe that while the average Foreign exporter productivity, terms of labor, and the share of imported goods in total available products in Home are positively related with the real exchange rate (i.e., an increase in these variables, ceteris paribus, depreciates the Home real exchange rate), average Home exporter productivity and the share of imported goods in total available products in Foreign are negatively related to $\tilde{Q}_t$.

As for the gas price, we can build intuition on the determinants of the real exchange rate by considering the log-linear version of equation (30). Letting NUM_t denote the numerator of the expression in equation (30) and DEN_t the denominator, it is:

$$\tilde{\mathsf{Q}}_t = \frac{\overline{NUM} \cdot \mathrm{d}DEN_t - \overline{DEN} \cdot \mathrm{d}NUM_t}{(\theta - 1) \cdot \overline{NUM} \cdot \overline{DEN}} \tag{31}$$

where d is the differentiation operator. Hence, up to the constant $[(\theta - 1) \cdot \overline{NUM} \cdot \overline{DEN}]^{-1}$, the behavior of $\tilde{\mathsf{Q}}_t$ is determined by $\overline{NUM} \cdot \mathrm{d}DEN_t - \overline{DEN} \cdot \mathrm{d}NUM_t$. We show in Appendix 4 that:

$$\begin{aligned}
&\overline{NUM} \cdot \mathrm{d}DEN_t - \overline{DEN} \cdot \mathrm{d}NUM_t \\
=&(\theta - 1)(\zeta_1 - \zeta_2)[(1 - \alpha)\mathsf{TOL}_t + \alpha(\mathsf{Z_t} - \mathsf{Z^*_t} - \hat{\tau}_{G,t})] \\
&+ (\theta - 1)\left[(\zeta_2 + \zeta_4)(\tilde{\mathsf{z}}^*_{\mathsf{X,t}} - \hat{\tau}^*_t) - (\zeta_2 + \zeta_3)(\tilde{\mathsf{z}}_{\mathsf{X,t}} - \hat{\tau}_t)\right] \\
&+ \zeta_1[\mathsf{N_{D,t}} - \mathsf{N_t} - (\mathsf{N^*_{D,t}} - \mathsf{N^*_t})] + \zeta_2[\mathsf{N^*_{X,t}} - \mathsf{N_t} - (\mathsf{N_{X,t}} - \mathsf{N^*_t})] \\
&- \zeta_3[\mathsf{N_{X,t}} - \mathsf{N^*_t} - (\mathsf{N_{D,t}} - \mathsf{N_t})] + \zeta_4[\mathsf{N^*_{X,t}} - \mathsf{N_t} - (\mathsf{N^*_{D,t}} - \mathsf{N^*_t})],
\end{aligned} \tag{32}$$

where $\zeta_1 \equiv \chi_1(\bar{N}_D/\bar{N})^2(\overline{TOL}^{1-\alpha}\bar{\tau}_G^{-\alpha})^{1-\theta} > 0$, $\zeta_2 \equiv \gamma\chi_1(\bar{N}^*_X/\bar{N})^2(\overline{TOL}^{1-\alpha}\bar{\tau}_G^{-\alpha}\chi_2)^{1-\theta} > 0$, $\zeta_3 \equiv \gamma\chi_1(\bar{N}_D\bar{N}^*_X/\bar{N}^2)(\chi_2\bar{\tau}^*\tilde{z}_D/\bar{\tilde{z}}^*_X)^{1-\theta} > 0$, $\zeta_4 \equiv \chi_1(\bar{N}_D\bar{N}^*_X/\bar{N}^2)(\bar{\tau}^*\tilde{z}_D/\bar{\tilde{z}}^*_X)^{1-\theta}(\overline{TOL}^{1-\alpha}\bar{\tau}_G^{-\alpha})^{2(1-\theta)} > 0$, and we assumed $\bar{Z} = \bar{Z}^* = 1$. In the expressions above, the parameters χ_1, χ_2, and γ are defined implicitly by:

$$\frac{\bar{N}^*_D}{\bar{N}^*} = \chi_1\frac{\bar{N}_D}{\bar{N}}, \quad \frac{\bar{N}_X}{\bar{N}^*} = \gamma\chi_1\frac{\bar{N}^*_X}{\bar{N}}, \quad \text{and} \left(\frac{\bar{\tau}\tilde{z}_D}{\bar{\tilde{z}}_X}\right)^{1-\theta} = \left(\chi_2\frac{\bar{\tau}^*\tilde{z}_D}{\bar{\tilde{z}}^*_X}\right)^{1-\theta}.$$

Equation (32) or, more precisely, the equation that follows from combining equations (31) and (32), is a more complicated version of the log-linear equation that is central to understanding real exchange rate dynamics in GM. Our version of the equation is more complicated because of the two-sector structure of production in each country and the fact that the steady-state of the model is not symmetric. Nevertheless, it is still possible to obtain an equation that, to a first order, disentangles the different determinants of the real exchange rate that are at work in our model.

Consider the effect of TOL_t. We show in Appendix 4 that $\zeta_1 - \zeta_2 > 0$ when $\bar{\tau}\bar{\tau}^* > 1$. It follows that, ceteris paribus, appreciation of the terms of labor (a downward movement in TOL_t) causes appreciation of the data-consistent real exchange rate (negative $\tilde{\mathsf{Q}}_t$) as in GM. Furthermore, higher average productivity of Foreign exporters (higher $\tilde{\mathsf{z}}^*_{X,t}$) causes $\tilde{\mathsf{Q}}_t$ to depreciate because it implies a lower domestic price index $\tilde{P}_t$, as more productive Foreign exporters charge lower prices.

The last four parts of equation (32) capture the effects of changes in the composition of consumption baskets in Home and Foreign. The first term measures the relative share of domestic goods in the total numbers of products available in Home and Foreign. The second term measures the relative share of imported goods in the total numbers of products available in Home and Foreign. If the share of imported goods in total Home variety rises relative to Foreign, the real exchange rate depreciates. An increase in Foreign exporter representation in the Home consumption basket relative to Home exporter representation in the Foreign consumption basket implies a lower price level $\tilde{P}_t$ in Home and a higher price level $\tilde{P}^*_t$ in Foreign because, on average, exporters charge lower prices. Hence, depreciation of $\tilde{\mathsf{Q}}_t$. The third and fourth terms measure the relative share of imported goods in total available variety versus domestic goods in total variety abroad in the two countries. If this share rises for Home, the real exchange rate depreciates; if it rises for Foreign, the real exchange rate appreciates. Consider, for example, the third term: If imported products representation in total variety available in Foreign rises relative to domestic products representation in total variety available in Home, $\tilde{P}^*_t$ falls and $\tilde{P}_t$ rises because, on average, exporters charge lower prices than non-exporters. Similarly, but with opposite effects on $\tilde{\mathsf{Q}}_t$ for the fourth term.

The results in the previous paragraphs help us understand the results of policy actions (including sanctions) that cause changes in the determinants of the real exchange rate. We use these results and those for the price of gas above to guide our interpretation of the numerical exercises in the next section.

4. Model Calibration and Dynamics

This section calibrates and solves the model numerically, and it illustrates its functioning by studying the responses to productivity shocks. We solve the model as a nonlinear, forward-looking, deterministic system using Dynare's nonlinear equation solver with line search.

4.1. Calibration

An essential feature of our model involves the asymmetry between the two countries. We depart from the symmetric two-country standard parameterization by emphasizing comparative advantages and (long-run) imbalanced capital flows. In this setting, the sanctioned country exhibits a comparative advantage in the gas sector and maintains positive net foreign assets (NFAs).

We assume that the countries have different consumption good sector productivities and natural gas endowments. Home is endowed with smaller natural gas resources than Foreign ($G_N < G_N^*$). Home is a gas importer, while Foreign is a gas exporter. The consumption goods-producing sector in Home is more efficient than in Foreign, characterized by higher productivity and lower cost of firm entry ($Z_0 > Z_0^*$ and $f_E/Z_0 < f_E^*/Z_0^*$). Specifically, we set $Z_0 = 1.5$ and $G_N^* = 1.5$, where $Z_0^* = 1$ and $G_N = 1$ are normalized. This calibration implies that Home GDP is about 53% larger than Foreign GDP in the initial steady-state, i.e., without sanctions.

We calibrate the initial value of Foreign Households' holdings of Home bonds to be 118% of Foreign GDP, and the Foreign NFA position to 38% of GDP. This is accomplished by setting $B_H = -5$ and $B_H^* = 3$.

The remaining parameters are identical across the two countries and calibrated using widely accepted values in the literature, minimizing the risk of our results being influenced by an unconventional calibration. Each period represents a quarter. The discount factor (β) and firm exit rates (δ) are set at 0.99 and 0.025, respectively. The disutility parameters from working in the consumption good and gas sectors (κ and κ_G) are both set to 0.75, normalizing the labor supply in the consumption good sector to 1. The scale parameter for the costs of adjusting bond and share holdings (η) is set at 0.0025, a value with negligible impact on model dynamics, except for pinning down the non-stochastic steady-state and ensuring mean reversion after transitory shocks.

Following Ghironi and Melitz (2005), entry costs (f_E) are normalized to 1, and the elasticity of substitution across varieties (θ) is set to 3.8. Firm-level productivity (z) follows a Pareto distribution with a lower bound ($z_{\min}$) of 1 and a shape parameter (κ) of 3.4. This calibration for the firm-level productivity distribution results in a Pareto shape parameter of 1.21 for the (domestic) sales distribution. Additionally, the top 5 percent exporters (top 1 percent firms) contribute to 60% of total exports when 20% of firms export. This aligns with empirical observations, as reported by Mayer and Ottaviano (2008) for various countries.[9] The fixed cost of exporting (f_X) is set at 0.0085, ensuring that in the initial steady-state, 18% and 24% of Home and Foreign firms export their goods, respectively. This calibration ensures that the lower bound of firm-level productivity ($z_{\min}$) is smaller than the exporter cutoff ($z_{X,t}$). Following Kim, Ozhan, and Schembri (2021), the share of gas in consumption good production (α) is set at 0.1. Iceberg costs for consumption good trade are set at 30% ($\tau = \tau^* = 1.3$), while there is no iceberg cost for gas trade ($\tau_G = 1$), suggesting relatively smoother international transactions for gas in the absence of sanctions.

4.2. Effects of a Change in Aggregate Home Productivity

We begin our analysis by examining the model's response to a permanent positive change in Home's consumption goods sector productivity. This allows us to draw comparisons with existing models in the literature on international trade and macroeconomic dynamics. Notably, our model includes an energy sector, introducing a distinctive feature often absent in prior works, and considers asymmetry between countries.

To facilitate comparison, simulations are conducted with varying shares of gas in the consumption goods production sector. Figure 4 and A2 present the responses to a 10% permanent increase in Home's consumption goods sector aggregate productivity. The blue, green, and red lines represent simulations with gas shares of 20%, 10%, and 1%, respectively (i.e., $\alpha \in \{0.2, 0.1, 0.01\}$). The different gas shares influence model dynamics quantitatively but do not alter the overall dynamics qualitatively.

Following a permanent productivity increase, Home becomes a more attractive business environment, leading to increased entry in the consumption goods producing sector. The long-run decrease in marginal costs of production in this sector is driven by high productivity. As a result, consumption goods producers demand more labor to expand production, leading to higher wages for labor in the consumption goods sector. The temporary increase in marginal costs is a consequence of higher factor demands immediately following the shock. In the short run, the cutoff productivity decreases for the least productive exporter, and more producers become productive

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[9] According to Mayer and Ottaviano (2008), the share of top 5 percent exporters in total exports is 81, 73, 69, 59, 73, and 81 percent in Germany, France, UK, Italy, Belgium, and Norway, respectively.

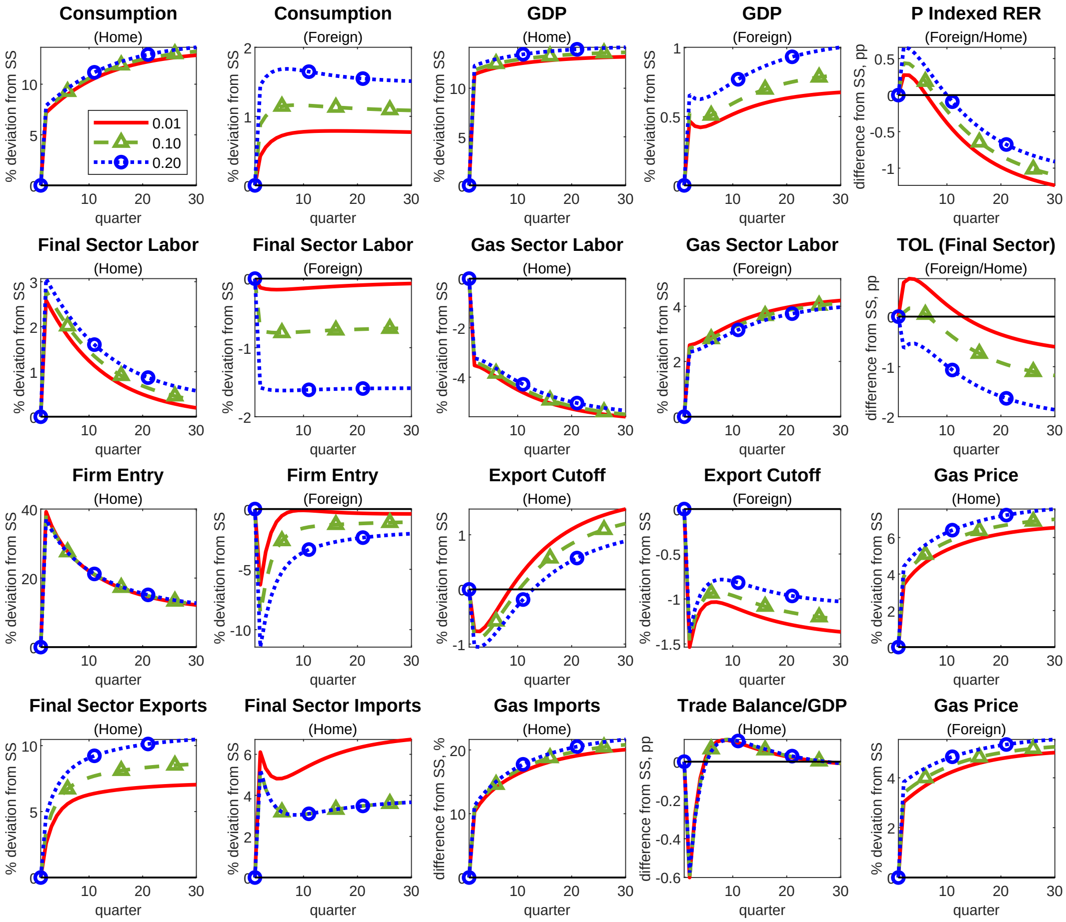

Figure 4: Responses to a 10% Permanent Increase in Home Productivity

Notes: The red solid lines, green dashed lines with triangles, and blue dashed lines with circles represent the model's transition dynamics when the aggregate productivity of the Home consumption goods sector increases by 10% at $t = 1$, with gas cost shares (α) set at 1%, 10%, and 20%, respectively. All deviations, except for the figures titled P indexed RER, TOL (Final Sector), and Trade Balance/GDP, are expressed in units of percent deviation from the initial steady-state ($t = 0$), i.e., $100 \times (x_t/x_0 - 1)$. The figures titled P indexed RER, TOL (Final Sector), and Trade Balance/GDP are presented in units of percentage points difference from the initial steady-state, i.e., $100 \times (x_t - x_0)$. Refer to Figure A2 for the responses of other variables.

enough to cover export costs.

On the household side, higher wages for workers employed in the consumption goods production sector prompts an expansion of labor supply to that sector but a reduction in labor supply to the gas-producing sector. Consequently, the amount of domestically produced gas diminishes. This reduction is compensated by an increase in the import of Foreign gas. The heightened demand for gas by consumption goods firms raises the Home gas price more than the Foreign gas price.

The responses of gas and real exchange rate to productivity shocks align with our analytical analysis in Section 3. As illustrated in equation (25) and Figure 2, Home gas price ($\hat{\rho}_{G,t}$) positively related with Home consumption (C_t) but negatively with the real exchange rate and Foreign consumption (Q_t and C_t^*). In response to a positive Home consumption sector productivity shock, consumption in Home and Foreign increase, but the rise of consumption in Home is more pronounced than in Foreign. Therefore, Home gas price increases following the productivity shock in the consumption sector.

The real exchange rate depreciates in the short run and appreciates in the long run. The short run depreciation is dependent on how much gas is used as an input in production of consumption goods in Home. If the gas share is small, there is little demand on Foreign gas for the production of Home consumption goods, and therefore, Foreign reallocation of production towards gas sector is less pronounced (red lines in Figure 4). Smaller slowdown in entry in Foreign implies higher average prices in Foreign, implying a depreciation from Home perspective.

We use equation (32) and Figure 3 to understand the dynamics of the data-consistent real exchange rate ($\tilde{\mathsf{Q}}_t$) in response to a permanent 10% increase in Home productivity. While $\mathsf{Z}_t > 0$ directly feeds into Home real exchange rate depreciation ($\tilde{\mathsf{Q}}_t > 0$) according to equation (32), other channels contribute to appreciation following the productivity increase.

