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MISALLOCATION AND ASSET PRICES

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ABSTRACT

We develop an endogenous growth model with heterogeneous firms facing financial frictions, where misallocation emerges explicitly as a crucial endogenous state variable and plays a significant role in driving economic growth through the valuation channel. The model illustrates that transient macroeconomic shocks affecting misallocation can yield persistent effects on aggregate growth. In equilibrium, slow-moving misallocation endogenously generates long-run uncertainty about economic growth by distorting innovation decisions. When agents hold recursive preferences, misallocation-driven low-frequency growth fluctuations result in substantial risk premia in capital markets and large losses in consumer welfare. Employing a misallocation measure motivated by the model, we substantiate our findings with empirical evidence showing that misallocation effectively captures low-frequency fluctuations in both aggregate growth and asset returns.

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A data appendix is available at http://www.nber.org/data-appendix/w32147

1 Introduction

Misallocation is important to understanding economic growth, both during economic transitions (e.g., Buera and Shin, 2013, 2017; Moll, 2014) and in long-run steady states (e.g., Jovanovic, 2014; Acemoglu et al., 2018; Peters, 2020). Various measures of cross-sectional dispersion indicate that the allocation efficiency of capital displays strong pro-cyclical patterns (e.g., Eisfeldt and Rampini, 2006; Bloom, 2009; Kehrig, 2015; Bloom et al., 2018). The link between capital misallocation and growth prospects can potentially shed light on the fundamental forces that drive low-frequency growth fluctuations. These fluctuations in growth constitute a systematic risk capable of quantitatively rationalizing numerous asset pricing phenomena (e.g., Bansal and Yaron, 2004; Hansen, Heaton and Li, 2008) and helping justify the substantial welfare costs associated with economic fluctuations. By introducing a novel misallocation-based asset pricing mechanism, we underscore the critical role of the valuation channel in conveying the substantial impact of capital misallocation on economic growth prospects.

This paper quantitatively explores the relationship between misallocation, growth prospects, and the systematic risk that shapes asset prices in capital markets. To achieve this, we develop an analytically tractable general equilibrium model with heterogeneous firms and endogenous stochastic growth. In our model, the misallocation of production capital is endogenously slow-moving and causes low-frequency fluctuations in economic growth. A shock that increases the misallocation of production capital results in a prolonged elevation in the level of misallocation. This, in turn, leads to a persistent decline in incentives for innovation (R&D), thereby affecting economic growth adversely. At the heart of this mechanism is the impact of production capital misallocation on the marginal q of intangible capital, which is the present value of marginal profits derived from intangible capital (e.g., Crouzet and Eberly, 2023). A reduced marginal q of intangible capital leads to weakened incentives for innovation, with increased misallocation depressing the marginal q of intangible capital through two channels. First, it reduces the rents of innovation by decreasing the aggregate demand for the goods licensed by innovation outputs. Second, when investors exhibit recursive preferences, it elevates the risk premium required to discount the future rents of innovation, due to heightened volatility in growth prospects, acting as a "leverage effect" of financial frictions. This second channel magnifies the impact of capital misallocation on economic growth prospects, establishing what we term the "valuation channel." Through this channel, fluctuations in misallocation have amplified effects on variations in low-frequency economic growth.

Our model builds upon the framework established by Moll (2014), where misallocation endogenously emerges as an outcome of financial frictions. We extend Moll (2014)'s framework in three ways, all the while preserving its analytical tractability. First, we model

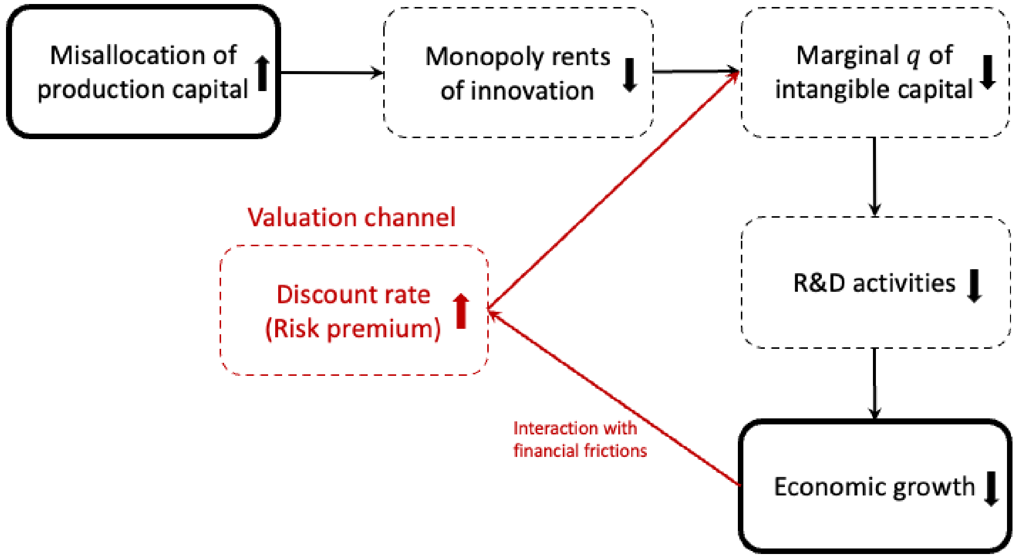


Figure 1: Our model elucidates a mechanism that quantitatively links capital misallocation to economic growth via a valuation channel.

heterogeneous firms engaged in the production of final goods. These firms are typically publicly traded, owned by a diverse group of shareholders with homogeneous recursive preferences. However, they are managed by corporate managers with objectives that differ from those of their shareholders. This agency conflict leads to the financial frictions faced by these firms. Second, in addition to the final goods sector, our model incorporates both intermediate goods and R&D sectors. R&D activities, which contribute to expanding the variety of intermediate goods utilized in the production of final goods, are pivotal in driving technological progress and, consequently, endogenous economic growth, as articulated by Romer (1986, 1990) and Jones (1995). Third, we incorporate aggregate shocks that drive slow-moving misallocation.

The combination of these three components enables us to illustrate a novel and relevant economic mechanism, as illustrated in Figure 1. When the misallocation of production capital within the final goods sector increases, the aggregate demand for the goods produced by the intermediate goods sector declines. In this sector, each producer holds monopoly power over a specific type of intermediate goods by acquiring the blueprint from the R&D sector. The lower demand for intermediate goods leads to lower monopoly rents, and thus reduces the value of the blueprints invented within the R&D sector. This chain of events then leads to a decline in the marginal q of intangible capital across the economy. As a result, the incentive for innovation within the R&D sector declines, which ultimately leads to a decrease in equilibrium economic growth prospects.

Importantly, there is more to the above mechanism, as highlighted in red color in Figure 1. The endogenously persistent and slow-moving misallocation of production capital implies that the aggregate shocks influencing misallocation can lead to low-frequency fluctuations in economic growth. When agents have recursive preferences, these low-frequency growth fluctuations, as a fundamental source of systematic risk, play a crucial role in determining the discount rate (particularly, the risk premium). Importantly, due to the interaction with financial frictions, the economic growth rate is not only low but also highly volatile in periods with high misallocation, leading to a higher risk premium in economic downturns. This further depresses the marginal q of intangible capital across the economy, amplifying the impact of production capital misallocation on economic growth prospects through the "valuation channel."

Our theoretical framework, which links misallocation to growth, bears resemblances to the model by Peters (2020), wherein the innovation rates of firms are negatively impacted by the misallocation of production labor. However, our model is different in three significant ways: (i) the source of misallocation in our model is financial frictions arising from agency conflicts, rather than product market imperfections; (ii) our model focuses on the dynamics of stochastic growth, rather than deterministic steady-state growth; and (iii) our model emphasizes the "valuation channel," a crucial aspect absent in the model of Peters (2020). Our mechanism is also different from that of Acemoglu et al. (2018), who emphasize the role of misallocation of R&D inputs, rather than the misallocation of production capital, in determining equilibrium economic growth.

Below, we elaborate on the key elements of our model. Our economic model encompasses three sectors. First, the R&D sector serves as the engine of knowledge creation, leveraging final goods and the existing reservoir of knowledge to generate new blueprints. Second, the intermediate goods sector capitalizes on these blueprints, in conjunction with final goods, to produce a variety of differentiated intermediate goods. In this sector, there is a continuum of producers. Each producer holds a monopoly over a specific type of intermediate good, with the monopoly power secured by the blueprint acquired from the R&D sector. Finally, the final goods sector uses production capital, labor, and intermediate inputs to produce the ultimate end products. There is a representative agent who owns firms in all sectors, a continuum of heterogeneous firms in the final goods sector, and homogeneous firms in the intermediate goods and R&D sectors.

Firms in the final goods sector differ in both productivity levels and their stock of production capital. Yet, due to agency conflicts, they are subjected to both collateral constraints on borrowing and equity market constraints relating to payouts and issuances. These financial frictions contribute to the misallocation of production capital among firms operating within the final goods sector. Increased misallocation leads to a decline in aggregate productivity within the final goods sector, wihch reduces the aggregate demand for intermediate goods. This, in turn, adversely affects the profitability of innovation and thus reduces the incentive of innovators to generate new blueprints for expanding the variety of intermediate goods, ultimately resulting in a lower growth rate for the economy.

There is one aggregate shock — the production capital depreciation shock — that drives the aggregate fluctuation of the economy. Modeling aggregate shocks to production capital depreciation rates follows Storesletten, Telmer and Yaron (2007), Gourio (2012), Brunnermeier and Sannikov (2017), among others. Firms endogenously choose their capacity utilization intensity. Higher intensity allows firms to produce more outputs at the cost of a higher production capital depreciation rate. In equilibrium, firms with higher productivity deploy their production capital more intensively, making them more vulnerable to the aggregate capital depreciation shock than less productive firms. As a result, this aggregate shock leads to fluctuations in the misallocation of production capital within the economy, which in turn results in fluctuations in the economy's growth rate.

Our model represents a general equilibrium framework featuring heterogeneous firms and aggregate fluctuations. The standard approach to solving such models typically involves numerical approximations based on key moments summarizing the cross-sectional distribution of firms. We depart from the standard approach by proposing a parametric approximation of the distribution of log productivity and log capital using a bivariate normal distribution. This parametric approximation approach offers two major advantages. First, it allows us to derive a closed-form expression for the misallocation of capital in the final goods sector, which emerges as a crucial and explicit endogenous state variable that summarizes the cross-sectional distribution of firms. Specifically, in our model, misallocation is characterized by the covariance between the log marginal revenue product of capital (MRPK) and log capital, normalized by the variance of log MRPK. This covariance-based measure of misallocation is intuitive and aligns with similar metrics used in empirical studies for assessing capital allocation efficiency (e.g., Olley and Pakes, 1996; Bartelsman, Haltiwanger and Scarpetta, 2009, 2013). Second, our proposed parametric approximation makes the model highly tractable and transparent. It allows the evolution of the model economy to be analytically characterized by two endogenous state variables — misallocation and the knowledge stock-capital ratio. This approach enables an analytical exploration of the relationship between the dynamics of misallocation and the dynamics of aggregate growth.[1]

To illustrate the key theoretical mechanism, we begin our analysis by focusing on the deterministic balanced growth path in the absence of aggregate shocks. We show that a one-time shock that increases misallocation can exert a persistently adverse effect on economic growth. Specifically, due to financial frictions, the reallocation of capital across

[1]We justify the validity of this approximation using the Berry-Esseen bound (Tikhomirov, 1980; Bentkus, Gotze and Tikhomoirov, 1997) under certain conditions. Our parametric approximation yields results that are close to those obtained via standard numerical approximation methods, particularly under baseline calibration, as detailed in Online Appendix III.

firms takes time. As a result, the shock not only escalates misallocation at the moment of impact but also prolongs this heightened level into the long-term future. Therefore, through its influence on the marginal q of intangible capital, and consequently on R&D incentives, what begins as a temporary shock to misallocation can result in a prolonged downturn in economic growth. This underscores the profound and lasting effects that misallocation, stemming from agency conflicts in the corporate sector, has on the economy. Furthermore, we show that the persistence of both misallocation and economic growth is closely related to the persistence of firms' idiosyncratic productivity. This augments the key insight from Moll (2014), which states that an increase in the persistence of firms' idiosyncratic productivity leads to a longer time for the economy to reach its steady state. In our model, the persistence of idiosyncratic productivity emerges as a crucial determinant of the persistence of aggregate economic growth. This is primarily because misallocation naturally adjusts more slowly when idiosyncratic productivity becomes more persistent.

Building on this mechanism, we show that in the full model with aggregate shocks, misallocation evolves slowly, leading to low-frequency fluctuations in economic growth. In quantitative terms, the annual autocorrelation of misallocation stands at 0.73, while that of consumption growth is 0.46, both closely mirroring empirical measures. Consequently, our model demonstrates a novel mechanism linking misallocation fluctuations to low-frequency growth fluctuations.[2] Central to this mechanism is the valuation channel, which significantly magnifies the effects of production capital misallocation in the final goods sector on economic growth. In particular, during economic downturns, characterized by heightened misallocation and reduced growth, firms in the final goods sector face greater financial constraints. In such periods, the economic growth rate is not only low but also highly volatile. Consequently, low expected consumption growth typically coincides with high macroeconomic volatility, leading to an elevated risk premium. As a result, the marginal q of intangible capital suffers a double impact: it is depressed not only because of reduced profits, but also because future profits are discounted at a higher rate owing to the increased risk premium.

Furthermore, we show that our model not only rationalizes several crucial asset pricing moments but also suggests significant welfare costs associated with misallocation fluctuations. Specifically, the model implies a high Sharpe ratio of 0.39 for the aggregate consumption claim, accompanied by a low and stable risk-free rate, aligning with empirical observations. The representative agent would experience a welfare gain of about 10% if consumption fluctuations are eliminated. The large quantitative effects generated by misallocation fluctuations hinge on two properties of the model, the low-frequency

[2]Our model provides a misallocation-based explanation for the observed low-frequency covariation in the time series of consumption growth and output growth (e.g., Bansal, Dittmar and Lundblad, 2005; Hansen, Heaton and Li, 2008; Müller and Watson, 2008, 2018).

growth fluctuations driven by slow-moving misallocation, and the recursive preference of the representative agent. We show that if misallocation does not affect economic growth or if it is not sufficiently slow moving to generate low-frequency growth fluctuations, the quantified Sharpe ratio and welfare gain would be very small. Moreover, we demonstrate that if the representative agent possesses time-separable preferences with constant relative risk aversion (CRRA), the observed effects would be markedly diminished. The crux of this phenomenon lies in the recursive preferences, which ensure that the representative agent's marginal utility today is influenced not just by current consumption growth, but more importantly, by expectations of future consumption growth. Consequently, fluctuations in anticipated consumption growth can exert significant valuation effects via the stochastic discount factor (SDF). Furthermore, given the persistent nature of consumption growth, even a transient shock can lead to enduring future effects. This persistence markedly heightens the influence of future consumption growth on current marginal utility, thereby amplifying the impact of capital misallocation on economic growth.

