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**ABSTRACT**

We introduce safe asset demand for dollar-denominated bonds into a tractable incomplete-market model of exchange rates. The convenience yield on dollar bonds enters as a stochastic wedge in the Euler equations for exchange rate determination. This wedge reduces the pass-through from marginal utility shocks to exchange rate movements, resolving the exchange rate volatility puzzle. The wedge also exposes the dollar's exchange rate to convenience yield shocks, giving rise to exchange rate disconnect from macro fundamentals and a quantitatively important driver of currency risk premium. This endogenous exposure identifies a novel safe-asset-demand channel by which the Fed's QE impacts the dollar and long-term U.S. Treasury bond yields.

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# 1 Introduction

We introduce safe asset demand for dollar bonds into an otherwise standard two-country, incomplete-market economy. The model makes progress on outstanding exchange rate puzzles that complete-market models cannot address. The analysis also uncovers a novel convenience yield mechanism through which quantitative easing affects U.S. Treasury bond yields and the dollar exchange rate.

We focus on safe asset demand for dollar-denominated assets for three reasons. First, there is strong empirical evidence connecting movements in the dollar exchange rate and measures of the convenience (safety/liquidity) services that foreign investors attach to U.S. dollar safe bonds (see recent work by Avdjiev, Du, Koch, and Shin, 2019; Jiang, Krishnamurthy, and Lustig, 2021; Engel and Wu, 2021). Second, recent work on market segmentation and intermediation has found that Euler equation wedges can help to address exchange rate puzzles. In particular, in the models developed by Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2021), the exchange rate is determined by the Euler equation of a specialized FX intermediary. Intermediation frictions give rise to wedges in the Euler equations of standard investors who do not operate in foreign exchange markets and/or foreign bond markets, and this research shows that these wedges can resolve exchange rate puzzles. The convenience yields we study are a form of the stochastic wedges, and they can be measured from the Covered Interest Rate Parity (CIP) violations in government bond markets. In doing so, we bring data to discipline the Euler equation wedges. Third, there is a well-documented channel by which quantitative easing (QE) affects the exchange rate (see Neely, 2015; Krishnamurthy and Lustig, 2019) as well as interest rates (Krishnamurthy and Vissing-Jorgensen, 2011). We will show that the safe asset demand offers a natural connection from bond quantities to bond prices and exchange rates, and our convenience yield model can jointly explain these patterns.

We study a two-country, incomplete-market economy. Investors in both countries derive convenience utility on their holdings of U.S. Treasurys. Markets are not segmented. All investors can buy U.S. Treasurys and foreign risk-free bonds, while the financial markets for other contingent claims may or may not open, which allows us to model a flexible degree of market incompleteness. We characterize the exchange rate processes that satisfy the four Euler equations for the home and foreign investors (2 investors $\times$ 2 bonds). These Euler equations encapsulate investors' attitudes towards exchange rate risk and their special desire for owning dollar-denominated U.S. Treasury bonds. In particular, we posit a pair of U.S. (dollar) and foreign log SDFs and a stochastic convenience yield process. This approach allows us to derive a tractable expression for the exchange rate as a function of the histories of U.S. and foreign SDF shocks and USD convenience yield shocks.

By focusing only on the four Euler equations, our approach characterizes a family of solutions for the exchange rate process. We study the properties of the exchange rate process in this family and characterize its performance vis-à-vis exchange rate puzzles. In this sense, our approach is in the tradition of Hansen and Jagannathan (1991) and the preference-free approach to FX markets (Backus, Foresi, and Telmer, 2001; Lustig, Roussanov, and Verdelhan, 2011; Chernov and Creal, 2018; Lustig and Verdelhan, 2019; Chernov, Haddad, and Itskhoki, 2023) rather than the international macro-finance models such as Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2021). While the four Euler equations hold in most international macro models, these models also imply additional restrictions on the joint dynamics of the exchange rate and the trade balance which we do not impose. Furthermore, we allow for an arbitrary correlation structure of U.S. and foreign SDF shocks and USD convenience yield shocks when solving for the exchange rate. Including other macro considerations will restrict the correlation structure. In Online Appendix OA, we present a two-period international macro model to explain these points further. Our minimalist approach allows us to characterize how far convenience yields in an incomplete-market setting can go towards resolving exchange rate puzzles in a large class of models.

Next, we discuss the four key findings of our paper in detail. First, we make progress on the exchange rate volatility puzzle. In our model, the equilibrium log exchange rate is less volatile than the difference between the U.S. and foreign log SDFs. This result, which is a convenience-yield variant of the result derived in Lustig and Verdelhan (2019), helps to resolve the volatility of exchange rates vis-à-vis stock prices (Brandt, Cochrane, and Santa-Clara, 2006). In our closed-form characterization, the covariance between the SDF shocks and the exchange rate is tightly connected to the covariance between the exchange rate and the convenience yield. In the complete-market benchmark model without convenience yields, exchange rates have to fully close the gap between the SDFs, absorbing all of the shocks. In our model with convenience yields, the convenience yield can partially act as a shock absorber, too. To calibrate the model, we match the comovement of convenience yields and exchange rates reported by Jiang, Krishnamurthy, and Lustig (2021). Our calibrated model manages to match the volatility of the exchange rate in the data, because the convenience yield drives a wedge between the volatility of the exchange rate and that of the SDFs.

Second, we make progress on the exchange rate disconnect puzzle (Backus and Smith, 1993; Kollmann, 1995). Convenience yield shocks impact exchange rates in our model.

The equilibrium exchange rate reflects expected future interest rate spreads, currency risk premia, and USD convenience yields. In particular, the dollar appreciates when dollar bonds carry a higher convenience yield. In the case of a foreign flight to the safety of U.S. Treasurys, the foreign convenience yield on U.S. bonds increases and causes the dollar to appreciate. This convenience yield channel counteracts the standard complete markets channel through which the foreign currency appreciates as foreign investors experience a higher marginal utility growth in this episode. As a result, the dollar can appreciate against the foreign currency in foreign recessions. We explore these countervailing forces in a calibrated version of our model. The baseline model generates a roughly acyclical exchange rate. However, we also prove that in our model it is not possible to change the sign and deliver a pro-cyclical exchange rate. This theoretical result holds in a larger class of incomplete-market models.

Third, our model generates sizable deviations from the Uncovered Interest Rate Parity (UIP). Foreign investors earn a negative excess return on dollars because the dollar has a positive convenience yield and because the dollar endogenously appreciates when the foreign SDF is high, thereby providing a hedge. In this way, the convenience yield endogenously generates a currency risk premium for the dollar, allowing our model to deliver plausible currency returns while matching the volatility of exchange rates. In stark contrast, Lustig and Verdelhan (2019) show that these moments cannot be matched jointly in a generic incomplete-market model without any Euler equation wedges or convenience yields. In their setting, while market incompleteness reduces the exchange rate volatility and cyclicality, it also shrinks the currency risk premium towards zero. As a result, resolving the risk premium puzzle immediately deepens the other exchange rate puzzles.

Fourth, our model sheds light on the connection between QE, exchange rates, and interest rates. Let $r_{t,T-t}$ and $r^*_{t,T-t}$ denote the log home and foreign interest rates with maturity $T-t$. In a large class of models with complete markets and stationary exchange rates, the current exchange rate in deviation from its mean depends only on the home-foreign spread in the long rates:

$$s_t - \bar{s} = \lim_{T\to\infty} (T-t)(r_{t,T-t} - r^*_{t,T-t}). \tag{1}$$

This result is because stationary exchange rates imply no long-run exchange rate risk, which further implies that U.S. and foreign bonds are perfect substitutes over long holding periods for a long-horizon investor.

Recently, Gourinchas, Ray, and Vayanos (2021); Greenwood, Hanson, Stein, and Sunderam (2020) develop equilibrium models of the joint pricing of bonds and currencies to elucidate the workings of QE. In their models, the exchange rate is stationary, so that a version of Eq. (1) applies. QE in their model lowers the risk premium on long-term bonds and thus lowers long yields and via (1), depreciates the dollar exchange rate. In our model, Eq. (1) fails because the exchange rate is also affected by the convenience yield, which introduces a wedge in the exchange rate determination. In particular, holding fixed the differences in the long rates, the dollar appreciates when the convenience yields that foreign investors earn on U.S. Treasurys increase.

We show that the convenience yield channel is needed to explain why Fed QE actions can simultaneously lower long-term U.S. interest rates and appreciate the dollar. Focusing only on (1), we see that QE which lowers long-term U.S. Treasury yields should depreciate the dollar. Given the evidence that QE lowers long-term U.S. Treasury yields (Krishnamurthy and Vissing-Jorgensen, 2011), it is puzzling that QE often also appreciates the dollar. On the other hand, in our model with the convenience yield, if a QE also raises current and future convenience yields, the dollar can appreciate despite the dollar yields decreasing.

We present evidence from QE event dates that both of these channels, via the U.S. interest rates and via the convenience yield, are operational. Quantitatively, using only movements in U.S. Treasury yields to explain dollar exchange rate changes around QE event dates gives an $R^2$ of 12%. Conditioning on both changes in U.S. Treasury yields and changes in the government bond CIP basis (which measures the convenience yield) increases the $R^2$ to 73%. We then show how our calibrated model can match this evidence. Our paper is the first to examine this distinct convenience yield channel of QE in the foreign exchange markets, which is separate from the bond risk premium channel.

Our paper is most closely related to Lustig and Verdelhan (2019) who examine an incomplete market setting where investors in home and foreign can invest in bonds in both countries and study the extent to which market incompleteness can help resolve exchange rate puzzles. We study the same four Euler equations as in Lustig and Verdelhan (2019) but introduce bond convenience yields, thereby demonstrating the importance of the convenience yield ingredient. Our paper is also related to Jiang, Krishnamurthy, and Lustig (2023). The current paper explores how far convenience yields can go in terms of resolving the exchange rate puzzles by characterizing and solving for equilibria where convenience yields alter the four Euler equations. Jiang et al. (2023) provides a general characterization of exchange rate cyclicality and predictability under a broad class of Euler equation wedges, which include convenience yields as well as home bias and transaction costs.

Alvarez, Atkeson, and Kehoe (2002); Gabaix and Maggiori (2015); Dou and Verdelhan (2015); Itskhoki and Mukhin (2021); Chien, Lustig, and Naknoi (2020); Sandulescu, Trojani, and Vedolin (2020) develop international macro models with segmented markets to resolve the exchange rate disconnect puzzle. Their models sever the equilibrium exchange rate from its macro-fundamentals by introducing market segmentation and deliver a pro-cyclical exchange rate based on the model's assumed patterns in the arbitrageur's portfolio. For example, Chien et al. (2020) consider a model in which only a small pool of investors arbitrage between domestic and foreign securities. As a result, the real exchange rate is disconnected from the differences in aggregate consumption growth between U.S. and foreign. Amador, Bianchi, Bocola, and Perri (2020) also study segmented markets in the international context. Their model shows how capital flows can lead to violations of covered interest rate parity when markets are segmented. Our model does not rely on market segmentation and instead delivers results from the assumption of convenience yields on dollar bonds.

Our paper adds to the recent international finance literature exploring the macro and financial market implications when international investors earn convenience yields on foreign bonds (Valchev (2020); Jiang, Krishnamurthy, and Lustig (2020); Kekre and Lenel (2021)). Our model also fits into the literature on portfolio-balance models of exchange rates (Kouri (1975); Hau and Rey (2006); Pavlova and Rigobon (2008)), which studies the effect of portfolio holdings and flows on exchange rates, as well as the literature on frictional foreign exchange markets (Du, Tepper, and Verdelhan (2018b); Augustin, Chernov, Schmid, and Song (2020)). We focus on how special demand for dollar safe assets affects equilibrium exchange rates.

Investors seem to face an incomplete menu of assets in international financial markets, either because of transactions and capital controls or because of other frictions (Lewis, 1995). Recent theoretical contributions on market incompleteness and exchange rates include the work by Chari, Kehoe, and McGrattan (2002), Bacchetta and van Wincoop (2006), Corsetti, Dedola, and Leduc (2008), Alvarez, Atkeson, and Kehoe (2009), and Pavlova and Rigobon (2010). Our work shows that the convenience yield mechanism can be fruitfully incorporated into all of these incomplete market models.

The rest of the paper proceeds as follows. Section 2 presents our minimalist model of exchange rate determination with convenience yields. This section presents a series of propositions that theoretically characterize the impact of convenience yields on exchange rate moments. Section 3 turns to quantifying the forces identified in the propositions of Section 2. We parameterize and calibrate the model and examine its implications for a common set of exchange rate puzzles. Section 4 discusses how quantitative easing impacts currency and bond markets through the lens of our model and provides empirical support for the nuanced results of our model. Proofs of all propositions are in the Appendix. A separate Online Appendix presents a two-period international macro setting with convenience yields to elucidate the deeper foundations of our minimalist model.

# 2 Model and Equilibrium Characterization

We develop an incomplete-market model of exchange rates in continuous time. Our economy has an infinite horizon. We fix a probability space $(\Omega, \mathcal{F}, P)$ and a given filtration $\mathbb{F} = \{\mathcal{F}_0 : t \geq 0\}$ satisfying the usual conditions. We assume that all stochastic processes are adapted to this filtration. There are two countries, the U.S. and foreign. Let $s_t$ denote the log real exchange rate. A higher $s_t$ means a stronger U.S. currency (dollar, USD). The key input into the model is that the USD is special in that U.S. and foreign investors earn a convenience yield on USD bonds.

## 2.1 Investor Preferences

The U.S. plays a unique role in the international financial system as the world's provider of dollar-denominated safe assets, as analyzed by Caballero, Farhi, and Gourinchas (2008), Caballero and Krishnamurthy (2009), Maggiori (2017), He, Krishnamurthy, and Milbradt (2018). We formalize this by assuming that U.S. and foreign investors derive utility from their holdings of the U.S. risk-free bond (U.S. Treasurys). Let $c_t$ denote the U.S. households' consumption, and let $q_{H,t}$ denote the U.S. households' holding in the U.S. risk-free bond. The investors' utility is derived over consumption and the dollar value of U.S. bond holdings:

$$u(c_t, q_{H,t}) = w(c_t) + v(q_{H,t}; \theta_t),$$

where $\theta_t$ is a time-varying demand shifter for U.S. bonds. We assume that the utility is increasing in the consumption and the holding in the U.S. bonds, i.e. $w'(c_t) > 0$ and $v'(q_{H,t}; \theta_t) > 0$ , and the marginal utility for holding U.S. bonds is decreasing in quantity, i.e., $v''(q_{H,t}; \theta_t) < 0$. In this way, the U.S. risk-free bond carries a convenience yield which captures its non-pecuniary benefits to U.S. and foreign investors, and is decreasing in the quantity held. We also assume that the exponentially discounted utility functions $w(\cdot)$ and $v(\cdot; \theta)$ are integrable.

The expected lifetime utility for U.S. investors is

$$\mathbb{E}_0\left[\int_{t=0}^{\infty} e^{-\rho t} u(c_t, q_{H,t}) dt\right] = \mathbb{E}_0\left[\int_{t=0}^{\infty} e^{-\rho t} (w(c_t) + v(q_{H,t}; \theta_t)) dt\right].$$

Similarly, for foreign investors, we assume their expected lifetime utility is

$$\mathbb{E}_0\left[\int_{t=0}^{\infty} e^{-\rho t} u(c_t^*, q_{H,t}^*) dt\right] = \mathbb{E}_0\left[\int_{t=0}^{\infty} e^{-\rho t} (w(c_t^*) + v(q_{H,t}^* \exp(s_t); \theta_t^*)) dt\right],$$

where $c_t^*$ denotes their aggregate consumption, $\theta_t^*$ is a time-varying demand shifter for U.S. bonds, and $q_{H,t}^*$ denotes the foreign investors' holdings of the U.S. risk-free bond in foreign currency terms. The product $q_{H,t}^* \exp(s_t)$ converts the holdings into dollar terms.

## 2.2 A Quartet of Euler Equations

Markets are not segmented. We assume that all investors can trade both U.S. and foreign risk-free bonds. The model analysis is centered around four Euler equations, 2 for the U.S. investors and 2 for the foreign ones. The asset markets for other risky claims, such as equity and long-term debt, may or may not be open to foreign investors. This approach allows us to model a general form of market incompleteness.

The U.S. (instantaneous) risk-free bond has a constant price $P_t = 1$ and an interest rate $r_t$, and the foreign risk-free bond has a constant price $P_t^* = 1$ and an interest rate $r_t^*$. These interest rates are determined in equilibrium from the Euler equations. We start by characterizing the optimality conditions for U.S. households.

**Lemma 1.** *The first-order conditions for the U.S. investor with respect to holdings in the U.S. and foreign risk-free bonds are given by*[1]

$$0 = \mathbb{E}_t\left[\frac{dM_t}{M_t}\right] + r_t + \frac{v'(q_{H,t}; \theta_t)}{w'(c_t)} \tag{2}$$

$$0 = \mathbb{E}_t\left[\frac{d(M_t \exp(-s_t))}{M_t \exp(-s_t)}\right] + r_t^* \tag{3}$$

*where $M_t = e^{-\rho t} w'(c_t)$ is the SDF of the U.S. investor.*

See Online Appendix OB.1 for the proof. To interpret this result, we note that the

[1] With some abuse of notation we use the notation $\mathbb{E}_t[dX_t]$ to represent the infinitesimal generator of a stochastic process $X_t$. The formal notation, which is adopted in the proof, is $\mathcal{A}[X_t]$.

discrete-time counterparts to these equations are given by the following expressions:

$$1 - \frac{v'(q_{H,t};\theta_t)}{w'(c_t)} = \mathbb{E}_t\left[\frac{M_{t+1}}{M_t}\exp(r_t)\right] \tag{4}$$

$$1 = \mathbb{E}_t\left[\frac{M_{t+1}}{M_t}\exp(r_t^* - \Delta s_{t+1})\right] \tag{5}$$

Given $v'(q_{H,t};\theta_t) > 0$ and $w'(c_t) > 0$, the left-hand side of Eq. (4) is less than 1. U.S. investors are willing to accept a lower risk-neutral expected return in exchange for holding the U.S. risk-free bond, whereas the risk-neutral expected return on the foreign risk-free bond is exactly 1. We refer to the gap $\frac{v'(q_{H,t};\theta_t)}{w'(c_t)}$ as the convenience yield that U.S. households attach to dollar bonds.

Similarly, the foreign households' asset pricing conditions for the foreign bond and for U.S. Treasurys, respectively, are given by:

$$0 = \mathbb{E}_t\left[\frac{dM_t^*}{M_t^*}\right] + r_t^* \tag{6}$$

$$0 = \mathbb{E}_t\left[\frac{d(M_t^*\exp(s_t))}{M_t^*\exp(s_t)}\right] + r_t + \frac{v'(q_{H,t}^*\exp(s_t);\theta_t^*)}{w'(c_t^*)} \tag{7}$$

Let us define $\tilde{\lambda}_t^h = \frac{v'(q_{H,t};\theta_t)}{w'(c_t)}$ as the convenience yield earned by U.S. investors on their dollar bond holdings. Likewise, define $\tilde{\lambda}_t^f = \frac{v'(q_{H,t}^*\exp(s_t);\theta_t^*)}{w'(c_t^*)}$ as the foreign investors' convenience yield on dollar bonds. Then, we can rewrite the Euler equations as:

$$0 = \mathbb{E}_t\left[\frac{dM_t}{M_t}\right] + r_t + \tilde{\lambda}_t^h, \qquad 0 = \mathbb{E}_t\left[\frac{d(M_t\exp(-s_t))}{M_t\exp(-s_t)}\right] + r_t^*,$$

$$0 = \mathbb{E}_t\left[\frac{d(M_t^*\exp(s_t))}{M_t^*\exp(s_t)}\right] + r_t + \tilde{\lambda}_t^f, \qquad 0 = \mathbb{E}_t\left[\frac{dM_t^*}{M_t^*}\right] + r_t^*.$$

These four equations arise in a large class of international macro models that may have different specifications of preferences, spanning of tradable assets, and frictions, as long as these models permit agents to trade, in an unconstrained fashion, in risk-free bonds and derive convenience yields on dollar safe assets.