In the long run, Home terms of labor (consumption sector) appreciates ($\mathsf{TOL}_t < 0$) due to new entry and production, thus contributing to Home real exchange rate appreciation ($\tilde{\mathsf{Q}}_t < 0$). Moreover, high and low export cutoffs in Home and Foreign ($\mathsf{z}_{X,t} > 0$ and $\mathsf{z}_{X,t}^* < 0$) decrease Foreign average imported price and increase Home average imported price, resulting in Home exchange rate appreciation. When a Home favorable shock occurs, the Home boom attracts more Foreign exporters and lowers their export cutoff ($\mathsf{z}_{X,t}^* < 0$). Additionally, increased Home firms and heightened Home factor market competition raise Home firms' production costs, leading to an increase in the Home export cutoff ($\mathsf{z}_{X,t} > 0$). Finally, the composition of consumption baskets in Home and Foreign reacts to the shocks. High Home productivity prompts Foreign consumers to rely more on imported products ($\mathsf{N}_{X,t} > 0$) from Home, while fewer domestically produced goods are available ($\mathsf{N}^{*}_{D,t} < 0$). Simultaneously, increased Home demand encourages Foreign firms to export more, resulting in more imported products ($\mathsf{N}^{*}_{X,t} > 0$) in the Home market. Hence, the compositional effect contributes to Home real exchange rate appreciation, albeit sluggishly changing due to the gradual adjustment of the number of products and firms. In the short run, this channel is relatively small compared to the long run, explaining the positive impulse response of $\tilde{\mathsf{Q}}_t$ at the beginning of periods in Figure 4.

It is also useful to note that the model's response when the gas share in the production is small ($\alpha = 0.01$) is in line with the GM.[10] When the gas share is large ($\alpha = 0.2$), Home consumption and labor responses resemble those with a negligible gas share ($\alpha = 0.01$), while there are notable increases in Foreign consumption and decreases in Foreign consumption sector labor. Additionally, terms of labor (consumption goods sector) significantly depreciate more. This happens because higher productivity in Home leads to more firm entry and production, increasing input demands and prices in the Home consumption-good sector. Higher gas price boosts the Foreign economy through an increase in revenues from gas exports. Consequently, Foreign income and consumption increase, but consumption-sector labor supply decreases, a mechanism that becomes stronger when consumption good production requires more gas. This example illustrates how comparative advantage in gas production amplifies the international propagation mechanisms of technology shocks.

5. The Sanctions

Sanctions are assumed to be imposed by Home and are of three types: consumption goods trade sanctions, financial sanctions, and gas trade sanctions. The numerical exercises align with the mechanisms outlined in our analytical findings, providing an in-depth understanding of the effect of each type of sanction in the short, medium, and long term.

The introduction of consumption goods trade sanctions involves preventing trade for consumption goods producers with firm-specific productivity above a certain threshold. The underlying idea is that sanctions lead to a reduction in the trade activities of larger producers. In the case of financial market sanctions, a fraction of Foreign households is initially excluded from international

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[10] While the original GM and many other papers assume balanced trade and capital flows in the long run, we introduce long-run imbalances. However, this departure does not alter the significant differences in dynamics to aggregate productivity shocks in our model.

bond trading, with the possibility of extending the exclusion of all Foreign households in the limit (financial autarky). For gas trade sanctions, we study the effects of a ban on gas trade, implemented in the first period. In our model, this is equivalent to the imposition of a price cap that is below the marginal cost of Foreign gas production. The simulations track the system's response to the shocks until it reaches a new steady-state ($t = 201$). Simulating the model for 200 periods is sufficient for the economy to reach a new steady-state.

5.1. Consumption Good Trade Sanctions

We implement sanctions on consumption goods trade by introducing another productivity cutoff for Home or Foreign exporters, $z^{s}_{X,t}$ or $z^{s*}_{X,t}$, respectively. These sanctions take two forms: Home consumption goods producers with productivity levels higher than the sanction cutoff ($z^{s}_{X,t}$) cease exporting to Foreign and/or Home stops importing from the most productive Foreign producers (those with productivity above $z^{s*}_{X,t}$). Then, export sanctions imply the modification of the average Home exporter productivity in equation (7) as follows:

$$\tilde{z}_{X,t} \equiv \left[\frac{1}{\Phi(z^{s}_{X,t}) - \Phi(z_{X,t})} \int_{z_{X,t}}^{z^{s}_{X,t}} z^{\theta-1} \mathrm{d}\Phi(z) \right]^{\frac{1}{\theta-1}}. \tag{33}$$

Similarly, the average productivity of Foreign exporters under import sanctions is given by:

$$\tilde{z}^{*}_{X,t} \equiv \left[\frac{1}{\Phi(z^{s*}_{X,t}) - \Phi(z^{*}_{X,t})} \int_{z^{*}_{X,t}}^{z^{s*}_{X,t}} z^{\theta-1} \mathrm{d}\Phi(z) \right]^{\frac{1}{\theta-1}}. \tag{34}$$

In our simulations, we determine the cutoff by assuming that the top 1 percent most productive consumption goods producers cease exporting. Although these top 1 percent Home firms represent around 60 percent of aggregate Home exports to Foreign, the export ban on them increases the export participation of other (less productive) firms, resulting in a decrease in aggregate final goods export amount that is less than 60 percent.[11]

Figures 5 and A3 depict the dynamics after the introduction of consumption goods trade sanctions. Green lines represent simulations after the introduction of export sanctions (EXS) where the top 1 percent productive Home firms cease exporting to Foreign. Blue lines indicate simulations after the introduction of import sanctions (IMS) where the top 1 percent productive Foreign firms

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[11] See Ghironi, Kim, and Ozhan (2024) for comparison of this productivity-based sanction to an alternative type of sanction in which firms are excluded randomly, independent of their productivity.

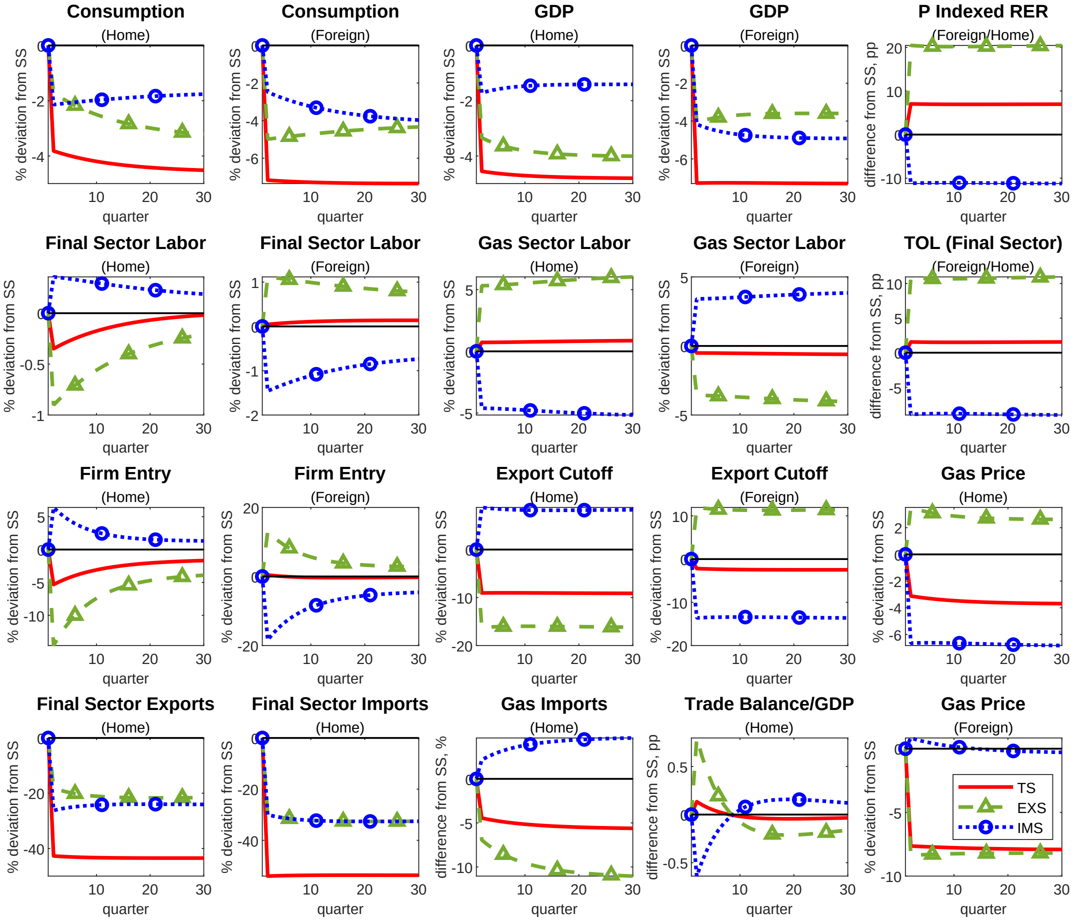

Figure 5: Transition Dynamics after Consumption Good Export and Import Sanctions

Notes: The red solid lines, green dashed lines with triangles, and blue dashed lines with circles represent the transitional dynamics when consumption goods export (EXS), import (IMS), and trade sanctions (TS, both export and import) are imposed at $t = 1$. All deviations, except for the figures titled P indexed RER, TOL (Final Sector), and Trade Balance/GDP, are measured in units of percent deviation from the initial steady-state without sanctions ($t = 0$), i.e., $100 \times (x_t/x_0 - 1)$. The figures titled P indexed RER, TOL (Final Sector), and Trade Balance/GDP express deviations in units of percentage points from the initial steady-state, i.e., $100 \times (x_t - x_0)$. Refer to Figure A3 for responses of other variables.

cannot export to Home. Red lines indicate simulations when both import and export sanctions are in place simultaneously (TS).

Following export sanctions, the most productive producers in the Home export market cease international trade, but the Home economy still faces external demand due to its comparative advantage in producing consumption goods. After the exclusion of the most productive Home exporters, the productivity cutoff level for the least Home exporter falls and less productive Home producers join the export market. Therefore, the average price of Home exports increases, and the Home real exchange rate depreciates. The Foreign economy reallocates production resources towards the consumption goods sector to compensate for the lost imports from Home. Moreover, the exclusion of the most productive Home exporters from international trade implies less gas usage in Home and a decrease in gas imports from Foreign, resulting in a decrease in the price of gas in Foreign.

In response to import sanctions, the top 1 percent productive Foreign consumption goods producers cease international trade. Lower consumption-good sector labor demand causes lower wages and reduces the consumption value of the fixed export cost in Foreign. This leads to a fall in the cutoff productivity level of the least productive Foreign exporter and less productive Foreign producers start exporting. The number of exporters in Foreign increases, and Foreign consumption exports become more expensive on average due to the fall in the average productivity of Foreign exporters. This implies an appreciation of the Home real exchange rate. A shrinking number of consumption goods producers implies less demand for labor in the consumption sector in Foreign. Increased entry in the Home consumption sector to compensate for the lost imports from Foreign implies more gas demand from the Home consumption sector, inducing the Foreign economy to reallocate production to the gas sector and the price of gas in Foreign to increase.

Joint introduction of both export and import sanctions yields responses in trade and macroeconomic variables similar to those of export sanctions rather than those of import sanctions. This phenomenon is attributed to the asymmetry between the two countries, particularly the relative advantage of Home in producing consumption goods, which causes export sanctions to dominate over import sanctions when variables move in opposite directions.

5.2. Financial Sanctions

In this subsection, we first outline the changes in the equilibrium conditions of the model implied at the time of sanction introduction by the exclusion of Foreign agents from trading in international financial markets. Subsequently, we explore the simulations conducted under financial sanctions.

When Home imposes financial sanctions on Foreign, a fraction $\lambda > 0$ of Foreign households is excluded from participating in international financial markets. After the imposition of sanctions, these households can only trade Foreign bonds and shares with other Foreign households. When the entire Foreign economy is subject to financial sanctions with $\lambda = 1$, Foreign operates under financial autarky.

Once financial sanctions are imposed, the Foreign population is divided into two groups of households: λ who are subject to the sanctions and $1 - \lambda$ who are not. The budget constraint for the representative sanctioned household becomes:

$$\begin{aligned} C^*_{S,t} + B^*_{S,F,t+1} + \frac{\eta}{2}(B^*_{S,F,t+1} - B^*_{S,F})^2 + \frac{\eta}{2}\tilde{\upsilon}^*_t N^*_t (x^*_{S,t+1} - 1)^2 + \tilde{\upsilon}^*_t N^*_t x^*_{S,t+1} \\ = (1 + r^*_t)B^*_{S,F,t} + w^*_{G,t}L^*_{S,G,t} + w^*_t L^*_{S,t} + (\tilde{d}^*_t + \tilde{\upsilon}^*_t)N^*_{D,t}x^*_{S,t} + T^{*f}_{S,t}, \end{aligned} \tag{35}$$

for periods $t = 1, 2, \ldots, \infty$. Here, the subscript S denotes households that are subject to sanctions. The sanctioned households lose the Home-issued bonds ($B_{S,F,1} = B_F = -B_H > 0$), and thus, they cannot receive any returns from them at $t = 1$. Additionally, they cannot trade Home bonds for the entire duration after the sanctions, i.e., $B_{S,F,t+1} = 0$ for $\forall t \geq 1$. However, they can still trade Foreign bonds with unsanctioned Foreign households, but their terminal steady-state bond holding is zero, i.e., $B^*_{S,F} = 0$.

After the imposition of financial sanctions, the budget constraint for the representative non-sanctioned household remains unchanged:

$$\begin{aligned} C^*_{NS,t} + \frac{B_{F,t+1}}{Q_t} + \frac{\eta}{2Q_t}(B_{NS,F,t+1} - B_{NS,F})^2 + B^*_{NS,F,t+1} + \frac{\eta}{2}(B^*_{NS,F,t+1} - B^*_{NS,F})^2 \\ + \frac{\eta}{2}\tilde{\upsilon}^*_t N^*_t (x^*_{NS,t+1} - 1)^2 + \tilde{\upsilon}^*_t N^*_t x^*_{NS,t+1} \\ = (1 + r_t)\frac{B_{NS,F,t}}{Q_t} + (1 + r^*_t)B^*_{NS,F,t} + w^*_{G,t}L^*_{NS,G,t} + w^*_t L^*_{NS,t} + (\tilde{d}^*_t + \tilde{\upsilon}^*_t)N^*_{D,t}x^*_t + T^{*f}_{NS,t}, \end{aligned} \tag{36}$$

where the subscript NS denotes non-sanctioned households, who retain the ability to trade bonds during the transition. In the terminal steady-state, the bond holdings of non-sanctioned Foreign households remain unchanged after financial sanctions, specifically $B_{NS,F} = B_F = -B_H$ and $B_{NS,F} = B^*_F = -B^*_H$.

The market clearing conditions for bonds and shares in the presence of financial market sanc-

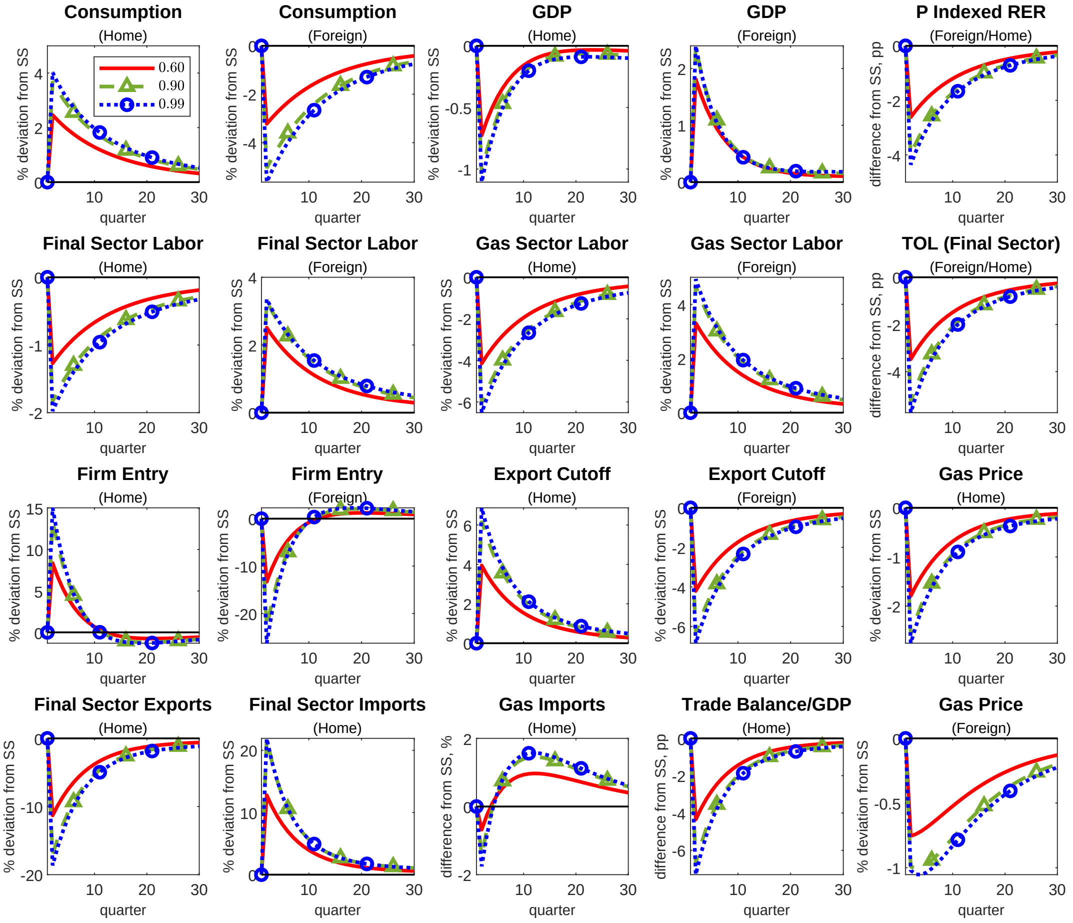

Figure 6: Transition Dynamics after Financial Sanctions

Notes: The red solid lines, green dashed lines with triangles, and blue dashed lines with circles depict the model transition dynamics when financial sanctions are imposed at $t = 1$, with the fraction of sanctioned Foreign households (λ) set at 60%, 90%, and 99%, respectively. All deviations, except for the figures titled P indexed RER, TOL (Final Sector), and Trade Balance/GDP, are measured in units of percent deviation from the initial steady-state without sanctions ($t = 0$), i.e., $100 \times (x_t/x_0 - 1)$. The figures titled P indexed RER, TOL (Final Sector), and Trade Balance/GDP express deviations in units of percentage points from the initial steady-state, i.e., $100 \times (x_t - x_0)$. All Foreign variables are aggregates. Refer to Figure 7 for sanctioned and unsanctioned households' variables. See Figure A4 for responses of other variables.

