NBER WORKING PAPER SERIES

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Working Paper No. 3195

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
December 1989

Alberto Giovannini was partially supported by a CEPR grant from the Ford Foundation and the Alfred P. Sloan Foundation, Philippe Jorian was supported by the Columbia Business School Faculty Research Fellowship. This paper is part of NBER's research program in Financial Markets and Monetary Economics. Any opinions expressed are those of the authors not those of the National Bureau of Economic Research.

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ABSTRACT

This paper provides two alternative estimation and testing procedures of a representative-agent model of asset pricing which relies on a particular parametrization of non-expected-utility preferences. The first is based on maximum-likelihood estimates, supplemented with an explicit model of time varying first and second moments (where the time-variation of second moments in modelled with an ARCH-Autoregressive Conditionally Heteroskedastic-process); the second is based on generalized-method-of moments estimates. We perform our tests on a data set that includes monthly observations of rates of return on US stock prices and US consumption of nondurables and services. Our results are directly comparable to a test of the dynamic capital asset pricing model performed by Hansen and Singleton (1983), and to a recent test of the model studied here performed by Epstein and Zin (1989).

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1 Introduction

A popular model of asset pricing, developed by Lucas (1978) and Breeden (1979) assumes that a single, infinitely-lived, "representative" consumer-investor chooses her consumption plan and portfolio composition to maximize expected utility. Equilibrium returns on individual assets are determined by the covariance of those assets' payoffs with the marginal rate of substitution in consumption, a function of consumption growth and a taste parameter. The poor empirical performance of this model[1] has led researchers to question a number of its assumptions, and—as a result—to develop a number of alternative theories.

The specification of preferences is a crucial building block of the representative-consumer asset pricing model. Typically, preferences are assumed to be of the time-separable, isoelastic family, characterized by a single parameter. This specification has been recently criticized mainly on two grounds. First, the property of time-separability makes the marginal rate of substitution in consumption independent of past consumption experiences. By contrast, non-separable utility functions can induce, in equilibrium, smooth consumption paths that might resemble the data more closely. Bergman (1985), Sundaresan (1989) and Constantinides (1989) explore the effects of allowing for non-separable preferences on equilibrium asset returns.

A second type of criticism regards the interpretation of the elasticity parameter in the utility function. In the absence of uncertainty, that parameter represents the elasticity of intertemporal substitution in consumption, while in a static, stochastic setting it measures Arrow-Pratt's coefficient of relative risk aversion. In the

[1]See, for a recent comprehensive study, Breeden, Gibbons and Litzenberger (1989).

stochastic and dynamic asset pricing model that parameter is, however, difficult to interpret: it has been interchangeably labelled risk aversion and intertemporal substitution. A number of recent papers have helped clarifying this problem. Epstein and Zin (1989a), Farmer (1988) and Weil (1988) have developed isoelastic preferences that separate attitudes towards risk from attitudes towards allocation of consumption over time, by relaxing the expected-utility restriction. These preferences generalize the time-separable expected utility functionals originally postulated in dynamic asset pricing models.

The purpose of this paper is to test alternative specifications of an asset pricing model that relies on the isoelastic non-expected-utility preferences developed by the above-mentioned authors. Since these preferences subsume the standard time-separable expected utility function applied in earlier statistical tests, we are able to compare the empirical performance of both specifications of preferences. We apply the model to monthly US data on consumption and stock returns.

Our results are directly comparable with two other tests of asset pricing models. Like Hansen and Singleton (1983) we test the restrictions imposed by the model on the relation between (conditional) expected returns on individual assets and their (conditional) covariance with consumption growth, and, given the preference we postulate, the rate of return on the market portfolio. This test is based on a time-series extension of the cross-section tests performed by Giovannini and Weil (1989). We generalize Hansen and Singleton's tests by allowing time variation in conditional second moments, which we assume follow an Autoregressive Conditionally Heteroskedastic (ARCH) scheme, developed by Engle (1982). In addition, we also study the general—and less restricted—specification used by Epstein and Zin (1989b), which exploits the orthogonality restrictions implicit in the first-order conditions of the investor's optimal plan.[2] In both cases, we are interested to verify whether the more general specification of non-expected-utility preferences results in any appreciable improvement in the statistical performance of the asset pricing model.

The paper is organized as follows. Sections 2 and 3 describe the model and the data set, respectively. Sections 4 and 5 discuss the results, while section 6 contains a few concluding remarks.