Although our main contribution is theoretical, we empirically test the main predictions of our model. Motivated by our theory, we construct a misallocation measure based on the covariance between log MRPK and log capital using U.S. Compustat data. We show that the misallocation measure is persistent, with a yearly autocorrelation of 0.75. Moreover, the value of our empirical measure of misallocation increases during economic downturns. We show that an increase in misallocation predicts declines in R&D intensity and reductions in the growth of aggregate consumption and output over long horizons. Moreover, we provide direct causal evidence to support the model's mechanism that misallocation drives long-run growth through its impact on R&D. We consider the policy shock from the American Jobs Creation Act (AJCA) passed in 2004, which relaxes financial constraints of firms with pre-tax income from abroad. By exploiting industries' differential exposure to this policy shock in a difference-in-differences (DID) setting, we find that the AJCA considerably reduces industry-level misallocation and raises R&D expenditure in treated industries. We further show that the impact of the AJCA on industry-level R&D expenditure becomes statistically insignificant after controlling for its impact on industry-level misallocation. In Section I of the Online Appendix, we provide additional cross-sectional evidence to support the main theoretical mechanism, suggesting that the proposed misallocation measure captures a quantity-based macroeconomic asset pricing factor.

Related Literature. Our paper is related to several strands of the literature. First, our work contributes to the enduring yet rapidly expanding body of literature emphasizing the importance of misallocation in driving economic growth and development. In terms of economic growth, it relates to the research by Banerjee and Duflo (2005), Jones (2013), Jovanovic (2014), Acemoglu et al. (2018), Peters (2020), König et al. (2022), and Glode and

Ordonez (2023), among others. Regarding economic development, it relates to studies such as those by Foster, Haltiwanger and Syverson (2008), Restuccia and Rogerson (2008), Hsieh and Klenow (2009), Jones (2011), Bartelsman, Haltiwanger and Scarpetta (2013), Asker, Collard-Wexler and Loecker (2014), David, Hopenhayn and Venkateswaran (2016), and David and Venkateswaran (2019). Our paper is particularly related to the literature on financial frictions and misallocation. Most of this literature focuses on the long-run total factor productivity (TFP) and welfare losses due to misallocation in the deterministic steady state (e.g., Amaral and Quintin, 2010; Greenwood, Sanchez and Wang, 2010, 2013; Caselli and Gennaioli, 2013; Midrigan and Xu, 2014; Buera, Kaboski and Shin, 2015), while a few papers also analyze transitional dynamics (e.g., Jeong and Townsend, 2007; Buera and Shin, 2013; Moll, 2014; Gopinath et al., 2017).[3] Our paper develops a stochastic growth model in which slow-moving misallocation endogenously drives low-frequency growth cycles. We show that when agents have recursive preferences, the misallocation caused by financial frictions can generate large risk premia and welfare losses through endogenous low-frequency growth fluctuations. Moreover, the persistence of firm-level idiosyncratic productivity plays an important role in generating slow-moving misallocation, which, in turn, generates low-frequency growth fluctuations.[4] Our results complement the key insight of Moll (2014), who shows that as idiosyncratic productivity becomes increasingly persistent, the transition speed from a distorted initial state to the steady state slows down, resulting in potentially large welfare losses during transitions.

Relatively few studies in the finance literature have concentrated on the role of misallocation, especially when compared to the development and growth literature. This discrepancy highlights a potential research gap that could be pivotal in understanding the broader impacts of misallocation on financial systems and markets. Existing advances include Eisfeldt and Rampini (2006, 2008*b*), Rampini and Viswanathan (2010), Fuchs, Green and Papanikolaou (2016), van Binsbergen and Opp (2019), Ai, Li and Yang (2020), Ai et al. (2020), and Lanteri and Rampini (2021), among others. Our paper is most related to David, Schmid and Zeke (2022), who analyze the implications of macroeconomic risk for misallocation. Nevertheless, our paper examines the reverse relationship — the consequences of misallocation on macroeconomic risk. Specifically, our model reveals that misallocation within the economy can itself act as a macroeconomic risk factor of asset pricing. This occurs as

[3]Several papers measure the importance of financing costs in generating misallocation. For example, Gilchrist, Sim and Zakrajsek (2013) find that the costs of debt play a limited role in generating misallocation based on a sample consisting of about 500 (mostly large) firms that issue corporate bonds. David, Schmid and Zeke (2022) find that the costs of equity are important in generating misallocation. Whited and Zhao (2021) find significant variations in the costs of debt and equity across U.S. firms.

[4]By connecting the persistence in idiosyncratic productivity with the persistence in aggregate consumption growth, our model implies that low-frequency growth fluctuations can be identified using granular firm-level data, which potentially helps address the issues of weak identification in the asset pricing literature (Chen, Dou and Kogan, 2022; Cheng, Dou and Liao, 2022).

misallocation influences the investors' SDF through its effect on low-frequency consumption growth. In contrast, the model of David, Schmid and Zeke (2022) adopts an exogenous SDF to assess how macroeconomic risk impacts the economy's misallocation.

Our paper also contributes to the asset pricing literature. Various theoretical studies provide micro foundations to justify low-frequency growth fluctuations (e.g., Ai, 2010; Kaltenbrunner and Lochstoer, 2010; Nicolae, Panageas and Yu, 2012; Croce, 2014; Kung and Schmid, 2015; Collin-Dufresne, Johannes and Lochstoer, 2016; Ai, Li and Yang, 2020; Gârleanu and Panageas, 2020; Croce, Nguyen and Raymond, 2021). Our paper is most closely related to the work of Kung and Schmid (2015), who illustrate that R&D endogenously drives a small, persistent component of productivity, generating long-run uncertainty about economic growth. The primary distinction of our model from theirs lies in incorporating the cross-sectional misallocation of production capital, which influences both the aggregate TFP and the total demand for intermediate inputs. Importantly, due to the interaction with financial frictions, the economic growth rate is not only reduced but also subject to greater volatility in times of economic downturns, which elevates the risk premium, thereby amplifying the effect of production capital misallocation on economic growth prospects through the "valuation channel." This difference allows our theory to rationalize low-frequency growth fluctuations through the equilibrium interactions between endogenous slow-moving misallocation, marginal q of intangible capital, and R&D activities — a mechanism supported by the data.

Finally, our paper is related to the literature on the welfare cost of economic fluctuations (e.g., Lucas, 1987). In particular, the following papers are closely related to our work. Barlevy (2004) shows that economic fluctuations at business cycle frequencies can cause substantial welfare losses by influencing the growth rate of consumption via capital investment. Furthermore, Alvarez and Jermann (2004) use asset prices to estimate that the welfare gains from eliminating all consumption fluctuations are significantly greater than those from just eliminating the fluctuations at business cycle frequencies. Our model, which focuses on slow-moving misallocation, driven endogenously by transient shocks, explains the occurrence of low-frequency growth fluctuations. As a result, it underscores the significant welfare costs associated with these misallocation fluctuations, particularly through their impact on low-frequency growth. This is especially pertinent in scenarios where the representative agent exhibits recursive preferences.

2 Model

There are three sectors: a final goods sector with heterogeneous firms, an intermediate goods sector, and an R&D sector. A representative agent holds ownership in firms across all these sectors.

2.1 Final Goods Sector

In the final goods sector, there is a continuum of firms of measure one, indexed by $i \in \mathcal{I} \equiv [0,1]$ and operated by managers. Firms are different from each other in their idiosyncratic productivity $z_{i,t}$ and capital $a_{i,t}$. The distribution of firms at t is characterized by the joint probability density function (PDF), $\varphi_t(a,z)$.

The firm produces output at intensity $y_{i,t}$ over $[t, t+\mathrm{d}t)$ using a technology with constant returns to scale (CRS):

$$y_{i,t} = \left[(z_{i,t}u_{i,t}k_{i,t})^{\alpha}\ell_{i,t}^{1-\alpha}\right]^{1-\varepsilon} x_{i,t}^{\varepsilon}, \quad \text{with } \alpha,\ \varepsilon \in (0,1), \tag{1}$$

where labor $\ell_{i,t}$ is hired in a competitive labor market at the equilibrium wage w_t. The variable $k_{i,t} = a_{i,t} + \widehat{a}_{i,t}$ is the capital installed in production, which includes the firm's own capital $a_{i,t}$ and the leased capital $\widehat{a}_{i,t}$ borrowed from a competitive rental market at the equilibrium risk-free rate $r_{f,t}$.[5] The final goods are the numeraire.

As specified in (1), the firm's output $y_{i,t}$ increases with its idiosyncratic productivity $z_{i,t}$ and endogenous choice of capacity utilization intensity $u_{i,t} \in [0,1]$. Utilizing capital at intensity $u_{i,t}$ leads to depreciation of $u_{i,t}k_{i,t}\mathrm{d}\Delta_t$ over $[t, t+\mathrm{d}t)$. In this expression, $\mathrm{d}\Delta_t = \delta_k \mathrm{d}t + \sigma_k \mathrm{d}W_t$ represents the stochastic depreciation rate, where δ_k and σ_k are positive constants. In our framework, the standard Brownian motion, denoted as W_t, is employed to represent the aggregate capital depreciation shock. This method of modeling the aggregate capital depreciation shock is in line with the approaches used in existing studies, such as Storesletten, Telmer and Yaron (2007), Albuquerue and Wang (2008), and Gourio (2012).

The firm's own capital stock evolves according to

$$\mathrm{d}a_{i,t} = -\delta_a a_{i,t}\mathrm{d}t + \sigma_{a,t}a_{i,t}\mathrm{d}W_t + \mathrm{d}I_{i,t}, \tag{2}$$

where $\delta_a > 0$ is the constant depreciation rate and $\sigma_{a,t}a_{i,t}\mathrm{d}W_t$ captures shocks to capital efficiency. We assume that a single aggregate shock $\mathrm{d}W_t$ affects both capital depreciation and efficiency for tractability. This implies that an improvement in the efficiency of new capital is associated with the depreciation of existing capital, capturing the displacement effect of new capital (e.g., Gârleanu, Kogan and Panageas, 2012; Kogan et al., 2017; Kogan, Papanikolaou and Stoffman, 2020).[6] The firm's investment over $[t, t+\mathrm{d}t)$, denoted by $\mathrm{d}I_{i,t}$,

[5] The capital leasing market is relevant for firms' production and financial decisions (e.g., Eisfeldt and Rampini, 2008a; Rampini and Viswanathan, 2013; Li and Tsou, 2021).

[6] The modeling of capital efficiency shocks is widely adopted in the literature (e.g., Sundaresan, 1984; Cox, Ingersoll and Ross, 1985; Kogan, 2001, 2004; Gourio, 2012; Di Tella, 2017; Dou, 2017).

is defined in equation (17) below. As we show in Section 3.2, the aggregate shock $\mathrm{d}W_t$ generates time variations in the misallocation of production capital.[7]

The composite $x_{i,t}$ in equation (1) consists of differentiated intermediate goods, given by the constant elasticity of substitution (CES) aggregation:

$$x_{i,t} = \left(\int_0^{N_t} x_{i,j,t}^{\nu} \mathrm{d}j \right)^{\frac{1}{\nu}}, \tag{3}$$

where $x_{i,j,t}$ is the quantity of intermediate goods $j \in [0, N_t]$. The elasticity of substitution among differentiated intermediate goods is $1/(1-\nu) > 0$. At any given time t, the stock of knowledge in the economy, encapsulated in the variety of intermediate goods, is quantified as N_t. It is through the expansion of N_t that technological advances occur and drive economic growth.

The firm's idiosyncratic productivity $z_{i,t}$ evolves according to

$$\mathrm{d}\ln z_{i,t} = -\theta \ln z_{i,t} \mathrm{d}t + \sigma\sqrt{\theta}\mathrm{d}W_{i,t}, \tag{4}$$

where the standard Brownian motion $W_{i,t}$ captures idiosyncratic productivity shocks. The parameter θ determines the persistence of $z_{i,t}$.

2.2 Intermediate Goods Sector

There is a continuum of homogeneous intermediate goods producers indexed by $j \in [0, N_t]$. Each producer j holds monopoly power in pricing a specific type of intermediate good, a power that is guaranteed by the blueprint obtained from the R&D sector. These producers purchase final goods and convert them into intermediate goods following the blueprints. In this process, one unit of final goods is utilized to produce one unit of intermediate goods. Let $p_{j,t}$ denote the price of intermediate good j. The producer solves the following problem to maximize monopoly profit:

$$\pi_{j,t} = \max_{p_{j,t}} p_{j,t} e_{j,t} - e_{j,t}, \tag{5}$$

subject to the downward-sloping demand curve:

$$e_{j,t} = \left(\frac{p_{j,t}}{p_t} \right)^{\frac{1}{\nu-1}} X_t, \ \ \text{with } p_t = \left(\int_0^{N_t} p_{j,t}^{\frac{\nu}{\nu-1}} \mathrm{d}j \right)^{\frac{\nu-1}{\nu}}, \tag{6}$$

[7] Other aggregate shocks can also generate time variations in misallocation, such as the aggregate shocks to firms' financial constraints. However, Jermann and Quadrini (2012) show that financial shocks cannot generate persistent macroeconomic effects unless the shocks themselves are calibrated to be persistent. The aggregate shock $\mathrm{d}W_t$ in our model directly affects firms' capital. Because capital accumulation is a gradual process, even independent and identically distributed shocks to firms' capital can generate persistent macroeconomic effects.

where $X_t \equiv \int_{i \in \mathcal{I}} x_{i,t} \mathrm{d}i$ is the aggregate demand for the composite of intermediate goods.

Let $q_{j,t}$ be the value of owning the exclusive rights to produce intermediate good j. Because intermediate-good producers are homogeneous, in a symmetric equilibrium, it must hold that $q_{j,t} \equiv q_t$ and $\pi_{j,t} \equiv \pi_t$, for all producers $j \in [0, N_t]$. Intermediate good producers, while engaging in monopolistic competition in intermediate goods markets dealing with final goods firms, operate under perfect competition in the blueprint market with innovators. As a result, the price of a blueprint, q_t, equates to the present value of future monopoly rents that a blueprint can generate, discounted by the SDF of the representative agent. Thus, the value of q_t satisfies the Hamilton-Jacobi-Bellman equation:

$$0 = \Lambda_t \left(\pi_t - \delta_b q_t\right) \mathrm{d}t + \mathbb{E}_t \left[\mathrm{d}(\Lambda_t q_t)\right], \tag{7}$$

where Λ_t is the SDF of the representative agent, as specified in (13), and δ_b is the patent obsolescence rate. The variable q_t can be interpreted as the marginal q of intangible capital in the economy.

