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QUALITY LADDERS AND PRODUCT CYCLES

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ABSTRACT

We develop a two-country model of endogenous innovation and imitation in order to study the interactions between these two processes. Firms in the North race to bring out the next generation of a set of technology-intensive products. Each product potentially can be improved a countably infinite number of times, but quality improvements require the investment of resources and entail uncertain prospects of success. In the South, entrepreneurs invest resources in order to learn the production processes that have been developed in the North. All R&D investment decisions are made by forward looking, profit maximizing entrepreneurs. The steady-state equilibrium is characterized by constant aggregate rates of innovation and imitation. We study how these rates respond to changes in the sizes of the two regions and to policies in each region to promote learning.

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I. Introduction

The life history of a technology-intensive commodity can be quite complex. Consider the recent life of the personal computer (PC). Ever since IBM introduced the 8088-based IBM PC in the early 1980's, we have witnessed a more or less continuous race between IBM, Compaq, and others to provide products of ever higher quality. This quality competition has led to the almost complete replacement of the 8088-based computers by machines housing an 80286 processor, and more recently these latter PC's are losing their place in the market to computers based on the 80386 processor. At the same time, other firms, many of them located in low wage countries such as Taiwan and Korea, have strived to copy the state-of-the-art machines and to come to market with competitively priced "clones". As a result, market shares have fluctuated for innovators and imitators, and for last generation and next generation products.

The complex dynamics of this example and others like it (e.g., semiconductors, consumer electronics, etc.) stem from the interaction between two concurrent and interrelated stochastic processes. The first is the common process of quality upgrading. Scherer (1980) cites evidence that improvements in product quality account for more than one half of all industrial innovation. The second process is that of imitation. Latecomers seek to exploit the public good nature of technology by mimicking the designs and prototypes developed by others. Both of these processes are characterized by fundamental uncertainties. Even when firms know ahead of time the attributes of the good that they hope to invent or mimic, they often cannot be sure *ex ante* exactly how long it will take to develop a marketable product. Thus, the dynamic evolution of the market shares and trade pattern for any given product is bound to be complex.

Elsewhere, we have studied these two components of the innovation process in isolation. In Grossman and Helpman (1989a), we developed a model of endogenous product cycles. Building on work by Krugman (1979) on the dynamics of trade with exogenous innovation and exogenous diffusion, and by Romer (1988) and ourselves (1989c) on endogenous technical change, we described a process whereby new, differentiated products are first developed in the industrialized North, and then, after a (random) period of production there, are subject to imitation by entrepreneurs in the less developed South. While useful for studying the implications of imitation in the South for incentives to innovate in the North, our model does have the somewhat unrealistic property that, once the locus of production for some good shifts to the South, that good is produced there forever after.

In Grossman and Helpman (1989d) we considered the process of quality upgrading. Drawing building blocks from the works of Aghion and Howitt (1989) and Segerstrom, Anant and Dinopoulos (1988), we constructed a model of quality ladders wherein firms race to improve each of a continuum of industrial products. We took the increments to quality to be exogenously given, and studied the determinants of the average rate of innovation. In that paper, we did not allow for the possibility of profitable imitation.

Our goal here is to analyze the nature of the interactions between these two processes. In doing so, we hope to gain insights into an important aspect of North-South trade. We construct a model in which Northern firms race to improve a given set of technology-intensive products while Southern entrepreneurs strive to learn the production technologies developed in the North. The learning decisions by each set of agents are affected by the choices of the others. All rates of innovation and imitation are jointly determined in the dynamic general equilibrium. We study how the sizes of the two regions and the levels of support that the two governments provide for research and development affect the long-run rates of innovation and imitation and the length of the average product cycle.

Before proceeding, we relate our work to some other recent papers on North-South trade and the product cycle. Krugman (1979) was the first to attempt to formalize some of the ideas contained in Vernon's (1966) seminal article. However, his model and the extension by Dollar (1986) did not incorporate endogenous determination of the rates of innovation and imitation by profit-maximizing agents. Jensen and Thursby (1988a, 1988b) introduced a decision-theoretic framework into the Krugman model, but they allowed innovation by only one firm and imitation only by a utility-maximizing central planner in the South. The papers most similar in spirit to ours are those by Segerstrom, Anant and Dinopoulos (1988) and Dinopoulos, Oehmke and Segerstrom (1989). These authors also study quality upgrading for a range of industrial products and our utility function is borrowed from them. Two salient features distinguish our work from theirs. First, they assume that patent races take place in one industry at a time, and so products are improved in sequence. By contrast, we allow for simultaneous research activities in all sectors of the economy. Second, we incorporate imitation as an economic activity that requires resources and responds to profitability considerations, whereas they assume that diffusion is automatic and costless once a patent of fixed duration has expired. Tangentially related to our work are the papers by Flam and Helpman (1987) and Stokey (1989). These authors also study quality upgrading in the context of North-South trade. But they do not introduce explicit dynamics, nor do they incorporate R&D as a distinct economic activity. Rather, they assume that all goods can be produced from the beginning of time, but that only a subset of goods is produced at any moment depending upon demand and factor endowment conditions. Product improvements are studied via exogenous variations in income levels, income distributions, factor productivities, and factor supplies.

The remainder of this paper is organized as follows. In the next section we specify preferences and technologies and describe optimal behavior by individual agents. Conditions for a steady-state equilibrium are developed in Section III. We find that two different equilibrium regimes are possible, depending upon the gap in R&D efficiency between the agents who have been successful in developing the latest generation of a product and all others. We study the determinants of innovation and imitation in these alternative regimes in Sections IV and V. Section VI contains a summary of findings.

II. The Model

We study innovation, imitation, and trade in a world economy comprising two regions. The regions -- which we refer to as "North" and "South" -- are distinguished by their abilities to conduct state-of-the-art research and development (R&D). Resources in the North are assumed to be considerably more productive in undertaking innovation than are resources in the South. We focus on steady-state equilibria in which all innovative activity takes place in the North.

Innovation in this economy takes the form of improvements in the qualities of a fixed set of goods. We take the set of goods to be continuous and index its elements by $\omega \in [0,1]$. Each good potentially can be improved an infinite number of times. These improvements require resources and entail uncertain prospects, as we shall describe below. The increments to quality are common to all products and exogenously given by a parameter $\lambda>1$. We denote by $q_0(\omega)=1$ the initial (lowest) quality of each good. After j improvements of product ω, its highest available quality is given by $q_j(\omega)=\lambda^j$.

Consumers worldwide share identical preferences. They seek to maximize an additively separable intertemporal utility function of the form

$$(1) \qquad U = \int_0^\infty \exp(-\rho t) \log u(t)dt,$$

where ρ is the common subjective discount rate and log u(t) represents instantaneous utility at time t. We specify

$$(2) \qquad \log u(t) = \int_0^1 \log \left[\Sigma_j q_j(\omega) d_{jt}(\omega)\right] d\omega,$$

where $d_{jt}(\omega)$ denotes consumption of quality j of good ω at time t.

The representative consumer maximizes utility by choosing an optimal time path for nominal spending and by allocating spending optimally at each point in time. Given prices $p_{jt}(\omega)$ and the level of spending $E(t)= \int_0^1 [\Sigma_j p_{jt}(\omega) d_{jt}(\omega)] d\omega$, the composition of spending that maximizes (2) is attained when the consumer allocates an equal expenditure share to every product type ω and when he chooses for every ω the single variety that offers the lowest quality adjusted price $p_{jt}(\omega)/q_j(\omega)$. We shall see that in equilibrium it is always the highest available quality that provides the lowest quality adjusted price Let $q_t(\omega)$ denote the quality of the state-of-the-art variety of good ω at time t. Substituting the optimal allocation into (2), and the result into (1), we obtain the indirect utility function

$$(3) \qquad U = \int_0^{\infty} \exp(-\rho t)\{\log E(t) - \int_0^1 \log[p_t(\omega)/q_t(\omega)]d\omega\}dt,$$

where $p_t(\omega)$ represents the price of the state-of-the-art product.