2 Asset Pricing with Non-Expected-Utility Preferences

We consider an infinitely-lived consumer-investor, who chooses her consumption and portfolio composition to maximize utility. There is only one good in the economy, but N nonreproducible and nondepreciable assets which generate the consumption good stochastically. The shares of the N assets in the investor's portfolio are $\alpha_i, i = 1, \ldots, N$ and are arranged in the vector α. We assume that preferences are represented by the isoelastic utility function independently proposed by Epstein and Zin (1989a) and Weil (1988).[3] Formally the investor problem is:

$$\max_{C_t,\alpha_t} U[C_t, E_t V(w_{t+1}, s_{t+1})] =$$
$$\left\{(1-\delta)C_t^{1-\rho} + \delta\left(E_t V(w_{t+1}, s_{t+1})\right)^{\frac{1-\rho}{1-\gamma}}\right\}^{\frac{1-\gamma}{1-\rho}}, \tag{1}$$

[2]Hall (1988), Zin (1987) and Attanasio and Weber (1989) study a similar model, the two-period "Ordinal Certainty Equivalent" model of Selden (1978).

[3]This is a tractable parametrisation of the preferences originally developed by Kreps and Porteus (1979).

subject to:

$$w_{t+1} = R_{m,t+1}(w_t - C_t) \tag{2}$$

$$R_{m,t+1} = \sum_{i=1}^{N} \alpha_{i,t} R_{i,t+1} \tag{3}$$

$$\sum_{i=1}^{N} \alpha_{i,t} = 1 \tag{4}$$

where the indirect utility function, V is defined as follows:

$$V(w_t, s_t) = \max_{C_t, \alpha_t} U[C_t, E_t V(w_{t+1}, s_{t+1})]$$

with w_t representing the investor's total wealth at the beginning of time t, and s_t the state of the economy at t. C_t is consumption at t and R_i are (1 plus) the rates of return on the assets. Equation (2) describes the evolution of the investor's wealth, equation (3) defines the rate of return on the portfolio and equation (4) defines the portfolio shares.

The parameter $\gamma \geq 0$ ($\gamma \neq 1$) can be interpreted as the Arrow-Pratt coefficient of relative risk aversion, while the parameter $1/\rho \geq 0$ ($\rho \neq 1$) represents the elasticity of intertemporal substitution, and $\delta \in (0, 1)$ is the subjective discount factor.[4]

The first-order conditions for the problem (1)-(4) are, together with (2)-(4):

$$1 = E_t \left[U_{2,t} \frac{U_{1,t+1}}{U_{1,t}} R_{i,t+1} \right], \quad i = 1, \ldots, N. \tag{5}$$

where $U_{1,t}$ and $U_{2,t}$ represent the partial derivatives of the "aggregator" function U at time t with respect to its first and second argument, respectively. To solve for U_1 and U_2 in terms of the preference parameters we postulate, and verify, that $V(w_t, s_t) = \Phi(s_t) w_t^{1-\gamma}$ and $c_t = \mu(s_t) w_t$. The result is:

$$1 = E_t \left\{ \left[\delta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \right]^{\frac{1-\gamma}{1-\rho}} \left[\frac{1}{R_{m,t+1}} \right]^{1-\frac{1-\gamma}{1-\rho}} R_{i,t+1} \right\}, \tag{6}$$

[4]See Weil (1988) for a good illustration of the properties of the functional form adopted here.

Equation (6) has been extensively discussed in the papers we cited above, where it is stressed that both the rate of growth of consumption and the market rate of return determine the equilibrium returns on individual assets. We just note, following Giovannini and Weil (1989), that in this model myopia in consumption-savings decisions—that is a consumption function that does not depend on expectations of future variables, but is simply a constant times current wealth—is the result of a unit-elastic intertemporal substitution but that myopia in portfolio allocation decisions—portfolio allocation rules that depend only on asset returns at the current time—arises when the coefficient of relative risk aversion is unity, for any values of ρ. In other words, the static CAPM arises from unit risk aversion, and not from offsetting income and substitution effects in the consumption-savings problem.[5] Equations (6) can be exploited in the empirical analysis, once market equilibrium conditions are solved jointly with the optimal decision rules of the investor. These conditions state that the representative investor has to consume all output produced (since the utility functions we adopt displays non-satiation, and output is perishable) and hold all available assets. Then $R_{m,t}$ represents the return on the market portfolio, and the vector α_t represents the shares of all assets in the market portfolio. Hence equations (6) have to be satisfied in equilibrium, and represent a testable constraint in the joint stochastic process of consumption and asset returns. These equations have been estimated and tested by Epstein and Zin (1989b).

[5]When intertemporal substitution equals 1 and asset returns are lognormal, however, equation (6) becomes:

$$E_t\left[R_{i,t+1}R_{m,t+1}^{-(\gamma-\eta)}\right] = E_t\left[R_{i,t+1}R_{m,t+1}^{1-(\gamma-\eta)}\right]$$

with $\eta = (1-\gamma)\pi\delta/(1-\pi\delta)$ where π is the autoregressive coefficient for the process followed by the log of R_m.See Giovannini and Weil (1989) and Giovannini and Jorion (1989).