2.3 R&D Sector

Innovators in the model are atomistic, with each one capable of inventing a single blueprint through an R&D experiment over $[t, t + \mathrm{d}t)$ with a success rate $\vartheta_t > 0$. Each R&D experiment requires the use of final goods as R&D expenditure with unity intensity over $[t, t + \mathrm{d}t)$. Each innovator in the model can optimally decide to engage in an R&D experiment without incurring any entry costs. Let S_t represent the total number of innovators who choose to participate over $[t, t + \mathrm{d}t)$. As a result, the total number of newly created blueprints over $[t, t + \mathrm{d}t)$ is given by $\vartheta_t S_t \mathrm{d}t$, which contributes to the evolution of the aggregate knowledge stock, N_t, as follows:

$$\mathrm{d}N_t = \vartheta_t S_t \mathrm{d}t - \delta_b N_t \mathrm{d}t. \tag{8}$$

Importantly, the success rate of R&D experiments, ϑ_t, is influenced by both the aggregate stock of knowledge N_t and the total R&D expenditure S_t. In line with Comin and Gertler (2006), we model the success rate as $\vartheta_t = \chi \left(N_t / S_t\right)^h$, where $h \in (0, 1)$. This formulation captures the positive spillover effect of the aggregate knowledge stock, N_t, as emphasized by Romer (1990) and Jones (1995), and the congestion or competition effect of the total R&D activities, S_t, in the success rate.[8]

In equilibrium, the free-entry condition dictates that the expected return from R&D for the marginal innovator choosing to engage in an R&D experiment must be equal to the

[8]The production of non-rival knowledge stock through R&D is the core engine of long-run growth (Romer, 1986, 1990; Jones, 1995). Recently, Crouzet et al. (2022) develop a model to show that the degree of nonrivalry in intangible capital has non-monotonic effects on growth.

expenditure incurred for the R&D experiment. This implies that

$$q_t \vartheta_t = 1. \tag{9}$$

The free-entry condition implies an investment-q relation for intangible capital at the aggregate level (e.g., Peters and Taylor, 2017; Crouzet and Eberly, 2023) as follows:

$$q_t = \chi^{-1} \left(S_t / N_t\right)^h. \tag{10}$$

2.4 Agents

There is a continuum of agents, including workers and managers. Each manager operates a firm in the final goods sector that is subject to agency problems.[9] Workers in the model lend funds to firms and additionally hold equity claims on all of them. We assume the existence of a complete set of Arrow-Debreu securities, allowing agents to fully insure against idiosyncratic consumption risks, ensuring the existence of a representative agent. The aggregate labor supply is inelastic and normalized to be $L_t \equiv 1$.

Preferences. The representative agent has stochastic differential utility as in Duffie and Epstein (1992):

$$U_0 = \mathbb{E}_0 \left[\int_0^\infty f(C_t, U_t) \mathrm{d}t \right], \tag{11}$$

where

$$f(C_t, U_t) = \left(\frac{1-\gamma}{1-\psi^{-1}} \right) U_t \left[\left(\frac{C_t}{[(1-\gamma)U_t]^{1/(1-\gamma)}} \right)^{1-\psi^{-1}} - \delta \right]. \tag{12}$$

This preference is a continuous-time version of the recursive preferences proposed by Kreps and Porteus (1978), Epstein and Zin (1989), and Weil (1990). The felicity function f is an aggregator over the current consumption rate C_t of final goods and future utility level U_t. The coefficient δ is the subjective discount rate, the parameter ψ is the elasticity of intertemporal substitution (EIS), and the parameter γ captures risk aversion.

The representative agent's SDF is

$$\Lambda_t = \exp \left[\int_0^t f_U(C_s, U_s) ds \right] f_C(C_t, U_t). \tag{13}$$

Limited Enforcement. Constraints in the equity market for payouts/issuances and in the credit market for borrowing emerge endogenously due to limited enforcement problems

[9] The managers, who may be executives, directors, or entrepreneurs, can also broadly include controlling shareholders. These shareholders have control over the firms' investment and payout policies (e.g., Albuquerque and Wang, 2008).

associated with equity and debt contracts.

Manager i extracts pecuniary rents $\tau a_{i,t}\mathrm{d}t$ over $[t, t+\mathrm{d}t)$ while running firm i.[10] In line with the corporate finance literature (e.g., Myers, 2000; Lambrecht and Myers, 2008, 2012), we conceptualize rents primarily as cash compensation, although, in practice, managerial rents can take various forms, extending to real resources appropriated by a broad coalition of managers and staff. These forms include above-market salaries, generous pensions, perks, and enhanced job security. Shareholders have the option to intervene and take control of the firm by replacing the manager. However, this intervention is costly due to the need for collective action, as noted by Myers (2000), and it can also damage the firm's talent-dependent customer capital, as detailed in Dou et al. (2020).

In particular, we assume that upon shareholder intervention, a fraction τ/ρ of the capital $a_{i,t}$ is lost, with $\tau < \rho$, and the shareholders then become the firm's new manager. In equilibrium, to prevent such an intervention, the manager optimally pays out dividends at the minimum amount necessary to dissuade shareholders from intervening. This leads to a payout intensity policy of $d_{i,t} = \rho a_{i,t}$ over $[t, t+\mathrm{d}t)$.[11]

Moreover, the manager can divert a fraction $1/\lambda$ of leased capital $\widehat{a}_{i,t}$ with $\lambda \geq 1$. As a punishment, the firm would lose its own capital $a_{i,t}$. In equilibrium, the manager is able to borrow up to the point where he has no incentive to divert leased capital, implying a collateral constraint of $\widehat{a}_{i,t} \leq \lambda a_{i,t}$, as in Buera and Shin (2013) and Moll (2014).

The financial frictions described above are formally encapsulated in the following proposition, with its proof provided in the Online Appendix II.A.

Proposition 1. *Because of the agency problem with limited enforcement, the firm's payout/issuance policy is subject to the following equity market constraint:*

$$d_{i,t} = \rho a_{i,t}. \tag{14}$$

Moreover, the firm's leased capital is subject to the following collateral constraint:

$$-a_{i,t} \leq \widehat{a}_{i,t} \leq \lambda a_{i,t}. \tag{15}$$

Several points are worth further discussion. First, there are other agency problems that can lead to the equity market and collateral constraints (e.g., Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011). Second, the equity market constraint, widely studied in the

[10]Managers are able to extract rents due to the imperfections in corporate governance. In practice, preventing managers from diverting cash flows for their private benefit is challenging for shareholders, despite the observability of cash flows and the protection of shareholders' property rights to firm assets.

[11]Technically, since the dividend intensity is a constant fraction of the firm's capital, the model has linear solutions and tractable aggregation. A similar feature is observed in the model of Moll (2014), which results from the logarithmic preferences of entrepreneurs and the presence of CRS technology.

corporate finance literature (Myers, 2000; Lambrecht and Myers, 2008, 2012, e.g.,), essentially means that firms cannot freely move funds in and out of themselves. Third, our model's formulation of capital market imperfections, which is analytically tractable, captures the fact that firms face restrictions and costs in accessing external funds. Fourth, one specific interpretation of interfirm borrowing and lending is through a competitive rental market, where firms can rent capital from each other (e.g., Jorgenson, 1963; Hall and Jorgenson, 1969; Buera and Shin, 2013; Moll, 2014).

Managers' Problem. The manager of firm i makes decisions for all $s \geq t$ to maximize the present value $J_{i,t}$ of future managerial rents, as in Lambrecht and Myers (2008, 2012),

$$J_{i,t} = \max_{\widehat{a}_{i,s}, u_{i,s}, \ell_{i,s}, \{x_{i,j,s}\}_{j=0}^{N_t}} \mathbb{E}_t \left[\int_t^\infty \frac{\Lambda_s}{\Lambda_t} \tau a_{i,s} \mathrm{d}s \right], \tag{16}$$

subject to the equity market constraint (14), the collateral constraint (15), and the intertemporal budget constraint (2) with $\mathrm{d}I_{i,t}$ given by

$$\mathrm{d}I_{i,t} = y_{i,t}\mathrm{d}t - \int_0^{N_t} p_{j,t} x_{i,j,t} \mathrm{d}j\mathrm{d}t - w_t \ell_{i,t}\mathrm{d}t - u_{i,t} k_{i,t} \mathrm{d}\Delta_t - r_{f,t} \widehat{a}_{i,t}\mathrm{d}t - d_{i,t}\mathrm{d}t, \tag{17}$$

where profits are reinvested, similar to Pástor and Veronesi (2012).

The SDF Λ_t evolves according to

$$\frac{\mathrm{d}\Lambda_t}{\Lambda_t} = -r_{f,t}\mathrm{d}t - \eta_t \mathrm{d}W_t, \tag{18}$$

where η_t is the endogenous market price of risk. Given that the technology, the budget constraint, and the collateral constraint are all linear in $a_{i,t}$, the value $J_{i,t}$ is also linear in $a_{i,t}$:

$$J_{i,t} \equiv J_t(a_{i,t}, z_{i,t}) = \xi_t(z_{i,t}) a_{i,t}, \tag{19}$$

where $\xi_{i,t} \equiv \xi_t(z_{i,t})$ captures the marginal value of capital to the manager, which depends on $z_{i,t}$ and the aggregate state of the economy. The variable $\xi_{i,t}$ evolves as follows:

$$\frac{\mathrm{d}\xi_{i,t}}{\xi_{i,t}} = \mu_{\xi,t}(z_{i,t})\mathrm{d}t + \sigma_{\xi,t}(z_{i,t})\mathrm{d}W_t + \sigma_{w,t}(z_{i,t})\mathrm{d}W_{i,t}, \tag{20}$$

where $\mu_{\xi,t}(z_{i,t})$, $\sigma_{\xi,t}(z_{i,t})$, and $\sigma_{w,t}(z_{i,t})$ are endogenously determined in equilibrium.

By exploiting the homogeneity of $J_{i,t}$ in capital $a_{i,t}$, we derive the manager's optimal decisions. These are summarized in Lemma 1, with the proof provided in the Online Appendix II.B.

Lemma 1. *There is a cutoff $\underline{z}_t$ for being active, and factor demands are linear in capital $k_t(a,z)$:*

$$u_t(z) = \begin{cases} 1, & z \geq \underline{z}_t \\ 0 & z < \underline{z}_t \end{cases}, \qquad k_t(a,z) = \begin{cases} (1+\lambda)a, & z \geq \underline{z}_t \\ 0 & z < \underline{z}_t \end{cases}, \tag{21}$$

$$\ell_t(a,z) = (\varepsilon/p_t)^{\frac{\varepsilon}{\alpha(1-\varepsilon)}} h_t^{\frac{1}{\alpha}} z u_t(z) k_t(a,z), \text{ and} \tag{22}$$

$$x_{j,t}(a,z) = \left(p_{j,t}/p_t\right)^{\frac{1}{\nu-1}} (\varepsilon/p_t)^{\frac{1-(1-\alpha)(1-\varepsilon)}{\alpha(1-\varepsilon)}} h_t^{\frac{1-\alpha}{\alpha}} z u_t(z) k_t(a,z), \text{ for } j \in [0, N_t], \tag{23}$$

where $h_t = (1-\alpha)(1-\varepsilon)/w_t$. The productivity cutoff $\underline{z}_t$ is determined by:

$$\underline{z}_t \kappa_t = r_{f,t} + \delta_k + \sigma_k[\sigma_{\xi,t}(\underline{z}_t) - \eta_t], \text{ with } \kappa_t = \alpha(1-\varepsilon)\,(\varepsilon/p_t)^{\frac{\varepsilon}{\alpha(1-\varepsilon)}} h_t^{\frac{1-\alpha}{\alpha}}. \tag{24}$$

At time t, only firms with $z_{i,t} \geq \underline{z}_t$ produce, and these firms rent the maximal amount $\widehat{a}_{i,t} = \lambda a_{i,t}$ allowed by the collateral constraint. In equation (24), the cutoff $\underline{z}_t$ is determined such that the marginal return $\underline{z}_t \kappa_t$ is equal to the marginal cost of leased capital, which includes the locally deterministic user cost of capital $r_{f,t} + \delta_k$ and a stochastic term $\sigma_k\left[\sigma_{\xi,t}(\underline{z}_t) - \eta_t\right]$, reflecting the firm's exposure to aggregate risk.

Using Lemma 1, equation (17) can be simplified as[12]

$$\mathrm{d}I_{i,t} = (1+\lambda)\left(\kappa_t z_{i,t}\mathrm{d}t - \mathrm{d}\Delta_t - r_{f,t}\mathrm{d}t\right) a_{i,t} \mathbb{1}_{z_{i,t} \geq \underline{z}_t} + (r_{f,t} - \rho) a_{i,t}\mathrm{d}t. \tag{25}$$

2.5 Equilibrium and Aggregation

The dividend intensity D_t is given by

$$D_t = \rho A_t + \int_{j=0}^{N_t} \pi_{j,t}\mathrm{d}j - S_t, \tag{26}$$

where A_t is the aggregate capital held by firms in the final goods sector, given by

$$A_t = \int_0^\infty \int_0^\infty a\varphi_t(a,z)\mathrm{d}a\mathrm{d}z. \tag{27}$$

Definition 2.1 (Competitive Equilibrium)**.** *At any given time t, the competitive equilibrium of the economy is defined by a set of prices w_t, $r_{f,t}$, and $\{p_{j,t}\}_{j=0}^{N_t}$, along with their corresponding quantities, such that*

(i) each firm i in the final goods sector maximizes (16) by choosing $\widehat{a}_{i,t}$, $u_{i,t}$, $\ell_{i,t}$, and $\{x_{i,j,t}\}_{j=0}^{N_t}$,

[12]Similar to the approach in Moll (2014), the drift term in the capital accumulation equation is proportional to the firm's capital $a_{i,t}$. This relationship directly results from the constant payout ratio as specified in equation (14) and the CRS production technology, outlined in equation (1), given a specific N_t.

subject to (14), (15), and (17), given the equilibrium prices;

(ii) each intermediate goods producer j maximizes (5) by choosing $p_{j,t}$ for $j \in [0, N_t]$;

(iii) the equilibrium R&D expenditure S_t is determined by equation (9);

(iv) the SDF Λ_t is given by equation (13) and the risk-free rate $r_{f,t}$ is determined by

$$r_{f,t} = -\frac{1}{dt}\mathbb{E}_t\left[\frac{d\Lambda_t}{\Lambda_t}\right]; \tag{28}$$

(v) the labor market-clearing condition determines w_t:

$$L_t = \int_{\underline{z}_t}^{\infty}\int_0^{\infty} \ell_t(a,z)\varphi_t(a,z)dadz; \tag{29}$$

(vi) the leased capital market-clearing condition determines the representative agent's bond holdings B_t:

$$B_t = \int_0^{\infty}\int_0^{\infty} \widehat{a}_t(a,z)\varphi_t(a,z)dadz. \tag{30}$$

The aggregate capital K_t is the sum of capital in the final goods sector A_t and bonds B_t

$$K_t = \int_0^{\infty}\int_0^{\infty} k_t(a,z)\varphi_t(a,z)dadz = A_t + B_t. \tag{31}$$

(vii) the resource constraint is satisfied because of Walras's law.

Because firms' problem is linear in capital $a_{i,t}$, we introduce the capital share $\omega_t(z)$ to fully characterize the distribution of firms in the final goods sector:

$$\omega_t(z) \equiv \frac{1}{A_t}\int_0^{\infty} a\varphi_t(a,z)\mathrm{d}a. \tag{32}$$

Intuitively, the capital share $\omega_t(z)$ plays the role of a density and captures the share of firms' capital held by each productivity type z. We define the analogue of the corresponding cumulative distribution function (CDF) as follows

$$\Omega_t(z) \equiv \int_0^{z} \omega_t(z')\mathrm{d}z'. \tag{33}$$

To ensure well-behaved equilibrium growth, as in standard growth models, we need output Y_t to be homogenous of degree one in the accumulating factors N_t and K_t, i.e., $\frac{(1-\nu)\varepsilon}{\nu(1-\varepsilon)} + \alpha = 1$, as in Kung and Schmid (2015). For the remainder of the paper, we assume this parameter restriction.