We assume that the consumer can borrow or lend freely on a capital market with instantaneous (and riskless) rate of return r. Consumers take this interest rate as given, though its value must be determined in the general equilibrium. The optimal time profile for nominal spending is that which maximizes (3) subject to an intertemporal budget constraint. The constraint limits the present value of the infinite stream of expenditures to the present value of income plus the value of initial asset holdings. As is well known (see, for example, Grossman and Helpman (1989c)), the solution to this problem yields the following differential equation for spending:

$$(4) \qquad \dot{E}/E = r - \rho .$$

The consumer-investor also must solve a portfolio allocation problem. He may choose among shares in a variety of profit-earning firms and among interest-bearing bonds. Claims on particular firms are risky assets, as we shall see. However, the risk attached to any one equity is idiosyncratic, so the investor can earn a sure rate of return by holding a diversified portfolio of shares. It follows that, in asset-market equilibrium, the consumer will be indifferent between holding bonds and stocks, and between holding different diversified portfolios of stocks. Although the individual's portfolio allocation problem has no unique solution, the associated no-arbitrage conditions allow us to value the various profit-making entities.

Consider then a firm that earns a profit stream $\pi(\tau)$ for $\tau \geq t$. Below we show that profits accrue only to firms that are the lowest cost producers of state-of-the-art products. The stream of profits of such a producer continues until the time that another firm succeeds in copying the product or improving upon it. Then the value of its shares falls to zero. Recognizing this risk of total capital loss, we can calculate the expected return to any equity as follows. If v(t) is the value of a firm at time t, $(\pi/v)dt$ is the dividend rate in a time interval of length dt, and $(\dot{v}/v)dt$ is the rate of capital gain. But with probability fdt the shareholders will suffer a capital loss of v at the end of interval dt. Summing these components of the expected return and equating the result to the sure rate of return on bonds, we have

$$(5) \qquad \pi/v + \dot{v}/v - f = r.$$

This equation implicitly determines the value of any firm as a function of its profit rate, the interest rate, the rate of capital gain and the relevant value for f. In what follows, we shall link f to the activities that competitors undertake in equilibrium in order to supplant the profit earner.

We shall need to distinguish three types of profit-making firms that may exist in equilibrium. These firms earn different rates of profit and face different risks of loss of their market power. Consequently, their stock-market values differ. They are (i) Northern firms that have exclusive ability to produce some state-of-the-art product and that compete with another Northern firm that can produce the second-to-top quality; (ii) Northern firms that have exclusive ability to produce a state-of-the-art product and that compete with a Southern firm that can produce the second-to-top quality; and (iii) Southern firms that are able, via imitation, to produce the state-of-the-art quality. We shall refer to the measure of firms in each of these categories as n_{NN}, n_{NS}, and n_S, respectively. As we shall see, these firm types are exhaustive and each product is manufactured in equilibrium by exactly one firm. Therefore, we have $n_{NN}+n_{NS}+n_S=1$.

Let us derive now the profit rates for each of these types of firms. We will show at the same time that no other type of firm can earn positive profits. Consider first a firm in the South that is the only one there to have successfully imitated the top-of-the-line quality for some product ω. We assume that this firm can manufacture (a flow of) one unit of output using one unit of Southern labor, regardless of ω or $q_t(\omega)$. We assume as well that Northern firms with knowledge to produce this or other qualities of product ω require one unit of Northern labor in order to manufacture one unit of output. Finally, we assume that all firms with the know-how to produce some quality of good ω compete as price-setting (Bertrand) oligopolists. Marginal costs of production then are w_S in the South and w_N in the North, where w_i is the wage rate in region i, i=N,S. In all of the equilibria that we study below, the wage in the South is less than that in the North. The Southern manufacturer maximizes profit by charging a price equal to the Northern wage rate (or ϵ below it).[1] It thereby captures the entire market, makes sales of E/w_N, where E now represents aggregate spending (recall that consumers spend equal shares on all products), and earns instantaneous profits

[1] Each product ω has a unitary price elasticity of demand. Therefore, the Southern firm would never wish to set a price discretely below that of its Northern rival.

(6) $\pi_S = (w_N - w_S)E/w_N.$

It is clear that when the Southern firm charges this price, all suppliers of qualities below $q_t(\omega)$, as well as any Northern suppliers of $q_t(\omega)$, make zero sales and earn zero profits.

If two Southern firms are able to produce the same top-of-the-line product $q_t(\omega)$ for some ω, then both price at marginal cost and neither earns positive profits in the resulting Bertrand competition. Such a situation never arises in equilibrium, because once some Southern firm has succeeded in copying $q_t(\omega)$ no other entrepreneur is willing to invest costly resources in order to learn the technology for that same product.[2]

Consider next the case of a Northern firm that has successfully improved a product most recently manufactured in the South. Suppose that the quality of the Southern product is exactly one increment below that of the Northern leader's product, as is always the case in equilibrium (see below). The Northern leader maximizes profits in this instance by setting a quality adjusted price equal to (or ϵ below) the Southern firm's marginal cost of production. This price of λw_S yields the Northern firm a 100 percent market share for product group ω and sales of $E/\lambda w_S$. Its instantaneous profits are given by[3]

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[2] The probability that two Southern firms will succeed in their efforts to imitate the same Northern product at precisely the same moment is of order $(dt)^2$, and so can be ignored.

[3] Naturally, the expression for profits given in (7) applies only when $w_S/w_N < \lambda$; i.e., when the wage differential between North and South is not too large. Otherwise, $\pi_{NS}=0$ and the Northern firm will have no incentive to improve the quality of a product manufactured in the South.

$$(7) \qquad \pi_{NS} = (\lambda w_S - w_N)E/\lambda w_S \ .$$

Consider finally a Northern firm with the exclusive ability to produce a top-of-the-line product whose nearest competitor on the quality ladder is another Northern firm one step behind. This firm maximizes profit by charging a price of (or ϵ below) λw_N. This price once again ensures the leader of a 100 percent market share and thus a flow of sales equal to $E/\lambda w_N$. The flow of profits in this case is given by

$$(8) \qquad \pi_{NN} = (\lambda w_N - w_N)E/\lambda w_N \ .$$

In the event that two Northern firms are able to produce the same top-of-the-line product, both earn zero profits. This never occurs in equilibrium, since imitation requires resources and so is not a viable activity in the North.

Our last tasks in describing the economy are to specify the technologies for innovation and imitation and to ascertain what is optimal behavior by entrepreneurs in regard to these activities. We suppose that innovation and imitation entail uncertainty for the individual entrepreneurs. When an investor devotes resources to one of these activities for an interval of time, she purchases a probability of success during that interval that is proportional to the intensity of effort. The probability of success during any time interval does not depend upon the resources that have been spent in previous (unsuccessful) periods. Thus, individual research success is a continuous Poisson process, as in Lee and Wilde (1980). Free entry characterizes both the innovation and imitation activities. Accordingly, we model a continuum of product-specific "patent races".

