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ABSTRACT

This paper extends the “dynamic monopsony” Burdett-Mortensen model of wage posting and on-the-job search to incorporate granular employers with decreasing returns to scale. We provide a complete analytical characterization of the resulting equilibrium and show how to allow for firm heterogeneity, additional inputs, and product market power. As an application, we study noncompete agreements theoretically and quantitatively. A US ban would yield wage gains typically in the range of 0.8 3% depending on local conditions, with a baseline estimate of 0.9%, but these come with higher worker turnover and a mild decline in aggregate welfare.

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1 Introduction

A large academic literature now studies competition in the labor market. At the same time, antitrust regulators in the US and elsewhere have started paying much more attention to the labor market. The questions are often granular in nature: How does market structure affect pay and employment? How do mergers, employer collusion, or labor market policies affect workers when only a few firms compete for them locally?

This paper integrates a finite number of large employers with decreasing returns to scale technology into a model of a frictional labor market with wage posting and on-the-job search, akin to the canonical framework of Burdett and Mortensen (1998, BM). Despite these additions, the model remains analytically tractable.

As a concrete application, we use our framework to evaluate the consequences of banning noncompete agreements in the United States. Such a policy was proposed by the Federal Trade Commission (FTC) in 2023, although it was never implemented. We find that a ban would raise average wages by 0.9%, with substantial heterogeneity depending on local market conditions.

The "dynamic monopsony" environment of BM is widely considered a workhorse model in the literature on imperfect competition in the labor market (Manning, 2003). We extend it along three key dimensions. First and most importantly, our model has a finite number of employers, all large with respect to the labor market. This allows us to capture the granular nature of many local labor markets, where anticompetitive practices might have particularly adverse impacts on workers. Second, it has decreasing returns and hence can endogenize firm size, aggregate employment, and market structure more flexibly than the BM model, which is restrictive since firms can adjust employment only through wages. Third, the model assumes a linear hiring technology which significantly simplifies the analysis. It also captures that the most substantive costs associated with turnover are due to hiring and training, rather than merely locating workers (Manning, 2011; Blatter et al., 2012). These elements yield a unified and tractable framework that endogenizes firm size and aggregate employment, making the framework suitable for analyzing general-equilibrium effects of shocks and policies in a granular labor market.

We describe how to integrate these additions into an otherwise canonical environment where workers search for better pay on and off the job and firms commit to pay posted wages. In laying out the basic environment, we emphasize three modeling choices: (i) Firms and workers take each other's actions as given, as in a standard Nash equilibrium. (ii) Firms choose a distribution of wage offers and a contact rate at which workers encounter these offers. Firms do so at no cost, but pay a hiring cost whenever they add a worker. (iii) The baseline studies "timeless" no-discounting equilibria where firms just maximize steady-state flow profits, as is standard in the BM literature.

We then characterize worker optimality which has two pieces: a reservation wage adjusted to a granular setting and an acceptance rule whereby employed workers accept any higher wage offer. We then establish a key lemma that allows for a dual formulation of the firm's problem. Under the dual, firms directly choose employment and a distribution of pay. This formulation yields a simple and intuitive characterization of firm optimality.

The main theoretical result in Proposition 1 then establishes existence and uniqueness of an equilibrium and characterizes it. The equilibrium is symmetric, with all firms posting uniformly distributed wage offers. This ensures that the user cost of labor is equated across all posted wages. Firm-level employment is such that the user cost of labor equals its marginal revenue product, which uniquely determines the equilibrium contact rate. All relevant equilibrium objects are linked to primitives via simple closed forms that can be solved in a block-wise fashion. We give intuition for a few key properties such as symmetry and the absence of gaps and mass points in the wage offer distribution.

The equilibrium is next shown to be constrained efficient in the sense that a planner would choose the same employment level that arises in equilibrium and would be indifferent to the shape of any atomless wage offer distribution. The analysis then covers the limit of our setting with a continuum of firms and shows that wages remain dispersed in that limit. It next explains how to additionally modify the hiring technology so as to recover the BM allocation.

We next use the framework to theoretically study noncompete agreements, modeled as wage offers that prohibit job-to-job transitions. We show that firms with access to noncompetes offer a mass of identical jobs, all of which deliver the lowest possible acceptable value to workers. In the BM model such mass gets eliminated by competition as marginally higher pay yields discretely lower turnover, creating incentives to deviate. Noncompetes break this mechanism—workers bound by them cannot be poached. As more firms adopt noncompetes, competition along the job ladder weakens and, in the limit where all firms use noncompetes, the Diamond (1971) Paradox reemerges with wages collapsing to the reservation value. While this sharply reduces wages, it increases welfare as it eliminates socially costly worker turnover. In equilibrium, firms with noncompetes are larger because they have lower worker turnover and hence a lower user cost of labor. This is constrained efficient: a planner who takes firms' use of noncompetes as given would choose the same firm-level employment that occurs in equilibrium.

We then revisit the baseline model and consider several extensions. We show how to allow for differences in productivity across firms and, additionally, in hiring costs. We characterize the resulting equilibria and show that these extensions generate endogenous sorting of firms along the wage ladder: the hiring cost governs average pay because firms with low cost care less about turnover. Productivity in turn governs size, with larger firms satisfying their hiring needs by posting a wider range of wages. We present simple algorithms to solve the models with heterogeneity numerically.

The subsequent quantitative analysis considers homogeneous firms, but extends the model along the three following dimensions: (i) a market-level product demand curve such that granular firms have both labor and product market power, to speak to settings where adjustment to shocks operates primarily through prices and not employment and output, (ii) additional inputs such as capital in the production function, to allow for richer patterns of input substitution, (iii) time discounting on the firm side, allowing the environment to be integrated into other dynamic frameworks. With these, we study a US ban of noncompetes quantitatively.

Our baseline analysis suggests that such a policy would mildly increase wages by 0.9%, as a consequence of the rise in competition. This comes with a large rise in worker churn, and strong wage spillovers to firms that did not initially use noncompetes. The rise in worker turnover, however, drives up the user cost of labor which leads firms to pull back on labor demand. This negative general equilibrium effect leads to a mild fall in aggregate employment and output.

We conduct substantive heterogeneity analysis with regard to (local) labor market features. The wage gains are larger in markets with widespread initial use of noncompetes, inelastic product demand, high employment concentration, a low labor share, and in settings that mimic the "tech" sector, as opposed to the "fast food" sector. Wage gains are typically in the range of $0.8 - 3\%$.

Worker churn jumps in response to a noncompete ban which is wasteful from an aggregate perspective. The associated output decline additionally exacerbates product market distortions due to product market power. We consequently find mild negative aggregate welfare effects but point to several assumptions that might overturn this. We also briefly discuss how the policy might redistribute across workers.

1.1 Relation to Literature

It has long been recognized that labor markets are imperfectly competitive. Robinson (1933) was the first to formulate a notion of monopsony in labor markets. Similarly, work in the search tradition has long emphasized frictions that lead to rents and market power. However, only in recent years has a literature specifically focused on the origins and consequences of employer labor market power taken off.

An early, important contribution to this literature by Manning (2003) conceptualized labor market power through the lens of the BM model. In this "dynamic monopsony" framework, labor market power is rooted in search frictions. In equilibrium, employers post wages below the marginal product since it takes time for workers to find alternatives.

A more recent wave of papers takes a "neoclassical" approach, starting with Card et al. (2018), who build on a static model of monopsonistic competition. This is a frictionless approach in which market power derives from employers being differentiated from the perspective of the workforce, yielding an upward-sloping labor supply curve at the employer level. Berger et al. (2022) extend this approach so that it can connect with (locally) granular labor markets, with a finite number of large employers that strategically compete for workers. Jungerman (2024) adds human capital dynamics.

This paper revisits the "dynamic monopsony" BM search perspective but extends it to allow for granular employers and an endogenous market structure. Doing so allows this approach to connect with a new range of questions related to barriers to competition in the labor market where worker mobility and labor market structure play a key role.

Jarosch et al. (2024) first modeled a granular market structure in a frictional labor market. They do so in the context of the canonical random search model with bargaining, the Diamond–Mortensen–Pissarides model. We introduce similar considerations into the canonical random search model with wage posting, the BM model. In Jarosch et al. (2024), competition for workers operates through outside options. Here, outside competition works along the job ladder and more competition leads to more quits, which drives up wages. In addition, we endogenize firm size and market structure while they treat it as exogenous. Bagga (2023) studies the same setting as Jarosch et al. (2024) with offer matching and on-the-job search as in Cahuc et al. (2006). Berger et al. (2023) additionally integrate dispersed amenities into that setting.

Other related work includes Deb et al. (2024) and Trottner (2023) who study product and labor market power in an integrated way. Relative to these, we microfound labor market competition and wage mark-downs via on-the-job search in the BM tradition. Gouin-Bonenfant (2022) uses a BM setting to explore the fanning-out of the firm productivity distribution, which might lead to less local competition for workers, depressing wages. Our setting shares the emphasis on competition in a BM setting but focuses on a granular market structure. The paper also relates to the recent literature on models with random on-the-job search and decreasing returns to scale (Bilal et al., 2022; Elsby and Gottfries, 2022; Lentz and Mortensen, 2022). The key difference is that we study markets with a finite number of granular employers rather than a continuum of small firms.

Potter et al. (2024) model noncompetes as a decrease in the efficiency of on-the-job search in a wage posting model. Finally, Shi (2023) is closely related and studies noncompetes in a frictional labor market with bargaining. She argues that worker-firm pairs can use noncompetes to extract rents from outside employers that poach workers. Noncompetes in her setting are thus used to extract rents from a third party, outside employers, while they are used by employers to reduce competition and extract rents from workers in our setting, representing a different channel.

2 Baseline model

This section first lays out the basic model environment. The following subsection sets up the problems of workers and firms and characterizes optimal behavior. We then define an equilibrium, establish existence and uniqueness and offer a complete characterization. The following subsections consider efficiency and the relation to the textbook BM framework. We then discuss several key assumptions in a final subsection.

2.1 Basic Environment

There is a unit measure of workers and a finite number $M > 1$ of firms.[1] Time is continuous. Firms and workers have linear preferences over income and discount the future at rate r. We restrict attention to stationary equilibria.

Employed workers earn a contracted, fixed wage w. They lose their jobs at rate δ and become unemployed. Unemployed workers receive flow utility b. Both unemployed and employed workers search for (better) jobs in a random, undirected fashion. The employed search with relative efficiency $s \in (0, 1]$.

Firms operate a decreasing-returns-to-scale production function xn^{α} with $\alpha \in (0, 1)$, with n denoting firm-level employment and x denoting firm-level productivity. To hire workers, firms make two choices. First, they choose a Poisson contact rate $\psi \in [0, \infty)$ $(s\psi)$ at which unemployed (employed) workers receive a job offer from them. Generating these contacts is costless to firms. Second, firms choose a distribution of posted wages F from which each of their job offers is drawn.[2] Firms commit to paying the offered wage for the duration of the match. For each new hire, they pay a hiring/training cost c, as in Elsby and Gottfries (2022). This has two important implications. First, the cost associated with adding workers is linear. Second, worker turnover is costly to firms because of hiring costs (as in Coles and Mortensen (2016)), not because searching for workers is expensive per se.

The set of firms in the market $\mathbb{M} \equiv \{1, 2, \ldots, M\}$ is assumed to be fixed. The state variable of an employed worker is $\{i, w\}$ where $i \in \mathbb{M}$ indicates the employer and w the wage; denoting the state of the unemployed with 0, a worker's state space is $\Omega \equiv (\mathbb{M} \times \mathbb{R}_+) \cup \{0\}$.

A worker's sole action is to choose an acceptance policy $\mathcal{A}$ which specifies the set of wage offers she accepts from employer $j \in \mathbb{M}$ given her current state. We later verify that these policies are summarized by a simple threshold rule $\mathcal{A} : \Omega \times \mathbb{M} \to \{[\tilde{w}, \infty) : \tilde{w} \in \mathbb{R}\}$. Workers are assumed to move whenever they are indifferent about the offer.

A finite number of firms and random search implies that employed workers sometimes encounter jobs posted by their own employers. To the extent that an employer posts a mix

[1]The full monopsonist case with $M = 1$ is not particularly interesting since that firm simply posts wages equal to workers' flow income in unemployment. We therefore focus on $M > 1$.

[2]Firms may choose distributions with arbitrary mass points (including degenerate distributions), gaps, and segments with densities. To avoid cumbersome notation, we exclude only distributions with singular continuous components (e.g., Cantor distribution)—continuous functions that have zero derivative almost everywhere.

of wages, this gives rise to the possibility of an internal transition to a higher wage. We assume that, in such an event, the firm does not need to pay the hiring cost again.

We assume that both workers and firms know all the primitives and form rational expectations over the equilibrium when making their choices. When making their choices, agents hold other agents' choices fixed. In this sense, we consider a Nash equilibrium.

2.2 Worker and Firm Problems and Optimality

This section sets up the worker problem and characterizes optimal behavior. It then establishes a key lemma that allows to formulate the firm problem in a simple and tractable way and then characterizes firm optimality.

Worker Problem and Optimality

Random search, along with the assumption that firms do not discriminate between internal and external workers in their job offers, implies that search opportunities are the same in any firm. The identity of their employer is thus irrelevant to employed workers, with the relevant state given solely by the current wage. This also implies that workers transition whenever they receive a weakly higher wage offer, including from their own employer. It further implies that the unemployed accept any wage above a single reservation wage which is the same across firms.

A worker's value of unemployment and employment at wage w thus satisfy

$$rU = b + \sum_j \psi_j \int_{w_r}^{\infty} \left(W\left(\tilde{w}\right) - U\right) dF_j\left(\tilde{w}\right), \tag{1}$$

$$rW\left(w\right) = w + \delta\left(U - W\left(w\right)\right) + \sum_j s\psi_j \int_{w}^{\infty} \left(W\left(\tilde{w}\right) - W\left(w\right)\right) dF_j\left(\tilde{w}\right), \tag{2}$$

with $W(w_r) = U$ by definition. This latter identity can be used to derive an expression for the reservation wage.[3] We provide details in Appendix A.1 and collect all the relevant

[3] The derivations impose that $\lim_{w\to\infty} \psi_j(1 - F_j(w))w = 0$ for all $j \in \mathrm{M}$ by assumption. This is a restriction on the fatness of the tail which is, for instance, satisfied by any Pareto distribution with a finite mean. We proceed like this to keep things streamlined but have a proof that this condition is satisfied under firm optimality.

conditions for worker optimality as

$$
\begin{aligned}
\mathcal{A}(\{i,w\},j) &= [w,\infty) \quad \forall i,j \in \mathrm{M}, && (3a)\\
\mathcal{A}(0,j) &= [w_r,\infty) \quad \forall j \in \mathrm{M}, && (3b)\\
w_r &= b + (1-s)\int_{w_r}^{\infty} \frac{\sum_j \psi_j \left(1-F_j(\tilde{w})\right)}{r+\delta+\sum_j s\psi_j \left(1-F_j\left(\tilde{w}\right)\right)} d\tilde{w}. && (3c)
\end{aligned}
$$

We note that the expression for the reservation wage is just standard BM, but extended to a setting with granular employers.

Dual Representation of Firm Choices

Let x^- denote the left hand limit of x. Given workers' optimal policy (3), we can compute firm-level employment n_i and a distribution of *paid* wages within the firm G_i as a function of its choices $\{\psi_i, F_i\}$, given policies at other employers $\{\psi_j, F_j\}_{j\neq i}$ and workers' reservation wage w_r. This gives rise to a useful dual representation of the firm's choices which we summarize in the following Lemma.

LEMMA 1. Fix w_r and $\{\psi_j, F_j\}_{j\neq i}$. Each firm policy $\{\psi_i, F_i\}$ implies a unique and distinct $\{n_i, G_i\}$, with the mapping given by

$$
\begin{aligned}
G_i(w) &= \frac{\int_{w_r}^{w} \frac{1}{\delta+s\sum_j \psi_j\left(1-F_j(\tilde{w}^-)\right)} \frac{1}{\delta+s\sum_j \psi_j\left(1-F_j(\tilde{w})\right)} dF_i(\tilde{w})}{\int_{w_r}^{\infty} \frac{1}{\delta+s\sum_j \psi_j\left(1-F_j(\tilde{w}^-)\right)} \frac{1}{\delta+s\sum_j \psi_j\left(1-F_j(\tilde{w})\right)} dF_i(\tilde{w})}, && (4a)\\
n_i &= \int_{w_r}^{\infty} \frac{\psi_i}{\delta+s\sum_j \psi_j\left(1-F_j(w^-)\right)} \frac{\delta+s\sum_j \psi_j}{\delta+s\sum_j \psi_j\left(1-F_j(w)\right)} \frac{\delta}{\delta+\sum_j \psi_j} dF_i(w). && (4b)
\end{aligned}
$$

The result allows to cast the firm problem in *dual* form, namely as one of directly choosing the level of employment n_i and G_i.[4] The proof in Online Appendix B provides a simple way to invert (4) so as to construct the contact rate and offer distribution $\{\psi_i, F_i\}$ that implement a particular policy $\{n_i, G_i\}$.

[4]We normalize ψ_i to include only offers weakly above the reservation wage w_r since any lower offers are rejected. There are no such offers if there is an arbitrarily small cost of making an offer.

Firm Problem and Optimality

We next formally set up the firm problem. In order to minimize the distance to BM and to maximize tractability, we do so under the assumption that firms maximize steady state flow income. This is accurate when $r \to 0$. Section 4.2 shows how to formulate and solve the firm problem under discounting.

Flow profits consist of gross revenue net of the wage bill and turnover costs which reflect that replacing departing workers is costly because of the hiring cost. As workers accept any job with a weakly higher wage (3a), turnover costs take a very simple form. In its dual formulation, the firm problem is given by

$$\max_{\{n_i, G_i\}} \left\{ x n_i^{\alpha} - n_i \int_{w_r}^{\infty} \left(w + c \left(\delta + s \sum_{j \neq i} \psi_j \left(1 - F_j(w^-)\right) \right) \right) dG_i(w) \right\}, \tag{5}$$

$$\text{subject to:} \quad G_i(w_r^-) = 0, \ \lim_{w \to \infty} G_i(w) = 1, \text{ and } G_i(w_1) \leq G_i(w_2) \text{ for all } w_1 < w_2.$$

Firm i takes workers' optimal behavior (3) and their competitors' actions—contact rates $\{\psi_j\}_{j \neq i}$ and offer distributions $\{F_j\}_{j \neq i}$—as given when making their decisions.[5]

We henceforth refer to the term under the integral as the *user cost of labor*. It consists of the wage, along with turnover costs, which are wage-specific because higher wages come with a lower quit rate.

A firm's optimal policy needs to satisfy two conditions. First, it must be that the user cost of labor is equated across all wages offered by a firm. If a firm offers a mix of wages, this implies that the additional costs of a higher wage must be exactly offset by the associated reduction in turnover cost. If this condition did not hold the firm could increase profits by shifting jobs to the wage with the lowest user cost while keeping employment n_i constant.[6] The second condition is standard, the marginal revenue product must equal

[5]This considers a relaxed problem that does not impose that the firms choices $\{n_i, G_i\}$ need to be implementable via some $\{\psi_i, F_i\}$. This is redundant given our equilibrium definition below.

[6]This condition is stated for the *relaxed* firm problem that does not impose implementability and so one might wonder whether the corresponding deviation—moving mass to the wage with a lower user cost—is always feasible in the primal formulation. Using the the proof of Lemma 1, we formally establish that this is the case in Online Appendix C.

the user cost of labor. Formally, this is summarized by the following two conditions,

$$\int_{w_r}^{\infty}\left(w - cs\sum_{j\neq i}\psi_j F_j\left(w^-\right) - \min_{\tilde{w}\geq w_r}\left\{\tilde{w} - cs\sum_{j\neq i}\psi_j F_j\left(\tilde{w}^-\right)\right\}\right)dG_i(w) = 0, \tag{6a}$$

$$\alpha x n_i^{\alpha-1} = \min_{w\geq w_r}\left\{w - c\sum_{j\neq i}s\psi_j F_j\left(w^-\right)\right\} + c\left(\delta + \sum_{j\neq i}s\psi_j\right). \tag{6b}$$

We note that the first line omits the constant from the user cost for brevity.

Equilibrium

We next define an equilibrium. Given the objects in the definition, any additional objects such as mean wages, flow profits, worker values are straightforward to compute.

DEFINITION 1. An equilibrium consists of employment, contact rates, cumulative wage distributions and cumulative wage offer distributions $\{n_j, \psi_j, G_j, F_j\}_{j=1}^{M}$, along with an acceptance policy $\mathcal{A}$ such that:

(i) Worker optimality: workers' job acceptance policy $\mathcal{A}$ satisfies (3);

(ii) Firm optimality: employment and wages paid $\{n_i, G_i\}$ satisfy (6) for all firms i.

(iii) Consistency: $\{\psi_i, F_i\}$ and $\{n_i, G_i\}$ satisfy (4) for all firms i.

We note that this definition imposes that F_i and G_i are cumulative distribution functions. We also note that condition (iii) guarantees implementability which is why we have relaxed the firm problem above.

2.3 Equilibrium Characterization

This section offers a complete closed-form characterization of the equilibrium. It then discusses a few steps in the proof of the characterization since those convey some key economics of the environment.

PROPOSITION 1 (Equilibrium Characterization).

i) There exists a unique equilibrium. The equilibrium is symmetric and firm policies are the same across firms $\{\psi_i, F_i\} = \{\psi, F\}$ for all $i \in \{1, 2, \ldots, M\}$.

ii) Given the equilibrium contact rate ψ, the worker acceptance policy is given by (3), with the reservation wage simplifying to

$$w_r = b + c\frac{1-s}{s}\frac{M-1}{M}\left(sM\psi - (r+\delta)\log\left(\frac{r+\delta+sM\psi}{r+\delta}\right)\right), \tag{7}$$

the wage offer distribution is uniform,

$$F(w) = \frac{w - w_r}{(M-1)\, s\psi c}, \tag{8}$$

the distribution of wages paid is

$$G(w) = \frac{F(w)}{1 + \frac{sM\psi}{\delta}(1 - F(w))}, \tag{9}$$

and firm level employment is

$$n = \frac{\psi}{M\psi + \delta}. \tag{10}$$

iii) The common offer rate ψ solves

$$\alpha x \left(\frac{\psi}{M\psi + \delta}\right)^{\alpha - 1} = b + c\left(\delta + (M-1)\psi - \frac{M-1}{M}\frac{1-s}{s}(r+\delta)\log\left(\frac{r+\delta+sM\psi}{r+\delta}\right)\right). \tag{11}$$

Proof. See Appendix A.2. □

The proposition fully characterizes the equilibrium in terms of five equations in five unknowns $\{w_r, F, G, n, \psi\}$. We note that the characterization has a block like structure with all equilibrium objects computable given a contact rate ψ which in turn is determined by (11). To underscore this useful feature we broke the mathematical characterization into the two separate blocks.

With these in hand we can compute additional equilibrium objects that might be of interest. Appendix D provides closed forms for the highest wage, mean wages, and the worker values of unemployment and employment at each wage.

Discussion

To obtain the characterization in Proposition 1, we rely on a few simple results which are proved formally in Appendix A.2. We briefly discuss some of these here informally to illustrate key features of the equilibrium.