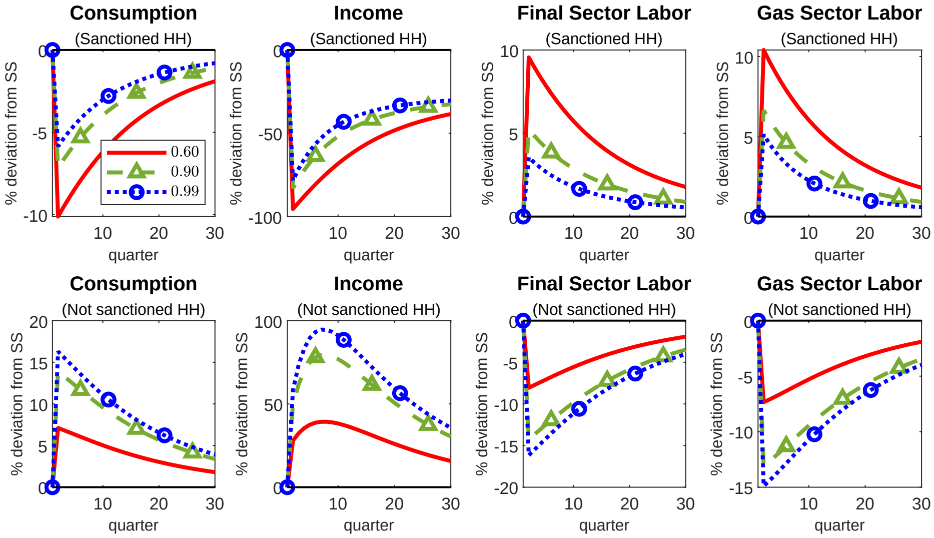

Figure 7: Financially Sanctioned vs. Unsanctioned Foreign Households

Notes: The red solid lines, green dashed lines with triangles, and blue dashed lines with circles illustrate the model transition dynamics when financial sanctions are imposed at $t = 1$, with the fraction of sanctioned Foreign households (λ) set at 60%, 90%, and 99%, respectively. The first and second rows depict the responses of sanctioned and unsanctioned households, respectively. All deviations are measured in units of percent deviation from the initial steady-state without sanctions ($t = 0$), i.e., $100 \times (x_t/x_0 - 1)$. Refer to Figures 6 and A4 for the aggregate variables' responses.

tions are as follows:

$$B_{H,t+1} + (1-\lambda)B_{NS,F,t+1} = 0 = B_{H,t} + (1-\lambda)B_{NS,F,t} \tag{37}$$

$$B^*_{H,t+1} + (1-\lambda)B^*_{NS,F,t+1} + \lambda B^*_{S,F,t+1} = 0 = B^*_{H,t} + (1-\lambda)B^*_{NS,F,t} + \lambda B^*_{S,F,t} \tag{38}$$

$$x_{t+1} = 1 = x_t \tag{39}$$

$$\lambda x^*_{S,t+1} + (1-\lambda)x^*_{NS,t+1} = 1 = \lambda x^*_{S,t} + (1-\lambda)x^*_{NS,t}. \tag{40}$$

Because financial sanctions enforce zero bond holdings for sanctioned Foreign households, the new steady-state bond holdings for Home and Foreign are $-(1-\lambda)B_{NS,F}$ and $-(1-\lambda)B^*_{NS,F}$, respectively.

Figures 6, 7, and A4 present transition dynamics under financial sanctions. The figures plot transition dynamics from the initial steady-state in which Foreign has a positive NFA position to the terminal steady-state in which Foreign has a zero NFA position. The blue, green, and red lines show simulations for this transition behavior when 99%, 90%, and 60% of Foreign households are excluded from international financial transactions, respectively (i.e., $\lambda \in \{0.99, 0.9, 0.6\}$).

Immediate observation highlights that the impact of financial sanctions on Foreign consumption is more pronounced in the short to medium term when a larger proportion of Foreign households is subjected to these sanctions. In our model, only a fraction of the Foreign population is affected by financial sanctions, providing an avenue for the sanctioned fraction to alleviate the effects through transactions with those in Foreign who still have access to international financial markets.

Upon the imposition of financial sanctions, our initial observation reveals a sharp short-term decline in Foreign consumption, surpassing its long-term reduction. This behavior is primarily attributed to wealth effects due to the loss of their holdings of Home bonds. When sanctions come into effect, Foreign sanctioned households experience a negative transitory income shock, and also their opportunities for savings through Home bonds are limited. As a result, Foreign consumption demand decreases, including its demand for imports from Home. The reduction in Foreign imports leads to a smaller number of Home exporters accessing the Foreign market, elevating their export threshold. This, in turn, lowers the average Home export price and causes an appreciation of the Home real exchange rate.

Figure 7 illustrates significant heterogeneity between sanctioned and unsanctioned households in Foreign. Sanctioned households respond to the fall in their financial income by increasing their labor supply to both the consumption goods production and gas production sectors. This surge in labor supply results in lower wages, contributing to the depreciation of the Foreign terms of labor and real exchange rate, along with an increase in the trade surplus (exports in both sectors rise after sanctions). In contrast, unsanctioned households decrease their labor supply in both sectors and benefit from non-labor incomes by trading bonds with Home households and sanctioned households. This result indicates that (almost) all Foreign households should be sanctioned for financial sanctions to be effective.

5.3. Gas Sanctions

We explore gas sanctions by implementing a permanent ban on gas trade starting from period $t = 1$. This action is equivalent to imposing a permanent price cap on Home gas imports from Foreign, effective from period $t = 1$ onward. The price cap is set below Foreign marginal costs of gas production, causing Foreign to cease gas exports to Home in response to the gas sanction.

Under gas sanctions, a ban on gas trade ($g^*_{H,t}(z) = 0$ and $G^*_{H,t} = 0$) implies that gas price equalization no longer holds ($\rho G, t \neq \tau_{G,t} Q_t \rho_{G,t}$ for $t \geq 1$). In the new equilibrium, Home and Foreign gas markets are separated, and market clearing conditions in Home and Foreign are $G_N L_{G,t} = G_{H,t}$ and $G^*_N L^*_{G,t} = G^*_{F,t}$, respectively.

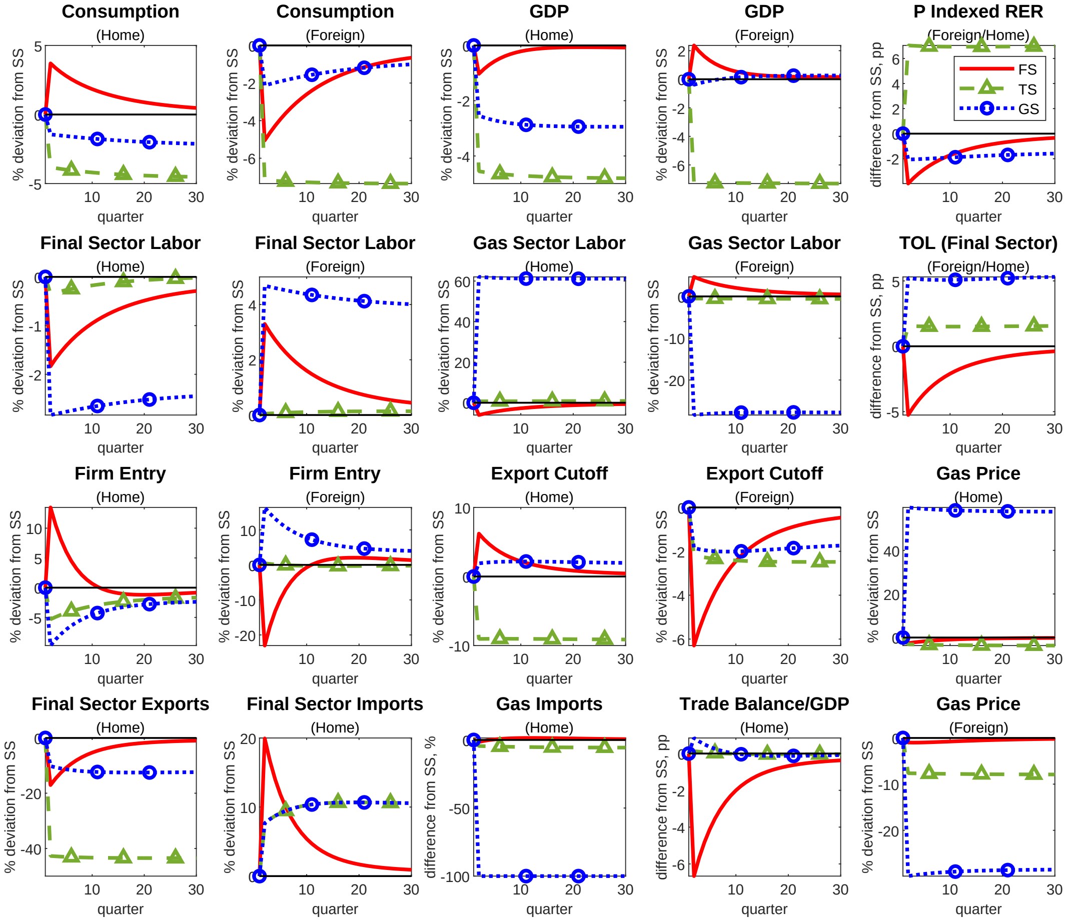

Figure 8: Transition Dynamics after Trade, Financial, and Gas Sanctions

Notes: The red solid lines, green dashed lines with triangles, and blue dashed lines with circles depict the transitional dynamics when financial (FS), consumption good sector trade (TS), and gas sanctions (GS), respectively, are imposed at $t = 1$. All deviations, except for the figures titled P indexed RER, TOL (Final Sector), and Trade Balance/GDP, are measured in units of percent deviation from the initial steady-state without sanctions ($t = 0$), i.e., $100 \times (x_t/x_0 - 1)$. The figures of ratio, titled P indexed RER, TOL (Final Sector), and Trade Balance/GDP, are measured in units of percent point difference from the initial steady-state, i.e., $100 \times (x_t - x_0)$. Refer to Figure A5 for the responses of other variables.

Figures 8 and A5 illustrate the dynamics in response to the implementation of gas sanctions (blue lines, GS). For comparative analysis, we include the dynamics under trade sanctions (green lines, TS) and financial sanctions (red lines, FS). Gas sanctions, while not as impactful as the combined import and export sanctions on consumption goods trade in reducing Foreign consumption, still lead to a more significant immediate drop in Foreign consumption compared to Home consumption. This occurs even without the ability of the Home economy to substitute Foreign gas with imports from other countries.

The decrease in demand for Foreign gas results in a reduction in gas production in Foreign and a subsequent increase in the price of gas in Home. The gas price increase is driven by consumption goods producing firms, which demand more domestic gas to compensate for the lost imported gas. As the price of gas increases, the marginal cost of production in the Home consumption goods sector also rises. This cost escalation is reflected in fewer entrants in Home, leading to a decline in the total number of producers. Home households respond by increasing labor supply to gas production and decreasing labor supply to the consumption goods production sector.

In contrast, the Foreign economy undergoes a rebalancing in the opposite direction. To offset the loss of gas exports, the economy shifts towards increased production in the consumption goods sector and higher imports of consumption goods. Consumption goods producers in Foreign increase their demand for labor, resulting in rising wages. Concurrently, the decrease in gas production reduces the need for labor in the gas sector, leading to a decline in wages. This economic shift encourages more entrants into the consumption goods sector. Consequently, the number of producers in the Foreign consumption goods sector increases. To counter the loss of gas exports, more firms in the consumption goods sector begin exporting, and the cutoff productivity level for the least efficient exporter in Foreign decreases. This adjustment in exporter productivity cutoff translates into higher average Foreign export prices, appreciating the Home real exchange rate.

5.4. Combinations of Sanctions

In this subsection, we present the combined impact of several sanctions that are introduced simultaneously. In particular, we consider three cases: (1) combination of financial sanctions and consumption good import and export sanctions (FS&TS), (2) combination of financial and gas sanctions (FS&GS) and, (3) combination of financial, consumption good trade, and gas sanctions, altogether (FS&TS&GS).

Figures 9, 10, and A6 show dynamics when several combination of sanctions are in place. The

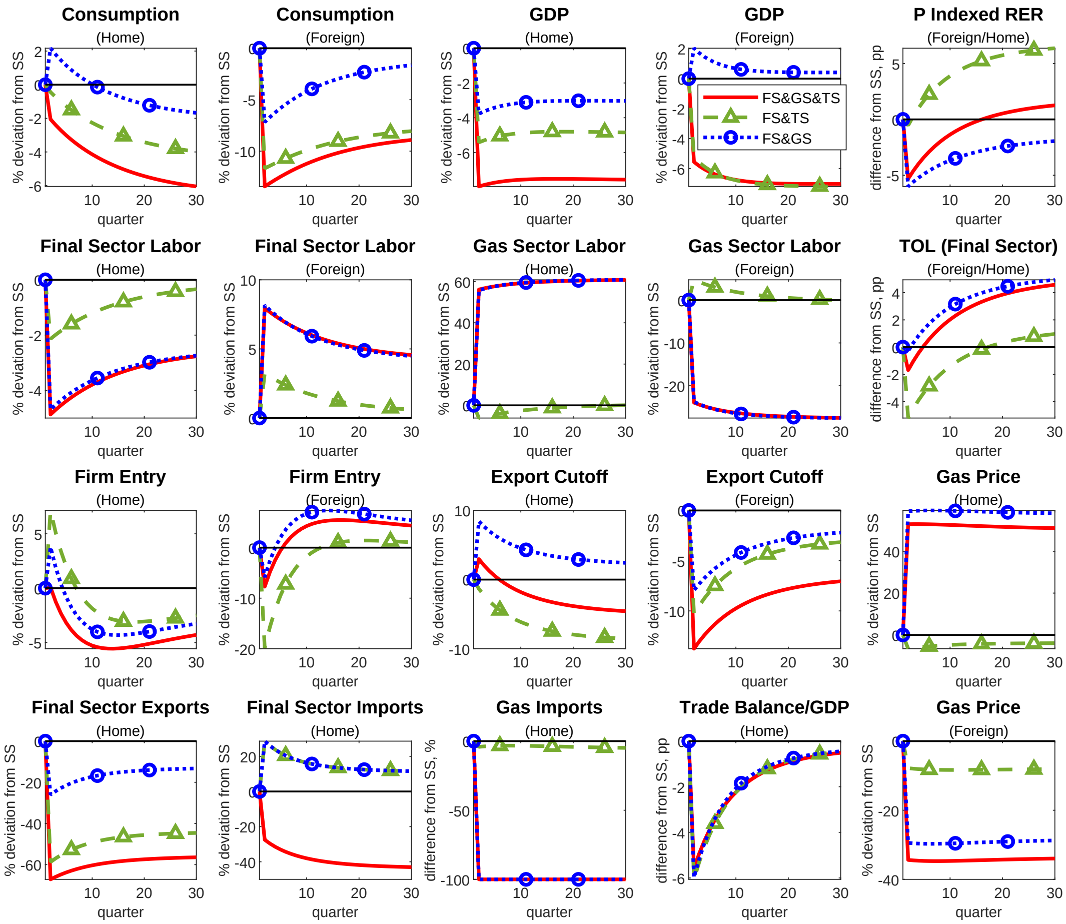

Figure 9: Transition Dynamics with Combinations of Sanctions

Notes: The red solid lines depict the transitional dynamics when all sanctions (FS&GS&FS) are imposed at $t = 1$. The green dashed lines with triangles illustrate the transitional dynamics when financial and consumption good sector trade sanctions (FS&TS) are imposed at $t = 1$. The blue dashed lines with circles show the transitional dynamics when financial and gas sanctions (FS&GS), respectively, are imposed at $t = 1$. All deviations, except for the figures titled P indexed RER, TOL (Final Sector), and Trade Balance/GDP, are measured in units of percent deviation from the initial steady-state without sanctions ($t = 0$), i.e., $100 \times (x_t/x_0 - 1)$. The figures of ratio, titled P indexed RER, TOL (Final Sector), and Trade Balance/GDP, are measured in units of percent point difference from the initial steady-state, i.e., $100 \times (x_t - x_0)$. Refer to Figure 10 for sanctioned and unsanctioned households' variables. See Figure A6 for the responses of other variables.