Our approach is to solve for the family of equilibrium exchange rate dynamics that are consistent with these four Euler equations. In doing so, we provide a general characterization of the exchange rate dynamics in this large class of international macro models. If we were to further specify the macroeconomic environment, e.g., to derive the dynamics of the current account, the model would yield additional restrictions on the exchange rate process. Our minimal approach allows us to explore the extent to which convenience yields in an incomplete market setting can address puzzling aspects of the behavior of exchange rates.

In Online Appendix OA, we present a two-period international model with convenience yields on dollar bonds and trade in both countries' goods and bonds. The model reproduces the four Euler equations we study. At the same time, the model imposes additional restrictions on the relation between convenience yields and the trade balance, and it pins down a unique equilibrium from the family of solutions we characterize in the main text. We show that the two-period model qualitatively produces the forces present in our analysis, although obviously, that model cannot be explored quantitatively.

## 2.3 Equilibrium Forces

Before diving into the model solution, we work through a thought experiment that helps elucidate the restrictions these Euler equations impose on equilibrium exchange rates.

Suppose that at time $t$, there is an exogenous increase in $\tilde{\lambda}_t^h$, i.e., the foreign households' convenience yield on the dollar safe assets. For the sake of this argument, we will assume the U.S. and foreign SDFs and the U.S. households' convenience yield remain unaffected. In the next section, we specify the joint dynamics of the SDFs and the convenience yield shocks.

This increase in foreign households' convenience yield sets off the following chain of events. First, consider the U.S. households' Euler equation for holding domestic bonds, reproduced in the discrete-time form below,

$$1 - \tilde{\lambda}_t^h = \mathbb{E}_t \left[ \frac{M_{t+1}}{M_t} \exp(r_t) \right].$$

Since the U.S. households' SDF and convenience yield are assumed to be unaffected by the shock, this Euler equation implies that the dollar risk-free rate $r_t$ does not change.

Second, from the foreign households' Euler equation for holding U.S. bonds, reproduced in discrete-time form below,

$$1 - \tilde{\lambda}_t^f = \mathbb{E}_t \left[ \frac{M_{t+1}^*}{M_t^*} \exp(r_t + \Delta s_{t+1}) \right],$$

an increase in their convenience yield $\tilde{\lambda}_t^f$ lowers their risk-neutral expected return on holding dollar safe bonds. Since the dollar risk-free rate $r_t$ does not change, the exchange rate has to adjust to equilibrate this Euler equation. In particular, the dollar needs to appreciate today and create an expected depreciation to generate the lower expected return.

Lastly, if we examine the U.S. households' Euler equation for holding foreign bonds, reproduced in discrete-time form below,

$$1 = \mathbb{E}_t \left[ \frac{M_{t+1}}{M_t} \exp(r_t^* - \Delta s_{t+1}) \right],$$

we learn that the dollar exchange rate movement also raises the expected return on purchasing foreign currency bonds from the U.S. perspective. Since the U.S. households do not derive a convenience yield on foreign bonds that can adjust, all adjustments must happen in the dollar's currency risk premium. In our equilibrium, this happens via endogenous changes in the cyclicality and volatility of the dollar exchange rate. Thus, these four Euler equations require endogenous responses in both first moments (i.e., exchange rate level and expected return) as well as second moments (i.e., currency cyclicality and volatility) in response to the convenience yield shock. Solving the full model involves deriving the endogenous exchange rate process that respects these forces. Section 3.5 revisits this section's experiment in the context of the solved model.

## 2.4 SDF and Convenience Yield Dynamics

We posit a pair of U.S. (dollar) and foreign log SDFs. Let $m_t = \log(M_t)$ and $m_t^* = \log(M_t^*)$ denote the log SDFs. We posit that the log SDFs have the following dynamics:

$$dm_t = -\mu_t dt - \sigma_t dZ_t, \qquad dm_t^* = -\mu_t^* dt - \sigma_t dZ_t^*,$$

where the Brownian increments $dZ_t$ and $dZ_t^*$ represent shocks to the marginal utilities of home and foreign households, which may originate from aggregate shocks to consumption or output in fully specified models. To simplify the notation, we assume that the home and foreign SDF volatilities are identical.

We assume that the convenience yields derived by U.S. and foreign investors can be different. As we show later, the difference in these convenience yields is all that matters for exchange rate dynamics. We denote the difference between the two convenience yields as $\tilde{\lambda}_t = \tilde{\lambda}_t^f - \tilde{\lambda}_t^h$, and parameterize the difference as follows:

$$\tilde{\lambda}_t = \ell \frac{\exp(\lambda_t)}{\exp(\lambda_t) + \exp(\eta)},$$

which is bounded between 0 and $\ell$. The auxiliary state variable $\lambda_t$ satisfies

$$d\lambda_t = -\theta \lambda_t dt + \nu dX_t,$$

where $dX_t$ is a standard Brownian motion and $\nu > 0$.

Finally, with slight abuse of notation, let $[dX_t, dY_t]$ denote the instantaneous conditional covariance between two diffusion processes $X_t$ and $Y_t$. We assume that the convenience yield shock and the SDF shocks can be pairwise correlated:

$$[dZ_t, dX_t] = \rho, \qquad [dZ_t^*, dX_t] = \rho^*, \qquad [dZ_t, dZ_t^*] = \zeta.$$

Note that the home and foreign SDFs load negatively on the $dZ$ or $dZ^*$ shocks. We assume that $\rho^* < 0$, so that the foreign households' convenience yield on the dollar safe bonds tends to increase when their marginal utilities rise. This correlation captures foreigners' "flight-to-Treasurys" during their recessions. When global volatility in financial markets increases, the convenience yield on U.S. Treasurys tends to increase. Similarly, we assume $\rho > 0$. That is, we posit that a rising convenience yield $\tilde{\lambda}_t^h$ from the U.S. perspective is associated with an increase in U.S. marginal utility.

These correlations and their signs are the natural ones that would emerge in an international macroeconomic model. In Online Appendix OA we develop a two-period international macro model where both U.S. and foreign households earn convenience yields on their holdings of the U.S. Treasury bond, trade both home and foreign countries' bonds and goods. We require that the net flows of capital and goods balance at the equilibrium exchange rate. In the model, an increase in foreign demand for U.S. Treasury bonds leads to foreign households cutting back on current consumption (hence higher marginal utility or SDF) in order to purchase these bonds, with the U.S. households accommodating by selling U.S. Treasury bonds, increasing holdings of foreign bonds, and increasing current consumption (hence lower marginal utility or SDF). Thus, although we work at a high level with the dynamics of the SDF in our derivations, these dynamics can be consistent with the implications from a fully specified macroeconomic model.

## 2.5 Equilibrium Exchange Rate

We posit that the real exchange rate has the following dynamics,[2]

$$ds_t = \alpha_t dt + \beta_t \sigma_t (dZ_t^* - dZ_t) + \gamma_t \nu dX_t, \tag{8}$$

[2]Equation (8) covers the set of equilibria in which the exchange rate is exposed to only the SDF shocks and the convenience yield shock. There exists additional exchange rate solutions that are exposed to additional shocks: $ds_t = \alpha_t dt + \beta_t \sigma_t (dZ_t^* - dZ_t) + \gamma_t \nu dX_t + \omega_t dW_t$, where $dW_t$ is orthogonal to $dX_t$. These additional solutions generate higher exchange rate volatility that worsens the puzzles we explore in the next section.

where $\alpha_t$, $\beta_t$, and $\gamma_t$ are $\mathbb{F}$-adapted stochastic processes that we must solve for. $\beta_t$ governs the distance from complete markets. When $\beta_t \equiv 1$, and $\gamma_t \equiv 0$, we are back in the benchmark complete markets case: $ds_t^{cm} = dm_t - dm_t^*$. That is, the change in the log exchange rate equals the difference in the log SDFs. The complete-market exchange rate adjusts to absorb all SDF shocks.

Our objective is to characterize a solution to (8) that satisfies the four pricing conditions (2), (3), (6) and (7). In our incomplete-market setting, there are many candidate solutions. We obtain the following family of solutions that are consistent with the four pricing conditions.

**Proposition 1.** *There is a class of solutions indexed by a positive real number* $k_t \in \left[\frac{\ell-(\rho^*-\rho)^2\sigma_t^2/4}{1-(\rho^*-\rho)^2/(2(1-\zeta))}, \frac{\sigma_t^2(1-\zeta)}{2}\right]$ *so that,*

$$\beta_t = \frac{1}{2} \pm \frac{1}{2}\sqrt{\frac{\sigma_t^2(1-\zeta)-2k_t}{\sigma_t^2(1-\zeta)}}, \tag{9}$$

$$\gamma_t = \frac{(\rho^*-\rho)\sigma_t(1-2\beta_t) \pm \sqrt{(\rho^*-\rho)^2\sigma_t^2(1-2\beta_t)^2+4(k_t-\tilde{\lambda}_t)}}{2\nu}. \tag{10}$$

*The log exchange rate movement satisfies:*

$$ds_t = \left(-\frac{1}{2}\tilde{\lambda}_t - (\mu_t - \mu_t^*) + \frac{1}{2}\sigma_t\gamma_t\nu(\rho+\rho^*)\right)dt + \gamma_t\nu dX_t + \beta_t\sigma_t(dZ_t^* - dZ_t), \tag{11}$$

*which loads on both the SDF shocks* $dZ_t$ *and* $dZ_t^*$ *and the convenience yield shock* $dX_t$.

We present the proof in Appendix A. To avoid imaginary roots in equation (9) for $\beta_t$ and equation (10) for $\gamma_t$ for all values of $\lambda_t$, $k_t$ has to take values in the specified range.

$\beta_t$ captures the degree to which the SDF shocks drive exchange rate movements. For a given $k_t$, there are two solutions for $\beta_t$. One root is between 1/2 and 1, and the other is between 0 and 1/2. As $k_t$ can in principle vary over time, $\beta_t$ can also vary over time.

As for $\gamma_t$, we pick the greater root $\gamma_t = \frac{(\rho^*-\rho)\sigma_t(1-2\beta_t)+\sqrt{(\rho^*-\rho)^2\sigma_t^2(1-2\beta_t)^2+4(k_t-\tilde{\lambda}_t)}}{2\nu}$, so that for $k_t > \tilde{\lambda}_t$, we can guarantee $\gamma_t > 0$. We do so to arrive at the natural result that the U.S. dollar appreciates when the foreign convenience yield for dollar bonds rises. Finally, note that $\gamma_t$ varies with the convenience yield.

**Corollary 1.** *For any* $\ell < k_t < \frac{\sigma_t^2(1-\zeta)}{2}$, *we have* $0 < \beta_t < 1$ *and the greater root of* $\gamma_t > 0$.

We focus on this range of $k_t$ in the rest of the analysis.

## 2.6 Exchange Rate Moments

Now, we are in a position to analytically characterize the exchange rate volatility, cyclicality, expected return, and level decomposition. We present the proofs in Appendix A.

**Proposition 2. (Volatility)**
*Given that the real exchange rate follows*

$$ds_t - \mathbb{E}_t[ds_t] = \gamma_t \nu dX_t + \beta_t \sigma_t (dZ_t^* - dZ_t),$$

*the exchange rate volatility can be expressed as*

$$[ds_t, ds_t] = 2(1-\zeta)\beta_t^2\sigma_t^2 + \gamma_t^2\nu^2 + 2\gamma_t\nu\beta\sigma_t(\rho^* - \rho), \tag{12}$$

*whereas under complete markets, it is*

$$[ds_t^{cm}, ds_t^{cm}] = 2(1-\zeta)\sigma_t^2.$$

If we compare equation (12) with the complete-market solution, we note that the incomplete-market case reduces volatility in our model through two channels. First, the pass-through coefficient $\beta_t$ can be smaller than one. This result is related to Lustig and Verdelhan (2019), who show that incomplete markets introduce a wedge in the exchange rate movement and this wedge is always negatively correlated with the SDF differential. As a result, it offsets the exchange rate movements induced by the SDF shocks and lead to a less volatile exchange rate movement. In our model, this wedge coincides with the convenience yield, which we will calibrate in the next section based on the empirical analysis in Jiang et al. (2021), and thus pin down $\beta_t$. This allows us to go further than Lustig and Verdelhan (2019) and nail down the extent of incomplete pass-through.

Second, if $\rho^* - \rho < 0$, the last term in (12) is negative and further reduces the exchange rate volatility. In economic terms, the countercyclical convenience yield drives countercyclical exchange rate movements, which partially offsets the exchange rate movements induced by the SDF shocks. This additional channel goes beyond the first channel that arise in any incomplete markets.

The Backus-Smith puzzle is the observation that the correlation between consumption growth and real exchange rate movement is close to zero or negative. This correlation is usually inferred from the slope coefficient in a projection of the exchange rate changes on the relative log SDF differential.

**Proposition 3. (Cyclicality)**

*The correlation between exchange rate changes and changes in the relative log SDF is:*

$$\frac{[ds_t, dm_t - dm_t^*]}{[dm_t - dm_t^*, dm_t - dm_t^*]} = \beta_t + \frac{\gamma_t \nu(\rho^* - \rho)}{2\sigma_t(1-\zeta)}. \tag{13}$$

When the markets are complete and there are no convenience yields, $\beta_t = 1$ and $\gamma_t = 0$, and this coefficient is therefore

$$\frac{[ds_t^{cm}, dm_t - dm_t^*]}{[dm_t - dm_t^*, dm_t - dm_t^*]} = 1.$$

However, in the data, using consumption growth as a proxy for the SDF shocks, the coefficient is close to zero or negative.

Similar to the case of exchange rate volatility, the cyclicality is lessened when the markets are incomplete for two reasons. First, market incompleteness shrinks the $\beta_t$ coefficient from 1 towards 0, lowering the first term in (13). This is the channel due to market incompleteness that is highlighted by Lustig and Verdelhan (2019).

Second, convenience yield shocks also impact the dollar's exchange rate and hence affect its cyclicality. Intuitively, as the dollar's convenience yield increases in foreign high marginal utility states, the dollar appreciates and partially offsets the foreign currency appreciation driven by the high realization of the foreign SDF. When $\rho^* < \rho$, i.e., the foreign country's SDF is more exposed to the convenience yield shock than the U.S., the second term in equation (13) is negative, which further reduces the slope coefficient.

However, while the incomplete market and convenience yield channels reduce the exchange rate cyclicality, they cannot reduce the covariance $[ds_t, dm_t - dm_t^*]$ below zero in our setting. Our approach provides a useful, albeit negative, result.

The expected excess return to a foreign investor of going long U.S. bonds relative to foreign bonds is:

$$(\mathbb{E}_t[ds_t] + r_t - r_t^*) + \frac{1}{2}[ds_t, ds_t] = -\tilde{\lambda}_t^f - [dm_t^*, ds_t],$$

where the right-hand side reflects the compensation the investor requires for exposure of this currency trade to the SDF as well as the convenience benefit. We can likewise write this expression in terms of the U.S. investor's return for going long foreign bonds relative to U.S. bonds:

$$(-\mathbb{E}_t[ds_t] - r_t + r_t^*) + \frac{1}{2}[ds_t, ds_t] = \tilde{\lambda}_t^h + [dm_t, ds_t].$$

The terms in parentheses in these two equations are equal but have opposing signs: they reflect the carry trade returns alternately from the foreign and the U.S. standpoints. That is, if the foreign investor is receiving an expected log currency return of 2% on going long the dollar, the U.S. investor must be receiving an expected log currency return of −2% on going long the foreign currency.

We can combine these equations to find:

$$-\left([dm_t, -ds_t] + \frac{1}{2}[ds_t, ds_t]\right) + \tilde{\lambda}_t^h = \left([dm_t^*, ds_t] + \frac{1}{2}[ds_t, ds_t]\right) + \tilde{\lambda}_t^f.$$

Rearranging terms, we obtain,

**Proposition 4. (Cyclicality Bound)**
*The covariation between shocks to the SDF differential and exchange rates is:*

$$[dm_t - dm_t^*, ds_t] = [ds_t, ds_t] + \tilde{\lambda}_t > 0.$$

The left-hand side is the numerator in our exchange rate cyclicality calculation in equation (13). Therefore, regardless of our choice in the family of exchange rate equilibria, as long as the bond convenience yield differential $\tilde{\lambda}_t$ is positive, the exchange rate cyclicality $\frac{[ds_t, dm_t - dm_t^*]}{[dm_t - dm_t^*, dm_t - dm_t^*]}$ will be positive. In particular, when we consider an incomplete-market model without convenience yields, $\tilde{\lambda}_t = 0$, then, the exchange rate cyclicality has to be non-negative as $[ds_t, ds_t] > 0$. This result puts a lower bound on the exchange rate cyclicality in all possible equilibria in the class of general diffusion models in which the investors can trade home and foreign risk-free bonds.

The result is perhaps surprising. While it is the case that convenience yield shocks dampen the impact of the SDF shocks, it cannot be the case that the typical shock looks like a convenience yield shock. The home and foreign investors' conditional Euler equations impose a straightjacket on the comovement between the SDF and exchange rates.

As such, our results offer only a partial resolution on the comovement. The convenience yield mechanism reduces the cyclicality of the exchange rate, but does not change the sign of the cyclicality. The argument of this section is also broader than convenience yield models. Any model operating through the SDF equations we have written down must satisfy Proposition 4. If we take the further step of associating SDF shocks with shocks to aggregates such as consumption, our result shows that it is not possible to change the sign on the consumption-exchange rate correlation in the class of models encompassed by our four Euler equations.[3]

[3] Another solution is to break the link between aggregate consumption and the SDF of the trader driving exchange rate behavior. This is the approach of market segmentation models such as Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2021).

We next consider expected returns:

**Proposition 5. (Expected Returns)**
*The foreign investors' expected excess return on going long U.S. government bonds relative to foreign government bonds is given by*

$$\Pi_t^f = \frac{\mathbb{E}_t\left[d\exp(r_t + s_t - r_t^*)\right]}{\exp(r_t + s_t - r_t^*)} = -\tilde{\lambda}_t^f - [dm^*, ds_t] = -\tilde{\lambda}_t^f + \beta_t \sigma_t^2 (1-\zeta) + \sigma_t \gamma_t \nu \rho^*. \quad (14)$$

The first term $-\tilde{\lambda}_t^f$ captures the convenience yield: when the foreign households have a higher convenience yield on the dollar safe bonds, they are willing to accept a lower expected return. The second term $-[dm_t^*, ds_t]$ captures the standard risk premium: if the dollar tends to appreciate in high foreign marginal utility states, the foreign households are also willing to accept a lower expected return. In comparison, the expected return under complete markets is

$$\Pi_t^{f,cm} = \sigma_t^2 (1-\zeta),$$

which does not contain the convenience yield term $-\tilde{\lambda}_t^f$, nor the limited pass-through coefficient $\beta_t$.