Key Equilibrium Properties The proof sequentially establishes four key properties of equilibrium firm behavior. First, the user cost of labor must be equalized across any two (hence, all) firms. If not, the firm with the higher user cost could offer just above the competitor's highest wage, thereby reducing its user cost. Second, if a firm posts offers at or below some wage w, it must post at least as many offers above w as any competitor; otherwise the competitor would enjoy strictly lower turnover, and hence user cost, at that w. Together, these arguments imply that firms must be symmetric: the lowest wage offered is the same across firms, which forces identical contact rates and, by extension, identical wage offer distributions. Finally, symmetry also rules out mass points (a firm could reduce its user cost by paying marginally above) or gaps in the offer distribution (a firm could lower wages without increasing turnover).

Bounded Labor Demand and Equilibrium Uniqueness Given symmetry, when labor demand is sufficiently low ($\psi \to 0$), decreasing returns imply that the marginal product of labor exceeds its user cost which is just $b + c\delta$ in this limit. Similarly, when labor demand becomes sufficiently large (and the economy approaches full employment, i.e., $\psi \to \infty$), the marginal product approaches $\alpha x(1/M)^{\alpha-1}$ while turnover rises without bound. This yields the existence of an equilibrium with strictly positive unemployment. Uniqueness follows because the user cost is monotonically increasing in the offer rate ψ while the marginal product is declining in ψ (firm employment is $\frac{\psi}{\delta+M\psi}$). (11) gives the condition for marginal product to equal the user cost.

Derivations and Intuition Employment per firm (10) follows from a simple flow balance: inflow $(1 - Mn)M\psi$ equals outflow δMn. The same logic links $F(w)$ and $G(w)$ in (9): the inflow $(1 - Mn)M\psi F(w)$ equals the outflow $(\delta + sM\psi(1 - F(w)))\, MnG(w)$.

A uniform wage offer distribution ensures that higher pay is exactly offset by the associated reduction in turnover costs (in (6a)). This reflects that in our setting firms care only about the retention rate. By contrast, firms in the BM model additionally care about the pace of hiring, resulting in a convex density of wage offers.

Finally, the optimal hiring condition (6b) implies $\alpha x n^{\alpha-1} - w = c(\delta + s\psi(M-1)(1-F(w)))$, so the endogenous wage markdown must cover the annuitized hiring cost. Faster worker churn thus requires larger markdowns in equilibrium, the opposite of the neoclassical, frictionless prediction.

2.4 Efficiency

This section asks whether the economy is constrained efficient in the following sense. We consider a planner that maximizes steady state outcomes using symmetric, time-invariant firm policies. In particular, the constrained planner chooses common contact rates and wage offers $\{\psi, F\}$ to maximize welfare subject to workers' job acceptance decisions.

To begin with, note that we can rule out mass in the constrained efficient distribution of pay. Any mass point would increase turnover, since workers move when indifferent. The planner can strictly reduce turnover by spreading such mass over a small interval. Hence, the constrained-efficient distribution is atomless.

Utilitarian flow welfare corresponds to total output, home production of the unemployed, net of the cost of worker turnover

$$rV = Mxn^{\alpha} + b\,(1-Mn) - Mn\int_{w_r}^{\infty} c\left(\delta + (M-1)s\psi\left(1-F\left(w^{-}\right)\right)\right)dG(w). \tag{12}$$

Using symmetry, the optimally atomless distribution, and the mapping between G and F in (9) implies that the average job-to-job rate can be written as

$$\int_{w_r}^{\infty}(M-1)s\psi\left(1-F\left(w^{-}\right)\right)dG(w) = \frac{M-1}{M}\delta\left(\frac{\delta+sM\psi}{sM\psi}\log\left(\frac{\delta+sM\psi}{\delta}\right)-1\right). \tag{13}$$

Substituting for (13), (12) simplifies to

$$rV = Mxn^{\alpha} + b(1-Mn) - c\delta n - cn(M-1)\delta\frac{\delta+sM\psi}{sM\psi}\log\left(\frac{\delta+sM\psi}{\delta}\right). \tag{14}$$

Welfare therefore depends only on ψ and not F. This reflects that the planner is indifferent across any atomless distribution of wages since the shape of the wage offer distribution does not affect the amount of job-to-job transitions. We can thus remove F from the planning problem.

Similar to the decentralized setting, we can let the planner directly choose employment n (rather than ψ). The first-order condition with respect to n, using $\frac{\partial\psi}{\partial n} = \frac{\psi^2}{\delta n^2}$, is, after substituting back again for ψ,

$$\begin{aligned}\frac{\partial rV}{\partial n} &= M\left[\alpha x\left(\frac{\psi}{\delta+M\psi}\right)^{\alpha-1} - b - c\left(\delta+(M-1)\psi - \frac{M-1}{M}\frac{1-s}{s}\delta\log\left(\frac{\delta+sM\psi}{\delta}\right)\right)\right] \\ &= 0. \end{aligned} \tag{15}$$

This is the same condition as (11) when $r \to 0$. Since the second derivative is negative, this constitutes a global maximum which implies that the equilibrium is constrained efficient.[7] Intuitively, aggregate employment is efficient due to the linear hiring technology that does not feature any congestion jointly with the take-it-or-leave-it nature of wage posting and the absence of markups in the product market.[8]

2.5 Continuum of firms and relation to BM

This section shows that the framework naturally nests the standard case with small firms by studying the limit where $M \to \infty$. It then shows how to additionally alter the hiring technology to recover the equilibrium wage offer distribution from the BM model.

Denote the total offer arrival rate by $\lambda \equiv M\psi$. We take the limit holding $Mx^{\frac{1}{1-\alpha}} = \bar{x}^{\frac{1}{1-\alpha}}$ fixed to ensure that aggregate productivity is unchanged. As $M \to \infty$, the equilibrium characterization in Proposition 1 simplifies to the following system of three equations in three unknowns (λ, F, w_r)

$$w_r = b + \frac{1-s}{s}c\left(s\lambda - \delta\log\left(\frac{\delta+s\lambda}{\delta}\right)\right), \tag{16a}$$

$$\alpha\bar{x}\left(\frac{\lambda}{\lambda+\delta}\right)^{\alpha-1} = w_r + (\delta+s\lambda)c, \tag{16b}$$

$$F(w) = \frac{w-w_r}{s\lambda c}. \tag{16c}$$

[7]See Gautier et al. (2010) and Cai (2020) for studies of efficiency of the canonical BM model.

[8]This section focuses on symmetric policies since this is a feature of the equilibrium. We also point out that the planner can reduce aggregate turnover (and thereby increase welfare) with asymmetric policies, e.g. by having each firm post a unique wage. Such an allocation would however not be an equilibrium. A planner could of course also trivially do better if they can restrict firms to only hire from unemployment.

Vacancy cost

Next, in this limit, suppose firms hire by posting vacancies at flow cost c_v. Workers that are paid w need to be replaced at rate $\delta + s\lambda(1 - F(w^-))$. The vacancy filling rate at wage w is the vacancy contact rate q times the conditional probability that the contact results in a match. Writing the latter as the fraction of wage w offers that are accepted and simplifying gives

$$\frac{u + s(1-u)G(w)}{u + s(1-u)} = \frac{u + s(1-u)\frac{F(w)}{1+s\lambda/\delta(1-F(w))}}{u + s(1-u)} = \frac{\delta}{\delta + s\lambda(1 - F(w))},$$

where u denotes the unemployment rate. The cost incurred per hire at wage w is c_v divided by the vacancy filling rate at w. Taken together (and using that there are no mass points), the equilibrium user cost of labor at wage w is $w + (\delta + s\lambda(1 - F(w)))^2 \frac{c_v}{q\delta}$.

In any equilibrium, the user cost of labor has to be the same across all wages offered and equal to the marginal product of labor, here denoted m. As before, the reservation wage must be the lowest wage. This implies that

$$w + (\delta + s\lambda(1 - F(w)))^2 \frac{c_v}{q\delta} = m, \tag{17}$$

$$w_r + (\delta + s\lambda)^2 \frac{c_v}{q\delta} = m. \tag{18}$$

Combine and solve for the wage offer distribution to get the standard BM result,

$$F(w) = \frac{\delta + s\lambda}{s\lambda}\left(1 - \sqrt{\frac{m - w}{m - w_r}}\right). \tag{19}$$

The corresponding set of equations to (17) and (18) in our model with $M \to \infty$ are

$$w + (\delta + s\lambda(1 - F(w)))\, c = m, \tag{20}$$

$$w_r + (\delta + s\lambda)\, c = m. \tag{21}$$

The sole difference is whether $F(w)$ enters quadratic or linearly, hence the square root in BM and the uniform shape in our setting. The economics is simply that higher pay leads to both lower turnover and cheaper hiring in BM while in our setting it only affects turnover.

2.6 Discussion of Assumptions

We discuss several key assumptions in turn.

Linear hiring technology: The assumption that firms pay a fixed cost c per hire and can locate workers for free reflects the view that hiring and training, rather than locating workers, is the primary cost associated with worker turnover (Manning, 2011; Coles and Mortensen, 2016). In addition, the fact that firms frequently manage to expand employment very rapidly (Davis and Haltiwanger, 1992) favors the linear perspective on the costs of hiring taken here. The setup disconnects wages from size in the sense that a higher wage comes with lower turnover but size can be achieved independently just via hiring. This implies that size is fundamentally governed by needs of production (and product demand) rather than the labor supply curve as in models in the neoclassical tradition. We note that, as already discussed above, equilibrium labor demand remains strictly below one even with linear hiring, since turnover (and thus the user cost) grows without bound as the market approaches full employment.

The previous subsection showed how to work with the canonical vacancy-cost approach. Alternatively, a nonlinear hiring cost could also be incorporated without losing tractability. In the baseline (no-discounting) case, the only change in Proposition 1 is that the (constant marginal) hiring cost c is replaced with the marginal hiring cost.[9]

Nash Equilibrium: We consider a Nash equilibrium where firms, when choosing their actions, take their competitors' and workers' choices as given. Similarly, workers take firms' choices as given when choosing their job acceptance policy.

Perhaps the least palatable aspect of this is that large firms treat the reservation wage w_r as given. If workers instead chose their acceptance policies after observing firms' choices, large firms would recognize that their choices affect the reservation wage. Under such a scenario, firms have an incentive to make fewer and lower paying job offers. Our approach is consistent with a setting where workers do not observe the full set of firms' actions and interpret any lack of high wage offers as coming from chance. Arguably, most workers

[9] In the model with discounting (Section 4.2), either alternative would lead to the additional complexity of a slow transition. By contrast, the linear hiring technology allows an instantaneous transition to steady state, which is why we stick to it throughout. Tractability could otherwise be preserved by assuming the economy starts in steady state.

only observe their own offer history which only entails a few data points over the course of a career. More generally, one can motivate our approach assuming that workers interpret out-of-equilibrium observations as the result of noise (measurement error, trembling hand, etc.) rather than from an actual deviation. Importantly, all these considerations disappear with a binding minimum wage or when $s = 1$ (as assumed in, e.g., Shimer (2006) and Coles and Mortensen (2016)) since then the lowest wage is fixed and the acceptance policy within a job (3a) remains optimal irrespective of firms' choices.

Of course, an alternative approach could consider the sequential dynamic interactions across large firms and between large firms and workers in the labor markets. This is an interesting avenue to explore but beyond the scope of this paper.

Internal Raises: We assume firms do not incur hiring costs when a worker receives a higher wage offer from within the firm. This reflects the idea that the primary cost associated with hiring is to train a worker. Of course, since firms are indifferent across any wage they post in equilibrium, they are thus also indifferent about granting an internal raise while workers strictly benefit.

Always move when indifferent: What matters economically is that workers move with strictly positive probability when indifferent, ruling out equilibria where all firms pay the same wage and no worker mobility occurs. Unlike standard BM models where firms care about both hiring pace and retention, our setting focuses solely on retention, making such degenerate equilibria possible if indifferent workers never moved. Assuming that workers always move when indifferent is notationally convenient without changing the economics.[10] Our assumption is common in the literature (e.g., Shimer (2006) and Coles and Mortensen (2016)).

3 Noncompete Agreements

We now use the model to theoretically analyze the general equilibrium impact of noncompetes between workers and firms. We revisit these in Section 5.2 for a quantitative assessment.

[10] One caveat is that above we show that a planner never wants mass in the job offer distribution in order to minimize turnover. This would change when the probability of a move is below one.

We assume that (the first) K out of the M employers in a labor market have access to a legal technology that allows them to implement and enforce noncompetes. We model these as part of the take-it-or-leave-it offer that is posted by the firm. Instead of just posting a wage, the firm posts a contract that stipulates a wage and a covenant that prohibits the worker from transitioning directly to another firm.[11] From the worker's perspective, signing a noncompete thus eliminates the option to search for preferred jobs. Let variables associated with noncompetes be indicated by an *nc* superscript (subscript were notationally convenient). We then have

$$rW^{nc}(w) \;=\; w + \delta\left(U - W^{nc}(w)\right). \tag{22}$$

Consider the lowest acceptable wage w_r^{nc} under a noncompete, such that $W^{nc}(w_r^{nc}) = W(w_r) = U$. We will see that $w_r^{nc} > w_r$ because it includes a compensating differential for the foregone option value of on-the-job search. At the same time $w_r^{nc} < w_u$ since it offers only the reservation value. Specifically, w_r^{nc} represents the average discounted wage that a worker receives over an employment spell started at w_r. This is higher than w_r due to on-the-job search and the difference is larger when job offers arrive often (s high) and when the employment spells are long (δ low).

From the firm's perspective, workers under a noncompete are shielded from outside competition. Consequently, firms with access to noncompetes offer only w_r^{nc}. The user cost of labor is therefore lower for a firm with noncompetes since it allows the firm to pay a lower wage yet have the same turnover as a firm that posts the highest wage w_u. The difference in the user cost is given by the gap $w_u - w_r^{nc} > 0$. Any firm that can do so will therefore adopt noncompete contracts since these allow the firm to avoid the costs associated with turnover.[12] Firms with access to noncompetes optimally select a contact rate such that the marginal product under the implied employment equals the user cost

[11]It is straightforward to limit the scope of these agreements to restrict only transitions to a subset of firms. In fact, if all firms have access to noncompetes that block direct movement of workers to K other firms, the offer distribution takes the same form as reported below. Alternatively, one could model limited enforcement by letting workers under noncompetes accept outside job offers with some fixed probability.

[12]This raises the question why not all firms adopt noncompete agreements. Arguably, the required legal resources to set up and enforce such agreements are costly and so not all firms opt in. The choice of contract could straightforwardly be modeled as an upfront investment decision. We also note that, when the hiring costs are heterogeneous, then low-cost firms might not want to adopt noncompetes since the compensating wage differential might outweigh the lower turnover cost.

with noncompetes $w_r^{nc} + c\delta$.

With this in hand, it is straightforward to prove existence and uniqueness of an equilibrium and to derive closed form expressions for all equilibrium objects following the proof of Proposition 1.[13] Online Appendix E offers a complete characterization of the equilibrium with noncompetes, here we only report two results. First, the two reservation wages are related as follows,

$$w_r = (1-s)w_r^{nc} + sb. \tag{23}$$

This shows that firms with noncompetes pay a premium above the lowest wage. Second, when $K < M-1$, the wage offer distribution by firms without noncompetes is

$$F(w) = \frac{w - w_r}{(M-K-1)\, s\psi c}. \tag{24}$$

In turn, it is degenerate at w_r when $K \geq M-1$.

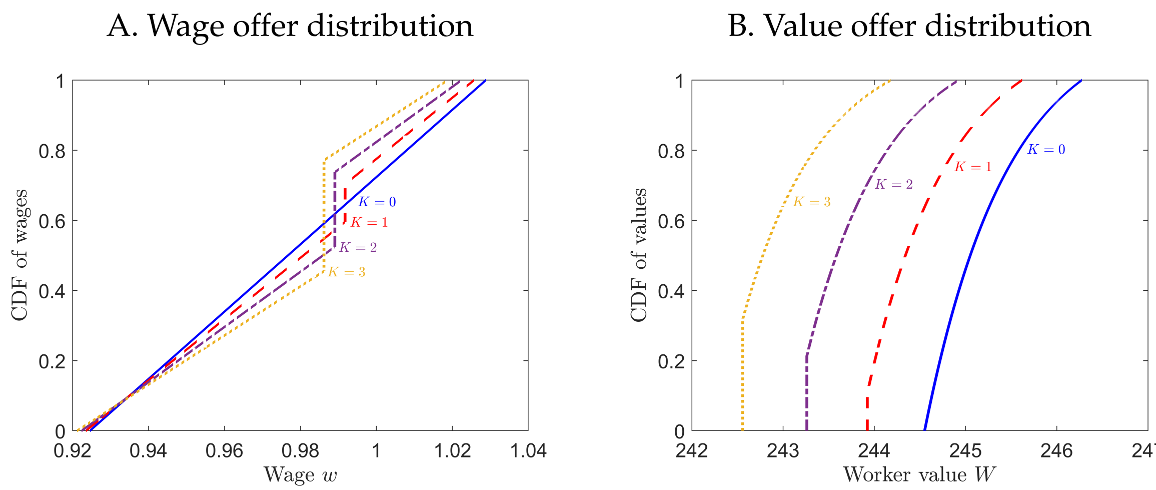
Importantly, this latter case shows that noncompetes can fully unravel competition in this labor market. To see why, it is useful to revisit the so-called Diamond Paradox. Diamond (1971) argued that, in an equilibrium model of wage posting, no firm should post any wage above the reservation wage, which will hence equal the flow value of unemployment. BM overcomes this by incorporating competition for workers along the job ladder. What undoes the Diamond equilibrium is a deviation argument that encapsulates job ladder competition. A firm offering pay marginally higher has discretely lower user cost. This ultimately undoes all mass in the wage offer distribution and shifts it outward, so competition among employers leads to gains for workers. Noncompetes can unravel this because, when at least all but one employer have noncompetes, everyone pays $w_r^{nc} = w_r = b$ and the classical BM deviation yields no gains since workers under noncompetes cannot be poached.[14]

For the case with $K < M-1$, comparing (24) with the equilibrium wage offer distribution in the baseline shows that only $M-K$ firms effectively compete for workers along the

[13]This requires that $K < M-1$ or that $x(1/M)^{\alpha-1} < b + c\delta$ to ensure aggregate labor demand is bounded below unity. This is required since the firms with noncompetes face no turnover cost. A similar condition arises in the heterogeneous firm setting discussed below.

[14]Strictly speaking, in the equilibrium when $K = M-1$, the sole firm without a noncompete offers pay marginally above w_r to avoid losing indifferent workers to competitors. This is mathematically awkward to handle since (w_r, ∞) has no minimum element. To formally address this, a tie-breaking assumption that workers without noncompetes do not move into jobs with noncompetes when indifferent suffices.

Figure 1: Impact of Noncompetes



Notes: The left figure plots the wage offer distribution as K rises. The right figure plots the corresponding distribution of posted values. For each wage w, we plot the CDF value $F(w)$ against the corresponding worker value $W(w)$. The parameters correspond to the baseline calibration Table 1 where $M = 10$ employers.

job ladder while those employers with noncompetes start piling up jobs at the very bottom of the job ladder. To illustrate this, we plot the equilibrium distribution of posted wages and values in Figure 1. The left panel shows that firms with noncompetes post a mass of wages at w_r^{nc}. While $w_r^{nc} > w_r$, these jobs all offer the value of unemployment and are the least desirable jobs in the labor market, as can be seen from the right panel. As K rises, the wage and value distributions shift to the left and conditions for workers deteriorate. The reason is the associated decline in competition for workers and the drop in the reservation wage.

We emphasize, however, that these are partial equilibrium considerations that do not account for a countervailing general equilibrium force. Noncompetes reduce socially wasteful turnover and hence increase desired employment, driving up wages. The overall effect of noncompetes on workers is thus ambiguous, which is why we revisit the question in the quantitative section.

Misallocation and Efficiency Noncompetes introduce misallocation of workers to firms in the sense that the marginal product of labor differs across firms with and without noncompetes. To see why, consider the user cost of labor at employers without noncompetes. Since they are indifferent across the wages they post it is given by $w_u + c\delta$. For those employers with noncompetes, it is given by $w_r^{nc} + c\delta$ which is smaller. Firms with noncompetes are thus larger than those without, despite operating the same decreasing-returns technology.

To understand the aggregate efficiency properties of noncompetes, Online Appendix E considers a planning problem akin to that studied in Section 2.4. The planner chooses contact rates and wage offer distributions for the firms with and without noncompetes, otherwise taking the forces shaping the equilibrium, including the use of noncompetes by the K employers as given. We show that the equilibrium with noncompetes is constrained efficient in this sense. The level of employment and the size differential is optimal given the differences in the (social) user cost of labor brought about by the use of noncompetes.

Online Appendix E further shows that, as M grows large, treating K as a continuous variable, the first order impact of an increase in K on welfare is given by difference in profits for the firms with noncompetes compared with those without,

$$\frac{\partial rV}{\partial K} = (1-\alpha)x\left(n_{nc}^{\alpha} - n^{\alpha}\right). \tag{25}$$

This is strictly positive owing to the lower costly turnover in firms with noncompetes. The simple expression is due to an envelope logic that reflects the constrained efficiency of the equilibrium. We will revisit this and show that it works well quantitatively when studying the labor market impact of noncompetes.[15]

4 Model Extensions

This section shows how to allow for firm heterogeneity in productivity and hiring costs, making the framework amenable to empirical settings where heterogeneity is of first order importance. While we subsequently focus on the homogeneous firms case, the final

[15]Given the usefulness of (25), we point out that it can easily be quantified in terms of observables. The marginal product $\alpha x n^{\alpha-1}$ is equal to the user cost which is the same for all wages offered by a firm, which we can use to measure the welfare impact of noncompetes via

$$\frac{drV}{dK} = \frac{1-\alpha}{\alpha}\left(n_{nc}\left(\mathrm{E}(w^{nc}) + c\cdot\text{avg. separations}^{nc}\right) - n\left(\mathrm{E}(w) + c\cdot\text{avg. separations}\right)\right), \tag{26}$$

where avg. separations refers to the average rate at which workers leave the firm . These objects can straightforwardly be measured.

part shows how to allow for product market power, capital, and time discounting for our quantitative application.

4.1 Firm Heterogeneity

This section considers two extensions of the model. We show how to allow for firm heterogeneity to make the framework amenable to empirical settings where heterogeneity is key, such as merger analysis (e.g. in Berger et al. (2025)). We first introduce heterogeneity in productivity only and then turn to the case where firms differ in both productivity and hiring costs. We relegate details and formal derivations to Online Appendices F and G.

In terms of the exposition, the worker problem is altogether unaffected by this and both the value functions (1) and (2) and the optimality conditions (3) remain unchanged. The difference comes in the firm problem where the optimality conditions (6) are adjusted to account for the added firm heterogeneity by exchanging x for x_i and c with c_i. An equilibrium satisfies Definition 1 subject to this change.

Heterogeneous Productivity

There are M firms, which differ in terms of productivity x_i with $x_1 \geq x_2 \geq \ldots x_{M-1} \geq x_M$. Firms still choose a distribution of posted wages. In addition, we allow firms to choose a wage-specific contact rate.[16]

Online Appendix F establishes uniqueness of the equilibrium and a condition for existence. The equilibrium takes the following form. Partition the support of the wage distribution into M contiguous intervals, from the reservation wage up to the maximum wage. Firm M posts only on the highest interval, firm $M-1$ posts on both the highest and second-highest intervals, and so on. Firm 1 posts on all $M-1$ intervals and, in addition, posts a mass of jobs at the reservation wage, the last interval. Posted wages on any interval are uniformly distributed. All firms posting wages on a given interval pick the same, interval-specific contact rate. As a result, total employment on that interval is the same across all firms.[17]

[16] This is merely expositional and allows the construction and intuitive explanation of the equilibrium. The resulting firm choices can always be re-cast as a single contact rate with associated wage offer distribution.