As we noted at the outset, entrepreneurs in the South are quite inefficient at innovation. We do not specify a technology for this activity, but simply suppose that no innovation takes place there. We distinguish two technologies for innovation in the North. An entrepreneur who has achieved the most recent research success for some product ω is likely to have acquired substantial product specific information by dint of her successful effort. This information may be valuable in developing the next generation product and not all such information will be readily apparent to outsiders who observe only the final product and not the process that led to its discovery. Thus, we specify one technology for product improvement by "leaders" -- firms that successfully developed quality $q_t(\omega)$ -- and another for all "followers", including firms that may have developed previous generations of product ω, firms that may have tried to develop $q_t(\omega)$ but failed, and all new potential entrants into industry ω.

A Northern firm that undertakes innovative activities at intensity ι for an interval of time dt succeeds in developing the next generation of the targeted product with probability ιdt. This effort requires $a_{DL}\iota$ units of labor per unit of time for a leader, and $a_{DF}\iota$ for a follower, with $a_{DF}>a_{DL}$. Imitation similarly requires resources and entails uncertain success. In the South, imitation at intensity μ for a time interval of length dt requires $a_M\mu dt$ units of Southern labor and yields results with probability μdt. We need not specify a technology for imitation in the North, because we have seen that this activity is never profitable there.

Having specified the various research technologies, we turn now to individual behavior. Consider first those research efforts in the North targeted at improving a good momentarily produced in the South. Suppose that a profit opportunity exists for a successful Northern innovator; i.e., π_{NS} in (7) is positive. Either the leader or one or more followers might contemplate undertaking this activity. All face the same cost of funds in the capital market and all stand to gain the same expected profit stream if successful. However, the leader alone enjoys a cost advantage in innovation by dint of her superior product-specific knowledge. With a linear R&D technology, only the Northern leaders (if anyone) ever engage in efforts to improve products that are being manufactured in the South.

The profit opportunity for any Northern leader that successfully recaptures its market share from an imitator in the South is the same, regardless of the product ω or its current quality level. We suppose, therefore, that all such Northern firms undertake innovative activities with equal intensity. Let ι_S denote this intensity level. Successful ones among those who undertake such R&D will attain a stock market value of v_{NS}. So each such firm can achieve an expected gain of $v_{NS}\iota_S dt$ at cost $w_N a_{DL} \iota_S dt$ by targeting for further improvement the product that it previously developed and undertaking R&D at intensity ι_S for an interval dt. Value maximization by leaders requires that the expected gain not exceed the cost, with equality holding if any innovation activity takes place at these firms. Thus we have

(9) $$v_{NS} \leq a_{DL} w_N , \qquad \text{with equality for } \iota_S > 0.$$

Now consider research efforts aimed at improving a good momentarily manufactured by a firm in the North. Again, these efforts might conceivably be undertaken by any of a large number of followers, or by the industry leader itself. The followers can achieve an expected gain of $v_{NN}\iota dt$ by bearing research cost $w_N a_{DF}\iota$ for an interval dt. Value maximization again requires that the expected gain not exceed the cost, with equality if any R&D is conducted by followers. We suppose that all Northern products are targeted by followers for improvement to the same (aggregate) extent. Let ι_N denote the total of such research activity by followers for each ω produced by a Northern firm. Then we have

(10) $v_{NN} \leq a_{DF}w_N$, with equality for $\iota_N > 0$.

In Appendix A, we show that $a_{DF}/a_{DL} < 2 - 1/\lambda$ is sufficient to rule out any efforts by extant Northern producers to improve their own products. Intuitively, these firms stand to gain less from product improvement than followers. The benefit to a leader from a research success is the difference between the profit stream that accrues to a producer two steps ahead of its nearest rival, and that which is earned by a producer one step ahead of its nearest rival. A follower stands to gain a profit stream of π_{NN} in place of zero. The latter increment is always larger than the former, so the followers have greater incentive to conduct R&D unless their cost disadvantage is severe. We shall henceforth assume that $a_{DF}/a_{DL} < 2 - 1/\lambda$ and thereby exclude the possibility that leaders engage in R&D unless and until their products have been imitated by a firm in the South.

Southern entrepreneurs that target for imitation a product manufactured in the North stand to reap an expected gain in firm value of $v_S\mu dt$ for a cost of $\mu a_M w_S dt$. We assume that all Northern products are targeted by imitators in the South to the same (aggregate) extent μ. By familiar reasoning, value maximization by Southern entrepreneurs implies

$$v_S \leq a_M w_S \text{ , } \qquad \text{with equality for } \mu>0. \tag{11}$$

We return finally to the no-arbitrage conditions. A Northern producer -- be it one that faces competition from another Northern firm as its closest rival or one that competes with a Southern firm as its closest rival -- may forfeit its earnings potential in one of two ways. Its product might be improved upon by another Northern entrepreneur, or it might be successfully copied by a firm in the South. The probabilities that these events occur in a time interval of length dt are $\iota_N dt$ and μdt, respectively. Using (5), then, with $f=\iota_N+\mu$, we obtain

$$\pi_{NS}/v_{NS} + \dot{v}_{NS}/v_{NS} = r + \mu + \iota_N \tag{12}$$

and

$$\pi_{NN}/v_{NN} + \dot{v}_{NN}/v_{NN} = r + \mu + \iota_N . \tag{13}$$

Each Southern firm, on the other hand, faces a probability $\iota_S dt$ of losing its earning potential during a time interval of length dt. This event occurs if a Northern leader succeeds in improving once again the product that the Southern firm had earlier copied. Applying (5) once more, this time with $f=\iota_S$, we have

$$\pi_S/v_S + \dot{v}_S/v_S = r + \iota_S . \tag{14}$$

Equations (12), (13), and (14) link the value of existing firms to the intensities of innovation and imitation that take place in equilibrium. With these equations, we have completed the description of behavior by individual agents in our economy. We turn in the next section to the characterization of steady-state equilibria.

III. Steady-State Equilibrium

We study steady-state equilibria in which the rates of imitation and innovation, the measure of firms of each type, and all relative prices are constant. In these equilibria, growth takes the form of an ongoing process of product improvements. Each product follows a stochastic life cycle. An individual product might not be improved or copied for a period, or it might be improved several times in succession by various Northern firms, or else it might be copied in the South and produced there for a while before being upgraded again in the North. Despite the many possible life histories for individual products, the rate of increase of an aggregate utility indicator is constant in the steady state.

Our first equilibrium conditions ensure that labor markets clear in each region. Labor supplies L_j, j=N,S, are fixed and constant. Labor demand arises from innovators and manufacturing enterprises in the North, and from imitators and manufacturing in the South. The South targets each of n_N Northern products for imitation at aggregate intensity μ and so employs $a_M \mu n_N$ units of labor in imitation. Manufacturing employment there is $n_S E/w_N$. Thus, labor market clearing in the South requires

(15) $$a_M \mu n_N + n_S E/w_N = L_S .$$

In the North, followers employ $a_{DF}\iota_N n_N$ units of labor in efforts to improve products, while leaders employ $a_{DL}\iota_S n_S$ units of labor in this activity. Manufacturing requires $n_{NN}E/\lambda w_N + n_{NS}E/\lambda w_S$ units of labor. Consequently, labor market equilibrium in the North entails

(16) $$a_{DF}\iota_N n_N + a_{DL}\iota_S n_S + n_{NN}E/\lambda w_N + n_{NS}E/\lambda w_S = L_N.$$

These labor market clearing conditions apply of course not only in steady states, but out of steady state as well.