As Giovannini and Weil (1989) show, additional assumptions allow to solve out explicitly for equilibrium excess returns. Consider the case where the logarithm of the return on on the market and individual assets, as well as the growth of consumption are—conditionally on information available at time t—jointly normal, with mean

$$\xi_{i,t} = (\bar{r}_{i,t}, \bar{r}_{m,t}, \bar{c}_t)' \tag{7}$$

and variance-covariance matrix

$$\Sigma_{i,t} = \begin{pmatrix} \sigma^2_{i,t} & \sigma_{im,t} & \sigma_{ie,t} \\ \sigma_{im,t} & \sigma^2_{m,t} & \sigma_{me,t} \\ \sigma_{ie,t} & \sigma_{me,t} & \sigma^2_{e,t} \end{pmatrix}, \tag{8}$$

where c denotes the logarithm of (1 plus) the rate of growth of consumption, and the subscript t indicates that the moments are conditional on time t information, while returns are realized at time $t+1$. Of course, given the definition of the market return (3), the assumption of joint log-normality is, strictly speaking, not correct: a linear combination of log-normal variables is not log-normal. Yet joint log-normality of returns can hold exactly in continuous time, and might be approximately correct in discrete time, since returns are numbers that do not deviate much from 1. In addition, the appropriateness of this approximation can be tested, and we do so in section 4.

The first-order condition (6) can now be rewritten, using our distributional assumption, for any asset i and for the return (at time $t+1$) on a riskfree bond, $r_{f,t}$ whose value is known with certainty at time t. The result is:

$$\bar{r}_{i,t} + \sigma^2_{i,t}/2 = r_{f,t} + \rho\frac{1-\gamma}{1-\rho}\sigma_{ie,t} + \frac{\gamma-\rho}{1-\rho}\sigma_{im,t} \tag{9}$$

where:

$$r_{f,t} \;\; = \;\; -ln\delta + \rho\frac{1-\gamma}{1-\rho}\bar{c}_t + \frac{\gamma-\rho}{1-\rho}\bar{r}_{mt} + \rho\frac{1-\gamma}{1-\rho}\frac{\gamma-\rho}{1-\rho}\sigma_{me,t}$$

$$-\frac{1}{2}\left[\rho^2\left(\frac{1-\gamma}{1-\rho}\right)^2\sigma^2_{c,t}+\left(\frac{\gamma-\rho}{1-\rho}\right)^2\sigma^2_{m,t}\right] \tag{10}$$

Equation (9) is just a generalization of the one originally estimated by Hansen and Singleton (1983). If preferences are like those assumed by Hansen and Singleton the coefficient of relative risk aversion γ equals the inverse of the elasticity of intertemporal substitution ρ, and the equilibrium return on an asset depends only on the riskfree rate, and its covariance with the rate of growth of consumption:

$$\bar{r}_{i,t}+\sigma^2_{i,t}/2-r_{f,t}=\rho\sigma_{ic,t}$$

If the coefficient of relative risk aversion equals unity, equation (9) implies

$$\bar{r}_{i,t}+\sigma^2_{i,t}/2-r_{f,t}=\sigma_{im,t},$$

the standard static asset pricing equation with logarithmic utility. In the more general model we estimate, both the covariance with the market rate of return and the covariance with the rate of growth of consumption affect equilibrium returns on individual assets.

3 The Data

One strong prediction of the model is that first-order conditions and asset pricing equations hold for any assets available to the consumer-investor. We carry out our tests on five industry indices computed from stocks listed on the New York Stock Exchange (NYSE), jointly with the value-weighted NYSE market index. These data are obtained from the Center for Research in Security Prices of the University of Chicago (CRSP).

The data are monthly and cover the period from January 1953 to December 1987. The value-weighted industry indices are constructed from an exhaustive classification of the market into five industry groups: primary, manufacturing, tranportation, trade, finance and services.[6] Real returns are measured using the nodurables consumption price deflator.

The consumption measure is real *per capita* consumer expenditure in nondurables and services (measured in terms of nondurables) from the US National Income Accounts. Because some of the instruments consist of lagged values, the estimation starts in April 1953.

Table I contains summary statistics for our data set. The table reports means, standard deviations, and autocorrelation coefficients at lags 1 to 12 months of all series used in the empirical tests. The standard error of the autocorrelation coefficients is approximately 0.054 under the null hypothesis of no autocorrelation. Notice that the first-order autocorrelation coefficient for consumption is negative and significantly different from zero. This result is similar, but not identical due to our longer sample, to that reported by Breeden, Gibbons and Litzenberger (1989). As these authors point out, this evidence is, *prima facie*, inconsistent with the presence of time-aggregation biases in the data, and is perhaps suggestive of the presence of errors in measurement of consumption. Other noteworthy features of the data include the difference in the standard deviations of consumption and asset returns (the standard errors of asset returns are about 10 times the standard error of consumption growth) and the significantly positive autocorrelation coefficient of two indices of returns (trade and finance and services). The low volatility

[6]These were obtained by aggregating according to the first two digits of the SIC number 1-19, 20-39, 40-49, 50-59, 60-79).

of consumption growth relative to the market rate of return, and relative to the returns of individual portfolios, suggests that the covariance of individual asst returns with the market could contain valuable information in predicting asset returns, and hence that models aimed at explaining fluctuations in asset returns in terms of their covariation with both a market index and consumption could perform better than models relying exclusively on the covariance with consumption growth. This conjecture is tested in the following sections.