Moreover, the convenience yield also drives an endogenous currency risk premium. As the foreign households' convenience yield tends to increase during foreign recessions, the dollar becomes a better hedge from the perspective of the foreign households. As a result, the convenience yield not only affects the dollar's expected return directly through the $-\tilde{\lambda}_t^f$ term but also indirectly through the currency risk premium term $\sigma_t \gamma_t \nu \rho^*$.

Similarly, the foreign currency's expected return $\Pi_t^h$ from the perspective of the U.S. households reflects the U.S. households' convenience yield $\tilde{\lambda}_t^h$ and the covariance between the U.S. households' SDF and the exchange rate movement, $[dm_t, ds_t]$:

$$\Pi_t^h = \frac{\mathbb{E}_t\left[d\exp(-s_t - r_t + r_t^*)\right]}{\exp(-s_t - r_t + r_t^*)} = \tilde{\lambda}_t^h + [dm_t, ds_t] = \tilde{\lambda}_t^h + \beta \sigma^2 (1-\zeta) - \sigma_t \gamma_t \nu \rho. \quad (15)$$

As the foreign household's convenience yield $\tilde{\lambda}_t^f$ increases, the excess return on the dollar falls, while as the home household's convenience yield $\tilde{\lambda}_t^h$ increases, the excess return on foreign currency rises. This behavior of the currency risk premium is also driven by the combination of market incompleteness and the cyclicality of the convenience yield.[4]

The result in Proposition 5 resolves a tension that constitutes the key result in Lustig and Verdelhan (2019) in their incomplete-market setting without convenience yields: more market incompleteness helps to reduce exchange rate volatility and cyclicality, diminishing these puzzles, but it also shrinks the currency risk premia towards zero. In our model, by incorporating the convenience yield, we avoid this tension and can generate reasonable risk premia. The next section quantifies these points.

Finally, the forward premium puzzle refers to the empirical regularity that the interest rate differential captures the conditional variation in the currency excess return. Typically, the forward premium puzzle is explained by time-varying SDF volatility, which moves the interest rate differential and the currency expected return in the same direction.

While this risk-premium channel is also present in our model, the convenience yield induces an additional mechanism to resolve the forward premium puzzle. Specifically, if the U.S. and foreign convenience yields ($\tilde{\lambda}_t^f$ and $\tilde{\lambda}_t^h$) on the U.S. bond are positively correlated, then, a higher convenience yield leads to a lower interest rate differential and a lower expected excess return on the dollar.

## 2.7 Exchange Rate Level

We close our analytical results with a characterization of the exchange rate level.

**Proposition 6. (Level)**
*If the long-run expectation of the exchange rate level exists and is finite:*

$$\bar{s} = \lim_{T\to\infty} \mathbb{E}_t[s_T],$$

*the exchange rate level $s_t$ has a forward-looking representation (Campbell and Clarida, 1987; Froot and Ramadorai, 2005; Jiang et al., 2021):*

$$s_t = \bar{s} + \lim_{T\to\infty} \mathbb{E}_t \int_t^T (r_u - r_u^*)du + \lim_{T\to\infty} \mathbb{E}_t \int_t^T \tilde{\lambda}_u^f du - \lim_{T\to\infty} \mathbb{E}_t \int_t^T rp_u du. \quad (17)$$

[4] Alternatively, we can also derive the dollar's expected excess return in log, which is given by

$$\pi_t^f = \mathbb{E}_t[d \log \exp(r_t + s_t - r_t^*)] = \mathbb{E}_t[ds_t] + r_t - r_t^* = -\frac{1}{2}(\tilde{\lambda}_t^f + \tilde{\lambda}_t^h) + \frac{1}{2}\sigma_t\gamma_t\nu(\rho + \rho^*), \quad (16)$$

while the foreign currency's expected log excess return take the opposite value:

$$\pi_t^h = \mathbb{E}_t[d \log \exp(-r_t - s_t + r_t^*)] = -\mathbb{E}_t[ds_t] - r_t + r_t^* = -\pi_t^f.$$

On the right-hand side, $\lim_{T\to\infty} \mathbb{E}_t \int_t^T (r_u - r_u^*)du$ captures expected future short rate differences, $\lim_{T\to\infty} \mathbb{E}_t \int_t^T \tilde{\lambda}_u^f du$ captures expected future convenience yields earned by the foreign investors, and $-\lim_{T\to\infty} \mathbb{E}_t \int_t^T rp_u du$ captures expected future currency risk premia from the foreign perspective.

This decomposition in equation (17) is the equivalent of a Campbell-Shiller decomposition for exchange rates. The exchange rate level today reflects future interest rate differences (cash flows), future convenience yields, minus future risk premia (discount rates). This expression is forward-looking, which complements the backward-looking expression for the exchange rate level in equation (8). The dollar appreciates when future U.S. short rates increase and dollar currency risk premia decline. Jiang et al. (2021) derive a version of this decomposition that allows for convenience yields. When foreign investors expect to earn larger convenience yields on U.S. bonds, the dollar appreciates in spot markets.

Alternatively, we can define a U.S. bond yield without convenience yield, $r_t^{xcy} = r_t + \tilde{\lambda}_t^h$. Then, the exchange rate decomposition becomes

$$s_t = \bar{s} + \lim_{T\to\infty} \mathbb{E}_t \int_t^T (r_u^{xcy} - r_u^*)du + \lim_{T\to\infty} \mathbb{E}_t \int_t^T \tilde{\lambda}_u du - \lim_{T\to\infty} \mathbb{E}_t \int_t^T rp_u du, \tag{18}$$

in which case the first term becomes the interest rate differential of U.S. and foreign bonds without convenience yields, and the second term becomes the differential in the convenience yields from foreign and U.S. investors' perspectives, $\tilde{\lambda}_t = \tilde{\lambda}_t^f - \tilde{\lambda}_t^h$.

We contrast this expression to the case when markets are complete and there are no convenience yields. We use $r_{t,T-t}^{xcy}$ and $r_{t,T-t}^{*,xcy}$ to denote the long-term bond yields that do not carry a convenience yield. They are directly implied from the home and foreign SDFs:

$$\exp(-(T-t)r_{t,T-t}^{xcy}) = \mathbb{E}_t[M_{t,T}], \qquad \exp(-(T-t)r_{t,T-t}^{*,xcy}) = \mathbb{E}_t[M_{t,T}^*].$$

Backus, Boyarchenko, and Chernov (2018); Lustig, Stathopoulos, and Verdelhan (2019) show that if the exchange rate is stationary and markets are complete, the exchange rate reflects differences in long yields:

$$s_t - \bar{s} = \lim_{T\to\infty} (T-t)(r_{t,T-t}^{xcy} - r_{t,T-t}^{*,xcy}). \tag{19}$$

The intuition is simple: because there is no difference in riskiness between holding a U.S. and a foreign bond over long holding periods, these investments have to carry the same risk premium in the limit. This logic results in the UIP condition with long-term bond yields. However, as we will show in our numerical section, this long-run UIP condition will fail in the presence of convenience yields.

# 3 Quantitative Implications of Convenience Yields for Exchange Rates

In this section, we provide a quantitative evaluation of our characterization in the last section, focusing on the extent to which the convenience yield channel resolves (1) the comovement between dollar exchange rate and flight-to-safety as in Jiang et al. (2021), (2) the partial SDF-FX pass-through and the Brandt et al. (2006) puzzle, (3) the Backus-Smith puzzle, and (4) currency risk premium in the short and long run.

Our analytical results characterize a family of equilibria indexed by $k_t$. In our quantitative evaluation, we choose one equilibrium in this family, with $k_t = k$, a constant. Below, we explain how we settle on the value for $k$. We emphasize that our objective in this section is to understand how far the convenience yield/incomplete market model can go in resolving exchange rate puzzles.

## 3.1 Calibration Choices

We calibrate the model at the annual frequency and report our parameter values in Table 1.

TABLE 1–PARAMETER CHOICES

| Symbol | Interpretation | Value | Calibration Target |
|---|---|---|---|
| | *Panel A: SDF and FX* | | |
| $\mu$ | SDF drift | 0 | symmetry |
| $\sigma$ | SDF shock volatility | 1.5 | max Sharpe ratio |
| $\zeta$ | SDF shock correlation | 0.32 | consumption growth correlation |
| $\phi$ | exchange rate persistence | 0.135 | half life of exchange rate shock |
| $\rho$ | U.S. SDF loading on convenience shock | 0.3 | see text |
| $\rho^*$ | foreign SDF loading on convenience shock | $-0.34$ | see text |
| | *Panel B: Convenience Yield* | | |
| $\ell$ | convenience yield level | 15% | Treasury basis mean |
| $\eta$ | convenience yield shape | 15 | Treasury basis mean |
| $\nu$ | convenience yield volatility | 30 | Treasury basis volatility |
| $\theta$ | convenience yield persistence | 1.5 | Treasury basis persistence |

**Convenience Yield** We target the moments of the implied 1-year convenience yield. Specifically, let $p_t^{xcy}(1)$ denote the price of a one-period bond without convenience yield:

$$p_t^{xcy}(h) = \mathbb{E}_t[M_{t,t+h}], \quad h = 1.$$

Let $p_t(1)$ denote the price of a one-period bond with convenience yield, whose price solves the following (for $h = 1$):

$$\exp(-\tilde{\lambda}_t dt) = \mathbb{E}_t \left[ M_{t,t+dt} \frac{p_{t+dt}(h - dt)}{p_t(h)} \right].$$

The 1-year convenience yield is given by the log difference between the two bond prices: $\tilde{\lambda}_t^{1y} = \log(p_t(1)/p_t^{xcy}(1))$.

Jiang et al. (2021) show that the 1-year convenience yield can be measured from the deviation from CIP for government bonds ("the Treasury basis"), defined as the difference between the yield on a cash position in U.S. Treasurys $y_t^{\$}$ and the synthetic dollar yield constructed from a cash position in a foreign government bond.[5] Denote $x_t^{Treas} = y_t^{\$} + (f_t^1 - s_t) - y_t^*$ as the Treasury basis, where $f_t^1$ is the log forward rate. Then Jiang et al. (2021) show that the convenience yield is proportional to the basis:

$$(1 - \beta^{basis})\tilde{\lambda}_t^{1y} = -x_t^{Treas},$$

where $\beta^{basis}$ measures the fraction of the convenience yield earned on a synthetic U.S. Treasury constructed from a foreign currency denominated safe government bond. Jiang et al. (2021) estimate the constant of proportionality $\beta^{basis}$ to be 0.9 so that the mean Treasury basis of 0.22 gives a mean convenience yield of $0.22/(1 - 0.9) = 2.2\%$. The standard deviation of the Treasury basis of 0.23 implies a standard deviation of the convenience yield of $0.23/(1 - 0.9) = 2.3\%$. We estimate an AR(1) model of the Treasury basis and find the estimated persistence coefficient to be close to 0.5.

We pick the appropriate parameter values for $\ell$, $\nu$, $\theta$, and $\eta$ to match these empirical properties. Figure 1 reports the unconditional distribution of the 1-year convenience yield in our model. It has a mean of 2.06% and a standard deviation of 1.97%, and an

[5] Since the Great Financial Crisis, sizable deviations from Covered Interest Parity have opened up in LIBOR markets (Du et al., 2018b), but even before the GFC, there were large, persistent deviations from CIP in government bond markets (see Du, Im, and Schreger, 2018a; Jiang et al., 2021; Du and Schreger, 2021). U.S. Treasurys are always expensive relative to synthetic Treasurys constructed from foreign bonds. In Jiang et al. (2021), we estimate that foreign investors earn convenience yields of around 200 basis points, significantly larger than the CIP deviations. Using a demand-system-based approach, Koijen and Yogo (2020) report similar estimates of the convenience yields.

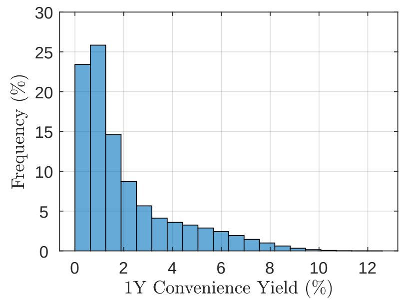



FIGURE 1. THE DISTRIBUTION OF 1-YEAR CONVENIENCE YIELD.

autocorrelation of 0.63 at quarterly frequency. The skewness is 1.52 which is consistent with the fact that the convenience yield is small on average but jumps up in periods of global distress.

**SDF** We posit that the log SDFs have the following dynamics:

$$dm_t = -\mu dt - \sigma dZ_t, \qquad dm_t^* = \phi s_t dt - \sigma dZ_t^*.$$

First, we note that the volatility is a constant. While time-varying volatility generates time variation in currency risk premium, we choose this simpler specification because it is enough to illustrate the new channel that highlights the role of the convenience yield.

The SDF volatility $\sigma$ is calibrated to 1.5 per annum, which implies that the maximal annual Sharpe ratio permitted by either country's SDF is roughly 1.5 as well. Moreover, we set $\zeta = [dZ_t, dZ_t^*]$, the correlation between U.S. and foreign SDF shocks, to 0.32, which is the average correlation between U.S. consumption growth and other G10 countries' consumption growth, using annual data from 1970 to 2018.[6]

In terms of the correlations between the SDF shocks and the convenience shock, intuitively, each country's demand for dollar safe assets should increase when its marginal utility is higher, i.e., $[dZ_t, d\lambda_t^h] < 0$ and $[dZ_t^*, d\lambda_t^f] < 0$. Since $dX_t$ is the shock to the convenience yield differential $(\lambda_t^f - \lambda_t^h)$, we expect $\rho = [dZ_t, dX_t] > 0$ and $\rho^* = [dZ_t^*, dX_t] < 0$. While this logic determines the signs, it does not pin down values for the correlation. We calibrated the specific values of $\rho = 0.3$ and $\rho^* = -0.34$ to match the empirical moments as we will see in Table 2. We also report alternative choices of $(\rho = 0.5, \rho^* = -0.5)$ and $(\rho = 0, \rho^* = 0)$ to illustrate how these parameters affect results.

[6] Alternatively, we can calibrate this parameter using the average correlation between the change in the U.S. stock log price-to-dividend ratio and other G10 countries', which yields a slightly higher value of 0.48.

Second, the drifts in the SDFs are specified such that the foreign interest rate is decreasing in the level of the dollar exchange rate. With $\phi > 0$, this implicit monetary policy rule leads to mean reversion in the exchange rate level. In fact, if markets are complete, the log exchange rate $s_t^{cm}$ is a simple stationary process:

$$ds_t^{cm} = dm_t - dm_t^* = (-\mu - \phi s_t^{cm})dt + \sigma(dZ_t^* - dZ_t). \tag{20}$$

The adjustment in interest rate in response to the exchange rate level is governed by the parameter $\phi$, which we set to 0.135. This parameter value implies that the half-life of the variation in a shock to the real exchange rate is $\log(2)/\phi = 5.13$ years. In the data, we estimate an AR(1) model of the log dollar index and find the estimated model to have a half-life of 5.18 years.

**Pass-through coefficient $\beta$** Our objective is to characterize a solution to (8) that satisfies the four pricing conditions (2), (3), (6) and (7). In our incomplete-market setting, there are many candidate solutions. In principle, the loading $\beta_t$ of the exchange rate on the SDF shocks may vary over time. In our calibration exercise, we consider the family of solutions in which $\beta_t$ is time-invariant (or equivalently, $k_t$ is constant), which allows us to derive a closed-form solution for the exchange rate level. With a constant $\beta$, we obtain a closed-form expression for the log of the real exchange rate.

**Proposition 7. (Exchange Rate Level)**
*(1) The exchange rate $s_t$ can be expressed as*

$$s_t = f(\lambda_t) + H_t + \beta s_t^{cm}. \tag{21}$$

*The first term $f(\lambda_t)$ is a function of the current convenience yield $\lambda_t$, the second term $H_t$ captures the history of past convenience yields:*

$$H_t = e^{-\phi t} H_0 + \int_0^t e^{-\phi(t-u)} h(\lambda_u) du,$$

*and the third term is the pass-through $\beta$ times the real exchange rate $s_t^{cm}$ under complete markets whose dynamics is given by (20).*

*(2) The exchange rate's long-run expectation $\lim_{T\to\infty} \mathbb{E}_t[s_T]$ is also well-defined:*

$$\bar{s} \equiv \lim_{T\to\infty} \mathbb{E}_t[s_T] = \frac{1}{\phi}\left(-\frac{1}{2}\lim_{T\to\infty} \mathbb{E}_0[\tilde{\lambda}_T] - \mu + \frac{1}{2}\sigma \lim_{T\to\infty} \mathbb{E}_0[\gamma_T]\nu(\rho+\rho^*)\right).$$

The proof and the expressions of functions $f$ and $h$ are in Appendix OB.2. This proposition shows that the real exchange rate level is determined by not only the SDF differential, as summarized by the real exchange rate $s_t^{cm}$ under complete markets, but also the current convenience yield and the history of the convenience yields $\lambda_t$.

## 3.2 Exchange Rate, Flight-to-Safety, and the Choice of $k$

Table 2 presents regression results from data generated by simulating the calibrated model for different values of $k$. Panel A reports the benchmark calibration with flight to safety, as explained below. Panel B reports the results for the calibration without flight to safety. Panel C reports the same moments in the data. We discretize the model by a time increment of $\Delta t = 0.001$ period and simulate $50,000$ periods. As we have noted, our model generates a family of solutions indexed by $k$. Thus, in the table, we reported simulation results for a range of $k$.

Panel A reports results for the benchmark calibration of $\rho = 0.3$ and $\rho^* = -0.34$, in which case positive SDF shocks in each country raise their demand for dollar safe assets. We first note that for all values of $k$, we have that $\beta < 1$. Lustig and Verdelhan (2019) show that when markets are incomplete, shocks to the SDF will pass through less than one-for-one to the exchange rate.[7] Thus, the value of $\beta < 1$ is a manifestation of their result in our setting. Market incompleteness reduces the exchange rate's exposure to fundamentals. However, the Lustig and Verdelhan (2019) approach does not allow one to pin down $\beta$, while our approach does.

We pin down the value of $k$, and hence the values of $\beta$ and $\gamma$ from the regression coefficient in column (3). This is a regression of the exchange rate movement on the change in the 1-year bond convenience yield:

$$\Delta s_t = \alpha + \beta \Delta \tilde{\lambda}_t^{1y} + \varepsilon_t.$$

Jiang et al. (2021) show that the dollar's real exchange rate is increasing in the convenience yield that foreign investors assign to the dollar risk-free bond. Specifically, when the U.S. Treasury's 1-year convenience yield increases by one standard deviation (0.23% as measured by Treasury basis), the dollar appreciates by 2.35%. In the post-2008 sample, the one-standard-deviation shock leads to a dollar appreciation of 3.28%. These results indicate a regression coefficient of the exchange rate movement $\Delta s_t$ on the change in the convenience yield $\Delta \tilde{\lambda}_t$ of between 1.02 and 1.49 (see Panel B). We run the same regression

[7] In the two-period macroeconomic model in Online Appendix OA, we show that $\beta < 1$ in equilibrium and that the $R^2$ of SDF shocks on exchange rate innovations is also less than 100%.