In a steady state, all nominal values grow at a common rate. In particular $\dot{v}_j/v_j = \dot{E}/E$ for firms with positive value, $j \in \{NN, NS, S\}$. Imposing these conditions, using (4), and combining the respective asset-pricing equations with the corresponding value-maximization conditions (for example, (9) and (12)) we derive the following steady-state relationships:

(17) $$\pi_S/a_M w_S \le \rho + \iota_S , \qquad \text{with equality for } \mu > 0;$$

(18) $$\pi_{NS}/a_{DL} w_N \le \rho + \mu + \iota_N , \qquad \text{with equality for } \iota_S > 0;$$

(19) $$\pi_{NN}/a_{DF} w_N \le \rho + \mu + \iota_N , \qquad \text{with equality for } \iota_N > 0.$$

The remaining steady-state conditions ensure that the measures of products manufactured in the North and in the South, as well as the composition of Northern products, remain constant through time. At each moment, the South successfully copies $\mu(n_{NN}+n_{NS})$ products and takes over production of these goods. At the same time, $\iota_S n_S$ products revert to Northern leaders who succeed in their upgrading efforts. The measure of products manufactured in the South remains constant if and only if

$$\mu n_N = \iota_S n_S \, , \tag{20}$$

where $n_N \equiv n_{NN}+n_{NS}$. Within the group of Northern produced goods, the sub-group with a Northern firm as closest competitor grows in some cases in which a follower succeeds in her research efforts; i.e., at rate $\iota_N n_{NS}$. This sub-group shrinks when one of its members is copied by a Southern firm, which happens at rate μn_{NN}. The composition of products (and therefore the allocation of labor) in the North remains constant if and only if the inflows match the outflows, or

$$\mu n_{NN} = \iota_N n_{NS} \, . \tag{21}$$

This completes the description of the steady-state equilibrium.

Our model admits several types of steady-state equilibria. These equilibria differ in terms of which innovation and imitation activities are actively undertaken in the long run. Parameter values determine which one applies. Two types of equilibria may arise when innovation or imitation are quite costly, one in which no innovation or imitation takes place in the steady state, and another in which innovation does occur but imitation does not. Our main interest lies in the interaction between imitation and innovation, when product cycles co-exist with quality ladders. For this reason, we concentrate on steady states with both imitation and innovation.

Even so, two different types of equilibria may arise. In one type, leaders enjoy a large cost advantage over followers in product improvement, in which case only the leaders engage in R&D in the steady state. Then equilibrium involves alternating phases of Northern and Southern production of each good, with active R&D by Northerners after their product has been copied, and active imitation by Southerners after the North has reclaimed a product by improving it. In a second type of equilibrium, both leaders and followers undertake R&D. This case arises when followers are relatively efficient at innovation, though still less so than leaders. In this type of equilibrium, the path followed by any particular good can be complex, since at any time it may pass from one Northern firm to another, or from North to South.

Consider the latter type of equilibrium first, wherein equations (17) through (19) all hold with equality. We can derive a useful, five equation, reduced-form system as follows. Define $\iota \equiv \iota_N n_N + \iota_S n_S$, the aggregate (or average) rate of product improvement, and $\eta \equiv \mu n_N$, the aggregate rate at which goods flow from North to South. These variables summarize the extent of innovation and imitation in the world economy, and so will be the focus of the analysis that follows. Define also $\delta \equiv 1/\lambda$, $w \equiv w_N/w_S$, and $e \equiv E/w_N$. Then substituting (6)-(8) and (20)-(21) into the labor-market clearing conditions (15) and (16) and the no-arbitrage conditions (17)-(19), we derive

$$a_M \eta + (1 - n_N)e = L_S , \tag{22}$$

$$a_{DF}(\iota - \eta) + a_{DL}\eta + \delta e n_N(1 - \eta/\iota) + \delta e n_N w \eta/\iota = L_N , \tag{23}$$

$$(24) \qquad (w - 1)e(1 - n_N) = a_M[\rho(1 - n_N) + \eta] \ ,$$

$$(25) \qquad (1 - \delta w)en_N = a_{DL}(\rho n_N + \iota) \ ,$$

$$(26) \qquad (1 - \delta)en_N = a_{DF}(\rho n_N + \iota) \ .$$

Using (25) and (26), we can solve immediately for the relative wage rate that prevails in a regime with "efficient followers". We find

$$(27) \qquad (1 - \delta w)/(1 - \delta) = a_{DL}/a_{DF} \ .$$

In this case, the steady-state relative wage does not vary with the sizes of the two regions. The same can be said of the terms of trade, which depend only on w and λ. The relative wage of the North (and the South's terms of trade) are higher the more efficient are leaders at R&D relative to followers.

In a regime with "inefficient followers", $\iota_N = 0$. Then (20) and (21) imply $n_{NN} = 0$ and $\eta = \iota$. In a steady state with no active followers, the measure of products improved per unit time just matches the measure of products that flow from North to South. Now (19) holds as an inequality, and (15)-(18) can be written (after substituting (6)-(8)) as:

$$(28) \qquad a_M\iota + (1 - n_N)e = L_S \ ,$$

$$(29) \qquad a_{DL}\iota + \delta w e n_N = L_N \ ,$$

$$(30) \qquad (w - 1)e(1 - n_N) = a_M[\rho(1 - n_N) + \iota] \ ,$$

$$(31) \qquad (1 - \delta w)en_N = a_{DL}(\rho n_N + \iota) \ .$$

As is evident, the relative wage does depend upon the sizes of the two regions in this case. We discuss in the following two sections the determinants of the steady-state rates of imitation and innovation, and of the average length of a product cycle, in each of the two regimes.

IV. Efficient Followers

We investigate the determinants of the aggregate rate of innovation (ι), the aggregate rate of imitation (η), and the average length of a product cycle ($1/\mu$). This last variable tells us how long, on average, a product will be produced in the North before being successfully copied by a firm in the South. We perform comparative statics with respect to changes in the sizes of the two regions, and with respect to two policy parameters reflecting government support for research activities. To perform the calculations, we substitute (27) into (22)-(25), then apply familiar techniques. These calculations are of little interest and so are relegated to Appendix B.

Several of our conclusions may hinge on an intermediate result that in principle can go either of two ways. Consider labor demand in the North (the left-hand side of (23)). Suppose that, for some reason, the intensity of R&D per product undertaken by followers (ι_N) were to increase, with ι_S, μ, n_N (hence η), e and w constant. The direct effect of this would be to increase demand for Northern labor in proportion to a_{DF}. But an indirect effect operates in the opposite direction. With n_N constant and ι_S having increased, the steady-state composition of Northern products shifts in favor of goods for which the nearest competitor is another Northern firm; i.e., n_{NN} rises, while n_{NS} falls. The former goods bear a higher price and thus involve fewer sales than the latter, so the change in composition entails a fall in demand for Northern labor by manufacturing establishments. To keep our discussion from becoming taxonomic, we shall assume in reporting our results that the direct effect dominates.[4] In all cases in which we rely upon it, this assumption is sufficient but not necessary for the result that we report.

Consider first the long-run effects of an expansion in the size of the North. As in other contexts with endogenous growth (for example, Römer (1988), Grossman and Helpman (1989a,b) and Aghion and Howitt (1989))[5], a larger resource base means that more resources may be deployed in R&D, and so the rate of innovation is higher. Expansion in the North induces an increase in the aggregate extent of imitation by the South (η increases), but the average span of the product cycle for any given product may rise or fall. The latter ambiguity stems from the fact that the measure of products manufactured in the North rises with L_N, so the expanded Southern imitation effort must be spread over a larger set of goods.