4 Empirical Tests: Log-Normal Model

Equation (9) implies the following expression for the conditional expectation of the *log of* the real return on an asset i:

$$\bar{r}_{i,t} = r_{f,t} - \sigma^2_{i,t}/2 + \rho\frac{1-\gamma}{1-\rho}\sigma_{ic,t} + \frac{\gamma-\rho}{1-\rho}\sigma_{im,t}, \tag{11}$$

Substituting on the left-hand side of (11) the *realized* value of the log of asset i's return, amounts to adding a disturbance term on the right-hand side. Since the disturbance term represents rate-of-return innovations at time $t+1$, it is orthogonal to all variables on the right-hand side. We estimate equilibrium returns equations jointly with a reduced-form equation explaining consumption growth. The general form of our system is:

$$r_{it+1} = \bar{r}_{it} + \epsilon_{it+1}, \quad i = 1, ..., N, \tag{12}$$

$$r_{mt+1} = \bar{r}_{mt} + \epsilon_{mt+1} \tag{13}$$

$$c_{t+1} = \bar{c}_t + \epsilon_{ct+1}, \tag{14}$$

The expected returns $\bar{r}_{it}$ and $\bar{r}_{mt}$ are given by equation (9), which we estimate under different assumptions on conditional moments. Notice that we impose the same model on the five industry indices *together with* the market index which at every time t is just a weighted average of the industry indices. Since however the weights vary substantially over time (the market index is value-weighted), it turns out that the covariance matrix of $r_i, i = 1, \ldots, N, m$ does not suffer from multicollinearity. The condition number of the 6×6 unconditional covariance matrix computed over the period from April 1953 to December 1987 is only 4745. As a reference, the condition number of the 5×5 matrix obtained from dropping the first industry is equal to 3006, which is of the same order of magnitude.

In order to achieve consistency and maximum efficiency, we apply the maximum likelihood procedure, and, in particular, we impose that the conditional variances and covariances in equation (11) are precisely the elements of the covariance matrix of the estimated residuals in the system (12)–(14).

Given the above distributional assumptions, the conditional log-likelihood function for the residuals is

$$\begin{aligned} \ell(\theta) &= \ell(\rho, \gamma, \Sigma_t, \ldots) \\ &= -\sum_{t=1}^{T} \left[\frac{N+2}{2} ln(2\pi) + \frac{1}{2} ln \mid \Sigma_t \mid + \frac{1}{2} \epsilon_t' \Sigma_t^{-1} \epsilon_t \right] \end{aligned} \tag{15}$$

The parameters of interest are found by maximizing ℓ over the parameter space Θ. At the maximum, an estimate of the covariance matrix of the estimated parameters is obtained from the inverse of the sum of the outer product of the score vectors.

4.1 Constant Conditional Second Moments

If second moments are assumed constant over time, the model reduces to

$$r_{it+1} = r_{f,t} - \sigma_i^2/2 + \rho \frac{1-\gamma}{1-\rho} \sigma_{ie} + \frac{\gamma - \rho}{1-\rho} \sigma_{im} + \epsilon_{it+1} \tag{16}$$

for $i = 1, ..., N, m$ and

$$c_{t+1} = \bar{c}_t + \epsilon_{ct+1}, \tag{17}$$

where:

$$\bar{c}_t = \begin{cases} \bar{c} \\ a_0 + a_1 c_t \end{cases} \tag{18}$$

Equation (18) displays the stochastic processes we postulate for $\bar{c}_t$. The first assumes that consumption growth is serially uncorrelated, and hence $\bar{c}_t$ is a constant; however, given the evidence reported in Table I—showing significant negative autocorrelation in consumption growth—we also assume that consumption growth follows a first-order autoregressive process. Notice, using equation (10), that with constant conditional second moments the fluctuations in expected returns are due exclusively to fluctuations in the riskfree rate, which in turn fluctuates with the conditional expectation of consumption growth. The implication is that with constant conditional second moments risk premia are constant and all ex-ante rates of return are perfectly correlated. When consumption growth is assumed to be uncorrelated both first and second conditional moments of asset returns are constant.

When consumption growth is uncorrelated, the parameters to be estimated are δ (or, equivalently, r_f), $\rho, \gamma, \bar{c}$ plus the $(N+2) * ((N+2)+1)/2$ elements of the covariance matrix of innovations—28 elements with $N = 5$—for a total of 32 parameters. When consumption growth follows a first-order autoregressive process the parameters to be estimated are $\delta, \rho, \gamma, a_0, a_1$ plus the $(N+2) * ((N+2)+1)/2$ covariance matrix elements, for a total of 33 parameters.