TABLE 2–SIMULATED MOMENTS

| | | | | | |
|---|---|---|---|---|---|
| *Panel A: $\rho = 0.3$ and $\rho^* = -0.34$* | | | | | |
| (1) | (2) | (3) | (4) | (5) | (6) |
| $k$ | $\beta$ | FX-Conv Yield Coef | FX Vol (%) | SDF-FX Pass-Thru | Exp. Log Return (%) |
| 0.00 | 0.00 | -1.58 | 8.53 | 0.01 | -1.31 |
| 0.12 | 0.04 | 0.55 | 9.49 | 0.01 | -1.70 |
| 0.13 | 0.04 | 0.76 | 10.26 | 0.01 | -1.74 |
| 0.14 | 0.05 | 0.96 | 11.09 | 0.01 | -1.78 |
| **0.16** | **0.05** | **1.17** | **11.96** | **0.02** | **-1.82** |
| 0.17 | 0.06 | 1.36 | 12.86 | 0.02 | -1.86 |
| 0.18 | 0.06 | 1.56 | 13.78 | 0.02 | -1.90 |
| 0.77 | 0.50 | 8.34 | 81.71 | 0.23 | -4.19 |
| *Panel B: Data* | | | | | |
| (1) | (2) | (3) | (4) | (5) | (6) |
| $k$ | $\beta$ | FX-Conv Yield Coef | FX Vol (%) | SDF-FX Pass-Thru | Exp. Log Return (%) |
| - | - | 1.02—1.49 | 10.00 | $< 0$ | -1.89 |
| *Panel C: Complete-Market Model* | | | | | |
| (1) | (2) | (3) | (4) | (5) | (6) |
| $k$ | $\beta$ | FX-Conv Yield Coef | FX Vol (%) | SDF-FX Pass-Thru | Exp. Log Return (%) |
| 0.77 | 1.00 | -0.17 | 173.97 | 1.00 | -0.48 |
| *Panel D: $\rho = 0.5$ and $\rho^* = -0.5$* | | | | | |
| (1) | (2) | (3) | (4) | (5) | (6) |
| $k$ | $\beta$ | FX-Conv Yield Coef | FX Vol (%) | SDF-FX Pass-Thru | Exp. Log Return (%) |
| 0.00 | 0.00 | -0.89 | 4.99 | 0.01 | -1.43 |
| 0.27 | 0.10 | 0.24 | 10.01 | 0.01 | -1.45 |
| 0.77 | 0.50 | 2.10 | 46.07 | 0.08 | -1.54 |
| *Panel E: $\rho = \rho^* = 0$* | | | | | |
| (1) | (2) | (3) | (4) | (5) | (6) |
| $k$ | $\beta$ | FX-Conv Yield Coef | FX Vol (%) | SDF-FX Pass-Thru | Exp. Log Return (%) |
| 0.15 | 0.05 | 6.16 | 35.39 | 0.05 | -1.41 |
| 0.21 | 0.08 | 8.99 | 44.15 | 0.07 | -1.44 |
| 0.77 | 0.50 | 19.47 | 121.45 | 0.50 | -1.65 |

*Notes:* Calibration of parameters in Table 1. We vary the value for the remaining parameter $k$ and report several moments from the simulated model. Column (1) reports the value of $k$. (2) reports the implied $\beta$ parameter in the exchange rate process $ds_t = \alpha_t dt + \beta\sigma(dZ_t^* - dZ_t) + \gamma_t \nu dX_t$. (3) reports the slope coefficient in regression of $\Delta s_t$ on $\Delta\tilde{\lambda}_t$. (4) reports annual FX volatility. (5) reports the slope coefficient in regression of $\Delta s$ on $\Delta m - \Delta m^*$. (6) reports the annual expected log excess return on long position in the U.S. dollar. The regressions are run at quarterly frequency. Our simulation is based on a long sample of $T = 50,000 \times 1000$ time intervals.

in our simulated data and report the results for different values of $k$ in column (3). When $k$ is 0.16, the regression coefficient is 1.17 and in the range of the data. Once $k$ is chosen, the values for $\beta$ as well as other regression results are pinned down and reported in the table in the rest of the columns.

To illustrate these forces, Figure 2 plots $\beta$ against $\gamma_t$ at $\lambda_t = 0$, for the family of solutions indexed by $k$, by varying $k$. This plot is for the principal calibration of $\rho = 0.3$ and $\rho^* = -0.34$. Over most of the range of $\beta$, the convenience yield loading $\gamma_t$ of the exchange rate increases as the SDF loading $\beta$ on fundamentals increases. Our calibration pins down $\gamma$ by matching the regression coefficient in column (3) in Table 2 and then the logic of the model pins down $\beta$.

Panel B reports these moments in the data, based on the sample of the dollar exchange rates relative to other G10 countries. In the next sections, we will further discuss how our model-implied moments compare to these empirical moments.

Panel C reports the moments implied by a complete-market model with the same parameter values. In this complete-market model, the exchange rate movement is fully determined by the SDF shocks, and the implied moments miss the empirical moments by a large margin.

We next consider alternative values of $\rho$ and $\rho^*$, which govern the correlations between the convenience yield shock and the SDF shocks. In Panel D, we consider a greater magnitude for the correlation between the convenience yield shock and the SDF shocks by setting $\rho = 0.5$ and $\rho^* = -0.5$. In this parameterization, the response of the dollar appreciation to the convenience yield is dampened. To understand this result, we note that the exchange rate movement can be expressed by $ds_t = \alpha_t dt + \beta\sigma(dZ_t^* - dZ_t) + \gamma_t \nu dX_t$.

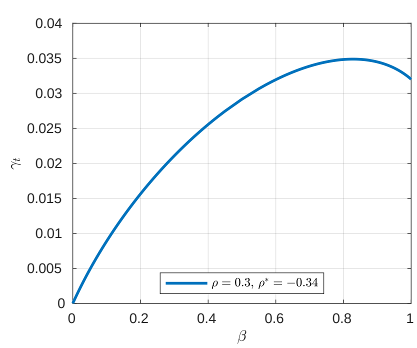



FIGURE 2. EXCHANGE RATE LOADINGS ON THE SDF AND THE CONVENIENCE YIELD SHOCKS.

*Notes:* We plot the coefficients $\beta$ and $\gamma_t$ of the exchange rate process $= \alpha_t dt + \beta\sigma(dZ_t^* - dZ_t) + \gamma_t \nu dX_t$ by varying $k$ and setting $\lambda_t = 0$. Calibration in Table 1.

Its response to the convenience yield shock is governed by both the SDF component $\beta\sigma(dZ_t^* - dZ_t)$ and the convenience yield component $\gamma_t \nu dX_t$. Specifically, a positive $dX$ shock to the convenience yield differential always increases $\gamma_t \nu dX_t$ and appreciates the dollar. However, because the SDF shocks and the convenience yield shock are correlated, a positive $dX$ shock also lowers the U.S. marginal utility and increases the foreign marginal utility, which weakens the dollar through the SDF component $\beta\sigma(dZ_t^* - dZ_t)$. When the correlations between the convenience yield shock and the SDF shocks are high enough, the SDF channel offsets the convenience yield channel, leading to a lower or even negative regression coefficient in column (3) as we see in this panel. In comparison, our preferred choice of $\rho = 0.3$ and $\rho^* = -0.34$ manages to generate this regression coefficient that is consistent with the data.

In Panel E, we turn off the flight-to-safety channel by setting $\rho = \rho^* = 0$, which makes the convenience yield acyclical with respect to the SDFs. Now the FX volatility is too high relative to the data; the convenience yield is not acting as a shock absorber, and the model generates too high a regression coefficient of the exchange rate movement on the convenience yield innovation.

## 3.3 Backus-Smith Puzzle and Exchange Rate Cyclicality

Column (5), Table 2 reports the Backus-Smith coefficient in (13), reproduced below:

$$\frac{[ds_t, dm_t - dm_t^*]}{[dm_t - dm_t^*, dm_t - dm_t^*]} = \beta + \frac{\gamma_t \nu(\rho^* - \rho)}{2\sigma(1-\zeta)}.$$

This puzzle is lessened when the markets are incomplete. In our preferred calibration, this coefficient is 0.02, i.e., nearly acyclical. This result is driven by both market incompleteness and the convenience yield. First, the $\beta$ coefficient is 0.05 in our calibration whereas it is 1 in the complete-market case.

Second, the convenience yield shocks also impact the dollar's exchange rate cyclicality, generating a negative term of $\frac{\gamma_t \nu(\rho^* - \rho)}{2\sigma(1-\zeta)} = -0.04$ which further reduces the slope coefficient. Figure 3 illustrates this point. We simulate the impact of a shock to the convenience yield, traced out in the first panel. The second panel plots the paths of the SDFs $m_t$ and $m_t^*$, whose innovations are correlated with the convenience yield shock under our parameterization. The foreign SDF rises, reflecting that bad news for the foreign economy is correlated with an increase in the foreign demand for U.S. dollar bonds. The home SDF decreases in our calibration since $\rho > 0$. The third panel plots the exchange rate under the complete markets benchmark. We see that the home currency (dollar) depreciates when the home SDF declines, reflecting the Backus-Smith puzzle. The last panel plots the exchange rate in our incomplete markets convenience yield model. The home currency appreciates, with the convenience yield shock offsetting the change in the SDF.

That said, as proved in Proposition 4, our convenience yield model still generates a non-negative Backus-Smith coefficient. While it is the case that convenience yield shocks dampen the impact of the SDF shocks and generates a dollar appreciation in good U.S. states, Proposition 4 shows that it cannot be the case that the typical shock looks like a convenience yield shock. The home and foreign investors' conditional Euler equations impose a straightjacket on the comovement between the SDF and exchange rates.

## 3.4 Exchange Rate Volatility Puzzle and Partial SDF-FX Pass-through

Column (4), Table 2 reports the conditional volatility of the exchange rate movement in (12), reproduced below:

$$[ds_t, ds_t]^{1/2} = \left(2(1-\zeta)\beta^2\sigma^2 + \gamma_t^2\nu^2 + 2\gamma_t\nu\beta\sigma(\rho^* - \rho)\right)^{1/2}.$$

In our preferred calibration, the exchange rate volatility is 12%, which is again due to two channels. First, the pass-through coefficient $\beta = 0.05$ shrinks the first term $2(1-\zeta)\sigma^2$ by $\beta^2$. In comparison, when the markets are complete, this term implies a much higher exchange rate volatility of $(2(1-\zeta)\sigma^2)^{1/2} = 173\%$. We also note that higher values of $k$ lead to higher values of $\beta$ and higher exchange rate volatility. This partial SDF-FX pass-through result helps resolve the volatility puzzle of Brandt et al. (2006), as the complete

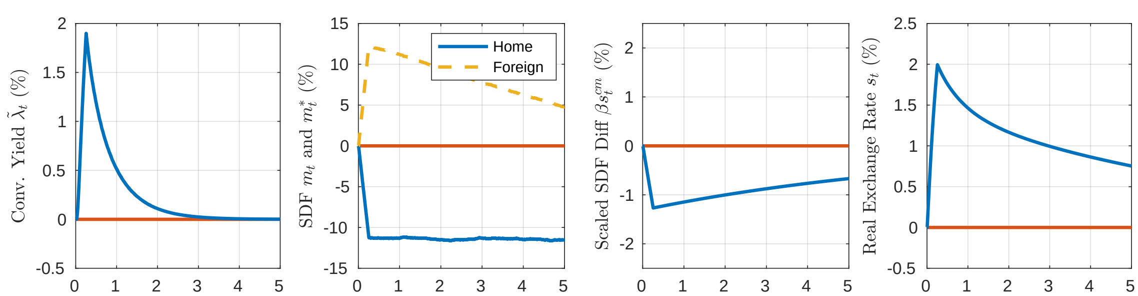



FIGURE 3. IMPULSE RESPONSE TO A CONVENIENCE YIELD SHOCK

*Notes:* We report the average difference between simulations in which the convenience yield $\lambda_t$ jumps up by 1 standard deviation in period $(0, 0.25]$ and simulations in which all shocks have zero means. The shock to the convenience yield (top-left panel) also leads to a correlated change in the SDFs (top-right panel).

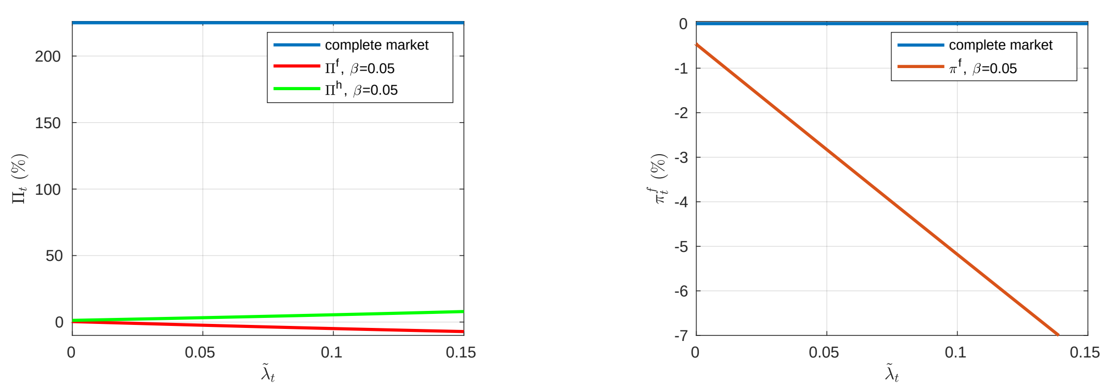
markets $dm - dm^*$ is more volatile than $ds$.

Second, the countercyclical convenience yield further reduces the volatility of exchange rates, which goes beyond the first channel that arises in an incomplete markets model. Specifically, a negative $(\rho^* - \rho)$, which captures a countercyclical convenience yield, generates a negative last term of $2\gamma_t \nu \beta \sigma(\rho^* - \rho)$ in the volatility formula and further lowers the volatility.

## 3.5 Currency Risk Premium Puzzles

Next, we report the model-implied currency expected returns $\Pi_t^f$ in (14), which reflects the foreign investors' expected excess return on going long U.S. government bonds relative to foreign government bonds. Figure 4(a) plots the dollar's expected excess returns ($\Pi_t^f$, in red) for different values of the convenience yield differential $\tilde{\lambda}_t$. On average, the dollar has a negative expected excess return from the foreign perspective. We also note that these expected excess returns vary with the state variable $\tilde{\lambda}_t$: a higher convenience yield differential on dollar safe assets leads to an even lower expected return on the dollar.

Similarly, Figure 4(a) also plots this foreign currency's expected return from the U.S. perspective, which is positive on average ($\Pi^h$, in green). As $\gamma_t$ is decreasing in the convenience yield differential $\tilde{\lambda}_t$, the foreign currency's expected return is increasing in the foreign convenience yield on dollar safe assets. This result echoes the intuition in our discussion of equilibrium forces in Section 2.3: U.S. households do not derive convenience yields on foreign bonds. As a result, to enforce the U.S. households' Euler equation for holding foreign bonds with a higher expected return on the foreign currency, the foreign




*Panel (a) Expected Excess Return Levels $\Pi_t^f$ and $\Pi_t^h$* *Panel (b) Expected Log Excess Return $\pi_t^f$*

FIGURE 4. CURRENCY EXPECTED EXCESS RETURN

currency's cyclicality and volatility have to adjust to generate an endogenous currency risk premium. Specifically, when $\tilde{\lambda}_t$ is high, $\gamma_t$ is low, and hence the exchange rate movement loads less on the convenience yield shock. From the U.S. households' perspective, since $\rho > 0$, a higher U.S. marginal utility is associated with a lower convenience yield differential and a weaker dollar/stronger foreign currency. As a lower $\gamma_t$ weakens this hedging property, the foreign currency becomes riskier and hence has a higher risk premium.

Figure 4(b) plots the dollar's expected log excess return $\pi_t^f$ as defined in (16), whereas the foreign currency's expected log excess return take the opposite value: $\pi_t^h = -\pi_t^f$. As $\tilde{\lambda}_t$ increases, the log excess return on the dollar falls, while the log excess return on foreign currency rises. This behavior of the log currency risk premium is also driven by the combination of market incompleteness and the cyclicality of the convenience yield.

In Table 2, our preferred calibration implies that the expected log return is $-1.82\%$ on average. For comparison, in Jiang et al. (2021), we compute the returns for a foreign investor owning the entire U.S. Treasury bond index relative to their U.S. government bond index over a sample from 1980 to 2019. We report that the dollar Treasury return is 1.89% lower than the foreign bond return, which is close to the model-implied estimate. According to equation (16), given an average convenience yield of $\mathbb{E}[\tilde{\lambda}_t] = 2.0\%$, our model indicates that about $\frac{1}{2}\mathbb{E}[\tilde{\lambda}_t] = 1.0\%$ in the expected log return is attributable to the dollar's convenience yield, and the remaining $1.82\% - 1.00\% = 0.82\%$ is attributable to the dollar's currency risk premium.

For comparison, if markets are complete, since the U.S. and the foreign SDFs have the same volatilities, the log currency risk premium on USD is zero, and the risk premium in levels equals the variance of the SDF (Bansal, 1997; Backus et al., 2001).

$$\begin{aligned}\pi_t^{cm,f} &= \pi_t^{cm,h} = 0, \\ \Pi_t^{cm,f} &= \tilde{\Pi}_t^{cm,h} = (1-\zeta)\sigma^2.\end{aligned}$$

In this case, the log currency risk premium is too small relative to the data, whereas the level of currency risk premium is too large. These values are represented by the blue lines in Figure 4.

This result resolves a tension that constitutes the key result in Lustig and Verdelhan (2019) in an incomplete-market models without convenience yields: more market incompleteness helps to reduce exchange rate volatility and cyclicality, but it also shrinks currency risk premia towards 0. In our model, by incorporating the convenience yield, we can escape this trade-off and provide a joint resolution to all these puzzles.

### 3.6 Exchange Rate Level

Finally, we discuss the implication of the calibrated model for the exchange rate level characterized by Proposition 6. The convenience yield introduces a wedge not only in the investors' Euler equations determining the currency return, but also in the determination of the exchange rate level. In particular, we compare the exchange rate level in our calibrated model with the benchmark case with complete markets and stationary exchange rates. In the latter case, the bond yield differential between the U.S. and foreign converges to zero:

$$0 = \lim_{T\to\infty} r^{xcy}_{t,T-t} - r^{*,xcy}_{t,T-t},$$

and the difference in the log bond prices is equal to the exchange rate level, as we note in the long-run UIP condition (19),

$$s^{cm}_t - \bar{s}^{cm} = \lim_{T\to\infty} (T-t)(r^{xcy}_{t,T-t} - r^{*,xcy}_{t,T-t}).$$

We make two observations. First, in our calibrated convenience yield model, even when the exchange rate level has a well-defined long-run mean, the infinite-horizon yield differential does not converge to zero. Figure 5 plots the bond yield differential between the home and foreign countries as we vary the bond maturity from 1 year to 1000 years. For maturities above 200 years, the home bond yield is about 6% lower than the foreign bond yield, which implies that the right-hand side of the long-run UIP condition (19) is

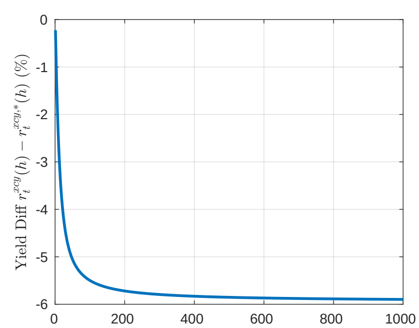



FIGURE 5. HOME AND FOREIGN LONG-TERM YIELD DIFFERENTIAL.