An expansion of the labor supply in the South effects greater aggregate learning by the South (η rises), and a greater intensity of imitation targeted at each Northern product (μ rises). The latter implies a fall in the average length of time that a Northern firm can expect to earn profits (<u>ceteris</u>

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[4] More precisely, we assume $a_{DF}\iota > \delta e n_N \eta(w-1)/\iota$. It should be noted that none of our results concerning the determination of the aggregate rate of innovation, ι, rely on this assumption.

[5] But see Grossman and Helpman (1989d) for a discussion of the lack of a general positive relationship between resources and growth in models with endogenous innovation.

paribus). This effect serves to dampen the incentive to innovate. However, the profit rate in the North may rise, and so aggregate innovation in the North may rise or fall. This result stands in contrast with what we found in our paper "Endogenous Product Cycles" (and also with the results for the inefficient-follower case reported in Section V below). There, a larger South always leads to faster world growth. In those other circumstances, we find that when Northern firms earn higher profits for a shorter (expected) period due to an increase in μ, the net effect is a greater incentive to conduct R&D. But the necessary dominance of the profit effect over the risk effect does not extend to the current environment.

Next we consider subsidies to innovation and imitation. These exercises are intended to capture the effects of a broad range of measures that governments might take in support of learning and research activities within their borders. Government support for innovation in the North can be represented by a parameter s_D that reflects the fraction of R&D costs borne by the government. Then we must multiply the right-hand sides of (25) and (26) by $1-s_D$. Similarly, we can multiply the right-hand side of (24) by $1-s_M$ to represent a policy whereby the government of the South bears a fraction s_M of research costs there.

We consider first a small increase in s_D above zero. This augments incentives for R&D and so increases ι. Again, this finding is reminiscent of others in the literature. At the same time, the subsidy reduces the measure of products copied by the South in a given interval of time. So aggregate learning in the South is adversely affected by policies that promote research in the North. The average length of a product cycle may rise or fall, as Southern entrepreneurs devote fewer total resources to copying what, in the steady state, ends up being a smaller set of Northern goods.

A small subsidy to imitation in the South expands the aggregate amount of resources devoted to learning there (η rises). But this policy unambiguously slows the rate of world growth (ι falls). Again we find an inverse relationship between policies that support learning in one region and the equilibrium rate of learning in the other. And again this result stands in contrast with the findings from our earlier (1989a) paper.

We find that a subsidy to imitation, while it augments aggregate imitation, may reduce the intensity of innovation targeted at any given Northern product and so increase the average length of the product cycle. The ambiguity stems from the fact that, holding constant the intensity of imitation μ and real spending e, an increase in n_S may either expand or reduce the demand for Southern labor. Labor demand tends to rise with n_S, because manufacturing employment in the South rises, but tends to fall with n_S, because employment in imitation, $a_M\mu(1-n_S)$, declines. If the former effect dominates, then a subsidy to imitation must increase μ; otherwise, μ may fall.

We summarize our findings for the case of efficient followers in Table 1. The results that we wish to emphasize are those that concern the feedback links between the regions. We have found that, holding the resource bases constant, the greater are incentives to undertake research activities in one region, the slower will be the steady-state rate of learning in the opposite region. We have identified, therefore, a fundamental tension in the learning processes of the North and the South that may exist due to the knowledge spillover and trade links between these different economic regions.

V. Inefficient Followers

With inefficient followers, $\iota_N = 0$. Then $\iota = \iota_S n_S$, which in turn is equal to $\eta = \mu n_N$ by (21). So the steady-state rates of aggregate innovation and aggregate imitation are equal in this case. We may treat the case diagrammatically by further reducing our system to two equations that represent the no-arbitrage conditions for Northern (leader) and Southern firms. To this end, we substitute the labor-market conditions (28) and (29) into (30) and (31), to obtain

$$\left[\frac{L_N - a_{DL}\iota}{\delta\iota} - \frac{L_S - a_M\iota}{\mu - \iota} \right] \frac{\mu}{a_M} = \rho + \frac{\iota\mu}{\mu - \iota} , \tag{32'}$$

$$\left[\frac{L_S - a_M\iota}{\mu - \iota} - \frac{L_N - a_{DL}\iota}{\iota} \right] \frac{\mu}{a_{DL}} = \rho + \mu . \tag{33'}$$

The left-hand sides of (32') and (33') represent the profit rates for Southern and Northern (leader) firms, respectively, while the right-hand sides represent the risk adjusted interest rates faced by each of these types of firms. The Northern firms, of course, face a constant risk of imitation, while the Southern firms face the risk that the Northern leaders will succeed at taking another step up the quality ladder. As is evident, the respective terms reflecting the risk factors appear on both sides of (32') and (33'). Canceling these terms, we may rewrite the two equations as

$$\left[\frac{L_N - a_{DL}\iota}{\delta\iota} - \frac{L_S}{\mu - \iota} \right] \frac{\mu}{a_M} = \rho , \tag{32}$$

$$(33) \qquad \left[\frac{L_S - a_M \iota}{\mu - \iota} - \frac{L_N}{\iota} \right] \frac{\mu}{a_{DL}} = \rho \; .$$

We depict the combinations of ι and μ that satisfy (32) by the curve SS in the two panels of Figure 1. The curve NN in the figure shows the combinations of ι and μ that satisfy (33). The SS curve is upward sloping, since the left-hand side of (32) increases in both the rate of innovation and the rate of imitation. The NN curve may slope in either direction. Panel a shows the downward-sloping case. If the curve slopes upward, it must be steeper then the SS curve, as in panel b (see Appendix C).[6] The intersection point A in each panel represents the initial steady-state equilibrium.

Now suppose that L_N increases. This shifts both curves to the left. In Appendix C we show that, at constant ι, the SS curve must shift by more. As can be seen from the figure, the net result is an increase in the aggregate rate of innovation. The per-product intensity of imitation may rise or fall, though it necessarily falls in the downward-sloping case. Of course, the average length of the product cycle moves inversely with μ.

An expansion of the resource base of the South shifts both curves to the right (not shown in the figure). But the NN curve shifts further with changes in L_S (see Appendix C). This implies an increase in both the rate of innovation and the intensity of imitation. The average length of the product cycle falls. These results are similar to our findings in Grossman and Helpman (1989a), but differ, as we noted above, from those for the case of

[6] We argue in the appendix that the upward-sloping case has greater empirical relevance.

efficient followers.[7]

We introduce subsidies to learning, as before, in the form of two parameters that represent the share of R&D costs borne by the respective governments. With small subsidies to imitation and innovation of s_M and s_D, respectively, we multiply the right side of (32') by $1-s_M$ and the right side of (33') by $1-s_D$. An increase in s_D from zero shifts the NN curve to the right without affecting the SS curve. The result is an increase in the rate of innovation and per-product intensity of imitation, and a fall in the average length of the product cycle. A small subsidy to learning in the South shifts the SS curve to the left. This increases the intensity of imitation in the upward-sloping case, while reducing it in the downward-sloping case. The former result is more likely to apply (see footnote 6). In any event, the rate of innovation always accelerates. Again, we note the contrast with our findings for the case of efficient followers.[8]

We record our results for comparison purposes in Table 1. In this case of inefficient followers, we find no conflict between the incentives for learning in the North and in the South. Evidently, structural conditions of the world economy help to determine whether the two learning processes are mutually reinforcing or not.

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[7] We show in Appendix C, however, that resource expansions affect relative wages differently than in Grossman and Helpman (1989a).

