A FRAMEWORK FOR GEOECONOMICS

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ABSTRACT

Governments use their countries' economic strength from existing financial and trade relationships to achieve geopolitical and economic goals. We refer to this practice as geoeconomics. We build a framework based on three core ingredients: limited contract enforceability, input-output linkages, and externalities. Geoeconomic power arises from the ability to jointly exercise threats across separate economic activities. A hegemon, like the United States, exerts its power on firms and governments in its economic network by asking these entities to take costly actions that manipulate the world equilibrium in the hegemon's favor. We characterize the optimal actions and show that they take the form of mark-ups on goods or higher rates on lending, but also import restrictions and tariffs. Input-output amplification makes controlling some sectors more valuable for the hegemon since changes in the allocation of these strategic sectors have a larger influence on the world economy. This formalizes the idea of economic coercion as a combination of strategic pressure and costly actions. We apply the framework to two leading examples: national security externalities and the Belt and Road Initiative.

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Governments use their countries' economic strength from existing financial and trade relationships to achieve geopolitical and economic goals. We refer to this practice as geoeconomics. We build a framework to understand the role of geoeconomics in shaping global real and financial activity. Our model conceptualizes how the great powers use their financial and economic strength to extract economic and political surplus from countries around the world.

Geoeconomic power is a form of soft indirect power. It is not as blunt as the direct threat to go to war, as it operates through commercial channels like the interruption of the supply or purchase of goods, the sharing of technology, or financial relationships and services. At the other extreme, this power operates in areas in which complete contracts are not feasible either because of limited enforceability or because for political and legal reasons formal contracts are unpalatable. For example, government to government relationships take this nature due to the limited presence of courts with the power to adjudicate disputes.

We consider a collection of countries and productive sectors with an input-output network structure. We think of sectors as collections of firms operating in a specific country and industry (e.g. Russian oil extraction and American oil extraction are two distinct sectors). The model features limited enforceability of contracts, as well as externalities both in production functions and in the objective functions of country-level representative agents. Geoeconomic power arises from the ability of a country to consolidate threats across multiple economic relationships, often with some of the threats carried out by third party entities also being pressured, to induce a target to take a desired action. We refer to countries that exert such power as hegemons. We characterize when these threats are valuable and how a hegemon extracts value from the targeted entities.

We model threats as trigger strategies that firms and governments can employ to punish other entities for deviating from contracts. For example, a supplier of a good might refuse to supply the good again to a buyer that did not pay for an earlier shipment. A lender might withhold future financing from a borrower that defaulted on a loan. Joint threats are trigger strategies in which the trigger can be based on multiple economic relationships. In our model, a hegemon is a country that is able to coordinate many such threats both via its national entities and via their economic network abroad. For example, a hegemon can threaten to withhold future financing if a recipient country either defaults on a loan or breaches the contract for importing intermediate goods.

While the hegemon can potentially make many threats, some are either not feasible or not valuable in equilibrium. A threat may not be feasible in the sense that a hegemon does not control the economic relationship either directly or indirectly. Even if the threat is feasible, it might not be valuable. The hegemon making the threat might be offering an input that can easily be sourced elsewhere, i.e. substituted with a similar input not controlled by the hegemon. Joint threats are particularly effective because they use the economic value of each activity as an endogenous cost of default on the other activities. For example, sovereign lending might be hard to sustain on its own given the lack of legal enforceability, but might be more sustainable if occurring jointly with manufacturing exports or military provisions, even if the latter are themselves subject to

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expropriation risk.

The hegemon's threats can generate value in the presence of limited enforceability of contracts for the targeted entities because they increase the penalty for deviating from contracts. The hegemon uses this value to demand costly actions from entities in its economic network. In our model targeted entities could be either firms or governments. The costly actions can take many forms: monetary transfers, mark-ups on goods, surcharges on loans, import-export restrictions, and political concessions. We characterize the hegemon's optimal use of these instruments and show how they are used to manipulate the world equilibrium in the hegemon's favor.

The general equilibrium of the model features endogenous transmission of sectoral production decisions via the input-output structure. We introduce production externalities whereby an individual sector's productivity can depend of what other sectors are producing both within and across countries. These can capture both traditional economic forces such as external economies of scale, network effects, and also externalities that are outside the traditional focus of economics such as national security. We show that the input-output network propagates the production externalities. For example, changes in the exogenous productivity of a sector propagate through the network. One sector producing more, might make another more productive, that sector producing more affects the productivity of another sector, and so on. We show that this propagation can be summarized by a Leontief inverse matrix based on the production externalities. We also allow for the more traditional propagation via equilibrium prices of the intermediate goods.

We allow for direct externalities on consumers, appearing in countries' representative consumer utility function. This can capture political affinity between citizens of any two countries, or the political preferences of their respective governments. In particular, they capture the idea that the size of various sectors around the world may make citizens of one country feel less secure. For instance, the development of a cutting edge semiconductor or AI sector for military use in a country's geopolitical rival may directly lower this country's utility above and beyond any effect on the profits of the country's own firms.

We show that the hegemon builds as much power as possible by making all joint threats at its disposal. To the extent that these threats generate value for the targeted entities, the hegemon is able to demand two general classes of costly actions.[1] First, it demands transfers from the targeted entities (e.g. monetary transfers, but also mark-ups). Second, it imposes restrictions on each entity bilaterally sourcing inputs from other entities. Formally, these restrictions are revenue-neutral wedges on bilateral input purchases and can be specialized to capture import quantity restrictions that are good and destination specific, as well as tariffs and price caps. These are common tools in the implementation of sanctions, as well as international economic policy more generally. These restrictions generate no direct revenue for the hegemon and are instead used to manipulate general equilibrium quantities and prices in the hegemon's favor.

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[1]In most of the paper we consider the presence of a single hegemon, but in Appendix A.3 we explore how multiple hegemons compete with each other in the geoeconomic arena.

Transfers lower the profits of targeted entities and, therefore, tighten their incentive constraints since profitability is a source of commitment. For this reason, the hegemon never demands transfers from domestic entities since it directly cares about their profits. By contrast, the hegemon does not directly care about the profitability of foreign entities while it does value receiving transfers from them. When interacting with foreign entities, the hegemon trades off asking for transfers versus imposing input restrictions (wedges). The hegemon's optimal wedges trade off the benefit it receives from changing the target's actions against the cost of tightening the target's constraints. The benefit the hegemon receives accrues either directly because the hegemon's representative consumer utility is affected by the target sector production, or indirectly because the target's production choices impact the activities of sectors that the hegemon values.

Our framework shows that a sector can be strategic in two dimensions. First, because the hegemon can use it to form threats on other entities, thus building the hegemon's power. Second, because demanding costly actions from this sector is particularly effective at shaping the world equilibrium in the hegemon's favor. We define these two dimensions as Micro-Power and Macro-Power.

Micro-Power arises when the hegemon's joint threats increase the value of the targeted entity, taking as given all equilibrium aggregate quantities and prices. Micro-Power measures the private value to the targeted entity of the hegemon's joint threat. The source of Micro-Power is the increase in the loss in continuation value for the targeted entity from losing access to a larger set of inputs. We show that strategic sectors in this micro-sense are those that supply inputs that are widely used, with high value added for targets, and with poor substitutes. Some goods may have these properties due to physical constraints: rare earths, oil and gas. Others have them in equilibrium due to increasing returns to scale and natural monopolies. For example, the dollar-based financial infrastructure of payment and clearing systems (like SWIFT) is a strategic asset that the US often uses in geoeconomic threats.

Macro-Power arises when the hegemon collectively asks the targeted entities for costly actions that shape equilibrium aggregate quantities and prices in the hegemon's favor. The propagation and amplification through the network structure is key to this effect. In this macro sense, strategic sectors tend to be those that have a high influence on world output due to endogenous amplification (in the Leontief-inverse). Sectors like research and development, and information technology are good candidates for being strategic in this sense. A hegemon particularly values having Micro-Power over sectors that increase its Macro-Power because it can exploit the difference between the private costs to targeted entities and the social benefit to itself. In accepting the hegemon's demands, the targeted entities consider only their private costs, but the hegemon enjoys the social benefits of the outcomes of these action.

We show that allocations with a hegemon are constrained inefficient from a global perspective. Although threats are a positive-sum by increasing enforcement and therefore economic activity, the actions that the hegemon demands from targeted firms can be negative-sum. Transfers are

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distortionary because they lower profits thereby worsening incentives of the targeted entities. In addition, the global planner and the hegemon do not value externalities in the same way since the planner cares about the effect of externalities on all countries, not only on the hegemonic country. We show that the global planner would impose no transfers and, in general, a different set of wedges than the hegemon does.

After characterizing the general model, we focus on two leading applications. In the first application, we show how production and national security externalities can interact and lead the hegemon to coerce third party countries to restrict the use of inputs of an hostile country. An example is the US demand to European governments and firms that they stop using information technology (IT) infrastructure produced by China's Huawei. We think of a world composed of three regions: the US hegemon, third party countries, and China. China has a sector producing IT goods that the rest of the world firms use as an input. We assume that this IT infrastructure has external economies of scale so that more firms using that input makes a firm more productive in using that same input. We also assume that the US experiences a direct utility loss, which we refer to as a national security externality, from the size of China's exports of the technology.

We show that in this application it is optimal for the US hegemon to demand governments and firms in third party countries that it can pressure to curb their imports of Chinese technology. The extent of the requested import restrictions is higher than the direct perceived national security externality because the hegemon internalizes the amplification effect of the sanctions. As the firms in its network use this technology less, using the technology becomes less attractive also for firms that the hegemon cannot directly pressure. This Leontief-type of linkage also feeds back to the firms accepting the sanctions: knowing that other firms will not be using the technology either, makes complying with U.S. led sanctions easier on the margin.

Our second application focuses on the Belt and Road Initiative by China. We model it as a sovereign lending program that aims to join borrowing and trade decisions. We illustrate how sovereign debt can be represented in the form of a productive input in our framework, and show that a country's borrowing capacity increases when the hegemon lender, in this case China, is able to consolidate threats in the sovereign lending arena with activity in export markets. Even if sovereign lending is legally unenforceable, so that as an isolated activity only limited lending would take place, we show that profitable trade relationships can act as an endogenous cost of default. Targeted countries voluntarily increase their endogenous cost of default in order to be able to borrow more from Chinese lenders. The optimal contract extracts surplus for China in one of three forms: as a mark-up on the price of the exports, as a higher return on loans, or as a political concession. In practice, it seems the latter has been the dominant form of request by the Chinese hegemon. More generally, the application shows the futility of assessing the success of the Belt and Road Initiative lending or infrastructure investment in isolation. The sustainability of the debt and the return of the program are inextricably linked with other economic and political activities.

Literature Review. In two landmark contributions Hirschman (1945, 1958) relates the structure of international trade to international power dynamics and sets up forward and backward linkages in input-output structures as a foundation for structural economic development. Much of our model is inspired by this work and aims to provide a formal framework for the power structures. In doing so, we connect to three broad strands of literature.

First, the paper connects to the literature in political science on economic statecraft. The notion of economic statecraft, or the use of economic means for political ends, was explored in depth by Baldwin (1985) and the subsequent literature. A particular tool of economic statecraft, economic and financial sanctions, is a focus of this political science literature, including such contributions as Lindsay (1986), Kirshner (1997), Drezner (2003), and Mulder (2022). Blackwill and Harris (2016) explore the rise of geoeconomics, that is the use of economic power for geopolitical goals. Farrell and Newman (2019) and Drezner et al. (2021) introduce the idea of "weaponized interdependence" whereby governments can use the increasingly complex global economic network to influence and coerce other governments.[2]

Second, the paper relates to the literature on networks, industrial policy, and trade. There is a growing literature on networks in economics including Gabaix (2011), Acemoglu, Carvalho, Ozdaglar and Tahbaz-Salehi (2012), Jones (2011), Blanchard et al. (2016), Bigio and La'O (2020), Baqaee and Farhi (2019), Baqaee and Farhi (2020, 2022), Liu (2019), Liu and Ma (2021), and Elliott, Golub and Leduc (2022). Bachmann et al. (2022) and Moll et al. (2023) use this class of models to find limited impact for Germany of a stop of energy imports from Russia. Our notion of friends and enemies of the hegemon is related to the work of Kleinman, Liu and Redding (2020) who explore whether countries become more politically aligned as they trade more with each other. In trade we relate to the study of global value chains (Antràs and Staiger (2012); Caliendo and Parro (2015); Grossman et al. (2021); Antràs and Chor (2022)) as well as the study of optimal tariffs and trade agreements (Bagwell and Staiger (1999, 2001, 2004); Grossman and Helpman (1995); Ossa (2014)). Our supplier-buyer relationship also encompasses forms of trade credit (Schmidt-Eisenlohr (2013); Bocola and Bornstein (2023)). Antràs and Miquel (2011, 2023) explore how foreign influence affects tariff and capital taxation policy. Bartelme, Costinot, Donaldson and Rodriguez-Clare (2019) and Ottonello, Perez and Witheridge (2023) estimate sector-level economies of scale to quantify the expected gains from industrial policy.[3] At the intersection with political economy, Berger, Easterly, Nunn and Satyanath (2013) demonstrate that countries where the CIA intervened during the Cold War imported more from the United States. Kuziemko and Werker (2006) document that a country that rotates on the UN Security council experiences an increase in foreign aid. Juhász, Lane, Oehlsen and Pérez (2022) use textual analysis to measure industrial policy interventions around the world. Juhász, Lane and Rodrik (2023) surveys the recent literature on industrial policy.

Third, the paper uses several tools developed in economic theory and macroeconomics. We

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[2]Mangini (2022) studies how states' attempts to use economic coercion interact with domestic political constraints.

[3]Camboni and Porcellacchia (2021) use a gravity framework to test for geopolitical competition.

employ grim trigger strategies to build a subgame perfect equilibrium building on Abreu et al. (1986, 1990).[4] Our notion of joint triggers relates to the literature on multitasking (Holmstrom and Milgrom (1991)) and multi-market contact (Bernheim and Whinston (1990)) in which the presence of multiple activities or tasks can help to provide higher powered incentives. We introduce externalities a la Greenwald and Stiglitz (1986) and our study of the hegemon optimal usage of wedges and transfers is related to the analysis of inefficiency in the presence of externalities (Geanakoplos and Polemarchakis (1985)) and the macro-prudential tools that can be used to improve welfare (Farhi and Werning (2016)).