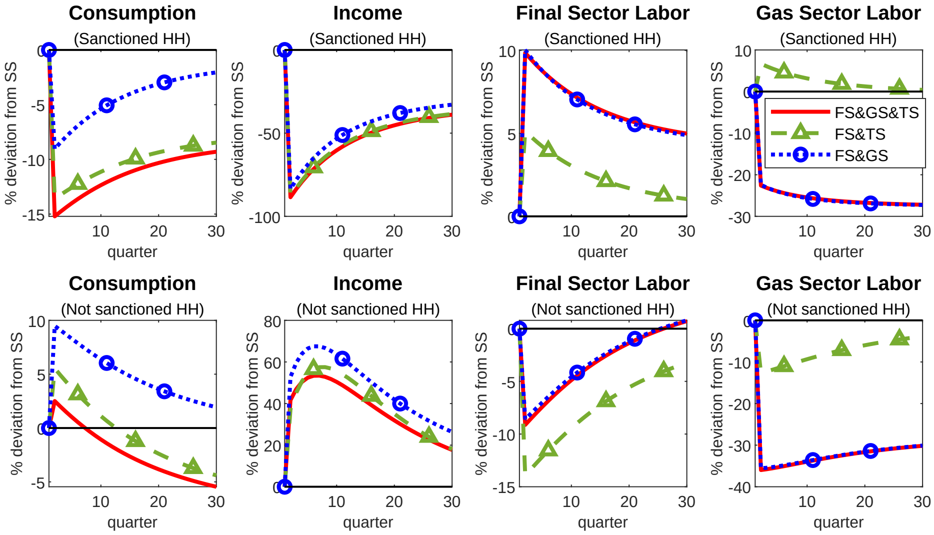

Figure 10: Financially Sanctioned vs. Unsanctioned Foreign Households (under Combinations of Sanctions)

Notes: The red solid lines depict the transitional dynamics when all sanctions (FS&GS&FS) are imposed at $t = 1$. The green dashed lines with triangles illustrate the transitional dynamics when financial and consumption good sector trade sanctions (FS&TS) are imposed at $t = 1$. The blue dashed lines with circles show the transitional dynamics when financial and gas sanctions (FS&GS), respectively, are imposed at $t = 1$. The first and second rows plot the responses of sanctioned and unsanctioned households, respectively. All deviations are in units of percent deviation from the initial steady-state without sanctions ($t = 0$), i.e., $100 \times (x_t/x_0 - 1)$. Refer to Figures 9 and A6 for the aggregate variables' responses.

negative impact on Foreign consumption is amplified if all sanctions are applied simultaneously. The quantitative impact is similar when all sanctions are applied simultaneously (FS&TS&GS, red solid lines) or when only consumption good trade and financial sanctions (FS&TS, green dashed lines with triangles) are applied simultaneously. It is also important to note that all of the sanctions generate a fall in Home consumption and GDP, although the fall is not as large as in Foreign. The impact of consumption good export sanctions generates the most pronounced fluctuations. Therefore, the Home real exchange rate depreciates when trade sanctions are combined with any other sanction.

Gas sanctions damage gas exporters (Foreign) more than gas importers (Home). The Foreign economy rebalances toward consumption good production and, therefore, Foreign GDP stays stable. Gas sanctions contribute to a long-term drop in Home consumption whereas the long-term impact on Foreign consumption is relatively small. The combination of financial and consumption good trade sanctions (FS&TS) dampens the negative impact on the Home economy while amplifying the negative impact on Foreign economy, compared to the other combination of sanctions. On the other hand, Figures 9, 10, and A6 show that cessation of gas trade could also be used by Foreign to generate a fall in Home GDP. To be more concrete on the evaluation of the success of sanctions, we provide welfare results in the next subsection.

5.5. Welfare

In this subsection, we explore the welfare effects of sanctions on both the imposing and targeted economies. To measure welfare, we consider lifetime utility from consumption and disutility from labor. The effect of sanctions is incorporated from the first period ($t = 1$), and we analyze the transition dynamics until $t = 200$, accounting for the terminal impact at $t = 201$.

The lifetime utility used to measure welfare without sanctions is given by:

$$\mathcal{U}_0 = \frac{1}{1-\beta}\left(\log C_0 - \frac{\kappa}{2}L_0^2 - \frac{\kappa_G}{2}L_{G,0}^2\right), \tag{41}$$

where C_0, L_0, and $L_{G,0}$ are Home household's consumption, consumption goods labor supply, and gas sector labor supply, respectively, at the (initial) steady-state without sanctions. Similarly, the lifetime utility at the new steady-state after sanctions is:

$$\mathcal{U}_{201}^s = \frac{1}{1-\beta}\left(\log C_{201} - \frac{\kappa}{2}L_{201}^2 - \frac{\kappa_G}{2}L_{G,201}^2\right), \tag{42}$$

where C_{201}, L_{201}, and $L_{G,201}$ are Home household's consumption, consumption goods labor supply, and gas sector labor supply, respectively, at the (terminal) steady-state with sanctions. Then, the welfare with sanctions and transition paths can be expressed as:

$$\mathcal{U}_0^s = \sum_{t=0}^{200}\beta^t\left(\log C_t - \frac{\kappa}{2}L_t^2 - \frac{\kappa_G}{2}L_{G,t}^2\right) + \beta^{201}\mathcal{U}_{201}^s. \tag{43}$$

To measure aggregate Foreign welfare under sanctions, we calculate the weighted average of welfare for sanctioned and non-sanctioned households: $\mathcal{U}_0^{s*} = \lambda\mathcal{U}_{S,0}^{s*} + (1-\lambda)\mathcal{U}_{NS,0}^{s*}$, where the subscripts S and NS indicate financially sanctioned and unsanctioned Foreign households, respectively.

We then calculate the welfare gain in initial consumption-equivalent terms:

$$\mathcal{U}_0^s = \frac{1}{1-\beta}\left\{\log[(1+\Delta)C_0] - \frac{\kappa}{2}L_0^2 - \frac{\kappa}{2}L_{G,0}^2\right\}, \tag{44}$$

where Δ measures lifetime welfare gains in (initial steady-state) consumption-equivalent terms.

After some algebra, it can be expressed as $\Delta = \exp[(1-\beta)(\mathcal{U}_0^s - \mathcal{U}_0)] - 1$.[12] To compare welfare with and without dynamics, we calculate the welfare gain in consumption-equivalent terms from comparative statistics between the initial and new steady-states:

$$\mathcal{U}_{201}^s = \frac{1}{1-\beta}\left\{\log[(1+\Delta_{ss})C_0] - \frac{\kappa}{2}L_0^2 - \frac{\kappa}{2}L_{G,0}^2\right\}, \tag{45}$$

where Δ_{ss} can be expressed as $\Delta_{ss} = \exp[(1-\beta)(\mathcal{U}_{201}^s - \mathcal{U}_0)] - 1$.[13]

Table 1 provides the calculated welfare effects in Home and Foreign resulting from different sanctions. Consumption goods trade sanctions lead to the most substantial welfare loss in both economies, with a more pronounced impact on Foreign. Gas sanctions, while causing more significant welfare losses in Home compared to Foreign, have a smaller quantitative impact than consumption goods trade sanctions. Perhaphs, surprisingly, financial sanctions generate a small welfare gain in Home. This happens because Home transitions from a negative NFA position to having no external debt.

As discussed in the previous section, the economies' rebalancing toward different sectors plays a crucial role in the effect of sanctions. Sanctioning the less efficient sector in Foreign results in the most considerable welfare losses in Foreign because the Foreign economy reallocates resources toward this sector. Combining consumption goods trade sanctions with financial sanctions adds little additional loss in Foreign.

The welfare analysis in Table 1 emphasizes the importance of considering transition paths when evaluating the effect of sanctions. Comparing the first and second columns to the third and fourth columns reveals that Home welfare losses are overvalued, while Foreign welfare losses are undervalued, when transition paths are ignored. These discrepancies arise due to the sluggish convergence of Home consumption to the new steady-states and the short-term sharp decline in Foreign consumption, driven by the loss of income from Home bonds at $t = 1$, as illustrated in Figures 6 and 10.

The mismeasurements are even more profound when focusing on financially unsanctioned Foreign households. Table 2 shows that comparative statistics between the initial and new steady-states tend to overestimate their welfare losses by more than half. For instance, combined gas and financial sanctions lead to welfare gains for unsanctioned households ($\Delta_{NS}^* = 4.56\%$), while the

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[12] The aggregate Foreign welfare gains are calculated by $\Delta^* = \exp[(1-\beta)(\mathcal{U}_0^{s*} - \mathcal{U}_0^*)] - 1$, where $\mathcal{U}_0^{s*} = \lambda\mathcal{U}_{S,0}^{s*} + (1-\lambda)\mathcal{U}_{NS,0}^{s*}$ and $\mathcal{U}_0^* = \lambda\mathcal{U}_{S,0}^* + (1-\lambda)\mathcal{U}_{NS,0}^*$.

[13] The aggregate Foreign welfare gains are calculated by $\Delta_{ss}^* = \exp[(1-\beta)(\mathcal{U}_{201}^{s*} - \mathcal{U}_0^*)] - 1$, where $\mathcal{U}_{201}^{s*} = \lambda\mathcal{U}_{S,201}^{s*} + (1-\lambda)\mathcal{U}_{NS,201}^{s*}$ and $\mathcal{U}_0^* = \lambda\mathcal{U}_{S,0}^* + (1-\lambda)\mathcal{U}_{NS,0}^*$.

Table 1: Change in Welfare after Sanctions

	Welfare Gain (%, consumption)			
	with transition paths (Δ)		without transition paths (Δ_{ss})	
Type of Sanctions	Home	Foreign	Home	Foreign
Gas sanction	–2.24	–0.80	–2.44	–0.52
C-good export sanction	–3.20	–4.28	–3.54	–4.12
C-good import sanction	–1.73	–3.94	–1.63	–4.30
C-good trade sanction	–4.47	–7.27	–4.64	–7.35
Financial sanction	0.82	–1.25	0.27	–0.40
Gas + Financial sanctions	–1.44	–2.05	–2.16	–0.94
C-good trade + Financial sanctions	–3.64	–8.50	–4.39	–7.72
Gas + C-good trade + Financial sanctions	–5.81	–9.17	–6.68	–8.19

Notes: The first two columns present the welfare (lifetime utility) gains of sanctions in terms of consumption, Δ, defined in equation (44). In the last two columns, we calculate the welfare gains of sanctions through comparative statistics (ignoring transition paths) between the initial point ($t = 0$) and the terminal point ($t = 201$) in terms of consumption, Δ_{ss}, defined in equation (45). The Foreign welfare gain measures (Δ^* and Δ^*_{ss}) are calculated from the weighted sum of financially sanctioned (S) and unsanctioned (NS) Foreign households' welfare ($\mathcal{U}_t^{s*} = \lambda \mathcal{U}_{S,t}^{s*} + (1-\lambda)\mathcal{U}_{NS,t}^{s*}$ and $\mathcal{U}_t^{*} = \lambda \mathcal{U}_{S,t}^{*} + (1-\lambda)\mathcal{U}_{NS,t}^{*}$ for $t = 0, 201$). See Table 2 for the welfare of financially sanctioned and unsanctioned Foreign households.

Table 2: Welfare Changes of Sanctioned vs. Unsanctioned Foreign Households

	Welfare Gain (%, consumption)			
	with transition paths (Δ)		without transition paths (Δ_{ss})	
Type of Sanctions	Sanctioned	Unsanctioned	Sanctioned	Unsanctioned
Financial sanction	–1.88	4.56	–0.45	0.00
Gas &Financial sanctions	–2.61	3.17	–0.98	–0.51
C-good trade + Financial sanctions	–9.06	–3.22	–7.76	–7.37
Gas + C-good trade + Financial sanctions	–9.71	–4.17	–8.23	–7.79

Notes: The first two columns present the welfare (lifetime utility) gains of sanctions in terms of consumption, Δ, defined in equation (44). In the last two columns, we calculate the welfare gains of sanctions through comparative statistics (ignoring transition paths) between the initial point ($t = 0$) and the terminal point ($t = 201$) in terms of consumption, Δ_{ss}, defined in equation (45). See Table 1 for the aggregate welfare of Foreign.

welfare measure without transition paths indicates a decrease ($\Delta^*_{NS,ss} = -0.51\%$). This discrepancy arises because comparative statistics cannot account for short-run advantages through the ability to trade Home and Foreign bonds in domestic and international markets, advantages that diminish in the long run (see Figures 7 and 10 for the transition dynamics, particularly the short-run increases in unsanctioned Foreign households' consumption and income).

6. International Coordination with a Third Country

Our analysis, thus far, has not accounted for the potential substitution to a third country in response to sanctions. To address this, we extend our model to a three-country setting in this section.[14]

Incorporating a third country, referred to the Rest of the World (RoW), allows us to explore the effect of international coordination when sanctioning Foreign. For clarity, we assume that Home and RoW are symmetric, sharing similar characteristics. Specifically, Home and RoW are subject to smaller entry costs in the consumption goods sector and have smaller gas endowment than Foreign.[15] The calibration follows the numerical values set in Section 4, with adjustments for financial variables. Introducing a third country increases export productivity cutoffs; the percentage of Home firms exporting their goods to RoW and vice versa is 17%. For RoW firms, the percentages exporting to Foreign and from Home to RoW are 14%, while for Foreign firms, the percentages exporting to Home and from Foreign to RoW are 19%, all in the initial steady-state.

The assumption of symmetry between Home and RoW implies a net foreign asset position of zero between them. To maintain a positive net foreign position for Foreign, we set Home (as well as RoW) households' initial holdings of Home and Foreign bonds to $B_H = -5/2$ and $B^*_H = 3/2$, respectively (half of the value in the two-country model). This calibration implies that the value of Foreign households' initial holdings of Home and RoW bonds is 108%, and the Foreign NFA position is 36% of its GDP. See Table A1 for the details of initial and terminal bond holdings.

Figures 11 and A7 present the responses of the three-country model to the simultaneous introduction of financial, consumption good, and gas sanctions against Foreign. Green lines indicate dynamics when only Home sanctions Foreign (i.e., uncoordinated sanctions), whereas red lines indicate dynamics when both Home and RoW sanction Foreign (i.e., coordinated sanctions).

Under uncoordinated sanctions, Home and Foreign consumption fall, while RoW increases.

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[14] We provide a quantitative assessment of the three-country version of our model in our follow-up work, Ghironi, Kim, and Ozhan (2024).

[15] See Ghironi, Kim, and Ozhan (2024) for the details for three country model setup.

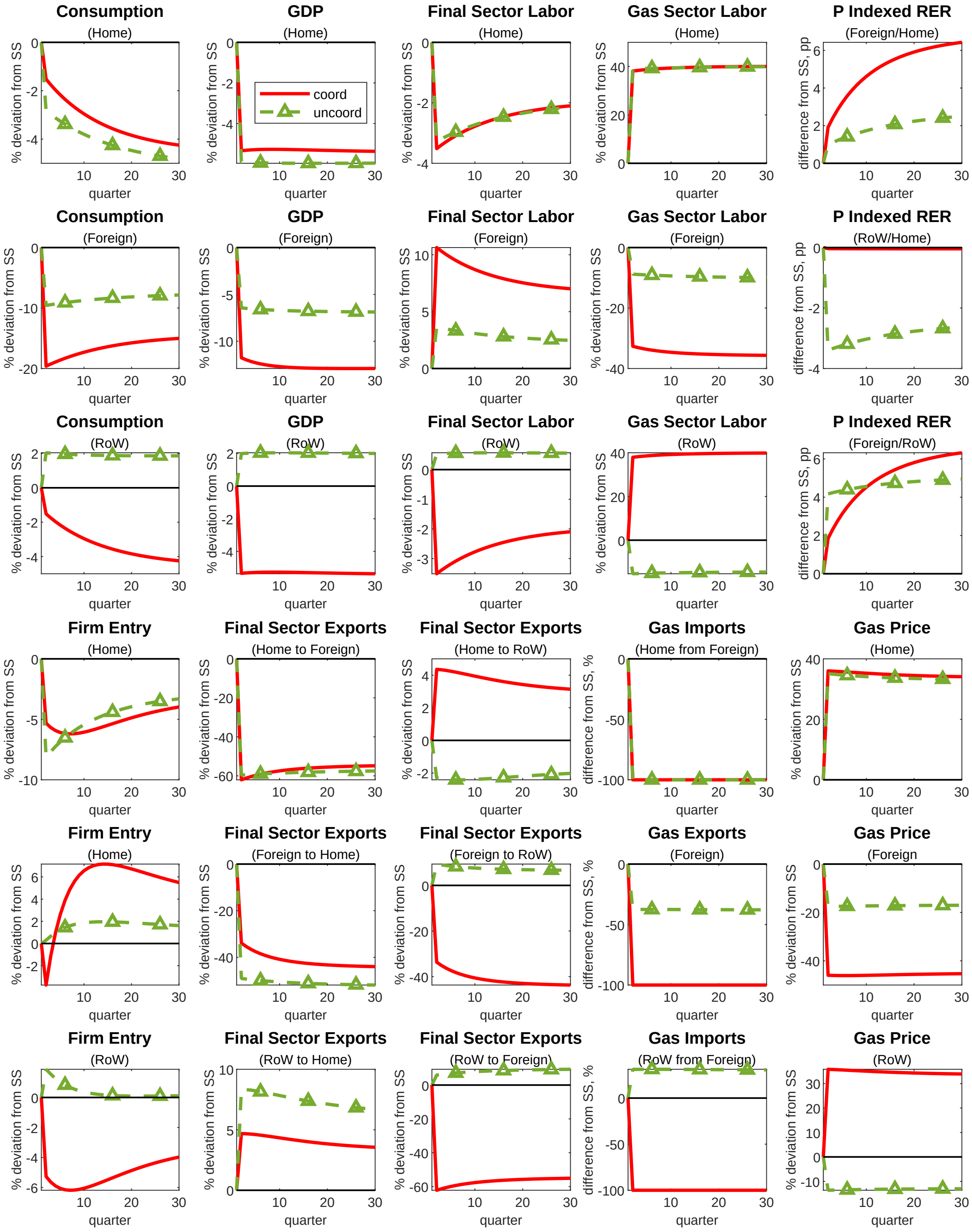

Figure 11: Transition Dynamics from the Three-Country Model with or without Coordination

Notes: The red solid lines (labeled 'coord.') plot the three-country model transition dynamics when all sanctions are imposed by Home and RoW at $t = 1$. The green dashed lines (labeled 'uncoord.') with triangles plot the three-country model transition dynamics when all sanctions are imposed by Home at $t = 1$, while RoW does not participate in sanctions. All deviations, except for the figures titled 'P indexed RER,' are in units of percent deviation from the initial steady-state without sanctions ($t = 0$), i.e., $100 \times (x_t/x_0 - 1)$. The figures of the ratio, titled 'P indexed RER,' are in units of percent point difference from the initial steady-state, i.e., $100 \times (x_t - x_0)$. See Figure A7 for the other variables' responses.