[8] As a further point, we note that a subsidy to R&D in the North increases the steady-state relative wage of Northern workers, and deteriorates the long-run terms of trade of the North. A subsidy to imitation has the opposite effect on these variables. We establish these claims in Appendix C.

VI. Conclusions

We have developed a model of quality competition with simultaneous, stochastic innovation and imitation. In our model, each product exists on a quality ladder. Entrepreneurs in the developed "North" devote resources to R&D in an attempt to upgrade the quality of a product. If an entrepreneur is successful in her research efforts, then she acquires market power in production of that good for a period of time. In the less developed "South", entrepreneurs strive to learn the production processes developed in the North. These efforts, too, are costly and involve uncertain prospects. When successful, a Southern entrepreneur can take advantage of the favorable manufacturing cost conditions in the South to earn monopoly profits until the next product improvement takes place.

The steady-state equilibrium is characterized by continuous product upgrading and product cycles (migration in the locus of production of particular goods from North to South and back again) of varying durations. Each individual product may exhibit a complex life history, with alternating periods of technological stagnation, rapid innovation, and with many or few shifts in its direction of trade. In the aggregate, the average rates of imitation and innovation are constant in the steady state, as are the fraction of products manufactured in the South, the North-South terms of trade, and the average length of a product cycle.

We have analyzed two distinct equilibrium regimes that may arise under different parameter configurations. In one such regime, the most recent Northern innovators for each product enjoy a substantial productivity advantage over potential entrants in generating the next product improvement. In this case of "inefficient followers", a fixed set of Northern firms conducts all innovative activity. These firms engage in R&D following every instance of loss of market share to Southern imitators. When the productivity gap between leaders and potential displacers is less extreme, entry by new firms is possible. In the regime with "efficient followers", the Northern leaders conduct R&D in order to re-capture products copied by the South, while inactive producers in the North attempt quality improvements in the hope of stealing business away from the extant industry leaders.

We studied the implications of region size and of government technology policy for the steady-state rates of innovation and imitation. We found a rich set of possible interactions. When followers are efficient, the feedbacks between the learning processes in the North and in the South generally are negative. Northern policy to promote innovation causes the steady-state rate at which goods flow from North to South to decline. Similarly, government subsidies to imitation in the South effect a decline in the average rate at which goods climb up the quality ladder. Expansion in the size of the North causes innovation to accelerate, whereas expansion in the size of the South may (but need not) have the opposite effect.

The results for the regime with inefficient followers are quite different. In this case, the two learning processes generally are mutually reinforcing. Northern subsidies to innovation increase the rate at which the South learns to produce new goods and shorten the average length of a product cycle. Southern promotion of imitation leads to faster steady-state innovation in the North, and may increase or decrease the average length of the product cycle. Also, an expansion in the size of either region causes the rates of innovation and imitation to increase.

We have so far been unsuccessful in our attempts to attach normative significance to these suggestive findings. In our earlier work on quality ladders in a single country (Grossman and Helpman, 1989d; see also Aghion and Howitt, 1989) we have shown that innovation in an environment such as this one may be too fast or too slow. Welfare analysis in the present circumstances is complicated by the terms of trade adjustments attendant to any policy change, and by the complex transition path that may link steady states. It may prove necessary to resort to simulation techniques in order to resolve whether governments in the North and South face conflicting or coincident incentives where technology policy is concerned. We leave this important issue for future research.

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APPENDIX A

In this appendix we prove that, in a steady state, if $a_{DF}/a_{DL} < 2 - 1/\lambda$, no Northern leader with positive sales will undertake efforts to improve its own product. We do this in two steps. First we show that no Northern firm whose nearest competitor is another Northern firm (an NN firm) will improve its own product. Then we show that the same is true for a Northern firm whose nearest competitor is a Southern firm (an NS firm).

Let v_{N_2} be the value of a Northern firm that is two steps ahead of its nearest (Northern) rival (an N2 firm). An N2 firm charges a price $\lambda^2 w_N$. It is easy to calculate that it earns profits $\pi_{N_2} = (\lambda+1)\pi_{NN}/\lambda$. The no-arbitrage condition for shares in N2 firms and that for shares in NN firms in equation (13) imply, in a steady state, that $v_{N_2} = \pi_{N_2} v_{NN}/v_{NN}$. Then

$$(A1) \qquad v_{N_2} - v_{NN} = v_{NN}/\lambda.$$

An NN firm that improves its own product will achieve a capital gain of $v_{N_2} - v_{NN}$. The cost per unit intensity of effort is $w_N a_{DL}$. Equations (10) and (A1) imply $v_{N_2} - v_{NN} \leq w_N a_{DF}/\lambda$. If $a_{DF}/a_{DL} < \lambda$, the expected gain from research effort by an NN firm cannot justify the cost. But $a_{DF}/a_{DL} < 2 - 1/\lambda$ $\Rightarrow a_{DF}/a_{DL} < \lambda$. So R&D is not profitable for NN leaders.

Next consider an NS firm. If this firm undertakes research and succeeds, it becomes an N2S firm, two steps ahead of its nearest (Southern) rival. An N2S firm would charge a price $\lambda^2 w_S$ and earn a flow of profits $\pi_{N_2S} = 1 - w_N/\lambda^2 w_S$. We calculate

(A2) $$\pi_{N_2S} - \pi_{NS} = w_N/\lambda w_S - w_N/\lambda^2 w_S.$$

An NS firm that conducts research stands to gain $v_{N_2S} - v_{NS}$. Using the no-arbitrage conditions and the steady-state requirement that all nominal values increase at the same rate, we find

(A3) $$v_{N_2S} - v_{NS} = (\pi_{N_2S} - \pi_{NS})v_{NS}/\pi_{NS}.$$

From (11) and $\iota_S > 0$,

(A4) $$v_{NS}/\pi_{NS} = w_N a_{DL}/(1 - w_N/\lambda w_S).$$

Now (10), (11), (13) and (14) imply, in a steady state with $\iota_S > 0$ that

(A5) $$1 - 1/\lambda \leq (a_{DF}/a_{DL})(1 - w_N/\lambda w_S)$$

Then $a_{DF}/a_{DL} < 2 - 1/\lambda$ and (A5) imply $w_N/\lambda w_S - w_N/\lambda^2 w_S < 1 - 1/\lambda$. Finally, this inequality, together with (A2), (A3) and (A4) implies $v_{N_2S} - v_{NS} < a_{DF}w$ Thus, research efforts by NS firms are not profitable.