[17] The existence condition, akin to the one in the setting with noncompetes above, ensures that aggregate labor demand does not exceed the unit mass of workers. This can occur when a single firm is far more productive than the rest. As long as productivity is sufficiently similar, turnover skyrockets as employment approaches

What explains those equilibrium features? First, the marginal revenue product is equated across firms. The reason, as before, is that the user cost must optimally be equated within firms and equal to the marginal revenue product and, since all firms can post the highest wage, the user cost must be equated across firms.

The equilibrium features guarantee that workers at the same wage receive more attractive offers at the same frequency, independently of their employer. This allows for the user cost to be equated within and across firms, yet highly productive firms to be larger. This rules out symmetric wage posting across firms, since smaller firms would face higher turnover from more external poaching. It also rules out cases where small firms primarily recruit at the bottom of the wage distribution.

The resulting size-wage gradient is negative, at odds with stylized empirical facts. To overcome this, we turn to an additional extension, heterogeneity in hiring costs.

Heterogeneity in Hiring Costs

In Online Appendix G, we consider the case in which the firms additionally differ in their hiring costs. Rank firms according to their hiring cost, with $c_1 \geq c_2 \geq \ldots c_{M-1} \geq c_M$ (allowing for arbitrary productivities x_i). We establish that any equilibrium takes the following form[18]: Again partition the wage distribution into M contiguous intervals. On the highest wage interval, firm 1 posts uniformly distributed wages. However, so does firm 2 and, possibly, firms 3, 4, ..., with the cutoff depending on their relative costs. The condition that determines which firms post in the highest interval is presented in Lemma 16 in the Online Appendix. These firms no longer pick identical contact rates, however. Instead, the firms with higher costs offer more jobs.

On the second interval, one firm drops out. The cutoff is determined by the desired size of that firm. All but one of the firms posting on the first interval also post on the second. In addition, one or more firms that do not post on the top interval might be added, in order of their hiring cost (again determined by Lemma 16). This continues until only one firm remains. That firm posts a mass-point at the reservation wage which is at the lower end of the $M-1$th interval.

What gives rise to these features? Uniformity within intervals is still required to equal-

unity which ensures aggregate labor demand never exceeds it.

[18] We have not formally established existence or uniqueness for this case.

ize user costs. Firms that post on the same interval now have different contact rates because they trade off outside competition differently. Turnover must fall by more for firms with lower c to warrant an additional dollar of pay. This creates a sorting pattern. Among firms posting on the same interval those with higher hiring cost thus need to account for more offers. In addition, the relative cost also determines the set of firms that post on any given interval. An instructive example considers two very high-c firms posting on a given interval. For them to be indifferent across uniformly distributed wages, outside competition must be very low. However, this makes it impossible for a firm that cares little about turnover to also be indifferent across the same wages. As a consequence, that firm locates only on lower-paying intervals.

How, then, can firms with high productivity achieve large scale given these restrictions? They do so by posting wages further down on the job ladder, just as before. This is the force that was already present when firms have heterogeneous productivity. Here, however, what keeps low-c firms out of the high-wage intervals is the indifference requirement within the interval.

In sum, the hiring cost governs the vertical position of an employer on the job ladder. Productivity, in turn, governs the range of wages it posts in the same way it did before when firms differed only in terms of productivity.

Intuitively, if hiring is more expensive at highly productive firms, then the theory can generate a size wage premium. Firms with the highest hiring cost are most concerned with worker turnover and hence tend to locate at the top of the wage ladder. This case seems empirically relevant since a more advanced technology might require more upfront worker training.

Lemma 17 in Online Appendix G shows how to use this to characterize the equilibrium recursively, using only the highest wage w_u and the unemployment rate. This allows for a simple algorithm where one needs to iterate only over these two scalars (rather than $\{\psi_j, F_j\}_{j \in \mathrm{M}}$).

4.2 Discounting, Capital, and Product Demand

We now return to the homogeneous productivity case and offer three additional extensions that make the model amenable for our quantitative application. The three extensions i) generalize to inelastic demand in the product market, ii) add capital to the production function, and iii) introduce time discounting. We restrict attention to stationary equilibria.

Product Market Demand All employers produce an identical composite good with equilibrium price p. The (unit measure of) consumers of that good have quasi-linear utility,

$$v = \frac{\eta}{\eta - 1}\bar{Q}^{\frac{1}{\eta}} C^{\frac{\eta-1}{\eta}} + I - pC.$$

$\bar{Q}$ is a demand shifter, I denotes some flow income, and C denotes consumption of the composite good.[19] Assume that $\eta > \frac{1}{M}$. This results in an iso-elastic market-level inverse demand function,

$$p = \bar{Q}^{\frac{1}{\eta}} \left(\sum_{j=1}^{M} y_j \right)^{-\frac{1}{\eta}}. \tag{27}$$

This formulation distinguishes between consumers, who are affected by price changes, and workers, who are affected by wage changes. In practice, these groups often overlap substantially. In local markets for non-tradable goods, workers are also the consumers of the goods they produce. But even with tradable goods, when analyzing economy-wide policies that affect many different local labor markets that produce differentiated output, workers collectively consume the national tradable output mix and so in this sense are also the consumers that bear the cost of rising prices. Our quantitative application below considers a US-wide ban of noncompete agreements which is why we include the impact on consumers in our welfare calculations throughout. Additionally, we note that redistribution occurs across labor markets in such a setting with tradeables whenever the wage impact of a nationwide policy varies locally, a point we briefly revisit below.

Capital We extend the production function to include capital as a second input factor. This allows to speak to a wider range of empirical settings. When studying a ban of noncompetes below, a key consideration is that the rise in turnover costs leads firms to hire less which can undo much of the direct pro-competitive effects of a ban. This employment pull-back is naturally muted when there is other, imperfectly substitutable factors of production.

[19] We note that quasi-linear preferences shut down any income effects on consumption.

Output produced by firm i is denoted y_i, given by

$$y_i = x\left(\theta^{\frac{1}{\sigma}} n_i^{\frac{\sigma-1}{\sigma}} + (1-\theta)^{\frac{1}{\sigma}} k_i^{\frac{\sigma-1}{\sigma}}\right)^{\alpha\frac{\sigma}{\sigma-1}}, \tag{28}$$

where $\sigma > 0$ denotes the elasticity of substitution. Capital k can be freely adjusted. Denote by r_k the rental price of capital which we will also take to be the social cost when calculating welfare.

Firms output now depends on their choices in the labor market and in the capital market. Instead of treating capital as a choice variable, we instead assume that firms directly choose a timepath for output y_i, with capital adjusting as the residual input. As before, in doing so firms take each others choices as given, engaging in standard Cournot competition.[20] Rearranging (28) gives the level of capital at the firm as a function of output and employment

$$k^*(y,n) \equiv \left(\frac{\left(\frac{y}{x}\right)^{\frac{\sigma-1}{\alpha\sigma}} - \theta^{\frac{1}{\sigma}} n^{\frac{\sigma-1}{\sigma}}}{(1-\theta)^{\frac{1}{\sigma}}}\right)^{\frac{\sigma}{\sigma-1}}. \tag{29}$$

Discounting The challenge with allowing for full time discounting, including on the firm side, is usually that the stock of labor is a slow moving state variable and so firms might optimally make time-varying choices, depending on the initial state. However, the firm problem in our setting retains the same form as under the baseline, under a slight generalization of the firms' choice set and under a restriction on the initial state. The reason is the linear hiring technology as we show next.

Assume first that initially all workers are unemployed. Firms choose time paths for the wage offer distribution and the contact rate (and, now, output). In addition, assume firms can hire a mass of workers (from unemployment) with a desired distribution of pay—subject to workers reservation strategies—instantaneously at time 0. This approach allows employment to be discontinuous in time, making an instantaneous transition to a stationary allocation feasible.

We restrict attention to stationary equilibria in which the path for output, offer rates

[20] Assuming firms choose output directly avoids strategic complications because firms would recognize that, by choosing a higher ψ_i, its competitors choices would yield a different amount of employment and hence output. In a setting with product market power this in turn would have price impact. Our formulation further results in the standard markup formulation. This formulation can still capture firms that only use labor in production if there is perfect substitutability and r_k sufficiently large relative to the user cost of labor.

and offer distribution are constant. We do not directly impose stationarity onto the firm's choices but first verify that stationary policies are optimal if all competitors make stationary choices.[21]

Consider an unrestricted problem where firms can directly pick time paths for employment $n_{i,t}$, the distribution of pay $G_{i,t}$, and output $y_{i,t}$. These time paths are not generically implementable via the firms actual underlying choice sets so this unrestricted problem effectively drops the "duality constraint". We then show that, assuming all other firms make time-invariant choices, it is optimal for a firm to also make time-invariant choices. Crucially, these are implementable via the firms actual (now extended) choice set and so the duality constraint is satisfied (with slack).

To establish this, consider the unrestricted problem and also allow, for mathematical convenience, for firms to gain c whenever a worker is fired. This relaxed problem yields a weakly higher maximum value than the original problem. Defining $\mathbf{Y}_{-i,t} \equiv \{y_{j,t}\}_{j\neq i}$, the relaxed problem can be written as

$$\max_{\{n_{i,t},G_{i,t},y_{i,t}\}_{t\geq 0}} \quad \int_0^\infty e^{-rt}\Big[p\left(y_{i,t},\mathbf{Y}_{-i,t}\right)y_{i,t} - r_k k^*(y_{i,t},n_{i,t}) - n_{i,t}\int_{w_r}^\infty w dG_{i,t}(w)$$
$$-cn_{i,t}\int_{w_r}^\infty\Big(\delta + s\sum_{j\neq i}\psi_{j,t}(1-F_{j,t}(w^-))\Big)dG_{i,t}(w)\Big]dt - c\int_0^\infty e^{-rt}dn_{i,t}.$$

Flow profits are given by gross revenue net of the user cost of capital and labor. The second line breaks the hiring cost into two pieces. First, the cost to maintain the current workforce $n_{i,t}$ due to turnover to unemployment and the competition. Second, the cost for any net adjustment of the workforce. That last term also covers the cost of any initial hires at time 0. We emphasize that the output price is endogenous and given by (27) but that firms take their competitors' output choices $\mathbf{Y}_{-i,t}$ as given.

Using integration by parts, along with the assumption that all workers are initially unemployed ($n_{i,0^-} = 0$), on the last term gives $c\int_0^\infty e^{-rt}dn_{i,t} = rc\int_0^\infty e^{-rt}n_{i,t}dt$. The relaxed

[21] We conjecture that there are additional nonstationary equilibria, for example one where all firms post a unique, identical wage (fixed within a match) which declines continuously over time, e.g. $b + e^{-t}\epsilon$, such that there is no turnover.

problem becomes

$$\max_{\{n_{i,t},G_{i,t},y_{i,t}\}_{t\geq 0}} \int_0^\infty e^{-rt}\Big[p\left(y_{i,t},\mathbf{Y}_{-i,t}\right)y_{i,t}-r_k k^*(y_{i,t},n_{i,t})$$
$$-n_{i,t}\int_{w_r}^\infty \left(w+\left(r+\delta+s\textstyle\sum_{j\neq i}\psi_{j,t}(1-F_{j,t}(w^-))\right)c\right)dG_{i,t}(w)\Big]dt.$$

Since the objective function depends only on contemporaneous values $\{n_{i,t},G_{i,t},y_{i,t}\}$ at each point in time, a stationary policy $\{n_i,G_i,y_i\}$ is optimal given stationary policies of other firms $\{\psi_j,F_j,y_j\}_{j\neq i}$. Moreover, the optimal policy in that case involves no firing, so it remains feasible in the original problem and is optimal there as well.

We can thus reformulate the firm's problem as static optimization over constant employment, wage distribution, and output levels. The worker problem remains completely unchanged from the baseline model.

Extended Firm Problem and Characterization

$$\begin{aligned}\max_{\{n_i,G_i,y_i\}} \quad & \left\{p\left(y_i,\mathbf{Y}_{-i}\right)y_i-r_k k^*(y_i,n_i)-n_i\int_{w_r}^\infty\left(w+c\left(r+\delta+s\textstyle\sum_{j\neq i}\psi_j(1-F_j(w^-))\right)\right)dG_i(w)\right\},\\ \text{s.t.} \quad & G_i(w_r^-)=0,\ \lim_{w\to\infty}G_i(w)=1,\ \text{and } G_i(w_1)\leq G_i(w_2)\text{ for all } w_1<w_2,. \end{aligned} \tag{30}$$

In sum, firms make, at time 0, simultaneous, stationary choices in both the product and labor market. In this dual formulation, its choice variables in the labor market are employment n_i and a wage distribution G_i, even though firms implement these via a contact rate ψ_i and a wage offer distribution F_i as before, as well as an initial mass of hires with wages distributed according to G_i.

As before, firms choose their actions taking their competitors' choices $\{\psi_j,F_j,y_j\}_{j\neq i}$ and workers' acceptance rules $\mathcal{A}$ as given. This formulation ensures time consistency by accounting for the cost of the initial hires in annuitized form (hence the r in the user cost of labor).[22]

This problem is almost symmetric to the baseline firm problem and we can follow the same steps to establish existence, uniqueness, and characterization. The proof of Proposition 1 applies almost unchanged. In particular, the user cost of labor is again equated

[22] Not accounting for the cost of the transition (as in the timeless equilibrium above and in the BM literature), would lead to higher employment, akin to the "golden rule" savings case.

across and within firms, and so is, consequently, output and capital. The equilibrium is therefore symmetric.

Equation (11) which characterizes the equilibrium ψ takes a slightly generalized form which is derived in Online Appendix H. As before, all other equilibrium objects can be characterized given ψ and the equations are unchanged. In particular, w_r is still given by (7) and the offer and cross sectional distribution satisfy (8) and (9), respectively.

5 Quantitative Assessment of Noncompetes

We now put the framework to work to quantitatively assess the labor market impact of noncompete agreements. The FTC has recently proposed a blanket ban on noncompete agreements in the US. While this proposal is currently no longer under consideration, the potential consequences of such a move are still of interest, in particular in light of ongoing regulatory discussions in Europe and evolving state policies in the US.

We start with a basic "representative US labor market" calibration. We then focus on how local labor market features mediate the impact of noncompetes, as local conditions are likely to differ substantially across space. Throughout, we use the model with homogeneous firms, discounting, and a downward-sloping product demand curve. We introduce capital where explicitly mentioned.

5.1 Calibration

We calibrate at a monthly frequency. A set of parameters is set externally. The remaining parameters are calibrated in a moment-matching fashion.

Externally calibrated parameters The discount rate is set to $r = 0.004$ to match an annual discount rate of 5%. We set the curvature of the production function $\alpha = 0.64$, in line with the estimates of Cooper et al. (2007) and Cooper et al. (2015). The separation rate δ is set such that a monthly job finding rate of 25% (a targeted moment) implies an unemployment rate of 6%. We set the relative search efficiency s so as to obtain a monthly job-to-job transition rate of 2.5% (Fujita et al., 2024) in a competitive, thick labor market with $M \to \infty$ symmetric firms without noncompetes.[23] We calibrate η so as to capture

[23] That is, s is such that (13) is equal to 0.025 given $M\psi = 0.25$, $\delta/(\delta + M\psi) = 0.06$ and $M \to \infty$.

Table 1: Baseline Parameters

	Value	Reason / Moment	Model	Target
r	0.004	Annual discount rate	0.05	0.05
δ	0.016	Unemployment rate	0.06	0.06
s	0.581	E-to-E rate	0.025	0.025
x	0.705	U-to-E rate	0.25	0.25
c	1	Hiring costs / Monthly pay	1	1
b	0.875	Normalization	1	1
$\bar{Q}$	1.552	Normalization	1	1
α	0.64	Cooper et al. (2007, 2015)	—	—
η	1.3	Edmond et al. (2023)	—	—
M	10	HHI	1001	$\approx$ 1000
K	2	Noncompete share	0.214	$\approx$ 0.18

Notes: Monthly calibration. The rationale and source for each targeted moment are explained in the main text.

estimates for the (average) elasticity of demand at the sectoral level. We pick a baseline level of 1.3, which is in the middle of the range in Edmond et al. (2023), close to 1.2 used in De Loecker et al. (2021), and somewhat below the 1.8 used in Burstein et al. (2025). We consider a wide range of alternatives for η below. The (integer) number of overall employers M is set to 10 to get closest to an HHI of 1000 (in line with Berger et al. (2022) and Jarosch et al. (2024)) and the number of employers with noncompetes K is set to 2 to capture, subject to the integer constraint, that around 18% of US employees are under a noncompete as estimated by Starr et al. (2021).

Targeted moments. The target moments jointly determine the parameters, but we list them in a way that points to the most informative moment for each parameter.

We choose c to target the size of the hiring cost relative to the average wage. We use a value of one month's wages, in line with Manning (2011), but consider as alternative a much higher value of 6 months below.[24] We choose the demand shifter $\bar{Q}$ such that the (initial) price level is normalized to 1. We set the flow income in unemployment b such that the mean wage is normalized to 1. We pick the (common) level of productivity x such that the job-finding rate is 25%. This corresponds to the average monthly value reported in the Current Population Survey "gross flows" data.[25] The baseline strategy and the resulting

[24]Recall that c captures all costs associated with turnover, including training costs.
[25]https://www.bls.gov/cps/cps_flows.htm

calibration are summarized in Table 1.

5.2 The Impact of Banning Noncompetes in the US

To assess the impact of noncompetes, we ask what happens when they are banned. We assume that the labor market is in a steady-state equilibrium prior to the ban. We then compute a counterfactual steady-state equilibrium with $K = 0$ (all other parameters unchanged) and contrast it with the initial allocation. When considering alternative parameters or targets, we re-calibrate. We maintain the assumption that noncompetes rule out any job-to-job transitions. To the extent that they are imperfectly enforceable or of limited scope this would mute the results.

Baseline The first column in Table 2 reports our baseline results. Average wages rise by 0.9% when noncompetes are banned. The flipside of this is a 1.5% fall in employment. Employers pull back from hiring due to the rise in turnover costs given the sharp jump in worker churn (job-to-job transitions rise by almost a third). That wages rise despite this large negative general equilibrium employment effect showcases that noncompetes shift rents from workers to firms by reducing competition. We can also break down the wage response into a direct response at those employers that initially had noncompetes and a spillover response outside. The spillovers are substantial, with wages rising by 0.6% even at employers that are not directly affected by a ban.

We also report the change in welfare.[26] To do so, we compute the consumption-equivalent welfare change across the two allocations and then normalize that by total wage bill.

Aggregate welfare falls when noncompetes are banned. The direct reason is that there is now additional, socially inefficient worker turnover. As a consequence, output falls as

[26] Using that firms with noncompetes offer the lowest value, welfare in the full model is

$$rV = \frac{\eta}{\eta-1}\bar{Q}^{\frac{1}{\eta}}\left(\sum_j y_j\right)^{\frac{\eta-1}{\eta}} + b - \left(b + c\left(r+\delta\right)\right)\sum_j n_j - \sum_j k_j r_k - c\sum_{i>K} n_i \int_{w_r}^{\infty} \sum_{\substack{j>K \\ j\neq i}} s\psi_j\left(1 - F_j\left(w^-\right)\right) dG_i(w).$$

This is exactly analogous to (12) but allows for firms to be heterogeneous, accounts for the richer model of product demand, subtracts out the social cost of capital, and accounts for the fact that firms with noncompetes face no turnover to other employers. For simplicity, it also imposes the equilibrium feature, established in section 3, that no employed worker transitions to a firm with noncompetes.

employers pull back on demand due to a rise in the user cost of labor. Here, this exacerbates existing wedges due to product market power. In the theoretical section on noncompetes, we showed that the labor misallocation cost are of second order which explains why we find negative welfare consequences of a ban throughout. Interestingly, the welfare results suggest that the first order approximation in an efficient economy in (25) works quantitatively quite well since the markups are small. The welfare losses according to this are 0.64% (compared with actual losses of 0.78%).

We add two caveats. First, these welfare calculations include the utility loss of the consumers that are now subject to higher prices. In settings where the rise in costs are partially borne by other consumers, workers whose noncompetes are outlawed might well also benefit in utility terms. Second, the model assumes that training is fully firm-specific and that any training is lost when workers change jobs, making turnover very costly from a social perspective.[27]

Product Demand Elasticity We pay particular attention to the role of η for the consequences of a ban. This is because 1) we are uncertain about the right value and 2) it matters substantially for the quantitative results as it governs the general equilibrium employment effects caused by rising turnover costs. The second and third column in Table 2 thus entertain a low value of $\eta = 0.2$ and high value of $\eta = 5$ for the industry demand elasticity. These values are extreme with respect to the averages reported in, e.g. Edmond et al. (2023) (which range from 1 to 1.6). We imagine, however, that there are individual sectors such as energy, food, health care, or computing that have very inelastic demand while others might be closer to individual products in terms of their substitutability.[28]

When product market demand is elastic, then the wage gains from a ban fall substan-

[27] One could alternatively assume that there exist training firms that can retrain employed workers at cost $\kappa_e c$ where $\kappa_e < 1$, reducing the social cost of job-to-job transitions by a factor $1 - \kappa_e$. This would preserve all equilibrium conditions while reducing the welfare costs of turnover-increasing policies such as noncompete bans.

[28] Regarding the latter case, estimates for trade elasticities which are typically in the ballpark of 5 (Costinot and Rodríguez-Clare, 2014) can serve as a benchmark. Regarding the inelastic cases, Beaudry et al. (2018) estimate a city level wage elasticity of employment demand of 0.3. In our model (without noncompetes and capital), the market level demand elasticity with respect to the user cost of labor cost is $\frac{d\log(n)}{d\log(w_r + c(r+\delta+s(M-1)\psi))} = -\frac{1}{1-\alpha\frac{\eta-1}{\eta}}$; see (136) in the Online Appendix. Setting this equal to -0.3, approximating the wage by the user cost, and using our implied value of α gives $\eta = 0.22$. Ellis et al. (2017) report very low demand elasticities (e.g. 0.02 for prevention visits) in the health care sector where noncompetes are common.

Table 2: Banning Noncompetes à la FTC (values in percent)

	Baseline	$\eta = 0.2$	$\eta = 5$
Share non-comp.	21.3	20.5	21.4
$\Delta \log(\mathrm{E}[w])$	0.86	2.38	0.44
$\Delta \log(1-u)$	-1.51	-0.8	-1.73
$\Delta \log(\text{output})$	-0.96	-0.51	-1.1
Δ Welfare	-0.78	-1.5	-0.69
$\Delta \log(\text{jtj})$	27.4	31	26.2
$\Delta \log(w_{nc})$	1.96	3.49	1.53
$\Delta \log(w_{rest})$	0.57	2.09	0.14

Notes: Counterfactual results for $M = 10$ and $K = 2$, based on recalculating the equilibrium with $K = 0$. u and jtj denotes the unemployment rate and job-to-job rate, respectively. $\Delta \log(w_{nc})$ ($\Delta \log(w_{rest})$) denotes the wage change for the firms that initially used (did not use) noncompetes. ΔWelfare measured in consumption equivalent and normalized by initial wage bill.

tially.[29] The reason is that employers cannot pass much of the rise in cost into prices and hence strongly pull back on labor demand when costs rise. On the other hand, when the demand elasticity is low, then aggregate employment is largely unresponsive to the rise in cost associated with a noncompete ban. Consequently, there are far larger wage gains when banning noncompetes. This suggests an important role for product market demand for the quantitative wage impact of labor market policies such as noncompetes.