The estimation results are reported in Table II. The point estimates of both the reciprocal of the elasticity of intertemporal substitution and of the coefficient of relative risk aversion are positive. The reciprocal of the elasticity of intertemporal substitution is equal to 5.7 and 4.9 under the two alternative specifications for consumption growth, while the coefficient of relative risk aversion is 8.3 and 14.1 in the two different specifications. In all cases the standard errors of the estimates are very high, especially when consumption growth is assumed to be a white noise process. The inclusion of lagged consumption improves the efficiency of the estimates dramatically, without much affecting their values. Yet both parameters are still not significantly different from anything of interest. Not surprisingly, then, the hypothesis that $\rho = \gamma$, *i.e.* the Von-Neumann Morgenstern restriction, is not rejected in either case: the t statistic for the $\rho - \gamma$ is in both cases well below 1.

The restrictions imposed by the model can be tested against an alternative hypothesis that conditional expectations of returns are unrelated to the elements of the covariance matrix of disturbances. Under the assumption that conditional first moments are constant, the unrestricted regressions are just projections on a constant. The total number of parameters estimated with maximum likelihood is $(N + 2) + (N + 2) * ((N + 2) + 1)/2 = 35$ with $N = 5$. Under the assumption that expected returns are time varying, the unrestricted regressions project actual returns on a constant and the lagged growth rate of consumption. A total of $(N + 2) * 2 + (N + 2) * ((N + 2) + 1)/2 = 42$ parameters are estimated. The results of the tests are also reported also in Table II. The cross equation constraints from the model are in both cases rejected at the 5 percent level, but not at the 1 percent level.

The results in Table II are comparable to those obtained by Hansen and Singleton (1983) who apply maximum likelihood estimation to a lognormal model which is a restricted version of ours, with $\gamma = \rho$. They also use monthly data (from February 1959 to December 1978) on consumption of nondurables and services, although they apply their model to individual stock returns, rather than industry indices. Their estimate of $\gamma = \rho$ ranges from .507 to 4.106, two values which are of the same order of magnitude as those we report in Table II. They also reject the cross-equation restrictions of the model at any conventional significance level.[7]

4.2 Time-Varying Conditional Second Moments

Given the widespread evidence that conditional variances of asset returns change over time, the assumption of constant second moments may lead to unwarranted rejections of the model. It is therefore important to extend the model to time-varying second moments. A tractable specification is the Autoregressive Conditional Heteroskedastic (ARCH) proposed by Engle (1982), where conditional second moments are written as a deterministic function of lagged squared innovations and previous conditional covariances:

$$\Sigma_t = E_{t-1}[\epsilon_t \epsilon_t'] = \Gamma + A \bullet \epsilon_{t-1}\epsilon_{t-1}' + B \bullet \Sigma_{t-1}, \tag{19}$$

where $\bullet$ indicates element-by-element matrix multiplication. In practice the matrices Γ, A, B are constrained to be positive definite by estimating their Choleski factors.

The model then reduces to:

$$r_{it+1} = r_{f,t} - \sigma_{it}^2/2 + \rho\frac{1-\gamma}{1-\rho}\sigma_{ict} + \frac{\gamma-\rho}{1-\rho}\sigma_{imt} + \epsilon_{it+1} \tag{20}$$

for $i = 1, \ldots, N, m$. And

$$c_{t+1} = \bar{c}_t + \epsilon_{ct+1}, \tag{21}$$

[7] The cross-sectional regressions of Giovannini-Weil (1989), who use average returns and standard errors on 379 individual stocks over the period January 1959 to May 1987, also yield quite unprecise estimates of the relative risk aversion coefficient, but lead to a strong rejection of the Von Neumann-Morgenstern restriction.

where:

$$\bar{c}_t = \begin{cases} \bar{c} \\ a_0 + a_1 c_t \end{cases} \tag{22}$$

The riskfree rate of interest, defined in equation (10), is now time-varying both through the variation of expected consumption growth and the expected return on the market, and through the variation of conditional variances and covariances. Furthermore, in this case the variation of conditional second moment induces time-varying risk premia across all assets.

When consumption growth is assumed to follow a white noise process, the model involves a total of $4 + 3 * [(N+2) * (N+2+1)/2] = 88$ parameters ($\delta, \rho, \gamma, \bar{c}$ plus the elements of the three matrices in equation (19), Γ, A and B). When consumption growth is assumed to follow a first-order autoregressive process, the number of parameters to estimate increases to 89.

Table III reports the results from the estimation and test of this model. The point estimates of the two parameters are of the same order of magnitude as those obtained assuming conditional homoskedasticity, and here also the Von Neumann-Morgenstern restriction that the relative risk aversion coefficient equals the reciprocal of the elasticity of intertemporal substitution is not rejected. Even though the estimates of the taste parameters are very unprecise,[8] the standard errors of the coefficients are lower than in the case where conditional second moments, indicating that the assumption of conditionally heteroskedastic returns improves the fit of the model. This is confirmed by the test of homoskedasticity (a test of the restriction that the matrices A and B are jointly equal to zero), which shows a strong rejections under both assumptions for the consumption-growth process.