Table 3: Change in Welfare after Sanctions with International Coordination

	International	Welfare Gain, (%, consumption)		
Type of Sanctions	Coordination	Home	Foreign	RoW
Gas + Financial sanctions	Coordinated	–0.97	–3.16	–0.97
	Uncoordinated	–0.91	–1.73	0.75
C-good trade + Financial sanctions	Coordinated	–2.74	–13.30	–2.74
	Uncoordinated	–3.56	–6.85	1.15
Gas + C-good trade + Financial sanctions	Coordinated	–4.14	–15.13	–4.13
	Uncoordinated	–4.75	–7.88	1.89

Notes: The table presents the three-country model welfare (lifetime utility) gains of sanctions in terms of consumption, Δ defined in equation (44). Under the coordinated and uncoordinated sanctions, RoW participates and does not participate in sanctions imposed by Home, respectively. The Foreign welfare gain measures (Δ^*) are calculated from the weighted sum of financially sanctioned (S) and unsanctioned (NS) Foreign households' welfare ($\mathcal{U}_t^{s*} = \lambda \mathcal{U}_{S,t}^{s*} + (1-\lambda)\mathcal{U}_{NS,t}^{s*}$ and $\mathcal{U}_t^{*} = \lambda \mathcal{U}_{S,t}^{*} + (1-\lambda)\mathcal{U}_{NS,t}^{*}$ for $t = 0, 201$).

RoW GDP is also positively affected under uncoordinated sanctions because of substitution effects. RoW reallocates resources toward consumption good production to match the additional demand coming from Foreign after Home introduces sanctions. RoW expands exports to Foreign while increasing its gas imports from Foreign. Therefore, Home sanctions on Foreign generate a milder drop in Foreign consumption vis-a-vis the two-country model.

Under coordinated sanctions, the negative effects of sanctions on Home are dampened, while the effect on Foreign is amplified, nearly doubling the loss in Foreign consumption and GDP in both the short and long run. Coordinated sanctions also prove to be costly for RoW, as it similarly rebalances its economy toward gas production, mirroring Home's response. The identical responses of RoW to the sanctions stem from the assumed symmetry between Home and RoW in our calibration.

The effects of third country are seen most clearly when comparing welfare. Table 3 presents welfare changes from sanctions under coordinated and uncoordinated scenarios and for different combinations of sanctions. The immediate observation is in line with the analysis above: Coordinated sanctions generate more welfare losses in Foreign, but are costly for RoW. The effect of sanctions in Foreign is almost doubled under every scenario. In line with our analysis from the two-country model, consumption good trade sanctions generate the most welfare losses in Foreign. We observe that when there is no coordination between Home and RoW, consumption good trade sanctions can benefit RoW. Under the uncoordinated scenario, after Home introduces sanctions to Foreign, RoW welfare increases, accounting for the substitution of Foreign trade form Home to RoW. The main takeaway of this section is that coordinated sanctions result in the most notable welfare losses for the sanctioned country at lower costs for the sanctioning country, but the rest of the world has no economic incentive to coordinate.

7. Conclusions

We studied the effects of economic sanctions in the context of an international trade and macroeconomic model of countries that can have similar size. Our focus was on understanding how sanctions operate and the ripple effects they induce on relative prices, economic balances, and standard macroeconomic indicators. A distinctive aspect of our sanctions model is the implementation of sanctions as forced exit from the financial and goods markets, which sets it apart from traditional approaches that primarily manipulate the prices and adjustment costs of a given set of traded assets and goods at the intensive margin.

The transmission of sanctions in our model hinges at the extensive margin effects. For instance, when sanctions prohibit consumption goods exports from Home producers, the average price of Home exports rises because the most productive Home exporters must exit the Foreign market. Foreign households respond by shifting their demand to domestically produced goods, leading to a depreciation of the Home exchange rate. Conversely, gas sanctions, achieved by prohibiting the import of Foreign gas cause the Home real exchange rate to appreciate, because the Foreign economy reallocate resources to the consumption goods sector to export more goods. This results in an increase in the number of exporters in Foreign, but a decrease in their average productivity, driving up average export prices. As these examples illustrate, the exchange rate proves to be an unreliable metric for assessing the effectiveness of sanctions, as it can either depreciate or appreciate depending on the type of sanction and how economies rebalance resources between sectors.

The welfare analysis in our model reveals that consumption goods trade sanctions inflict the most significant welfare losses in the targeted country, because they force it to rebalance toward its less efficient sectors. Furthermore, extending our model to a three-country setting suggests that coordination with a third country when imposing significantly increases welfare losses in the targeted economy. However, this coordination comes at a cost for the third country, which loses the gains from substitution when sanctions are imposed only be one country. This result highlights the importance but also the difficulty of coordinating sanctions across countries.

The framework developed in this paper is amenable to extensions in several directions. The results contribute valuable insights to ongoing discussions on economic conflicts, geopolitical tensions, and their impacts on the global economy. Our analysis serves as a guide for comprehending the mechanisms underlying different types of sanctions. As a natural extension, future work could explore the effects of sanctions on the consolidated budget constraint of the targeted country's government sector, especially when fiscal revenues heavily rely on international trade. Other promising directions for future research include the study of optimal sanctions and responses and consideration of nominal rigidities and monetary policies. We plan to explore these issues in our future work. Economic sanctions and conflicts can induce inflation, making currency appreciation attractive to monetary policymakers. However, these same events can also lead to economic downturns, which may necessitate expansionary monetary policy responses. Additionally, long-lasting economic sanctions can influence neutral rates, inducing additional complexity to policymaking. We plan to explore these issues in our future work.

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Appendix

A. Mathematical Derivations

A.1. Gas Price Determination

Total demand for labor in the final Home sector can be written as $N_{D,t}L_t(\tilde{z}_D) + N_{X,t}L_t(\tilde{z}_{X,t})$. Since our assumptions are such that Home is the gas importer, total demand of gas by Home has to be equal to Home gas production ($G_N L_{G,t}$) plus Home imports of gas from Foreign ($G^*_{H,t}/\tau_{G,t}$). Optimal input demand conditions in Home then imply:

$$w_t \left[N_{D,t}l_t(\tilde{z}_D) + N_{X,t}l_t(\tilde{z}_{X,t})\right] = \left(\frac{1-\alpha}{\alpha}\right)\rho_{G,t}\left(G_N L_{G,t} + \frac{G^*_{H,t}}{\tau_{G,t}}\right). \tag{A1}$$

Using final sector production functions, this equation can be rewritten as:

$$\begin{aligned} & w_t \left\{ N_{D,t}\left[\frac{y_t(\tilde{z}_D)}{Z_t\tilde{z}_D}\right]^{\frac{1}{1-\alpha}}\left[g_t(\tilde{z}_D)\right]^{-\frac{\alpha}{1-\alpha}} + N_{X,t}\left[\frac{y_t(\tilde{z}_{X,t})}{Z_t\tilde{z}_{X,t}}\right]^{\frac{1}{1-\alpha}}\left[g_t(\tilde{z}_{X,t})\right]^{-\frac{\alpha}{1-\alpha}} \right\} \\ & \quad = \left(\frac{1-\alpha}{\alpha}\right)\rho_{G,t}\left(G_N L_{G,t} + \frac{G^*_{H,t}}{\tau_{G,t}}\right). \end{aligned} \tag{A2}$$

Next, note that optimal gas demand by a firm with productivity $\tilde{z}_D$ and market clearing for its output are such that $g_t(\tilde{z}_D) = [\alpha/(1-\alpha)]^{1-\alpha}(w_t/\rho_{G,t})^{1-\alpha}[y_t(\tilde{z}_D)]/(Z_t\tilde{z}_D)$ and $y_t(\tilde{z}_D) = [\rho_{H,t}(\tilde{z}_D)]^{-\theta}C_t$. Similarly, optimal gas demand by a firm with productivity $\tilde{z}_D$ and market clearing for its output satisfy $g_t(\tilde{z}_{X,t}) = [\alpha/(1-\alpha)]^{1-\alpha}(w_t/\rho_{G,t})^{1-\alpha}[y_t(\tilde{z}_{X,t})]/(Z_t\tilde{z}_{X,t})$ and $y_t(\tilde{z}_{X,t}) = [\rho_{H,t}(\tilde{z}_{X,t})]^{-\theta}C_t + \tau_t[\rho_{X,t}(\tilde{z}_{X,t})]^{-\theta}C^*_t$. Substituting these equations into equation (A2) and rearranging yields:

$$\begin{aligned} & \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha}\rho^{\alpha}_{G,t}w^{1-\alpha}_t\left\{N_{D,t}\frac{[\rho_{H,t}(\tilde{z}_D)]^{-\theta}C_t}{Z_t\tilde{z}_D} + N_{X,t}\frac{[\rho_{H,t}(\tilde{z}_{X,t})]^{-\theta}C_t + \tau_t[\rho_{X,t}(\tilde{z}_{X,t})]^{-\theta}C^*_t}{Z_t\tilde{z}_{X,t}}\right\} \\ = \; & \left(\frac{1-\alpha}{\alpha}\right)\rho_{G,t}\left(G_N L_{G,t} + \frac{G^*_{H,t}}{\tau_{G,t}}\right). \end{aligned} \tag{A3}$$

Optimal price setting by Home final sector firms and the expression for final sector marginal cost imply:

$$\rho_{H,t}(\tilde{z}_D) = \left(\frac{\theta}{\theta-1}\right)\frac{\rho_{G,t}^{\alpha} w_t^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}\tilde{z}_D Z_t}, \tag{A4}$$

$$\rho_{F,t}(\tilde{z}_{X,t}) = \frac{\tau_t}{Q_t}\frac{\tilde{z}_D}{\tilde{z}_{X,t}}\rho_{H,t}(\tilde{z}_D), \tag{A5}$$

$$\rho_{H,t}(\tilde{z}_{X,t}) = \frac{\tilde{z}_D}{\tilde{z}_{X,t}}\rho_{H,t}(\tilde{z}_D), \tag{A6}$$

Substituting equations (A4)–(A6) into equation (A3) and rearranging yields:

$$\begin{aligned}\alpha\left(\frac{\theta-1}{\theta}\right)^{\theta} Z_t^{\theta-1}\left\{N_{D,t}\tilde{z}_D^{\theta-1}C_t + N_{X,t}\tilde{z}_{X,t}^{\theta-1}\left[C_t + \left(\frac{\tau_t}{Q_t}\right)^{1-\theta} Q_t C_t^*\right]\right\}\left[\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\rho_{G,t}^{\alpha} w_t^{1-\alpha}}\right]^{\theta-1} \\ = \left(\frac{1-\alpha}{\alpha}\right)\rho_{G,t}\left(G_N L_{G,t} + \frac{G_{H,t}^*}{\tau_{G,t}}\right).\end{aligned} \tag{A7}$$

This equation can be solved for w_t as:

$$w_t = \left(\frac{\alpha}{\rho_{G,t}}\right)^{\frac{1+\alpha(\theta-1)}{(1-\alpha)(\theta-1)}}\left\{\frac{Z_t^{\theta-1}\left\{N_{D,t}\tilde{z}_D^{\theta-1}C_t + N_{X,t}\tilde{z}_{X,t}^{\theta-1}\left[C_t + (\tau_t/Q_t)^{1-\theta}Q_t C_t^*\right]\right\}}{[\theta/(\theta-1)]^{\theta}(G_N L_{G,t} + G_{H,t}^*/\tau_{G,t})}\right\}^{\frac{1}{(1-\alpha)(\theta-1)}}. \tag{A8}$$

Working in a similar way for the Foreign economy yields:

$$w_t^* = \left(\frac{\alpha\tau_{G,t}Q_t}{\rho_{G,t}}\right)^{\frac{1+\alpha(\theta-1)}{(1-\alpha)(\theta-1)}}\left\{\frac{Z_t^{*\theta-1}\left\{N_{D,t}^*\tilde{z}_D^{\theta-1}C_t^* + N_{X,t}^*\tilde{z}_{X,t}^{*\theta-1}\left[C_t^* + (\tau_t^* Q_t)^{1-\theta}C_t/Q_t\right]\right\}}{[\theta/(\theta-1)]^{\theta}G_N^* L_{G,t}^*}\right\}^{\frac{1}{(1-\alpha)(\theta-1)}}. \tag{A9}$$

To economize on notation in the following steps, rewrite the last two equations as:

$$w_t = f(\rho_{G,t}) \quad \text{and} \quad w_t^* = f^*(\rho_{G,t}). \tag{A10}$$

Equilibrium in the world market for gas requires total supply to be equal to demand. Hence, using production functions and optimal demand conditions:

$$\begin{aligned} & G_N L_{G,t} + G_N^* L_{G,t}^* \\ & = N_{D,t} \left(\frac{1-\alpha}{\alpha} \frac{w_t}{\rho_{G,t}} \right)^{1-\alpha} \frac{y_t(\tilde{z}_D)}{\tilde{z}_D Z_t} + N_{X,t} \left(\frac{1-\alpha}{\alpha} \frac{w_t}{\rho_{G,t}} \right)^{1-\alpha} \frac{\tau_t y_t(\tilde{z}_{X,t})}{\tilde{z}_{X,t} Z_t} \\ & \quad + N_{D,t}^* \left(\frac{1-\alpha}{\alpha} \frac{w_t^*}{\rho_{G,t}^*} \right)^{1-\alpha} \frac{y_t^*(\tilde{z}_D)}{\tilde{z}_D Z_t^*} + N_{X,t}^* \left(\frac{1-\alpha}{\alpha} \frac{w_t^*}{\rho_{G,t}^*} \right)^{1-\alpha} \frac{\tau_t^* y_t^*(\tilde{z}_{X,t}^*)}{\tilde{z}_{X,t}^* Z_t^*}. \end{aligned} \tag{A11}$$

Optimal labor supply for Home and Foreign gas production is given by, respectively:

$$L_{G,t} = \frac{w_{G,t}}{\kappa_G C_t} \quad \text{and} \quad L_{G,t}^* = \frac{w_{G,t}^*}{\kappa_G^* C_t^*}. \tag{A12}$$

Therefore, it is:

$$L_{G,t} = \frac{\rho_{G,t} G_N}{\kappa_G C_t}, \tag{A13}$$

and:

$$L_{G,t}^* = \frac{\rho_{G,t} G_N^*}{\kappa_G^* \tau_{G,t} Q_t C_t^*}. \tag{A14}$$

Substituting $w_t = f(\rho_{G,t})$, $w_t^* = f^*(\rho_{G,t})$, and equations (A13) and (A14) into equation (A11), using market clearing conditions for Home and Foreign final sector products, and rearranging yields:

$$\begin{aligned} & \rho_{G,t} \left(\frac{G_N^2}{\kappa_G C_t} + \frac{G_N^{*2}}{\tau_{G,t} \kappa_G^* Q_t C_t^*} \right) \\ = & N_{D,t} \left[\frac{\alpha}{1-\alpha} \frac{f(\rho_{G,t})}{\rho_{G,t}} \right]^{1-\alpha} \left[\tilde{z}_D Z_t \rho_{H,t}(\tilde{z}_D) \right]^{-\theta} \left(\tilde{z}_D Z_t \right)^{\theta-1} C_t \\ & + N_{X,t} \left[\frac{\alpha}{1-\alpha} \frac{f(\rho_{G,t})}{\rho_{G,t}} \right]^{1-\alpha} \tau_t \left[\tilde{z}_D Z_t \rho_{H,t}(\tilde{z}_D) \right]^{-\theta} \left(\tilde{z}_{X,t} Z_t \right)^{\theta-1} \left[C_t + \left(\frac{\tau_t}{Q_t} \right)^{1-\theta} Q_t C_t^* \right] \\ & + N_{D,t}^* \left[\frac{\alpha}{1-\alpha} \frac{f(\rho_{G,t})/X_t}{\rho_{G,t}/\left(\tau_{G,t} Q_t\right)} \right]^{1-\alpha} \left[\tilde{z}_D Z_t^* \rho_{F,t}^*(\tilde{z}_D) \right]^{-\theta} \left(\tilde{z}_D Z_t^* \right)^{\theta-1} C_t^* \\ & + N_{X,t}^* \left[\frac{\alpha}{1-\alpha} \frac{f(\rho_{G,t})/X_t}{\rho_{G,t}/\left(\tau_{G,t} Q_t\right)} \right]^{1-\alpha} \tau_t^* \left[\tilde{z}_D Z_t^* \rho_{F,t}^*(\tilde{z}_D) \right]^{-\theta} \left(\tilde{z}_{X,t}^* Z_t^* \right)^{\theta-1} \left[C_t^* + \left(\tau_t^* Q_t \right)^{1-\theta} \frac{C_t}{Q_t} \right], \end{aligned} \tag{A15}$$

In this equation, we used the fact that $f^*(\rho_{G,t}) = f(\rho_{G,t})/X_t$, with:

$$\begin{aligned} & X_t^{(1-\alpha)(\theta-1)} \left(\tau_{G,t} Q_t\right)^{1+\alpha(\theta-1)} \\ &= \left(\frac{Z_t}{Z_t^*}\right)^{\theta-1} \frac{N_{D,t}\tilde{z}_D^{\theta-1} C_t + N_{X,t}\tilde{z}_{X,t}^{\theta-1}\left[C_t + (\tau_t/Q_t)^{1-\theta} Q_t C_t^*\right]}{N_{D,t}^*\tilde{z}_D^{\theta-1} C_t^* + N_{X,t}^*\tilde{z}_{X,t}^{*\theta-1}\left[C_t^* + (\tau_t^* Q_t)^{1-\theta} C_t/Q_t\right]} \left(\frac{G_N^* L_{G,t}^*}{G_N L_{G,t} + G_{H,t}^*/\tau_{G,t}}\right). \end{aligned} \tag{A16}$$

Notice that X_t does not depend directly on $\rho_{G,t}$.