Proposition 2. *At time $t \geq 0$, given $\omega_t(z)$, the equilibrium aggregate output is*

$$Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha}, \tag{34}$$

where Z_t is the economy's TFP, given by

$$Z_t = (\varepsilon\nu)^{\frac{\varepsilon}{1-\varepsilon}} H_t N_t^{1-\alpha} \;\; with \;\; H_t = \left[\frac{\int_{\underline{z}_t}^{\infty} z\omega_t(z)dz}{1-\Omega_t(\underline{z}_t)} \right]^{\alpha}. \tag{35}$$

The variable H_t captures the endogenous productivity of the final goods sector. Factor prices are

$$p_{j,t} = 1/\nu \;\; for \;\; j \in [0, N_t], \;\; p_t = N_t^{\frac{\nu-1}{\nu}}/\nu, \;\; and \;\; w_t = (1-\alpha)(1-\varepsilon)Y_t/L_t. \tag{36}$$

The aggregate profits of the intermediate goods sector and R&D expenditure are, respectively,

$$N_t\pi_t = (1-\nu)\varepsilon Y_t \;\; and \;\; S_t = (\chi q_t)^{\frac{1}{h}} N_t. \tag{37}$$

Equation (35) shows that TFP depends on both the knowledge stock N_t and the final goods sector's productivity H_t, which is the average firm-level productivity z weighted by $\omega_t(z)$.[13] The value of H_t is higher when more productive firms are associated with more capital, which reflects a more efficient allocation of capital across firms.

3 Model Solution and Mechanism

In Section 3.1, we present a parametric approximation of the firm distribution. This approximation allows us to derive an endogenous state variable that captures the misallocation of production capital within the model economy. In Sections 3.2 and 3.3, we explore the economy's evolution under the influence of aggregate shocks and describe the deterministic balanced growth path in the absence of aggregate shocks, respectively. Finally, in Section 3.4, we leverage the deterministic balanced growth path to highlight the key theoretical mechanism underlying our model. We show that a one-time shock exerts an endogenous and lasting impact on misallocation, thereby engendering persistent transitional dynamics in the aggregate growth rate.

[13]Equation (35) is related to the industry-level TFP formula derived by Hsieh and Klenow (2009). The key difference is that in our model, firms in the final goods sector produce homogeneous goods, whereas firms in the model of Hsieh and Klenow (2009) produce differentiated goods. In Online Appendix IV, we show that by driving the elasticity of substitution among goods to infinity and wedges to 0, the industry-level TFP formula of Hsieh and Klenow (2009) coincides with our productivity H_t in equation (35).

3.1 Parametric Approximation: Misallocation as a State Variable

The capital share $\omega_t(z)$ is an infinite-dimensional object that evolves endogenously. This makes our general equilibrium model with aggregate shocks intractable. Rather than solving the model using the standard numerical approximation methods developed in the literature (e.g., Krusell and Smith, 1998), we propose a parametric approximation of $\omega_t(z)$, which serves three purposes. First, it yields a simple endogenous state variable that intuitively captures the misallocation of capital in the final goods sector. Second, it enables us to clearly illustrate the relationship between misallocation dynamics and aggregate growth dynamics, thereby making it easier to demonstrate the pivotal mechanism linking production capital misallocation with the low-frequency component of economic growth.[14] Third, It facilitates an analytical characterization of the model economy's evolution, rendering the computation of model dynamics highly tractable. In Online Appendix III, we assess the accuracy of our parametric approximation and offer in-depth discussions on its relationship with numerical approximation methods previously developed in the literature.[15]

Specifically, at any time $t \geq 0$, we approximate the distribution of log capital $\widetilde{a}_{i,t} = \ln a_{i,t}$ and log productivity $\widetilde{z}_{i,t} = \ln z_{i,t}$ across firms in the final goods sector using a bivariate normal distribution. This assumption is similar in spirit to the bivariate log-normal distribution of the skills of matched young and old agents in the model of Jovanovic (2014). With this parametric assumption, Jovanovic (2014) derives analytical transitional dynamics to cleanly characterize the link between misallocation in the labor market and growth.

This approximation is intuitive because according to equation (4), in the stochastic steady state, we have $\widetilde{z}_{i,t} \sim N(0, \sigma^2/2)$ in the cross section of firms. Moreover, using the Berry-Esseen bound, it holds heuristically that $\widetilde{a}_{i,t}$ across firms approximately follows a normal distribution (see Online Appendix III.A). This joint log-normal approximation enables the derivation of a closed-form formula for $\omega_t(z)$.

Proposition 3. *For any $t \geq 0$, the capital share $\omega_t(z)$ can be approximated by the PDF of a*

[14] The idea of using tractable parametric approximations to deliver key model mechanisms is similar in spirit to several influential works in the finance literature. For example, Campbell and Shiller (1988*b*) propose log-linear present value approximations to decompose the impact of discount-rate news and cash-flow news on stock valuations. Gabaix (2007, 2012) develops the class of "linearity-generating" processes to achieve analytical convenience when revisiting a set of macro-finance puzzles.

[15] Under the baseline calibration, our solutions closely match those obtained through standard numerical approximation methods, which directly track the evolution of $\omega_t(z)$ by utilizing higher-order moments in both the deterministic balanced growth path and the stochastic steady state. Our parametric approximation method shares a similar philosophy with these standard methods, in that it uses a selected number of moments to encapsulate the infinite-dimensional cross-sectional distribution of agents or firms. The key distinction, however, lies in our approach's direct application of a parametric functional form to delineate the distribution at any given time. This approach enables us to derive closed-form equations for the evolution of these moments. In contrast, with standard numerical approximation methods, the evolution of these moments cannot be characterized in closed form.

log-normal distribution,

$$\omega_t(z) = \frac{1}{z\sigma\sqrt{\pi}} \exp\left[-\frac{(\ln z + \sigma^2 M_t/2)^2}{\sigma^2}\right], \tag{38}$$

where $M_t \equiv -Cov(\widetilde{z}_{i,t}, \widetilde{a}_{i,t})/\mathrm{var}(\widetilde{z}_{i,t}) = -2Cov(\widetilde{z}_{i,t}, \widetilde{a}_{i,t})/\sigma^2$.

Intuitively, Proposition 3 implies that under our approximation, the endogenous state variable $M_t \equiv -\mathrm{Cov}(\widetilde{z}_{i,t}, \widetilde{a}_{i,t})/\mathrm{var}(\widetilde{z}_{i,t})$ is a sufficient statistic that characterizes the evolution of $\omega_t(z)$. We further characterize the economy's TFP Z_t in closed form.

Proposition 4. *Under our approximation, the aggregate TFP* Z_t *is*

$$Z_t = (\varepsilon\nu)^{\frac{\varepsilon}{1-\varepsilon}} N_t^{1-\alpha} \left[(1+\lambda)\frac{A_t}{K_t} \exp\left(-\frac{\sigma^2}{2}M_t + \frac{\sigma^2}{4}\right) \Phi\left(\Phi^{-1}\left(\frac{1}{1+\lambda}\frac{K_t}{A_t}\right) + \frac{\sigma}{\sqrt{2}}\right)\right]^{\alpha}, \tag{39}$$

where $\Phi(\cdot)$ *represents the CDF of a standard normal variable.*

Equation (39) shows that the economy's TFP, Z_t, strictly decreases with the endogenous state variable M_t, holding aggregate variables A_t, K_t, and N_t fixed. Thus, M_t reflects the degree of misallocation in our model economy. In fact, M_t also directly reflects the distribution of MRPK. To elaborate, substituting out labor and intermediate inputs in firms' technology using Lemma 1, we obtain

$$y_{i,t} = v_{i,t}k_{i,t}, \text{ with } v_{i,t} = (\varepsilon/p_t)^{\frac{\varepsilon}{\alpha(1-\varepsilon)}} h_t^{\frac{1-\alpha}{\alpha}} z_{i,t}. \tag{40}$$

Because final goods are the numeraire, $v_{i,t}$ measures firm i's MRPK at t. Define $\widetilde{v}_{i,t} = \ln v_{i,t}$. We obtain a theoretically motivated measure for misallocation:

$$M_t \equiv -\frac{\mathrm{Cov}(\widetilde{z}_{i,t}, \widetilde{a}_{i,t})}{\mathrm{var}(\widetilde{z}_{i,t})} = -\frac{\mathrm{Cov}(\widetilde{v}_{i,t}, \widetilde{a}_{i,t})}{\mathrm{var}(\widetilde{v}_{i,t})}, \ \forall\ t \geq 0. \tag{41}$$

Intuitively, in our model, the covariance between productivity and production capital, $\mathrm{Cov}(\widetilde{z}_{i,t}, \widetilde{a}_{i,t})$, is fundamentally akin to the covariance between MRPK and capital, $\mathrm{Cov}(\widetilde{v}_{i,t}, \widetilde{a}_{i,t})$, given that firms produce homogeneous goods using a CRS technology. A higher M_t reflects that firms with higher productivity ($z_{i,t}$) or MRPK ($v_{i,t}$) are linked to a lower level of production capital ($a_{i,t}$), which, according to Proposition 4, results in a diminished TFP.

Relation to Existing Empirical Measures of Misallocation. Our model-implied misallocation measure M_t is similar to the capital allocation efficiency measure based on the covariance between size and productivity (e.g., Olley and Pakes, 1996; Bartelsman, Haltiwanger and Scarpetta, 2009, 2013).[16] The state variable M_t constructed in equation (41) provides a theoretical justification for using the size-productivity covariance as a measure of capital allocation efficiency. In particular, our model analytically characterizes that a higher M_t (i.e., a lower covariance) reduces aggregate TFP (see equation (39)). Moreover, under the parametric approximation of our model economy, M_t sufficiently summarizes the cross-sectional distribution of firms, $\omega_t(z)$, thereby highlighting the significant role of production capital misallocation as an endogenous state variable.

The covariance-type measure for misallocation is fundamentally similar to the dispersion measures employed in the misallocation literature, such as the dispersion of revenue TFP or MRPK (e.g., Foster, Haltiwanger and Syverson, 2008; Hsieh and Klenow, 2009). These measures assess the effects of capital allocation efficiency on aggregate TFP and are grounded in the crucial assumption that allocation distortions occur when the marginal revenue product of a production factor diverges from its marginal cost. Bartelsman, Haltiwanger and Scarpetta (2013) offer evidence that the relationship between size and productivity is more resistant to multiplicative measurement errors compared to the dispersion measures. They argue that classical measurement errors typically inflate the standard deviation of measured MRPK but leave the measured covariance unaffected. Similarly, Eisfeldt and Shi (2018) contend that due to the noisiness of productivity dispersion measures, they may not be informative for capturing business cycle variations in misallocation frictions. This aspect is especially relevant to our research, as we seek to investigate the asset pricing implications of misallocation and its influence on economic growth fluctuations. In Section 4.1, we construct a model-consistent empirical measure of misallocation based on the covariance-type measure M_t, as outlined in equation (41).

3.2 Evolution of the Economy

Under the parametric approximation, the economy's transitional dynamics are characterized by the evolution of aggregate capital A_t in the final goods sector, the knowledge stock N_t, and misallocation M_t, as summarized in the proposition below.

Proposition 5. *Under our parametric approximation, for all $t \geq 0$, the economy is fully characterized*

[16] Olley and Pakes (1996) decompose total productivity into the unweighted average of plant-level productivities and the sample covariance between productivity and the share of output. They argue that the latter captures capital allocation efficiency because a higher covariance implies that a higher share of output goes to more productive firms. Bartelsman, Haltiwanger and Scarpetta (2009, 2013) generalize this measure by focusing on the covariance between firm-level log productivity and size, where productivity is measured by physical TFP, revenue TFP, or labor productivity and size is measured by the amount of physical output, revenue, or input. They show that the size-productivity relationship carries over to these alternative measures of size and productivity in a class of models.

by the evolution of A_t, N_t, and M_t, as follows

$$dA_t = \left[\alpha(1-\varepsilon)Y_t - \delta_k K_t - \delta_a A_t - r_{f,t}B_t - \rho A_t\right] dt - (\sigma_k K_t - \sigma_{a,t}A_t)dW_t, \tag{42}$$

$$dN_t = \chi\left(\chi q_t\right)^{\frac{1-h}{h}} N_t dt - \delta_b N_t dt, \tag{43}$$

$$dM_t = -\theta M_t dt - \; Cov(\widetilde{z}_{i,t}, d\widetilde{a}_{i,t})/\mathrm{var}(\widetilde{z}_{i,t}), \tag{44}$$

where $K_t = (1+\lambda)\left[1-\Omega_t(\underline{z}_t)\right] A_t$, $B_t = K_t - A_t$, and $Cov(\widetilde{z}_{i,t}, d\widetilde{a}_{i,t})$ is given by equation (IA.67) in Online Appendix II.F.

Define $E_t = N_t/A_t$ as the knowledge stock-capital ratio. Because the economy is homogeneous of degree one in A_t, the three state variables (A_t, N_t, M_t) can be further reduced to two state variables (E_t, M_t).

In equation (42), the last term $(\sigma_k K_t - \sigma_{a,t}A_t)\mathrm{d}W_t$ captures the variation in A_t due to aggregate shocks. As our aim is to theoretically demonstrate the asset pricing implications of misallocation, we seek to create a setting devoid of confounding effects from other channels. Hence, we adopt the technical specification $\sigma_{a,t} = K_t/A_t\sigma_k$. This specification ensures that the evolution of the aggregate capital stock is locally deterministic, allowing economic fluctuations in our model to be purely driven by the variation in misallocation M_t in equation (44). This setup enables a focused analysis on the role of M_t.[17]

Equation (44) shows that the evolution of M_t depends on two terms. The first term $-\theta M_t \mathrm{d}t$ is related to the evolution of idiosyncratic productivity $z_{i,t}$ (see equation (4)). Intuitively, a higher θ implies a less persistent $z_{i,t}$, which drives misallocation $M_t = -\mathrm{Cov}(\widetilde{z}_{i,t}, \widetilde{a}_{i,t})/\mathrm{var}(\widetilde{z}_{i,t})$ towards zero more rapidly, thereby making M_t less persistent. The second term, $\mathrm{Cov}(\widetilde{z}_{i,t}, \mathrm{d}\widetilde{a}_{i,t})/\mathrm{var}(\widetilde{z}_{i,t})$, captures the impact of capital accumulation, $\mathrm{d}\widetilde{a}_{i,t}$, as described by equation (2). A higher $\mathrm{Cov}(\widetilde{z}_{i,t}, \mathrm{d}\widetilde{a}_{i,t})$ implies that more productive firms accumulate their capital at a higher rate, which reduces misallocation M_t. The variable $\mathrm{Cov}(\widetilde{z}_{i,t}, \mathrm{d}\widetilde{a}_{i,t})$ negatively depends on the aggregate shock $\mathrm{d}W_t$ (see equation (IA.67) in Online Appendix II.F). Intuitively, a positive shock ($\mathrm{d}W_t > 0$) increases the depreciation rate of capital $k_{i,t}$, which reduces the capital accumulation of more productive firms (i.e., $z_{i,t} \geq \underline{z}_t$) but not that of less productive firms (i.e., $z_{i,t} < \underline{z}_t$), which do not produce (see equation (21)). As a result, a positive shock leads to a lower $\mathrm{Cov}(\widetilde{z}_{i,t}, \mathrm{d}\widetilde{a}_{i,t})$ and increases misallocation M_t, which, in turn, reduces aggregate output and consumption, indicating that M_t is countercyclical.