*Notes:* We plot the difference between home and foreign bond yields as a function of the bond maturity $h$. These bonds do not contain convenience yields. The bond yields are evaluated at $s_t = 0$ and $\lambda_t = 0$.

infinite,

$$\lim_{T\to\infty}(T-t)(r^{xcy}_{t,T-t}-r^{*,xcy}_{t,T-t})=-\infty,$$

and therefore cannot be equal to the exchange rate level.

To understand this result, recall Eq. (17) in Proposition 6, reproduced below,

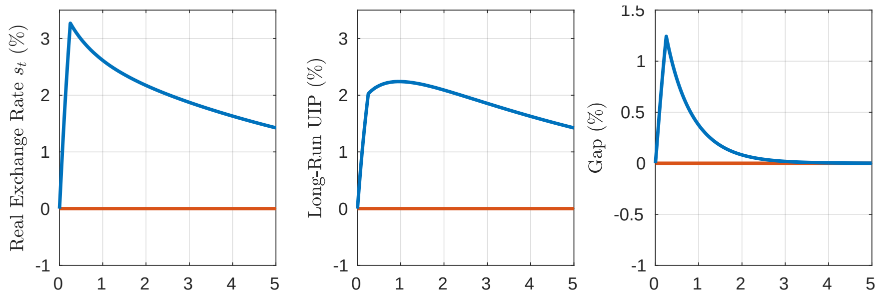
$$s_t=\bar{s}+\lim_{T\to\infty}\mathbb{E}_t\int_t^T(r_u-r_u^*)du+\lim_{T\to\infty}\mathbb{E}_t\int_t^T\tilde{\lambda}^f_u du-\lim_{T\to\infty}\mathbb{E}_t\int_t^T rp_u du.$$

On the right-hand side, the convenience yield differential $\tilde{\lambda}^f_u$ is nonzero and positive on average. Then, to generate a finite exchange rate on the left-hand side, the short rate differential and the risk premium terms on the right-hand side need to exactly offset the sum of convenience yield differentials, which leads to a permanent gap in the long-term bond yields between home and foreign countries.

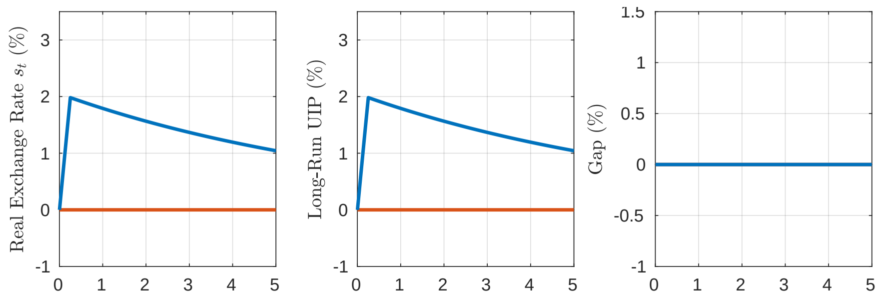
Second, we consider the deviation from the long-run UIP condition by removing the infinite mean of the right-hand side and evaluating the long-term bond yield at a very long maturity (1000 years). We find that the convenience yield shock drives an additional *conditional* wedge in the long-run UIP condition. In Figure 6(a), we simulate the model in the presence of a positive convenience yield shock in the first quarter. In response to this shock, the dollar appreciates (left panel), and the long-run UIP term, i.e., $\lim_{T\to\infty}(T-t)(r^{xcy}_{t,T-t}-r^{*,xcy}_{t,T-t})$, also increases relative to its mean because of the response of the foreign SDF to exchange rate movements (middle panel). However, the conditional increase in the long-run UIP term is not enough to explain the magnitude of dollar appreciation (right panel). The convenience yield shock drives additional dollar appreciation, and the long-run UIP condition (19) also fails conditionally.

In comparison, when we consider a foreign SDF shock in Figure 6(b), we find that the dollar appreciates, and the conditional increase in the long-run UIP term is enough to explain the magnitude of the dollar appreciation. As a result, the left-hand and the right-hand sides of (19) do not further deviate in response to the SDF shock.

This discussion shows that the long-run UIP condition which holds in the complete markets model does not hold in the convenience yield model. The convenience yield drives not only an unconditional failure of the UIP condition, but also conditionally appreciates the dollar more than what is warranted by the bond yield differential. The results are useful to understand the impact of quantitative easing on exchange rates, as we show next.




(A) IMPULSE RESPONSE TO A PURE CONVENIENCE YIELD SHOCK.




(B) IMPULSE RESPONSE TO A PURE SDF SHOCK.

FIGURE 6. IMPULSE RESPONSES OF EXCHANGE RATE AND LONG-RUN UIP TERM

*Notes:* Panel (a) reports the average difference between simulations in which the convenience yield $\lambda_t$ jumps up by 1 standard deviation in period $(0, 0.25]$ and simulations in which all shocks have zero means. Similarly, Panel (b) reports the average difference in the presence of the foreign SDF shock. The long-run UIP term is $\lim_{T\to\infty}(T-t)(r^{xcy}_{t,T-t} - r^{*,xcy}_{t,T-t})$. The gap is the difference between the series reported in the first two panels.

# 4 Quantitative Easing and the Convenience Yield Channel

Quantitative easing (QE) policies—that is, large-scale purchases of long-term bonds matched by increases in bank reserves—have been shown to affect exchange rates (Neely, 2015) and the convenience yield component of interest rates (Krishnamurthy and Vissing-Jorgensen, 2011). In this section, we show how our model can shed light on these connections.

## 4.1 Evidence for the Convenience Yield Channel

In a model without convenience yields, when long-term yields at home fall relative to foreign, the exchange rate depreciates, as we have explained in Section 3.6. Since quantitative easing lowers the long-term interest rates in the U.S. (Krishnamurthy and Vissing-Jorgensen, 2011), it will depreciate the dollar's exchange rate. This is the operative channel in the models of Gourinchas et al. (2021); Greenwood et al. (2020), where changes in bond supply affect the term-premium and long-term bond yields, and hence the exchange rate.[8]

The first column of Table 3 presents evidence consistent with this channel. We consider 14 QE event dates, using an event window from the close of trading on the day prior to the event date to the close of trading one day after the event date. Across the event dates, we measure the change in the government bond interest rate differential between the U.S. and average of other G10 countries' yields, for maturities ranging from 1 year to 10 years, and the change in USD exchange rate against the G10 countries. An increase in the 10-year yield differential between the U.S. and foreign countries appreciates the dollar. The coefficient estimate of 0.06 suggests that a 1% change in the yield differential moves the dollar by 6%. In complete-market models, we expect that the magnitude would be somewhat close to 10 if the product between the yield differential and tenor, i.e., $(T-t)(r_{t,T-t}-r^*_{t,T-t})$, evaluated at $T-t=10$ is close to this product evaluated at the infinite horizon $T-t \to \infty$.

Our work identifies a novel convenience yield channel through which large-scale asset purchases affect exchange rates. Shifts in the supply of dollar safe assets, as happens via QE, will change convenience yields on dollar safe bonds and exchange rates. The impact of QE on Treasury bond yields via a convenience yield channel is outlined in Krishnamurthy and Vissing-Jorgensen (2011). An important observation is that this bond channel works in the *opposite* direction as the long-run FX UIP channel. In the presence of bond convenience yields, the long-term interest rate is driven by not only the integral of the no-convenience short rate and the bond risk premium, but also a convenience yield term:

$$r_{t,T-t} = \mathbb{E}_t\left[\int_t^T (r_u^{xcy} - \tilde{\lambda}_u^h)du\right] + brp_{t,T-t}.$$

To consider an extreme case, suppose that quantitative easing increases the convenience yield on the long-term bond and thus lowers the long-term interest rate. In this case, according to our equation (18), reproduced below,

$$s_t - \bar{s} = \lim_{T\to\infty} \mathbb{E}_t \int_t^T (r_u^{xcy} - r_u^*)du + \lim_{T\to\infty} \mathbb{E}_t \int_t^T \tilde{\lambda}_u du - \lim_{T\to\infty} \mathbb{E}_t \int_t^T rp_u du$$

[8] Gourinchas et al. (2021) and Greenwood et al. (2020) bring an equilibrium model of the term structure with market segmentation along the lines of Vayanos and Vila (2021) to bear on FX markets. These authors explore the impact of downward sloping demand curves for Treasurys. A decrease in the net U.S. supply of long bonds, as would occur under a QE purchase by the Fed, causes U.S. arbitrageurs to lower their required bond risk premium on long USD bonds. As a result, policymakers can control long rates and thereby impact exchange rates via a bond risk-premium channel.

TABLE 3–QUANTITATIVE EASING, TREASURY BASIS, AND EXCHANGE RATE

| *Panel (a) Bivariate Regression* | | | | | | | |
|---|---|---|---|---|---|---|---|
| Basis Tenor $k$ | – | 10Y | 7Y | 5Y | 3Y | 2Y | 1Y |
| $\Delta$ 10-year $y$-diff | 0.06 | $0.11^{***}$ | $0.14^{***}$ | $0.09^{***}$ | $0.08^{***}$ | $0.09^{***}$ | $0.06^{**}$ |
| | (0.03) | (0.03) | (0.03) | (0.03) | (0.03) | (0.02) | (0.02) |
| $\Delta$ $k$-year Treasury Basis | | $-0.15^{**}$ | $-0.20^{***}$ | $-0.14^{***}$ | $-0.16^{***}$ | $-0.20^{***}$ | $-0.17^{***}$ |
| | | (0.05) | (0.05) | (0.04) | (0.05) | (0.04) | (0.03) |
| Num. of Obs. | 14 | 14 | 14 | 14 | 14 | 14 | 14 |
| Adj. $R^2$ (full model) | 0.12 | 0.48 | 0.59 | 0.55 | 0.54 | 0.70 | 0.73 |
| *Panel (b) Univariate Regression* | | | | | | | |
| Basis Tenor $k$ | – | 10Y | 7Y | 5Y | 3Y | 2Y | 1Y |
| $\Delta$ $k$-year Treasury Basis | | $-0.06$ | $-0.04$ | $-0.10^{*}$ | $-0.12^{*}$ | $-0.16^{**}$ | $-0.17^{***}$ |
| | | (0.06) | (0.06) | (0.05) | (0.06) | (0.06) | (0.04) |
| Num. of Obs. | | 14 | 14 | 14 | 14 | 14 | 14 |
| Adj. $R^2$ (full model) | | $-0.01$ | $-0.04$ | 0.16 | 0.18 | 0.31 | 0.56 |

*Notes:* In Panel (a), we report the bivariate regression of the dollar exchange rate movement (against G-10 average) on QE-induced changes in U.S. Treasury basis ($\Delta$ Basis) and in yield differential ($\Delta$ $y$-diff) between the U.S. and the G-10 average. In Panel (b), we report the univariate regression of the dollar exchange rate movement on QE-induced changes in U.S. Treasury basis. We include 14 QE event dates, and the event window is from the close of trading on the day prior to the event day to the close of trading 1 day after the event day. We vary the tenor of Treasury basis from 10 years to 1 year, and we keep the tenor of the yield differential at 10 years. $^{***}p < 0.01$; $^{**}p < 0.05$; $^{*}p < 0.10$.

the dollar's exchange rate should appreciate even though the long-term Treasury rate falls.

Column (2) of Table 3 presents a regression showing evidence in favor our model. We measure the change in the Treasury basis, against G10 countries, and include this change as independent variable along with the change in the 10-year bond yield differential. The 10-year Treasury basis in this regression is a proxy for the expected future convenience yields, $\mathbb{E}_t \int_t^T \tilde{\lambda}_u du$. We note that both terms have explanatory power for the dollar's exchange rate, and that the sign is as expected. That is, a less negative basis (lower convenience yield) depreciates the dollar, while a higher interest rate differential appreciates the dollar. Importantly, the adjusted $R^2$ in this regression rises from 12% in Column (1) to 48% in Column (2), suggesting that there are offsetting effects in the regression with only the 10-year yield, and that separating these effects, as our model suggests, provides far more explanatory power.

In Jiang et al. (2021), we argue that the 1-year Treasury basis is a better measure of the expected convenience yields, $\mathbb{E}_t \int_t^T \tilde{\lambda}_u du$, than the 10-year Treasury basis. This is because 10-year Treasury bonds are not as liquid and safe as 1-year bonds, and it is the liquidity and safety attributes of Treasurys that drive the convenience yield. In the last column of Table 3, we replace the 10-year Treasury basis with the 1-year Treasury basis. Consistent with this point, the adjusted $R^2$ rises to 73%. The coefficient estimate suggests that a 1% widening in the 1-year Treasury basis *around the QE windows* is associated with 17% appreciation of the dollar. In comparison, Jiang et al. (2021) find that a 1% widening in the 1-year Treasury basis is associated with a 10% to 15% appreciation of the dollar depending on the sample period.

Figure 7 presents a scatter plot of the change in 1-year Treasury basis against the dollar. Consistent with the tables, we see that the basis has considerable explanatory power for the change in the exchange rates. A second point from this figure is that the dollar appreciates in some of these events while it depreciates in others. If we focused purely on the impact on long-term interest rates, QE is known to reduce long-term rates via both a convenience yield channel and the bond risk premium channel of Vayanos and Vila (2021), and thus should be expected to depreciate the dollar if viewed through the lens of the long-run UIP condition. However, the convenience yield channel has a nuance. A swap of mortgage-backed securities for reserves likely increases the supply of safe assets, since reserves are a more convenient asset than mortgage-backed securities. A swap of Treasurys for reserves may increase or decrease the supply of safe assets depending on whether banks pass on the reserve expansion by expanding deposits and the relative convenience of these deposits and Treasurys. Thus, convenience yields can either rise or fall with QE, reconciling the patterns in Figure 7.

Furthermore, the convenience yield channel assigns a special role to the U.S., to the extent that the U.S. is the world's safe asset supplier. On the other hand, the bond risk-premium channel of Gourinchas et al. (2021); Greenwood et al. (2020) is symmetric; purchases by the ECB, BoJ, or BoE also affect bond risk premia, long yields, and the exchange rate. Evidence in favor of this special U.S. effect can be found in the impact of the Fed's dollar swap lines. While our model predicts that the association between QE and exchange rate changes should not always have the same sign, its deeper prediction is between changes in the supply of dollar safe assets and exchange rates. The Fed's dollar swap lines increase the supply of dollar safe assets abroad, which lowers the convenience yield that foreign investors impute on dollar safe assets. Through our expression of dollar exchange rate determination, this action supports foreign exchange rates. There is empirical evidence for this channel. Baba and Packer (2009); Aizenman, Ito, and Pasricha (2022) present evidence that the dollar swap lines between central banks depreciate the dollar. Kekre and Lenel (2023) show that the dollar swap line announcements reduce the Treasury basis and depreciate the dollar, with little impact on long-term Treasury yields. This last point identifies our U.S.-specific convenience yield channel.

## 4.2 QE Evaluation in the Model

We next turn to our model to see how well it can capture these patterns. We do not explicitly model the relation between the convenience yield $\lambda$ and the quantity of safe assets. Instead, we focus directly on inducing a shock to $\lambda$ and tracing out the impact of this shock on the exchange rate. Online Appendix OA shows how a change in the supply of safe assets affects $\lambda$ in the context of fully specified macro-finance model.

We simulate $dX_t$, $dZ_t$ and $dZ_t^*$ under the normal distribution with mean zero and standard deviation $\sqrt{\Delta t}$. We discretize the model by a time increment of $\Delta t = 0.0025$ and start the model at $t = 0$. For initial values, we set $s_0 = s_0^{cm} = \bar{s}$ and $\lambda_0 = 0$, and set $H_0$ to satisfy

$$s_0 = f(\lambda_0) + H_0 + \beta s_0^{cm}.$$

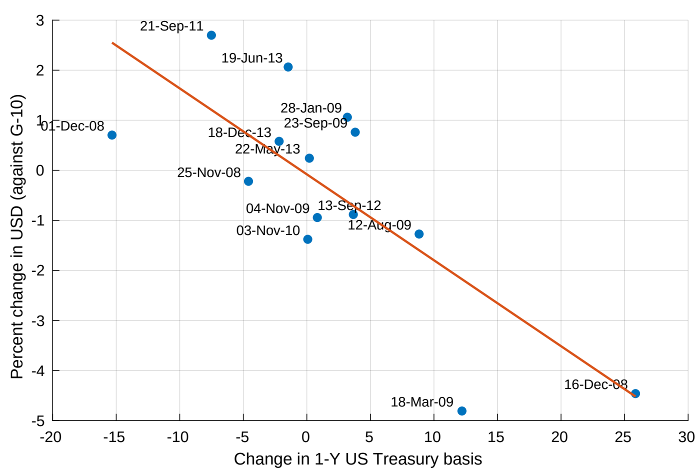



FIGURE 7. G-10 DOLLAR APPRECIATION AGAINST CHANGE IN BASIS AROUND QE EVENT DATES.

*Notes:* The first panel plots the change in the Treasury basis ($\Delta$ Basis) and the change in the dollar exchange rate from the close of trading on the day prior to the event day to the close of trading 2-days later. The second panel replaces the realized dollar exchange rate movement by the movement predicted by the long-run UIP condition. Our sample includes 14 QE event dates.

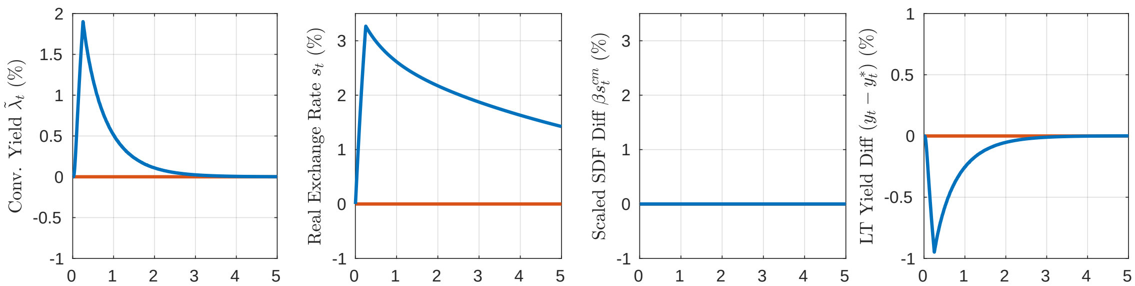



FIGURE 8. IMPULSE RESPONSE TO A PURE CONVENIENCE YIELD SHOCK.

*Notes:* We report the average difference between simulations in which the convenience yield $\lambda_t$ jumps up by 1 standard deviation in period $(0, 0.25]$ and simulations in which all shocks have zero means. Note that we assume that there is also no change in the SDFs.