The expressions for optimal $\rho_{H,t}(\tilde{z}_D)$ and $\rho_{F,t}^*(\tilde{z}_D)$ and tedious manipulation then make it possible to rewrite equation (A15) as:

$$\begin{aligned} & \rho_{G,t}\left[\frac{G_N}{\kappa_G C_t} + \left(\frac{G_N^*}{G_N}\right)\frac{G_N^*}{\tau_{G,t}\kappa_G^* Q_t C_t^*}\right] \\ =& \frac{\alpha}{G_N \rho_{G,t}}\left(\frac{\theta}{\theta-1}\right)^{-\theta}\left[\frac{\alpha^\alpha (1-\alpha)^{1-\alpha}}{\rho_{G,t}^\alpha f(\rho_{G,t})^{1-\alpha}}\right]^{\theta-1} \\ & \times \left\{ \begin{array}{l} N_{D,t}\left(\tilde{z}_D Z_t\right)^{\theta-1} C_t + N_{X,t}\tau_t\left(\tilde{z}_{X,t} Z_t\right)^{\theta-1}\left[C_t + (\tau_t/Q_t)^{1-\theta} Q_t C_t^*\right] \\ +\tau_{G,t} Q_t\left[\left(\tau_{G,t} Q_t\right)^\alpha X_t^{1-\alpha}\right]^{\theta-1} \left[\begin{array}{l} N_{D,t}^*\left(\tilde{z}_D Z_t^*\right)^{\theta-1} C_t^* + \\ N_{X,t}^*\tau_t^*\left(\tilde{z}_{X,t}^* Z_t^*\right)^{\theta-1}\left[C_t^* + (\tau_t^* Q_t)^{1-\theta} C_t/Q_t\right] \end{array} \right] \end{array} \right\}. \end{aligned} \tag{A17}$$

Equation (A8) implies:

$$f(\rho_{G,t})^{-(1-\alpha)(\theta-1)} = \left(\frac{\alpha}{\rho_{G,t}}\right)^{-[1+\alpha(\theta-1)]}\left(\frac{\theta}{\theta-1}\right)^\theta \frac{G_N L_{G,t} + G_{H,t}^*/\tau_{G,t}}{N_{D,t} Z_t^{\theta-1} A_t}. \tag{A18}$$

where $A_t \equiv \tilde{z}_D^{\theta-1} C_t + \tau_t (\nu z_{\min})^k \tilde{z}_{X,t}^{-[k-(\theta-1)]}[C_t + (\tau_t/Q_t)^{1-\theta} Q_t C_t^*]$. In this expression, we used the relation between $N_{X,t}$ and $N_{D,t}$ implied by the assumption of a Pareto distribution of firm-specific productivity draws: $N_{X,t} = (\nu z_{\min}/\tilde{z}_{X,t})^k N_{D,t}$.

It is also possible to verify that:

$$X_t^{(1-\alpha)(\theta-1)} = \left(\tau_{G,t} Q_t\right)^{-[1+\alpha(\theta-1)]}\left(\frac{N_{D,t} Z_t^{\theta-1} A_t}{G_N L_{G,t} + G_{H,t}^*/\tau_{G,t}}\right)\left(\frac{G_N^* L_{G,t}^*}{N_{D,t}^* Z_t^{*\theta-1} A_t^*}\right), \tag{A19}$$

where: $A_t^* \equiv \tilde{z}_D^{\theta-1} C_t^* + \tau_t^* (\nu z_{\min})^k \tilde{z}_{X,t}^{*-[k-(\theta-1)]}[C_t^* + (\tau_t^* Q_t)^{1-\theta} C_t/Q_t]$. Equation (A17) can be rewritten as:

$$\begin{aligned} & \rho_{G,t}\left[\frac{G_N}{\kappa_G C_t}+\left(\frac{G_N^*}{G_N}\right)\frac{G_N^*}{\tau_{G,t}\kappa_G^* Q_t C_t^*}\right] \\ = \quad & \frac{\alpha}{G_N\rho_{G,t}}\left(\frac{\theta}{\theta-1}\right)^{-\theta}\left[\frac{\alpha^\alpha(1-\alpha)^{1-\alpha}}{\rho_{G,t}^\alpha}\right]^{\theta-1} f(\rho_{G,t})^{-(1-\alpha)(\theta-1)} \\ & \times\left[N_{D,t}Z_t^{\theta-1}A_t+(\tau_{G,t}Q_t)^{1+\alpha(\theta-1)}\,X_t^{(1-\alpha)(\theta-1)}N_{D,t}^*Z_t^{*\theta-1}A_t^*\right]. \end{aligned} \tag{A20}$$

Then, substituting equations (A18) and (A19) into equation (A20) yields:

$$\rho_{G,t}\left[\frac{G_N}{\kappa_G C_t}+\left(\frac{G_N^*}{G_N}\right)\frac{G_N^*}{\tau_{G,t}\kappa_G^* Q_t C_t^*}\right]=\frac{(1-\alpha)^{(1-\alpha)(\theta-1)}}{G_N}\left(1+\frac{G_N^*L_{G,t}^*}{G_N L_{G,t}+G_{H,t}^*/\tau_{G,t}}\right). \tag{A21}$$

Finally, using $L_{G,t}=G_N\rho_{G,t}/(\kappa_G C_t)$ and $L_{G,t}^*=G_N^*\rho_{G,t}/(\tau_{G,t}\kappa_G^* Q_t C_t^*)$ and rearranging gives us:

$$\begin{aligned} & \rho_{G,t}\frac{G_N}{\kappa_G C_t}\left[1+\left(\frac{G_N^*}{G_N}\right)^2\left(\frac{\kappa_G}{\kappa_G^*\tau_{G,t}}\right)\left(\frac{C_t}{Q_t C_t^*}\right)\right] \\ = & \frac{(1-\alpha)^{(1-\alpha)(\theta-1)}}{G_N}\left[1+\left(\frac{G_N^*}{G_N}\right)^2\left(\frac{\kappa_G}{\kappa_G^*\tau_{G,t}}\right)\left(\frac{C_t}{Q_t C_t^*}\right)\left(\frac{\rho_{G,t}}{\rho_{G,t}+\kappa_G\tau_{G,t}^{-1}G_N^{-2}C_t G_{H,t}^*}\right)\right]. \end{aligned} \tag{A22}$$

Home imports of Foreign gas are given by:

$$G_{H,t}^*=G_N^*L_{G,t}^*-G_{F,t}^*=\frac{\rho_{G,t}G_N^{*2}}{\kappa_G^*\tau_{G,t}Q_t C_t^*}-G_{F,t}^*, \tag{A23}$$

where the second equality follows from using equation (A14).

Optimal input demands by Foreign final sector firms and the relation $N_{X,t}^*=(\nu z_{\min}/\tilde{z}_{X,t}^*)^k N_{D,t}^*$ imply:

$$G_{F,t}^*=N_{D,t}^*\left(\frac{\alpha}{1-\alpha}\frac{w_t^*}{\rho_{G,t}^*}\right)^{1-\alpha}\left[\frac{y_t^*(\tilde{z}_D)}{\tilde{z}_D Z_t^*}+\left(\frac{\nu z_{\min}}{\tilde{z}_{X,t}^*}\right)^k\tau_t^*\frac{y_t^*(\tilde{z}_{X,t}^*)}{\tilde{z}_{X,t}^*Z_t^*}\right]. \tag{A24}$$

Substituting market clearing conditions for Foreign final sector products and optimal price setting by Foreign firms into equation (A24) yields:

$$\begin{aligned} G_{F,t}^*= & \left(\frac{\theta-1}{\theta}\right)^\theta N_{D,t}^*\left(\frac{\alpha Q_t\tau_{G,t}}{\rho_{G,t}}\right)^{1+\alpha(\theta-1)}\left(\frac{1-\alpha}{w_t^*}\right)^{(1-\alpha)(\theta-1)}Z_t^{*\theta-1} \\ & \times\left\{\tilde{z}_D^{\theta-1}C_t^*+\tau_t^*(\nu z_{\min})^k\tilde{z}_{X,t}^{*-[k-(\theta-1)]}\left[C_t^*+(\tau_t^*Q_t)^{1-\theta}\frac{C_t}{Q_t}\right]\right\}. \end{aligned} \tag{A25}$$

Finally, substituting $N^*_{X,t} = (\nu z_{\min}/\tilde{z}^*_{X,t})^k N^*_{D,t}$ and equation (A14) into equation (A9), and plugging the resulting expression for w^*_t into equation (A25) makes it possible to obtain:

$$G^*_{F,t} = (1-\alpha)^{(1-\alpha)(\theta-1)} \frac{G^{*2}_N \rho_{G,t}}{\tau_{G,t}\kappa^*_G Q_t C^*_t}. \tag{A26}$$

Equations (A23) and (A26) then imply:

$$G^*_{H,t} = \left[1-(1-\alpha)^{(1-\alpha)(\theta-1)}\right] \frac{G^{*2}_N \rho_{G,t}}{\tau_{G,t}\kappa^*_G Q_t C^*_t}. \tag{A27}$$

This expression can be substituted into equation (A22). Then, rearranging the resulting equation and defining $\xi_t \equiv (G^*_N/G_N)^2[\kappa_G/(\kappa^*_G\tau_{G,t})[C_t/(Q_t C^*_t)]$, we have:

$$\rho_{G,t} = \frac{(1-\alpha)^{(1-\alpha)(\theta-1)}\kappa_G C_t}{G^2_N} \left\{ \frac{1+\xi_t+\tau^{-1}_{G,t}\left[1-(1-\alpha)^{(1-\alpha)(\theta-1)}\right]\xi_t}{\left[1+\tau^{-1}_{G,t}\left[1-(1-\alpha)^{(1-\alpha)(\theta-1)}\right]\xi_t\right](1+\xi_t)} \right\}. \tag{A28}$$

A.2. Gas Price and Gas Share

The gas price equation in the special case of complete markets, $\tau_{G,t}=1$, $G_N = G^*_N$, and $\kappa_G = \kappa^*_G$ is reproduced below for your convenience:

$$\rho_{G,t} = \frac{(1-\alpha)^{(1-\alpha)(\theta-1)}\kappa_G C_t}{G^2_N} \left\{ \frac{3-(1-\alpha)^{(1-\alpha)(\theta-1)}}{2\left[2-(1-\alpha)^{(1-\alpha)(\theta-1)}\right]} \right\}. \tag{A29}$$

Let $\psi \equiv (1-\alpha)^{(1-\alpha)(\theta-1)}$. The derivative of ψ with respect to α is given by:

$$\frac{\partial\psi}{\partial\alpha} = -(\theta-1)(1-\alpha)^{(1-\alpha)(\theta-1)}[1+\log(1-\alpha)]. \tag{A30}$$

Now, let $\Lambda_t \equiv \kappa_G C_t/G^2_N$. Then, equation (A29) can be rewritten as:

$$\rho_{G,t} = \psi\Lambda_t\left[\frac{3-\psi}{2(2-\psi)}\right]. \tag{A31}$$

Our interest is in determining how $\psi(3-\psi)/[2(2-\psi)]$ varies with α. Taking the derivative and rearranging yields:

$$\frac{\partial\psi\left[\frac{3-\psi}{2(2-\psi)}\right]}{\partial\alpha} = \left[\frac{2(3-2\psi)+\psi^2}{2(2-\psi)^2}\right]\frac{\partial\psi}{\partial\alpha}. \tag{A32}$$

The definition of ψ, $0 \leq \alpha \leq 1$, and $\theta > 1$ imply $3 > 2\psi$. Thus, the sign of the derivative we are interested in is determined by the sign of $\partial\psi/\partial\alpha$. Since $\theta > 1$, the sign of $\partial\psi/\partial\alpha$ depends on the sign of $[1 + \log(1-\alpha)]$. This expression is a monotonically decreasing function of α. It is positive if α is smaller than (approximately) 0.63. It is negative if α is higher than this number. It follows that $\psi(3-\psi)/[2(2-\psi)]$ is a monotonically decreasing function of α if $0 \leq \alpha \leq 0.63$, and it increases with α if $0.63 < \alpha \leq 1$. Since $\psi(3-\psi)/[2(2-\psi)] = 1$ when $\alpha = 0$ and $\alpha = 1$, the relation between the price of gas and its share in production of final goods when markets are complete, countries are fully symmetric, and there is no iceberg cost of gas trade is U-shaped.

A.3. The Log-Linear Gas Price Equation

The non-linear equation for the gas price $\rho_{G,t}$ is reproduced below for your convenience:

$$\rho_{G,t} = \frac{(1-\alpha)^{(1-\alpha)(\theta-1)}\kappa_G C_t}{G_N^2} \left\{ \frac{1 + \xi_t + \tau_{G,t}^{-1}\left[1-(1-\alpha)^{(1-\alpha)(\theta-1)}\right]\xi_t}{\left[1 + \tau_{G,t}^{-1}\left[1-(1-\alpha)^{(1-\alpha)(\theta-1)}\right]\xi_t\right](1+\xi_t)} \right\}. \tag{A33}$$

Taking logarithms and applying first-order Taylor approximation, the left hand side of the above equation yields

$$\log \bar{\rho}_G + \frac{1}{\bar{\rho}_G}(\rho_{G,t} - \bar{\rho}_G) \equiv \log \bar{\rho}_G + \hat{\rho}_{G,t}, \tag{A34}$$

where $\hat{\rho}_{G,t}$ is the percentage deviation of $\rho_{G,t}$ from the steady-state.

Proceeding similarly, taking logs and applying first-order Taylor approximation, the right hand side of equation (A33) becomes

$$\log \bar{\rho}_G + \mathsf{C}_t + \frac{(1+\bar{\tau}_G^{-1}A)(1+\bar{\tau}_G^{-1}A\bar{\xi})(1+\bar{\xi})\bar{\xi} - (1+\bar{\xi}+\bar{\tau}_G^{-1}A\bar{\xi})\left[\bar{\tau}_G^{-1}A(1+\bar{\xi}) + (1+\bar{\tau}_G^{-1}A\bar{\xi})\right]\bar{\xi}}{\left[(1+\bar{\tau}_G^{-1}A\bar{\xi})(1+\bar{\xi})\right]^2}\hat{\xi}_t, \tag{A35}$$

where $A \equiv 1-(1-\alpha)^{(1-\alpha)(\theta-1)}$, C_t is the percentage deviation of C_t from the steady-state: $\mathsf{C}_t \equiv \mathrm{d}C_t/\bar{C}$, d is the differentiation operator, and Sans Serif variables in equations below are defined similarly.

It is trivial to show that $\hat{\xi}_t = \mathsf{C}_t - \mathsf{Q}_t - \mathsf{C}_t^*$. Thus;

$$\hat{\rho}_{G,t} = \mathsf{C}_t + (\Gamma_1 - \Gamma_2)(\mathsf{C}_t - \mathsf{Q}_t - \mathsf{C}_t^*) \tag{A36}$$

with

$$\Gamma_1 \equiv \frac{(1+\bar{\tau}_G^{-1}A)(1+\bar{\tau}_G^{-1}A\bar{\xi})(1+\bar{\xi})\bar{\xi}}{(1+\bar{\xi}+\bar{\tau}_G^{-1}A\bar{\xi})(1+\bar{\xi})(1+\bar{\tau}_G^{-1}A\bar{\xi})} \quad \text{and} \quad \Gamma_2 \equiv \frac{\left[\bar{\tau}_G^{-1}A(1+\bar{\xi})+(1+\bar{\tau}_G^{-1}A\bar{\xi})\right]\bar{\xi}}{(1+\bar{\tau}_G^{-1}A\bar{\xi})(1+\bar{\xi})}.$$

It follows that $\hat{\rho}_{G,t}$ is positively related with both C_t and $\mathsf{Q}_t + \mathsf{C}_t^*$ if and only if $-1 < \Gamma_1 - \Gamma_2 < 0$.

A.3.1. On $\Gamma_1$and Γ_2

$\Gamma_1 - \Gamma_2 > -1$ if and only if:

$$-(1+\bar{\xi}+\bar{\tau}_G^{-1}A\bar{\xi}) < \bar{\xi}(1+\bar{\tau}_G^{-1}A\bar{\xi}) - \frac{\bar{\tau}_G^{-1}A\bar{\xi}(1+\bar{\xi}+\bar{\tau}_G^{-1}A\bar{\xi})}{1+\bar{\tau}_G^{-1}A\bar{\xi}} - \frac{\bar{\xi}(1+\bar{\xi}+\bar{\tau}_G^{-1}A\bar{\xi})}{1+\bar{\xi}} \tag{A37}$$

Tedious algebra shows that this inequality implies:

$$-1 < \bar{\xi}\left[1-(1+\bar{\xi}+\bar{\tau}_G^{-1}A\bar{\xi})(1+2\bar{\tau}_G^{-1}A\bar{\xi})-(\bar{\tau}_G^{-1}A)^2\bar{\xi}\right]. \tag{A38}$$

The term in the squared brackets is negative and scales $\bar{\xi}$ with less than unity for the parameterization in the paper.[16] $\Gamma_1 - \Gamma_2 < 0$ if and only if:

$$(\bar{\xi}\bar{\tau}_G^{-1}A)^2 + (1+\bar{\xi}+\bar{\tau}_G^{-1}A\bar{\xi})(1+2\bar{\tau}_G^{-1}A\bar{\xi}) > 0 \tag{A39}$$

which is ensured for $A, \bar{\xi}, \bar{\tau}_G > 0$.