[17] By excluding other channels, our technical specification might exaggerate the quantitative impact of misallocation. With this consideration in mind, we clarify that the aim of our quantitative analysis in Section 4 is not to precisely identify the contribution of misallocation to growth fluctuations. Rather, our objective is to show that, within a reasonably calibrated model, slow-moving misallocation can induce significant low-frequency fluctuations in growth. These fluctuations, in turn, can explain the high Sharpe ratio observed in the capital market and result in considerable welfare costs.

3.3 Deterministic Balanced Growth Path

To clearly illustrate the equilibrium relationship between misallocation and long-run growth, we characterize the economy's deterministic balanced growth path in the absence of aggregate shocks (i.e., $\mathrm{d}W_t \equiv 0$).

Proposition 6. *There is a deterministic balanced growth path on which $E_t \equiv E$, $M_t \equiv M$, and $H_t \equiv H$ are constant. The aggregate capital A_t, knowledge stock N_t, output Y_t, TFP Z_t, and consumption C_t grow at the same constant rate g, and their ratios are constant.*

The values of these variables and the growth rate g are determined by the system of equations presented in Online Appendix II.G. We highlight that g is directly related to the marginal q of intangible capital as follows:

$$g = \chi(\chi q)^{\frac{1-h}{h}} - \delta_b. \tag{45}$$

The next proposition clearly shows that on the deterministic balanced growth path, there is a negative relationship between misallocation M and the marginal q of intangible capital.

Proposition 7. *Under our parametric approximation, the marginal q of intangible capital is negatively related to misallocation M on the deterministic balanced growth path:*

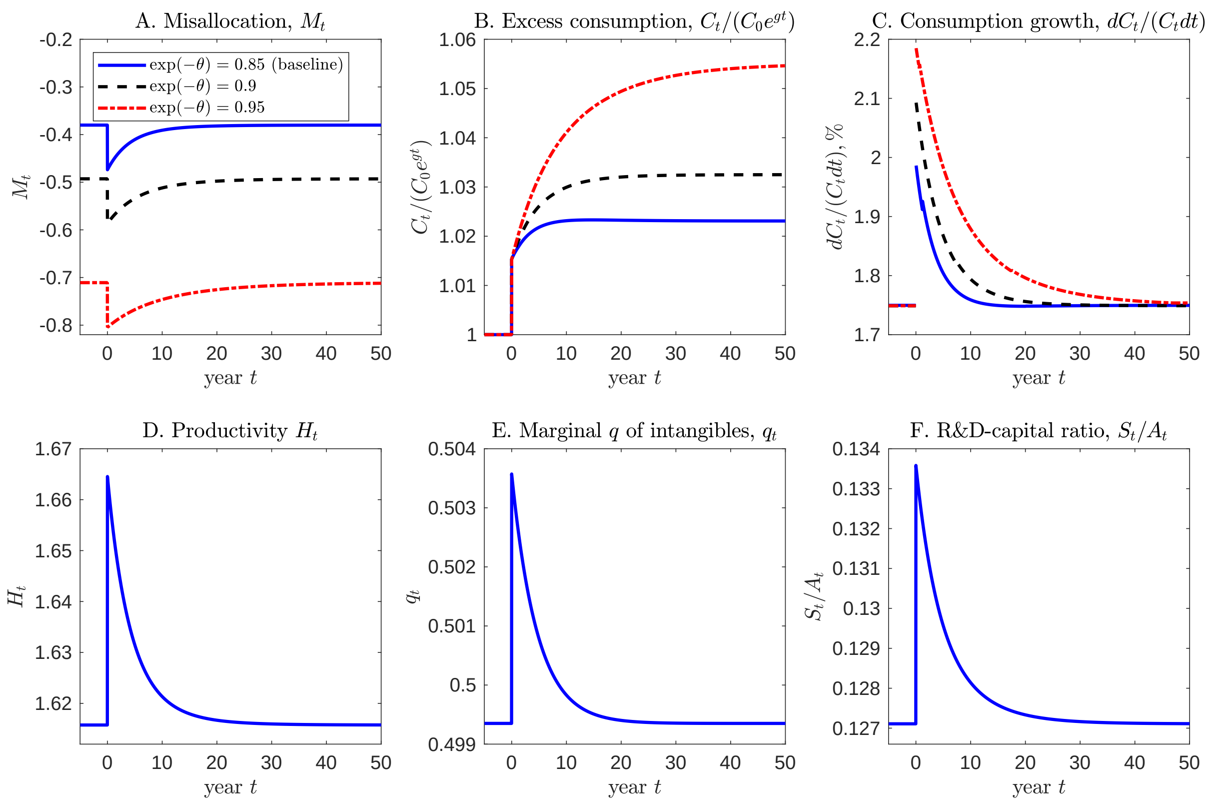
$$\begin{aligned}\ln q = &-\frac{\alpha\sigma^2}{2}M + \frac{\alpha\sigma^2}{4} + \ln\left[\frac{(1-\nu)\varepsilon(\varepsilon\nu)^{\frac{\varepsilon}{1-\varepsilon}}}{r_f+\delta_b}\right] + \alpha\ln(1+\lambda) - \alpha\ln E \\ &+ \alpha\ln\left[\Phi\left(\Phi^{-1}\left(\frac{K/A}{1+\lambda}\right) + \frac{\sigma}{\sqrt{2}}\right)\right],\end{aligned} \tag{46}$$

where K/A represents the constant ratio of K_t to A_t on the deterministic balanced growth path.

3.4 Key Mechanism: Persistence of Misallocation and Growth

In this section, we focus on the deterministic balanced growth path to illustrate the model's core mechanism. We show that a one-time shock, reducing the misallocation level at $t = 0$, induces an endogenous and persistent effect on misallocation M_t from $t = 0$ onwards. This effect, in turn, triggers a long-lasting influence on aggregate growth by affecting the marginal q of intangible capital (see Proposition 7), and consequently, the R&D-capital ratio – a critical driver of economic growth. Moreover, we show that the persistence of aggregate growth depends on the persistence of misallocation, which depends largely on the persistence of idiosyncratic productivity.

Impulse Response Function. Consider a scenario involving a one-time, unexpected shock that exogenously reduces misallocation M_t at $t = 0$. Consequently, the economy begins at



Note: Consider an unexpected shock that reduces misallocation M_t by $\sigma[M_t] = 0.09$ at $t = 0$. Panels A, B, and C plot the transitional dynamics of misallocation M_t, excess consumption $C_t/(C_0e^{gt})$, and the contemporaneous consumption growth rate $\mathrm{d}C_t/(C_t\mathrm{d}t)$ when θ is calibrated at different values. For each choice of θ, we recalibrate the parameter χ so that the consumption growth rate in the deterministic balanced growth path is the same as our baseline calibration. All other parameters are set according to our calibration in Table 1. Panels D, E, and F plot the transitional dynamics of the final goods sector's productivity H_t, the marginal q of intangible capital, q_t, and R&D-capital ratio, S_t/A_t, for the baseline calibration with $e^{-\theta} = 0.85$.

Figure 2: Transitional dynamics after a one-time shock to misallocation M_t.

an anomalously low level of production capital misallocation due to this shock. From $t = 0$ onward, it will gradually converge back to the deterministic balanced growth path. The blue solid lines in Figure 2 illustrate the transitional dynamics of several key variables from $t = 0$ onward, based on our benchmark calibration (see Table 1). To render the quantitative effects informative, the magnitude of the shock is set to 0.09, aligning with the standard deviation of M_t in our calibration. As depicted in Panel A, misallocation M_t will experience an extended endogenous transitional period, lasting about 20 years, before it reaches the level in the deterministic balanced growth path.

In the absence of aggregate shocks, aggregate consumption would follow C_0e^{gt}, growing at a constant annual rate of $g = 1.75\%$ for all $t \geq 0$. To remove the deterministic trend in C_t and focus on the fluctuation in growth rates, we consider excess consumption, defined as $C_t/(C_0e^{gt})$. The blue solid line in Panel B indicates that excess consumption $C_t/(C_0e^{gt})$

is 1 before the shock, jumps to approximately 1.015 at the moment the shock hits at $t = 0$, and gradually increases until reaching the level in the deterministic balanced growth path. Although the shock to misallocation is transitory, the economy shifts to a steady state with permanently higher consumption, driven by the endogenous accumulation of capital A_t and knowledge stock N_t. Panel C demonstrates a similar concept by displaying the contemporaneous consumption growth rate over the interval $[t, t + \mathrm{d}t)$, calculated as $\mathrm{d}C_t/(C_t\mathrm{d}t)$. The blue solid line illustrates that the consumption growth rate spikes to about 1.98% at the onset of the shock at $t = 0$ and then slowly adjusts to the level in the deterministic balanced growth path as misallocation persists.

The mechanism connecting misallocation to growth is depicted by the black arrows in Figure 1. Specifically, a decrease in misallocation, M_t, directly enhances the productivity, H_t, of the final goods sector, as shown in Panel D of Figure 2. An elevated H_t boosts aggregate output, Y_t, which in turn increases the marginal q of intangible capital (refer to Panel E of Figure 2 and equations (7) and (37)), encouraging more R&D activities (illustrated in Panel F of Figure 2). This chain of effects culminates in a higher rate of economic growth via the expansion of the knowledge stock, N_t.

Role of the Persistence of Idiosyncratic Productivity. As discussed, it is the persistence of misallocation M_t, particularly through its impact on R&D, that drives the persistent excess consumption growth relative to the deterministic balanced growth path. As shown in equation (44), the persistence of misallocation depends on θ, which governs the persistence of $z_{i,t}$. To further illustrate the relationship between the persistence of misallocation and the persistence of aggregate consumption growth, we study the transitional dynamics under different values of θ. Specifically, according to equation (4), the yearly autocorrelation in $\ln z_{i,t}$ is $e^{-\theta}$. In Panels A, B and C of Figure 2, we compare our baseline calibration of $e^{-\theta} = 0.85$ with two alternative calibrations in which the yearly autocorrelation in $\ln z_{i,t}$ is 0.9 (black dashed line) and 0.95 (red dash-dotted line), respectively.

Panel A demonstrates that calibrations with a higher persistence of $z_{i,t}$ result in lower misallocation M_t on the deterministic balanced growth path, aligning with the insights provided by Buera and Shin (2011) and Moll (2014). Crucially, the convergence speed of M_t to its deterministic balanced growth path slows as the persistence of $z_{i,t}$ increases. As a measure to capture this phenomenon, we compute the half-life of transitions, which is the time it takes for M_t to revert to half of its long-term value post-shock. The half-life of M_t is 3.0, 4.2, and 6.9 years for $e^{-\theta} = 0.85$, 0.9, and 0.95, respectively, indicating that misallocation becomes more persistent when idiosyncratic productivity is more persistent. Comparing the three curves in Panels B and C, it is clear that the economy with a higher persistence of $z_{i,t}$ has more persistent consumption growth after the shock to M_t.

Thus, our model suggests that the persistence of idiosyncratic productivity $z_{i,t}$ plays an important role in determining the persistence of the growth rate of aggregate consumption, $\mathrm{d}C_t/(C_t\mathrm{d}t)$. The persistence levels of these two variables are connected via the persistent endogenous misallocation M_t. This result generalizes the key insight of Moll (2014) to an economy with stochastic growth. In a model without long-run growth or aggregate shocks, Moll (2014) shows that the transition to steady states slows down as idiosyncratic productivity shocks become more persistent. Building on this insight, we additionally demonstrate that in a model with endogenous stochastic growth, the persistence of idiosyncratic productivity shapes the persistence of aggregate growth by affecting the persistence of endogenous misallocation.

3.5 Growth Fluctuations and Discount Rates

As illustrated by the black arrows in Figure 1 and the impulse responses in Figure 2, on the deterministic balanced growth path without aggregate shocks, misallocation affects growth through its impact on the marginal q of intangible capital, which determines aggregate R&D expenditure. In the full model with aggregate shocks, the link between the marginal q of intangible capital and growth is amplified by countercylical discount rates (risk premium) through the valuation channel, as illustrated by the red arrows in Figure 1.

In our model, recessions are the periods with high misallocation, during which firms in the final goods sector are financially constrained. The aggregate output is low and particularly volatile. Thus, recessions are times with low expected consumption growth and high macroeconomic uncertainty, which implies high risk premium. Specifically, the conditional volatility of the one-year growth rates of consumption C_t and that of SDF Λ_t are strongly positively correlated with misallocation M_t and negatively correlated with one-year expected consumption growth rate, with correlation coefficients given by:

$$corr[\sigma_t[\Delta \ln C_{t+1}], M_t] = 0.91 \quad \text{and} \quad corr[\sigma_t[\Delta \ln \Lambda_{t+1}], M_t] = 0.93,$$
$$corr[\sigma_t[\Delta \ln C_{t+1}], \mathbb{E}_t[\Delta \ln C_{t+1}]] = -0.88 \quad \text{and} \quad corr[\sigma_t[\Delta \ln \Lambda_{t+1}], \mathbb{E}_t[\Delta \ln C_{t+1}]] = -0.89,$$

where $\Delta \ln X_t = \ln X_t - \ln X_{t-1}$ represents the difference in $\ln X_t$ between year t and year $t-1$; the yearly value of X_t is computed by integrating $X_t\mathrm{d}t$ in continuous time. Thus, the model implies countercylical macroeconomic uncertainty and risk premium.

The countercyclical risk premium amplifies the variation in the marginal q of intangible capital. To see this, note that the marginal q of intangible capital, q_t, is determined by equation (7). During recessions with high misallocation, q_t is depressed not only because of reduced profits π_t but also because future profits are discounted at a higher discount rate due to increased macroeconomic uncertainty. This generates large variation in q_t, which in turn generates significant variation in aggregate consumption growth (see equation (45)), capturing the valuation channel illustrated in Figure 1.

Quantitatively, more than half of the volatility of q_t is attributed to the countercyclical risk premium while the remaining is due to procycical profits π_t. Following the theoretical mechanism elaborated in Section 3.4, this valuation channel is quantitatively significant because the fluctuations in misallcocation driven by the aggregate shocks $\mathrm{d}W_t$ are persistent, when the parameter θ is calibrated to match the persistence of idiosyncratic productivity shocks in the data (see Table 1). The slow-moving misallocation in turn generates low-frequency fluctuations in economic growth and macroeconomic uncertainty, thereby generating a high Sharpe ratio in the capital market (see Section 4.4) and significant welfare losses (see Section 4.5).