For the first quarter, i.e., periods $(0, 0.25]$, we introduce a positive impulse that raises all realizations of the shocks $dX_t$ by one standard deviation. This impulse simulates a positive convenience yield shock in the first quarter. Then, we average across 1,000,000 simulated paths of the shocks $(dX_t, dZ_t, dZ_t^*)$. In this way, we estimate the average response following a positive convenience yield shock at date 0. We also simulate a benchmark case in which we draw from the normal distribution with mean 0 for the entire period $t \in (0, T]$. As expected, the average responses of exchange rate and convenience yield are close to zero in this benchmark case. We report the difference between the average responses in the case of a convenience yield shock and the benchmark case.

Figure 8 reports the result. In the first panel, we shock the convenience yield $\tilde{\lambda}_t$ and then let the internal dynamics of mean reversion gradually bring the convenience yield to zero over the next 2 years. We can think of this shock as an announcement by the central bank to purchase assets at date 0, and then slowly unwind these purchases over the next 10 quarters.

The second panel plots the exchange rate dynamics. The increase in the convenience yield leads to an appreciation of the dollar, which is consistent with our regression results in Table 3. Quantitatively, in period 0.25, the increase in the instantaneous convenience yield corresponds to a 0.73% increase in the one-year convenience yield, which maps to a widening in the 1-year Treasury basis of 7.3 basis points. This convenience yield shock is associated with a 3.2% appreciation of the dollar. In our empirical analysis as well as our calibrated model, a 10 basis point widening of the Treasury basis is associated with a 1.0% to 1.7% appreciation of the dollar. The difference in these results is because in the calibrated model the SDFs also adjust to partially offset the dollar appreciation.

We also note that the dollar appreciation is more persistent than the convenience yield shock. The term $H_t$ representing the cumulative convenience yields captures this behavior. Since

$$H_t = \exp(-\phi t) H_0 + \int_0^t \exp(-\phi(t-u)) h(\lambda_u) du,$$

it aggregates the influence of past convenience yields with exponentially decaying weights. As a result, the half-life of the response in the real exchange rate is longer than the half-life of the response in the spot convenience yield.

The third panel of Figure 8 shows the "complete-market component" of the exchange rate which is equal to the scaled SDF differential under complete markets, $\beta s_t^{cm} = \beta(m_t^{cm} - m_t^{cm,*})$. The convenience yield shock does not impact the path of the SDFs under complete markets (see Eq. (20)), implying no contribution to the equilibrium exchange rate dynamics from this component.

The last panel shows the long-term yield differential, $r_{t,1000} - r^*_{t,1000}$. We consider the yield of a U.S. long-term bond that carries a convenience yield. We need to make two additional assumptions to map this variable to our model. First, we assume that the observed 1000-year dollar bond yield contains a convenience yield, and its level is equal to the convenience yield on the dollar short rate. From the U.S. investors' perspective,

$$r_{t,1000} = r^{xcy}_{t,1000} - \tilde{\lambda}^h_t.$$

In reality, the convenience yield on the long-term bond is likely smaller than that on the T-bill. An alternative approach is to assume that the 10-year bond's convenience yield is a fraction of that on short rate, and choosing the fraction appropriately gives similar results. Second, we assume that the convenience yields $\tilde{\lambda}^h_t$ and $\tilde{\lambda}^f_t$ from the U.S. and foreign perspectives are both scaled versions of the convenience yield differential $\tilde{\lambda}_t = \lambda^f_t - \lambda^h_t$:

$$\tilde{\lambda}^h_t = \psi \tilde{\lambda}_t, \qquad \tilde{\lambda}^f_t = (1+\psi)\tilde{\lambda}_t,$$

so that high convenience yield state corresponds to a state in which both home and foreign investors derive high convenience yields from holding the dollar bond. Krishnamurthy and Vissing-Jorgensen (2012) find that the U.S. domestic convenience yield averages 75 basis points historically, which is about $\psi/(1+\psi) = 1/3$ of the convenience yield from the foreign perspective $\tilde{\lambda}^f_t = 2\%$ based on Jiang et al. (2021). Thus, we set $\psi = 1/2$. Under these assumptions, the observed U.S.-foreign bond yield differential, which incorporates their convenience yields, declines as the dollar convenience yield increases.

Finally, we plot the impulse responses to a pure foreign SDF shock in Figure 9, and contrast this with the response to the convenience yield shock. In the case of the SDF shock, the shock lowers the foreign SDF and drives a dollar appreciation. As the convenience yield is not affected, the magnitude of the dollar appreciation is solely attributable to the SDF dynamics, which is fully captured by the scaled SDF differential $\beta s_t^{cm}$.

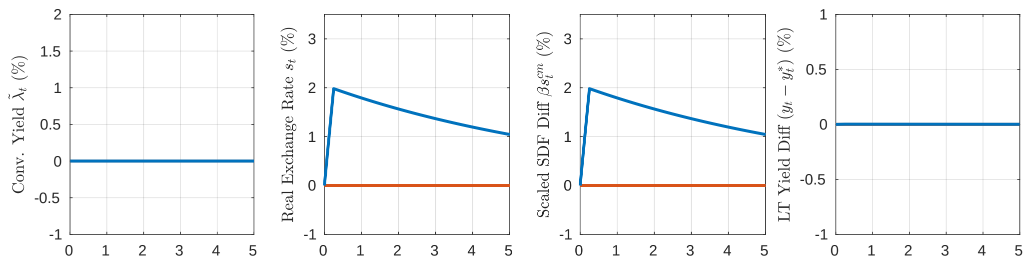



FIGURE 9. IMPULSE RESPONSE TO A PURE FOREIGN SDF SHOCK.

*Notes:* We report the average difference between simulations in which the foreign SDF shock $dZ_t$ jumps up by 1 standard deviation in period $(0, 0.25]$ and simulations in which all shocks have zero means. Note that we assume that there is also no change in the convenience yield.

# 5 Conclusion

We summarize our work by revisiting the equation describing exchange rate dynamics:

$$ds_t = \alpha_t dt + \beta\sigma(dZ_t^* - dZ_t) + \gamma_t \nu dX_t.$$

These exchange rate dynamics must be consistent with the four asset pricing conditions, for each of home and foreign investor in each of a home and foreign risk-free bond, where the home (dollar) bond offers convenience services to investors. The introduction of convenience yields leads to $\gamma_t > 0$ (dollar appreciates when foreign investor convenience for dollar bonds rises), and $\beta < 1$ (limited pass-through of marginal utility shocks to exchange rates). Our calibration demonstrates that convenience yields plus incomplete markets can help address exchange rate puzzles. Our joint modeling of bond and currency markets also shows how QE, as well as dollar swap lines, can affect exchange rates and resolves the puzzle of how QE can both appreciate the dollar and lower long-term U.S. Treasury yields.

These results highlight the significance of the worldwide demand for dollar safe assets in determining the international financial markets equilibrium. We conclude by noting that our analysis, by design, only models the asset pricing determination of exchange rates. We have solved for exchange rate dynamics that are consistent with four asset-pricing Euler equations. In doing so, we have sidestepped other salient aspects of equilibrium concerning quantities, especially the bond positions of foreign/home investors and the dynamics of the current account. Next steps in research may move in this direction. For these next steps, our research points out that including convenience yields and incomplete markets are important ingredients in a richer macroeconomic model. That is, as any macroeconomic model will include the four Euler equations we work with, the solution to the macroeconomic model will be among the class of solutions we have presented.

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# A Appendix

**Proposition 1.** Recall that the real pricing kernels are

$$dM_t = M_t(-\mu_t + \frac{1}{2}\sigma_t^2)dt - M_t\sigma_t dZ_t$$
$$dM_t^* = M_t^*(-\mu_t^* + \frac{1}{2}\sigma_t^2)dt - M_t^*\sigma_t dZ_t^*$$

Rewrite the first and the fourth Euler equations, we have

$$0 = \mathcal{A}[\int dM_t + M_t r_t dt + M_t \tilde{\lambda}_t^h dt] \quad \Rightarrow \quad r_t = \mu_t - \frac{1}{2}\sigma_t^2 - \tilde{\lambda}_t^h \tag{A.1}$$

$$0 = \mathcal{A}\left[\int dM_t^* + M_t^* r_t^* dt\right] \quad \Rightarrow \quad r_t^* = \mu_t^* - \frac{1}{2}\sigma_t^2 \tag{A.2}$$

Similarly, the second and the third Euler equations become

$$0 = -\mu_t + \frac{1}{2}\sigma_t^2 - \mathcal{A}[s_t] + \frac{1}{2}[ds_t, ds_t] + [-\sigma_t dZ_t, -ds_t] + r_t^* \tag{A.3}$$

$$0 = \tilde{\lambda}_t^f - \mu_t^* + \frac{1}{2}\sigma_t^2 + r_t + \mathcal{A}[s_t] + \frac{1}{2}[ds_t, ds_t] + [-\sigma_t dZ_t^*, ds_t] \tag{A.4}$$

Define $\tilde{\lambda}_t = \tilde{\lambda}_t^f - \tilde{\lambda}_t^h$. The sum of equations (A.3) and (A.4), together with equations (A.1) and (A.1), gives

$$-\tilde{\lambda}_t = \tilde{\lambda}_t^h - \tilde{\lambda}_t^f = [ds_t, ds_t] - \sigma_t[dZ_t^* - dZ_t, ds_t].$$

Plug in the conjecture (8), we obtain

$$-\tilde{\lambda}_t = \gamma_t^2 \nu^2 + 2\beta_t^2\sigma_t^2(1-\zeta) + 2\gamma_t\nu\beta_t(\rho^* - \rho)\sigma_t - 2\beta_t\sigma_t^2(1-\zeta) - (\rho^* - \rho)\sigma_t\gamma_t\nu. \tag{A.5}$$

Suppose for a certain value $k_t$,

$$-k_t = 2(1-\zeta)\beta_t^2\sigma_t^2 - 2(1-\zeta)\beta_t\sigma_t^2$$
$$k_t - \tilde{\lambda}_t = \gamma_t^2\nu^2 + 2\gamma_t\nu\beta_t(\rho^* - \rho)\sigma_t - (\rho^* - \rho)\sigma_t\gamma_t\nu$$

The solutions are

$$\beta_t = \frac{1}{2} \pm \frac{1}{2}\sqrt{\frac{\sigma_t^2(1-\zeta) - 2k_t}{\sigma_t^2(1-\zeta)}},$$
$$\gamma_t = \frac{(\rho^* - \rho)\sigma_t(1-2\beta_t) \pm \sqrt{(\rho^* - \rho)^2\sigma_t^2(1-2\beta_t)^2 + 4(k_t - \tilde{\lambda}_t)}}{2\nu},$$

which have real roots for all possible values of $\lambda_t$ if and only if

$$\frac{\ell - (\rho^* - \rho)^2\sigma_t^2/4}{1 - (\rho^* - \rho)^2/(2(1-\zeta))} \leq k_t \leq \frac{\sigma_t^2(1-\zeta)}{2}.$$

When the upper bound of $k_t$ is obtained, $\beta_t = 1/2$. When the lower bound of $k_t$ is obtained,

$$\beta_t = \frac{1}{2} \pm \frac{1}{2\sigma_t}\sqrt{\frac{\sigma_t^2(1-\zeta)-2\ell}{(1-\zeta)-(\rho^*-\rho)^2/2}}$$

which bounds the range of possible value of $\beta_t$.

Lastly, we also solve $\alpha_t$ from

$$\begin{aligned}
-\alpha_t &= \tilde{\lambda}_t^f - \mu_t^* + \frac{1}{2}\sigma_t^2 + r_t + \frac{1}{2}[ds_t, ds_t] + [-\sigma_t dZ_t^*, ds_t] \\
\alpha_t &= -\frac{1}{2}\tilde{\lambda}_t + \mu_t^* - \mu_t + \frac{1}{2}\sigma_t\gamma_t\nu(\rho+\rho^*)
\end{aligned}$$

**Volatility.** Given that real exchange rate follows $ds_t - \mathbb{E}_t[ds_t] = \gamma_t\nu dX_t + \beta_t\sigma_t(dZ_t^* - dZ_t)$, the exchange rate volatility is

$$\begin{aligned}
[ds_t, ds_t] &= [\gamma_t v dX_t + \beta_t\sigma_t(dZ_1^* - dZ_t), \gamma_t\nu dX_t + \beta_t\sigma_t(dZ_1^* - dZ_t)] \\
&= 2(1-\zeta)\beta_t^2\sigma_t^2 + \gamma_t^2\nu^2 + 2\gamma_t\nu\beta_t\sigma_t(\rho^*-\rho).
\end{aligned}$$

Under complete markets, $\beta_t = 1$, $\gamma_t = 0$, and $[ds_t^{cm}, ds_t^{cm}] = 2(1-\zeta_t)\sigma_t^2$.

**Cyclicality.** Notice that $[ds_t, dm_t - dm_t^*] = [\gamma_t\nu dX_t + \beta_t\sigma_t(dZ_t^* - dZ_t), \sigma_t(dZ_t^* - dZ_t)] = 2\beta_t\sigma_t^2(1-\zeta) + (\rho^*-\rho)\sigma_t\gamma_t\nu$ and $[dm_t - dm_t^*, dm_t - dm_t^*] = [\sigma_t(dZ_t^* - dZ_t), \sigma_t(dZ_t^* - dZ_t)] = 2\sigma_t^2(1-\zeta)$, we have

$$\frac{[ds_t, dm_t - dm_t^*]}{[dm_t - dm_t^*, dm_t - dm_t^*]} = \frac{2\beta_t\sigma_t^2(1-\zeta) + (\rho^*-\rho)\sigma_t\gamma_t\nu}{2(1-\zeta)\sigma_t^2} = \beta_t + \frac{\gamma_t v(\rho^*-\rho)}{2\sigma_t(1-\zeta)}.$$

**Expected return.** Plugging (A.1), (A.2), and the expression of $\alpha_t$ into the foreign investors' expected excess return on the U.S. bond relative to the foreign bond, we have

$$\begin{aligned}
\Pi_t^f &= \frac{\mathbb{E}_t[d\exp(r_t + s_t - r_t^*)]}{\exp(r_t + s_t - r_t^*)} = r_t + \mathbb{E}_t[ds_t] + \frac{1}{2}[ds_t, ds_t] - r_t^* \\
&= -\tilde{\lambda}_t^f + \beta_t\sigma_t^2(1-\zeta) + \sigma_t\gamma_t\nu\rho^* = -\tilde{\lambda}_t^f - [dm^*, ds_t].
\end{aligned}$$

Under complete markets, we have $\Pi_t^{f,cm} = \sigma_t^2(1-\zeta)$.

The dollar's expected excess return in log is

$$\pi_l^f = \mathbb{E}_t[d\log\exp(r_t + s_t - r_t^*)] = \mathbb{E}_t[ds_t] + r_t - r_t^* = -\frac{1}{2}\left(\tilde{\lambda}_l^f + \tilde{\lambda}_t^h\right) + \frac{1}{2}\sigma_t\gamma\nu(\rho+\rho^*).$$

# Online Appendix

# OA Two-Period International Macroeconomic Model

In this appendix, we develop a simple, fully specified, international macroeconomics model that nests the general characterization of the exchange rate dynamics in our main text. We use it as an example of a large class of international macroeconomic models that can be characterized by our approach.

There are two periods, indexed by $t = 0, 1$. We assume that home (U.S.) and foreign households have preferences over consumption and obtain convenience services from their holdings of the home country's bonds.

## OA.1 Home Households

There is a unit mass of identical price-taking households in each country. Let $c_{H,t}$ denote home households' consumption of a home composite good; let $c_{F,t}$ denote home households' consumption of a foreign composite good. We define $c_t = (c_{H,t})^{\alpha}(c_{F,t})^{1-\alpha}$ as the aggregated consumption bundle. Home households' lifetime utility is

$$u = \frac{1}{1-\gamma}c_0^{1-\gamma} + v(q_{H,0}\exp(-r_0)) + \delta\frac{1}{1-\gamma}c_1^{1-\gamma},$$

where $q_{H,0}$ is the notional amount of holdings of home bonds, and $r_0$ is the equilibrium interest rate of the home bonds.

The only tradable assets are the home and foreign risk-free bonds. Let $q_{F,0}$ denote home households' holding in foreign bonds. The home and foreign risk-free bonds are denominated in the unit of their composite bundles. The real exchange rate $s_t$ is also between the home composite bundle and the foreign composite bundle. Using this numeraire, the state-by-state budget constraint is

$$\exp(-r_0)\tau_1 + y_0 p_0 = c_0 + \exp(-r_0)q_{H,0} + \exp(-r_0^* - s_0)q_{F,0},$$
$$y_1 p_1 + q_{H,0} + \exp(-s_1)q_{F,0} = c_1 + \tau_1,$$

where $y_t$ denotes an exogenous endowment in home goods; $p_t$ is the price of the home good in the numeraire of the home consumption bundle. We assume that the home bonds are issued by the home country's government. $\tau_1$ is the total par value of the issuance. The proceeds from the issuance in period 0 (i.e., $\exp(-r_0)\tau_1$) are transferred to the home households, and the bonds are paid off in period 1 using taxes collected from the home households.

The Lagrangian is

$$\mathbb{E}_0[\frac{1}{1-\gamma}c_0^{1-\gamma} + v(q_{H,0}\exp(-r_0)) + \delta\frac{1}{1-\gamma}(y_1p_1 + q_{H,0} + \exp(-s_1)q_{F,0} - \tau_1)^{1-\gamma}]$$
$$+ \zeta_0\left[\exp(-r_0)\tau_1 + y_0p_0 - (c_0 + \exp(-r_0)q_{H,0} + \exp(-r_0^* - s_0)q_{F,0})\right].$$

**Inter-period solution** The first-order conditions for investment in home and foreign bonds give two Euler equations:

$$1 - c_0 v'(q_{H,0}\exp(-r_0)) = \mathbb{E}_0\left[\delta\frac{c_0}{c_1}\exp(r_0)\right],$$
$$1 = \mathbb{E}_0\left[\delta\frac{c_0}{c_1}\exp(-\Delta s_1 + r_0^*)\right].$$

**Intra-period solution** Let $p_t$ denote the price of the home good in units of the home consumption bundle, and let $p_t^*$ denote the price of foreign good in units of the foreign consumption bundle. Since we assume that the foreign consumption bundle is the foreign good, we have that $p_t^* = 1$. Then, the price of the consumption bundle is $c_t = p_t c_{H,t} + c_{F,t}\exp(-s_t)$. We substitute this expression into the Lagrangian:

$$1 - c_0^\gamma v'(q_{H,0}\exp(-r_0)) = \mathbb{E}_0\left[\delta(\frac{c_0}{c_1})^\gamma\exp(r_0)\right],$$
$$1 = \mathbb{E}_0\left[\delta(\frac{c_0}{c_1})^\gamma\exp(-\Delta s_1 + r_0^*)\right].$$

The first-order conditions for home and foreign consumption imply:

$$p_0\exp(s_0) = \frac{\alpha}{1-\alpha}\frac{c_{F,0}}{c_{H,0}},$$
$$p_1\exp(s_1) = \frac{\alpha}{1-\alpha}\frac{c_{F,1}}{c_{H,1}}.$$

## OA.2 Foreign Households

For tractability, we assume that foreign households only consume foreign goods. Their total consumption is $c^* = c_F^*$. Then foreign utility is,

$$u^* = \frac{1}{1-\gamma}(c_0^*)^{1-\gamma} + v(q_{H,0}^*\exp(-r_0 + s_0)) + \delta\frac{1}{1-\gamma}(c_1^*)^{1-\gamma}.$$

Using foreign goods as numeraire, the foreign budget constraint at each date are:

$$\exp(-r_0^*)\tau_1^* + y_0^* p_0^* = c_0^* + \exp(-r_0 + s_0)q_{H,0}^* + \exp(-r_0^*)q_{F,0}^*,$$
$$y_1^* p_1^* + q_{H,0}^* \exp(s_1) + q_{F,0}^* = c_1^* + \tau_1^*,$$

where $y_t^*$ denotes an exogenous endowment in foreign goods. Recall that $p_t^*$ is the price of the foreign good in the numéraire of the foreign composite bundle and that $p_t^* = 1$.