[8]This is consistent with the finding in Giovannini and Jorion (1989), who cannot obtain precise estimates of the risk aversion parameter in a asset pricing model estimated assuming conditionally-heteroskedastic returns using weekly data on stock returns and foreign exchange returns.

As above, the model can be tested against a more general alternative. In this case the alternative allows different cross-sectional means but preserves the type of time-variation in expected returns induced by second moments specified above:

$$r_{it+1} = k_{0i} + \bar{c}_t - \sigma^2_{i,t}/2 + k_1\sigma^2_{c,t} + k_2\sigma^2_{m,t} + k_3\sigma_{ic,t} + k_4\sigma_{im,t} + \epsilon_{it+1}, \tag{23}$$

for $i = 1, \ldots, N, m$. And

$$c_{t+1} = \bar{c}_t + \epsilon_{ct+1}, \tag{24}$$

where:

$$\bar{c}_t = \begin{cases} \bar{c} \\ a_0 + a_1 c_t \end{cases} \tag{25}$$

Given our specification of the alternative, the model can be directly tested using a chi-square statistic. In both cases, the restrictions are rejected strongly.

4.3 Tests of Joint Log-Normality

Finally, we verify the assumption that returns and consumption growth are jointly (conditionally) log-normally distributed, by performing tests on the estimated residuals of the equations in the models. Table IV reports the results. The table contains the values of the Kolmogorov-Smirnov statistic, which provides a test of the hypothesis that the data are a random sample from a normal distribution, by computing the largest absolute deviation between the sample and the theoretical cumulative distributions. The probability of obtaining the observed value under the null is computed under the Kolmogorov-Smirnov limiting distribution.

Alternatively, a chi-square goodness-of-fit statistic is obtained as follows. First the observations are sorted in order of increasing magnitude, and classified into n=20 equally sized groups. Knowing the lognormal density function allows to compute the theoretical number of observations for each group. The goodness of fit between the actual and theoretical distributions is tested by summing the squares of the differences between the observed and theoretical number of outcomes in each group. Asymptotically, this test statistic has a χ^2_{n-1} distribution.

Using both test statistics, and under both the homoskedastic and heteroskedastic specifications of the model, we find that the hypothesis of log-normal residual is not rejected. This result, consistently with the early findings by Fama (1976), suggests that with monthly data the log-normality assumption might be a satisfactory approximation.

5 Empirical Tests: Euler Equations

The maximum-likelihood-based tests reported in the previous section are the most powerful tests, since they fully exploit all the restrictions the model imposes on the data, *given* the maintained hypotheses on the process followed by the forcing variables (consumption, in our case) and the assumed dynamic structure of conditional second moments. A less powerful set of tests was recently performed by Epstein and Zin (1989b). These tests rely on the orthogonality restrictions implicit in the first-order, necessary conditions for optimization expressed in equation (6).

In this section we follow the same procedures as Epstein and Zin (1989b) to verify whether relaxing the assumptions about the time variation of consumption growth and conditional variances improves the empirical performace of the model relative to the more restrictive Von Neumann Morgenstern setup. The system of equations in (6) is estimated jointly using the generalized method of moments (GMM) developed by Hansen (1982) and Cumby, Huizinga and Obstfeld (1983). The model is tested by verifying the orthogonality of the instruments in excess of the parameters to be estimated with the estimated residuals from the equations. The test statistic, proposed by Hansen (1982) is $\epsilon' D\epsilon$, where $D = Z\hat{\Omega}^{-1}Z$, Z is a matrix of instruments, and $\hat{\Omega}$ is a consistent estimate of $\Omega = lim_{T\to\infty}(1/T)E(Z'\epsilon\epsilon' Z)$, obtained from sample moments. It is distributed as chi-square with degrees of freedom equal to the difference between the total number of instruments[9] and the number of parameters to be estimated.

The results are reported in Table V. We use four different sets of instruments, including:

- a constant;
- a constant plus lagged consumption growth and lagged return on the market;
- a constant plus lagged returns on industry 1 and 2;
- a constant plus lagged returns on industry 1 to 5.

Our point estimates of risk aversion and intertemporal substitution are similar to those reported by Epstein and Zin (1989b). They differ, but not significantly, from those obtained with maximum-likelihood under the log-normality assumption, where ρ was generally a lower number, while γ was much larger.

Another interesting difference with the maximum-likelihood estimates is that the Von Neumann-Morgenstern restriction is rejected in all cases, except when only a constant is used as an instrument (a case where also the parameter estimates are very unprecise).

In the table we report also estimates and tests of the restricted version of the model, where $\gamma = \rho$. Notice that whenever the overidentifying restrictions of the

[9] When the instruments used in each equation are the same, the number of instruments times the number of equations.

model are rejected, they are also rejected in the restricted, Von Neumann Morgenstern specification.