A.4. The Log-Linear Real Exchange Rate Equation

The non-linear equation for the data-consistent real exchange rate $\tilde{Q}_t$ is reproduced below for your convenience:

$$\tilde{Q}_t^{1-\theta} = \frac{\frac{N_{D,t}^*}{N_t^*}\left[TOL_t^{1-\alpha}\left(\frac{Z_t}{\tau_{G,t}Z_t^*}\right)^{\alpha}\right]^{1-\theta} + \frac{N_{X,t}}{N_t^*}\left[\frac{\tau_t\tilde{z}_D}{\tilde{z}_{X,t}}\right]^{1-\theta}}{\frac{N_{D,t}}{N_t} + \frac{N_{X,t}^*}{N_t}\left[TOL_t^{1-\alpha}\left(\frac{Z_t}{\tau_{G,t}Z_t^*}\right)^{\alpha}\frac{\tau_t^*\tilde{z}_D}{\tilde{z}_{X,t}^*}\right]^{1-\theta}}. \tag{A40}$$

Let NUM_t denote the numerator of this equation and DEN_t the denominator. Then, the

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[16] Taking a log on the definition of $\bar{\xi}$, we obtain $\log\bar{\xi} = 2\log(G^*/G_N) - \log(\kappa_G^*/\kappa_G) - \log\bar{\tau}_G - \log(\bar{Q}\bar{C}^*/\bar{C})$. Here, we assume that $\log(G^*/G_N) > 0$, and also $\log(\kappa_G^*/\kappa_G) = 0$. By the definition of gas trade costs, $\log\bar{\tau}_G > 0$. In the last term, $\log(\bar{Q}\bar{C}^*/\bar{C})$ is negative because of $Z^* < Z$.

log-linear version of equation (A40) can be written as:

$$\tilde{\mathsf{Q}}_t = \frac{\overline{NUM}\cdot \mathrm{d}DEN_t - \overline{DEN}\cdot \mathrm{d}NUM_t}{(\theta-1)\overline{NUMDEN}} \tag{A41}$$

Assume $\bar{Z}=\bar{Z}^*=1$. Differentiating NUM_t and using the definitions of log-linearized variables yields:

$$\begin{aligned}\mathrm{d}NUM_t =& \frac{\bar{N}_D^*}{\bar{N}^*}\left(\mathsf{N}_{\mathsf{D,t}}^*-\mathsf{N}_{\mathsf{t}}^*\right)\left(\overline{TOL}^{1-\alpha}\bar{\tau}_G^{-\alpha}\right)^{1-\theta}\\ &+(1-\theta)\frac{\bar{N}_D^*}{\bar{N}^*}\left[\mathsf{TOL}_{\mathsf{t}}\overline{TOL}^{1-\alpha}\bar{\tau}_G^{-\alpha}+\alpha\overline{TOL}^{1-\alpha}\bar{\tau}_G^{-\alpha}(\mathsf{Z}_{\mathsf{t}}-\mathsf{Z}_{\mathsf{t}}^*-\hat{\tau}_{G,t})\right](\overline{TOL}^{1-\alpha}\bar{\tau}_G^{-\alpha})^{-\theta}\\ &+\frac{\bar{N}_X}{\bar{N}^*}\left(\mathsf{N}_{\mathsf{X,t}}-\mathsf{N}_{\mathsf{t}}^*\right)\left(\frac{\bar{\tau}\tilde{z}_D}{\bar{\tilde{z}}_X}\right)^{1-\theta}+(1-\theta)\frac{\bar{N}_X}{\bar{N}^*}\frac{\bar{\tau}\tilde{z}_D}{\bar{\tilde{z}}_X}(\hat{\tau}_t-\tilde{\mathsf{z}}_{\mathsf{X,t}})\left(\frac{\bar{\tau}\tilde{z}_D}{\bar{\tilde{z}}_X}\right)^{-\theta},\end{aligned} \tag{A42}$$

or, after rearranging:

$$\begin{aligned}\mathrm{d}NUM_t =& \frac{\bar{N}_D^*}{\bar{N}^*}\left(\overline{TOL}^{1-\alpha}\bar{\tau}_G^{-\alpha}\right)^{1-\theta}\left\{\mathsf{N}_{\mathsf{D,t}}^*-\mathsf{N}_{\mathsf{t}}^*+(1-\theta)[(1-\alpha)\mathsf{TOL}_{\mathsf{t}}+\alpha(\mathsf{Z}_{\mathsf{t}}-\mathsf{Z}_{\mathsf{t}}^*-\hat{\tau}_{G,t})]\right\}\\ &+\frac{\bar{N}_X}{\bar{N}^*}\left(\frac{\bar{\tau}\tilde{z}_D}{\bar{\tilde{z}}_X}\right)^{1-\theta}\left[\mathsf{N}_{\mathsf{X,t}}-\mathsf{N}_{\mathsf{t}}^*+(1-\theta)(\hat{\tau}_t-\tilde{\mathsf{z}}_{\mathsf{X,t}})\right].\end{aligned} \tag{A43}$$

Proceeding similarly with DEN_t yields:

$$\begin{aligned}\mathrm{d}DEN_t =& \frac{\bar{N}_D}{\bar{N}}\left(\mathsf{N}_{\mathsf{D,t}}-\mathsf{N}_{\mathsf{t}}\right)+\frac{\bar{N}_X^*}{\bar{N}}\left(\overline{TOL}^{1-\alpha}\bar{\tau}_G^{-\alpha}\frac{\bar{\tau}^*\tilde{z}_D}{\bar{\tilde{z}}_X^*}\right)^{1-\theta}\left(\mathsf{N}_{\mathsf{X,t}}^*-\mathsf{N}_{\mathsf{t}}\right)\\ &+\frac{\bar{N}_X^*}{\bar{N}}\left(\overline{TOL}^{1-\alpha}\bar{\tau}_G^{-\alpha}\frac{\bar{\tau}^*\tilde{z}_D}{\bar{\tilde{z}}_X^*}\right)^{1-\theta}(1-\theta)\left[\hat{\tau}_t^*-\tilde{\mathsf{z}}_{\mathsf{X,t}}^*+(1-\alpha)\mathsf{TOL}_{\mathsf{t}}+\alpha(\mathsf{Z}_{\mathsf{t}}-\mathsf{Z}_{\mathsf{t}}^*-\hat{\tau}_{G,t})\right].\end{aligned} \tag{A44}$$

Let the parameters, χ_1, χ_2, and γ be defined implicitly by: $\bar{N}_D^*/\bar{N}^*=\chi_1\bar{N}_D/\bar{N}$, $\bar{N}_X/\bar{N}^*=\gamma\chi_1\bar{N}_X^*/\bar{N}$, and $(\bar{\tau}\tilde{z}_D/\bar{\tilde{z}}_X)^{1-\theta}=(\chi_2\bar{\tau}^*\tilde{z}_D/\bar{\tilde{z}}_X^*)^{1-\theta}$. Then, equation (A43) can be written as:

$$\begin{aligned}\mathrm{d}NUM_t =& \chi_1\frac{\bar{N}_D}{\bar{N}}\left(\overline{TOL}^{1-\alpha}\bar{\tau}_G^{-\alpha}\right)^{1-\theta}\left\{\mathsf{N}_{\mathsf{D,t}}^*-\mathsf{N}_{\mathsf{t}}^*+(1-\theta)[(1-\alpha)\mathsf{TOL}_{\mathsf{t}}+\alpha(\mathsf{Z}_{\mathsf{t}}-\mathsf{Z}_{\mathsf{t}}^*-\hat{\tau}_{G,t})]\right\}\\ &+\gamma\chi_1\frac{\bar{N}_X^*}{\bar{N}}\left(\chi_2\frac{\tau^*\tilde{z}_D}{\bar{\tilde{z}}_X^*}\right)^{1-\theta}\left[\mathsf{N}_{\mathsf{X,t}}-\mathsf{N}_{\mathsf{t}}^*+(\theta-1)(\hat{\tau}_t-\tilde{\mathsf{z}}_{\mathsf{X,t}})\right].\end{aligned} \tag{A45}$$

Substituting equations (A44) and (A45) and the expressions for $\overline{NUM}$ and $\overline{DEN}$ into $\overline{NUM}\cdot\mathrm{d}DEN_t-\overline{DEN}\cdot\mathrm{d}NUM_t$ (the numerator of the expression for $\tilde{\mathsf{Q}}_t$ in equation A41), and rearranging yields:

$$\begin{aligned}
&\overline{NUM}\cdot \mathrm{d}DEN_t-\overline{DEN}\cdot \mathrm{d}NUM_t\\
&=(\theta-1)(\zeta_1-\zeta_2)[(1-\alpha)\mathsf{TOL}_t+\alpha(\mathsf{Z_t}-\mathsf{Z_t^*}-\hat{\tau}_{G,t})]\\
&\quad+(\theta-1)\left[(\zeta_2+\zeta_4)(\tilde{z}^*_{X,t}-\hat{\tau}^*_t)-(\zeta_2+\zeta_3)(\tilde{z}_{X,t}-\hat{\tau}_t)\right]\\
&\quad+\zeta_1[\mathsf{N_{D,t}}-\mathsf{N_t}-(\mathsf{N^*_{D,t}}-\mathsf{N^*_t})]+\zeta_2[\mathsf{N^*_{X,t}}-\mathsf{N_t}-(\mathsf{N_{X,t}}-\mathsf{N^*_t})]\\
&\quad-\zeta_3[\mathsf{N_{X,t}}-\mathsf{N^*_t}-(\mathsf{N_{D,t}}-\mathsf{N_t})]+\zeta_4[\mathsf{N^*_{X,t}}-\mathsf{N_t}-(\mathsf{N^*_{D,t}}-\mathsf{N^*_t})
\end{aligned}\tag{A46}$$

where

$$\begin{aligned}
\zeta_1 &\equiv \chi_1\left(\frac{\bar{N}_D}{\bar{N}}\right)^2\left(\overline{TOL}^{1-\alpha}\bar{\tau}_G^{-\alpha}\right)^{1-\theta}>0,\\
\zeta_2 &\equiv \gamma\chi_1\left(\frac{\bar{N}^*_X}{\bar{N}}\right)^2\left(\overline{TOL}^{1-\alpha}\bar{\tau}_G^{-\alpha}\chi_2\right)^{1-\theta}\left(\frac{\bar{\tau}^*\tilde{z}_D}{\bar{\tilde{z}}^*_X}\right)^{2(1-\theta)}>0,\\
\zeta_3 &\equiv \gamma\chi_1\frac{\bar{N}_D\bar{N}^*_X}{\bar{N}^2}\left(\chi_2\frac{\bar{\tau}^*\tilde{z}_D}{\bar{\tilde{z}}^*_X}\right)^{1-\theta}>0,\\
\zeta_4 &\equiv \chi_1\frac{\bar{N}_D\bar{N}^*_X}{\bar{N}^2}\left(\frac{\bar{\tau}^*\tilde{z}_D}{\bar{\tilde{z}}^*_X}\right)^{1-\theta}\left(\overline{TOL}^{1-\alpha}\bar{\tau}_G^{-\alpha}\right)^{2(1-\theta)}>0.
\end{aligned}$$

To To demonstrate that $\zeta_1>\zeta_2$ when $\bar{\tau}\bar{\tau}^*>1.$ we start by expressing the ratio of ζ_1 to ζ_2 as follows:

$$\frac{\zeta_1}{\zeta_2}=\frac{1}{\gamma}\left(\frac{\bar{N}_D}{\bar{N}^*_X}\right)^2\left(\chi_2\frac{\bar{\tau}^*\tilde{z}_D}{\bar{\tilde{z}}^*_X}\right)^{-2(1-\theta)}.\tag{A47}$$

Using $(\bar{\tau}\tilde{z}_D/\bar{\tilde{z}}_X)^{1-\theta}=(\chi_2\bar{\tau}^*\tilde{z}_D/\bar{\tilde{z}}^*_X)^{1-\theta}$, we simplify the above ratio as

$$\frac{\zeta_1}{\zeta_2}=\frac{1}{\gamma}\left(\frac{\bar{N}_D}{\bar{N}^*_X}\right)^2\left(\frac{\bar{\tau}\tilde{z}_D}{\bar{\tilde{z}}_X}\right)^{-(1-\theta)}\left(\frac{\bar{\tau}^*\tilde{z}_D}{\bar{\tilde{z}}^*_X}\right)^{-(1-\theta)}.\tag{A48}$$

Given that $\bar{N}_X=\gamma\bar{N}^*_D\bar{N}^*_X$, we can rewrite:

$$\frac{\zeta_1}{\zeta_2}=(\bar{\tau}\bar{\tau}^*)^{\theta-1}\left(\frac{\bar{\tilde{z}}_X}{\tilde{z}_D}\right)^{\theta-1}\left(\frac{\bar{N}_D}{\bar{N}_X}\right)\left(\frac{\bar{\tilde{z}}^*_X}{\tilde{z}_D}\right)^{\theta-1}\left(\frac{\bar{N}^*_D}{\bar{N}^*_X}\right).\tag{A49}$$

Since $\bar{N}_X/\bar{N}_D$ and $\bar{N}^*_X/\bar{N}^*_D$ are between 0 and 1, and $\bar{\tilde{z}}_X$ and $\bar{\tilde{z}}^*_X$ to be not smaller than $\tilde{z}_D$ by their definitions, it follows that

$$\frac{\zeta_1}{\zeta_2} \geq (\bar{\tau}\bar{\tau}^*)^{\theta-1} > 1, \quad \text{when} \quad \bar{\tau}\bar{\tau}^* > 1. \tag{A50}$$

B. Additional Figures and Tables

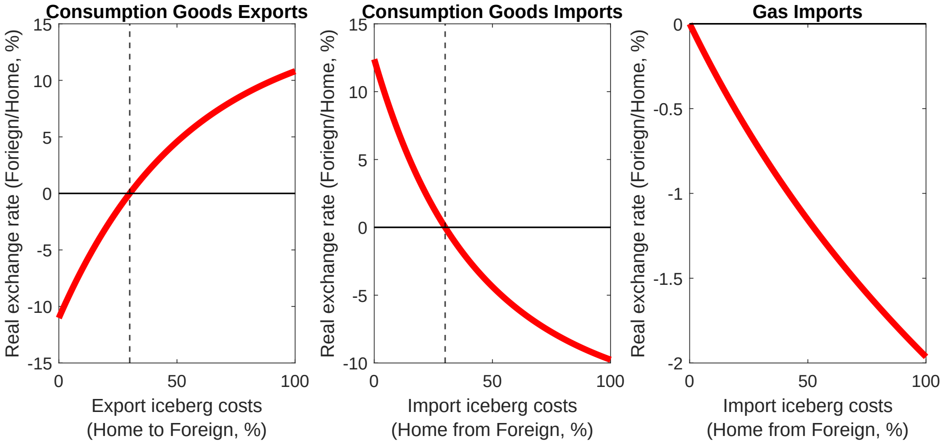

Figure A1: The Responses of Real Exchange Rate to Trade Costs, $\tilde{Q}$

Notes: The figures illustrate the response of the (data-consistent) real exchange rate (Foreign/Home, $\tilde{Q}$), as expressed in Equation (2), to changes in terms of consumption goods export costs (Home to Foreign, τ), import costs (Home from Foreign, τ^*), or gas trade costs (Home from Foreign, τ_G), ceteris paribus. On the y-axis of all figures, the real exchange rate is represented as a percentage deviation from the initial state. In the first figure, the x-axis indicates the iceberg costs of Home exporters, with the vertical dashed lines denoting the initial value (30%). The x-axis in the second figure represents how much Foreign exporters should pay iceberg trade costs, with the vertical dashed lines denoting the initial value (30%). In the third figure, the x-axis represents the gas import costs of Home from Foreign. For all figures, we set the initial state as $TOL = 1$, $\tilde{z}_X/\tilde{z}_D = \tilde{z}_X^*/\tilde{z}D = 0.3^{-1/k}$, and $N_X^*/N = N_X/N^* = 0.3$, with $\alpha = 0.1$, $Z = Z^* = 1.5$, $\tau = \tau^* = 1.3$, $\tau_G = 1$, $k = 3.4$, $\theta = 3.8$, and $\kappa_G = \kappa_G^* = 0.75$.