Then, the Lagrangian is

$$\mathbb{E}_0[\frac{1}{1-\gamma}(c_0^*)^{1-\gamma} + v(q_{H,0}^* \exp(-r_0 + s_0)) + \delta\frac{1}{1-\gamma}(y_1^* + q_{H,0}^* \exp(s_1) + q_{F,0}^* - \tau_1^*)^{1-\gamma}]$$
$$+ \zeta_0^*[\exp(-r_0^*)\tau_1^* + y_0^* - (c_0^* + \exp(-r_0 + s_0)q_{H,0}^* + \exp(-r_0^*)q_{F,0}^*)].$$

**Inter-period solution** The first-order conditions imply the Euler equations for foreign households

$$1 - (c_0^*)^\gamma v'(q_{H,0}^* \exp(-r_0 + s_0)) = \mathbb{E}_0\left[\delta(\frac{c_0^*}{c_1^*})^\gamma \exp(r_0 + \Delta s_1)\right],$$
$$1 = \mathbb{E}_0\left[\delta(\frac{c_0^*}{c_1^*})^\gamma \exp(r_0^*)\right].$$

## OA.3 Market Clearing

In the goods market:

$$y_t = c_{H,t},$$
$$y_t^* = c_{F,t} + c_{F,t}^*.$$

In the bond market:

$$\tau_1 = q_{H,0} + q_{H,0}^*$$
$$\tau_1^* = q_{F,0} + q_{F,0}^*.$$

## OA.4 Macro Synthesis

The set of known exogenous variables is

$$(y_0, y_0^*, \theta_0, \theta_0^*, \tau_1, \tau_1^*).$$

The set of stochastic exogenous variables is

$$(y_1, y_1^*).$$

The set of endogenous variables for home and foreign households is

$$(c_{H,0}, c_{F,0}, c_{H,1}, c_{F,1}, q_{H,0}, q_{F,0}, c^*_{F,0}, c^*_{F,1}, q^*_{H,0}, q^*_{F,0}, r_0, r_0^*, p_0, p_1, s_0, s_1).$$

There are 16 endogenous variables. Once we set the known exogenous variables, the endogenous variables at period 0 are pinned-down, and the endogenous variables at period 1 will be a function of the stochastic exogenous variables $(y_1, y_1^*)$.

The model implies the following 18 equations, two of which are redundant since the market clearing adds up to the sum of households' budget constraints. For the home households,

$$\begin{aligned}
\tau_1 \exp(-r_0) + y_0 p_0 &= (c_{H,0})^\alpha (c_{F,0})^{1-\alpha} + \exp(-r_0) q_{H,0} + \exp(-r_0^* - s_0) q_{F,0}, \\
(c_{H,0})^\alpha (c_{F,0})^{1-\alpha} &= p_0 c_{H,0} + c_{F,0} \exp(-s_0), \\
y_1 p_1 + q_{H,0} + \exp(-s_1) q_{F,0} &= (c_{H,1})^\alpha (c_{F,1})^{1-\alpha} + \tau_1, \\
(c_{H,1})^\alpha (c_{F,1})^{1-\alpha} &= p_1 c_{H,1} + c_{F,1} \exp(-s_1), \\
1 - \left( (c_{H,0})^\alpha (c_{F,0})^{1-\alpha} \right)^\gamma v'(q_{H,0} \exp(-r_0)) &= \mathbb{E}_0 \left[ \delta \left( \frac{(c_{H,0})^\alpha (c_{F,0})^{1-\alpha}}{(c_{H,1})^\alpha (c_{F,1})^{1-\alpha}} \right)^\gamma \exp(r_0) \right] \\
1 &= \mathbb{E}_0 \left[ \delta \left( \frac{(c_{H,0})^\alpha (c_{F,0})^{1-\alpha}}{(c_{H,1})^\alpha (c_{F,1})^{1-\alpha}} \right)^\gamma \exp(-\Delta s_1 + r_0^*) \right].
\end{aligned}$$

For the foreign households,

$$\begin{aligned}
\tau_1^* \exp(-r_0^*) + y_0^* &= c^*_{F,0} + \exp(-r_0 + s_0) q^*_{H,0} + \exp(-r_0^*) q^*_{F,0}, \\
y_1^* + q^*_{H,0} \exp(s_1) + q^*_{F,0} &= c^*_{F,1} + \tau_1^*, \\
1 - (c^*_{F,0})^\gamma v'(q^*_{H,0} \exp(-r_0 + s_0)) &= \mathbb{E}_0 \left[ \delta \left( \frac{c^*_{F,0}}{c^*_{F,1}} \right)^\gamma \exp(r_0 + \Delta s_1) \right], \\
1 &= \mathbb{E}_0 \left[ \delta \left( \frac{c^*_{F,0}}{c^*_{F,1}} \right)^\gamma \exp(r_0^*) \right].
\end{aligned}$$

Market clearing conditions are

$$y_0 = c_{H,0},$$

$$
\begin{aligned}
y_1 &= c_{H,1}, \\
\tau_1 &= q_{H,0} + q^*_{H,0}, \\
y^*_0 &= c_{F,0} + c^*_{F,0}, \\
y^*_1 &= c_{F,1} + c^*_{F,1}, \\
\tau^*_1 &= q_{F,0} + q^*_{F,0}.
\end{aligned}
$$

The prices and exchange rates can be pinned down by:

$$
\begin{aligned}
p_0 \exp(-s_0) &= \frac{\alpha}{1-\alpha} \frac{c_{F,0}}{c_{H,0}}, \\
p_1 \exp(-s_1) &= \frac{\alpha}{1-\alpha} \frac{c_{F,1}}{c_{H,1}}.
\end{aligned}
$$

## OA.5 Four Euler equations

We use $M_t$ and $M^*_t$ to denote the two households' marginal utility in period $t$. We then recover, in their discrete-time forms, the four Euler equations of the main text:

$$1 - c_0 v'(q_{H,0} \exp(-r_0)) = \mathbb{E}_0 \left[ \frac{M_1}{M_0} \exp(r_0) \right] \tag{OA.1}$$

$$1 = \mathbb{E}_0 \left[ \frac{M_1}{M_0} \exp(-\Delta s_1 + r^*_0) \right] \tag{OA.2}$$

$$1 - c^*_0 v^{*\prime}(q^*_{H,0} \exp(-r_0 + s_0)) = \mathbb{E}_0 \left[ \frac{M^*_1}{M^*_0} \exp(r_0 + \Delta s_1) \right], \tag{OA.3}$$

$$1 = \mathbb{E}_0 \left[ \frac{M^*_1}{M^*_0} \exp(r^*_0) \right] \tag{OA.4}$$

Thus as noted, these Euler equations which we study in the main text arise in the international macro model of this appendix. It should also be apparent that they will arise in most international macro models.

The two-period model adds two elements relative to the model of the main text. First, the $M$s and $\lambda$s (as reflected by the $v'$s) are endogenous objects. In the two-period model, they are driven by shocks to endowments and bond demand ($\theta$). In particular, the macro model indicates the correlation structure that will arise endogenously. In the model of the main text, we solve the model for an arbitrary correlation structure but then take a stand on the correlations when quantitatively evaluating the model. The next sections explain further these choices of correlation parameters. Second, the macro model imposes two further equations that must be satisfied in equilibrium. These equations are that trade in goods (and bonds) needs to be balanced in both periods. With these two equations, the model pins down both $s_0$ and the [stochastic] $s_1$. In our main text, we solve for a family of exchange rate solutions that solve the four Euler equations. The macro trade balance equation further restricts the possible equilibria within this family.

## OA.6 Further Simplification

To solve the model further, we assume that the foreign households' utility from holding the home bonds is

$$v^*(q^*_{H,0}\exp(-r_0+s_0)) = \theta^*_0 \frac{1}{1-\gamma}(q^*_{H,0}\exp(-r_0+s_0))^{1-\gamma}.$$

so that the demand for the home bonds is downward-sloping.

Also, for a positive $\varepsilon$, the home households' marginal utility from holding the home bonds is

$$v'(q_{H,0}) = \begin{cases} \frac{1}{c_0}(1-\frac{1}{\varepsilon}q_{H,0}) & \text{if } 0 \leq q_{H,0} \leq \varepsilon, \\ 0 & \text{if } q_{H,0} > \varepsilon. \end{cases}$$

That is,

$$v(q_{H,0}) = \begin{cases} \frac{1}{c_0}(q_{H,0}-\frac{1}{2\varepsilon}q^2_{H,0}) & \text{if } 0 \leq q_{H,0} \leq \varepsilon, \\ \frac{1}{2c_0}\varepsilon & \text{if } q_{H,0} > \varepsilon. \end{cases}$$

Then, we have a unique solution for $q_{H,0}$ that satisfies both equation (OA.1) and $q_{H,0} \in (0,\varepsilon)$:

$$q_{H,0} = \varepsilon \exp(r_0) E_0 \left[\frac{M_1}{M_0}\exp(r_0)\right].$$

We take the limit of $\varepsilon$ to 0 from above. Then, the model reduces to the case in which the home households hold a zero amount of home bonds in equilibrium, i.e.,

$$q_{H,0} \to 0$$
$$q^*_{H,0} \to \tau_1.$$

We further assume $y_0 = y^*_0 = 1$. Note that we still have that home and foreign endowments at time 1 are uncertain. Then, we obtain the following set of equations that characterize the equilibrium:

$$1 = \mathbb{E}_0\left[\delta\left(\frac{(c_{F,0})^{1-\alpha}}{(y_1)^\alpha(c_{F,1})^{1-\alpha}}\right)^\gamma \exp(r^*_0)\left(\frac{c_{F,0}y_1}{c_{F,1}}\right)^\alpha\right]$$

$$1-\theta_0^*\left(\frac{(1-\alpha)(1-c_{F,0})}{\tau_1\exp(-r_0)(c_{F,0})^\alpha}\right)^\gamma = \mathbb{E}_0\left[\delta\left(\frac{1-c_{F,0}}{y_1^*-c_{F,1}}\right)^\gamma \exp(r_0)\left(\frac{c_{F,1}}{c_{F,0}y_1}\right)^\alpha\right],$$

$$1 = \mathbb{E}_0\left[\delta\left(\frac{1-c_{F,0}}{y_1^*-c_{F,1}}\right)^\gamma \exp(r_0^*)\right]$$

$$c_{F,0} = \frac{1}{1-\alpha}(c_{F,0})^\alpha\exp(-r_0)\tau_1 - \exp(-r_0^*)q_{F,0}$$

$$c_{F,1} = -\frac{1}{1-\alpha}\frac{(c_{F,1})^\alpha}{(y_1)^\alpha}\tau_1 + q_{F,0}.$$

The solution to these equations pin down the five unknown endogenous variables

$$(c_{F,0}, c_{F,1}, q_{F,0}, r_0, r_0^*).$$

## OA.7 Parameterization, Solution, and Correlations

While it is possible to algebraically analyze the equilibrium as described, we will parameterize the model and illustrate the solution. We assume that both home and foreign endowments $y_1$ and $y_1^*$ follow a uniform distribution over $[0.9, 1.1]$ and we discretize this distribution at 0.01 increments. Other primitive parameters are given in Table OA.1(a).

The endogenous variables are given in Table OA.1(b). We note that the foreign households hold the entire outstanding amount of home bonds, i.e., $q_{H,0}^* = \tau_1 = 0.1$, whereas home households hold even more foreign bonds, i.e., $q_{F,0} = 0.3624 > q_{H,0}^*$. This saving by the home country allows the home households to purchase foreign goods for consumption in the 2nd period.

First, we compute the realized log exchange rate movement $\Delta s_1$ and the realized SDF differential $\Delta m_1 - \Delta m_1^*$. We run the regression

$$\Delta s_1 = \alpha + \beta(\Delta m_1 - \Delta m_1^*) + \varepsilon$$

and obtain $\beta < 1$. This partial pass-through is consistent with our characterization in the main text. We also obtain an $R^2 < 100\%$, consistent with the result in the main text regarding the presence of additional variation in exchange rates that arise from the incomplete-market wedge. For comparison, if markets are complete, $\beta$ should be equal to 1, and the $R^2$ should be 100%.

Second, we vary the $\theta_0^*$ parameter that governs the foreign investors' utility from holding home bonds and report the equilibrium dollar exchange rate and convenience yield in period 0 in Figure OA.1. Consistent with our characterization in the main text, a higher $\theta_0^*$ implies a stronger foreign demand for the U.S. bonds and hence a higher foreign convenience yield, which leads to a stronger dollar. We also report the interest rate differential between the U.S. and foreign bonds (in logs) and the expected dollar exchange rate movement in Figure OA.2. Also consistent with our results in the main text, as the foreigners' demand for U.S. bonds increases, the U.S. interest rate falls relative to the foreign interest rate (Figure OA.2, left panel), and the dollar's expected return falls (Figure OA.2, implied from the sum of both panels). Lastly, in Figure OA.3, we report the home and foreign households' marginal utilities in period 0 as functions of $\theta_0^*$. A stronger foreign demand for U.S. bonds results in a lower U.S. marginal utility and a higher foreign marginal utility. This happens because, in equilibrium, the higher demand causes the foreign households to sell foreign goods at date 0 to the U.S. households in order to purchase U.S. bonds (as well as causing foreign households to sell some of their foreign bonds to U.S. households to finance the U.S. bond purchase). This marginal-utility association justifies our assumption of $\rho > 0$ and $\rho^* < 0$ that is made in the main text.

Finally, we also consider a shock to the supply of U.S. safe bonds, as would occur under QE or the opening of dollar swap lines, by varying the $\tau_1$ parameter. As $\tau_1$ increases, the total supply of the U.S. safe bond increases, and, by our simplifying assumption, is entirely absorbed by the foreign households. This supply shock lowers the equilibrium convenience yield $\lambda_t$ and depreciates the dollar (Figure OA.4). Moreover, consistent with our results in the main text, the U.S. interest rate rises relative to the foreign interest rate, and the dollar's expected return also rises (Figure OA.5). The decline in the dollar bond's convenience yield also results in a higher U.S. marginal utility and a lower foreign marginal utility.

In sum, the model of this appendix illustrates that the forces governing exchange rate behavior that are embedded in our main text can arise in a fully-specified international macroeconomic model. The model also validates our assumptions about the correlations between marginal utilities and convenience yields. In particular, the reduced-form convenience yield shock $dX$ captures both demand and supply shocks in the U.S. safe bond market, and produces implications for the dollar's exchange rate and expected return that are consistent with the results in the fully-specified model.

TABLE OA.1–PRIMITIVE AND ENDOGENOUS VARIABLES

| | | |
|---|---|---|
| Panel (a) Primitive Variables | | |
| Subjective discount factor | $\delta$ | 0.95 |
| Degree of Relative Risk Aversion | $\gamma$ | 4 |
| Consumption home bias | $\alpha$ | 0.9 |
| Supply of home bonds | $\tau_1$ | 0.1 |
| Supply of foreign bonds | $\tau_1^*$ | 0.1 |
| Foreign investors' utility from holding home bonds | $\theta_0^*$ | 0.1 |
| Panel (b) Endogenous Variables | | |
| Home holdings of foreign bonds | $q_{F,0}$ | 0.3624 |
| Foreign holdings of home bonds | $q_{H,0}^*$ | 0.1 |
| Time-0 dollar exchange rate | $\exp(s_0)$ | 1.9959 |
| Avg time-1 dollar exchange rate | $\mathbb{E}\exp(s_1)$ | 1.9747 |
| Time-0 home consumption of home goods | $c_{H,0}$ | 1 |
| Avg time-1 home consumption of home goods | $\mathbb{E}c_{H,1}$ | 1 |
| Time-0 home consumption of foreign goods | $c_{F,0}$ | 0.1669 |
| Avg time-1 home consumption of foreign goods | $\mathbb{E}c_{F,1}$ | 0.1649 |
| Time-0 foreign consumption of foreign goods | $c_{F,0}^*$ | 0.8331 |
| Avg time-1 foreign consumption of foreign goods | $\mathbb{E}c_{F,1}^*$ | 0.8351 |
| SDF-FX pass-through | $\beta$ | 0.1789 |