1 Model Setup

Time is discrete and infinite, $t = 0, 1, \ldots$ Each period is a stage game, described below. All agents have subjective discount factor β.

1.1 Stage Game

There are N countries in the world. Each country n is populated by a representative consumer and a set of productive sectors $\mathcal{I}_n$, and is endowed with a set of local factors $\mathcal{F}_n$. We define $\mathcal{I}$ to be the union of all productive sectors across all countries, $\mathcal{I} = \bigcup_{n=1}^{N} \mathcal{I}_n$, and define $\mathcal{F}$ analogously. Each sector produces a differentiated good indexed by $i \in \mathcal{I}$ out of local factors and intermediate inputs produced by other sectors. Each sector is populated by a continuum of identical firms. The good produced by sector i is sold on world markets at price p_i. Local factor f has price p_f^ℓ. Local factors are internationally immobile. We take the good produced by sector 1 as the numeraire, so that $p_1 = 1$. We define the vector of all intermediate goods' prices as p, the vector of all local factor prices as p^ℓ, and the vector of all prices as $P = (p, p^\ell)$.

Representative Consumer. The representative consumer in country n has preferences $U_n(C_n) + u_n(z)$, where $C_n = \{C_{ni}\}_{i \in \mathcal{I}}$ and where z is a vector of aggregate variables which we use to capture externalities a la Greenwald and Stiglitz (1986). Consumers take z as given. We assume U_n is increasing, concave, and continuously differentiable. We assume that the representative consumer in each country owns all domestic firms and the endowments of local factors. The representative consumer of country n faces a budget constraint given by:

$$\sum_{i \in \mathcal{I}} p_i \, C_{ni} \leq \sum_{i \in \mathcal{I}_n} \Pi_i + \sum_{f \in \mathcal{F}_n} p_f^\ell \bar{\ell}_f,$$

where Π_i are the profits of sector i and $p_f^\ell \bar{\ell}_f$ is the compensation earned by the local factor of production f. We define the consumer's Marshallian demand function $C_n(p, w_n)$, where $w_n = $

$\sum_{i \in \mathcal{I}_n} \Pi_i + \sum_{f \in \mathcal{F}_n} p_f^\ell \bar{\ell}_f$, and the consumer's indirect utility function from consumption in the stage game as $W_n(p, w_n) = U_n(C_n(p, w_n))$. The consumer's total indirect utility in the stage game is $W_n(p, w_n) + u_n(z)$.

Firms. A firm in sector i located in country n produces output y_i using a subset $\mathcal{J}_i \subset \mathcal{I}$ of intermediate inputs and the set of local factors of country n, $\mathcal{F}_n$. Firm i's production is $y_i = f_i(x_i, \ell_i, z)$, where $x_i = \{x_{ij}\}_{j \in \mathcal{J}_i}$ is the vector of intermediate inputs used by firm i, x_{ij} is use of intermediate input j, $\ell_i = \{\ell_{if}\}_{f \in \mathcal{F}_n}$ is the vector of factors used by firm i, and ℓ_{if} is use of local factor f. Firms take the aggregate vector z as given. For expositional simplicity, we assume that for production functions that in principle can use both factors and intermediate inputs we have $f_i(0, \ell_i, z) = 0$, so that a firm that has no ability to source intermediate inputs cannot produce.[5] We further assume that f_i is increasing, strictly concave, satisfies the Inada conditions in (x_i, ℓ_i), and is continuously differentiable in (x_i, ℓ_i, z). The sector-specific production function f_i allows us to capture technology, but also transport costs and relationship specific knowledge.

The timing of the stage game includes three subperiods: Beginning, Middle, and End. Since each sector has a continuum of identical firms and we will study a symmetric equilibrium, it will end-up featuring a representative firm in each sector. We refer to firm i when clarity necessitates distinguishing an individual firm from the rest of the firms in the same sector, and sector i when describing representative firm outcomes. The game described below unfolds between an individual firm in sector i and the continuum of firms in sector j. We refer to the respective players as firm i and suppliers in sector j.

In the Beginning, firm i places an order x_{ij} to suppliers in sector $j \in \mathcal{J}_i$ and an order ℓ_i for local factors. The order x_{ij} is placed in equal proportion to each firm in sector j. Factor orders are always accepted and factors cannot be stolen.

In the Middle, each firm in sector j decides to Accept, $a_{ij} = 1$, or Reject, $a_{ij} = 0$, the order of firm i. We assume all firms within a given sector j play the same pure strategy. If the order x_{ij} is Rejected by suppliers in sector j, firm i receives none of that input and owes no payment to suppliers in sector j. If the order is Accepted by suppliers in sector j, the suppliers immediately deliver the entire order x_{ij} to firm i.

In the End, firm i owes the payment $p_j x_{ij}$ to suppliers in sector j. Firm i can choose to Pay suppliers in sector j, or Steal from them and not make the payment. If firm i chooses to Steal, suppliers in sector j are only able to recover an exogenous fraction $1 - \theta_{ij} \in [0, 1]$ of the sale order value $p_j x_{ij}$. We denote $S_i \subset \mathcal{J}_i$ the subset of sectors from which firm i steals. For example, $S_i = \{1, 2\}$ denotes the action of stealing inputs 1 and 2 and not any others, and $S_i = \emptyset$ denotes no stealing. The set of all possible stealing actions is $P(\mathcal{J}_i)$, where $P(.)$ denotes the power set, that is the set of all subsets of the firm's supplier relations.

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[5]We allow for the presence of sectors that simply repackage the factors and use no intermediate inputs. As we describe below, since factors cannot be stolen, these sectors are treated separately from the main analysis and only used in some examples to sharpen the characterization.

For an order (x_i, ℓ_i) in the Beginning, a vector $a_i \in \{0, 1\}^{J_i}$ of acceptance choices in the Middle, and a stealing action $S_i \in P(\mathcal{J}_i)$ in the End, the stage game payoff to firm i in its buyer relationships is given by

$$p_i f_i(x_i \cdot a_i, \ell_i, z) - \sum_{j \in \mathcal{J}_i} p_j a_{ij} x_{ij} - \sum_{f \in \mathcal{F}_n} p_f^\ell \ell_{if} + \sum_{j \in S} \theta_{ij} p_j a_{ij} x_{ij}.$$

The payoff to the sales side of suppliers in sector j is $-\mathbf{1}_{j \in S} \cdot \theta_{ij} p_j a_{ij} x_{ij}$. Firms make zero profits on their sales side for an order that is accepted and not stolen, or for an order that is rejected.[6] They make strictly negative profits for a positive accepted order if it is stolen.

The evolution of the stage game can capture many economic relationships that are based on repeated transactions and incomplete contracts. For example, it covers a lender/borrower relationship in finance, a supplier-customer relationship in the goods market, a service provider and customer relationship, and infrastructure building over multiple installments.

Supplier Beliefs.   In the Beginning, suppliers in sector j have a belief $B_{ij} \in \{0, 1\}$ about firm i. If $B_{ij} = 0$, suppliers in sector j Distrust firm i and believe that it will Steal from them this period with probability 1. If $B_{ij} = 1$, suppliers in sector j Trust firm i and believe it is possible for firm i not to steal this period. We denote $\mathcal{B}_i = \{j \mid B_{ij} = 1\}$ to be the set of supplying sectors that Trust firm i. The set $\mathcal{B}_i$ is common knowledge.

We define the profits under no stealing from the subset of suppliers that Trust firm i to be the function $\Pi_i(x_i, \ell_i, \mathcal{B}_i) = p_i f_i(x_i, \ell_i, z) - \sum_{j \in \mathcal{B}_i} p_j x_{ij} - \sum_{f \in \mathcal{F}_n} p_f^\ell \ell_{if}$, where we leave implicit that $x_{ij} = 0$ for $j \notin \mathcal{B}_i$.

1.2   Repeated Game

We study equilibria that are Markov in $\mathcal{B}_i$, and restrict attention to pure strategies that are symmetric within a sector. A strategy of firm i in the Beginning is $\sigma_i^-(\mathcal{B}_i) \in \mathbb{R}_+^{J_i + F_n}$, mapping its suppliers' beliefs into an order (x_i, ℓ_i). A strategy of suppliers in sector j in the Middle with regard to firm i is $\sigma_{ij}(x_i, \ell_i, \mathcal{B}_i) \in \{0, 1\}$, mapping an order size and supplier beliefs into an acceptance decision a_{ij}. A strategy of firm i in the End is $\sigma_i^+(a_i, x_i, \ell_i, \mathcal{B}_i) \in P(\mathcal{J}_i)$, mapping acceptance decisions of its suppliers, its order size, and supplier beliefs into stealing action S_i.

We conjecture and verify a value function $\mathcal{V}_i(\mathcal{B}_i)$ of firm i in the repeated game that is non-decreasing in $\mathcal{B}_i$, that is $\mathcal{V}_i(\mathcal{B}_i) \leq \mathcal{V}_i(\mathcal{B}_i')$ if $\mathcal{B}_i \subset \mathcal{B}_i'$. We build this function below starting from an exogenous continuation value $\nu_i(\mathcal{B}_i)$ assumed to be non-decreasing and with $\nu_i(\emptyset) = 0$.

1.2.1   Trigger Strategies and Incentive Compatibility

We assume trigger strategies that define the evolution of beliefs following a stealing action S_i of the firm. Triggers can take two forms: individual and joint. Appendix A.1.1 formally characterizes

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[6] This reflects that a firm always has the option to sell elsewhere, for example to a consumer, at the price p_j, and that individual firms take prices as given.

trigger strategies and we focus below on a more intuitive presentation.

An individual trigger is given by $B'_{ij}(S_i) = 0$ if $j \in S_i$, that is if firm i Steals from suppliers in sector j, then suppliers in sector j Distrust individual firm i in all future periods. That is $B'_{ij}(S_i)$ is the updated belief starting in the next stage game, following the firm stealing action S_i in the current stage game.

Consider $M_{ij} \subset \mathcal{J}_i$ to be the set of sectors that supply to firm i with which sector j has a direct joint trigger. As an example of direct joint trigger, consider $M_{ij} = \{k\}$ then j is triggered directly from firm i Stealing from k, that is $B'_{ij}(S_i) = 0$ if $k \in S_i$. We assume that joint triggers are symmetric: $k \in M_{ij}$ if and only if $j \in M_{ik}$.

Joint triggers, however, also occur indirectly; for example, suppliers in sector j could also be indirectly triggered by firm i Stealing from suppliers in sector h if suppliers in k have a direct joint trigger with those in h, and suppliers in j have a direct joint trigger with those in k. To formalize the full set of joint triggers, both direct and indirect, we proceed iteratively. We first consider all direct triggers, this generates sets of suppliers that are jointly triggered. Then we apply direct triggers to the sets defined in the previous step, and so on iteratively until convergence to a set K_{ij} that contains individual and all joint trigger relationships for suppliers in j with respect to firm i's stealing actions. The supplier beliefs about firm i following Stealing action S_i are therefore $\mathcal{B}'_i(S_i) = \mathcal{B}_i \backslash (\bigcup_{j \in S_i} K_{ij})$.

In building the incentive compatibility constraint for firm i, we know by backward induction that suppliers never accept an order that will be stolen since their payoff is strictly negative from doing so. We assume that firms never place an order that is rejected since there is no advantage from doing so. Hence, we focus on a constraint for orders that are placed, accepted, and not stolen. This leads to the following characterization of incentive compatibility.[7]

Lemma 1 *Let $\mathcal{S}_i(\mathcal{B}_i) = \bigcup_{j \in \mathcal{B}_i} \{K_{ij}\}$ and $\Sigma(\mathcal{S}_i) = \{\bigcup_{X \in \mathcal{X}} X \mid \emptyset \neq \mathcal{X} \subset \mathcal{S}_i\}$. The order (x_i, ℓ_i) is incentive compatible with respect to all stealing actions, $P(\mathcal{B}_i)$, if and only if it is incentive compatible with respect to $\Sigma(\mathcal{S}_i(\mathcal{B}_i))$. The incentive compatibility constraint for $S_i \in \Sigma(\mathcal{S}_i(\mathcal{B}_i))$ is*

$$\sum_{j \in S} \theta_{ij} p_j x_{ij} \leq \beta \left[\nu_i(\mathcal{B}_i) - \nu_i(\mathcal{B}_i \backslash S_i) \right] \tag{1}$$

Lemma 1 characterizes incentive compatibility (IC) as a trade-off between flow utility from stealing, and continuation value from maintaining access to intermediate inputs not stolen. Figure 1 Panel (a) illustrates the case of suppliers in sector j only having an individual trigger, then the resulting IC constraint is $\theta_{ij} p_j x_{ij} \leq \beta[\nu_i(\mathcal{B}_i) - \nu_i(\mathcal{B}_i \backslash \{j\})]$. Firm i is trading off the one-off Stealing gain of $\theta_{ij} p_j x_{ij}$ with the continuation value loss of not being able to use input j again.

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[7]Incentive compatibility must be defined for firm i for any belief $\mathcal{B}_i \in \Sigma(\mathcal{S}_i)$, which are the beliefs that can be reached by some Stealing action when a firm starts at $\mathcal{B}_i = \mathcal{J}_i$ given the configuration of trigger strategies. On the other hand, beliefs $\mathcal{B}_i \notin \Sigma(\mathcal{S}_i)$ are not reached by any Stealing action or sequence of Stealing actions.

In the presence of joint triggers, incentive compatibility depends on grouping of inputs. For example, if sectors j and k have a joint trigger with each other (and no one else), then $K_{ij} = K_{ik} = \{j, k\}$. Intuitively, firm i then never Steals from only one of sectors j or k, since both would retaliate anyway. This allows us to represent joint triggers as restrictions on the set of stealing actions of firm i, by eliminating trivially dominated stealing actions. This is illustrated in Panel (b) of Figure 1. The resulting IC is $\theta_{ij} p_j x_{ij} + \theta_{ik} p_k x_{ik} \leq \beta[\nu_i(\mathcal{B}_i) - \nu_i(\mathcal{B}_i \backslash \{j, k\})]$.