We note that the welfare approximation discussed above works very well when $\eta = 5$ (-0.67% versus -0.69%), but less so when $\eta = 0.2$ (-0.49% versus -1.5%), reflecting that banning noncompetes additionally exacerbates existing output distortions due to markups in the latter case.

Hiring Cost To investigate the role of the training cost we next increase its value from 1 to 6 months of average pay.[30] The results are reported in the first two columns of Table 3. Maybe surprisingly, this results in small wage reductions of about 0.1%. The reason is the wasteful nature of labor market churn which has now grown in size. The rise in

[29] Below we show that, even under our baseline demand elasticity of $\eta = 1.3$ noncompetes can lead to substantial wage losses in other settings.

[30] Blatter et al. (2012) report average hiring costs in the Swiss banking and insurance industry of 25,000 Swiss Francs already in 2000-2004.

Table 3: Banning Noncompetes as Local Conditions Vary (values in percent)

	high-c		concentration		high-K	model with capital			Tech	Fast Food
	$c = 6$ months		$K = 1$	$K = 20$	$K = 5$	$\sigma = .5$		$\sigma = .1$		
	$\eta = 1.3$	$\eta = 0.2$	$M = 5$	$M = 100$		θ high		θ low	see table notes	
Share non-comp.	27.2	22.8	21	21.7	51.4	20.9	20.4	20.2	36.4	14.5
$\Delta \log(\mathrm{E}[w])$	-0.1	4.47	0.95	0.8	2.9	1.04	1.54	2.66	3.25	0.84
$\Delta \log(1-u)$	-2.64	-1.71	-1.59	-1.46	-4.44	-1.4	-1.13	-0.66	-3.68	-0.55
$\Delta \log(\text{output})$	-1.36	-1.04	-1.01	-0.92	-2.83	-0.74	-0.15	-0.24	-2.14	-0.12
Δ Welfare	-4.38	-6.17	-0.92	-0.69	-1.69	-0.8	-0.85	-0.98	-6.66	-0.35
$\Delta \log(\text{jtj})$	31.8	28.9	29.9	25.9	93	27.5	28.4	31.4	48.4	20.4
$\Delta \log(w_{nc})$	5.98	11.08	1.9	2.03	3.41	2.15	2.66	3.78	8.16	1.48
$\Delta \log(w_{rest})$	-2.28	2.6	0.71	0.46	2.36	0.75	1.26	2.38	0.55	0.73

Notes: Parameters as in baseline with modifications listed in column header. θ high (θ low) calibrated such that cost share of labor is 2/3 (10%). For "Tech", $K = 3$, $c = 6$ months, $\eta = 0.5$. For "Fast Food", $K = 2$, $M = 14$, $\eta = \sigma = 0.5$, $c = 0.5$ months, and θ such that the cost share of labor income is 30%.

turnover resulting from a ban of noncompetes now results in larger cost increases and hence a larger employment pullback which more than offsets any direct wage gains due to a rise in competition.

To underscore this point, we revisit the low value of η considered above. In this case, larger hiring costs result in (twice) larger wage gains from a ban. The reason is that the low value of η shuts down most of the general equilibrium employment effects from the rise in cost, isolating the direct effect of a rise in competition. This direct effect is now larger because overall rents are larger due to the rise in frictions.

Market Concentration The next two columns in the table ask how the consequences of a ban vary with market concentration. To that end, we consider settings where $K = 1$ out of $M = 5$ or $K = 20$ out of $M = 100$ employers use noncompetes. The results show that the labor market impact of noncompetes gets amplified along all dimensions with, for instance, the wage gains from a ban about 20% larger in a concentrated setting. The reason is that the effective reduction in market-wide competition for workers from noncompetes is larger in a granular setting. With that said, the variation in impact is relatively small and even the baseline setting does not look too different from a market with effectively atomistic firms.

High Coverage We next ask how the impact of a ban changes when a larger share of workers is under a noncompete. We suspect that many labor markets have no noncompetes while they are ubiquitous in others, so we report results when the initial coverage is 50%. The results confirm the theoretical observations regarding the Diamond Paradox since the impact of a ban is larger, with wages rising by almost 3 percent, despite a sharp drop in employment due to a very large jump in turnover. The spillover effects are very strong, with similar wage gains in firms that had no noncompetes to begin with.

Capital We next consider the impact of noncompetes when labor and capital are imperfect substitutes as modeled in (28). As a baseline, we set the elasticity of substitution to $\sigma = 0.5$ (Chirinko, 2008; Oberfield and Raval, 2021) and θ such that mean labor income accounts for two thirds or firms' factor payments (not including the hiring costs). We then entertain cases where the labor (cost) share is small (0.1) and when additionally the factor substitutability is minimal ($\sigma = 0.1$).

The results show that the introduction of capital under the baseline calibration does little to the wage losses from noncompetes. However, when wages account for a small factor share and there is little room for substitutability the wage gains from banning noncompetes triple compared with baseline. The reason is a more muted labor demand response to a rise in the user cost of labor.

Two Prominent "Real World" Settings Finally, we consider two settings where the use of noncompete agreements has received widespread attention. The first is a "tech" setting. While California has long outlawed the use of noncompetes, they are widespread among Silicon Valley type occupations and industries, with 35% of US computer and math professionals and 36% of engineers under a noncompete according to Starr et al. (2021). The other is a low skilled service sector calibration that is motivated by the well-known case of Jimmy John's imposing noncompetes on its workers.[31]

We calibrate the tech industry to be concentrated with widespread noncompetes ($K = 3$, $M = 10$) to capture around 1/3 workers being bound by noncompetes, a relatively low

[31] https://www.forbes.com/sites/clareoconnor/2014/10/15/does-jimmy-johns-non-compete-clause-for-sandwich

demand elasticity ($\eta = 0.5$) and high training costs ($c = 6$ months of average pay) and use the baseline model without capital. In turn, we use the model with capital to capture the fast food industry, calibrating $K = 2$ and $M = 14$ in order to capture a lower share of noncompetes of around 14% in line with Starr et al. (2021) who report that 14.3% of US workers without a college degree are under a noncompete while 13.3% of workers with annual earnings $< \$40,000$ are covered by one. We set the cost share of labor to 0.3 (Aaronson and French, 2007) who study the restaurant employment response to the minimum wage. We set $\eta = 0.5$, as used by Aaronson and French (2007) and in the middle of the range of estimates for food demand elasticities reported in Andreyeva et al. (2010). We keep the elasticity of substitution between capital and labor at 0.5, consistent with Aaronson and French (2007). We choose a hiring cost of half the baseline value, corresponding to the lower end of Table 1 in Manning (2011).

The results are reported in the last two columns of Table 3 and are strikingly different. The tech calibration comes with a very large impact of a noncompete ban. Wages rise sharply and so does churn which in turn leads to a substantial drop in employment, output, and welfare. On the other hand, noncompetes have relatively little impact in our "fast food" calibration with an overall picture quite similar to that of our baseline US calibration.

The welfare calculations in the above tables implicitly equates tech workers with tech consumers. These groups are largely distinct, as with all sectors including fast food. In reality, of course, tech workers gain substantially from higher tech wages while bearing minimal costs from higher tech prices. This illustrates how a nationwide noncompete ban might redistribute: workers in high-wage-gain sectors benefit at the expense of workers in low-wage-gain sectors, since price increases from higher costs are borne equally by all workers who consume a similar mix of sectoral output.

Summary In summary, noncompetes decrease competition and usually lower wages, as the theoretical section on noncompetes suggested. The wage gains to workers heavily hinge on the product demand elasticity. The reason is that a ban raises turnover and hence the user cost of labor which in turn reduces labor demand. The strength of this negative general equilibrium labor demand effect is key for the overall wage impact as it works against the direct pro-competitive impact of a ban. This logic is also key in understanding how the results change when we introduce capital or increase the cost of hiring.

Banning noncompetes is hard to justify on pure efficiency grounds. The reason is that aggregate output and employment contract in all the settings we have considered, albeit often by a small amount. This reflects rising turnover costs, which are only partially offset by a reduction in misallocation. We again caution that our analysis omits several forces that might reduce or even reverse the output and welfare losses.

6 Conclusion

This paper integrates granular employers with decreasing returns into the canonical Burdett-Mortensen model of wage posting and on-the-job search. The resulting framework is analytically tractable and well-suited for analyzing competition in the labor market in general equilibrium, especially when labor mobility and market structure are central.

We apply the model to study noncompete agreements and find mild wage gains of around 0.9% from a nationwide ban in the US. A ban increases costly worker turnover, resulting in modest reductions in employment and welfare. The framework is applicable to other competition questions in labor markets, including merger analysis and collusive practices such as wage-fixing (Gottfries and Jarosch, 2025) or no-poach agreements (Krueger and Ashenfelter, 2022).

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APPENDIX

A Proofs and derivations

A.1 Derivation of Reservation Wage in (3c)

The value of a job to a worker depends only on the wage and it is strictly increasing in the wage. To see why, consider two jobs at employers i and j, with $(j, \tilde{w})$ and (i, w). The worker can always adopt the same acceptance strategy in both jobs which gives the worker value if the wage is the same, $\tilde{w} = w$, and a strictly higher value if the wage is higher, $\tilde{w} > w$. As a consequence, the worker accepts all jobs that pay weakly more (since by assumption workers accept jobs with equal value).

Differentiating the worker value in employment (2) with respect to the wage, we get

$$\left(r+\delta+\sum_j s\psi_j\left(1-F_j(w)\right)\right)W_w\left(w\right) \quad = \quad 1. \tag{31}$$

The reservation wage satisfies $W(w_r)=U$ which by rearranging (1) and (2) gives

$$\begin{aligned}
w_r-b &= (1-s)\sum_j \psi_j \int_{w_r}^{\infty}\left(W\left(\tilde{w}\right)-U\right)dF_j\left(\tilde{w}\right) \\
&= -(1-s)\sum_j \psi_j \left[\left(W\left(\tilde{w}\right)-U\right)\left(1-F_j\left(\tilde{w}\right)\right)\right]_{w_r^-}^{\infty} \\
&\quad +(1-s)\sum_j \psi_j \int_{w_r}^{\infty} W_w\left(\tilde{w}\right)\left(1-F_j\left(\tilde{w}\right)\right)dw \\
&= (1-s)\int_{w_r}^{\infty}\frac{\sum_j \psi_j\left(1-F_j(\tilde{w})\right)}{r+\delta+\sum_j s\psi_j\left(1-F_j(\tilde{w})\right)}d\tilde{w}.
\end{aligned}$$

The first step uses integration by parts. The second step uses that $W(w_r)=U$ and $\lim_{w\to\infty}(W(w)-U)(1-F_j(w))=0$. Since $W(w)-U$ is bounded above by $(w-b)/(r+\delta)$, it is sufficient that $\lim_{w\to\infty} w(1-F_j(w))=0$ for all $j\in\mathrm{M}$. The third step substitutes in using the above expressions for the derivative of worker value and gives the stated result.

A.2 Proof of Proposition 1

We proceed via a sequence of Lemmas.

LEMMA 2. The user cost of labor is equated across firms, i.e., for $i,z\in\mathrm{M}$

$$\min_{w\ge w_r}\left\{w-c\sum_{j\neq i}s\psi_jF_j\left(w^-\right)\right\}+c\left(\delta+\sum_{j\neq i}s\psi_j\right)=\min_{w\ge w_r}\left\{w-c\sum_{j\neq z}s\psi_jF_j\left(w^-\right)\right\}+c\left(\delta+\sum_{j\neq z}s\psi_j\right). \tag{32}$$

Proof. Let $\omega_i\equiv\min_{w\ge w_r}\left\{w-c\sum_{j\neq i}s\psi_jF_j\left(w^-\right)\right\}+c\left(\delta+\sum_{j\neq i}s\psi_j\right)$ denote the user cost of firm i. Assume that two firms i and z have different user costs, with $\omega_i>\omega_z$. Denote $\bar{w}_z\equiv\sup\{w:G_z(w)<1\}$ the "highest wage" posted by firm z. By condition (6a), the user cost is equated across all wages offered by a firm. Thus,

$$\omega_z = \bar{w}_z + c\left(\delta + \sum_{j \neq z} s\psi_j \left(1 - F_j\left(\bar{w}_z^-\right)\right)\right). \tag{33}$$

Next consider some wage $\bar{w}_z + \frac{\omega_i - \omega_z}{2}$. Since firm i is free to post there, (6a) implies that it must be that the user cost of firms i ω_i is weakly below the user cost associated with that wage. This implies

$$\begin{aligned}
\omega_i &\leq \bar{w}_z + \frac{\omega_i - \omega_z}{2} + c\left(\delta + \sum_{j \neq i} s\psi_j \left(1 - F_j\left(\left(\bar{w}_z + \frac{\omega_i - \omega_z}{2}\right)^-\right)\right)\right) \\
&= \frac{\omega_i - \omega_z}{2} + \bar{w}_z + c\left(\delta + \sum_{j \neq z} s\psi_j \left(1 - F_j\left(\left(\bar{w}_z + \frac{\omega_i - \omega_z}{2}\right)^-\right)\right)\right) \\
&\quad - cs\psi_i \left(1 - F_i\left(\left(\bar{w}_z + \frac{\omega_i - \omega_z}{2}\right)^-\right)\right) \\
&\leq \frac{\omega_i - \omega_z}{2} + \bar{w}_z + c\left(\delta + \sum_{j \neq z} s\psi_j \left(1 - F_j\left(\bar{w}_z^-\right)\right)\right) \\
&= \frac{\omega_i - \omega_z}{2} + \omega_z
\end{aligned} \tag{34}$$

where the first equality uses that $F_z\left(\left(\bar{w}_z + \frac{\omega_i - \omega_z}{2}\right)^-\right) = 1$ and the second inequality uses that, since F_j is a cdf, it is a weakly increasing function. This delivers a contradiction and proves that the user cost of labor must be the same across firms. □

LEMMA 3. If firm i offers a wage weakly below $\hat{w}$, i.e., $F_i(\hat{w}) > 0$, the amount of offers by i weakly above $\hat{w}$ is weakly larger than that of any other firm, i.e., $\psi_i(1 - F_i(\hat{w}^-)) \geq \max_{j \in M} \psi_j(1 - F_j(\hat{w}^-))$.

Proof. Assume to the contrary that there exist two firms i and z with $\hat{w}$ such that $\psi_z(1 - F_z(\hat{w}^-)) - \psi_i(1 - F_i(\hat{w}^-)) = \frac{\bar{\epsilon}}{sc} > 0$ and $F_i(\hat{w}) > 0$. Let ω denote the user cost of labor which by Lemma 2 is common across firms.

Since $F_i(\hat{w}) > 0$, firm i must make some offers at some other wage in $[w_r, \hat{w}]$. Denote $\bar{w} = \sup\{w : F_i(\hat{w}) - F_i(w^-) \geq \min\{\frac{\bar{\epsilon}}{3sc}, F_i(\hat{w})\}\}$. Fix $\epsilon = 0$ if $\bar{w} = \hat{w}$ and otherwise

$\epsilon \in (0, \min\{\frac{\bar{\epsilon}}{3}, \hat{w} - \bar{w}\})$. At $\bar{w} + \epsilon$, we have that

$$
\begin{aligned}
\omega &\leq \bar{w} + \epsilon + (r + \delta + sc \sum_{j \neq z} \psi_j (1 - F_j((\bar{w} + \epsilon)^-)) \\
&= \bar{w} + \epsilon + (r + \delta + sc \sum_{j \neq i} \psi_j (1 - F_j((\bar{w} + \epsilon)^-)) + sc \left(\psi_i (1 - F_i((\bar{w} + \epsilon)^-)) - \psi_z (1 - F_z((\bar{w} + \epsilon)^-)) \right) \\
&\leq \bar{w} + \epsilon + (r + \delta + sc \sum_{j \neq i} \psi_j (1 - F_j((\bar{w} + \epsilon)^-)) + sc \left(\psi_i (1 - F_i(\hat{w}^-)) - \psi_z (1 - F_z(\hat{w}^-)) \right) + \frac{\bar{\epsilon}}{3} \\
&= \bar{w} + \epsilon + (r + \delta + sc \sum_{j \neq i} \psi_j (1 - F_j((\bar{w} + \epsilon)^-)) - \frac{2}{3} \bar{\epsilon} \\
&< \bar{w} + \epsilon + (r + \delta + sc \sum_{j \neq i} \psi_j (1 - F_j((\bar{w} + \epsilon)^-))
\end{aligned} \tag{35}
$$

The first line uses that the user cost at $\bar{w}$ for firm z has to be weakly larger than ω. The second line rearranges this expression. The third line uses that, if $\bar{w} < \hat{w}$, firm i makes at most $\frac{\bar{\epsilon}}{3sc}$ offers over $(\bar{w}, \hat{w}]$, hence $cs\psi_i(F_i(\hat{w}^-) - F_i((\bar{w} + \epsilon)^-)) \leq \bar{\epsilon}/3$, and firm z might offer over the interval, $F_z(\bar{w}^-) \leq F_z((\bar{w} + \epsilon)^-)$. The fourth line uses that $\psi_z(1 - F_z(\hat{w}^-)) - \psi_i(1 - F_i(\hat{w}^-)) = \bar{\epsilon}/sc$. The last line implies that the user cost for firm i is strictly higher than ω at wage $\bar{w} + \epsilon$ (and when $\bar{w} < \hat{w}$, since this applies for any $\epsilon \in (0, \min\{\frac{\bar{\epsilon}}{3}, \hat{w} - \bar{w}\})$, it holds also at $\bar{w}$). We therefore get a contradiction to (6a) given that firm i offers a positive measure of jobs over $[\bar{w}, \hat{w}]$ yet the user cost is strictly larger than ω over this interval. □

The following is immediately implied.

COROLLARY 1. If firm i and z offer wage weakly lower than w, the amount of offers weakly above w is the same for i and z, i.e., $\psi_i(1 - F_i(w^-)) = \psi_z(1 - F_z(w^-))$.

LEMMA 4. All firms choose identical contact rates and wage offer distributions.

Proof. We first establish that the lowest wage must be identical across any two firms. Suppose this is not the same. Take the higher lowest wage. Both firms offer wages weakly below and therefore by Corollary 1 offer the same amount of jobs weakly above. Take any wage above the higher lowest wage. Again, both firms offer wages weakly below and therefore by Corollary 1 offer the same amount of jobs weakly above. Therefore, they act identically everywhere above the higher lowest wage. It follows that employment weakly above the highest lowest wage is the same across the two firms and that therefore the firm with the lower lowest wage is larger, contradicting Lemma 2 which together with (6b) implies that employment is the same across firms.

The final step establishes that if the lowest wage is the same, then the strategies are symmetric. We first establish that the offer rates are the same. Suppose they were not. Denote the two firms i and z. Assume first that the offer rates are different $\psi_i \neq \psi_z$. This however immediately contradicts Corollary 1 evaluated at the lowest wage.

Assume therefore instead that the offer distributions are different. This means that there exists some w between the lowest and highest wage such that $F_i(w^-) \neq F_z(w^-)$, implying that one firm places a larger fraction of their offers at wages weakly above w. We just established that the contact rates are the same and so this implies a contradiction to Corollary 1 at w. Thus, the offer distributions and contact rates are symmetric across firms. □

LEMMA 5. i) There cannot be mass points, ii) there cannot be gaps on the interior of the support, and iii) the lowest wage is equal to the reservation wage.

Proof. All three implications follow immediately from simple deviation arguments (contradicting (6a)) because of the symmetry established above. If there was mass, the user cost would be strictly lower right above the mass. If there were gaps then a firm right above the gap could lower its wage without increasing turnover. If the lowest wage was above the reservation wage, an employer could attain a strictly lower user cost by paying the reservation wage instead of the lowest wage (since turnover would be unchanged). □

We can now calculate the objects. Since we have symmetry, we remove the dependence on i and simply write $\{\psi, F, n, G\}$.

Offer distribution (8)**.** Since the reservation wage is offered and the equilibrium is symmetric, the user cost solves $\omega = w_r + (\delta + s(M-1)\psi)c$. Take (6a) and use the fact that the equilibrium is symmetric, that there are no gaps, that the reservation wage is offered, and $F(w_r) = 0$ to directly get (8)

$$F(w) = \frac{w - w_r}{s(M-1)\psi c}. \tag{36}$$

The highest wage solves $F(w_u) = 1$ and is therefore given by $w_u = w_r + s(M-1)\psi c$.

Reservation wage (7)**.** The reservation wage satisfies (3c) which, using symmetry and (8), simplifies to (7)

$$\begin{aligned} w_r &= b + (1-s)\int_{w_r}^{\infty} \frac{\psi M\left(1-F(\tilde{w})\right)}{r+\delta+sM\psi(1-F(\tilde{w}))} d\tilde{w} \\ &= b + \frac{(1-s)}{s}\int_{w_r}^{w_u} \frac{-(r+\delta)+r+\delta+sM\psi(1-\frac{\tilde{w}-w_r}{w_u-w_r})}{r+\delta+sM\psi(1-\frac{\tilde{w}-w_r}{w_u-w_r})} d\tilde{w} \\ &= b + \frac{(1-s)}{s}(w_u-w_r) - \frac{(w_u-w_r)(1-s)}{M\psi s^2}(r+\delta)\log\left(\frac{r+\delta+sM\psi}{r+\delta}\right) \\ &= b + c(1-s)(M-1)\psi - c\frac{M-1}{M}\frac{1-s}{s}(r+\delta)\log\left(\frac{r+\delta+sM\psi}{r+\delta}\right). \end{aligned} \tag{37}$$

Cross sectional wage distribution (9)**.** Flow balance for wages weakly below w, under symmetry, reads $M\psi F(w)u = (\delta + sM\psi(1-F(w)))(1-u)G(w)$. Rearranging gives (9).

Firm level employment (10)**.** Equating employment inflows and outflows gives $M\psi\left(1-Mn\right) = \delta Mn$ which can be rearranged to give (10).

Offer rate (11)**.** Take (6b) and use symmetry, that the reservation wage is offered, and $F(w_r) = 0$ to get $\alpha x n^{\alpha-1} = w_r + (\delta + s(M-1)\psi)c$. Use the expression for firm level employment (10) and (7) to get (11).

LEMMA 6. An equilibrium exists and is unique.

Proof. Rearranging (11), we get

$$0 = b + c\frac{M-1}{M}\left(M\psi - \frac{1-s}{s}(r+\delta)\log\left(\frac{r+\delta+sM\psi}{r+\delta}\right)\right) - \alpha x\left(\frac{\psi}{M\psi+\delta}\right)^{\alpha-1}. \tag{38}$$

The RHS is continuous and strictly increasing in ψ. As $\psi \to 0$, it tends to minus infinity and as $\psi \to \infty$, it tends to infinity. There thus exists a unique value of ψ satisfying (11).

This value of ψ, with corresponding reservation wage w_r (7), offer distribution F (8), firm level employment n (10), and cross sectional wage distribution G (9) constitutes the unique equilibrium. □

This concludes the proof.

ONLINE APPENDIX

B Proof of Lemma 1

The proof has two parts. First, it gives the mapping from $\{\psi_i, F_i\}$ to $\{n_i, G_i\}$ reported in (4). The second part proves the injectivity of the mapping.