APPENDIX B

In this appendix we provide calculations of comparative statics results for the efficient followers case. For this purpose we use equations (22)-(25) to calculate changes in e, n_N, η and ι, taking w from (27). In order to avoid a taxonomical treatment we assume that the condition from footnote 4 applies at the initial equilibrium point, i.e.,

$$\text{(B1)} \qquad a \equiv a_{DF} - \delta e n_N (w-1)\eta/\iota^2 > 0.$$

Our calculations proceed as follows. We multiply the right-hand side of (24) by $(1-s_M)$ and the right-hand side of (25) by $(1-s_D)$, with initial values $s_M = s_D = 0$. This way we account for innovation and imitation subsidies. Now total differentiation of (22)-(25), using L_S, L_N, s_M and s_D as shift parameters, yields:

$$\text{(B2)} \qquad A \begin{bmatrix} de \\ dn_N \\ d\eta \\ d_2 \end{bmatrix} = \begin{bmatrix} dL_S \\ dL_N \\ -b_M ds_M \\ -b_D ds_D \end{bmatrix},$$

where $b_M = a_M(\rho n_S + \eta)$, $b_D = a_{DL}(\rho n_N + \iota)$, and

$$A = \begin{bmatrix} n_S & e & a_M & 0 \\ \delta n_N \nu & \delta e \nu & (a_{DF} - a_{DL})\rho n_N/\iota & a \\ (w-1)n_S & -a_M \eta/n_S & -a_M & 0 \\ (1-\delta w)n_N & a_{DL}\iota/n_N & 0 & -a_{DL} \end{bmatrix},$$

where $\nu = 1 + (w-1)\eta/\iota > 0$, and use has been made of (24) in the simplification of a_{32}, of (25) in the simplification of a_{42}, of (27) and (26) in the simplification of a_{23}, and (B1) in the simplification of a_{24}. Developing the determinant of A by means of the last column, and making use of (24), it is straightforward to establish:

$$\Delta \equiv \det A > 0.$$

Next we calculate from (B2):

(B3) $$\frac{d\iota}{dL_S} = \frac{1}{\Delta}\{\delta a_M a_{DL} \nu\rho n_N - (a_{DF} - a_{DL})\rho n_N[(w-1)a_{DL} n_S/n_N + (1-\delta w)a_M \eta n_N/\iota n_S]\} \gtrless 0,$$

(B4) $$\frac{d\iota}{dL_N} = \frac{a_M}{\Delta}\{w a_{DL}\iota n_S/n_N + (1-\delta w)n_N(e + a_M \eta/n_S)\} > 0,$$

(B5) $$\frac{d\iota}{ds_M} = -\frac{b_M}{\Delta}\{\rho\delta\nu a_M a_{DL} n_N + (a_{DF} - a_{DL})\rho[a_{DL} n_S + (1-\delta w)e n_N^2/\iota]\} < 0,$$

(B6) $$\frac{d\iota}{ds_D} = \frac{b_D}{\Delta}\{a_M \delta\nu[(w-1)e n_S + a_M \eta n_N/n_S + e] + (a_{DF} - a_{DL})\rho n_N n_S a_M \rho/\iota\} > 0,$$

(B7) $$\frac{d\eta}{dL_S} = \frac{1}{\Delta}\{a_M \delta\nu[a_M \eta n_N/n_S + (w-1)e n_S] + a[(w-1)a_{DL}\iota n_S/n_N + (1-\delta w)a_M \eta n_N/n_S]\} > 0,$$

(B8) $$\frac{d\eta}{dL_N} = \frac{1}{\Delta} a_{DL} a_M \rho n_S > 0,$$

(B9) $$\frac{d\eta}{ds_M} = \frac{b_M}{\Delta}\{a_{DL}\delta\nu e + a[a_{DL}\iota n_S/n_N + (1-\delta w)en_N]\} > 0,$$

(B10) $$\frac{d\eta}{ds_D} = -\frac{1}{\Delta} b_D a a_M \rho n_S < 0,$$

where we have used (25) to derive (B3), and (B5) and (24) to derive (B6), (B8) and (B10).

In order to calculate the response of μ, recall that

$$\mu = \eta/n_N.$$

Similar calculations show that:

$$\frac{dn_N}{dL_S} < 0, \quad \frac{dn_N}{dL_N} > 0, \quad \frac{dn_N}{ds_M} \gtrless 0, \quad \frac{dn_N}{ds_D} < 0.$$

In addition:

$$\frac{d\mu}{dK} = \frac{d\eta}{dK} - \frac{\eta}{n_N^2}\frac{dn_N}{dK} \quad , \quad K = L_S,\ L_N,\ s_M,\ s_D.$$

An increase in L_S raises η and reduces n_N. Therefore

$$\frac{d\mu}{dL_S} > 0.$$

When Northern labor supply increases or the North subsidizes innovation η and n_N respond in the same directions. Consequently μ may increase or decline. When the South subsidizes imitation the ambiguity in the response of n_N leads to an ambiguity in the response of μ.

APPENDIX C

In this appendix we derive several properties of the SS and NN curves that are used in Section V. From (32) we calculate the slope of the SS curve,

(C1) $$\left.\frac{d\iota}{d\mu}\right|_{SS} = \frac{\iota}{\mu} \cdot \frac{(L_N - a_{DL}\iota)/\delta\iota^2 + L_S/(\mu-\iota)^2}{L_N/\delta\iota^2 + L_S/(\mu-\iota)^2} .$$

Since $(L_N - a_{DL}\iota) > 0$ (because it is equal to manufacturing employment in the North),

(C2) $$0 < \left.\frac{d\iota}{d\mu}\right|_{SS} < \iota/\mu.$$

We calculate the slope of NN from (33);

(C3) $$\left.\frac{d\iota}{d\mu}\right|_{NN} = \frac{\iota}{\mu} \cdot \frac{(L_S - a_M\iota)/(\mu-\iota)^2 + L_N/\iota^2}{(L_S - a_M\mu)/(\mu-\iota)^2 + L_N/\iota^2} .$$

Here the numerator is always positive, because $(L_S - a_M\iota)$ equals manufacturing employment in the South. If $(L_S - a_M\mu) > 0$ we also have a positive denominator, and since $\iota = \mu n_N < \mu$, the slope of the curve is positive and larger than ι/μ. In fact, whenever the denominator is positive the slope of the NN curve exceeds ι/μ. When the denominator is negative, however, the NN curve slopes downwards. Hence,

(C4) $$\left.\frac{d\iota}{d\mu}\right|_{NN} < 0 \quad \text{or} \quad \left.\frac{d\iota}{d\mu}\right|_{NN} > \iota/\mu.$$

These possibilities are depicted in Figure 1.

Although there exist two theoretical possibilities concerning the slope of the NN curve, casual empirical evidence suggests that the upward sloping case is more plausible. This argument is based on the observation that a large share of products actually is manufactured in the North (i.e., n_N is relatively large) while the South devotes only a small share of its resources to imitation (i.e., $a_M\iota/L_S$ is relatively small). If, as is plausible, the share of resources devoted to imitation in the South falls short of the share of products manufactured in the North (i.e., $n_N > a_M\iota/L_S$) then NN slopes upwards. The proof is straightforward. The condition $n_N > a_M\iota/L_S$ implies $L_S > a_M\mu$ (because $\iota = \mu n_N$), which implies in turn a positive denominator on the right-hand side of (C3).

Now consider an increase in L_N. For given ι we calculate from (32) the response of μ,

$$\text{(C5)} \qquad \left.\frac{d\mu}{dL_N}\right|_{SS} = - \frac{\mu/\delta\iota^2}{(L_N - a_{DL}\iota)/\delta\iota^2 + L_S/(\mu-1)^2} < 0.$$

This shows that the SS curve shifts leftward. We similarly calculate the horizontal shift of the NN curve from (33):

$$\text{(C6)} \qquad \left.\frac{d\mu}{dL_N}\right|_{NN} = - \frac{\mu/\delta\iota^2}{(L_S - a_M\iota)/(\mu-1)^2 + L_N/\iota^2} < 0.$$

Hence, the NN curve also shifts leftward. However, the difference in the denominator on the right-hand-side of (C6) and the denominator on the right-hand-side of (C5) equals:

$$\frac{(1-\delta)L_S - a_M\iota}{(\mu-1)^2} + a_{DL}/\iota.$$

On the other hand, (28) and (30) imply:

$$(1-\delta)L_S - a_M\iota = [\delta a_M\rho + (1-\delta w)e](1-n_N) > 0,$$

because $(1-\delta w) > 0$ (see (31)). Therefore, the denominator is larger in (C6) and the SS curve shifts leftwards by more than the NN curve.