In general, starting from all possible stealing actions $P(\mathcal{B}_i)$, we can use the logic in the example above to restrict our attention to a smaller subset of actions. Consider the set of the smallest undominated stealing actions $\mathcal{S}_i(\mathcal{B}_i) = \bigcup_{j \in \mathcal{B}_i} \{K_{ij}\}$, i.e., the union of the sets containing the joint triggers sets of suppliers that Trust firm i. Then consider the set of all supersets of this $\mathcal{S}_i(\mathcal{B}_i)$ defined to be $\Sigma(\mathcal{S}_i(\mathcal{B}_i))$. Lemma 1 shows that any order by firm i is incentive compatible if it is compatible with respect to the set of stealing actions $\Sigma(\mathcal{S}_i(\mathcal{B}_i))$. For expositional ease, henceforth we track the "action set (basis)" $\mathcal{S}_i(\mathcal{B}_i)$ directly, instead of tracking the sets K_{ij}. We use the shorthand $\mathcal{S}_i$ suppressing the dependency on $\mathcal{B}_i$ whenever all suppliers Trust the firm, i.e. $\mathcal{B}_i = \mathcal{J}_i$. Note that $\mathcal{S}_i$ is a partition of $\mathcal{J}_i$.

Returning to our illustrative examples. Panel (a) of Figure 1 considers the case in which suppliers in sectors j and k to firm i only have individual triggers. Therefore, $\mathcal{S}_i(\mathcal{B}_i) = \{\{j\}, \{k\}\}$ and $\Sigma(\mathcal{S}_i) = \{\{j\}, \{k\}, \{j, k\}\}$ and the resulting IC constraints are with respect to all possible stealing actions. Panel (b) of Figure 1 considers a joint trigger between sectors j and k. Therefore, $\mathcal{S}_i(\mathcal{B}_i) = \{\{j, k\}\}$ and $\Sigma(\mathcal{S}_i) = \{\{j, k\}\}$ and the only resulting IC constraint is with respect to stealing both j and k inputs together.

We are now ready to characterize the strategy of suppliers in sector j in the Middle. Suppliers in sector $j \in \mathcal{B}_i$ Accept the order if and only if equation (1) is satisfied for all $S \in \Sigma(\mathcal{S}_i(\mathcal{B}_i))$. Suppliers in sectors $j \notin \mathcal{B}_i$ reject any positive order.[8]

1.2.2 Firm i Optimal Production and Value Function

Since continuation value $\nu_i(\mathcal{B}_i)$ is non-decreasing, firm i's strategy in the Beginning is an order size (x_i, ℓ_i) to maximize its stage game payoff $\Pi_i(x_i, \ell_i, \mathcal{B}_i)$, subject to incentive compatibility of no stealing (equation (1)), and where $x_{ij} = 0$ for $j \notin \mathcal{B}_i$. Since Π_i is a concave function and equation (1) describes a convex set, the optimization problem of firm i is convex.

We complete construction of a subgame perfect equilibrium (SPE) for firm i by constructing the associated value function $\mathcal{V}_i(\mathcal{B}_i)$ at each point $\mathcal{B}_i \in \Sigma(\mathcal{S}_i)$. This construction follows an iterative process (Abreu et al. (1990)), which is derived in detail in Appendix A.2.1 and outlined here. First, we have $\mathcal{V}_i(\emptyset) = 0$.[9] The next step is to consider $\mathcal{B}_i \in \mathcal{S}_i$, so that firm i is Trusted by the smallest

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[8]If hypothetically suppliers in $j \notin \mathcal{B}_i$ Accepted a positive order, firm i would still believe that suppliers in j will reject every future order, given $B_{ij} = 0$. Firm i would then Steal from suppliers in j. Hence, suppliers in j reject the order. For $\theta_{ij} = 0$ this is an assumption given indifference for the suppliers, and otherwise a strict preference.

[9]In principle, one could allow for non-stationary (front-loaded) punishments in an attempt to worsen the

Figure 1: **Stealing, Action Sets, and Joint Threats**

(a) Unrestricted Stealing   (b) Joint Threat
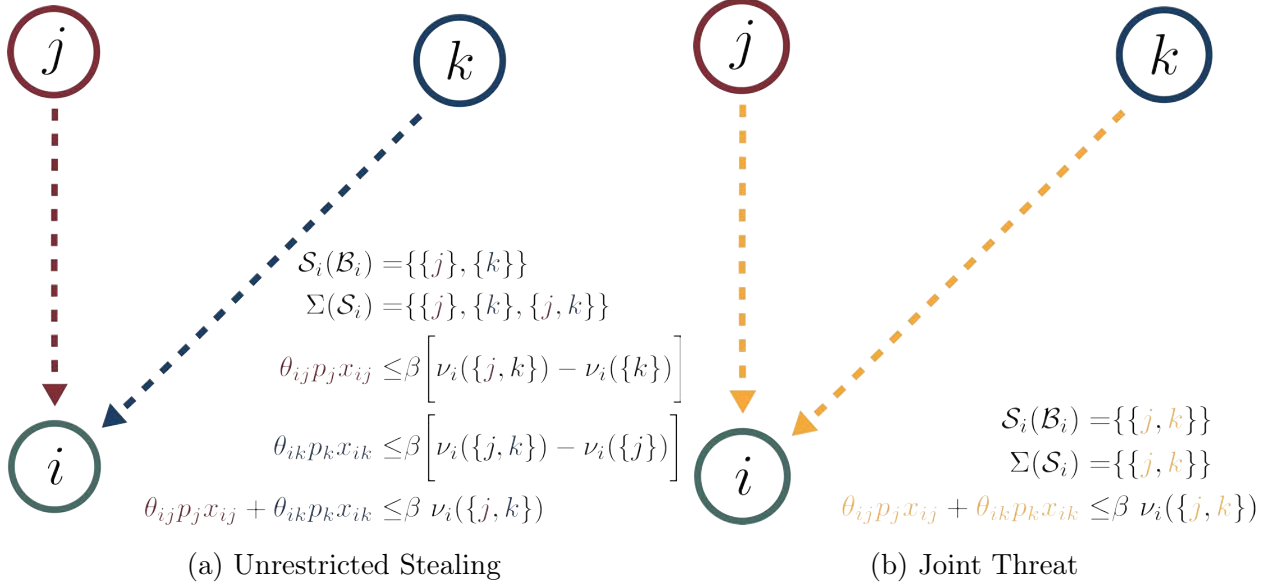
Notes: Panels focus on a firm in sector i with suppliers in sectors j and k. The action sets and related incentive constraints are from the perspective of firm i under different configurations. Panel (a) illustrates the case in which suppliers in sectors j and k have individual triggers only. Panel (b) illustrates the case in which suppliers in sectors j and k have a joint trigger, that is a joint threat.

subsets of suppliers that enter a joint trigger. We construct $\mathcal{V}_i(\mathcal{B}_i)$ using the fact that if firm i Steals it then reverts to $\mathcal{V}_i(\emptyset) = 0$. The iteration then progresses by constructing $\mathcal{V}_i(\mathcal{B}_i)$ for $\mathcal{B}_i = S_1 \cup S_2$ for $S_1, S_2 \in \mathcal{S}_i$, and so forth. In each step, the value function $\mathcal{V}_i(\mathcal{B}_i)$ is given as a fixed point of the equation

$$\mathcal{V}_i(\mathcal{B}_i) = \max_{x_i, \ell_i} \ \Pi_i(x_i, \ell_i, \mathcal{B}_i) + \beta \mathcal{V}_i(\mathcal{B}_i) \quad s.t. \quad \sum_{j \in S} \theta_{ij} p_j x_{ij} \leq \beta \left[\mathcal{V}_i(\mathcal{B}_i) - \mathcal{V}_i(\mathcal{B}_i \setminus S) \right] \quad \forall S \in \Sigma(\mathcal{S}_i(\mathcal{B}_i)).$$

In this iterative process, the value function constructed in the SPE with no stealing in steps $n = 0, \ldots, N$ is subsequently used as the off-path continuation values of the SPE at step $N + 1$, until the final step with $\mathcal{B}_i = \mathcal{J}_i$ is reached.[10]

off-path equilibrium and sustain a better equilibrium than Markov and potentially implement the Ramsey plan (Ray (2002); Acemoglu et al. (2008)). Our purpose is not to explore the best sustainable equilibrium but to focus on a simple Markov one that provides much economics while minimizing the theoretical complexity.

[10]If an element $\mathcal{B}_i \in \Sigma(\mathcal{S}_i)$ has no SPE associated with no stealing, then we assume that at the beginning of a period in which firm i faces beliefs $\mathcal{B}_i$, the beliefs of suppliers automatically update to an element $\hat{\mathcal{B}}_i \in \Sigma(\mathcal{S}_i(\mathcal{B}_i))$ such that $\hat{\mathcal{B}}_i$ results in an SPE with no stealing. As a result, $V_i(\mathcal{B}_i) = V_i(\hat{\mathcal{B}}_i)$. That is to say, suppliers understand that if beliefs were $\mathcal{B}_i$, the firm would in fact Steal from a subset with probability 1, and therefore suppliers update beliefs accordingly. We assume throughout the paper that $\mathcal{B}_i = \mathcal{J}_i$ has an SPE with no stealing.

Example: Two-Period Model and Nested CES Production. Assume there are only two periods and that in the second period there are no incentives problems (i.e. all θ's are set to zero in the second period). Each sector uses a two-tier nested constant elasticity of substitution (CES) production function. Firm i produces using input vector x_i with length $|\mathcal{J}_i|$, and for simplicity no local factors. The inputs are partitioned into bundles, where $\tilde{x} \in \tilde{X}_i$ denotes the varieties of inputs used in a given bundle, and $\tilde{X}_i$ is the set of all bundles. We assume each input only enters one bundle. The production function is then given by:

$$f_i(x_i) = \left(\sum_{\tilde{x} \in \tilde{X}_i} \tilde{\alpha}_{i\tilde{x}} \left(\sum_{j \in \tilde{x}} \alpha_{ij} x_{ij}^{\chi_{i\tilde{x}}} \right)^{\frac{\rho_i}{\chi_{i\tilde{x}}}} \right)^{\frac{\xi_i}{\rho_i}}.$$

We allow CES parameters $\chi_{i\tilde{x}}$ to vary across bundles. At time zero, the loss in continuation value arising from stealing variety k is given by:

$$\log \nu_i(\mathcal{B}_i) - \log \nu_i(\mathcal{B}_i \backslash \{k\}) = -\frac{\xi_i}{1 - \xi_i} \frac{1 - \rho_i}{\rho_i} \log \left[1 - \Omega_{i\tilde{x}_k} \left(1 - \left(1 - \omega_{ik} \right)^{\frac{1 - \chi_{i\tilde{x}_k}}{\chi_{i\tilde{x}_k}} \frac{\rho_i}{1 - \rho_i}} \right) \right], \quad (2)$$

where $\Omega_{i\tilde{x}_k}$ is the expenditure share of firm i on the bundle that contains input k denoted by $\tilde{x}_k$, and ω_{ik} is the expenditure share on input k within that bundle.[11]

All else equal, losing varieties with bigger expenditure shares leads to a greater loss. Intuitively, losing inputs that are cheap (low p_k) or are technologically a large fraction of production (i.e. high related α's) increases the loss. Losing a variety k is more costly the closer the production function is to constant returns to scale $\xi \uparrow 1$ because a more scalable production suffers more from one of its inputs being constrained at zero.

To understand the role of substitutability within and across buckets, consider the specific bucket that contains variety k. Fix a within-bundle expenditure share ω_{ik}. If that bucket has a parameter $\chi_{i\tilde{x}_k} \leq 0$ (i.e. more complementarity than Cobb-Douglas), then losing variety k amounts to the same as losing the entire bucket. Intuitively, this occurs because the absence of input k makes strictly positive production from that bucket impossible. For parameters $\chi_{i\tilde{x}_k} > 0$, the loss decreases the more the varieties are substitutable. A similar logic applies across baskets and is governed by the parameter ρ_i.

This example illustrates the role of "alternatives" in diminishing the value of threats to shut off a firm from a particular input. Intuitively, the existence of closely substitutable inputs or the fact that a particular input accounts for a small expenditure share, decreases this input's strategic value in threats. In principle, the values in equation (2) can be estimated in the data with standard methods for elasticities of nested CES production functions and given expenditure shares are observable.[12]

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[11]See Appendix A.4.1 for a derivation of equation (2) and definitions of the expenditure shares.

[12]Going back to Figure 1, this example can also be used to illustrate the pattern of binding constraints.

1.3 Market Clearing, Externalities, and Equilibrium

Denote $D_j = \{i \in \mathcal{I} \mid j \in \mathcal{J}_i\}$ the set of sectors that source from sector j, i.e. the sectors immediately downstream from j. Market clearing for good j is given by

$$\sum_{n=1}^{N} C_{nj} + \sum_{i \in D_j} x_{ij} = y_j.$$

Market clearing for factor f in country n is

$$\sum_{i \in \mathcal{I}_n} \ell_{if} = \bar{\ell}_f.$$

We assume that the vector of aggregates takes the form $z = \{z_{ij}\}$. In equilibrium $z_{ij}^* = x_{ij}^*$, where we use the $*$ notation to stress it is an equilibrium value. That is externalities are based on the quantities of inputs in bilateral sectors i and j relationships. This general formulation can be specialized to cover pure size externalities, in which it is the total output of a sector that matters, or export-import externalities, in which it is the fraction of output sold cross border that matters, but also thick market externalities, in which it is the extent to which an input is widely used by many sectors that matters.[13]

An equilibrium of the model is prices for goods and factors P and allocations $\{x_i, C_n, y_i, \ell_i, z_{ij}\}$ such that: (i) firms maximize profits, given prices; (ii) households maximize utility, given prices; (iii) markets clear.

1.4 Leading Simplified Environments

To build intuition for our model it will at times be useful to simplify the modeling environment by shutting off several channels. This will also be helpful in separately highlighting the driving forces behind the results. We consider three classes of simplifications going forward. First, a "constant prices" environment in which we switch off pecuniary externalities and terms-of-trade manipulation incentives. Second, a "no z-externalities" environment in which we switch off the dependency of utility functions and production functions on the aggregates vector z. Third, a "separable production" environment in which we restrict production functions to be separable by

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In Panel (a), if firm i's continuation values are sub-modular, that is $\nu_i(\mathcal{B}_i \backslash (S_1 \cup S_2)) + \nu_i(\mathcal{B}_i) \leq \nu_i(\mathcal{B}_i \backslash S_1) + \nu_i(\mathcal{B}_i \backslash S_2)$ for $S_1 \cap S_2 = \emptyset$, then the individual IC constraints for inputs j and k imply the joint constraint. Intuitively, this occurs because under submodularity the absence of one input weakly increases the firm demand for all other inputs. Submodularity is guaranteed by the restriction $\xi_i \leq \rho_i \leq \chi_{i\tilde{x}}$, i.e. that the goods are sufficiently substitutable. While we do not generally impose such restriction, the problem can be further simplified if this is imposed and in particular by focusing on separable production ($\xi = \rho_i = \chi_{i\tilde{x}}$).