From $\{\psi_i, F_i\}$ to $\{n_i, G_i\}$. The unemployment inflow is given by $\delta \sum_j n_j$. The outflow is given by $(1 - \sum_j n_j) \cdot \sum_j \psi_j$. Total employment in steady state thus solves

$$\sum_j n_j = \frac{\sum_j \psi_j}{\delta + \sum_j \psi_j}, \tag{39}$$

or, equivalently, unemployment is $u = \frac{\delta}{\delta + \sum_j \psi_j}$. The outflow of workers from employment at wages weakly lower than w is $\sum_j n_j G_j(w)(\delta + s \sum_j \psi_j \left(1 - F_j(w)\right))$ whereas the inflow is $u \sum_j \psi_j F_j(w)$, hence

$$\sum_j n_j G_j(w) = \frac{\sum_j \psi_j F_j(w)}{\delta + s \sum_j \psi_j \left(1 - F_j(w)\right)} u = \frac{1}{s} \frac{\delta + s \sum_j \psi_j}{\delta + s \sum_j \psi_j \left(1 - F_j(w)\right)} u - \frac{1}{s} u.$$

We next turn to flow balance at a particular firm and wage. We initially distinguish two cases and then work with a unified representation.

For wages where firm i's offer distribution is differentiable: Start with a wage w on any interval where where F_i is differentiable and $\{F_j\}_{j \neq i}$ are continuous, the flow into employment at wage w at firm i equals the flow out. The inflow consists of workers accepting wage w offers from firm i: $\psi_i f_i(w)$ times the measure of workers who would accept, namely the unemployed plus those employed at weakly lower wages. The outflow rate is $\delta + s \sum_j \psi_j (1 - F_j(w^-))$. Therefore,

$$\begin{aligned} n_i g_i(w) &= \frac{\psi_i f_i(w)}{\delta + s \sum_j \psi_j (1 - F_j(w^-))} \left(u + s \sum_j n_j G_j(w) \right) && (40) \\ &= \frac{\psi_i f_i(w)}{\delta + s \sum_j \psi_j (1 - F_j(w^-))} \frac{\delta + s \sum_j \psi_j}{\delta + s \sum_j \psi_j (1 - F_j(w))} \frac{\delta}{\delta + \sum_j \psi_j}. && (41) \end{aligned}$$

where the second line substitutes for u and $\sum_j n_j G_j(w)$ using the previous step.

For wages where firm i places a mass point: At wages where F_i has a discontinuity (mass point), the same steady-state logic applies, but we work with discrete masses rather than densities:

$$n_i \Delta G_i(w) = \frac{\psi_i \Delta F_i(w)}{\delta + s\sum_j \psi_j(1 - F_j(w^-))} \frac{\delta + s\sum_j \psi_j}{\delta + s\sum_j \psi_j(1 - F_j(w))} \frac{\delta}{\delta + \sum_j \psi_j} \tag{42}$$

where $\Delta F_i(w)$ and $\Delta G_i(w)$ denote the mass at wage w.

Both equations (41) and (42) have identical structure when we recognize that the Radon-Nikodym derivative $\frac{dG_i}{dF_i}(w)$ is, for continuous parts, $\frac{dG_i}{dF_i}(w) = \frac{g_i(w)}{f_i(w)} = \frac{dG_i/dw}{dF_i/dw}$ and, for discrete parts, $\frac{dG_i}{dF_i}(w) = \frac{\Delta G_i(w)}{\Delta F_i(w)}$. Thus, a unified representation of (41) and (42) is[32]

$$n_i \frac{dG_i}{dF_i}(w) = \frac{\psi_i}{\delta + s\sum_j \psi_j(1 - F_j(w^-))} \frac{\delta + s\sum_j \psi_j}{\delta + s\sum_j \psi_j(1 - F_j(w))} \frac{\delta}{\delta + \sum_j \psi_j}. \tag{43}$$

Integrating over all wages using the Lebesgue-Stieltjes integral gives (4b).

As G_i is the cumulative distribution function of wages at firm i, it can be expressed as $G_i(w) = \int_{w_r}^{w} \frac{dG_i}{dF_i}(\tilde{w})\, dF_i(\tilde{w})$. Substituting the Radon-Nikodym derivative from above gives

$$G_i(w) = \int_{w_r}^{w} \frac{1}{n_i} \frac{\psi_i}{\delta + s\sum_j \psi_j(1 - F_j(\tilde{w}^-))} \frac{\delta + s\sum_j \psi_j}{\delta + s\sum_j \psi_j(1 - F_j(\tilde{w}))} \frac{\delta}{\delta + \sum_j \psi_j}\, dF_i(\tilde{w}). \tag{44}$$

Use that $G_i(\infty) = 1$ by definition to eliminate the constants, yielding (4a).

We have thus derived the two expressions reported in the Lemma which give unique n_i and G_i for each $\{\psi_i, F_i\}$ given $\{\psi_j, F_j\}_{j \neq i}$ and w_r.

From $\{n_i, G_i\}$ to $\{\psi_i, F_i\}$. We now show that any $\{n_i, G_i\}$ with $G_i(w_r^-) = 0$ can be implemented by at most one firm policy $\{\psi_i, F_i\}$ which implies that the mapping (4) just derived is injective.

Proof Strategy: Fix $\{n_i, G_i\}$, $\{\psi_j, F_j\}_{j \neq i}$, and w_r. For each candidate ψ_i, we construct the corresponding F_i by solving backwards from the upper support $\overline{w}_i = \sup\{w : G_i(w) < 1\}$

[32] The same expression holds also for singular continuous distributions (such as the Cantor distribution) where F_i is continuous but not differentiable. In such cases, the Radon-Nikodym derivative $\frac{dG_i}{dF_i}(w)$ remains well-defined since (43) provides a bounded expression for it, which implies that G_i is absolutely continuous with respect to F_i.

with initial condition $F_i(\overline{w}_i) = 1$. We solve equation (43) until we reach a wage w where either $G_i(w^-) = 0$ or $F_i(w^-) = 0$. We do this separately for (i) the intervals where G_i is differentiable and strictly increasing with $\{F_j\}_{j\neq i}$ continuous, and (ii) the points where G_i is discontinuous. Since G_i is a nonsingular cdf and $\{F_j\}_{j\neq i}$ are cdfs, this leaves only the intervals where G_i is flat and a measure-zero set of remaining points on the support.

Steps 1-3 below provides details on this construction.

The key insight is that employment implied by this construction is strictly monotone and continuous in ψ_i which will imply a unique ψ_i yielding the target employment n_i. This is established in steps 4-7 below.

Step 1: Change of variables. We introduce $z = n_i(1 - G_i(w))$ and $q = \psi_i(1 - F_i(w))$, transforming the initial condition $F_i(\overline{w}_i) = 1$ to $q(0) = 0$. Since $\{F_j\}_{j\neq i}$ and G_i are CDFs, they are continuous almost everywhere.

Step 2: Mass Points At any mass point in G_i, the corresponding mass in F_i is uniquely determined by equation (42) which can be written as

$$n_i \Delta G_i(w) \left(\delta + s \sum_j \psi_j (1 - F_j(w)) + s \sum_{j\neq i} \psi_j \Delta F_j(w) + s\psi_i \Delta F_i(w) \right) = \frac{\delta + s \sum_j \psi_j}{\delta + s \sum_j \psi_j (1 - F_j(w))} \frac{\delta}{\delta + \sum_j \psi_j} \psi_i \Delta F_i(w). \quad (45)$$

We can rearrange to give an explicit solution for $\Delta F_i(w)$

$$\psi_i \Delta F_i(w) = \frac{n_i \Delta G_i(w) \left(\delta + s \sum_j \psi_j \left(1 - F_j(w)\right) + s \sum_{j\neq i} \psi_j \Delta F_j(w) \right)}{\frac{\delta}{\delta + s \sum_j \psi_j \left(1 - F_j(w)\right)} \frac{\delta + s \sum_j \psi_j}{\delta + \sum_j \psi_j} - s n_i \Delta G_i(w)}. \quad (46)$$

If the mass of wages $\Delta F_i(w)$ at w implied by (42) exceeds $F_i(w)$ (recalling that we are solving backwards toward lower wages), we set $F_i(\tilde{w}) = 0$ for $\tilde{w} < w$. In this case, (46) gives either a value for $\Delta F_i(w)$ in excess of $F_i(w)$ or a negative value (when the denominator is negative). Similarly if $G_i(w^-) = 0$, we set $F_i(\tilde{w}) = 0$ for $\tilde{w} < w$.

Step 3: Continuous Case Over any interval where G_i is differentiable and strictly increasing and $\{F_j\}_{j\neq i}$ are continuous, we can then write (43) as a simple ODE

$$\frac{dq(z)}{dz} = \frac{1}{\delta}\frac{\delta + \sum_j \psi_j}{\delta + s\sum_j \psi_j}\left(\delta + s\sum_{j\neq i}\psi_j\left(1 - F_j\left(G_i^{-1}\left(1 - \frac{z}{n_i}\right)\right)\right) + sq(z)\right)^2$$
$$\equiv \gamma(z, q(z), \psi_i), \tag{47}$$

using the relationship $w = G_i^{-1}\left(1 - \frac{z}{n_i}\right)$. By the Picard–Lindelöf theorem, since γ is continuous in z and Lipschitz continuous in q, there exists a unique solution to this ODE. We solve this function on the interval and if at some w either $F_i(w) = 0$ or $G_i(w) = 0$, we set $F_i(\tilde{w}) = 0$ for $\tilde{w} < w$.

In this way, step 2 and 3 constructs a unique distribution F_i for each ψ_i given $\{\psi_j, F_j\}_{j\neq i}$ and w_r.

Step 4: Monotonicity of $q(z) = \psi_i(1 - F_i(w))$ in ψ_i

We now show that $q(z)$, or equivalently $\psi_i(1 - F_i(w))$, is weakly increasing in ψ_i for all w, under the construction defined in Steps 2 and 3. This result is essential for showing that the employment level n_i implied by the constructed F_i is strictly increasing in ψ_i, which ensures that for any fixed n_i, a unique ψ_i can implement it.

Proof by contradiction. Fix two values $\overline{\psi}_i > \underline{\psi}_i$ and let $\overline{F}_i$ and $\underline{F}_i$ be the respective constructed distributions under these values. Define $\overline{q}(z) = \overline{\psi}_i(1 - \overline{F}_i(G_i^{-1}(1 - z/n_i)))$ and $\underline{q}(z) = \underline{\psi}_i(1 - \underline{F}_i(G_i^{-1}(1 - z/n_i)))$, where $z = n_i(1 - G_i(w))$.

Suppose for contradiction that $\overline{q}(z) < \underline{q}(z)$ for some z. Define the supremum of such points as

$$\hat{w} = \sup\left\{w : \overline{\psi}_i(1 - \overline{F}_i(w)) < \underline{\psi}_i(1 - \underline{F}_i(w))\right\}, \quad \hat{z} = n_i(1 - G_i(\hat{w})).$$

We distinguish two exhaustive cases based on the behavior at $\hat{w}$.

Case 1: Strict inequality at $\hat{w}^-$.

Suppose $\overline{\psi}_i(1 - \overline{F}_i(\hat{w}^-)) < \underline{\psi}_i(1 - \underline{F}_i(\hat{w}^-))$.

By construction, $\overline{\psi}_i(1 - \overline{F}_i(w)) \geq \underline{\psi}_i(1 - \underline{F}_i(w))$ for all $w > \hat{w}$. Hence, a mass point must occur in F_i at $\hat{w}$ under $\overline{\psi}_i$. The size of this mass is governed by equation (46), which gives

$$\psi_i \Delta F_i(w) = \frac{n_i \Delta G_i(w) \left(\delta + s\sum_j \psi_j (1 - F_j(w)) + s\sum_{j\neq i} \psi_j \Delta F_j(w)\right)}{\frac{\delta}{\delta + s\sum_j \psi_j(1-F_j(w))} \frac{\delta + s\sum_j \psi_j}{\delta + \sum_j \psi_j} - s n_i \Delta G_i(w)}. \tag{48}$$

Holding all other quantities fixed (including n_i, $\Delta G_i(w)$, and $\{\psi_j, F_j\}_{j\neq i}$), we observe that:

- The numerator is weakly larger under $\overline{\psi}_i$ compared with $\underline{\psi}_i$ since $\psi_i(1 - F_i(\hat{w}))$ enters positively and is (weakly) larger.
- The denominator is weakly smaller under $\overline{\psi}_i$ compared with $\underline{\psi}_i$ since $\frac{\delta + s\sum_j \psi_j}{\delta + \sum_j \psi_j}$ is (weakly) smaller given $s \in (0, 1]$ and $\psi_i(1 - F_i(\hat{w}))$ enters negatively and is (weakly) larger.

Therefore, $\psi_i \Delta F_i(w)$ is weakly increasing in ψ_i, and so we have

$$\overline{\psi}_i \Delta \overline{F}_i(\hat{w}) \geq \underline{\psi}_i \Delta \underline{F}_i(\hat{w}).$$

Adding this to the value at $\hat{w}$ gives

$$\overline{\psi}_i(1 - \overline{F}_i(\hat{w}^-)) = \overline{\psi}_i(1 - \overline{F}_i(\hat{w})) + \overline{\psi}_i \Delta \overline{F}_i(\hat{w}) \geq \underline{\psi}_i(1 - \underline{F}_i(\hat{w})) + \underline{\psi}_i \Delta \underline{F}_i(\hat{w}) = \underline{\psi}_i(1 - \underline{F}_i(\hat{w}^-)),$$

which contradicts our assumption that the strict inequality $\overline{\psi}_i(1 - \overline{F}_i(\hat{w}^-)) < \underline{\psi}_i(1 - \underline{F}_i(\hat{w}^-))$ holds at $\hat{w}^-$.

Case 2: Equality at $\hat{w}^-$. Suppose that $\overline{\psi}_i(1 - \overline{F}_i(\hat{w}^-)) = \underline{\psi}_i(1 - \underline{F}_i(\hat{w}^-))$.

Since $\{F_j\}_{j\neq i}$ and G_i are distributions, they are differentiable almost everywhere. Choose $\tilde{\epsilon} > 0$ small enough such that G_i and $\{F_j\}_{j\neq i}$ are differentiable on $(\hat{w} - \tilde{\epsilon}, \hat{w})$ and G_i is strictly increasing over this interval (such an interval exists since (43) otherwise implies that $\overline{\psi}_i(1 - \overline{F}_i(w))$ and $\underline{\psi}_i(1 - \underline{F}_i(w))$ are constant over this interval contradicting the definition of $\hat{w}$).

Define $h(z) \equiv \overline{q}(z) - \underline{q}(z)$. Then $h(\hat{z}) = 0$. By definition of $\hat{w}$ as the supremum, there exists $z_2 \in (\hat{z}, \hat{z} + \bar{\epsilon})$ such that $h(z_2) < 0$. Pick any such z_2 and denote z_1 such that $h(z) < 0$ for all $z \in (z_1, z_2)$, i.e., $z_1 = \sup\{z \leq z_2 : h(z) \geq 0\}$. Since $\{F_j\}_{j\neq i}$ are continuous and G_i is differentiable and strictly increasing over this interval, h is differentiable on (z_1, z_2) and $h(z_1) = 0$.

On the interval (z_1, z_2), $q(z)$ solves the ODE given in equation (47):

$$\frac{dq(z)}{dz} = \gamma(z, q(z), \psi_i). \tag{49}$$

This function γ is continuously differentiable in q and ψ_i, and hence Lipschitz in q. The Lipschitz constant is:

$$L = 2\frac{\delta + s\sum_j \psi_j}{\delta}\frac{\delta + \sum_j \psi_j}{\delta + s\sum_j \psi_j}, \tag{50}$$

since $q \in [0, \psi_i]$, $s \leq 1$ and $\sum_{j \neq i} \psi_j (1 - F_j(w)) \leq \sum_{j \neq i} \psi_j$. For $z \in (z_1, z_2)$, the derivative of $h(z)$ satisfies

$$\begin{aligned} h'(z) &= \gamma(z, \overline{q}(z), \overline{\psi}_i) - \gamma(z, \underline{q}(z), \underline{\psi}_i) \\ &= [\gamma(z, \overline{q}(z), \overline{\psi}_i) - \gamma(z, \overline{q}(z), \underline{\psi}_i)] + [\gamma(z, \overline{q}(z), \underline{\psi}_i) - \gamma(z, \underline{q}(z), \underline{\psi}_i)]. \end{aligned} \tag{51}$$

The first term is non-negative since γ is weakly increasing in ψ_i (as $\frac{\delta + \sum_j \psi_j}{\delta + s\sum_j \psi_j}$ is increasing in ψ_i given $s \leq 1$) and $\overline{\psi}_i > \underline{\psi}_i$. By the Lipschitz property of γ in q

$$|\gamma(z, \overline{q}(z), \underline{\psi}_i) - \gamma(z, \underline{q}(z), \underline{\psi}_i)| \leq L|\overline{q}(z) - \underline{q}(z)|. \tag{52}$$

Since $\underline{q}(z) > q(z)$, we have that $\gamma(z, \overline{q}(z), \underline{\psi}_i) \leq \gamma(z, \underline{q}(z), \underline{\psi}_i)$ which implies that

$$\gamma(z, \overline{q}(z), \underline{\psi}_i) - \gamma(z, \underline{q}(z), \underline{\psi}_i) \geq -L|\overline{q}(z) - \underline{q}(z)|. \tag{53}$$

Further using that $|\overline{q}(z) - \underline{q}(z)| = -h(z) > 0$, we get

$$\gamma(z, \overline{q}(z), \underline{\psi}_i) - \gamma(z, \underline{q}(z), \underline{\psi}_i) \geq Lh(z). \tag{54}$$

We therefore have that

$$h'(z) \geq Lh(z). \tag{55}$$

Applying Grönwall's inequality we get that for any $z \in (z_1, z_2)$

$$h(z) \geq \lim_{\tilde{z} \to z_1{}^+} e^{L(z - \tilde{z})} h(\tilde{z}) = 0, \tag{56}$$

since h is continuous and $h(z_1) = 0$. This contradicts $h(z) < 0$ on (z_1, z_2).

In summary, both cases lead to contradictions. We conclude that $\psi_i(1 - F_i(w))$ is weakly increasing in ψ_i for all w, as claimed.

Step 5: n_i is strictly increasing in ψ_i We want to prove that employment is strictly larger under $\overline{\psi}_i$ compared with $\underline{\psi}_i$. Assume to the contrary that employment is weakly larger under $\underline{\psi}_i$.

Denote by $\underline{w}_i = \inf\{w : \overline{F}_i(w) > 0\}$ the lowest wage offered under $\overline{\psi}_i$ which must be weakly larger than the lowest wage offered under $\underline{\psi}_i$ under our conjecture.

Under the conjecture, we have that employment in firm i strictly above $\underline{w}_i$ is the same. This is guaranteed by the construction of F in steps 1 (or 2) above since it is constructed so as to satisfy (43). Employment at $\underline{w}_i$ is weakly lower under $\overline{\psi}_i$ compared with $\underline{\psi}_i$. Total employment in firm i weakly above $\underline{w}_i$ is therefore weakly larger under the lower contact rate $\underline{\psi}_i$ compared with $\overline{\psi}_i$.

Total employment across all firms weakly above $\underline{w}_i$ on the other hand is strictly larger under $\overline{\psi}_i$ compared to $\underline{\psi}_i$. The reason is that the amount of offers weakly above $\underline{w}_i$ is strictly larger when i adopts a higher contact rate and given that all other firms act the same. In turn, the overall amount of offers strictly below $\underline{w}_i$ is weakly lower. It is straightforward to show that this implies that overall employment weakly above the threshold $\underline{w}_i$ is strictly larger under $\overline{\psi}_i$ compared with $\underline{\psi}_i$.

The previous two paragraphs imply that it must be the case that employment at firms $j \neq i$ must be strictly larger under the higher contact rate. We will now prove that the opposite is the case, implying a contradiction.

Take any wage $w \geq \underline{w}_i$, total outflow is weakly larger under $\overline{\psi}_i$, since $\psi_i(1 - F_i(w^-))$ is weakly larger for all $w \in [\underline{w}_i, \bar{w}_i]$ under $\overline{\psi}_i$ compared with $\underline{\psi}_i$ (due to the monotonicity established in step 4). To prove that employment weakly above $\underline{w}_i$ is smaller in firms $j \neq i$, it thus suffices to prove that the inflow is weakly smaller. The acceptance probability per offer at wage w is

$$u + s(1-u)G(w) = \frac{\delta}{\delta + \lambda}\left(1 + s\frac{\lambda - \bar{\psi}(w)}{\delta + s\bar{\psi}(w)}\right) = \frac{\delta + s\lambda}{\delta + \lambda}\frac{\delta}{\delta + s\bar{\psi}(w)}. \tag{57}$$

where we denote by $G(w)$ the market-wide distribution of pay. We will show that this is smaller under $\overline{\psi}_i$ compared with $\underline{\psi}_i$ which then proves the result.

Denote the total offer rate strictly above w by $\bar{\psi}(w) \equiv \sum_j \psi_j(1 - F_j(w))$ and the total offer rate to be $\lambda \equiv \sum_j \psi_j$. We know because our previous result (in step 4) that $\bar{\psi}(w)$ is weakly larger under $\overline{\psi}_i$. Similarly, λ is strictly larger since ψ_i is larger.

The acceptance probability (57) is decreasing in both λ and $\bar{\psi}(w)$ which implies that the acceptance probability is weakly lower for each wage w under $\overline{\psi}_i$ compared with $\underline{\psi}_i$ which completes the contradiction.

Step 6: n_i is continuous in ψ_i We next want to show that employment is continuous in ψ_i under our construction of F_i. Pick two offer rates ψ_i of $\overline{\psi}_i$ and $\overline{\psi}_i + \epsilon$. Solve for F_i under the two contract rates. By construction, total employment strictly above $\underline{w}_i$ (defined as the lowest wage offered under $\overline{\psi}_i$) is the same. In addition, total offers strictly above $\underline{w}_i$ are weakly larger under $\overline{\psi}_i + \epsilon$ (due to the monotonicity of q established in step 4). The additional measure of offers weakly below $\underline{w}_i$ under $\overline{\psi}_i + \epsilon$ is therefore bounded above by ϵ.[33]

The highest employment in firm i under $\overline{\psi}_i + \epsilon$ occurs if all of these additional offers are at $\overline{w}_i$ (because of job-to-job transitions). These offers are accepted with probability strictly less than 1 and have duration strictly shorter than $\frac{1}{\delta}$. It follows that that the difference in employment in firm i under offer rate $\overline{\psi}_i + \epsilon$ compared with $\underline{\psi}_i$ is strictly positive and bounded above by $\frac{\epsilon}{\delta}$. This implies that employment is continuous in ψ_i since this applies for any $\epsilon > 0$.

Step 7: Uniqueness of ψ_i As $\psi_i \to 0$ employment goes to 0 and as $\psi_i \to \infty$ employment in firm i is strictly positive. Given a level of employment n_i less than this limit, it then follows that there exists ψ_i such that employment using our construction of F_i is equal to n_i because of the continuity established in the previous step. And because of the strict monotonicity established in Step 5, there is a unique such value of ψ_i. If follows that the mapping from $\{n_i, G_i\}$ to $\{\psi_i, F_i\}$ is unique if it exists which implies that the mapping from $\{\psi_i, F_i\}$ to $\{n_i, G_i\}$ in (4) is injective.

This completes our proof of Lemma 1.

[33] Any mass of offers at $\underline{w}_i$ that is the same across the two contract rates results in smaller employment under $\overline{\psi}_i + \epsilon$ since turnover is higher (due to step 4) whereas recruitment is weakly lower.