Overall, the maximum likelihood estimation method, especially when allowing for time-variation in second moments, led to much stronger rejections of the model than the method of moments, using the same instruments. With the latter method, rejections only occur with a much larger set of instruments.

6 Concluding Remarks

In this paper we presented two alternative procedures to estimate and test a class of representative-agent asset pricing models which rely on a specification of preferences that explicitly distinguishes attitudes towards risk from attitudes towards intertemporal consumption smoothing, and subsumes the standard intertemporal CAPM as a special case.

The most important finding was that the relaxation of the constraint that the coefficient of relative risk aversion equals the inverse of the elasticity of intertemporal substitution (a constraint imposed by the standard consumption CAPM) does not improve the fit of the model. In the maximum-likelihood estimates we cannot reject the hypothesis that the two parameters are equal, but the model is rejected. In the generalized-method-of-moments estimates we find that whenever we reject the more general model we also reject the constrained one.

In general, it appears once again that it is extremely difficult to obtain precise estimates of the parameters: therefore the estimation of large-scale versions of the model could be of significant interest.

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Table I

Data Description:

April 1959-December 1987

Series	Mean	S.Dev.	Autocorrelation: Lags											
			1	2	3	4	5	6	7	8	9	10	11	12
C	0.00207*	0.0053	-.11*	0.10	0.05	-.02	0.07	0.04	0.07	0.12*	0.05	0.13*	0.07	-.04
M	0.00502*	0.0450	0.08	-.05	0.04	0.04	0.12*	-.05	-.08	-.07	0.00	-.01	-.01	0.04
Ind 1	0.00582*	0.0588	0.08	-.05	0.01	0.03	0.08	-.03	-.12	-.07	0.01	-.01	-.02	0.02
Ind 2	0.00536*	0.0467	0.07	-.04	0.04	0.05	0.10	-.04	-.09	-.05	-.01	-.02	0.00	0.03
Ind 3	0.00461*	0.0381	0.08	-.06	0.04	0.07	0.19*	-.07	-.10	-.05	0.06	0.05	0.05	0.05
Ind 4	0.00603*	0.0565	0.17*	-.01	-.01	0.02	0.12*	-.11	-.05	0.00	0.09	0.04	-.03	0.04
Ind 5	0.00568*	0.0531	0.14*	-.04	0.01	0.02	0.10	-.03	-.06	-.13	-.02	-.02	0.00	0.07

NOTES: Significance at the 5% level denoted by *. Under the null hypothesis of zero autocorrelation, the standard error of the aurocorrelation coefficients is about 0.054.

Definitions:

- C (consumption growth),
- M (value-weighted market return),
- Ind 1 (Primary industries),
- Ind 2 (Manufacturing industries),
- Ind 3 (Transportation),
- Ind 4 (Trade),
- Ind 5 (Finance and Services).

Table II

Maximum-Likelihood Tests: Constant Second Moments

$$
\begin{aligned}
r_{it} &= E_{t-1}[r_{it}] + \epsilon_{it}, && i = 1, \ldots, 5, \\
r_{mt} &= E_{t-1}[r_{mt}] + \epsilon_{mt} && \\
c_t &= E_{t-1}[c_t] + \epsilon_{ct}, && \\
\\
E_{t-1}[r_{it}] &= r_{Ft} - \sigma_i^2/2 + \rho\tfrac{1-\gamma}{1-\rho}\sigma_{ic} + \tfrac{\gamma-\rho}{1-\rho}\sigma_{iM}, && i = 1, \ldots, 5, m, \\
E_{t-1}[c_t] &= \bar{c}, \quad \text{or} \quad a_0 + a_1 c_{t-1} &&
\end{aligned}
$$

$$\Sigma_t = E_{t-1}[\epsilon_t \epsilon_t'] = \Sigma$$

April 1959-December 1987

Instruments	Model	Parameters ρ (s.e.)	γ (s.e.)	Test of VNM t-stat $\rho - \gamma = 0$	Log-lik.	D.F.	Test of Model Restrictions χ^2	P-value
Constant	Unrestricted	-	-	-	6750.63	35		
	Non-VNM	5.7 (68.9)	8.3 (116.6)	0.05	6746.02	32	9.2	0.026
Constant, C(-1)	Unrestricted	-	-	-	6751.53	42		
	Non-VNM	4.9 (6.0)	14.1 (23.1)	0.51	6746.81	33	19.4	0.022

NOTES: Asymptotic standard errors between parentheses.