[13]It is without loss of generality to assume that firm-to-firm sales, y_{ij}, do not cause externalities, since $x_{ji} = y_{ij}$ already captures such sales on the buyer side. It is straightforward to allow the z to also capture externalities coming from factor usage or consumption. In addition to externalities coming from the z the model features pecuniary externalities arising from prices in the constraints.

input. We briefly define each environment below so that it can easily be referred to when useful in the rest of the paper. Our main results do not use these simplified environments.

Definition 1 *Our **constant prices** environment assumes that consumers have identical linear preferences over goods, $U_n = \sum_{i \in \mathcal{I}} \tilde{p}_i C_{ni}$, and that each country has a local-factor-only firm with linear production $f_i(\ell_i) = \sum_{f \in \mathcal{F}_n} \frac{1}{\tilde{p}_i} \tilde{p}_f^\ell \ell_{if}$. We assume consumers are marginal in every good and factor-only firms are marginal in every local factor so that $p_i = \tilde{p}_i$ and $p_f^\ell = \tilde{p}_f^\ell$.[14]*

Definition 2 *Our **no z-externalities** environment assumes that $u_n(z)$ and $f_i(x_i, \ell_i, z)$ are constant in z.*

Definition 3 *Our **separable production** environment assumes that firms that use intermediate inputs have $f_i(x_i, \ell_i, z) = \sum_{j \in \mathcal{J}_i} f_{ij}(x_{ij}, z)$.*

2   Hegemonic Power

Our main analysis focuses on when and how a hegemon can build power and wield it to demand costly actions. We begin this section by defining and characterizing *pressure points* on firms, which denote a set of off-equilibrium-path threats on a firm that, when consolidated into a single joint threat, generate an increase in profits earned by that firm on the equilibrium path. We then introduce the problem of a hegemon country that is able to join together threats, and ask when and how the hegemon can create and extract value by doing so.

2.1   Joint Threats and Pressure Points

A joint threat in our model is a coordination of trigger strategies among multiple supplying sectors of the same firm. Figure 1 illustrates a simple example. Firms in sectors j and k are supplying inputs to sector i. A joint threat on a firm in sector i, in this example, is the suppliers in j and k adopting a joint trigger. Using Lemma 1, we define joint threats as restrictions on action sets.

Definition 4 *A **joint threat** $\mathcal{S}_i'$ is a partition of $\mathcal{J}_i$ such that $\mathcal{S}_i'$ is coarser than $\mathcal{S}_i$.*

As an example, consider the case of firm i sourcing from four sectors, and a starting configuration with only individual triggers. Then $\mathcal{S}_i = \{\{1\}, \{2\}, \{3\}, \{4\}\}$, i.e. the individual stealing decisions. A joint threat is a new partition $\mathcal{S}_i'$ of $\mathcal{J}_i$ such that every element of $\mathcal{S}_i$ is contained in exactly one element of $\mathcal{S}_i'$. For example, a joint threat among suppliers in sectors 1 and 2, and a separate one among suppliers in sectors 3 and 4, is represented by $\mathcal{S}_i' = \{\{1, 2\}, \{3, 4\}\}$. A joint threat among suppliers in sectors 1, 2, and 3 is represented by $\mathcal{S}_i' = \{\{1, 2, 3\}, \{4\}\}$.

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[14]For example, we can guarantee this by assuming consumers and the factor-only firms can short goods and factors.

Joint threats generically generate value for the firm being threatened because they relax incentive constraints. This is natural in set-ups in which trigger strategies can be used to threaten agents with punishments in order to induce good behavior. Consider a firm i that faces an exogenous continuation value function ν_i and is trusted by all of its suppliers, that is $\mathcal{B}_i = \mathcal{J}_i$. We define the firm's current value as a function of its action set $\mathcal{S}_i$ as[15]

$$V_i(\mathcal{S}_i) = \max_{x_i, \ell_i} \Pi_i(x_i, \ell_i, \mathcal{J}_i) + \beta\nu_i(\mathcal{J}_i) \quad s.t. \quad \sum_{j \in S} \theta_{ij} p_j x_{ij} \leq \beta\left[\nu_i(\mathcal{J}_i) - \nu_i(\mathcal{J}_i \backslash S)\right] \quad \forall S \in \Sigma(\mathcal{S}_i).$$

Then for any joint threat action set $\mathcal{S}_i'$ formed from $\mathcal{S}_i$, we have

$$V_i(\mathcal{S}_i') \geq V_i(\mathcal{S}_i).$$

Of course, in many cases the value creation is zero, for example when incentive constraints are all not binding, but our main interest is in the cases of strictly positive value. We define a pressure point for firm i as a joint threat that *strictly* increases the profits of firm i.[16]

Definition 5 *A **pressure point** of firm i is a joint threat $\mathcal{S}_i'$ that strictly increases firm i's profits, that is $V_i(\mathcal{S}_i') > V_i(\mathcal{S}_i)$.*

2.2 Hegemon Contract

We consider a single country m that has the opportunity to become a hegemon. At each date t, country m's government can pay a fixed utility cost $F_m \geq 0$ in order to become a hegemon for that date. For now, we think of all other countries' governments as facing arbitrarily large fixed costs, so that they do not become hegemons.[17] If m becomes a hegemon, it gains the ability to coordinate its firms ("collusion"), including the ability to create joint threats. It can then propose take-it-or-leave-it offers to *all* downstream sectors from $\mathcal{I}_m$, where contract terms specify joint threats, transfers, and restrictions on inputs purchased. Unlike individual firms and consumers, the hegemon internalizes how the terms of its contract affect the aggregates z and prices P.

Since we are focusing on Markov equilibria, the hegemon offers a contract only for the current stage game, and takes the future decisions of itself and of firms as given (i.e., the hegemon cannot commit to future contracts). As in Section 1.2, we start by taking $\nu_i(\mathcal{B}_i)$ to be an exogenous

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[15]Note that $V_i(\mathcal{S}_i)$ defines a value function of firm i over its action set $\mathcal{S}_i$ for an exogenous continuation value function and assuming the firm is trusted by all its suppliers ($\mathcal{B}_i = \mathcal{J}_i$), whereas $\mathcal{V}_i(\mathcal{B}_i)$ defines the equilibrium (fixed point) value function of firm i when Trusted by suppliers $\mathcal{B}_i$ and keeping constant the joint threats.

[16]In Appendix A.4.2, we show how to identify pressure points in our separable production environment (Definition 3).

[17]In Appendix A.3, we study competition between multiple hegemons.

continuation value function of firm i.[18]

Recalling that D_i is the set of sectors downstream from sector i, let $\mathcal{D}_m = \bigcup_{i \in \mathcal{I}_m} D_i \backslash \mathcal{I}_m$ denote the set of foreign sectors that source at least one input from the sectors in the hegemon's country. We assume that the hegemon can contract with all its domestic sectors and their foreign downstream sectors, and denote $\mathcal{C}_m = \mathcal{I}_m \cup \mathcal{D}_m$ to be this set. Let $\mathcal{J}_{im} = \mathcal{I}_m \cap \mathcal{J}_i$ denote the set of inputs that sector i sources from (sectors in) country m.

Hegemon m proposes a take-it-or-leave-it offer to each firm $i \in \mathcal{C}_m$. The contract offered to firm i has three terms: (i) a joint threat $\mathcal{S}'_i$; (ii) nonnegative transfers $\mathcal{T}_i = \{T_{ij}\}_{j \in \mathcal{J}_{im}}$ from firm i to the hegemon's representative consumer (with $T_{ij} > 0$ representing a payment to the hegemon associated with stealing decision j of firm i); (iii) revenue-neutral taxes $\tau_i = \{\{\tau_{ij}\}_{j \in \mathcal{J}_i}, \{\tau^\ell_{if}\}_{f \in \mathcal{F}_n}\}$ on purchases of inputs and factors, with equilibrium revenues $\tau_{ij} x^*_{ij}$ and $\tau^\ell_{if} \ell^*_{if}$ raised from sector i rebated lump sum to firms in sector i. Naturally, remitted revenues x^*_{ij} and ℓ^*_{if} are determined by the contract terms, as made clear below. We denote $\Gamma_i = \{\mathcal{S}'_i, \mathcal{T}_i, \tau_i\}$ the contract offered to firm $i \in \mathcal{C}_m$, and denote $\Gamma = \{\Gamma_i\}_{i \in \mathcal{C}_m}$.

Taxes adjust the effective price the firm faces in its relationship to $p_j + \tau_{ij}$ for inputs and $p^\ell_f + \tau^\ell_{if}$ for factors. Factor rebates occur regardless of Pay/Steal decisions since factors cannot be stolen. Transfers and input rebates occur contemporaneously with the Pay/Steal decision. Under the contract, if firm i Pays suppliers in sector j, then it pays $p_j x_{ij}$ to suppliers in sector j and pays $\tau_{ij}(x_{ij} - x^*_{ij}) + T_{ij}$ to the hegemon's consumer. If firm i Steals from suppliers in sector j, it makes no payments. In this case, suppliers in sector j only recover an amount $(1 - \theta_{ij})p_j x_{ij}$, while hegemon m's representative consumer recovers $(1 - \theta_{ij})\tau_{ij}(x_{ij} - x^*_{ij})$.

Transfers can cover different interpretations: direct monetary payments, a firm-specific mark-up charged by the hegemon on sales of its goods, or the extraction of value in some other action the firm takes on behalf of the hegemon (see later discussion of lobbying and political concessions).

The revenue-neutral taxes are a set of wedges in the problem of firm i that allow us to capture the ability of the hegemon to ask the firm to change its allocation of inputs and factors. Wedges of this type are typical in the macro-prudential literature that focuses on pecuniary and demand externalities (Farhi and Werning (2016)). This can capture either quantity restrictions or taxes/subsidies (see for example Clayton and Schaab (2022)). Importantly, we allow these instruments to target relationships between two sectors. This covers, for example, restricting energy imports from Russia but not from other countries; or tariffs and quantity restrictions on imports of Chinese goods.[19]

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[18]We provide the conditions for this value function and corresponding hegemon solution to be an equilibrium in Appendix A.2.2.

[19]We focus on restrictions, costly actions, imposed on firms on buying inputs from other suppliers. In principle, we could also allow for bilateral taxes on sales by firm i. In equilibrium, any sales taxes would be fully passed through to the buyer and, in this sense, would be captured by the input taxes that we already consider. However, a difference is that the input taxes on firm i that arise from sales taxes on firm j would not in principle require firm i to agree to the contract. Similarly, we could also allow bilateral taxes on sales by firm i to consumers.

Feasible Joint Threats. We restrict the joint threats that the hegemon can make to involve sectors that are at most one step removed from the hegemon, that is involving either the hegemon's sectors or their immediately downstream sectors. We impose this restriction to prevent unrealistic situations in which the hegemon threatens a firm that it has no (immediate) relationship with. Formally, we refer to the act of creating a joint threat from $S, S' \in \mathcal{S}_i$ as *consolidating* S and S', and define *direct transmission* of threats as follows.

Definition 6 *Hegemon m can consolidate $S \in \mathcal{S}_i$ under* direct transmission *if $\exists j \in S$ with either $j \in \mathcal{I}_m$ (direct control) or $j \in \mathcal{D}_m$ (indirect control). A joint threat is **feasible** if it can be achieved under direct transmission.*

Intuitively, Definition 6 says that the hegemon can create a joint threat using action $S \in \mathcal{S}_i$ if either the hegemon supplies a good $j \in S$ to firm i, or if the hegemon supplies a good to a sector $j \in \mathcal{D}_m$ that in turn is a supplier to firm i, that is $j \in S$. The former is a case of direct control: the hegemon coordinates a joint threat between two actions S and S' over which it has direct control by directly coordinating the trigger strategies of two or more firms, one in each action. The latter is a case of indirect control: the hegemon instead creates a joint threat via a downstream supplier, by requiring the downstream supplier, as part of its contract, to adopt the trigger strategy associated with the joint threat. Appendix Figure A.1 provides an illustration along the line of Figure 1 of which threats the hegemon can consolidate.

For each $i \in \mathcal{C}_m$, define the set of *direct transmission links* $\mathcal{S}_i^D \subset \mathcal{S}_i$ as the subset of elements $S \in \mathcal{S}_i$ that can be consolidated under direct transmission by the hegemon.[20] Observe that the ex-ante equilibrium can be implemented by a feasible contract, whereby the hegemon proposes the terms $\Gamma_i = \{\mathcal{S}_i, 0, 0\}$.

Firm Participation Constraint. Firm $i \in \mathcal{C}_m$ chooses whether or not to accept the take-it-or-leave-it offer made by the hegemon. If firm i rejects the hegemon's contract, it retains its original action set and achieves the value $V_i(\mathcal{S}_i)$. Firm i, being small, does not internalize the effect of its decision to accept or reject the contract on the prevailing aggregate vector z and prices. If instead firm i accepts the offer, it chooses allocations to maximize profits given the contract terms. Given

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[20]One can imagine threats being passed on over more than direct links, for example each firm passing on the threat to the next one over a chain. Further, one could imagine stipulating that the threats are agreed to be carried on with some probability less than one, so that at each link the threat becomes weaker in probability (decaying over the length of the chain). For simplicity, we keep the length of the chain to be 1 and the threat to be carried out for sure.

a contract Γ_i, the value to firm i of accepting the contract is given by[21]

$$V_i(\Gamma_i) = \max_{x_i,\ell_i} \Pi_i(x_i, \ell_i, \mathcal{J}_i) - \sum_{j\in\mathcal{J}_i}[\tau_{ij}(x_{ij} - x_{ij}^*) + T_{ij}] - \sum_{f\in\mathcal{F}_m} \tau_{ij}^\ell(\ell_{if} - \ell_{if}^*) + \beta\nu_i(\mathcal{J}_i) \tag{3}$$

$$s.t. \sum_{j\in S}\left[\theta_{ij}[p_j x_{ij} + \tau_{ij}(x_{ij} - x_{ij}^*)] + T_{ij}\right] \leq \beta\left[\nu_i(\mathcal{J}_i) - \nu_i(\mathcal{J}_i\backslash S)\right] \quad \forall S \in \Sigma(\mathcal{S}_i')$$

Recall that transfers and taxes are associated with the firm decision to Pay, and so enter the incentive constraint. Transfers T_{ij} tighten the incentive constraint, all else equal. At the level of the individual firm, taxes have two effects: (i) they affect the incentive constraint because they alter the perceived price of the input good; (ii) they affect the incentive constraint via loss of profits. In equilibrium, this latter effect washes out since taxes are rebated lump sum (i.e., $x_{ij} = x_{ij}^*$). The optimal allocation $x_{ij}^*(\Gamma_i)$, and hence remitted revenues, are defined implicitly as a function of contract terms by the above optimization problem.