Next we calculate horizontal shifts of the curves in response to an increase in L_S:

$$\text{(C7)} \qquad \left.\frac{d\mu}{dL_S}\right|_{SS} = \frac{\mu/\iota(\mu-\iota)}{(L_N - a_{DL}\iota)/\delta\iota^2 + L_S/(\mu-\iota)^2} > 0,$$

$$\text{(C8)} \qquad \left.\frac{d\mu}{dL_S}\right|_{NN} = \frac{\mu/\iota(\mu-\iota)}{(L_S - a_M\iota)/(\mu-\iota)^2 + L_N/\iota^2} > 0.$$

Hence, both curves shift to the right. The difference between the denominators on the right-hand-side of (C7) and (C8) equals

$$\frac{(1-\delta)L_N - a_{DL}\iota}{\delta\iota^2} + a_M\iota/(\mu-\iota)^2.$$

From (29) we obtain:

$$(1-\delta)L_N - a_{DL}\iota = \delta[w(1-\delta)en_N - a_{DL}\iota] > \delta[(1-\delta w)en_N - a_{DL}\iota] = \delta\rho n_N a_{DL} > 0.$$

The first inequality follows from the fact that $w > 1$, while the last equality results from (31). This shows that the denominator is larger on the right-hand side of (C7). Therefore the NN curves shifts further to the right than the SS curve.

Direct comparative statics calculations on system (28)-(31), augmented to include R&D subsidies (i.e., multiplying the right-hand side of (30) by $1-s_M$, where s_M is the imitation subsidy, and the right-hand-side of (31) by $1-s_D$, where s_D is the innovation subsidy) enables us to calculate also the response of relative wages and product shares, which have not been discussed in the main text. We obtain the following system of equations for comparative statics calculations (assuming 'small' subsidies):

$$\text{(C9)} \quad \begin{bmatrix} n_S & -e & 0 & a_M \\ \delta n_N w & \delta e w & \delta e n_N & a_{DL} \\ n_S w & -L_S/n_S & e n_S & 0 \\ n_N & L_N/n_N & 0 & 0 \end{bmatrix} \begin{bmatrix} de \\ dn_N \\ dw \\ d\iota \end{bmatrix} = \begin{bmatrix} dL_S \\ dL_N \\ -b_M ds_M \\ -b_D ds_D \end{bmatrix},$$

where $b_M = a_M(\rho n_S+\iota)$ and $b_D = a_{DL}(\rho n_N+\iota)$. The determinant of the matrix, denoted by Δ_2, is

$$\Delta_2 = -\ a_{DL} e n_S(n_S L_N/n_N + e n_N) - a_M \delta e n_N (w_N L_S/n_S + w e n_N) < 0.$$

Using this system it can be shown that an increase in L_N increases the share of products manufactured in the North and an increase in L_S increasees the share of products manufactured in the South. The effects of labor supply on relative wages are, however, ambiguous. This contrasts with our finding in

Grossman and Helpman (1989a) where an increase in a country's labor supply always raises its labor's relative reward.

The results for small R&D subsidies are, however, unambiguous:

$$\left.\frac{dn_N}{ds_M}\right|_{s_M=s_D=0} = \delta a_M e n_N^2/b_M(-\Delta) > 0,$$

$$\left.\frac{dw}{ds_M}\right|_{s_M=s_D=0} = [\delta a_M a_{DL} w\rho n_N + a_{DL}(n_S L_N/n_N + en_N)]/b_M\Delta < 0,$$

$$\left.\frac{dn_N}{ds_D}\right|_{s_M=s_D=0} = a_{DL} e n_S^2/b_D\Delta < 0,$$

$$\left.\frac{dw}{ds_D}\right|_{s_M=s_D=0} = [a_M a_{DL}\delta n_S + \delta a_M w(n_N L_S/n_S + wen_S)]/b_D(-\Delta) > 0,$$

where (30) and (31) have been used in the derivation of the second and fourth results, respectively. Thus, a subsidy to imitation expands the share of products manufactured in the North (because the South employs more resources in imitation and thereby contracts manufacturing employment) and reduces the North's relative wage. A subsidy to imitation has the opposite effects.

Table 1

Endogenous Variables		Exogenous Variables L_N	L_S	s_D	s_M	
	ι	+	+/-	+	-	Case of Efficient Followers
	η	+	+	-	+	
	μ	+/-	+	+/-	+/-	
	$\iota=\eta$	+	+	+	+	Case of Inefficient Followers
	μ	+/-	+	+/-	+/-	

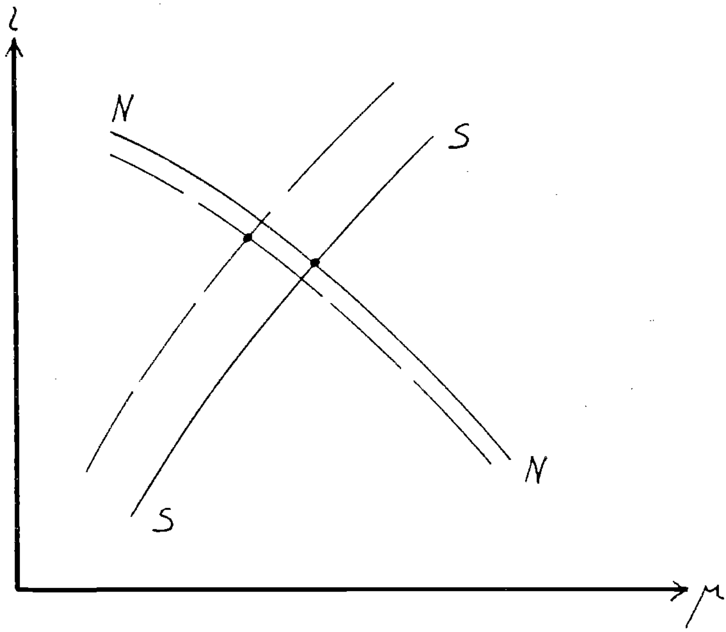

Figure 1a

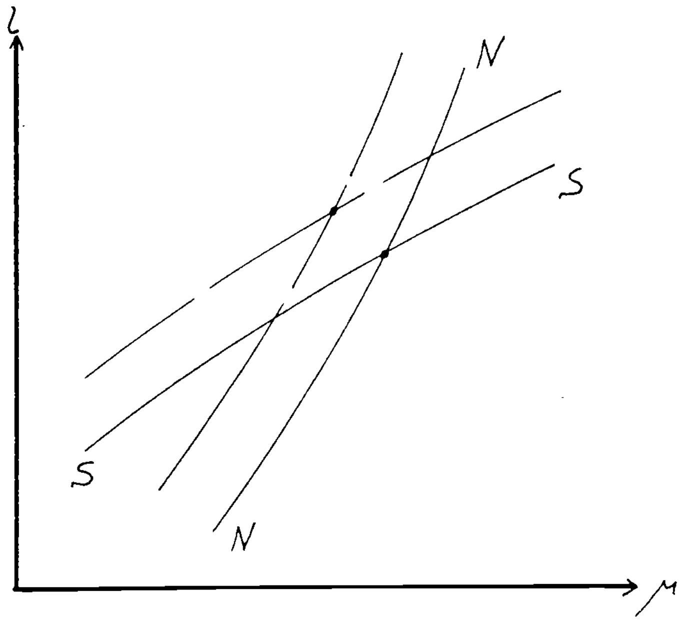

Figure 1b