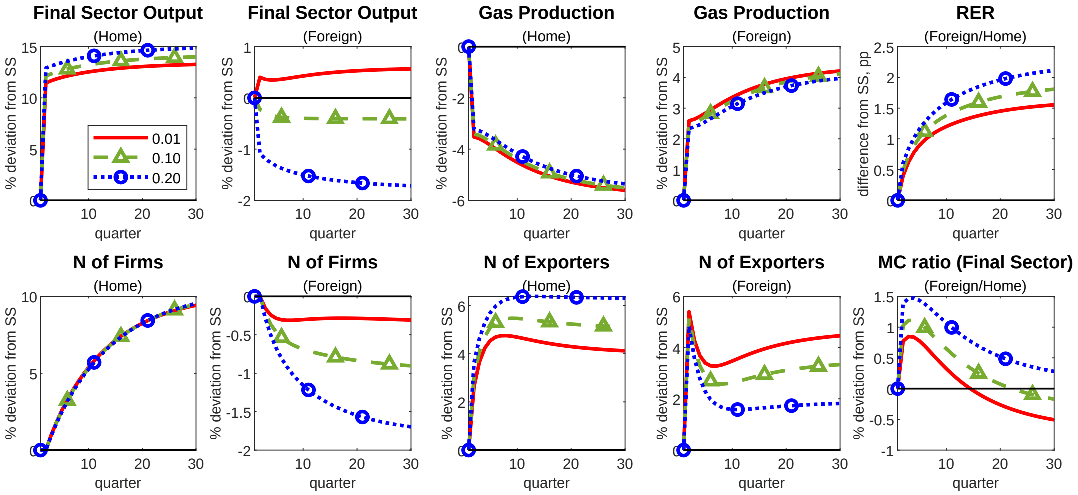

Figure A2: Responses to a 10% Permanent Increase in Home Productivity: Other Variables

Notes: The red solid lines, green dashed lines with triangles, and blue dashed lines with circles represent the model's transition dynamics when the aggregate productivity of the Home consumption goods sector increases by 10% at $t = 1$, with gas cost shares (α) set at 1%, 10%, and 20%, respectively. All deviations, except for the figures titled P indexed RER, TOL (Final Sector), and Trade Balance/GDP, are expressed in units of percent deviation from the initial steady-state ($t = 0$), i.e., $100 \times (x_t/x_0 - 1)$. The figures titled P indexed RER, TOL (Final Sector), and Trade Balance/GDP are presented in units of percentage points difference from the initial steady-state, i.e., $100 \times (x_t - x_0)$. Refer to Figure 4 for the responses of other variables.

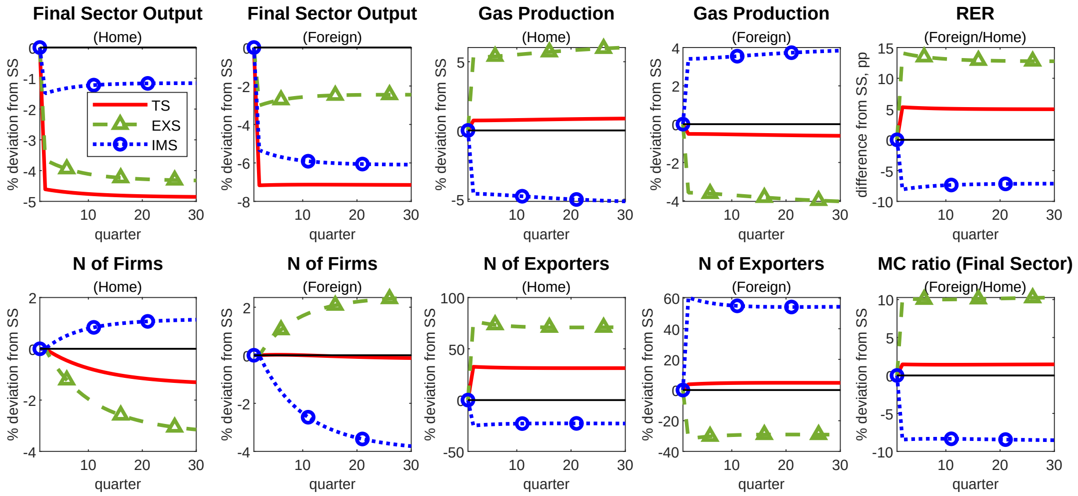

Figure A3: Transition Dynamics after Consumption Good Export and Import Sanctions: Other Variables

Notes: The red solid lines, green dashed lines with triangles, and blue dashed lines with circles represent the transitional dynamics when consumption goods export (EXS), import (IMS), and trade sanctions (TS, both export and import) are imposed at $t = 1$. All deviations, except for the figures titled P indexed RER, TOL (Final Sector), and Trade Balance/GDP, are measured in units of percent deviation from the initial steady-state without sanctions ($t = 0$), i.e., $100 \times (x_t/x_0 - 1)$. The figures titled P indexed RER, TOL (Final Sector), and Trade Balance/GDP express deviations in units of percentage points from the initial steady-state, i.e., $100 \times (x_t - x_0)$. Refer to Figure 5 for responses of other variables.

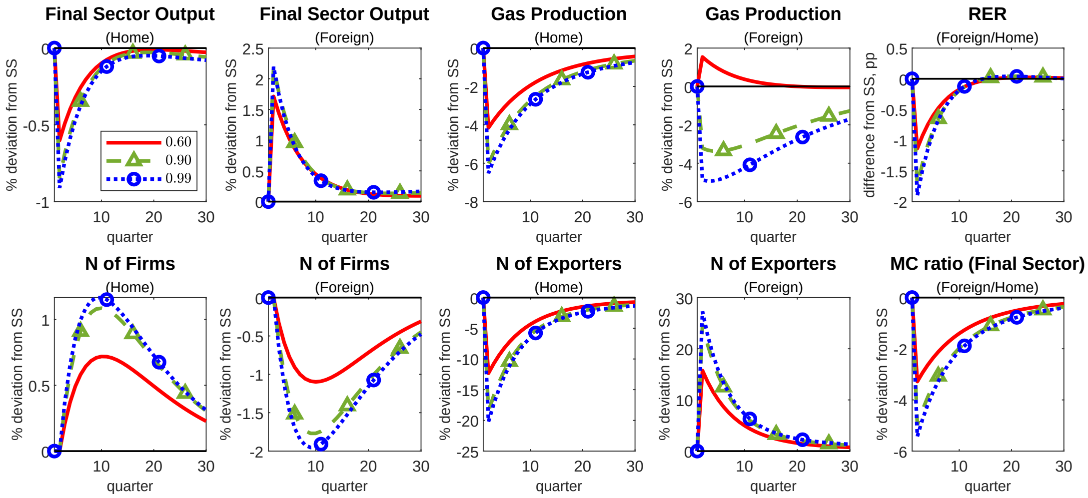

Figure A4: Transition Dynamics after Financial Sanctions: Other Variables

Notes: The red solid lines, green dashed lines with triangles, and blue dashed lines with circles depict the model transition dynamics when financial sanctions are imposed at $t = 1$, with the fraction of sanctioned Foreign households (λ) set at 60%, 90%, and 99%, respectively. All deviations, except for the figures titled P indexed RER, TOL (Final Sector), and Trade Balance/GDP, are measured in units of percent deviation from the initial steady-state without sanctions ($t = 0$), i.e., $100 \times (x_t/x_0 - 1)$. The figures titled P indexed RER, TOL (Final Sector), and Trade Balance/GDP express deviations in units of percentage points from the initial steady-state, i.e., $100 \times (x_t - x_0)$. All Foreign variables are aggregates. Refer to Figure 7 for sanctioned and unsanctioned households' variables. See Figure 6 for responses of other variables.

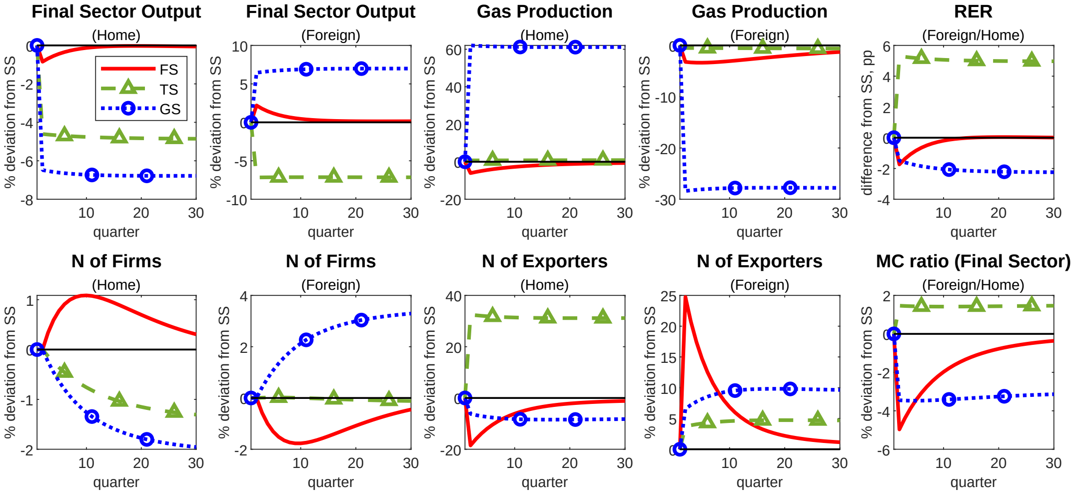

Figure A5: Transition Dynamics after Trade, Financial, and Gas Sanctions: Other Variables

Notes: The red solid lines, green dashed lines with triangles, and blue dashed lines with circles depict the transitional dynamics when financial (FS), consumption good sector trade (TS), and gas sanctions (GS), respectively, are imposed at $t = 1$. All deviations, except for the figures titled P indexed RER, TOL (Final Sector), and Trade Balance/GDP, are measured in units of percent deviation from the initial steady-state without sanctions ($t = 0$), i.e., $100 \times (x_t/x_0 - 1)$. The figures of ratio, titled P indexed RER, TOL (Final Sector), and Trade Balance/GDP, are measured in units of percent point difference from the initial steady-state, i.e., $100 \times (x_t - x_0)$. Refer to Figure 8 for the responses of other variables.

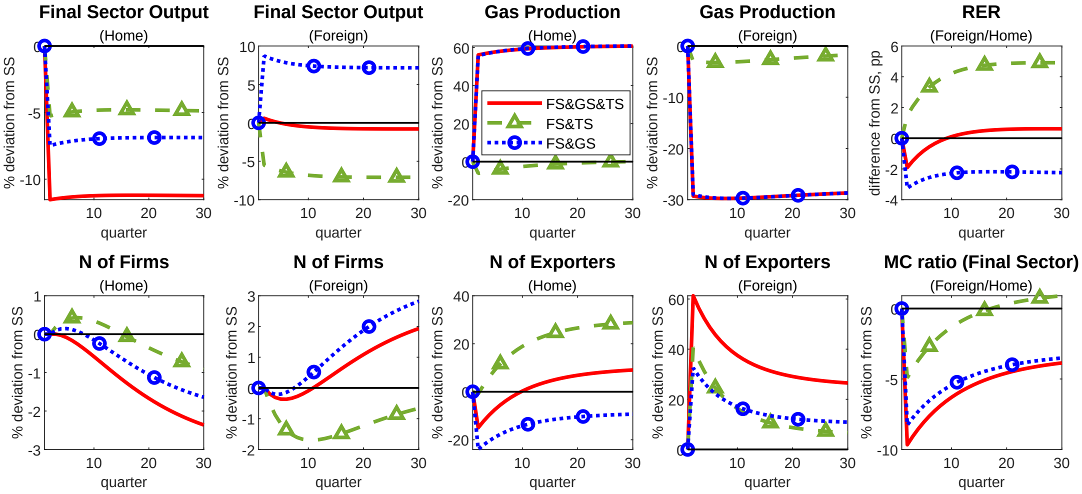

Figure A6: Transition Dynamics with Combinations of Sanctions: Other Variables

Notes: The red solid lines depict the transitional dynamics when all sanctions (FS&GS&FS) are imposed at $t = 1$. The green dashed lines with triangles illustrate the transitional dynamics when financial and consumption good sector trade sanctions (FS&TS) are imposed at $t = 1$. The blue dashed lines with circles show the transitional dynamics when financial and gas sanctions (FS&GS), respectively, are imposed at $t = 1$. All deviations, except for the figures titled P indexed RER, TOL (Final Sector), and Trade Balance/GDP, are measured in units of percent deviation from the initial steady-state without sanctions ($t = 0$), i.e., $100 \times (x_t/x_0 - 1)$. The figures of ratio, titled P indexed RER, TOL (Final Sector), and Trade Balance/GDP, are measured in units of percent point difference from the initial steady-state, i.e., $100 \times (x_t - x_0)$. Refer to Figure 10 for sanctioned and unsanctioned households' variables. See Figure 9 for the responses of other variables.

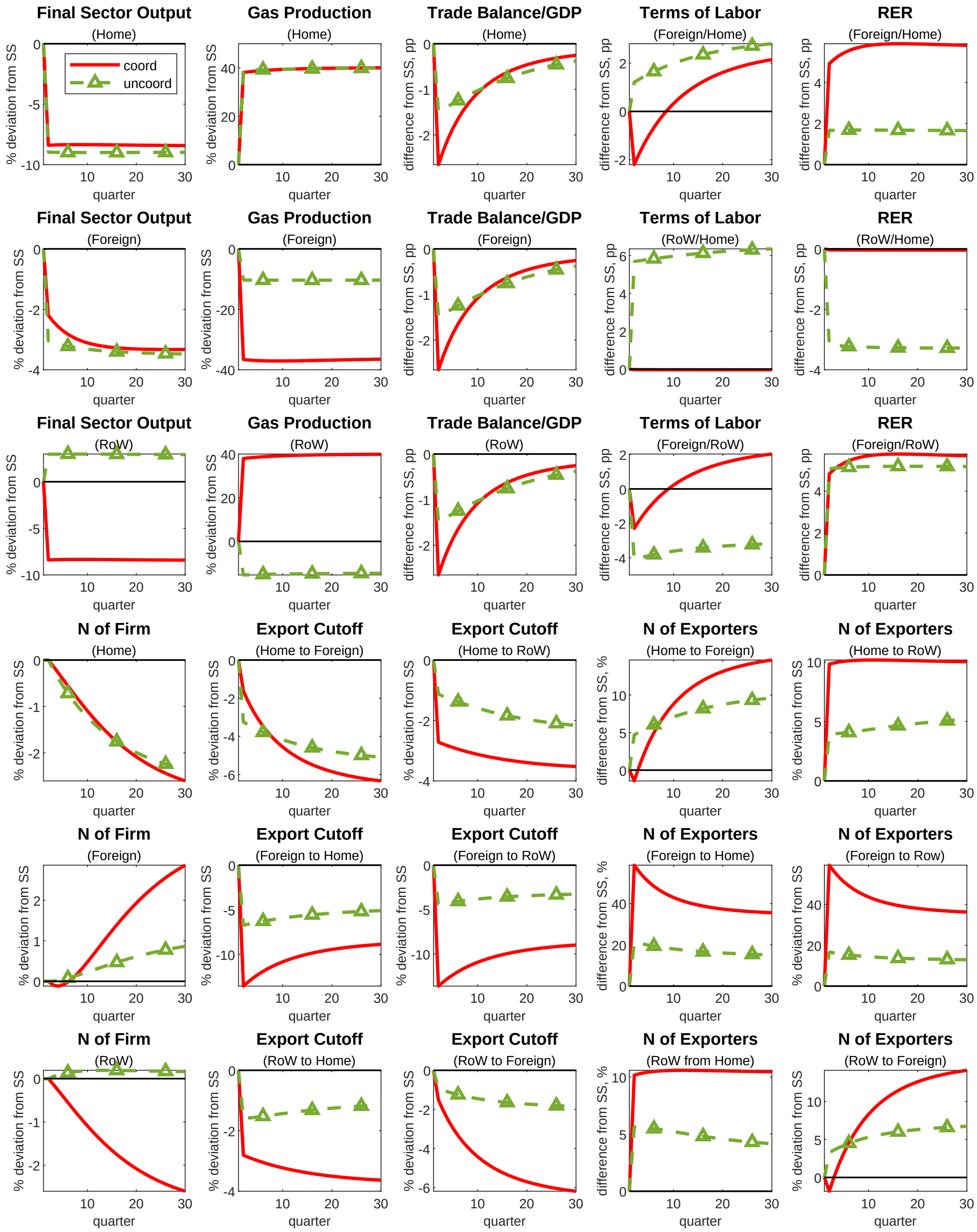

Figure A7: Transition Dynamics from the Three-Country Model with or without Coordination: Other Variables

Notes: The red solid lines (labeled 'coord.') plot the three-country model transition dynamics when all sanctions are imposed by Home and RoW at $t = 1$. The green dashed lines (labeled 'uncoord.') with triangles plot the three-country model transition dynamics when all sanctions are imposed by Home at $t = 1$, while RoW does not participate in sanctions. All deviations, except for the figures titled 'P indexed RER,' are in units of percent deviation from the initial steady-state without sanctions ($t = 0$), i.e., $100 \times (x_t/x_0 - 1)$. The figures of the ratio, titled 'P indexed RER,' are in units of percent point difference from the initial steady-state, i.e., $100 \times (x_t - x_0)$. See Figure 11 for the other variables' responses.

Table A1: Threey-Country Model: steady-state Bond Holdings

		No Financial	Financial Sanction	
Issued Country	Bond Holder	Sanction	Uncooridnated	Coordinated
Home Bonds	Home Households	–5/2	$-(1-\lambda)5/2$	$-(1-\lambda)5/2$
	RoW Households	0	0	0
	Foreign sanctioned Households	5/2	0	0
	Foreign unsanctioned Households	5/2	5/2	5/2
RoW Bonds	Home Households	0	0	0
	RoW Households	–5/2	–5/2	$-(1-\lambda)5/2$
	Foreign sanctioned Households	5/2	5/2	0
	Foreign unsanctioned Households	5/2	5/2	5/2
Foreign Bonds	Home Households	3/2	$(1-\lambda)3/2$	$(1-\lambda)3/2$
	RoW Households	3/2	3/2	$(1-\lambda)3/2$
	Foreign sanctioned Households	–3	–3/2	0
	Foreign unsanctioned Households	–3	–3	–3