For firm i to accept the contract, it must be better off under the contract than by rejecting it. This gives rise to the **participation constraint** of firm i,

$$V_i(\Gamma_i) \geq V_i(\mathcal{S}_i), \tag{4}$$

where recall that $\Gamma_i = \{\mathcal{S}_i', \mathcal{T}_i, \tau_i\}$ so that the participation constraint is comparing the hegemon's contract with joint threats, transfers, and wedges to the outside option. Slackness in this constraint when the hegemon demands no costly actions out of the target ($\Gamma_i = \{\mathcal{S}_i', \{0\}, \{0\}\}$) means that the hegemon has a pressure point on firm i (Definition 5). This is the source of hegemon's power over firm i.

Manipulating the Outside Option. In principle, we could have also allowed the hegemon to make threats conditional on a firm rejecting the contract. This amounts to manipulating the outside option of targeted entities by threatening to cut off access to any of the hegemon's inputs if the contract is rejected. The right hand side of the participation constraint (equation (4)) would then be the value $V_i(\mathcal{S}_i | \mathcal{B}_i = \mathcal{J}_i\backslash\mathcal{J}_{im})$, that is the value of firm i when it is no longer Trusted by any of the suppliers that the hegemons controls.[22]

Such a one-off threat at date t is (weakly) effective for the hegemon taking continuation values as given. However, in a Markov equilibrium the threat would be made in every period. Lowering

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[21]We extend the previous definition of firm i value function $V_i(\mathcal{S}_i)$ to incorporate the full terms of the hegemon contract $V_i(\Gamma_i)$ where $\Gamma_i = \{\mathcal{S}_i', \mathcal{T}_i, \tau_i\}$. We abuse notation and write $V_i(\mathcal{S}_i)$ as short hand for $V_i(\Gamma_i)$ when $\Gamma_i = \{\mathcal{S}_i, 0, 0\}$.

[22]Formally, we define

$$V_i(\mathcal{S}_i|\mathcal{B}_i) = \max_{x_i,\ell_i}\Pi_i(x_i, \ell_i, \mathcal{B}_i) + \beta\nu_i(\mathcal{B}_i) \quad s.t. \sum_{j\in S}\theta_{ij}p_j x_{ij} \leq \beta\left[\nu_i(\mathcal{B}_i) - \nu_i(\mathcal{B}_i\backslash S)\right] \quad \forall S \in \Sigma(\mathcal{S}_i(\mathcal{B}_i))$$

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the future value of retaining access to the hegemon's inputs tightens the incentive constraints of the targeted entity. This reduces the targeted entity's on-path production possibilities and tightens the left-hand side of their participation constraint. Therefore blunt threats to lower the targeted firm's outside option are in part self-defeating. We focus instead on the hegemon providing joint threats that increase the inside-option of the targeted entity in accepting the hegemon's contract.

Hegemon Maximization Problem. The hegemon's objective function is the utility of its representative consumer, to whom all domestic firm profits and all transfers accrue. As in Section 1.1, the consumer's wealth is

$$ w_m = \sum_{i \in \mathcal{I}_m} V_i(\Gamma_i) + \sum_{f \in \mathcal{F}_m} p_f^\ell \bar{\ell}_f + \sum_{i \in \mathcal{C}_m} \sum_{j \in \mathcal{J}_{im}} T_{ij}. $$

Note that because transfers from domestic sectors to the hegemon's consumer net out from the consumer's wealth, we need only keep track of operating profits $\Pi_i(\Gamma_i) = V_i(\Gamma_i) + \sum_{j \in \mathcal{J}_{im}} T_{ij}$ of the hegemon domestic sectors. Similarly, taxes on all sectors are revenue neutral for the hegemon, and therefore net out. However, transfers from foreign sectors do not net out, precisely because the hegemon's consumer has no claim to foreign sectors' profits. The hegemon objective function is then:

$$ \mathcal{U}_m = W_m(p, w_m) + u_m(z), \quad w_m = \sum_{i \in \mathcal{I}_m} \Pi_i(\Gamma_i) + \sum_{f \in \mathcal{F}_m} p_f^\ell \bar{\ell}_f + \sum_{i \in \mathcal{D}_m} \sum_{j \in \mathcal{J}_{im}} T_{ij}. \tag{5} $$

Conditional on entering, the hegemon's maximization problem is choosing a contract $\Gamma = \{\mathcal{S}_i', \mathcal{T}_i, \tau_i\}_{i \in \mathcal{C}_m}$ to maximize its consumer utility (equation (5)), subject to the participation constraints of firms (equation (4)), the feasibility of joint threats (Definition 6), the determination of aggregates $z_{ij}^* = x_{ij}^*(\Gamma_i)$, and determination of prices via market clearing. Given its optimal contract conditional on entry, the hegemon enters if $\mathcal{U}_m - F_m \geq \mathcal{U}_m^0$, where $\mathcal{U}_m^0$ is utility when not entering.

2.3   Optimality of Maximal Joint Threats

We solve the hegemon's problem in two steps. First, we prove that the hegemon offers a "maximal" joint threat that joins together all threats that it can consolidate, i.e. chooses the threat $\mathcal{S}_i'$ that is the coarsest feasible partition of $\mathcal{J}_i$. Second, we characterize transfers and wedges under the optimal contract.

For each $i \in \mathcal{C}_m$, define the maximal joint threat action set that is feasible under direct transmission as $\overline{\mathcal{S}}_i' = \{\cup_{S \in \mathcal{S}_i^D} S\} \cup (\mathcal{S}_i \backslash \mathcal{S}_i^D)$, which consolidates all $S \in \mathcal{S}_i^D$ into a single joint threat. We obtain the following result.

Lemma 2 *It is weakly optimal for the hegemon to offer a contract with maximal joint threats to every firm it contracts with, that is $\mathcal{S}_i' = \overline{\mathcal{S}}_i'$ for all $i \in \mathcal{C}_m$.*

Intuitively, Lemma 2 follows from the observation that joint threats expand the set of feasible allocations, and so weakly increase targeted entities' profits. Formally, a hegemon that chose a contract that did not involve maximal joint threats could always implement the same transfers and allocations while offering a contract with maximal joint threats. Hence offering maximal joint threats can increase value to the hegemon but cannot decrease it.[23]

Since the hegemon's contract involves all of its domestic sectors that supply to sector i entering a single joint threat, transfers can be tracked in total at the sector level, that is $\overline{T}_i = \sum_{j \in \mathcal{J}_{im}} T_{ij}$, rather than at the bilateral supplier level T_{ij}. We therefore abuse notation and track only $\overline{T}_i$ in the contract, rather than $\mathcal{T}_i$.

2.4   A First Pass: Optimal Contract

To build intuition for the optimal contract, we simplify the economic environment in this subsection to that described in Definition 1 and 2, that is an environment with constant equilibrium prices and no externalities arising from the vector of aggregates z. In this simplified environment, the proposition below characterizes the optimal contract offered by the hegemon, differentiating between domestic and foreign firms.

Proposition 1 *Conditional on entry, with constant prices (Definition 1) and no z-externalities (Definition 2), an optimal contract of the hegemon has the following terms:*

1. *All wedges are zero on all sectors, $\tau_{ij}^* = \tau_{if}^{\ell*} = 0$ for all $i \in \mathcal{C}_m$, $j \in \mathcal{J}_i$, $f \in \mathcal{F}_n$.*

2. *All transfers are zero for domestic sectors, that is $\overline{T}_i^* = 0$ for all $i \in \mathcal{I}_m$.*

3. *Foreign sector i is charged a positive transfer $\overline{T}_i^* > 0$ if and only if $\overline{\mathcal{S}}_i'$ is a pressure point on i. The transfers are then set so that the participation constraint binds, $V_i(\Gamma_i) = V_i(\mathcal{S}_i)$ and $\Gamma_i = \{\overline{\mathcal{S}}_i', \overline{T}_i^*, 0\}$.*

We define a sector to be an *extraction point* for the hegemon if, under the optimal contract, it makes a strictly positive transfer. Sectors are strategic for the hegemon if they build power by enabling the hegemon to make threats that increase extraction (either more extraction from an existing extraction point, or new extraction points). We return to this in Section 2.7 in the context of the general analysis.

To understand the hegemon's optimal contract, we focus first on domestic sectors. Since prices are fixed and there are no externalities from the vector of aggregates z, the hegemon's decision

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[23]Threats in this paper are off the equilibrium path, while costly actions (like sanctions) are carried out in equilibrium. We intentionally designed the threats to be "cheap" to make, with the advantage that we did not have to characterize which threats the hegemon chooses to make. What makes the threats cheap is that sellers can always find another buyer since buyers are atomistic. This can be relaxed, for example, by introducing a "trembling hand", to make threats take place on the equilibrium path and also by introducing costly threats whereby sellers cannot find alternative buyers to a positive mass of customers.

problem is equivalent to maximizing its representative consumer's wealth. Since the contribution of different sectors to hegemon wealth is separable, the hegemon's decision problem is separable across sectors. The hegemon's optimization problem for a domestic sector is

$$\max_{\overline{T}_i, \tau_i} \Pi_i(\Gamma_i) \quad s.t. \quad V_i(\Gamma_i) \geq V_i(\mathcal{S}_i).$$

The hegemon sets $\tau_i = 0$ because wedges, in the absence of z-externalities and with fixed prices, can only decrease the targeted entity's profits compared to its privately optimal decision, that is Π_i and V_i are maximized at $\tau_i = 0$. Similarly, positive transfers directly tighten the targeted entity's incentive constraint, and therefore reduce its profits. Since transfers from domestic sectors are a wash for the hegemon's representative consumer, it is optimal to set them to zero.[24] Domestic sectors, therefore, are never extraction points. If the hegemon's joint threat includes a pressure point on a domestic sector, the optimal contract features the threat, relaxes the targeted entity's incentive constraint, and expands its profits. The participation constraint is slack, and the hegemon receives the value of the increase in profits from the sector's payout to consumers.

For a foreign sector, the hegemon's decision problem is different, since the objective is to extract transfers rather than maximize the sector's profits. Therefore, the hegemon solves

$$\max_{\overline{T}_i, \tau_i} \overline{T}_i \quad s.t. \quad V_i(\Gamma_i) \geq V_i(\mathcal{S}_i).$$

For the same reason as for domestic sectors, the hegemon also sets $\tau_i = 0$ for foreign sectors. In contrast, while transfers do reduce sector profits, similarly to the domestic case, the hegemon's consumer has no claim to these profits. The hegemon therefore would like to charge transfers to foreign sectors. What limits the ability of the hegemon to do so is the participation constraint. If the joint threat that the hegemon offers does not include a pressure point, then the participation constraint binds even at no transfers. In this case, the hegemon has nothing of value to offer to the targeted entities, and so cannot extract any transfers. If instead the hegemon's threat includes a pressure point, then the hegemon extracts the entire increase in the target's value as a transfer. We conclude that the hegemon has an extraction point if and only if it has a pressure point on that sector.

2.5 Leontief Inverse and Network Propagation with Externalities

We now return to the general set up. We first show that our economy has an input-output structure in which amplification occurs not only via prices, but also via the z-externalities.

The equilibrium vector of aggregates, z^*, must satisfy $x_{ij}^*(\Gamma_i|z^*, P) = z_{ij}^*$. We derive an analysis

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[24]Recall that we ruled out negative transfers. As in the macro-prudential literature, the hegemon would want to use negative transfers (subsidies) to slacken incentive constraints of domestic sectors. Consistent with the literature, we have ruled out these subsidies.

allowing for stage game equilibria with (possibly suboptimal) contracts Γ. To clarify the ordering for matrix algebra, we have $z_i^* = (\ z_{i,\min \mathcal{J}_i}^*,\ \ldots,\ z_{i,\max \mathcal{J}_i}^*\ )^T$ which is a $|\mathcal{J}_i| \times 1$ vector, and $z^* = (\ z_1^{*T},\ \ldots,\ z_{|\mathcal{I}|}^{*T}\ )^T$, which is a $\sum_{i \in \mathcal{I}} |\mathcal{J}_i| \times 1$ vector. For compactness, we use $|z^*| = \sum_{i \in \mathcal{I}} |\mathcal{J}_i|$. We stack x^* starting from elements x_{ij}^* in the same manner.

Consider a generic exogenous variable e. To understand the impact that a change in e has on the entire input-output system, we derive a Leontief inverse based on the endogenous response of z^*. That is, we are interested in computing the vector $\frac{dz^*}{de}$, which is a $|z^*| \times 1$ vector. We start by totally differentiating $x_{ij}^*(\Gamma_i|z^*, P) = z_{ij}^*$ in e,

$$\frac{\partial x_{ij}^*}{\partial e} + \frac{\partial x_{ij}^*}{\partial P}\frac{dP}{de} + \frac{\partial x_{ij}^*}{\partial z^*}\frac{dz^*}{de} = \frac{dz_{ij}^*}{de},$$

where $\frac{\partial x_{ij}^*}{\partial z}$ is a $1 \times |z^*|$ vector. Stacking the system vertically, we write

$$\frac{\partial x^*}{\partial e} + \frac{\partial x^*}{\partial P}\frac{dP}{de} + \frac{\partial x^*}{\partial z^*}\frac{dz^*}{de} = \frac{dz^*}{de},$$

where $\frac{\partial x^*}{\partial e}$ is a $|z^*| \times 1$, and $\frac{\partial x^*}{\partial z^*}$ is a $|z^*| \times |z^*|$ matrix with each row corresponding to the vector $\frac{\partial x_{ij}^*}{\partial z^*}$. We can therefore express $\frac{dz^*}{de}$ as:

$$\frac{dz^*}{de} = \left(\mathbb{I} - \frac{\partial x^*}{\partial z^*}\right)^{-1}\left(\frac{\partial x^*}{\partial e} + \frac{\partial x^*}{\partial P}\frac{dP}{de}\right).$$

We define $\Psi^z = \left(\mathbb{I} - \frac{\partial x^*}{\partial z^*}\right)^{-1}$ and note that it is akin to a Leontief inverse matrix since it keeps track of all the successive amplification via the input-output structure of the original change in production.