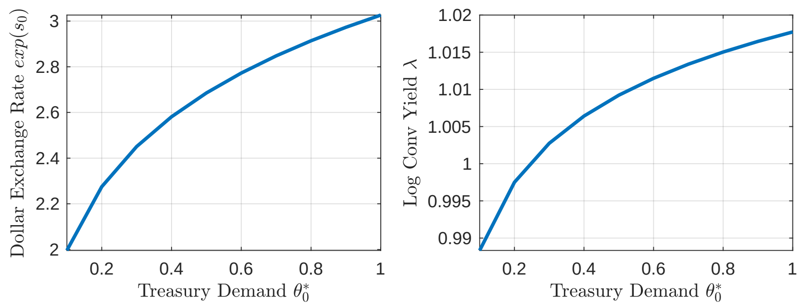


FIGURE OA.1. INITIAL DOLLAR EXCHANGE RATE AS A FUNCTION OF FOREIGN TREASURY DEMAND

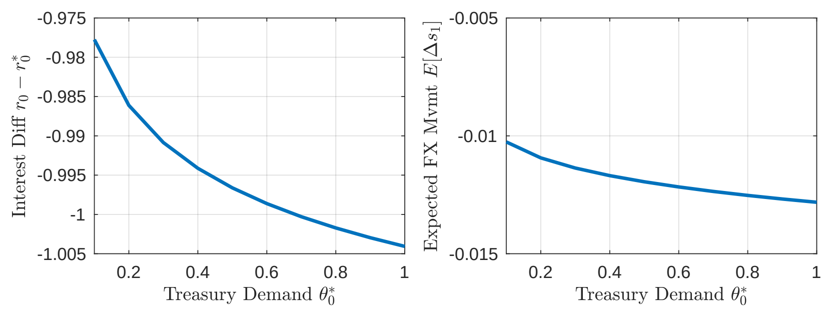


FIGURE OA.2. INTEREST DIFFERENTIAL AND EXPECTED DEPRECIATION AS FUNCTIONS OF FOREIGN TREASURY DEMAND

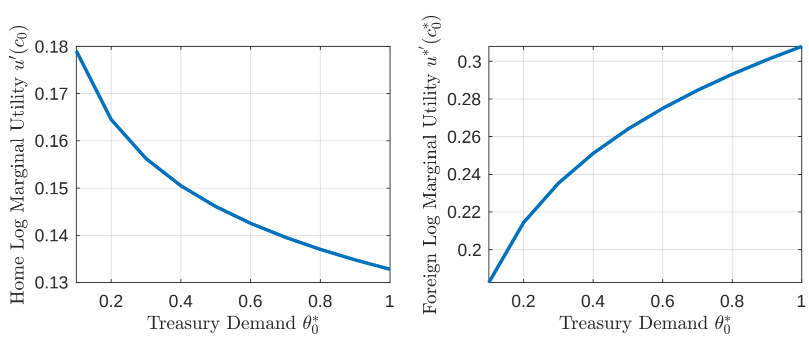


FIGURE OA.3. HOME AND FOREIGN MARGINAL UTILITIES AS FUNCTIONS OF FOREIGN TREASURY DEMAND

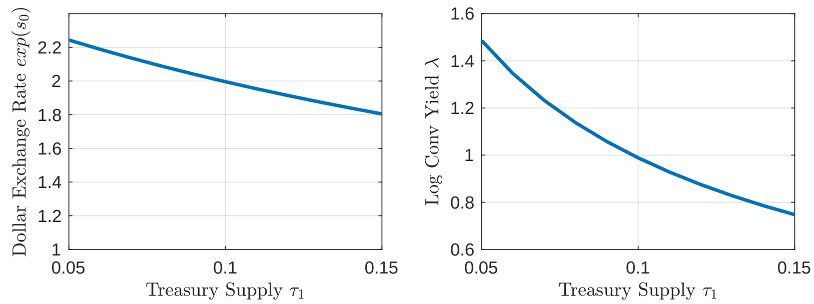


FIGURE OA.4. INITIAL DOLLAR EXCHANGE RATE AS A FUNCTION OF HOME TREASURY SUPPLY

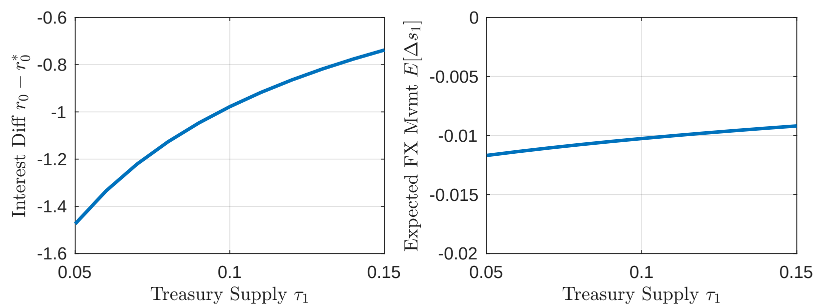


FIGURE OA.5. INTEREST DIFFERENTIAL AND EXPECTED DEPRECIATION AS FUNCTIONS OF HOME TREASURY SUPPLY

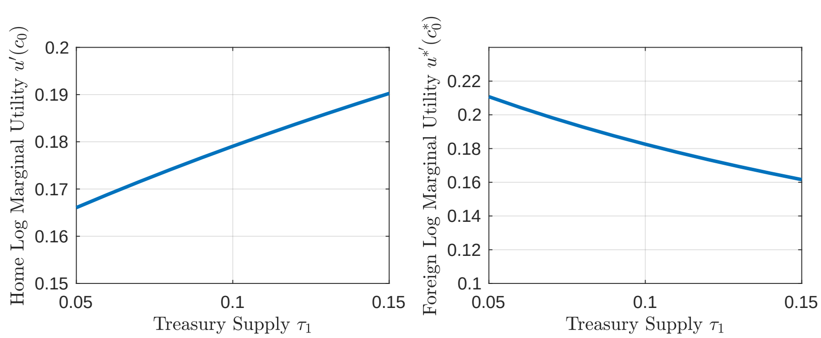


FIGURE OA.6. HOME AND FOREIGN MARGINAL UTILITIES AS FUNCTIONS OF HOME TREASURY SUPPLY

# OB Proof of Additional Results

## OB.1 Proof of Lemma 1

The U.S. households choose consumption and bond holding processes, $c$, $q_H$, $q_F$, to maximize their expected lifetime utility, given initial wealth $W(0) = W$. We have the problem,

$$V(W) = \sup_{c \geq 0; q_H; q_F} \mathbb{E}_0 \left[ \int_{t=0}^{\infty} e^{-\rho t} u(c_t, q_{H,t}; \theta_t) dt \right],$$
$$W(t) = W + \int_0^t (q_{H,s} r_s + q_{F,s} r_s^* S_s^{-1} - c_s) ds + \int_0^t q_{F,s} d(S_s^{-1}) \geq 0$$
$$W(t) = q_{H,t} + q_{F,t} S_t^{-1}$$

In order for $W(t)$ to remain nonnegative, admissible policies $c, q_F, q_H$ have the property that, for $t$ larger than the stopping time $\inf\{s : W_s = 0\}$, $q_{H,t} = q_{F,t} = 0$ and $c_t = W(t) = 0$. That is, nonzero investment and consumption are ruled out once there is no remaining wealth.

Suppose that the exchange rate $S_t = \exp(s_t)$ follows an Ito process, then so does $S_t^{-1}$. Therefore, the drift and volatility terms of $W(t)$ are:

$$\mu(W) = (q_H r + q_F r^* S^{-1} - c) + q_F \mu(S^{-1})$$
$$\sigma(W) = q_F \sigma(S^{-1})$$

To save notation, we have omitted the subscript $t$.

The Hamilton-Jacobi-Bellman equation of the household's problem is

$$\rho V(W) = \max_{c \geq 0; q_H; q_F} \{u(c, q_H; \theta) + \mathcal{D}V(W) + \lambda(W - q_H - q_F S^{-1})\} \tag{OA.5}$$

where

$$\mathcal{D}V(W) = V'(W)\mu(W) + \frac{1}{2}V''(W)(\sigma(W))^2 \tag{OA.6}$$
$$= V'(W)[q_H r + q_F r^* S^{-1} - c + q_F \mu(S^{-1})] + \frac{1}{2}V''(W) q_F^2 (\sigma(S^{-1}))^2$$

We take the first-order conditions inside $\max\{\cdot\}$ of Eq.(OA.5) with respect to $c, q_H, q_F, \lambda$

,

$$w'(c) = V'(W)$$

$$
\begin{aligned}
v'(q_H;\theta) - V'(W)r - \lambda = 0 \\
V'(W)[r^* S^{-1} + \mu(S^{-1})] + V''(W)q_F(\sigma(S^{-1}))^2 - \lambda S^{-1} = 0 \\
W - q_H - q_F S^{-1} = 0
\end{aligned}
$$

Assume we have an interior solution to the maximization problem Eq.(OA.5). Then, the optimal policies $\bar{c}$, $\bar{q_H}$, $\bar{q_F}$, and $\bar{\lambda}$ satisfy:

$$
w'(\bar{c}(W)) = V'(W) \tag{OA.7}
$$

$$
v'(\bar{q}_H(W);\theta) - V'(W)r = \bar{\lambda}(W) \tag{OA.8}
$$

$$
V'(W)[r^* S^{-1} + \mu(S^{-1})] + V''(W)\bar{q}_F(W)(\sigma(S^{-1}))^2 = \bar{\lambda}(W)S^{-1} \tag{OA.9}
$$

$$
W - \bar{q}_H(W) - \bar{q}_F(W)S^{-1} = 0 \tag{OA.10}
$$

We rewrite the HJB equation at its optimum,

$$
\begin{aligned}
\rho V(W) = {} & w'(\bar{c}(W)) + v(\bar{q}_H(W);\theta) \\
& + V'(W)[\bar{q}_H(W)r + \bar{q}_F(W)r^* S^{-1} - \bar{c}(W) + \bar{q}_F(W)\mu(S^{-1})] + \frac{1}{2}V''(W)(\bar{q}_F(W))^2(\sigma(S^{-1}))^2 \\
& + \bar{\lambda}(W)(W - \bar{q}_H(W) - \bar{q}_F(W)S^{-1})\}
\end{aligned} \tag{OA.11}
$$

Assume that all elements in the HJB equation are differentiable. Under this assumption, we take the derivative of Eq.(OA.11) with respect to the state variable $W$ and then substitute in the first-order conditions, Eq.(OA.7-OA.9), and the budget constraint, Eq.(OA.10), to find:

$$
\begin{aligned}
\rho V'(W) = {} & \bar{\lambda}(W) + V''(W)[\bar{q}_H(W)r + \bar{q}_F(W)r^* S^{-1} - \bar{c}(W) + \bar{q}_F(W)\mu(S^{-1})] \\
& + \frac{1}{2}V'''(W)(\bar{q}_F(W))^2(\sigma(S^{-1}))^2
\end{aligned} \tag{OA.12}
$$

Define the optimal wealth path $W^*(t)$ as

$$
\begin{aligned}
& W^*(0) = W_0^* \\
& W^*(t) = W^*(0) + \int_0^t (\bar{q}_H(W^*(s))r_s + \bar{q}_F(W^*(s))r^* S_s^{-1} - \bar{c}(W^*(s)))ds + \int_0^t \bar{q}_F(W^*(s))d(S_s^{-1}) \geq 0
\end{aligned}
$$

Note that, at any point along the optimal wealth path, Eq.(OA.12) holds.

By Ito's lemma, the drift of $V'(W^*(t))$ along the optimal path is

$$
\mathcal{A}V'(W^*(t)) = V''(W^*(t))\mu(W^*(t)) + \frac{1}{2}V'''(W^*(t))(\sigma(W^*(t)))^2 \tag{OA.13}
$$

where

$$\mu(W^*(t)) = \bar{q}_H(W^*(t))r + \bar{q}_F(W^*(t))r^*S^{-1} - \bar{c}(W^*(t)) + \bar{q}_F(W^*(t))\mu(S_t^{-1})$$
$$\sigma(W^*(t)) = \bar{q}_F(W^*(t))\sigma(S_t^{-1})$$

Substitute Eq.(OA.12) into Eq.(OA.13),

$$\mathcal{A}V'(W^*(t)) = \rho V'(W^*(t)) - \bar{\lambda}(W^*(t)) \tag{OA.14}$$

Substitute Eq.(OA.7) and Eq.(OA.8) into Eq.(OA.14),

$$\mathcal{A}[w'(\bar{c}(W^*(t)))] = (\rho - r_t)w'(\bar{c}(W^*(t))) - v'(W^*(t);\theta_t) \tag{OA.15}$$

which is the Euler equation of U.S. households investing in the U.S. bond.

By Ito's lemma, the drift of $V'(W^*(t)S_t^{-1})$ along the optimal path is

$$\mathcal{A}(V'(W^*(t)S_t^{-1}) = [V''(W^*(t))\mu(W^*(t)) + \frac{1}{2}V'''(W^*(t))(\sigma(W^*(t)))^2]S_t^{-1} + V'(W^*(t))\mu(S_t^{-1})$$
$$+ V''(W^*(t))\sigma(W^*(t))\sigma(S^{-1}) \tag{OA.16}$$

Substitute Eq.(OA.12) into Eq.(OA.16),

$$\mathcal{A}(V'(W^*(t))S_t^{-1}) = \rho V'(W^*(t))S_t^{-1} - \bar{\lambda}(W^*(t))S_t^{-1} + V'(W^*(t))\mu(S_t^{-1}) \tag{OA.17}$$
$$+ V''(W^*(t))\bar{q}_F(W^*(t))(\sigma(S_t^{-1}))^2$$

Substitute Eq.(OA.8) and Eq.(OA.7) into Eq.(OA.17),

$$\mathcal{A}[w'(\bar{c}(W^*(t)))S_t^{-1}] = \rho w'(\bar{c}(W^*(t)))S_t^{-1} - V'(W^*(t))r_t^*S_t^{-1} \tag{OA.18}$$

which is the Euler equation of U.S. households investing in the foreign bond.

Denote $M(t) = e^{-\rho t}w'(\bar{c}(W^*(t)))$. Then, Euler equations Eq.(OA.15) and Eq.(OA.18) can be rewritten as

$$0 = \mathbb{E}_t\left[\frac{dM_t}{M_t}\right] + r_t + \frac{v'(q_{H,t};\theta_t)}{w'(c_t)},$$
$$0 = \mathbb{E}_t\left[\frac{d(M_t\exp(-s_t))}{M_t\exp(-s_t)}\right] + r_t^*.$$

## OB.2 Proof of Proposition 7

(1) Recall the definition of the real exchange rate under complete markets, we have

$$d(s_t - \beta s_t^{cm}) = \left(-\frac{1}{2}\tilde{\lambda}_t - \phi(s_t - \beta s_t^{cm}) - (1-\beta)\mu + \frac{1}{2}\sigma\gamma_t\nu(\rho+\rho^*)\right)dt + \gamma_t\nu dX_t \tag{OA.19}$$

We conjecture

$$s_t - \beta s_t^{cm} = f(\lambda_t) + H_t$$
$$H_t = \exp(-\phi t)H_0 + \int_0^t \exp(-\phi(t-u))h(\lambda_u)du$$

which implies

$$\begin{aligned} dH_t &= \left(-\phi\exp(\phi(-t))H_0 + h(\lambda_t) - \phi\int_0^t \exp(\phi(u-t))h(\lambda_u)du\right)dt \\ &= \left(-\phi\exp(\phi(-t))H_0 + h(\lambda_t) - \phi(H_t - \exp(\phi(-t))H_0)\right)dt \\ &= \left(h(\lambda_t) - \phi H_t\right)dt \end{aligned}$$

We note

$$\begin{aligned} d(s_t - \beta s_t^{cm}) &= f' d\lambda_t + \frac{1}{2}f''[d\lambda_t, d\lambda_t]^2 dt + dH_t \\ &= f'(-\theta\lambda_t dt + \nu dX_t) + \frac{1}{2}f''\nu^2 dt + (h(\lambda_t) - \phi H_t)dt \end{aligned}$$

and this has to match equation (OA.19).

Matching the $dX_t$ term,

$$f' = \gamma_t = \frac{b + \sqrt{b^2 + 4(k - \tilde{\lambda}_t)}}{2\nu}$$

where $b = (\rho^* - \rho)\sigma(1 - 2\beta_t)$. Then,

$$\begin{aligned} f(\lambda) = \frac{1}{2\nu}\{ &-\sqrt{b^2+4k}\log\left(2e^{(\lambda-\eta)/2}\left(\cosh\left(\frac{\lambda-\eta}{2}\right)\left(\sqrt{b^2+4k}\sqrt{b^2+4k-2\ell\tanh\left(\frac{\lambda-\eta}{2}\right)-2\ell}+b^2+4k-\ell\right)-\ell\sinh\left(\frac{\lambda-\eta}{2}\right)\right)\right) \\ &+\sqrt{b^2+4k-4\ell}\log\left(2e^{(\lambda-\eta)/2}\left(\cosh\left(\frac{\lambda-\eta}{2}\right)\left(\sqrt{b^2+4k-4\ell}\sqrt{b^2+4k-2\ell\tanh\left(\frac{\lambda-\eta}{2}\right)-2\ell}+b^2+4k-3\ell\right)-\ell\sinh\left(\frac{\lambda-\eta}{2}\right)\right)\right) \\ &+(\lambda-\eta)\left(\sqrt{b^2+4k}+b\right)\} \end{aligned}$$

and

$$f''(\lambda) = \frac{\frac{\ell e^{2(\lambda-\eta)}}{\left(e^{(\lambda-\eta)}+1\right)^2} - \frac{\ell e^{(\lambda-\eta)}}{e^{(\lambda-\eta)}+1}}{\nu\sqrt{b^2 + 4\left(k - \frac{\ell e^{(\lambda-\eta)}}{e^{(\lambda-\eta)}+1}\right)}}$$

Matching the drift term,

$$h(\lambda_t) = -\frac{1}{2}\tilde{\lambda}_t - \phi f - (1-\beta)\mu + \frac{1}{2}\sigma\gamma_t\nu(\rho+\rho^*) + f'\theta\lambda_t - \frac{1}{2}f''\nu^2$$

Since $\gamma_t$ is also a function of $\lambda_t$, we confirm the conjecture that $h(\lambda_t)$ is a function only of $\lambda_t$:

$$s_t = f(\lambda_t) + H_t + \beta s_t^{cm}$$

(2) Since,

$$ds_t = \left(-\frac{1}{2}\tilde{\lambda}_t - \phi s_t - \mu + \frac{1}{2}\sigma\gamma_t\nu(\rho+\rho^*)\right)dt + \gamma_t\nu dX_t + \beta\sigma(dZ_t^* - dZ_t),$$

then,

$$d(e^{\phi t}s_t) = e^{\phi t}\left(-\frac{1}{2}\tilde{\lambda}_t - \mu + \frac{1}{2}\sigma\gamma_t\nu(\rho+\rho^*)\right)dt + e^{\phi t}\gamma_t\nu dX_t + e^{\phi t}\beta\sigma(dZ_t^* - dZ_t)$$

The solution of the above Stochastic Differential Equation is:

$$s_T = e^{-\phi T}s_0 + \int_0^T e^{\phi(t-T)}\left(-\frac{1}{2}\tilde{\lambda}_t - \mu + \frac{1}{2}\sigma\gamma_t\nu(\rho+\rho^*)\right)dt + \int_0^T e^{\phi(t-T)}\gamma_t\nu dX_t + \int_0^T e^{\phi(t-T)}\beta\sigma(dZ_t^* - dZ$$

Recall that

$$\gamma_t = \frac{(\rho^*-\rho)\sigma(1-2\beta_t) \pm \sqrt{(\rho^*-\rho)^2\sigma^2(1-2\beta_t)^2 + 4(k-\tilde{\lambda}_t)}}{2\nu},$$

$$|(1-2\beta_t)| = \sqrt{\frac{\sigma^2(1-\zeta)-2k}{\sigma^2(1-\zeta)}},$$

then $\gamma_t$ is bounded,

$$|\gamma_t| \leq \frac{|\rho^* - \rho|\sqrt{\frac{\sigma^2(1-\zeta)-2k}{(1-\zeta)}} + \sqrt{\frac{\sigma^2(1-\zeta)-2k}{(1-\zeta)}(\rho^* - \rho)^2 + 4k}}{2\nu}.$$

Hence, for $s_T$, the integrands in the stochastic integrals are all $\mathcal{H}^2$, and the stochastic integrals are Martingales with expectation 0. Then,

$$\begin{aligned}
\lim_{T\to\infty} \mathbb{E}_0[s_T] &= \lim_{T\to\infty} e^{-\phi T} s_0 + \lim_{T\to\infty} \mathbb{E}_0[\int_0^T e^{\phi(t-T)} \left(-\frac{1}{2}\tilde{\lambda}_t - \mu + \frac{1}{2}\sigma\gamma_t\nu(\rho+\rho^*)\right) dt] \\
&= \lim_{T\to\infty} \int_0^T e^{\phi(t-T)} \left(-\frac{1}{2}\mathbb{E}_0[\tilde{\lambda}_t] - \mu + \frac{1}{2}\sigma\mathbb{E}_0[\gamma_t]\nu(\rho+\rho^*)\right) dt \\
&= \frac{1}{\phi}\left(-\frac{1}{2}\lim_{T\to\infty} \mathbb{E}_0[\tilde{\lambda}_T] - \mu + \frac{1}{2}\sigma \lim_{T\to\infty} \mathbb{E}_0[\gamma_T]\nu(\rho+\rho^*)\right).
\end{aligned}$$