C Implementability of Deviation

The following establishes that the deviations used to establish firm optimality, namely moving a mass of jobs to a particular wage while holding overall employment constant, is always implementable in the primal formulation.

Step 1. If $\{n_i, G_i\}$ is implemented by some $\{\psi_i, F_i\}$, so is $\{\tilde{n}_i, G_i\}$ for any $\tilde{n}_i \in [0, n_i)$. To prove this, fix ψ_i and consider $\tilde{n}_i < n_i$. Under the construction of F_i in Appendix B, $\psi_i(1 - F_i(w))$ is strictly increasing in n_i, as (43) implies that $\frac{dF_i}{dG_i}(w)$ is strictly larger under n_i compared with $\tilde{n}_i$. This implies that $\psi_i(1 - F_i(w))$ is strictly larger for all $w \in (\inf\{w : G_i(w) > 0\}, \sup\{w : G_i(w) < 1\})$ under n_i. Following Step 5 in Appendix B, employment under ψ_i exceeds $\tilde{n}_i$. Step 7 in Appendix B therefore implies that there exists a $\tilde{\psi}_i < \psi_i$ which implements $\{\tilde{n}_i, G_i\}$.

Step 2. We will now show that, if there is a $\{\underline{\psi}_i, \underline{F}_i\}$ that implements $\{n_i, G_i\}$, then, for any $\hat{w} \geq w_r$, there exists an $\epsilon > 0$ and a policy $\{\tilde{\psi}_i, \tilde{F}_i\}$ that implements n_i with distribution $\tilde{G}_i(w) = (1 - \epsilon)G_i(w) + \epsilon 1(w \geq \hat{w})$.

We prove this in two steps. We first show that we can construct a $\bar{F}_i$ given some larger value of ψ_i that places additional mass at the reservation wage. In the second stage we then show how to instead place this mass at an arbitrary point in the distribution. Start with the $\{n_i, G_i\}$ which is implemented by $\{\underline{\psi}_i, \underline{F}_i\}$. Then proceed as follows.

Take a larger offer rate $\bar{\psi}_i > \underline{\psi}_i$. Construct $\bar{F}_i$ exactly as in the "backward procedure" in Steps 1-3 in Appendix B above, except do not impose $F_i(w^-) = 0$ if $G_i(w^-) = 0$. Because $\bar{\psi}_i > \psi_i$, the solution reaches w_r with positive residual mass. There is therefore a mass of additional employment at w_r relative to $\{n_i, G_i\}$ (by Step 5 in Appendix B). Denoting the amount of excess employment at w_r by $n_i \frac{\epsilon}{1-\epsilon} > 0$, total employment is $n_i + n_i \frac{\epsilon}{1-\epsilon} = n_i \frac{1}{1-\epsilon}$ and the distribution is $\bar{G}_i(w) = \frac{n_i \frac{\epsilon}{1-\epsilon} 1(w \geq w_r) + n_i G_i(w)}{n_i \frac{\epsilon}{1-\epsilon} + n_i} = \epsilon 1(w \geq w_r) + (1 - \epsilon)G_i(w)$.

To move this mass from the reservation wage to any point in the distribution proceed as follows: Construct $\tilde{F}_i$ as in Appendix B, using distribution $\tilde{G}_i(w) = \epsilon 1(w \geq \hat{w}) + (1 - \epsilon)G_i(w)$, employment $n_i \frac{1}{1-\epsilon}$, and offer rate $\bar{\psi}_i$. The RHS of (43) (at fixed $z = n_i(1 - G_i(w))$) is weakly larger under $\tilde{G}_i$ compared with $\bar{G}_i$, since $\tilde{G}_i$ first order stochastically dominates $\bar{G}_i$. The monotonicity of (43) implies that employment under $\{\bar{\psi}_i, \tilde{F}_i\}$ is weakly larger than $n_i \frac{1}{1-\epsilon}$. Step 6 in Appendix B then implies that there exists a $\tilde{\psi}_i \leq \bar{\psi}_i$ which, jointly with the

F_i constructed via the backward procedure, implements $\tilde{G}_i$ and employment $n_i \frac{1}{1-\epsilon}$. The result then follows immediately from step 1 above: Since $\{n_i \frac{1}{1-\epsilon}, \tilde{G}_i\}$ can be implemented, so can $\{n_i, \tilde{G}_i\}$.

D Additional Closed Forms for Equilibrium Objects

Let w_u denote the highest posted wage and use that $F(w_u) = 1$ to get

$$w_u = w_r + cs(M-1)\psi = b + c\frac{M-1}{M}\left(M\psi - \frac{1-s}{s}(r+\delta)\log\left(\frac{r+\delta+sM\psi}{r+\delta}\right)\right). \quad (58)$$

Integrating the distribution of paid wages yields that the average wage is given by

$$\begin{aligned}
\mathbf{E}(w) &= \int_{w_r}^{w_u} w dG(\tilde{w}) = -\left[w\left(1-G(\tilde{w})\right)\right]_{w_r^-}^{w_u} + \int_{w_r}^{w_u}\left(1-G(\tilde{w})\right)d\tilde{w} = w_r + \int_{w_r}^{w_u}\left(1-G(\tilde{w})\right)d\tilde{w} \\
&= = w_u - \frac{\delta}{sM\psi}\int_{w_r}^{w_u}\frac{1+\frac{sM\psi}{\delta} - \left(1+\frac{sM\psi}{\delta}\left(1-F(\tilde{w})\right)\right)}{1+\frac{sM\psi}{\delta}(1-F(\tilde{w}))}d\tilde{w} \\
&= w_u - (w_u - w_r)\frac{\delta}{sM\psi}\frac{\delta}{sM\psi}\left(1+\frac{sM\psi}{\delta}\right)\log\left(1+\frac{sM\psi}{\delta}\right) + (w_u - w_r)\frac{\delta}{sM\psi} \\
&= b + c\frac{M-1}{M}\left[\delta + M\psi - \frac{1-s}{s}(r+\delta)\log\left(\frac{r+\delta+sM\psi}{r+\delta}\right) - \delta\frac{sM\psi+\delta}{sM\psi}\log\left(\frac{\delta+sM\psi}{\delta}\right)\right].
\end{aligned}$$

Workers excess values solve

$$W(w) - U = \int_{w_r}^{w_u}\frac{1}{r+\delta+sM\psi\frac{w_u-w}{w_u-w_r}}dw = c\frac{M-1}{M}\log\left(\frac{r+\delta+sM\psi}{r+\delta++sM\psi\frac{w_u-w}{w_u-w_r}}\right), \quad (59)$$

while the value of unemployment is given by

$$\begin{aligned}
rU &= b + M\psi\int_{w_r}^{w_u}\left(W(w)-U\right)dF(\tilde{w}) \\
&= b - M\psi\left[\left(W(w)-U\right)\left(1-F(w)\right)\right]_{w_r^-}^{w_u} + \int_{w_r}^{w_u}\frac{\psi M\left(1-F(\tilde{w})\right)}{r+\delta+sM\psi(1-F(\tilde{w}))}d\tilde{w} \\
&= b + \int_{w_r}^{w_u}\frac{\psi M\left(1-F(\tilde{w})\right)}{r+\delta+sM\psi(1-F(\tilde{w}))}d\tilde{w} = b + \frac{1}{s}\int_{w_r}^{w_u}\frac{-(r+\delta)+r+\delta+sM\psi(1-\frac{\tilde{w}-w_r}{w_u-w_r})}{r+\delta+sM\psi(1-\frac{\tilde{w}-w_r}{w_u-w_r})}d\tilde{w} \\
&= b + \frac{1}{s}(w_u - w_r) - \frac{(w_u-w_r)}{M\psi s^2}(r+\delta)\log\left(\frac{r+\delta+sM\psi}{r+\delta}\right) \\
&= b + c\frac{M-1}{M}\left(M\psi - \frac{r+\delta}{s}\log\left(\frac{r+\delta+sM\psi}{r+\delta}\right)\right). \quad (60)
\end{aligned}$$

E Noncompete Agreements

E.1 Equilibrium Characterization

Using (24) to solve for the two reservation wages analogously to Appendix A.1 gives

$$w_r = b + c(1-s)\frac{M-K-1}{M-K}\left((M-K)\psi - \frac{r+\delta}{s}\log\left(\frac{r+\delta+s(M-K)\psi}{r+\delta}\right)\right), \quad (61)$$

$$w_r^{nc} = b + c\frac{M-K-1}{M-K}\left((M-K)\psi - \frac{r+\delta}{s}\log\left(\frac{r+\delta+s(M-K)\psi}{r+\delta}\right)\right). \quad (62)$$

The wage distribution among the firms without noncompetes is

$$G(w) = \frac{F(w)}{1+\frac{s(M-K)\psi}{\delta}(1-F(w))}, \quad (63)$$

and firm level employment is

$$n = \frac{\psi}{(M-K)\psi + K\psi^{nc} + \delta} \quad \text{and} \quad n_{nc} = \frac{\psi^{nc}}{(M-K)\psi + K\psi^{nc} + \delta}. \quad (64)$$

The offer rates therefore solve

$$\alpha x n^{\alpha-1} = w_r + c(r+\delta+s\psi(M-K)) \quad (65)$$

$$= b + c\delta + c\frac{M-K-1}{M-K}\left((M-K)\psi - \frac{1-s}{s}(r+\delta)\log\left(\frac{r+\delta+s(M-K)\psi}{r+\delta}\right)\right), \quad (66)$$

$$\alpha x n_{nc}^{\alpha-1} = w_r^{nc} + c(r+\delta) \quad (67)$$

$$= b + c\delta + c\frac{M-K-1}{M-K}\left((M-K)\psi - \frac{r+\delta}{s}\log\left(\frac{r+\delta+s(M-K)\psi}{r+\delta}\right)\right). \quad (68)$$

The derivations are equivalent to those for the model without noncompetes.

E.2 Efficiency

Flow welfare is given by total output net of turnover cost,

$$rV = (M-K)xn^{\alpha} + Kxn_{nc}^{\alpha} + b\left(1-(M-K)n - Kn_{nc}\right) - c\delta(M-K)n - c\delta K n^{nc}$$
$$-c(M-K)n\frac{M-K-1}{M-K}\delta\left(\frac{\delta+s(M-K)\psi}{s(M-K)\psi}\log\left(1+\frac{s(M-K)\psi}{\delta}\right)-1\right). \quad (69)$$

The last term captures the turnover cost due to job-to-job transitions and can be derived in analogy to (13).

First Order Conditions Differentiate flow welfare with respect to n using that $\psi = \frac{\delta n}{1-(M-K)n-Kn_{nc}}$ and therefore $\frac{\partial \psi}{\partial n} = \frac{\psi}{n}\left(1+\frac{(M-K)\psi}{\delta}\right)$ as well as $\frac{\partial \psi}{\partial n_{nc}} = \frac{\psi^2}{\delta n}K$. This gives

$$\frac{\partial rV}{\partial n} = (M-K)\left(\alpha x n^{\alpha-1} - b - c\delta - c(M-K-1)\psi + c\frac{M-K-1}{M-K}\frac{1-s}{s}\delta \log\left(1+\frac{s(M-K)\psi}{\delta}\right)\right), \quad (70)$$

which, if equated to zero, is the same condition as (66) when $r \to 0$. Similarly,

$$\frac{\partial rV}{\partial n_{nc}} = K\left(\alpha x n_{nc}^{\alpha-1} - b - c\delta - c(M-K-1)\left(\psi - \frac{\delta}{s(M-K)}\log\left(1+\frac{s(M-K)\psi}{\delta}\right)\right)\right), \quad (71)$$

which, if equated to zero, is the same condition as (66) when $r \to 0$.

Second Derivatives and Hessian The diagonal second derivatives are

$$\frac{\partial^2 rV}{\partial n^2} = -(M-K)\left(\alpha(1-\alpha)x n^{\alpha-2} + c(M-K-1)s\frac{1}{1+\frac{s(M-K)\psi}{\delta}}\frac{\psi}{n}\left(1+\frac{(M-K)\psi}{\delta}\right)^2\right),$$

$$\frac{\partial^2 rV}{\partial n_{nc}^2} = -K\left(\alpha(1-\alpha)x n_{nc}^{\alpha-2} + c(M-K-1)\frac{\frac{s(M-K)\psi}{\delta}}{1+\frac{s(M-K)\psi}{\delta}}\frac{\psi^2}{\delta n}K\right),$$

both strictly negative. The cross-partial derivative is

$$\frac{\partial^2 rV}{\partial n \partial n_{nc}} = -(M-K)c(M-K-1)s\frac{1+\frac{(M-K)\psi}{\delta}}{1+\frac{s(M-K)\psi}{\delta}}\frac{\psi^2}{\delta n}K,$$

which is also negative.

Since both diagonal elements are negative, the welfare function is concave along each dimension. For global concavity, the determinant of the Hessian must be positive, requiring

$$\frac{\partial^2 rV}{\partial n^2}\frac{\partial^2 rV}{\partial n_{nc}^2} > \left(\frac{\partial^2 rV}{\partial n \partial n_{nc}}\right)^2,$$

which holds given the expressions above. Therefore the Hessian is negative definite, and the welfare problem has a unique global maximum.

Because the first-order conditions of the planner coincide with the equilibrium conditions ((66) and (68)), any decentralized equilibrium allocation is constrained efficient and unique.

Existence and Interior Solution We further note that the solution for $\{n, n_{nc}\}$ is interior whenever $M - K > 1$, which implies existence. To see this, suppose $(M-K)n + Kn_{nc} \to 1$. Then $\psi \to \infty$, and since $M - K > 1$, the coefficient $(M - K - 1) > 0$ implies that the right-hand side of both (66) and (68) diverges to $+\infty$. However, at least one of the left-hand sides must remain bounded, because at least one of n or n_{nc} must be bounded below by $1/M$ for the sum to equal one. This contradiction implies that $(M-K)n + Kn_{nc} < 1$, which in turn guarantees $\psi < \infty$.

Moreover, as $n \to 0$ (or $n_{nc} \to 0$), the left-hand side $\alpha x n^{\alpha-1} \to \infty$ while the corresponding right-hand side remains finite. Thus both n and n_{nc} must be strictly positive. Together these arguments establish that the equilibrium allocation $\{n, n_{nc}\}$ is strictly interior and well-defined.

Impact of K on Welfare To analyze the impact of K on flow welfare rV, consider the case in which $M \to \infty$. Since the allocation with noncompetes is constrained efficient, it satisfies the first-order conditions

$$\frac{\partial rV}{\partial n} = \frac{\partial rV}{\partial n_{nc}} = 0. \tag{72}$$

To simplify the analysis of how K affects welfare, we will, with slight abuse of notation, define

$$N \equiv (M-K)n \quad \text{and} \quad N_{nc} \equiv Kn_{nc}. \tag{73}$$

These represent total employment in firms without noncompetes and in noncompete firms, respectively. To find the total effect of K on welfare, we apply the chain rule

$$\frac{drV}{dK} = \frac{\partial rV}{\partial K} + \frac{\partial rV}{\partial N}\frac{dN}{dK} + \frac{\partial rV}{\partial N_{nc}}\frac{dN_{nc}}{dK}. \tag{74}$$

A useful implication of the constrained efficient equilibrium is that there are no first-order indirect effects through N and N_{nc}. To see this, note that by the chain rule

$$\frac{\partial rV}{\partial n} = \frac{\partial rV}{\partial N}\frac{\partial N}{\partial n} = \frac{\partial rV}{\partial N}(M-K), \tag{75}$$

since $\frac{\partial rV}{\partial n} = 0$ at equilibrium, we have $\frac{\partial rV}{\partial N} = 0$. Similarly:

$$\frac{\partial rV}{\partial n_{nc}} = \frac{\partial rV}{\partial N_{nc}}\frac{\partial N_{nc}}{\partial n_{nc}} = \frac{\partial rV}{\partial N_{nc}}K, \tag{76}$$

which implies $\frac{\partial rV}{\partial N_{nc}} = 0$. The total derivative simplifies to just the direct partial derivative

$$\frac{drV}{dK} = \frac{\partial rV}{\partial K}, \tag{77}$$

Computing this direct effect (treating N and N_{nc} as parameters)

$$\frac{\partial rV}{\partial K} = (1-\alpha)xn_{nc}^{\alpha} - (1-\alpha)xn^{\alpha} + cn\frac{1}{M-K}\delta\left(\frac{\delta + s(M-K)\psi}{s(M-K)\psi}\log\left(1+\frac{s(M-K)\psi}{\delta}\right) - 1\right). \tag{78}$$

As $M \to \infty$, we get the condition in the main body,

$$\frac{\partial rV}{\partial K} = (1-\alpha)xn_{nc}^{\alpha} - (1-\alpha)xn^{\alpha}. \tag{79}$$

F Equilibrium with Firms that Differ in Productivity

The proof follows much the same structure as the proof of Proposition 1. Lemmas 2 and 3 as well as Corollary 1 go through unchanged. Lemma 4 gets replaced by the following (analogous) lemma:

LEMMA 7. If $x_j \geq x_i$ and $F_i(w) > 0$, $\psi_i(1-F_i(w)) = \psi_j(1-F_j(w))$.

Proof. Assume to the contrary that there exists some w for which $\psi_i(1-F_i(w)) \neq \psi_j(1-F_j(w))$ and $F_i(w) > 0$. We split this into two cases.

Case I $\psi_i(1-F_i(w)) < \psi_j(1-F_j(w))$ and $F_i(w) > 0$. In this case, we get an immediate contradiction of Lemma 3.

Case I $\psi_i(1-F_i(w)) > \psi_j(1-F_j(w))$ and $F_i(w) > 0$. In this case, Lemma 3 implies

$F_j(w) = 0$. Let $\hat{w}$ denote the lowest wage offered by j. Both firms offer wages weakly below $\hat{w}$ and therefore by Corollary 1 offer the same amount of jobs weakly above. It follows that employment weakly above $\hat{w}$ is the same across the two firms and firm i is thus larger. This contradicts Lemma 2 which together with (6b) implies that i should be weakly smaller since $x_j \geq x_i$. □

Lemma 5 established that there cannot be gaps on the interior of the (overall) wage offer distribution and that the lowest wage must equal the reservation wage. The identical arguments apply here. The Lemma also established that there cannot be mass points. Here, there cannot be mass except at the reservation wage (because if the mass was on the interior no firm would offer just below but there cannot be any gaps) and there can only be mass posted by a single firm at the reservation (because otherwise the user cost is strictly lower just above contradicting (6a)).

Equilibrium Characterization

We can now proceed to characterize the equilibrium given the highest wage w_u. We will then use the characterization to prove that there exists a unique highest wage and therefore a unique equilibrium.

Firm level employment. Given the highest wage w_u, the user cost of labor is $w_u + \delta c$ which (by Lemma 2) is common across firms. Optimal employment in firm i is thus

$$n_i = \left(\frac{\alpha x_i}{w_u + \delta c} \right)^{\frac{1}{1-\alpha}}. \tag{80}$$

Offer rates. We can now solve for the offer rates of each firm. We do so recursively. It is useful to introduce the variables $x_{M+1} = 0$, $n_{M+1} = 0$ and $\psi_{M+1} = 0$ to simplify the expressions.

Above the lowest wage posted by the least productive firm M all firms post at the same rate due to Lemma 7. The turnover between the firms within that range of wages therefore perfectly offsets. Similarly, the offer rates at higher wage are identical across firms $j \in \{1, \ldots, i\}$ above the lowest wage posted by firm i. It will therefore be useful to define the total offer rate between the lowest wage posted by firm i and the lowest wage posted by firm $i+1$ as λ_j. This satisfies $\lambda_M = M\psi_M$, $\lambda_{M-1} = (M-1)(\psi_{M-1} - \psi_M)$ and in general $\lambda_i = i\left(\psi_i - \psi_{i+1}\right)$. We will now characterize $\{\lambda_j\}_{j\in\mathrm{M}}$ from which one can recover $\{\psi_j\}_{j\in\mathrm{M}}$.

Employment weakly above the lowest wage offered by i is $\sum_{j=i}^{M} j\left(n_j - n_{j+1}\right) = n_i i + \sum_{j=i+1}^{M} n_j$. The total offer rate weakly above the lowest wage offered by i is $\sum_{j=i}^{M} \lambda_j$. The offer rate therefore satisfies

$$\sum_{j=i}^{M} \lambda_j \cdot \left(1 - \sum_{j=1}^{M} n_j + s\left(\sum_{j=1}^{i} n_j - i n_i\right)\right) = \delta\left(n_i i + \sum_{j=i+1}^{M} n_j\right). \tag{81}$$

Expanding and rearranging gives

$$\sum_{j=i}^{M} \lambda_j = \frac{\delta\left(n_i i + \sum_{j=i+1}^{M} n_j\right)}{1 - (1-s)\sum_{j=1}^{M} n_j - s\left(n_i i + \sum_{j=i+1}^{M} n_j\right)}. \tag{82}$$

Subtracting the expression evaluated at $i+1$ from that evaluated at i gives

$$\lambda_i = \frac{\delta\left(n_i i + \sum_{j=i+1}^{M} n_j\right)}{1 - (1-s)\sum_{j=1}^{M} n_j - s\left(n_i i + \sum_{j=i+1}^{M} n_j\right)} - \frac{\delta\left(n_{i+1}(i+1) + \sum_{j=i+2}^{M} n_j\right)}{1 - (1-s)\sum_{j=1}^{M} n_j - s\left(n_{i+1}(i+1) + \sum_{j=i+2}^{M} n_j\right)}. \tag{83}$$

It follows directly from (80) that all the λ_i can thus be constructed in terms of w_u only.

We will later use that this expression is strictly decreasing in w_u provided that $x_i > x_{i+1}$ and zero otherwise. To see this, factorize to get

$$\lambda_i = \frac{\delta i\left(n_i - n_{i+1}\right)}{1 - (1-s)\sum_{j=1}^{M} n_j - s\left(n_i i + \sum_{j=i+1}^{M} n_j\right)} \frac{1 - (1-s)\sum_{j=1}^{M} n_j}{1 - (1-s)\sum_{j=1}^{M} n_j - s\left(n_{i+1}(i+1) + \sum_{j=i+2}^{M} n_j\right)}. \tag{84}$$

First note that (80) implies that employment at all firms is strictly decreasing in w_u. It follows that the first denominator is strictly increasing. The numerator of the first term is strictly decreasing since $n_i - n_{i+1} = \alpha^{\frac{1}{1-\alpha}}\left(x_i^{\frac{1}{1-\alpha}} - x_{i+1}^{\frac{1}{1-\alpha}}\right)(w_u + \delta c)^{-\frac{1}{1-\alpha}}$ and $\left(x_i^{\frac{1}{1-\alpha}} - x_{i+1}^{\frac{1}{1-\alpha}}\right) > 0$ when $x_i > x_{i+1}$. Write the second term as $1 + \frac{s\left(n_{i+1}(i+1) + \sum_{j=i+2}^{M} n_j\right)}{1-(1-s)\sum_{j=1}^{M} n_j - s\left(n_{i+1}(i+1) + \sum_{j=i+2}^{M} n_j\right)}$ to see that it is decreasing in w_u. Together this implies that each λ_i is strictly decreasing in w_u when $x_i > x_{i+1}$ (and otherwise zero when $x_i = x_{i+1}$).

Wages. The fact that the user cost must be the same across all posted wages determines the offer distribution. The offer distribution is uniform over $(w_{i-1}, w_i]$ with interval boundaries that solve

$$cs\frac{i-1}{i}\lambda_i = w_i - w_{i-1}. \tag{85}$$

It follows from this that the lowest interval $i = 1$ is a masspoint at w_r.