Table III

Maximum-Likelihood Tests: Time-Varying Second Moments

$$
\begin{aligned}
r_{it} &= E_{t-1}[r_{it}] + \epsilon_{it}, \qquad i = 1,\ldots,5,\\
r_{mt} &= E_{t-1}[r_{mt}] + \epsilon_{mt}\\
c_t &= E_{t-1}[c_t] + \epsilon_{ct},
\end{aligned}
$$

$$
\begin{aligned}
E_{t-1}[r_{it}] &= r_{Ft} - \sigma_{it}^2/2 + \rho\tfrac{1-\gamma}{1-\rho}\sigma_{ict} + \tfrac{\gamma-\rho}{1-\rho}\sigma_{iMt}, \qquad i = 1,\ldots,5,m,\\
E_{t-1}[c_t] &= \bar{c}, \quad \text{or} \quad a_0 + a_1 c_{t-1}
\end{aligned}
$$

$$
\Sigma_t \;=\; E_{t-1}[\epsilon_t\epsilon_t'] \;=\; \Gamma + A \bullet \epsilon_{t-1}\epsilon_{t-1}' + B \bullet \Sigma_{t-1},
$$

April 1959-December 1987

Instruments	Model	Parameters ρ (s.e.)	Parameters γ (s.e.)	Test of VNM t-stat $\rho-\gamma=0$	Log-lik.	D.F.	Test of Model Restrictions χ^2	Test of Model Restrictions P-value
Constant	Unrestricted	-	-	-	6959.22	95		
	Non-VNM	3.9 (19.5)	11.8 (73.8)	0.14	6945.39	88	27.7**	0.0002
		Test of Homoskedasticity: χ^2_{56}=398.7**, p-val=0						
Constant, C(-1)	Unrestricted	-	-	-	6961.22	102		
	Non-VNM	3.1 (5.3)	10.6 (25.2)	0.38	6956.53	89	28.8**	0.007
		Test of Homoskedasticity: χ^2_{56}=419.4**, p-val=0						

NOTES: Asymptotic standard errors between parentheses. Significance at the 1% level denoted by **.

Table IV

Tests of Lognormality:

Residuals from Maximum-Likelihood Estimation

April 1959-December 1987

Series	Homoskedastic Model				Heteroskedastic Model			
	Kolmogorov-Smirnov		Chi-square		Kolmogorov-Smirnov		Chi-square	
	Statistic	P-value	Statistic	P-value	Statistic	P-value	Statistic	P-value
C	0.0241	0.99	9.75	0.96	0.0241	0.99	9.75	0.96
M	0.0569	0.21	26.30	0.07	0.0544	0.26	24.78	0.17
Ind 1	0.0667	0.09	34.84	0.01	0.0703	0.07	39.86**	0.004
Ind 2	0.0554	0.24	28.73	0.07	0.0548	0.25	29.14	0.06
Ind 3	0.0592	0.18	23.77	0.21	0.0547	0.25	20.87	0.34
Ind 4	0.0623	0.14	25.58	0.14	0.0583	0.19	21.53	0.31
Ind 5	0.0685	0.08	33.29	0.02	0.0676	0.08	31.44	0.04

NOTE: ** Significant at the 1% level.

The Kolmogorov-Smirnov statistic provides a test of the hypothesis that the data are a random sample from a normal distribution, by computing the largest absolute deviation between the sample and the theoretical cumulative distributions. The chi-square goodness6in of fit statistic is obtained from sorting the observations into $N = 20$ equally sized groups, and then computing the differences between the observed and theoretical number of outcomes in each group.

Table V

Euler Equations GMM Tests:

$$E_t\{MRS_{t+1}\ \ R_{i,t+1}\} = 1,$$

$$MRS_{t+1} = \left[\delta\left(\frac{c_{t+1}}{c_t}\right)^{-\rho}\right]^{\frac{1-\gamma}{1-\rho}} \left[\frac{1}{R_{m,t+1}}\right]^{1-\frac{1-\gamma}{1-\rho}}, \gamma = CRRA, \rho = EIS$$

April 1959-December 1987

Model	Instruments	ρ (s.e.)	γ (s.e.)	t-stat (VNM)	χ^2	D.F.	P-value
Non-VNM	Constant	9.7 (196.7)	-2.93 (24.08)	0.5	0.22	2	.897
	1,C(-1),M(-1)	26.7 (765.6)	1.13 (0.86)	30.3*	13.36	12	.344
	1,I1(-1),I2(-1)	5.2 (1.6)	0.58 (0.55)	8.5*	25.89*	12	.011
	1,I1 to I5(-1)	13.5 (21.4)	0.79* (0.33)	37.7*	60.20*	27	.00025
VNM	Constant	38.4 (72.7)	-	-	0.46	3	.927
	1,C(-1),M(-1)	-2.17 (2.20)	-	-	13.10	13	.440
	1,I1(-1),I2(-1)	3.56 (2.00)	-	-	26.41*	13	.015
	1,I1 to I5(-1)	2.28 (1.70)	-	-	68.72*	28	.00003

NOTES: Standard errors corrected for heteroskedasticity between parentheses. Significance at the 5% level denoted by *. The t-statistic tests the hypothesis that $(1-\gamma)/(1-\rho) = 1$, which is implied by the VNM expected utility model. The chi-square statistic tests the overidentifying restrictions of the model.