4 Quantitative Analysis

In this section, we explore the quantitative effects of misallocation on economic growth, asset prices, and welfare. Section 4.1 develops an empirical measure of misallocation, motivated by our theoretical model. Section 4.2 focuses on calibrating and validating the model against observed macroeconomic and asset pricing moments. Section 4.3 presents evidence that misallocation is a significant predictor of R&D expenditure and future economic growth, both in the data and in our model. Section 4.4 investigates how misallocation influences asset pricing. Finally, Section 4.5 calculates the welfare costs resulting from growth fluctuations driven by misallocation.

4.1 Data and Empirical Measures

We obtain annual consumption and GDP data from the U.S. Bureau of Economic Analysis (BEA) and stock return data from the Center for Research in Security Prices (CRSP). Consumption and output growth are measured by the log growth rate of per-capita real personal consumption expenditures on nondurable goods and services and the log growth rate of per-capita real GDP. The nominal variables are converted to real terms using the consumer price index (CPI). We obtain data on private business R&D investment from the National Science Foundation (NSF) and on R&D stock from the Bureau of Labor Statistics (BLS). These two time series are considered empirical counterparts for S_t and N_t, respectively. The ratio of the two (i.e., S_t/N_t) is our empirical measure of R&D intensity. The risk-free rate is constructed using the yield of 3-month Treasury Bills, obtained from CRSP. Firms' dividend yield is computed as the ratio of total dividends over market capitalization, obtained from Compustat.

Model-Consistent Empirical Measure of Misallocation. We construct a model-consistent empirical measure of misallocation according to equation (41), $M_t = -\mathrm{Cov}(\widetilde{v}_{i,t}, \widetilde{a}_{i,t})/\mathrm{var}(\widetilde{v}_{i,t})$. Specifically, we regress the empirical measure of log capital $\widetilde{a}_{i,t}$ on log MRPK $\widetilde{v}_{i,t}$ using the cross section of firms in each year t in U.S. Compustat data from 1965 to 2016:[18]

$$\widetilde{a}_{i,t} = \alpha_t + \beta_t \widetilde{v}_{i,t} + \varepsilon_{i,t}, \tag{47}$$

where the estimated coefficient $\widehat{\beta}_t$ directly captures $\mathrm{Cov}(\widetilde{v}_{i,t}, \widetilde{a}_{i,t})/\mathrm{var}(\widetilde{v}_{i,t})$. The empirical measure of M_t is constructed using the HP-filtered time series of $-\widehat{\beta}_t$ from 1965 to 2016 with a smoothing parameter of 100 (Backus and Kehoe, 1992). The HP filter allows us to extract the cyclical component of capital misallocation fluctuations, following the literature (e.g., Eisfeldt and Rampini, 2006). In the regression specification (47), the empirical measure of log capital $\widetilde{a}_{i,t}$ is constructed using the average log capital of firm i over the past T years, i.e., $\widetilde{a}_{i,t} \equiv T^{-1} \sum_{\tau=1}^{T} \ln\left(production\_capital_{i,t+1-\tau}\right)$, with $T = 3$. The empirical results are robust to alternative choices of T. Firm i's production capital is measured by its net property, plant and equipment (PPENT), and thus, $production\_capital_{i,t} = ppent_{i,t}$.[19] We construct the empirical measure for $\widetilde{v}_{i,t}$ using the average log $MRPK_{i,t}$ of firm i over the past T years, that is, $\widetilde{v}_{i,t} \equiv T^{-1} \sum_{\tau=1}^{T} \ln\left(MRPK_{i,t+1-\tau}\right)$. In accordance with equation (40), we define $MRPK_{i,t}$ as $MRPK_{i,t} = sale_{i,t}/(ppent_{i,t} + rented\_capital_{i,t})$. This formulation closely follows our theoretical model, where the total production capital is measured by the sum of the firm's own production capital ($ppent_{i,t}$) and rented capital ($rented\_capital_{i,t}$). The quantity of rented capital is determined by capitalizing rental expenses, adhering to established accounting conventions and literature precedents (e.g., Rauh and Sufi, 2011; Rampini and Viswanathan, 2013). Specifically, the rented capital for firm i in year t is calculated as its total rental expenses for the year, multiplied by a factor of 10, and is limited to a maximum of 0.25 times $ppent_{i,t}$.[20]

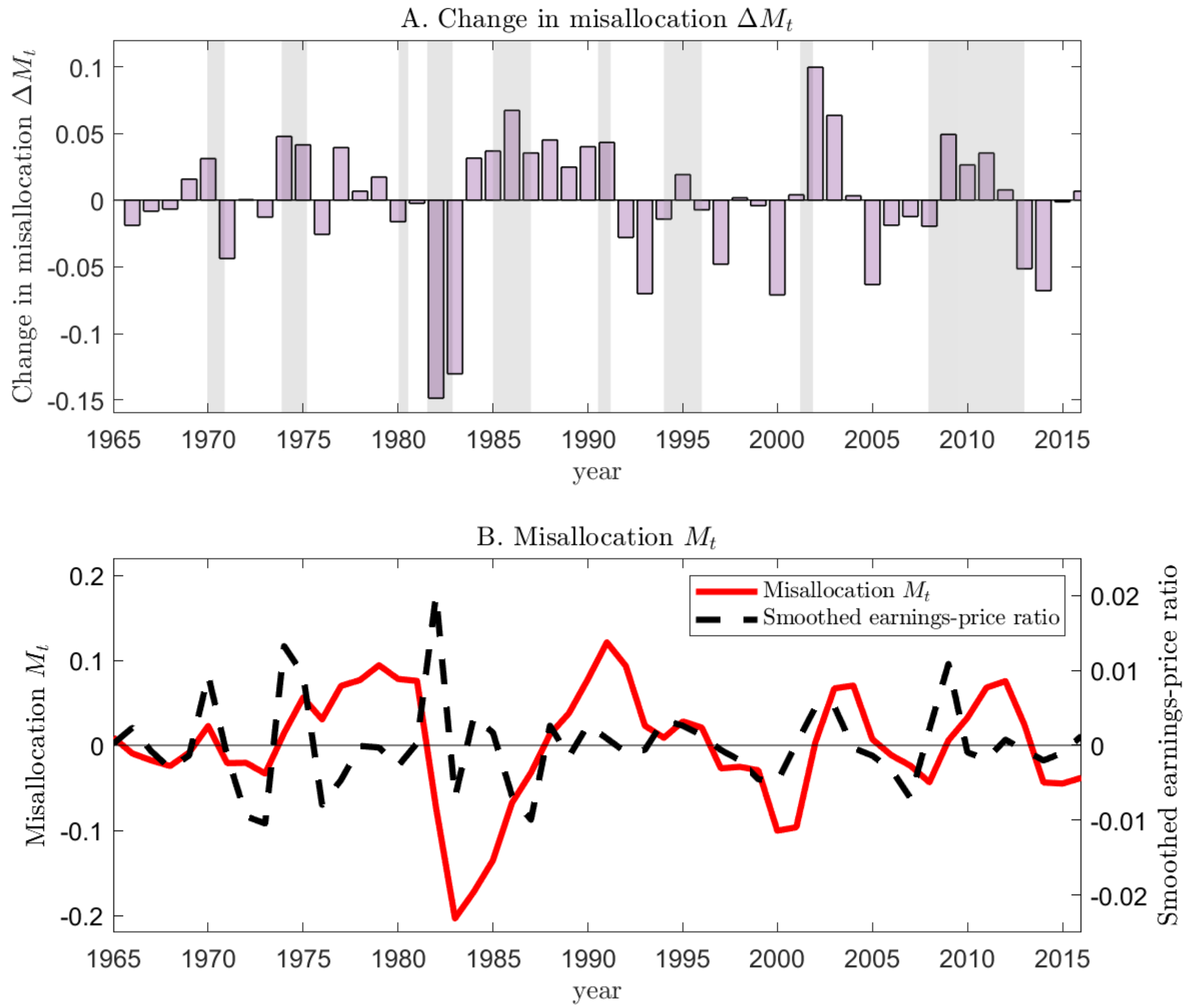
Panel A of Figure 3 plots the time series of year-on-year changes in the empirical measure of misallocation, denoted as ΔM_t. The shaded areas represent periods of economic downturns, including economic recessions and three financial crises.[21] Aligned with our the-

[18]Because our theory mainly applies to manufacturing firms, we exclude firms from financial, utility, public administration, and non-tradable industries, where non-tradable industries are defined according to Mian and Sufi (2014). The empirical results are robust if non-tradable industries are included in the sample.

[19]The robustness of our empirical findings is maintained when using a firm's tangible net worth as a proxy for its production capital, that is, $production\_capital_{i,t} = tangible\_net\_worth_{i,t}$. A firm's tangible net worth is constructed as $tangible\_net\_worth_{i,t} = ppent_{i,t} + current\_assets_{i,t} + other\_assets_{i,t} - total\_liabilities_{i,t}$. Highlighting the relevance of this measure, Chava and Roberts (2008) point out that lenders often rely on a firm's tangible net worth to evaluate its capacity to service and repay debt. Tangible net worth, serving as a key indicator of a firm's borrowing capacity, is frequently included in loan agreements, as noted by research including DeAngelo, DeAngelo and Wruck (2002), Roberts and Sufi (2009), Sufi (2009), and Prilmeier (2017).

[20]All empirical findings are consistent when using a capitalization factor of 5, 6, or 8 for rental expenses, or when capping the capitalized amount at different fractions (0, 0.5, 1, or 2) of $ppent_{i,t}$.

[21]The three crises are the savings and loan crisis from January 1986 to December 1987, the Mexican peso



Note: Panel A plots the year-on-year changes in the empirical measure of misallocation, i.e., ΔM_t. The shaded areas represent recessions or severe financial crises. Panel B plots the time series of M_t (left y-axis) and the smoothed earnings-price ratio (right y-axis) proposed by Campbell and Shiller (1988*a*).

Figure 3: Time-series plot of the empirical measure of misallocation M_t.

oretical framework and empirical evidence from the literature, capital misallocation typically escalates during economic downturns. Our empirical measure of misallocation significantly increases in seven out of the nine economic downturns we examined. This stylized pattern is consistent with the model's prediction that misallocation typically increases during a period involving macroeconomic recessions or financial turmoil.

In Panel B of Figure 3, we present a comparison between the empirical measure of misallocation M_t (illustrated by the red solid line) and the smoothed earnings-price ratio (shown as the black dashed line) introduced by Campbell and Shiller (1988*a*). The smoothed

crisis from January 1994 to December 1995, and the European sovereign debt crisis from September 2008 to December 2012.

Table 1: Parameter calibration and targeted moments.

Panel A: Externally determined parameters					
Parameter	Symbol	Value	Parameter	Symbol	Value
Capital share	α	0.33	Capital depreciation rate	δ_k, δ_a	0.03
Share of intermediate inputs	ε	0.5	$1-$ R&D elasticity	h	0.17
EIS	ψ	1.85	Risk aversion	γ	8
Patent obsolescence rate	δ_b	0.15	Volatility of idio. productivity	σ	1.39
Inverse markup	ν	0.6	Rent extraction rate	τ	0.01
Collateral constraint	λ	1.1	Persistence of idio. productivity	θ	0.1625

Panel B: Internally calibrated parameters and targeted moments					
Parameter	Symbol	Value	Moments	Data	Model
Subjective discount rate	δ	0.01	Real risk-free rate (%)	1.11	1.58
R&D productivity	χ	1.35	Consumption growth rate (%)	1.76	1.75
Capital depreciation shock	σ_k	0.19	Consumption growth vol. (%)	1.50	1.67
Dividend payout rate	ρ	0.037	Dividend yield (%)	2.35	2.14

earnings-price ratio and its variants are frequently employed as empirical proxies for the aggregate discount rate (e.g., Gourio, 2012; Hall, 2017; Dou, Ji and Wu, 2021, 2022). The time-series variation of this ratio typically aligns with the frequency of business cycles. Clearly, the empirical measure of misallocation, M_t, exhibits greater persistence compared to the smoothed earnings-price ratio, despite the two time series exhibiting positive comovement. The yearly autocorrelation of M_t is 0.75, which is close to the calibrated persistence of 0.77 that Bansal and Yaron (2004) find for the predictable component of consumption growth. If misallocation M_t affects economic growth, as suggested by our model, the highly persistent and volatile M_t appears to capture the low-frequency growth fluctuations, referred to as the medium-term business cycle by Comin and Gertler (2006) or the growth cycle by Kung and Schmid (2015). The observed positive comovement aligns with the model-implied interaction between misallocation and the discount rate.

4.2 Calibration and Validation of the Model

Panel A of Table 1 presents the externally calibrated parameters. Following standard practice, we set the capital share in production technology at $\alpha = 0.33$. We set the yearly capital depreciation rates at $\delta_k = \delta_a = 3\%$. We set the share of intermediate inputs at $\varepsilon = 0.5$ according to the estimates of Jones (2011, 2013). The inverse markup is set at $\nu = \epsilon/(\epsilon + (1-\alpha)(1-\epsilon)) = 0.6$ to guarantee the existence of a balanced growth path. Following standard practice in the asset pricing literature, we set risk aversion at $\gamma = 8$. Following Kung and Schmid (2015), we set the EIS at $\psi = 1.85$, the patent obsolescence rate

Table 2: Untargeted moments in the data and model.

Moments	Data	Model	Moments	Data	Model
		Panel A: Consumption moments			
$AC1(\Delta \ln C_t)$ (%)	0.44	0.46	$AC2(\Delta \ln C_t)$ (%)	0.08	0.28
$AC5(\Delta \ln C_t)$ (%)	-0.01	0.00	$AC10(\Delta \ln C_t)$ (%)	0.06	-0.06
$VR2(\Delta \ln C_t)$ (%)	1.52	1.46	$VR5(\Delta \ln C_t)$ (%)	2.02	2.21
		Panel B: Other moments			
$AC1(\Delta \ln S_t)$ (%)	0.30	0.42	$AC1(M_t)$ (%)	0.75	0.73
$SR[R_{m,t}]$	0.36	0.39	$\sigma[r_{f,t}]$ (%)	2.06	0.47

Note: With slight abuse of notations, $\Delta \ln X_t = \ln X_t - \ln X_{t-1}$ represents the difference in $\ln X_t$ between year t and year $t-1$, where the yearly value of X_t is computed by integrating $X_t \mathrm{d}t$ in continuous time. $ACk(\Delta \ln C_t)$ refers to the autocorrelation of log consumption growth with a k-year lag. $VRk(\Delta \ln C_t)$ refers to the variance ratio of log consumption growth with a k-year horizon. $AC1(\Delta \ln S_t)$ is the yearly autocorrelation of log private business R&D investment growth. $AC1(M_t)$ is the yearly autocorrelation of misallocation M_t. $SR[R_{m,t}] = \mathbb{E}[R_{m,t} - r_{f,t}]/\sigma[R_{m,t} - r_{f,t}]$ is the Sharpe ratio of the consumption claim.

at $\delta_b = 15\%$, and $h = 0.17$ so that the elasticity of new blueprints with respect to R&D is 0.83. We set the volatility of idiosyncratic productivity at $\sigma = 1.39$ according to the calibration of Moll (2014). The persistence of idiosyncratic productivity is set at $\theta = 0.1625$, which implies that log idiosyncratic productivity has a yearly autocorrelation of $e^{-\theta} = 0.85$, consistent with the estimate of Asker, Collard-Wexler and Loecker (2014) based on U.S. census data, as well as with the calibration in the macroeconomics literature (e.g., Khan and Thomas, 2008; Moll, 2014). We set the collateral constraint parameter at $\lambda = 1.1$, which is within the range of calibration values in the macroeconomics literature (e.g., Jermann and Quadrini, 2012; Buera and Shin, 2013; Midrigan and Xu, 2014; Moll, 2014; Dabla-Norris et al., 2021). The rent extraction rate τ is a scaling parameter, the value of which does not affect firm decisions. We normalize it at $\tau = 1\%$.