We can verify that this (uniquely) ensures that the user cost of labor is identical across all posted wages as follows. A worker with a wage w on the interval $(w_{i-1}, w_i]$ in a firm $z \leq i$ receives outside job offers from a higher interval at rate $s\sum_{j=i+1}^{M}\frac{j-1}{j}\lambda_j$. She receives outside job offers with a higher wage from the same interval at rate $s\frac{i-1}{i}\lambda_i\frac{w_i-w}{w_i-w_{i-1}}$. The quit rate for that worker thus can be written as

$$\begin{aligned}\sum_{j\neq z} s\psi_j(1-F_j(w)) &= s\frac{i-1}{i}\lambda_i\frac{w_i-w}{w_i-w_{i-1}} + s\sum_{j=i+1}^{M}\frac{j-1}{j}\lambda_j \\ &= \frac{w_i-w}{c} + \sum_{j=i+1}^{M}\frac{w_j-w_{j-1}}{c} = \frac{w_M-w}{c}.\end{aligned}$$

where the second equality uses (85). The user cost of labor at any wage is thus

$$w + \sum_{j\neq z} s\psi_j(1-F_j(w))c + \delta c = w_M + \delta c,$$

which is therefore equated across all posted wages and across all firms.

Reservation wage. For the reservation wage, we use (3c) derived in A.1 to get

$$\begin{aligned}w_r - b &= \frac{1-s}{s}\sum_{i=1}^{M}\int_{w_{i-1}}^{w_i}\frac{r+\delta-(r+\delta)+s\left(\sum_{j=i+1}^{M}\lambda_j+\lambda_i\frac{w_i-\tilde{w}}{w_i-w_{i-1}}\right)}{r+\delta+s\left(\sum_{j=i+1}^{M}\lambda_j+\lambda_i\frac{w_i-\tilde{w}}{w_i-w_{i-1}}\right)}d\tilde{w} \\ &= \frac{1-s}{s}\left(w_u - w_r - (r+\delta)\sum_{i=1}^{M}\int_{w_{i-1}}^{w_i}\frac{1}{r+\delta+s\left(\sum_{j=i+1}^{M}\lambda_j+\lambda_i\frac{w_i-\tilde{w}}{w_i-w_{i-1}}\right)}d\tilde{w}\right) \\ &= \frac{1-s}{s}\left(w_u - w_r + \sum_{i=1}^{M}(r+\delta)\left[\frac{w_i-w_{i-1}}{s\lambda_i}\log\left(r+\delta+s\left(\sum_{j=i+1}^{M}\lambda_j+\lambda_i\frac{w_i-\tilde{w}}{w_i-w_{i-1}}\right)\right)\right]_{w_{i-1}}^{w_i}\right) \\ &= \frac{1-s}{s}\left(w_u - w_r - c\sum_{i=1}^{M}(r+\delta)\frac{i-1}{i}\log\left(\frac{r+\delta+s\sum_{j=i}^{M}\lambda_j}{r+\delta+s\sum_{j=i+1}^{M}\lambda_j}\right)\right).\end{aligned}$$

Therefore,

$$w_r = sb + (1-s)w_u - (1-s)c\sum_{i=1}^{M}(r+\delta)\,\frac{i-1}{i}\log\left(\frac{r+\delta+s\sum_{j=i}^{M}\lambda_j}{r+\delta+s\sum_{j=i+1}^{M}\lambda_j}\right).$$

This gives an expression for the highest wage using (85)

$$\begin{aligned}
w_u &= cs\sum_{j=1}^{M}\frac{j-1}{j}\lambda_j + w_r \\
&= cs\sum_{j=1}^{M}\frac{j-1}{j}\lambda_j + sb + (1-s)w_u - (1-s)c\sum_{i=1}^{M}(r+\delta)\,\frac{i-1}{i}\log\left(\frac{r+\delta+s\sum_{j=i}^{M}\lambda_j}{r+\delta+s\sum_{j=i+1}^{M}\lambda_j}\right) \\
&= b + c\sum_{j=1}^{M}\frac{j-1}{j}\lambda_j - c\frac{1-s}{s}\sum_{i=1}^{M}(r+\delta)\,\frac{i-1}{i}\log\left(\frac{r+\delta+s\sum_{j=i}^{M}\lambda_j}{r+\delta+s\sum_{j=i+1}^{M}\lambda_j}\right). \qquad (86)
\end{aligned}$$

The derivative with respect to λ_z is

$$\begin{aligned}
\frac{\partial w_u}{\partial \lambda_z} &= c\frac{z-1}{z}\left(1-(1-s)\frac{r+\delta}{r+\delta+s\sum_{j=z}^{M}\lambda_j}\right) \\
&\quad + c(1-s)\sum_{i=1}^{z-1}(r+\delta)\,\frac{i-1}{i}\left(\frac{1}{r+\delta+s\sum_{j=i+1}^{M}\lambda_j} - \frac{1}{r+\delta+s\sum_{j=i}^{M}\lambda_j}\right). \qquad (87)
\end{aligned}$$

The first term is strictly positive for $z > 1$ and zero for $z = 1$. The second term is weakly positive for all $z \geq 1$. Above, we showed that all the λ_j are functions of the highest wage only and strictly decreasing in it (if $x_j > x_{j+1}$). It follows that the right hand side of (86) is continuous and strictly decreasing in w_u which implies that any highest wage is unique.

As $w_u \to \infty$, the right-hand side of (86) approaches b. Hence an equilibrium exists if, as w_u approaches its lowest feasible value, the right-hand side of (86) lies above that lowest feasible value. Inverting the unit-mass constraint of workers gives the lowest feasible highest wage, $w_u^{\min} \equiv \left(\sum_{i=1}^{M}(\alpha x_i)^{\frac{1}{1-\alpha}}\right)^{1-\alpha} - c\delta$. Therefore, the condition for existence is that, as $w_u \to w_u^{\min}$, the right-hand side of (86) exceeds $w_u^{\min}$.

G Characterization with heterogeneous costs

The analogous optimality condition for firm i is

$$\alpha x_i n_i^{\alpha-1} = \min_{w \geq w_r} \left\{ w - c_i \sum_{j \neq i} s\psi_j F_j\left(w^-\right) \right\} + c_i \left(\delta + \sum_{j \neq i} s\psi_j \right), \tag{88a}$$

$$\int_{w_r}^{\infty} \left(w - c_i s \sum_{j \neq i} \psi_j F_j\left(w^-\right) - \min_{\tilde{w} \geq w_r} \left\{ \tilde{w} - c_i s \sum_{j \neq i} \psi_j F_j\left(\tilde{w}^-\right) \right\} \right) dG_i(w) = 0. \tag{88b}$$

The proof proceeds with a number of Lemmas.

LEMMA 8. (i) For all $j \in \mathrm{M}$ and $w > w_r$, the offer distribution F_j is continuous (i.e., there are no mass points above the reservation wage). (ii) At most one firm can have a mass point at the reservation wage w_r.

Proof. Suppose firm i has a mass point at $w > w_r$, i.e., $\Delta F_i(w) \equiv F_i(w) - F_i(w^-) > 0$. Let $\underline{c} \equiv \min_{j \in \mathrm{M}} c_j$ and define $\varepsilon \equiv \frac{s\underline{c}\psi_i \Delta F_i(w)}{3}$. Consider any firm $z \neq i$. Compare offering a wage $\tilde{w} \in (w - \varepsilon, w]$ versus offering $w + \varepsilon$, the difference in user cost is:

$$\tilde{w} - w - \varepsilon + c_z s \sum_{j \neq z} \psi_j \left[F_j\left((w+\varepsilon)^- \right) - F_j(\tilde{w}^-) \right].$$

We have $\tilde{w} - w > -\varepsilon = -\frac{s\underline{c}\psi_i \Delta F_i(w)}{3}$. Since $\tilde{w} \leq w < w + \varepsilon$ and firm i has mass $\Delta F_i(w)$ at w:

$$\sum_{j \neq z} \psi_j \left[F_j\left((w+\varepsilon)^- \right) - F_j(\tilde{w}^-) \right] \geq \psi_i \Delta F_i(w). \tag{89}$$

Therefore, since $c_z \geq \underline{c}$, the difference in user cost is:

$$\begin{aligned} \tilde{w} - w - \varepsilon + c_z s \sum_{j \neq z} \psi_j \left[F_j\left((w+\varepsilon)^- \right) - F_j(\tilde{w}^-) \right] &\geq -2\frac{s\underline{c}\psi_i \Delta F_i(w)}{3} + c_z s \psi_i \Delta F_i(w) \\ &\geq -\frac{s\underline{c}\psi_i \Delta F_i(w)}{3} + \underline{c} s \psi_i \Delta F_i(w) = \frac{s\underline{c}\psi_i \Delta F_i(w)}{3} > 0. \end{aligned}$$

By condition (88b), this means no firm $z \neq i$ offers wages in $(w - \varepsilon, w]$.

Now consider firm i. Since no firm $j \neq i$ offers wages in $(w - \varepsilon, w]$, for any $w' \in$

$(w-\varepsilon, w)$

$$w' - c_i s \sum_{j\neq i} \psi_j F_j((w')^-) < w - c_i s \sum_{j\neq i} \psi_j F_j(w^-), \tag{90}$$

because $w' < w$ while $F_j((w')^-) = F_j(w^-)$ for all $j \neq i$.

This contradicts condition (88b).

Part (ii): At most one firm has a mass point at w_r. Suppose firms i and j both have mass points at w_r. For firm i, comparing w_r with $w_r + \epsilon$ for small $\epsilon > 0$:

$$\left[w_r + \epsilon - c_i s \sum_{k\neq i} \psi_k F_k((w_r+\epsilon)^-)\right] - \left[w_r - c_i s \sum_{k\neq i} \psi_k F_k(w_r^-)\right]$$
$$= \epsilon - c_i s \sum_{k\neq i} \psi_k [F_k((w_r+\epsilon)^-) - F_k(w_r^-)] = \epsilon - c_i s \psi_j [F_j(w_r) - F_j(w_r^-)] < 0,$$

for sufficiently small ϵ, since firm j has a mass point at w_r. This contradicts condition (88b) since $G_i(w_r) > 0$. □

LEMMA 9. The support of the wage distribution has no gaps above w_r. That is, if $G_i(w_2^-) < 1$ with $w_2 > w_r$ for some i, then for any $w_1 < w_2$ there exists a j such that $G_j(w_2^-) > G_j(w_1)$.

Proof. Suppose for contradiction that (w_1, w_2) is a gap with $w_2 > w_1 \geq w_r$. Define $\bar{\varepsilon} = \max_j \psi_j [1 - F_j(w_2^-)]$. Since $w_2 \in \bigcup_{j\in \mathrm{M}} \mathrm{supp}(G_j)$, we have $\bar{\varepsilon} > 0$. Let $\bar{c} = \max_j c_j$ and choose $\epsilon \in \left(0, \min\left\{\frac{w_2 - w_1}{3s(|\mathrm{M}|-1)\bar{c}}, \bar{\varepsilon}\right\}\right)$. Among firms with sufficient mass above w_2, let firm z be the one offering the lowest wages for mass ϵ

$$z = \underset{j:\psi_j[1-F_j(w_2^-)]>\epsilon}{\arg\min} \inf\{w \geq w_2 : \psi_j [F_j(w) - F_j(w_2^-)] \geq \epsilon\}. \tag{91}$$

Let $\hat{w} = \inf\{w \geq w_2 : \psi_z [F_z(w) - F_z(w_2^-)] \geq \epsilon\}$ denote this infimum for firm z. Since firm z has mass ϵ over $[w_2, \hat{w}]$, it follows that $G_z(\hat{w}) - G_z(w_2^-) > 0$. Consider another wage $\tilde{w} = w_1 + \frac{w_2 - w_1}{3}$. Comparing any wage in $[w_2, \hat{w}]$ to $\tilde{w}$, the wage decreases by at least $\frac{2(w_2 - w_1)}{3}$. Since (w_1, w_2) is a gap, for all $j \neq z$: $F_j(\tilde{w}^-) = F_j(w_1) = F_j(w_2^-) \leq F_j(\hat{w}^-)$. Therefore, the turnover cost increases by at most $c_z s \sum_{j\neq z} \psi_j [F_j(\hat{w}^-) - F_j(w_2^-)] \leq c_z s \epsilon$. By our choice of ϵ, the change in turnover is bounded above by $c_z s \epsilon < \frac{w_2 - w_1}{3}$. Therefore $w - c_z s \sum_{j\neq z} \psi_j F_j(w^-)$ is strictly higher over $[w_2, \hat{w}]$ than at $\tilde{w}$, contradicting (88b) since G_z places positive probability on $[w_2, \hat{w}]$. □

LEMMA 10. If firm i satisfies $G_i(w_1) - G_i(w_1 - \epsilon) > 0$ with $w_1 > w_r$ for all $\epsilon > 0$ but G_i is constant on (w_1, w_2) (i.e., $G_i(w_1) = G_i(w_2^-)$), then no firm j with $c_j \leq c_i$ places positive probability on (w_1, w_2).

Proof. Suppose for contradiction that firm z with $c_z \leq c_i$ satisfies $G_z(w_2^-) - G_z(w_1) > 0$. Since G_z increases on (w_1, w_2), pick any $\hat{w} \in (w_1, w_2)$ such that $G_z(\hat{w}) > G_z(w_1)$. By the optimality condition from (88b), there exists $w^* \in (w_1, \hat{w}]$ where G_z increases such that:

$$w^* - c_z s \sum_{k \neq z} \psi_k F_k((w^*)^-) = \min_{w \geq w_r} \left\{ w - c_z s \sum_{k \neq z} \psi_k F_k(w^-) \right\}. \tag{92}$$

Note that $F_z(w^*) = F_z(w^{*-}) > F_z(w_1)$ since $w^* > w_1$, G_z increased somewhere in $(w_1, w^*]$, and by Lemma 8 there are no mass points above w_r. Therefore, for any $\tilde{w} \geq w_r$:

$$w^* - c_z s \sum_{k \neq z} \psi_k F_k((w^*)^-) \leq \tilde{w} - c_z s \sum_{k \neq z} \psi_k F_k(\tilde{w}^-). \tag{93}$$

For firm i: Since G_i is constant on (w_1, w_2), $F_i((w^*)^-) = F_i(w_1)$. Since $G_i(w_1) - G_i(w_1 - \epsilon) > 0$ for all small $\epsilon > 0$, by optimality condition (88b), there exists $\tilde{w} \in (w_1 - \epsilon, w_1]$ such that:

$$\tilde{w} - c_i s \sum_{k \neq i} \psi_k F_k(\tilde{w}^-) = \min_{w \geq w_r} \left\{ w - c_i s \sum_{k \neq i} \psi_k F_k(w^-) \right\}, \tag{94}$$

which implies:

$$\tilde{w} - c_i s \sum_{k \neq i} \psi_k F_k(\tilde{w}^-) \leq w^* - c_i s \sum_{k \neq i} \psi_k F_k((w^*)^-). \tag{95}$$

Combining (93) and (95) gives:

$$c_i s \sum_{k \neq i} \psi_k \left[F_k((w^*)^-) - F_k(\tilde{w}^-) \right] \leq c_z s \sum_{k \neq z} \psi_k \left[F_k((w^*)^-) - F_k(\tilde{w}^-) \right]. \tag{96}$$

Using $c_z \leq c_i$

$$c_z s \sum_{k \neq z} \psi_k \left[F_k((w^*)^-) - F_k(\tilde{w}^-) \right] \leq c_i s \sum_{k \neq z} \psi_k \left[F_k((w^*)^-) - F_k(\tilde{w}^-) \right]. \tag{97}$$

Taking the limit as $\epsilon \to 0$ (so $\tilde{w} \to w_1$):

$$c_i s \sum_{k \neq i} \psi_k \left[F_k((w^*)^-) - F_k(w_1^-) \right] \leq c_i s \sum_{k \neq z} \psi_k \left[F_k((w^*)^-) - F_k(w_1^-) \right]. \tag{98}$$

This gives a contradiction since $F_z((w^*)^-) - F_z(w_1) > 0$ whereas $F_i((w^*)^-) - F_i(w_1) = 0$ (noting that there are no mass points by Lemma 8). □

LEMMA 11. If firm i offers wages over an interval (w_1, w_2) (i.e., $G_i(w_2^-) > G_i(w_1)$) with $w_1 \geq w_r$, then at least one other firm must also offer wages in this interval.

Proof. Assume for contradiction that no other firm offers wages in (w_1, w_2). Take any $\tilde{w} \in (w_1, w_2)$ with $G_i(w_2^-) > G_i(\tilde{w})$. Since no other firm offers wages in (w_1, w_2), for all $j \neq i$

$$F_j(((w_1 + \tilde{w})/2)^-) = F_j(w_1) = F_j(w_2^-). \tag{99}$$

Therefore

$$\begin{aligned}
&\int_{\tilde{w}}^{w_2} \left[w - c_i s \sum_{j \neq i} \psi_j F_j(w^-) - \min_{\underline{w} \geq w_r} \left\{ \underline{w} - c_i s \sum_{j \neq i} \psi_j F_j(\underline{w}^-) \right\} \right] dG_i(w) \\
&\geq \int_{\tilde{w}}^{w_2} \left[w - c_i s \sum_{j \neq i} \psi_j F_j(w^-) - \left((w_1 + \tilde{w})/2 - c_i s \sum_{j \neq i} \psi_j F_j(((w_1 + \tilde{w})/2)^-) \right) \right] dG_i(w) \\
&= \int_{\tilde{w}}^{w_2} \left[w - (w_1 + \tilde{w})/2 \right] dG_i(w) > 0.
\end{aligned} \tag{100}$$

This contradicts the optimality condition (88b) which requires that the integral is equal zero. □

LEMMA 12. The total offer rate over any interval $(w_1, w_2) \subseteq \cup_j \text{supp}(G_j)$ is bounded below by $\frac{w_2 - w_1}{s c_1}$ where $c_1 = \max_j c_j$.

Proof. Suppose for contradiction that the total offer rate over (w_1, w_2) satisfies

$$\sum_j \psi_j [F_j(w_2^-) - F_j(w_1)] = \frac{w_2 - w_1}{s c_1} - \bar{\epsilon}. \tag{101}$$

for some $\bar{\epsilon} > 0$.

Since $(w_1, w_2) \subseteq \cup_j \text{supp}(G_j)$ and there are no gaps (Lemma 9), for any $\epsilon \in (0, sc_1\bar{\epsilon})$ there exists firm i with $G_i(w_2) - G_i(w_2 - \epsilon) > 0$.

By optimality condition (88b)

$$
\begin{aligned}
0 &= \int_{w_2-\epsilon}^{w_2} \left(w - c_i s \sum_{j\neq i} \psi_j F_j(w^-) - \min_{\underline{w}\geq w_r} \left\{ \underline{w} - c_i s \sum_{j\neq i} \psi_j F_j(\underline{w}^-) \right\} \right) dG_i(w) \\
&\geq \int_{w_2-\epsilon}^{w_2} \left(w - c_i s \sum_{j\neq i} \psi_j F_j(w^-) - \left(w_1 - c_i s \sum_{j\neq i} \psi_j F_j(w_1^-) \right) \right) dG_i(w) \\
&\geq \int_{w_2-\epsilon}^{w_2} \left(w - w_1 - c_i s \left(\frac{w_2 - w_1}{sc_1} - \bar{\epsilon} \right) \right) dG_i(w) \geq (sc_1\bar{\epsilon} - \epsilon) \left(G_i(w_2) - G_i(w_2 - \epsilon) \right). \quad (102)
\end{aligned}
$$

This yields a contradiction of (88b) given our ϵ and the fact that $G_i(w_2) > G_i(w_2 - \epsilon)$. □

LEMMA 13. The total offer rate over any interval $[w_1, w_2) \subseteq \cup_j \text{supp}(G_j)$ with $w_1 > w_r$ is bounded above by $\frac{w_2 - w_1}{s} \left(\frac{1}{c_M} + \frac{1}{c_{M-1}} \right)$.

Proof. Assume for contradiction that the total offer rate over $[w_1, w_2)$ satisfies

$$
\gamma = \sum_j \psi_j [F_j(w_2^-) - F_j(w_1^-)] = \frac{w_2 - w_1 + \bar{\varepsilon}}{s} \left(\frac{1}{c_M} + \frac{1}{c_{M-1}} \right), \quad (103)
$$

for some $\bar{\varepsilon} > 0$.

Choose $\epsilon \in (0, \min\{\bar{\varepsilon}, w_1 - w_r\})$. Let $M_1 = \{j : G_j(w_1) - G_j(w_1 - \epsilon) > 0\}$ be the set of firms placing positive probability on $(w_1 - \epsilon, w_1]$. For any firm $i \in M_1$, by optimality condition (88b)

$$
\begin{aligned}
0 &= \int_{w_1-\epsilon}^{w_1} \left[w - c_i s \sum_{j\neq i} \psi_j F_j(w^-) - \min_{\underline{w}\geq w_r} \left\{ \underline{w} - c_i s \sum_{j\neq i} \psi_j F_j(\underline{w}^-) \right\} \right] dG_i(w) \\
&\geq \int_{w_1-\epsilon}^{w_1} \left[w - c_i s \sum_{j\neq i} \psi_j F_j(w^-) - \left(w_2 - c_i s \sum_{j\neq i} \psi_j F_j(w_2^-) \right) \right] dG_i(w) \\
&\geq [w_1 - \epsilon - w_2 + c_i s(\gamma - \kappa_i)] \left(G_i(w_1) - G_i(w_1 - \epsilon) \right), \quad (104)
\end{aligned}
$$

where $\kappa_i = \psi_i [F_i(w_2^-) - F_i(w_1)]$ is firm i's offer rate over $(w_1 - \epsilon, w_2)$ gives

$$
\kappa_i \geq \gamma - \frac{w_2 - w_1 + \epsilon}{sc_i}. \quad (105)
$$

Summing over all firms in M_1 gives

$$\sum_{i \in M_1} \kappa_i - \gamma \geq |M_1 - 1|\gamma - \sum_{i \in M_1} \frac{w_2 - w_1 + \epsilon}{sc_i} = |M_1 - 1| \left(\gamma \frac{w_2 - w_1 + \epsilon}{s} - \frac{\sum_{i \in M_1} \frac{1}{c_i}}{|M_1 - 1|} \right). \tag{106}$$

We will derive a contradiction by showing that the right hand side is negative. By Lemma 11, $|M_1| \geq 2$. The sum $\frac{\sum_{i \in M_1} \frac{1}{c_i}}{|M_1 - 1|}$ is maximized when $M_1 = \{M-1, M\}$. To see this, note that for fixed size of the set M_1, the summation is largest when the smallest values of c_i are included. Second, adding firm j with larger c_j to a set $M_1 = \{c_{j+1}, \ldots, c_M\}$ decreases the average

$$\frac{1}{|M_1|} \left(\sum_{i \in M_1} \frac{1}{c_i} + \frac{1}{c_j} \right) - \frac{1}{|M_1 - 1|} \sum_{i \in M_1} \frac{1}{c_i} = \frac{1}{|M_1||M_1 - 1|} \left(|M_1 - 1| \frac{1}{c_j} - \sum_{i \in M_1} \frac{1}{c_i} \right) < 0, \tag{107}$$

since $c_j \geq c_i$ for all $i \in M_1$. Therefore, with $M_1 = \{M-1, M\}$ and

$$\sum_{i \in M_1} \kappa_i \geq 2\gamma - \frac{w_2 - w_1 + \epsilon}{s} \left(\frac{1}{c_M} + \frac{1}{c_{M-1}} \right). \tag{108}$$

Since $\gamma = \frac{w_2 - w_1 + \bar{\varepsilon}}{s} \left(\frac{1}{c_M} + \frac{1}{c_{M-1}} \right)$

$$\sum_{i \in M_1} \kappa_i \geq \gamma + \frac{\bar{\varepsilon} - \epsilon}{s} \left(\frac{1}{c_M} + \frac{1}{c_{M-1}} \right) > \gamma, \tag{109}$$

where the last inequality follows because $\epsilon < \bar{\varepsilon}$. But $\sum_{i \in M_1} \kappa_i \leq \gamma$ by definition, giving a contradiction. □

The bounded offer rates imply each F_i is absolutely continuous. Therefore, if $F_i(w) < 1$, firm i must have strictly positive density on some interval(s) (w_1, w_2) above w.