If prices P were fixed as in the environment in Definition 1, then the term $\frac{\partial x^*}{\partial P}\frac{dP}{de}$ would be zero and amplification would only occur via the z-externalities: $\frac{dz^*}{de} = \Psi^z \frac{\partial x^*}{\partial e}$. If prices are also reacting endogenously, then the change in demand x^* fed through the externality-based Leontief inverse is not only the direct change, but also the change that arises through total changes in equilibrium prices. Conversely, if we switched-off the z-externalities as in Definition 2, then the matrix Ψ^z would reduce to the identity matrix, and the only amplification would occur via prices: $\frac{dz^*}{de} = \frac{\partial x^*}{\partial e} + \frac{\partial x^*}{\partial P}\frac{dP}{de}$. To provide a full characterization that incorporates these price changes, we define the excess demand for good i as[25]

$$ED_i = \sum_{n=1}^{N} C_{ni}(p, w_n(\Gamma|z^*, P)) + \sum_{j \in \mathcal{D}_i} x_{ji}(\Gamma_i|z^*, P) - y_i(\Gamma_i|z^*, P),$$

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[25]To simplify notation and avoid having to write multiple distinct summations, for sectors $i \notin \mathcal{C}_m$ we define their contribution to excess demand as a function of the trivial contract $\Gamma_i = \{\mathcal{S}_i, 0, 0\}$ that is equivalent to their outside option.

and analogously define the excess demand for factor f as

$$ED_f^\ell = \sum_{i \in \mathcal{I}_n} \ell_{if}^*(\Gamma_i | z^*, P) - \bar{\ell}_f.$$

Finally, we define the $(|\mathcal{I}| + |\mathcal{F}|) \times 1$ column vector $ED = (ED_1, \ldots, ED_{|\mathcal{I}|}, ED_1^\ell, \ldots, ED_{|\mathcal{F}|}^\ell)^T$.

In a manner parallel to the derivations above, we totally differentiate the system $ED(\Gamma, z^*, P) = 0$ to obtain price responses (see the proof of Proposition 2). The following proposition characterizes the change in both aggregate quantities z^* and prices P following an exogenous perturbation.

Proposition 2 *The aggregate response of z^* and P to a perturbation in exogenous variable e is*

$$\frac{dz^*}{de} = \Psi^z \left(\frac{\partial x^*}{\partial e} + \frac{\partial x^*}{\partial P} \frac{dP}{de} \right)$$

$$\frac{dP}{de} = -\left(\frac{\partial ED}{\partial P} + \frac{\partial ED}{\partial z^*} \Psi^z \frac{\partial x^*}{\partial P} \right)^{-1} \left(\frac{\partial ED}{\partial e} + \frac{\partial ED}{\partial z^*} \Psi^z \frac{\partial x^*}{\partial e} \right)$$

Consider the response of prices to a perturbation in e. If there are no z-externalities (Definition 2), it collapses to a standard equation $\frac{dP}{de} = -\left(\frac{\partial ED}{\partial P} \right)^{-1} \frac{\partial ED}{\partial e}$. Intuitively, the perturbation to e changes excess demand in each market as a result of reoptimization by firms and consumers. These changes in excess demand must then be counteracted through price changes to equilibrate markets, with $\frac{\partial ED}{\partial P}$ giving the response of excess demand to prices.

When there are externalities from aggregates z^*, both the effects of the perturbation on excess demand and the effects of price perturbations on excess demand now also operate through z-externalities. Considering first the direct impact on excess demand, the perturbation affects firm demand $\frac{\partial x^*}{\partial e}$, then it affects the vector of aggregates throughout the system via z-externalities, as represented by the inverse Ψ^z. This total change in aggregate quantities $\Psi^z \frac{\partial x^*}{\partial e}$ then affects excess demand, $\frac{\partial ED}{\partial z^*}$. The effects via price perturbations are analogous.

2.6 General Analysis: Optimal Contract and Efficiency

In characterizing the hegemon's optimal contract, we set up the following notation (see the proof of Proposition 3 for details). Letting $\mathcal{L}_m$ be the hegemon's Lagrangian, we denote $\eta_i \geq 0$ the Lagrange multiplier on the participation constraint of firm i, and $\Lambda_{iS} \geq 0$ the Lagrange multiplier on the incentive constraint of firm i for stealing action S. We also define $\overline{\Lambda}_{ij} = \sum_{S \in \Sigma(\overline{\mathcal{S}}_i') | j \in S} \Lambda_{iS}$, which sums all multipliers involving the action of stealing good j by firm i, and $\overline{\Lambda}_{iS_i^D} = \sum_{S \in \Sigma(\overline{\mathcal{S}}_i') | S_i^D \subset S} \Lambda_{iS}$, which sums all multipliers involving the action of stealing inputs contained in the hegemon's maximal joint threat. We define $\mathcal{E}_{ij} = \frac{\partial \mathcal{L}_m}{\partial z_{ij}^*}$ to be the hegemon's perceived externalities from an increase in z_{ij}^*, and $\Xi_{mn} = \frac{\partial \mathcal{L}_m}{\partial z^*} \left[\frac{dz^*}{dw_m} - \frac{dz^*}{dw_n} \right] + \frac{\partial \mathcal{L}_m}{\partial P} \left[\frac{dP}{dw_m} - \frac{dP}{dw_n} \right]$ to be the hegemon's perceived externalities

from a transfer of wealth from consumers in country n to consumers in country m. An optimal contract is characterized by the proposition below.[26]

Proposition 3 *Conditional on entry, an optimal contract of the hegemon has the following terms:*

1. *For domestic firms $i \in \mathcal{I}_m$, if $\overline{\mathcal{S}}'_i$ is a pressure point on i:*

   (a) *Input wedges satisfy:* $(\frac{\partial W_m}{\partial w_m} + \eta_i + \theta_{ij}\overline{\Lambda}_{ij})\tau^*_{ij} = -\mathcal{E}_{ij}.$

   (b) *Transfers are zero:* $\overline{T}^*_i = 0.$

2. *For foreign firms $i \in \mathcal{D}_m$ located in country n, if $\overline{\mathcal{S}}'_i$ is a pressure point on i:*

   (a) *Input wedges satisfy:* $(\eta_i + \theta_{ij}\overline{\Lambda}_{ij})\tau^*_{ij} = -\mathcal{E}_{ij}.$

   (b) *Transfers satisfy:* $\overline{\Lambda}_{iS^D_i} + \eta_i \geq \frac{\partial W_m}{\partial w_m} + \Xi_{mn}$, *with equality if* $\overline{T}^*_i > 0.$

3. *If $\overline{\mathcal{S}}'_i$ is not a pressure point of firm i, then $\overline{T}_i = 0$ and $\tau_i = 0$.*

Intuitively, for a domestic firm and in the presence of z-externalities and endogenous prices, the hegemon no longer finds it optimal to impose zero wedges because it uses wedges to manipulate externalities. Activities that generate positive externalities $\mathcal{E}_{ij} > 0$ are subsidized, while activities that generate negative externalities $\mathcal{E}_{ij} < 0$ are taxed. The externalities captured in $\mathcal{E}_{ij}$ are not only z-externalities, but also pecuniary externalities and terms-of-trade manipulation. Indeed, $\mathcal{E}_{ij}$ is composed of three terms (see proof of Proposition 3 for definitions):

$$\mathcal{E}_{ij} = \underbrace{\varepsilon^z_{ij}}_{\text{Direct Impact}} + \underbrace{\overbrace{\varepsilon^{zNC}\frac{dz^{*NC}}{dz_{ij}}}^{\text{Aggregate Quantities}} + \overbrace{\varepsilon^{Pm}\frac{dP^m}{dz_{ij}}}^{\text{Prices}}}_{\text{Indirect Impact: Input-Output Amplification}} \tag{6}$$

where ε^z_{ij} measures the direct value to the hegemon of increasing sector i's use of input j, $\varepsilon^{zNC}\frac{dz^{*NC}}{dz_{ij}}$ measures the indirect value of altering production via input-output amplification in sectors that the hegemon does not control, and $\varepsilon^{Pm}\frac{dP^m}{dz_{ij}}$ is the indirect value of the induced changes in equilibrium prices. From Proposition 2 the term $\frac{dz^{*NC}}{dz_{ij}}$ contains the Leontief inverse amplification. In setting the wedges the hegemon takes into account not only the direct effect of altering quantity z_{ij}, but also the indirect effect that this activity has on both other aggregate quantities, via the input-output amplification, and on equilibrium prices.

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[26]Proposition 3 provides necessary conditions for optimality. Formally, if for a foreign firm i we have $\eta_i = 0$ and $\Lambda_{iS} > 0$, it instead characterizes the limit of a sequence of wedges, each of which is part of a (different) optimal contract (see the proof for details). For technical reasons, we assume that if $\overline{\mathcal{S}}'_i$ is not a pressure point on firm i at the optimal (z^*, P), then it is also not a pressure point on i in a neighborhood of (z^*, P). Finally to streamline analysis we assume that every foreign country contains at least one firm that the hegemon cannot contract with, meaning that the hegemon cannot directly mandate factor prices in foreign countries. This final assumption is convenient but easily relaxed (see the proof).

The wedges interact with both the incentive constraint and the participation constraint. If the constraints are tighter, i.e. higher Lagrange multipliers, the subsidies and taxes shrink towards zero. The hegemon trades off distorting private production decisions, which tightens the constraints, against the benefit of the distortion arising from externalities. Tighter constraints make this trade-off put more weight on private costs (for fixed externalities).

Familiar from Proposition 1, domestic firms are never charged transfers. However, this result is no longer immediate: in the presence of externalities, in principle the hegemon might want to use transfers to reduce firms' capacity to engage in negative-externality activities. However, in the presence of complete wedges, the hegemon can instead use wedges to achieve this goal, and so no transfers are charged.

Consider next a foreign firm. The hegemon's optimal wedge formula is almost identical to that for domestic firms, except that the magnitude of wedges (whether tax or subsidy) is higher. Intuitively, this occurs because the hegemon does not (directly) value the profits of foreign firms, as it does for domestic firms. Indeed, the term $\frac{\partial W_m}{\partial w_m}$ is missing in 2 (a) compared to 1 (a). As a result, the hegemon is more willing to impose higher corrective wedges, even though they erode operating profits.

While the hegemon still has an incentive to extract transfers from foreign firms in the presence of externalities there are countervailing forces. As in Proposition 1 charging a higher transfer to a firm has a cost of tightening both the participation constraint and the incentive constraint, valued by the multipliers $\eta_i + \overline{\Lambda}_{iS_i^D}$. In the context of Proposition 1, this cost has to be balanced with the benefit to the hegemon consumer of receiving the transfer. With constant prices that marginal benefit is constant at 1 and therefore we had $\eta_i + \Lambda_{iS_i^D} \geq 1$, holding with equality for $\bar{T}_i^* > 0$.

In the general set-up, the marginal benefit to the hegemon of the transfer is more complex. First, the direct marginal benefit is not constant at 1, but given by the marginal value of wealth $\frac{\partial W_m}{\partial w_m}$. Second, there is an indirect (externality) term Ξ_{mn} because reallocating wealth from consumers in country n to those in the hegemon country m, alters equilibrium prices and aggregates z as long as these consumers have different marginal expenditures.

Much of the trade and international macroeconomics literature has focused on terms of trade manipulation as the motive for imposing tariffs, capital controls, and entering multilateral trade agreements. These motives are present in our general analysis, but they are not our main focus, and can be switched off by considering the constant prices environment in Definition 1. In that environment, $\Xi_{mn} = 0$, $\frac{\partial W_m}{\partial w_m} = 1$, and Leontief amplification occurs only via the z-externalities. The hegemon uses wedges to manipulate externalities in its favor and exploits the endogenous Leontief amplification that generates a gap between the private costs to the firms accepting the contract and the social (hegemon) value of the costly actions.

2.6.1 Classifying Friends and Enemies

Our framework provides a classification of "friends and enemies" of the hegemon based on externalities. This terminology and notion is related to Kleinman et al. (2020) who base it on a country's real income response to a foreign country's increase in productivity. Foreign sector i is friendly, neutral, or unfriendly based on the externalities that sector has from the hegemon's perspective.

Definition 7 *Under the hegemon's optimal contract, foreign sector i is:*

1. ***Unfriendly*** *to the hegemon if $\mathcal{E}_{ij} \leq 0$ for all $j \in \mathcal{J}_i$, with strict inequality for at least one j.*

2. ***Neutral*** *to the hegemon if $\mathcal{E}_{ij} = 0$ for all $j \in \mathcal{J}_i$.*

3. ***Friendly*** *to the hegemon if $\mathcal{E}_{ij} \geq 0$ for all $j \in \mathcal{J}_i$, with strict inequality for at least one j.*

Definition 7 delineates three types of relationships: friendly sectors that have only (weakly) positive spillovers from the hegemon's perspective, neutral sectors with no spillovers, and unfriendly sectors with only (weakly) negative spillovers. Of course, sectors can in general have some activities that generate positive spillovers and some activities that generate negative ones. We leave those sectors unclassified in the definition above, as mixed sectors.

The notion of friendship that we develop is both theoretically grounded and relevant for understanding how the hegemon interacts with these sectors in its optimal contract. For example, a friendly sector i has its strictly positive-externality activities subsidized, while an unfriendly sector has its strictly negative-externality activities taxed. A neutral sector, in contrast, is neither taxed nor subsidized as long as at least one constraint binds ($\overline{\Lambda}_{ij} + \eta_i > 0$), consistent with Proposition 1, in which all sectors were neutral.

Equation (6) shows that whether a sector is friendly does not only depend on its direct impact, but also on how its actions are amplified in equilibrium via the input-output network. For example, even a domestic sector might end up being unfriendly in equilibrium if its output leads to a large increase in production from foreign sectors unfriendly to the hegemon (i.e. the second and third terms in Equation (6) dominate the first one).