The remaining parameters are calibrated by matching the relevant moments summarized in Panel B of Table 1. When constructing the model moments, we simulate a sample for 1,000 years with a 100-year burn-in period, which is long enough to guarantee the stability of these moments. The discount rate is set at $\delta = 0.01$ to generate a real risk-free rate of 1.58%. R&D productivity is set at $\chi = 1.35$ to generate an average consumption growth rate of 1.75%. Following Storesletten, Telmer and Yaron (2007), we calibrate $\sigma_k = 0.19$ so that the model-implied volatility of consumption growth is 1.67%. We set the payout rate at $\rho = 3.7\%$ so that the dividend yield is 2.14%.

Table 2 presents the untargeted moments. Panel A shows that the moments reflecting the persistence of consumption growth implied by the model are roughly consistent with those in the data, even though these moments are not directly targeted in our calibration. Panel B shows that the yearly autocorrelation of R&D expenditure growth $\Delta \ln S_t$ and misallocation

M_t have comparable values in the model and data. The model implies a smooth risk-free rate and a high Sharpe ratio of the consumption claim, consistent with the Sharpe ratio of the market portfolio in our data sample. We discuss the mechanisms that generate asset pricing implications in Section 4.4.

4.3 Misallocation, R&D, and Growth

In this section, we illustrate that, within the framework of the model and the actual data, misallocation M_t successfully captures the low-frequency growth fluctuations. This conclusion is supported by predictive regressions over long horizons, showcasing M_t's capacity in reflecting long-term growth trends. In Panel A of Table 3, we study the relationship between misallocation M_t and R&D intensity. In both the actual data and model (i.e., the simulated data), we regress R&D intensity in the current year (t) and the next year ($t+1$) on misallocation M_t, as follows:

$$\frac{S_{t+h}}{N_{t+h}} = \alpha + \beta M_t + \varepsilon_{t+h}, \quad \text{with } h = 0, 1. \tag{48}$$

The results indicate that higher misallocation is associated with a decline in contemporaneous R&D intensity and predicts a lower R&D intensity in the next year.

Next, we examine whether misallocation M_t covaries with the slow-moving component of expected growth by testing whether misallocation negatively predicts future consumption growth in the data and model. We run the following regression:

$$\Delta \ln C_{t,t+1} + \cdots + \Delta \ln C_{t+h-1,t+h} = \alpha + \beta M_t + \varepsilon_{t,t+h}, \tag{49}$$

where $h = 1, \cdots, 5$ and $\Delta \ln C_{t+h-1,t+h}$ is the one-year log consumption growth from year $t+h-1$ to $t+h$. Panel B of Table 3 presents the results of projecting future consumption growth over horizons of 1 to 5 years on misallocation M_t. In both the data and model, the slope coefficients are negative and statistically significant. The coefficients are more negative for longer horizons because consumption growth is persistent. Our estimates indicate that misallocation M_t comoves with the slow-moving component of expected consumption growth. We further run regressions similar to (49) using future log output growth as the dependent variable. Panel C of Table 3 presents the results of projecting future output growth over horizons of 1 to 5 years on misallocation M_t. The patterns are similar to those of consumption growth in Panel B.

Taken together, we find empirical evidence that the aggregate growth rates of consumption and output can be predicted by our empirical measure of misallocation M_t, especially over long horizons. Our findings lend empirical support to the notion of misallocation-

Table 3: Misallocation, R&D, and growth in the data and model.

Panel A: R&D intensity (S_t/N_t)

	t		$t+1$	
	Data	Model	Data	Model
β	−0.106	−0.039	−0.094	−0.042
	(0.028)	(0.004)	(0.030)	(0.004)

Panel B: Consumption growth ($\Delta \ln C_t$)

	$t \to t+1$		$t \to t+2$		$t \to t+3$		$t \to t+4$		$t \to t+5$	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
β	−0.083	−0.140	−0.138	−0.201	−0.178	−0.246	−0.207	−0.275	−0.227	−0.276
	(0.027)	(0.017)	(0.043)	(0.033)	(0.053)	(0.047)	(0.056)	(0.064)	(0.060)	(0.080)

Panel C: Output growth ($\Delta \ln Y_t$)

	$t \to t+1$		$t \to t+2$		$t \to t+3$		$t \to t+4$		$t \to t+5$	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
β	−0.094	−0.109	−0.139	−0.243	−0.163	−0.218	−0.193	−0.225	−0.218	−0.233
	(0.046)	(0.032)	(0.065)	(0.037)	(0.083)	(0.054)	(0.084)	(0.064)	(0.088)	(0.075)

Note: The data sample is yearly and spans the period from 1965 to 2016. In the model, we simulate a sample of 52 years as in the data. Robust standard errors are reported in brackets.

driven low-frequency growth fluctuations. In the simulated data of our model, similar patterns are observed due to the key mechanism elaborated in Section 3.4. Thus, our model helps rationalize and identify misallocation as an economic source of low-frequency growth fluctuations in the data.

4.4 Asset Pricing Implications of Misallocation

Table 4 shows the asset pricing implications of misallocation. Column (1) presents the implications in the baseline model. The aggregate consumption claim has a high Sharpe ratio of 0.39, which is similar to that of the market portfolio in the data. Because the model is calibrated to match an annualized volatility of consumption growth of 1.5%, the excess return of the consumption claim has an annualized volatility of only 1.39%. Thus, the average excess return is low due to low volatility. The risk-free rate has an average value of 1.58% and low volatility, as in the data. We also compute the ratio of the volatility of 1-year SDF to its mean, $\frac{\sigma[\Lambda_{t+1}/\Lambda_t]}{\mathbb{E}[\Lambda_{t+1}/\Lambda_t]}$, which determines the maximal Sharpe ratio in the model. The baseline calibration implies a high value of 0.61.

Next, we study different model specifications. In column (2), we exogenously fix misallocation M_t at its long-run mean $\mathbb{E}[M_t]$. The volatility of the consumption claim's

Table 4: Asset pricing implications under different model specifications.

	(1) Baseline	(2) $M_t \equiv \mathbb{E}[M_t]$	(3) $\mathrm{d}N_t \equiv 0$	(4) $e^{-\theta}$ $= 0.2$	(5) $e^{-\theta}$ $= 0.45$	(6) CRRA ($\gamma = 1/\psi$) $= 1.5$	(7) CRRA ($\gamma = 1/\psi$) $= 3$
$\mathbb{E}[R^e_{m,t}]$ (%)	0.54	0	0.02	0.01	0.08	0.02	0.02
$\sigma[R^e_{m,t}]$ (%)	1.39	0	0.72	1.17	1.09	1.01	0.57
$SR[R_{m,t}]$	0.39	–	0.02	0.01	0.08	0.02	0.04
$\mathbb{E}[r_{f,t}]$ (%)	1.58	1.87	0.98	1.93	1.88	3.60	6.17
$\sigma[r_{f,t}]$ (%)	0.47	0	0.34	0.33	0.41	0.47	0.57
$\frac{\sigma[\Lambda_{t+1}/\Lambda_t]}{\mathbb{E}[\Lambda_{t+1}/\Lambda_t]}$	0.61	0	0.03	0.06	0.10	0.03	0.05

Note: In the table, $R^e_{m,t} = R_{m,t} - r_{f,t}$ is the consumption claim's return $R_{m,t}$ in excess of the risk-free rate $r_{f,t}$; $SR[R_{m,t}] = \mathbb{E}[R^e_{m,t}]/\sigma[R^e_{m,t}]$ is the Sharpe ratio of the consumption claim; and $\sigma[\Lambda_{t+1}/\Lambda_t]/\mathbb{E}[\Lambda_{t+1}/\Lambda_t]$ is the ratio of the volatility of 1-year SDF to its mean. Column (1) presents the results under the baseline calibration. In column (2), we adopt the same baseline calibration but eliminate fluctuations in misallocation by imposing $M_t \equiv \mathbb{E}[M_t]$ exogenously. In column (3), we adopt the same baseline calibration but eliminate the growth of knowledge stock N_t by imposing $\mathrm{d}N_t \equiv 0$ exogenously. In columns (4) and (5), we use alternative values of parameter θ. In columns (6) and (7), we impose $\gamma = 1/\psi$ and set different values of parameter γ. For columns (4) to (7), we calibrate χ and σ_k to generate the same model-implied average consumption growth rate and volatility as those reported in Panel B of Table 1. Other parameters are set at the same values as the baseline calibration.

excess returns falls to 0 and the Sharpe ratio is not defined. This occurs because, within our model, the aggregate shock $\mathrm{d}W_t$ drives economic fluctuations purely through its effect on M_t, with the dynamics of A_t and N_t being locally deterministic (see Proposition 5).[22]

To study the role of economic growth, we consider an alternative specification with no economic growth in column (3), setting $\mathrm{d}N_t \equiv 0$.[23] Compared with the baseline model in column (1), the volatility of the consumption claim's excess returns drops by about half, from 1.39% to 0.72%. The average excess return declines even more significantly, resulting in a Sharpe ratio of only 0.02.

In columns (4) and (5), we further show that fluctuations in economic growth are not sufficient to rationalize a high Sharpe ratio; it is important for misallocation fluctuations to generate low-frequency growth fluctuations. Specifically, following the insight illustrated in Figure 2, the persistence of idiosyncratic productivity determines the persistence of growth. In columns (4) and (5), we set $e^{-\theta}$ at 0.2 and 0.45, respectively, which results in a lower yearly autocorrelation of consumption growth than that in the baseline calibration, where $e^{-\theta} = 0.85$. Compared with column (1), the Sharpe ratio of the consumption claim

[22] This property differentiates our theoretical mechanism from those of Kaltenbrunner and Lochstoer (2010) and Kung and Schmid (2015), whose models generate low-frequency growth fluctuations through time-varying aggregate capital stock or R&D expenditure, rather than the covariance between capital and productivity across firms (i.e., M_t).

[23] Under this specification, the economy's aggregate output and consumption still fluctuate due to aggregate shocks. However, there is no long-run growth as the average growth rates of Y_t and C_t are 0.

drops significantly when idiosyncratic shocks are not persistent. These results highlight the importance of low-frequency growth fluctuations in amplifying the impacts of misallocation fluctuations on risk premia. Our findings complement the main insights of Buera and Shin (2011) and Moll (2014), who analyze the impacts of the persistence of idiosyncratic productivity on TFP, welfare, and the speed of transition through the self-financing channel.

In columns (6) and (7), we adopt a model specification where the representative agent is characterized by CRRA preferences, setting $\gamma = 1/\psi$. In this setup, the Sharpe ratio predicted by the model turns out to be notably low, whereas the risk-free rate is exceptionally high, a consequence of the low EIS. When considering a (non-recursive) CRRA preference structure, the valuation effects of low-frequency fluctuations in consumption growth are negligible. This occurs because the representative agent effectively prices the risk of the shock driving expected future consumption growth at zero.

4.5 Welfare Costs of Misallocation-Driven Growth Fluctuations

In our model, consumption fluctuations are almost entirely driven by fluctuations in misallocation. Therefore, by evaluating the welfare costs associated with consumption fluctuations, we are able to offer a quantitative analysis of the welfare implications of misallocation-driven growth fluctuations within our theoretical framework. It is acknowledged that, in real-world scenarios, consumption fluctuations may result from a variety of aggregate variables. Bearing this in mind, our objective is not to precisely isolate the welfare costs directly attributable to misallocation fluctuations. Rather, we aim to demonstrate that fluctuations in misallocation have the potential to inflict significant welfare costs by causing consumption fluctuations, within a model that is calibrated to align with observed aggregate consumption moments (see in Panel A of Table 2).

Specifically, we solve a similarly parameterized model without aggregate shocks (i.e., $\sigma_k = 0$) and compare the representative agent's utility gain relative to the model with aggregate shocks. Column (1) of Table 5 reports that the welfare gain from removing all consumption fluctuations is 10.34% under the baseline calibration. Moreover, in columns (2) through (6) of Table 5, we compute the welfare gains from removing consumption fluctuations under different specifications, similar to those in Table 4. Columns (2) through (4) show that the welfare gains will be small if misallocation cannot affect economic growth (i.e., setting $\mathrm{d}N_t \equiv 0$) or if misallocation is not persistent enough to generate low-frequency growth fluctuations (i.e., $e^{-\theta} = 0.2$ or $e^{-\theta} = 0.45$).[24] Columns (5) and (6) show that if the

[24]Columns (3) and (4) show that as idiosyncratic productivity becomes more persistent (i.e., higher $e^{-\theta}$), the welfare gain from removing consumption fluctuations increases. This finding is related to the key insight of Moll (2014), who shows that as the persistence of idiosyncratic productivity increases, the transition speed from a distorted initial state to the steady state slows down, resulting in potentially larger welfare losses during transitions. In our model with stochastic growth, the slow "transition" in response to aggregate

Table 5: Welfare gains from removing consumption fluctuations.

	(1) Baseline	(2) $dN_t \equiv 0$	(3) $e^{-\theta}$ $= 0.2$	(4) $e^{-\theta}$ $= 0.45$	(5) CRRA ($\gamma = 1/\psi$) $= 1.5$	(6) CRRA ($\gamma = 1/\psi$) $= 3$
Welfare gains (%)	10.34	0.33	0.24	0.98	0.58	0.65

Note: The welfare gains from removing consumption fluctuations are computed using $\overline{U}_0/U_0 - 1$, where U_0 ($\overline{U}_0$) represents the representative agent's utility at $t = 0$, with (without, i.e., setting $\sigma_k = 0$) consumption fluctuations. When computing $\overline{U}_0$, the parameter χ is recalibrated to have the same average consumption growth rate while all other parameter values remain unchanged. The specification in each column is described in Table 4.

agent's preference is non-recursive (i.e., setting $\gamma = 1/\psi$), the welfare gains are also small.