LEMMA 14. If for all $j \in M_k$ but no other firm wages over a region are offered $(w_{k-1}, w_k) \subseteq \text{supp}(G_j)$, the amount of offers for $i \in M_k$ and $w \in (w_{k-1}, w_k)$ solves

$$s\psi_i[F_i(w) - F_i(w_{k-1})] = (w - w_{k-1}) \left(\frac{1}{|M_k| - 1} \sum_{j \in M_k} \frac{1}{c_j} - \frac{1}{c_i} \right). \tag{110}$$

Proof. For $i \in M_k$ to be indifferent between all wages in this region, the optimality condition (6) implies that for all $w, w' \in (w_{k-1}, w_k)$

$$w - c_i s \sum_{j \neq i} \psi_j F_j(w^-) = w' - c_i s \sum_{j \neq i} \psi_j F_j(w'^-), \tag{111}$$

Rearranging this expression gives

$$w - w' = c_i s \sum_{j \neq i} \psi_j [F_j(w^-) - F_j(w'^-)]. \tag{112}$$

Dividing this expression by c_i and summing over all firms $i \in \mathrm{M}_k$

$$\begin{aligned}
(w - w') \sum_{i \in \mathrm{M}_k} \frac{1}{c_i} &= s \sum_{i \in \mathrm{M}_k} \sum_{j \neq i} \psi_j [F_j(w^-) - F_j(w'^-)] && (113) \\
&= s \sum_{i \in \mathrm{M}_k} \sum_{j \in \mathrm{M}_k \setminus \{i\}} \psi_j [F_j(w^-) - F_j(w'^-)] && (114) \\
&= s(|\mathrm{M}_k| - 1) \sum_{j \in \mathrm{M}_k} \psi_j [F_j(w^-) - F_j(w'^-)], && (115)
\end{aligned}$$

where the second equality uses that $F_j(w^-) = F_j(w'^-)$ for all $j \notin \mathrm{M}_k$ (no offers in this interval), and the third follows since each firm $j \in \mathrm{M}_k$ appears $(|\mathrm{M}_k| - 1)$ times in the summation. We therefore get

$$\sum_{j \in \mathrm{M}_k} s\psi_j [F_j(w) - F_j(w')] = \frac{w - w'}{|\mathrm{M}_k| - 1} \sum_{j \in \mathrm{M}_k} \frac{1}{c_j}. \tag{116}$$

Subtracting equation (111) divided by c_i from (116)

$$s\psi_i [F_i(w) - F_i(w')] = (w - w') \left(\frac{1}{|\mathrm{M}_k| - 1} \sum_{j \in \mathrm{M}_k} \frac{1}{c_j} - \frac{1}{c_i} \right). \tag{117}$$

Setting $w' = w_{k-1}$ gives the result. Note that equation (117) implies $F_i(w)$ is linear in w over (w_{k-1}, w_k), so each firm $i \in \mathrm{M}_k$ offers wages uniformly distributed over this interval. □

LEMMA 15. If firm i offers wages in (w_1, w_2) but not in (w_2, w_3) with $w_3 > w_2$, then firm i does not offer wages above w_3, i.e., $F_i(w_3) = 1$.

Proof. Suppose for contradiction that firm i offers wages in (w_1, w_2) and (w_3, w_4) but not in (w_2, w_3).

Step 1. Since $G_i(w_2) = G_i(w_3^-)$ but $G_i(w_2) - G_i(w_2 - \epsilon) > 0$ for small $\epsilon > 0$, by Lemma 10, no firm j with $c_j \leq c_i$ offers in (w_2, w_3). Let $\mathrm{M}_3 = \{j : G_j(w_3^-) - G_j(w_2) > 0\}$ denote all firms offering somewhere in (w_2, w_3) which therefore all have $c_j > c_i$.

Step 2. If any firm $j \in \mathrm{M}_3$ does not offer in $(\underline{w}, \overline{w}) \subseteq (w_3, w_4)$, then since $G_j(w_3^-) - G_j(w_2) > 0$ and G_j is constant on $(\underline{w}, \overline{w})$, by Lemma 10, no firm with $c_k < c_j$ could offer in $(\underline{w}, \overline{w})$. But firm i with $c_i < c_j$ does offer there, contradiction. Therefore $\mathrm{M}_3 \subseteq \mathrm{M}_4$ where M_4 denotes firms offering everywhere in (w_3, w_4).

Step 3. Suppose firm j with $c_j < c_i$ offers in $(w_3, w_3 + \epsilon)$, i.e., $G_j((w_3 + \epsilon)^-) - G_j(w_3) > 0$, but not in (w_2, w_3). For firm i's indifference between (w_1, w_2) and (w_3, w_4), we have

$$w_3 - w_2 = c_i s \sum_{k \in \mathrm{M}_3} \psi_k [F_k(w_3^-) - F_k(w_2^-)]. \tag{118}$$

For $c_j < c_i$ and sufficiently small $\epsilon > 0$,

$$w_3 - w_2 - \epsilon > c_j s \sum_{k \in \mathrm{M}_3} \psi_k [F_k((w_3 + \epsilon)^-) - F_k(w_2^-)], \tag{119}$$

since $F_k((w_3 + \epsilon)^-) - F_k(w_3^-)$ is bounded above by a constant times ϵ by Lemma 13. Firm j strictly prefers wages at w_2 over wages above w_3, contradicting that $G_j((w_3 + \epsilon)^-) - G_j(w_3) > 0$ for j such that $c_j < c_i$.

Step 4. The interval (w_2, w_3) may consist of sub-intervals where different subsets $\mathrm{M}_{3,h} \subseteq \mathrm{M}_3$ are active. For each sub-interval h of length $w_{3,h} - w_{2,h}$, equation (116) gives

$$\sum_{j \in \mathrm{M}_{3,h}} s \psi_j [F_j(w_{3,h}) - F_j(w_{2,h})] = (w_{3,h} - w_{2,h}) \frac{1}{|\mathrm{M}_{3,h}| - 1} \sum_{j \in \mathrm{M}_{3,h}} \frac{1}{c_j}. \tag{120}$$

Summing over all sub-intervals and using that $\sum_h (w_{3,h} - w_{2,h}) = w_3 - w_2$, we obtain

$$\sum_h (w_{3,h} - w_{2,h}) \frac{1}{|\mathrm{M}_{3,h}| - 1} \sum_{j \in \mathrm{M}_{3,h}} \frac{1}{c_j} = \sum_{j \in \mathrm{M}_{3,h}} s \psi_j [F_j(w_3) - F_j(w_2)] = (w_3 - w_2) \frac{1}{c_i}, \tag{121}$$

where the second inequality comes from the fact that firm i offers over both (w_1, w_2) and (w_3, w_4). The weighted $((w_{3,h} - w_{2,h})/(w_3 - w_2))$ average is therefore equal to $\frac{1}{c_i}$.

Step 5. We cannot have $\frac{1}{|\mathrm{M}|-1} \sum_{j \in \mathrm{M}} \frac{1}{c_j} > \frac{1}{c_i}$ while $\frac{1}{|\mathrm{M}|-2} \sum_{j \in \mathrm{M} \setminus \{z\}} \frac{1}{c_j} \leq \frac{1}{c_i}$ for $c_z \geq c_i$.

From the first inequality,

$$\sum_{j \in \mathrm{M} \setminus \{z\}} \frac{1}{c_j} > (|\mathrm{M}|-1)\frac{1}{c_i} - \frac{1}{c_z} \geq (|\mathrm{M}|-2)\frac{1}{c_i}.$$

Dividing by $(|\mathrm{M}|-2)$ contradicts the second inequality.

Step 6. For firm i to have positive offers in $(w_3, w_3+\epsilon)$ with $\mathrm{M}_\epsilon = \{j : G_j((w_3+\epsilon)^-) - G_j(w_3) > 0\}$, we need (from (110))

$$\frac{1}{|\mathrm{M}_\epsilon|-1} \sum_{j \in \mathrm{M}_\epsilon} \frac{1}{c_j} > \frac{1}{c_i}. \tag{122}$$

But $\mathrm{M}_3 \subseteq \mathrm{M}_\epsilon$ (from Step 2), where all $j \in \mathrm{M}_3$ have $c_j > c_i$ (from Step 1) and all $k \in \mathrm{M}_\epsilon \setminus \mathrm{M}_3$ have $c_k \geq c_i$ (from Step 3). From Step 4, subsets of M_3 achieve weighted average $\frac{1}{c_i}$. By Step 5, forming M_ϵ by adding firms with $c_k \geq c_i$ to this subset of M_3 cannot produce an average exceeding $\frac{1}{c_i}$, contradicting the requirement for positive offers. □

We will now show how we can recursively solve for the equilibrium starting from the highest wage. Take a wage w_k and denote $\bar{\mathrm{M}}_k$ the set of firms that post at or below w_k.

LEMMA 16. Given $\bar{\mathrm{M}}_k$, the set of firms posting on interval k is the unique set $\mathrm{M}_k \subseteq \bar{\mathrm{M}}_k$ such that

$$\frac{1}{|\mathrm{M}_k|-1} \sum_{j \in \mathrm{M}_k} \frac{1}{c_j} > \max_{j \in \mathrm{M}_k} \left\{ \frac{1}{c_j} \right\}, \tag{123}$$

and

$$\frac{1}{|\mathrm{M}_k|-1} \sum_{j \in \mathrm{M}_k} \frac{1}{c_j} \leq \min_{j \in (\bar{\mathrm{M}}_k \setminus \mathrm{M}_k)} \left\{ \frac{1}{c_j} \right\}. \tag{124}$$

Proof. By Lemma 10, if firm i posts in interval k, all firms with $c_j \leq c_i$ also post in this interval.

For equilibrium, condition (123) must hold, otherwise equation (117) implies negative offer rates for the lowest-cost firm in M_k.

Similarly, (124) must hold, otherwise by equation (116), the highest-cost firm in $\bar{\mathrm{M}}_k \setminus \mathrm{M}_k$ would prefer posting at the top of interval k rather than below.

To show uniqueness, suppose two different sets M_k and M'_k both satisfy these conditions. Without loss of generality, assume that $|\mathrm{M}'_k| > |\mathrm{M}_k|$. By Lemma 10, we know that if there is a firm j which is not in M_k, there cannot be a higher cost firm in M_k. This implies that within $\bar{\mathrm{M}}_k$, the firms posting in interval k must be those with the highest costs. By definition, all the firms in $\bar{\mathrm{M}}_k$ post wages lower than w_k. M_k therefore contains the $|\mathrm{M}_k|$ highest cost firms from $\bar{\mathrm{M}}_k$ and similarly M'_k contains the $|\mathrm{M}'_k|$ highest cost firms from $\bar{\mathrm{M}}_k$. Define $\bar{c} \equiv \min_{j \in M'_k} c_j$. Since $|\mathrm{M}'_k| > |\mathrm{M}_k|$ and both contain the highest-cost firms, we have $\mathrm{M}_k \subset \mathrm{M}'_k$ and $\bar{c} \in \mathrm{M}'_k \setminus \mathrm{M}_k$. We get a contradiction since under

$$\frac{1}{|M'_k| - 1} \sum_{j \in \mathrm{M}'_k} \frac{1}{c_j} \quad > \quad \frac{1}{\bar{c}}, \tag{125}$$

which implies that

$$\sum_{j \in \mathrm{M}_k} \frac{1}{c_j} \quad > \quad (|M'_k| - 1)\frac{1}{\bar{c}} - \sum_{j \in \mathrm{M}'_k \setminus \mathrm{M}_k} \frac{1}{c_j} \geq (|M_k| - 1)\frac{1}{\bar{c}}, \tag{126}$$

since $\frac{1}{\bar{c}} \geq \frac{1}{c_j}$ for $j \in \mathrm{M}'_k$. This contradicts condition (124), since $\bar{c} \in \bar{\mathrm{M}}_k \setminus \mathrm{M}_k$ and the average exceeds $\frac{1}{\bar{c}}$. □

We can now construct the equilibrium given any conjectured highest wage and unemployment rate. One then iterates over this guess to find the equilibrium. We do so by using the previous results to characterize the equilibrium given the conjectured unemployment u and highest wage w_u. The equilibrium will have (up to) M intervals and we will recursively construct the equilibrium starting from the highest interval.

We define four objects:

- The highest wage in interval k: w_k (where $w_M = w_u$)
- The total offer rate in each interval: λ_k (where $\lambda_{M+1} = 0$ for notational simplicity)
- The set of firms posting at or below interval k: $\bar{\mathrm{M}}_k$ (initially $\bar{\mathrm{M}}_M = \mathrm{M}$, all firms)
- Employment of firm i in interval k: $\hat{n}_{i,k}$
- Total employment of firm i above interval k: $\bar{N}_{i,k} = \sum_{\tilde{k} > k} \hat{n}_{i,\tilde{k}}$

LEMMA 17. Given $\{w_j, \bar{\mathrm{M}}_j, \lambda_j, \{\hat{n}_{i,j}\}_{i \in \mathrm{M}}\}_{j \geq k}$, all variables can be uniquely determined for interval k.

Proof. Given $\bar{\mathrm{M}}_k$, the set of firms in this interval is characterized in Lemma 16. Denote this set by M_k. The user cost of labor for each firm $i \in \mathrm{M}_k \setminus \mathrm{M}_{k+1}$ is

$$\omega_i = w_k + c_i \left(\delta + s \sum_{\tilde{k} > k} \lambda_{\tilde{k}} \right). \tag{127}$$

Employment for each firm $i \in \mathrm{M}_k$ satisfies $n_i = \left(\frac{\alpha x_i}{\omega_i} \right)^{\frac{1}{1-\alpha}}$. The remaining employment in regions k or below for firm i is $n_i - \bar{N}_{i,k}$. From equations (117) and (116), firm i's share of employment in the interval is

$$v_i = \frac{\frac{1}{|\mathrm{M}_k|-1} \sum_{j \in \mathrm{M}_k} \frac{1}{c_j} - \frac{1}{c_i}}{\frac{1}{|\mathrm{M}_k|-1} \sum_{j \in \mathrm{M}_k} \frac{1}{c_j}}. \tag{128}$$

The firm that first reaches its desired employment is

$$z = \arg\min_{i \in \mathrm{M}_k} \frac{n_i - \bar{N}_{i,k}}{v_i}. \tag{129}$$

Total employment in the interval is therefore

$$N_k = \frac{n_z - \bar{N}_{z,k}}{v_z}. \tag{130}$$

The flow balance equation gives λ_k

$$\lambda_k \left(u + s \left(1 - u - \sum_j \bar{N}_{j,k} - N_k \right) \right) = \left(\delta + s \sum_{j > k} \lambda_j \right) N_k. \tag{131}$$

From equation (116), we get w_{k-1} via

$$\lambda_k = s(w_k - w_{k-1}) \frac{1}{|\mathrm{M}_k| - 1} \sum_{j \in \mathrm{M}_k} \frac{1}{c_j}. \tag{132}$$

Employment for firm $j \in \mathrm{M}_k$ in interval k, i.e., $\hat{n}_{j,k}$, is given by

$$\hat{n}_{j,k} = v_j N_k. \tag{133}$$

Lastly, we update $\bar{\mathrm{M}}_{k-1} = \bar{\mathrm{M}}_k \setminus \{i \in \mathrm{M}_k : n_i = \bar{N}_{i,k} + \hat{n}_{i,k}\}$. □

H Equilibrium under the model in Section 4.2

The proof of common user cost in Lemma 2 goes though. Since the user cost of labor and the rental cost of capital is the same across firms, the level of employment and output are also equated across firms. The steps to show that the equilibrium is symmetric in Proposition 1 go through. We will therefore drop the i subscripts in the derivations. Denote the common user cost of labor by $\omega = w_r + (r + \delta + s(M-1)\psi)c$. The first order condition of (30) with respect to n_i (evaluated at n) is given by

$$
\begin{aligned}
\frac{\theta^{\frac{1}{\sigma}} n^{\frac{\sigma-1}{\sigma}-1}}{(1-\theta)^{\frac{1}{\sigma}}} \left(\frac{\left(\frac{y}{x}\right)^{\frac{\sigma-1}{\alpha\sigma}} - \theta^{\frac{1}{\sigma}} n^{\frac{\sigma-1}{\sigma}}}{(1-\theta)^{\frac{1}{\sigma}}} \right)^{\frac{\sigma}{\sigma-1}-1} r_k &= \omega, \\
n^{\frac{\sigma-1}{\sigma}} &= \frac{\theta^{\frac{\sigma-1}{\sigma}} \left(\frac{1}{\omega}\right)^{\sigma-1} \left(\frac{y}{x}\right)^{\frac{\sigma-1}{\alpha\sigma}}}{(1-\theta)\left(\frac{1}{r_k}\right)^{\sigma-1} + \theta\left(\frac{1}{\omega}\right)^{\sigma-1}},
\end{aligned}
$$

and the second derivative is negative. Rearranging gives

$$
n = \frac{\theta\left(\frac{1}{\omega}\right)^{\sigma}\left(\frac{y}{x}\right)^{\frac{1}{\alpha}}}{\left((1-\theta)\left(\frac{1}{r_k}\right)^{\sigma-1} + \theta\left(\frac{1}{\omega}\right)^{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}}}. \tag{134}
$$

The first order condition of (30) with respect to y_i after rearranging reads

$$
p\left(y_i, \mathbf{Y}_{-i}\right)\left(1 - \frac{1}{\eta}\frac{y_j}{\sum_j y_j}\right) = r_k \left(\frac{\left(\frac{y_i}{x}\right)^{\frac{\sigma-1}{\alpha\sigma}} - \theta^{\frac{1}{\sigma}} n^{\frac{\sigma-1}{\sigma}}}{(1-\theta)^{\frac{1}{\sigma}}} \right)^{\frac{\sigma}{\sigma-1}-1} \frac{1}{\alpha} \frac{\frac{1}{x}\left(\frac{y_i}{x}\right)^{\frac{\sigma-1}{\alpha\sigma}-1}}{(1-\theta)^{\frac{1}{\sigma}}},
$$

where again the second derivative is negative. After substituting for $p\left(y, \mathbf{Y}_{-i}\right)$ from (27) and evaluating at a symmetric equilibrium the first order condition becomes

$$
\begin{aligned}
\bar{Q}^{\frac{1}{\eta}} (My)^{-\frac{1}{\eta}} \left(1 - \frac{1}{\eta}\frac{1}{M}\right) &= r_k \left(\frac{\left(\frac{y}{x}\right)^{\frac{\sigma-1}{\alpha\sigma}} - \theta^{\frac{1}{\sigma}} n^{\frac{\sigma-1}{\sigma}}}{(1-\theta)^{\frac{1}{\sigma}}} \right)^{\frac{\sigma}{\sigma-1}-1} \frac{1}{\alpha} \frac{\frac{1}{x}\left(\frac{y}{x}\right)^{\frac{\sigma-1}{\alpha\sigma}-1}}{(1-\theta)^{\frac{1}{\sigma}}} \\
&= \left(\frac{1}{(1-\theta)\left(\frac{1}{r_k}\right)^{\sigma-1} + \theta\left(\frac{1}{\omega}\right)^{\sigma-1}} \right)^{\frac{1}{\sigma-1}} \frac{1}{\alpha}\frac{1}{x}\left(\frac{y}{x}\right)^{\frac{1-\alpha}{\alpha}},
\end{aligned}
$$

where the second line substitutes for n from (134). Rearranging gives output as

$$\frac{y}{x}=\left(\frac{\alpha x\bar{Q}^{\frac{1}{\eta}}\left(M\right)^{-\frac{1}{\eta}}\left(1-\frac{1}{\eta}\frac{1}{M}\right)}{\left(\frac{1}{(1-\theta)\left(\frac{1}{r_k}\right)^{\sigma-1}+\theta\left(\frac{1}{\omega}\right)^{\sigma-1}}\right)^{\frac{1}{\sigma-1}}}\right)^{\frac{1}{\frac{1-\alpha}{\alpha}+\frac{1}{\eta}}}.$$

Using (134) for n and substituting for y using the above equation and rearranging gives

$$\begin{aligned} n &= \frac{\theta\left(\frac{1}{\omega}\right)^{\sigma}}{\left((1-\theta)\left(\frac{1}{r_k}\right)^{\sigma-1}+\theta\left(\frac{1}{\omega}\right)^{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}}}\left(\frac{\alpha x\bar{Q}^{\frac{1}{\eta}}\left(M\right)^{-\frac{1}{\eta}}\left(1-\frac{1}{\eta}\frac{1}{M}\right)}{\left(\frac{1}{(1-\theta)\left(\frac{1}{r_k}\right)^{\sigma-1}+\theta\left(\frac{1}{\omega}\right)^{\sigma-1}}\right)^{\frac{1}{\sigma-1}}}\right)^{\frac{1}{1-\alpha+\frac{\alpha}{\eta}}} && (135)\\ &= \frac{\left(\frac{1}{\omega}\right)^{\sigma}\theta\left(\alpha x\bar{Q}^{\frac{1}{\eta}}\left(M\right)^{-\frac{1}{\eta}}\left(1-\frac{1}{\eta}\frac{1}{M}\right)\right)^{\frac{1}{1-\alpha+\frac{\alpha}{\eta}}}}{\left((1-\theta)\left(\frac{1}{r_k}\right)^{\sigma-1}+\theta\left(\frac{1}{\omega}\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}\left(\sigma-\frac{1}{1-\alpha+\frac{\alpha}{\eta}}\right)}}. && (136)\end{aligned}$$

After further substituting for $\omega = w_r + (r+\delta+s(M-1)\psi)c$ and $n=\frac{\psi}{\delta+M\psi}$, we get

$$\frac{\psi}{\delta+M\psi}=\frac{\left(\frac{1}{w_r+(r+\delta+s(M-1)\psi)c}\right)^{\sigma}\theta\left(\alpha x\bar{Q}^{\frac{1}{\eta}}\left(M\right)^{-\frac{1}{\eta}}\left(1-\frac{1}{\eta}\frac{1}{M}\right)\right)^{\frac{1}{1-\alpha+\frac{\alpha}{\eta}}}}{\left((1-\theta)\left(\frac{1}{r_k}\right)^{\sigma-1}+\theta\left(\frac{1}{w_r+(r+\delta+s(M-1)\psi)c}\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}\left(\sigma-\frac{1}{1-\alpha+\frac{\alpha}{\eta}}\right)}}. \qquad (137)$$

The LHS is increasing in ψ whereas the right-hand side is decreasing given $\sigma,\eta,\alpha>0$. Both sides are continous and the LHS goes to $1/M$ as $\psi\to\infty$ whereas the RHS goes to zero. As $\psi\to 0$, the LHS goes to 0 whereas the RHS remains finite. An unique value of ψ satisfying the equation therefore exists.