Proposition 3 1(a) shows that wedges are also applied to domestic firms. These wedges are akin to domestic industrial policy, and in our framework this policy can be driven by domestic (e.g. education and R&D) or foreign considerations (e.g. national security). Through our framework, one can understand recent U.S. export restrictions on U.S. semiconductor firms (such as Nvidia and Intel) selling their output to certain Chinese sectors. While the U.S. may value a larger American semiconductor industry, American policymakers may find that the downsides of providing Chinese firms access to these firms' output outweigh the benefits.

Friendship is also an important driver of which sectors are held to their participation constraints and achieve no surplus under the optimal contract. Despite the hegemon having all the bargaining power, in the presence of externalities the optimal contract might leave surplus to the foreign sectors

(slack participation constraint) whenever the indirect benefits to the hegemon from these sectors not shrinking are sufficiently strong. For example, a hegemon might leave surplus to a friendly sector in order to maximize the benefits arising indirectly from its positive externalities.

An analogous concept of friends and enemies can be extended to foreign representative consumers based on the sign of Ξ_{mn}. If this term is positive, the foreign representative consumer is an enemy in the sense that removing wealth from that consumer and reallocating it to the hegemon consumer increases hegemon's welfare beyond the direct effect of the transfer. Neutral and friendly are then defined accordingly.

2.6.2 Efficient Allocations

We provide an efficiency benchmark by taking the perspective of a global planner that has exactly the same powers and constraints as the hegemon, but cares about global welfare. Formally, the planner chooses a contract $\Gamma = \{\Gamma_i\}_{i \in \mathcal{C}_m}$ to maximize global welfare,

$$\sum_{n=1}^{N} \Omega_n \left[W_n(p, w_n) + u_n(z) \right], \quad w_n = \sum_{i \in \mathcal{I}_n} V_i(\Gamma_i) + \sum_{f \in \mathcal{F}_i} p_f^\ell \overline{\ell}_f + \mathbf{1}_{n=m} \sum_{i \in \mathcal{C}_m} \sum_{j \in \mathcal{J}_{im}} T_{ij}, \tag{7}$$

subject to the participation constraints of firms (equation (4)), the feasibility of joint threats (Definition 6), the determination of aggregates, and the determination of prices via market clearing. The Pareto weight placed on the welfare of country n's consumer is Ω_n. As is common in the literature, we mute the planner's motive to redistribute wealth between countries by setting the welfare weights to equalize the social marginal value of wealth across consumers. The following proposition characterizes the global planner's solution.

Proposition 4 *An optimal contract of the hegemon from the global planner's perspective features maximal joint threats $\mathcal{S}_i' = \overline{\mathcal{S}}_i'$, zero transfers $\overline{T}_i = 0$, and wedges given by $(\Omega_n \frac{\partial W_n}{\partial w_n} + \eta_i + \theta_{ij} \overline{\Lambda}_{ij}) \tau_{ij}^* = -\mathcal{E}_{ij}^p$ for all sectors $i \in \mathcal{C}_m$ on which the hegemon has a pressure point. Wedges and transfers are zero if $\overline{\mathcal{S}}_i'$ is not a pressure point on i.*

The planner and the hegemon agree that supplying maximal joint threats is optimal since it relaxes the targeted entities' incentive problems and in principle allows more economic activity to take place. The planner and the hegemon, however, disagree on the value of transfers and on the optimal wedges to be applied.

Both the planner and the hegemon understand that the transfers are a negative-sum globally since they tighten incentive problems. The planner, therefore, chooses never to demand transfers. The hegemon, instead, values receiving positive transfers from foreign firms.[27]

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[27]If we allowed hegemon consumers to own foreign sectors this would contribute to aligning the hegemons' incentives with those of the planner by making the hegemon care about the profits of foreign sectors that it owns. Exogenous ownership of foreign sectors would be easy to introduce in this framework.

Both the global planner and the hegemon want to use the wedges in equilibrium to affect externalities. However, the global planner implements wedges that are different from those implemented by the hegemon. In general, the global planner does not perceive friends and enemies the same way as the hegemon does. Intuitively, a sector might have a negative externality on the hegemon country but a positive one on other countries. Formally, this can be seen in the proposition above in which $\mathcal{E}_{ij}^p$ tracks the impact of activity x_{ij} on the planner's Lagrangian rather than the hegemon's one.

Proposition 4 highlights some crucial features of our model. The presence of geoeconomic power in our framework is not zero-sum. Geoeconomic power has the potential to improve global outcomes, making everyone weakly better off, but the benefits accrue disproportionately to the hegemon. One negative-sum aspect arises from the transfers that destroy value at the global level, while transferring wealth from foreign sectors to the hegemon. Another arises from the hegemon manipulating externalities in its favor rather than in the global benevolent perspective.

Intuitively, the geoeconomic supply of threats expands the global Pareto frontier, but the hegemon generally chooses a contract to the inside of this frontier to maximize its own welfare.

2.7 Strategic Sectors and The Nature of Geoeconomic Power

Controlling, defending from foreign influence, and growing strategic sectors is a core government policy in democracies and autocracies alike. While governments frequently protect or control industries claiming they are strategic for "national interest", there is a concern that the "strategic" label is in reality a cover for protectionism or for subsidies to politically connected entities. This ambiguity is possible because of a lack of clarity on what it means for an economic activity to be strategic and a clear framework against which policies are to be evaluated.[28]

In our framework, a sector is strategic in two dimensions. First, because the hegemon can use it to form (off-path) threats on other entities. Second, because the hegemon can demand (on-path) costly actions from this sector that shape the world equilibrium in the hegemon's favor. Control, either directly via ownership or indirectly via other economic relationships, of a sector enables the hegemon to build power by making joint threats. We distinguish two notions of power that are what makes sectors strategic: Micro-Power and Macro-Power.

Micro-Power arises when the hegemon's threats increase the value of the targeted entity, that is when the hegemon has a pressure point on firm i (Definition 5). The amount of Micro-Power is given by $V_i(\overline{\mathcal{S}}_i') - V_i(\mathcal{S}_i)$, taking as given all equilibrium aggregate quantities and prices. Micro-Power measures the private value to the targeted entity of the hegemon's threat. Macro-Power arises when the hegemon collectively asks the targeted entities for costly actions that shape equilibrium

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[28]See Baldwin (1985)["Strategic Goods" section, pages 223-233] for a review of many informal definitions of strategic goods, including a quote from Soviet leader Nikita Khrushchev: "Anything one pleases can be regarded as strategic material, even a button, because it can be sewn onto a soldier's pants. A solider will not wear pants without buttons, since otherwise he would have to hold them up with his hands. And then what can he do with his weapon?".

aggregate quantities and prices in the hegemon's favor. The presence of Micro-Power is a necessary condition for the hegemon to exert Macro-Power.

Micro-Power: Strategic Sectors in Threatening Target Output. Joint threats can increase commitment for the targeted entity by reducing its continuation value from stealing (the term $\nu_i(\mathcal{J}_i) - \nu_i(\mathcal{J}_i \backslash S)$ in the IC constraint in equation (3)), which increases the targeted entity's value $(V_i(\Gamma_i))$ and slackens the participation constraint (equation (4)). The slack in the participation constraint when the hegemon demands no costly actions corresponds to the amount of Micro-Power in a pressure point $(V_i(\overline{\mathcal{S}}_i') - V_i(\mathcal{S}_i))$. The hegemon builds as much Micro-Power as it can by making maximal joint threats (Lemma 2), and then wields this power to demand costly actions to maximize its objective function (equation (5)). Consider a hegemon that can use one more sector to form joint threats; that additional sector generates more Micro-Power to the extent its use in joint threats is valuable for the targeted entities.[29] Proposition 1 studies Micro-Power. It switches off Macro-Power by studying a set-up with constant prices and no z-externalities. The hegemon uses its power to extract monetary transfers and keeps all foreign entities at their participation constraint.

Section 1.2.2 formalizes the source of Micro-Power as the loss in continuation value for the targeted entity from losing access to a set of inputs. The nested CES example shows how the value added of these inputs and their micro-substitutability affect the continuation value loss. Typical examples of goods that are strategic in this micro-sense are those widely used, with high value added for targets, and with poor substitutes. Some goods have these properties due to physical constraints: rare earths and oil. Others have them in equilibrium due to increasing returns to scale and natural monopolies. For example, the dollar-based financial infrastructure of payment and clearing systems (like SWIFT) is a strategic asset that the US often uses in geoeconomic threats. These financial services platforms have a thick-market externality, whereby each marginal user finds the service more valuable the more other users are on the same platform. The dollar system is so dominant that non-dollar based alternatives are poor substitutes on the margin.

In identifying Micro-Power it is necessary, but not sufficient, to know the parameters of the production function (e.g. the various elasticities of substitution in a nested CES). It is also necessary to know which inputs the hegemon controls.[30] Consider for example oil as an input. Returning to the nested CES example of Section 1.2.2, let us consider a sector i for which the top-tier of production is Leontief $(\rho_i \downarrow -\infty)$ in oil and other inputs, and for which oil is itself a second-tier CES basket of different types of oil produced in different countries. We assume these varieties of oil are close to perfect substitutes with each other (χ_i close to 1 for the oil basket). Whether oil is a strategic input for the hegemon in targeting sector i depends on its control over the various forms

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[29]Formally, this is a vector of values. Each element is the increase in value $V_i(\overline{\mathcal{S}}_i') - V_i(\mathcal{S}_i)$ for targeted sector i in the hegemon contracting set $\mathcal{C}_m$. Since transfers and wedges are distortionary, the distribution of this power over the entities and not just its total (the sum of the vector elements) matters for hegemon's welfare.

[30]As emphasized by Schelling (1958), the notion of strategic has to be defined in the context of an equilibrium, and cannot be determined solely from ex-ante characteristics of a sector.

of oil. Controlling one variety of oil is ineffective since the high degree of substitutability in the oil basket makes the losses in equation (2) small from being cut off from any one variety. However, if the hegemon controls a joint threat among all varieties of oil, that threat is very valuable since equation (2) then implies that the withdrawal of the oil basket makes all production by sector i (using any other inputs) not viable.

The logic above equally applies to joint threats for inputs that might seem rather unrelated without guidance from a theoretical framework. For example, the joint threat might involve financing loans and manufacturing inputs, rather than different types of oil. Section 3.2 provides an application along these lines for China's Belt and Road Initiative.

Macro-Power: Strategic Sectors in General Equilibrium. Macro-power arises from the hegemon's ability to extract value from the world economy indirectly, via shaping the equilibrium. By collectively asking entities that it can pressure to take costly actions, such as curbing the usage of some inputs, the hegemon indirectly influences a larger part of the input-output network than what it directly controls. The propagation and amplification through the network structure (our externality based Leontief-inverse) is key to this effect. In this macro sense, strategic sectors tend to be those that have a high influence on world output due to endogenous amplification (in the Leontief-inverse). Sectors like research and development, and information technology are good candidates for being strategic.

Proposition 3 shows that the marginal value to the hegemon of having more power over sector i is given by the Lagrange multiplier η_i on that sector's participation constraint. This multiplier reflects the benefit to exerting both Micro- and the Macro-Power over sector i.[31]

A hegemon particularly values having Micro-Power over sectors that increase its Macro-Power because it can exploit the difference between the private costs to targeted entities and the social benefit to itself. In accepting the hegemon's demands the targeted entities consider only their private costs, but the hegemon internalizes the social benefits of the outcomes of these actions. Formally, starting from Proposition 3 2(a) we have that optimal wedges are set at $\tau_{ij}^* = -\frac{1}{(\eta_i + \theta_{ij}\bar{\Lambda}_{ij})}\mathcal{E}_{ij}$. Expanding the term $\mathcal{E}_{ij}$ (recall equation (6)), we have

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[31]Sectors with high multipliers should be candidates for foreign acquisitions by a hegemon. For example, in implementing its global ambitions, the Chines government formulated an official "Going Out" strategy and in 2006 provided a set of "Guidance Policies for Overseas Investment". Overseas investment projects were categorized as encouraged, permitted, or prohibited. Encouraged projects would receive full government support: "corresponding policy support in aspects such as macroeconomic regulation, multi-bilateral economic and trade policies, diplomacy, finance, taxation, foreign exchange, customs, resource information, credit, insurance, as well as bilateral and multilateral cooperation and foreign affairs." The policy included a detailed "Guidance Catalog for Overseas Investment Industries" that categorized industries. Encouraged foreign sector acquisitions included: Exploration and development of mining industries; Manufacturing of chemical products with advanced technology that cannot be obtained domestically, including engineering plastics, specialty chemical raw materials; and Development and processing production of electronic information products, including the development and processing production of communication equipment and products. Source: http://jldrc.jl.gov.cn/jgcs/wzc/gzdt/201310/t20131031_5227130.html, last downloaded Jan 2 2023, translation using ChatGPT referred to in quotes above.

$$\tau_{ij}^* = -\frac{1}{(\eta_i + \theta_{ij}\overline{\Lambda}_{ij})}\left[\varepsilon_{ij}^z + \varepsilon^{zNC}\frac{dz^{*NC}}{dz_{ij}} + \varepsilon^{P^m}\frac{dP^m}{dz_{ij}}\right], \tag{8}$$

All else equal, the hegemon demands more action (higher wedges) out of sectors that have higher influence on either equilibrium quantities or prices (the last two terms in the equation). Section 3.1 further explores these incentives in an application focusing on telecommunication infrastructure and national security, and characterizes how the hegemon can extract value indirectly by using network amplification to contain an hostile country. Equation (8) shows that the value of power over a sector, η_i, is related to the ratio of how much the hegemon wants to control activities in that sector, $\mathcal{E}_{ij}$, versus how much the hegemon actually controls activities in that sector, τ_{ij}^*. When desired control $\mathcal{E}_{ij}$ is high relative to actual control τ_{ij}^*, the hegemon has little correction in place over an activity that it perceives to have high general equilibrium influence. Macro-Power is thus highly valuable in such circumstances.

3 Applications

We show how the model can be specialized to capture leading applications in geoeconomics. In the interest of brevity, we only cover two prominent applications.