Taken together, our findings suggest that the model posits significant welfare costs arising from misallocation-driven consumption fluctuations, attributable to a combination of two distinct properties. First, as elaborated in Section 3.4, the model is able to generate low-frequency growth fluctuations through slow-moving misallocation. Second, given the representative agent's recursive preferences, news about future consumption growth impacts his current marginal utility. As illustrated in Table 4, these two properties also allow the model to account for the observed high Sharpe ratio in the capital markets. Within our model framework, there is a direct link between the welfare costs associated with consumption fluctuations and the Sharpe ratio observed in the capital markets. Intuitively, both metrics are elevated when variations in the representative agent's marginal utility in response to aggregate shocks are more pronounced. This connection is exploited by Alvarez and Jermann (2004) to estimate the welfare gains from eliminating all consumption fluctuations by directly applying the no-arbitrage principles on financial market data without specifying consumer preferences. We implement the method proposed by Alvarez and Jermann (2004) in our 1965-2016 sample and estimate that the welfare gain from eliminating all consumption fluctuations ranges from 6.03% to 23.97%, which nests the value implied by our structural model.[25]

The results in Tables 4 and 5 show that misallocation-driven growth fluctuations can have significant implications for asset prices and welfare. As misallocation arises from firms' financial constraints in our model, our results are related to the literature on the connection between financial frictions and misallocation (e.g., Buera and Shin, 2013; Midrigan and Xu, 2014; Moll, 2014). A direct comparison of our model's quantitative implications with these

shocks generates endogenous low-frequency growth fluctuations, which result in large welfare costs under the recursive preference of the representative agent.

[25] Alvarez and Jermann (2004) propose different estimation methods to demonstrate robustness. We use their first method, which projects consumption growth onto the payoff space spanned by a set of tradable assets. The estimates and implementation details of other methods are reported in Section 2.3 of the online note on additional materials.

models in the literature is difficult due to the differences in model setups. For example, our model involves stochastic growth driven by misallocation fluctuations, whereas these models quantify losses from misallocation in steady states or transitions, without aggregate shocks. In addition, although our model incorporates both the final goods and intermediate goods sectors, we only consider misallocation in the final goods sector.

Despite the differences in model setups, our findings in Table 5 are broadly consistent with the literature. For example, consistent with the calibration of Buera and Shin (2013) and Moll (2014), our calibration of large idiosyncratic shocks implies that firm-level productivity is not very persistent. As a result, purely through the variation in misallocation M_t, the model is able to generate a TFP volatility of 2.48%, as in the data. This result is consistent with the finding of Buera and Shin (2013) that misallocation resulting from financial frictions can generate sizable TFP losses.

While Buera and Shin (2013) focus on quantifying misallocation across the intensive margin (that is, differences in MRPK among active firms due to financial frictions), other research (e.g., Banerjee and Moll, 2010; Buera, Kaboski and Shin, 2011; Midrigan and Xu, 2014) underscores the significance of misallocation at the extensive margin (that is, productive firms may stay inactive or refrain from entering the market due to financial frictions). Depending on the calibration and model setup, Buera, Kaboski and Shin (2011) quantify that both extensive and intensive margins are important, whereas Midrigan and Xu (2014) estimate large TFP losses through the extensive margin rather than the intensive margin. In our model, misallocation due to financial frictions reduces the final goods sector's productivity H_t, which captures the intensive margin effect. A lower H_t, in turn, reduces the profits of innovators. Through the free-entry condition (9), this further leads to a lower growth rate of the variety of intermediate goods, $\mathrm{d}N_t/N_t$ (see equation (8)), which can be seen as capturing the extensive margin effect.[26] The results in column (3) of Table 4 and column (2) of Table 5 indicate that the extensive margin plays a crucial role in rationalizing the high Sharpe ratio in the capital market and in generating a large welfare cost of misallocation-driven growth fluctuations. These findings support the significant role of extensive-margin misallocation quantified by Midrigan and Xu (2014).

5 Empirical Tests on the Core Mechanism

In our model, financial frictions lead to misallocation, which in turn affects long-run growth by influencing R&D investment. This section presents empirical evidence supporting this mechanism, focusing on industry-level reactions to a policy shock designed to ease firms'

[26]There is no misallocation through the intensive margin in the intermediate goods sector because producers are homogeneous.

financial constraints.

The American Jobs Creation Act (AJCA) passed in 2004 allows domestic firms in the U.S. to repatriate their foreign profits at a tax rate of 5.25%, rather than the 35% tax rate that applied before the AJCA. This policy change effectively relaxes the financial constraints of treated firms and substantially boost the investments of financially constrained firms (Faulkender and Petersen, 2012). Our model suggests that easing financial constraints reduces misallocation and boosts firms' motivation to engage in R&D activities. To verify this hypothesis, we assess the effect of the AJCA on industry-level misallocation and R&D-capital ratio by exploiting industries' differential exposure to the AJCA using a difference-in-differences (DID) method.

Specifically, we construct industry-level measures of misallocation, the R&D-capital ratio, and exposure to the AJCA using U.S. Compustat data. We use three-digit Standard Industrial Classification codes to define industries. To ensure accurate estimation of industry-level misallocation, we exclude industries with a median number of firms below 10. We construct our industry-level measures of misallocation following the procedures described in Section 4.1, except that we run regression (47) based on firms within each industry. The industry-level R&D-capital ratio is constructed as the ratio of total R&D expenditure to total capital of firms within the industry. To capture an industry's exposure to the AJCA, we construct an industry-level measure of foreign business intensity, which is the proportion of firms in the industry whose pre-tax income from abroad exceeds 5% during the 3-year period before the AJCA (i.e., from 2001 to 2003). We consider industries with foreign business intensity above (below or equal to) 33% as treated (untreated) industries. Treated industries are matched with untreated industries using the nearest neighbor matching method based on six industry-level characteristics.[27] All industry-level characteristics are averaged over the 3-year period before the AJCA.

We run the following regression using industry-year observations for the period from 2000 to 2007:

$$Y_{j,t} = \alpha \times Treat_j \times Post_t + \beta_1 \times Treat_j + \beta_2 \times Post_t + \varepsilon_{j,t}, \tag{50}$$

where $Treat_j = 1$ if industry j is a treated industry, and 0 otherwise, and the variable $Post_t$ is an indicator that equals 1 for years 2004 and onwards. The coefficient of interest, α, estimates the average treatment effect of the AJCA on the outcome variable, $Y_{j,t}$, for treated industries. The outcome variable of interest, $Y_{j,t}$, is either industry-level misallocation ($M_{j,t}$) or industry-level R&D-capital ratio ($RD_{j,t}$). The estimated coefficients are presented in column (1) of Panels A and B in Table 6. Our results indicate that the AJCA results in

[27]The six industry-level characteristics are the means and standard deviations of firms' sales, profit margins, and Tobin's Q. We construct a firm's (net) profit margin using its income before extraordinary items divided by its sales following Dou, Ji and Wu (2021), and a firm's Tobin's Q as $Tobin\_Q_{i,t} = (total\_assets_{i,t} + market\_equity_{i,t} - book\_equity_{i,t})/total\_asset_{i,t}$, following Gompers, Ishii and Metrick (2003).

Table 6: Impacts of the AJCA on misallocation and R&D.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Industry-level misallocation							
α	α_{-4}	α_{-3}	α_{-2}	α_0	α_1	α_2	α_3
-0.470	-0.211	-0.250	-0.046	-0.404	-0.584	-0.773	-0.627
(0.174)	(0.319)	(0.193)	(0.172)	(0.192)	(0.221)	(0.283)	(0.319)
Panel B: Industry-level R&D-capital ratio							
α	α_{-4}	α_{-3}	α_{-2}	α_0	α_1	α_2	α_3
0.018	-0.013	-0.005	-0.002	0.010	0.013	0.015	0.014
(0.006)	(0.009)	(0.005)	(0.003)	(0.005)	(0.006)	(0.007)	(0.009)
Panel C: Industry-level R&D-capital ratio controlling for misallocation							
α	α_{-4}	α_{-3}	α_{-2}	α_0	α_1	α_2	α_3
0.013	-0.013	-0.005	-0.002	0.007	0.009	0.009	0.010
(0.007)	(0.010)	(0.006)	(0.003)	(0.005)	(0.007)	(0.009)	(0.010)

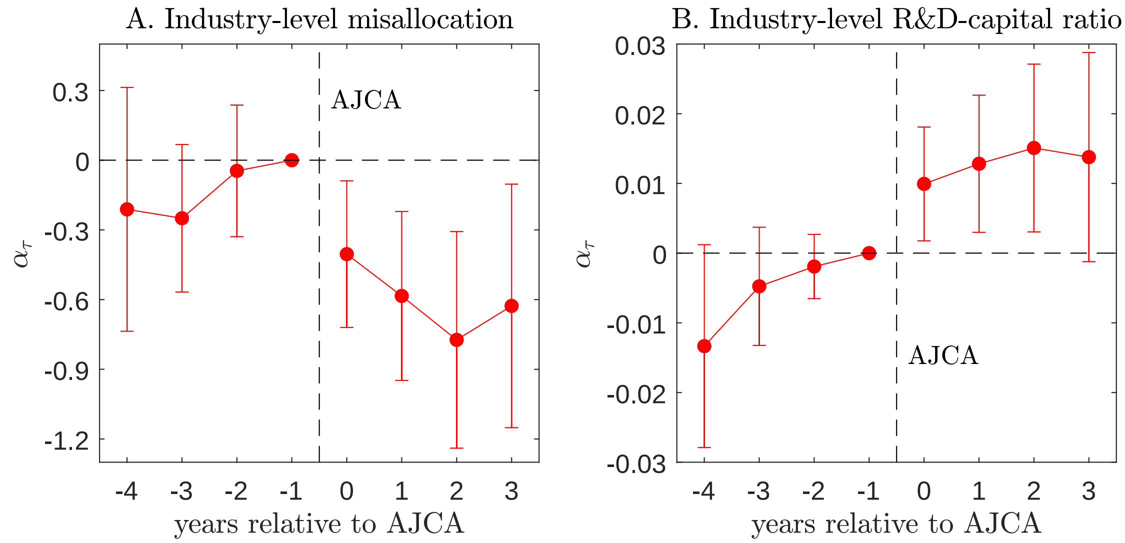
Note: Panel A estimates the impacts of the AJCA on industry-level misallocation. Column (1) reports the estimated $\widehat{\alpha}$ in specification (50). Columns (2) to (8) report the estimated α_τ in specification (51) for $\tau = -4, -3, -2, 0, 1, 2, 3$. All coefficients are normalized relative to $\tau = -1$. Panel B estimates the impacts of the AJCA on the industry-level R&D-capital ratio. Panel C estimates the impacts of the AJCA on industry-level R&D-capital ratio, controlling for industry-level misallocation. Standard errors clustered at the industry level are reported in brackets.

significantly lower misallocation and higher R&D-capital ratios in treated industries.

Next, we conduct an event study analysis centered around 2004, employing a dynamic DID regression approach to investigate the time-series evolution of the AJCA's treatment effect on the industry-level misallocation and R&D-capital ratio over the years:

$$Y_{j,t} = \sum_{\tau=-4}^{3} \alpha_\tau \times Treat_j \times Year_{t-\tau} + \beta_1 \times Treat_j + \sum_{\tau=-4}^{3} \beta_{2,\tau} \times Year_{t-\tau} + \varepsilon_{j,t}, \tag{51}$$

where $Year_{t-\tau}$ is an indicator set to 1 for the year $t-\tau$ being 2004 (the year the AJCA was enacted) and 0 otherwise. To circumvent collinearity in categorical regressions, we impose constraints: $\alpha_{-1} = \beta_{2,-1} = 0$, thereby designating the year immediately preceding the passage of the AJCA as the reference period. The estimated effects on industry-level misallocation and the R&D-capital ratio are detailed in columns (2) to (8) of Panels A and B, respectively, in Table 6 and are graphically depicted in Figure 4. The estimated coefficients, α_{-4}, α_{-3}, and α_{-2}, are near zero and not statistically significant, indicating that the parallel trend assumption holds in the years leading up to 2004. In the three years subsequent to 2004, our estimates reveal that the AJCA induces significant and enduring negative impacts on industry-level misallocation and significant positive impacts on the industry-level R&D-capital ratio.



Note: The solid lines visualize the empirical estimates in columns (2) to (8) of Panels A and B in Table 6, respectively. All coefficients are normalized relative to $\tau = -1$. The vertical bars represent the corresponding 90% confidence intervals.

Figure 4: Impacts of the AJCA on misallocation and R&D.

Furthermore, we present evidence suggesting that the positive effect of the AJCA on the industry-level R&D-capital ratio is primarily realized through its influence on industry-level misallocation. Specifically, we modify specification (50) as follows:

$$\begin{aligned} RD_{j,t} = {} & \alpha \times Treat_j \times Post_t + \beta_1 \times Treat_j + \beta_2 \times Post_t \\ & + \beta_3 \times Treat_j \times M_{j,t} + \beta_4 \times M_{j,t} + \varepsilon_{j,t}, \end{aligned} \tag{52}$$

which controls for industry-level misallocation $M_{j,t}$ and its interaction term with $Treat_j$. The estimated coefficient is displayed in column (1) of Panel C in Table 6, showing that after accounting for industry-level misallocation, the AJCA has a much less significant impact on the R&D-capital ratio.

We further employ a dynamic DID regression approach to demonstrate the time-series progression of the AJCA's treatment effects on the industry-level R&D-capital ratio over the years, while accounting for industry-level misallocation. This is achieved by executing the following regression:

$$\begin{aligned} RD_{j,t} = {} & \sum_{\tau=-4}^{3} \alpha_\tau \times Treat_j \times Year_{t-\tau} + \beta_1 \times Treat_j + \sum_{\tau=-4}^{3} \beta_{2,\tau} \times Year_{t-\tau} \\ & + \beta_3 \times Treat_j \times M_{j,t} + \beta_4 \times M_{j,t} + \varepsilon_{j,t}. \end{aligned} \tag{53}$$

Columns (2) to (8) of Panel C in Table 6 report the estimates for each year. Compared with Panel A, it is clear that the impacts of the AJCA on the industry-level R&D-capital ratio become statistically insignificant after controlling for industry-level misallocation.

6 Conclusion

This paper develops an analytically tractable general equilibrium model with heterogeneous firms and endogenous stochastic growth to quantitatively explore the relationship between misallocation, growth prospects, and the systematic risk that shapes asset prices in capital markets. In our model, increased misallocation reduces economic growth by depressing the marginal q of intangible capital and thus R&D incentives. Misallocation evolves slowly, leading to low-frequency fluctuations in economic growth. Central to this mechanism is the valuation channel, which significantly magnifies the effects of production capital misallocation in the final goods sector on economic growth. When agents have recursive preferences, the low-frequency growth fluctuations driven by slow-moving misallocation not only rationalize several crucial asset pricing moments but also suggest significant welfare costs associated with misallocation fluctuations.

In the data, we construct a misallocation measure motivated by our theory and provide supporting evidence for the model predictions. We show that the value of our empirical measure of misallocation is persistent and increases during economic downturns. Moreover, an increase in misallocation predicts declines in R&D intensity and reductions in the growth of aggregate consumption and output over long horizons. Finally, by exploiting a policy shock from the AJCA passed in 2004, we provide direct causal evidence to support the model's mechanism that misallocation drives long-run growth through its impact on R&D.

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