In the first, we focus on a hegemon blocking third party countries from using a technology input provided by an unfriendly country. We assume the unfriendly technology is a national security threat for the hegemon, but a positive externality for production by firms in third party countries. This helps us capture bans on emerging technology such as semiconductors or the 5G telecommunication infrastructure provided by Huawei.[32]

In the second, we show how the hegemon can combine lending and manufacturing activities to extract political concessions, which helps capture in the model programs such as China's Belt and Road Initiative. Dreher et al. (2022), Gelpern et al. (2022), Horn et al. (2021), Horn et al. (2023), and Liu (2023) document and analyze the rise of China as a global development and project finance lender.
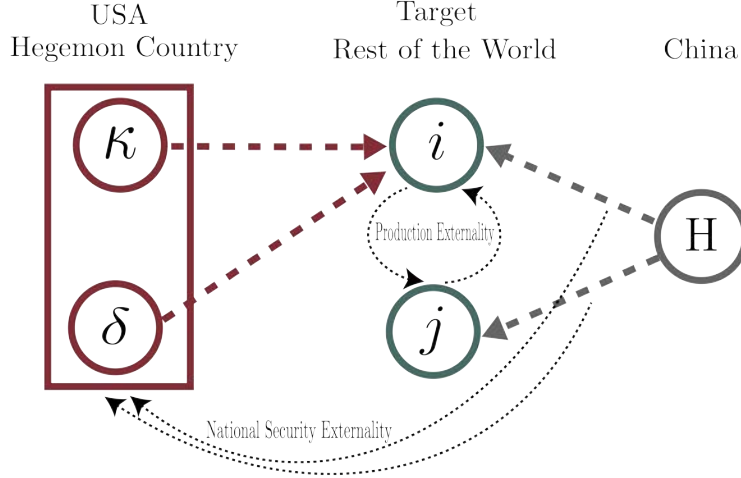
3.1 National Security Externalities

There are three regions: the hegemon country m, a hostile foreign country h, and "rest of world" *RoW* which may comprise multiple countries. Figure 2 illustrates the set-up of this application. We assume constant prices (Definition 1).

The hostile foreign country h has a single sector, which we denote by H. We take the output of this sector to be the numeraire, $p_H = 1$. Sector H and sectors in the hegemon country are not subject to externalities from z, that is $f_H(x_H, \ell_H, z)$ and $f_k(x_k, \ell_k, z)$ for $k \in \mathcal{I}_m$ are constant in z. For simplicity, we assume that firms in the hegemon country do not source from the hostile

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[32]See the discussions in Miller (2022) and Farrell and Newman (2023).

Figure 2: **Application: National Security Externality**

country's sector H and vice-versa, which ensures that H cannot be used by the hegemon as part of a joint threat.

The main action in this application comes from rest-of-world sectors, $i \in \mathcal{I}_{RoW}$. We assume that all rest-of-world sectors source from H, and define $z^H \equiv \{z_{iH}\}_{i \in \mathcal{I}_{RoW}}$ to be the vector of purchases by rest-of-world sectors of input H. We assume sectors in the rest of the world have production that is separable in H: $f_i(x_i, \ell_i, z) = f_{i,-H}(x_{i,-H}, \ell_i) + f_{iH}(x_{iH}, z^H)$, where $x_{i,-H}$ denotes the vector of all inputs except input H. We introduce external economies of scale by setting:

$$f_{iH}(x_{iH}, z^H) = A_{iH}(z^H)g_{iH}(x_{iH}). \tag{9}$$

We assume that $\frac{\partial A_{iH}}{\partial z_{jH}} > 0$ for all $i, j \in \mathcal{I}_{RoW}$, so that there are positive spillovers from greater usage of H.[33] We further assume that $A_{iH}(z^H)g_{iH}(z_{iH})$ is concave in z^H. Observe that $f_{i,-H}$ is constant in z. Finally, for simplicity we assume $\theta_{iH} = 0$, so that firm i is unconstrained in its use of input H. Finally, we assume that in absence of a hegemon, there are no joint triggers.[34]

Hegemon Negative Externality from H. We assume that the hegemon's representative consumer's utility function has a negative externality from rest-of-world production using H, that is $u_m(z) = u_m(z^H)$ and $\frac{\partial u_m}{\partial z_{iH}} < 0$ for all $i \in \mathcal{I}_{RoW}$. There are no other externalities in the utility function of consumers. From Lemma 2, maximal joint threats are optimal for the hegemon. Since there are no externalities associated with production by domestic firms and since prices are constant,

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[33]Note that sector i's purchases z_{iH} appear in z^H, so aggregate purchases by own sector affect own sector productivity.

[34]That is, $\mathcal{S}_i = \{\{j\}\}_{j \in \mathcal{J}_i}$.

Proposition 3 tells us $\overline{T}_i = 0$ and $\tau_i = 0$ is an optimal contract for all domestic sectors. Therefore, we focus on characterizing optimal contracts for foreign sectors in the rest of the world. The relevant part of the objective function (equation (5)) related to these foreign sectors is

$$\mathcal{U}_m = u_m(z^H) + \sum_{i \in \mathcal{D}_m} \overline{T}_i. \tag{10}$$

Externality Leontief Inverse. Consider a rest-of-world firm that the hegemon cannot contract with, $i \notin \mathcal{D}_m$. Firm i's demand for input H is given by the first-order condition

$$p_i A_{iH}(z^H) g'_{iH}(x_{iH}) = 1. \tag{11}$$

From here, let $z^{H,NC}$ be the subset of allocations z_{iH} of sectors that do not contract with the hegemon. Employing Proposition 2, we construct the endogenous response $\frac{dz^{H,NC}}{de} = \Psi^{z,NC} \frac{\partial x^{H,NC}}{\partial e}$ of rest-of-world sectors that the hegemon cannot contract with to changes in exogenous variable e resulting from a change in the hegemon's contract. Since from Proposition 2 we have $\Psi^{z,NC} = \left(\mathbb{I} - \frac{\partial x^{H,NC}}{\partial z^{H,NC}} \right)^{-1}$, the key objects of interest take the form $\frac{\partial x^*_{iH}}{\partial z_{jH}}$. Differentiating equation (11) in z_{jH}, we obtain

$$\frac{\partial x^*_{iH}}{\partial z_{jH}} = \frac{x^*_{iH}}{z_{jH}} \frac{\xi_{ij}}{\gamma_i},$$

where $\xi_{ij} = \frac{z_{jH}}{A_{iH}(z^H)} \frac{\partial A_{iH}(z^H)}{\partial z_{jH}}$ is the elasticity of productivity A_{iH} with respect to the externality z_{jH}, and where $\gamma_i = \frac{-x^*_{iH} g''_{iH}(x^*_{iH})}{g'_{iH}(x^*_{iH})}$.

Optimal Contract. The hegemon chooses transfers $\overline{T}_i$ and wedges τ_{iH} for all firms $i \in \mathcal{D}_m$ to maximize its utility, equation (10), subject to firms' participation constraints. In doing so, the hegemon accounts for the endogenous response of rest-of-world sectors that the hegemon does not contract with.

We can capture interesting economics of the application considering only two sectors in the rest of world: one sector, which we denote i, that the hegemon can contract with; and one sector, which we denote j, that the hegemon cannot contract with. In this environment, Proposition 3 yields an optimal tax formula given by[35]

$$\tau_{iH} = - \overbrace{\frac{1}{\eta_i} \frac{\partial u_m}{\partial z_{iH}}}^{\text{Direct Externality}} - \overbrace{\frac{1}{\eta_i} \frac{\partial u_m}{\partial z_{jH}} \frac{\xi_{ji}}{\gamma_j - \xi_{jj}} \frac{z_{jH}}{z_{iH}}}^{\text{Network Amplification}} \tag{12}$$
$$+ \underbrace{p_i A_{iH}(z^H) \left[g_{iH}(x_{iH}^{Outside}(z^H)) - g_{iH}(x^*_{iH}) \right] \left(\xi_{ii} + \xi_{ij} \frac{\xi_{ji}}{\gamma_j - \xi_{jj}} \right) \frac{1}{z_{iH}}}_{z\text{-Externalities via Participation Constraint}}$$

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[35]Appendix A.1.7 shows how the general tax formula in equation (8) specializes to the expression below.

If there were no externalities from z^H, then this tax formula would collapse to $\tau_{iH} = 0$, consistent with Proposition 1. In the presence of national security externalities, the optimal tax is positive, $\tau_{iH} > 0$, reflecting the hegemon's desire to mitigate the negative externality. Three key forces underlie the tax formula.

The first term in the tax formula is the direct externality from an increase in z_{iH} on representative consumer m. The negative externality contributes to a positive tax. This tax is upweighted when η_i is lower, that is when the marginal value of slack in the participation constraint is lower. This first term reflects standard Pigouvian correction when the planner has complete instruments on all agents.

The second term is the indirect effect from the production externality: as sector i usage of input H falls, that is z_{iH} falls, the productivity A_{jH} of firms in sector j for usage of input H also falls, prompting firms in sector j to reduce the use of H. This leads to a fall in z_{jH}, which has a positive externality effect on the hegemon consumer. The term $\frac{\xi_{ji}}{\gamma_j - \xi_{jj}}$ captures the magnitude of this response. This effect contributes towards an even higher tax rate, since reducing demand by sector i for input H has a positive externality to the hegemon by also reducing demand by sector j for input H.

Finally, the third term captures the effect of externalities on the participation constraint of firms in sector i. In particular, it captures the change in profits of the outside option for the firm relative to the inside option. This effect is positive, with $g_{iH}(x_{iH}^{Outside}(z^H)) - g_{iH}(x_{iH}^*) \geq 0$ representing the cost of foregone production from accepting the positive tax. Intuitively, a corrective tax on sector i reduces sector i's usage of input H, which reduces productivity A_{iH}, which in turn reduces the desired scale of a firm in sector i that chose not to subject itself to the corrective tax by rejecting the hegemon's contract. As a result, the temptation of firms in sector i to deviate to the outside option also falls. This amplification occurs not only through usage of input H by sector i, the term ξ_{ii}, but also propagates via decreases in usage of input H by sector j, the term $\xi_{ij}\frac{\xi_{ji}}{\gamma_j - \xi_{jj}}$. Therefore, the hegemon wishes to overshoot Pigouvian correction, and employ a higher-magnitude tax in order to reduce incentives of individual firms to reject the contract.

3.2 Official Lending, Infrastructure Projects, and Political Concessions

We specialize the model to the configuration in Figure 3. The hegemon country, in this application China, has two sectors: sector k is a lender and sector j is a manufacturer. For simplicity, both sectors produce only using local factors. The target country, in this application a developing economy, has a single sector i that uses both inputs from China to produce. To focus the application on the essentials, we further assume constant prices (Definition 1), no z-externalities (Definition 2), and

Figure 3: **Application: Belt and Road Initiative**
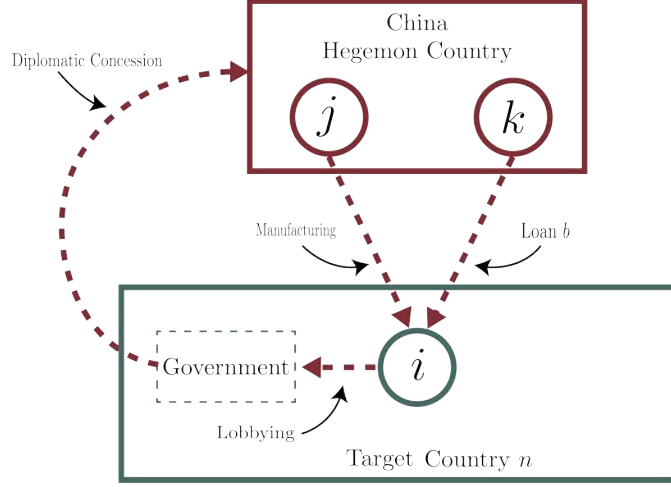
that sector i has a separable production function (Definition 3).[36]

We think of the lending sector, k, as providing a loan to or buying a bond issued by sector i. The loan is for amount $x_{ik} = b$ and the gross interest rate is $p_k = R$. Much like in the sovereign default literature, we assume that the loan is not legally enforceable, so that $\theta_{ik} = 1$.

If there are only individual triggers on j and k, lending can be sustained by the future surplus of the lending relationship, along the lines of the sovereign default model of Eaton and Gersovitz (1981). In particular, we have $Rb \leq \beta \left[\mathcal{V}_i(\{j,k\}) - \mathcal{V}_i(\{j\}) \right] = \beta \mathcal{V}_i(\{k\})$, where the latter equality follows from the separable production function and individual triggers. We can solve for the Markov equilibrium value of $\mathcal{V}_i(\{k\}) = \frac{p_i f_{ik}(b^*) - Rb^*}{1-\beta}$ which is the present discounted value of all future borrowing by sector i. Solving for the borrowing limit, we obtain $b \leq \left(\beta \frac{p_i}{R} \right)^{\frac{1}{1-\xi}}$ under the assumption that $f_{ik}(b) = b^{\xi}$ for $\xi \in (0,1)$. The IC (borrowing limit) binds whenever $\xi > \beta$, which we assume to be the case.

To sharpen the application, we assume that $\theta_{ij} = 0$ so that firms in sector i can never steal input j. Thus the incentive constraint for stealing j does not bind. Without a hegemonic China, the equilibrium features limited lending and an unconstrained manufacturing relationship.

China can, as a hegemon, impose a joint threat that links together the provision of lending and manufacturing goods. If the target country defaults on either input, both are withdrawn in the future. Under the joint threat the incentive constraint of the target country sector i is:

$$Rb \leq \beta \mathcal{V}_i(\{j,k\}) = \frac{p_i f_{ik}(b^*) - Rb^*}{1-\beta} + \frac{p_i f_{ij}(x_{ij}^*) - p_j x_{ij}^*}{1-\beta}.$$

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[36]The production function is $f_i(x_i) = f_{ij}(x_{ij}) + f_{ik}(x_{ik})$ with both f_{ij} and f_{ik} increasing, concave, and subject to Inada conditions.

The present value of the manufacturing relationship provides additional incentives to repay the debt in the joint threat, an endogenous cost of default on the loan. Under the joint threat, the equilibrium features the same level of manufacturing activity but an increase in the borrowing. The surplus can be extracted by China via positive transfer $\overline{T}_i^* > 0$.

Our mechanism is related to that proposed in Bulow and Rogoff (1989), whereby lenders seize the exports of a country conditional on a default, thereby generating a cost of default.[37] Mendoza and Yue (2012) consider a quantitative sovereign debt model in which a country faces an endogenous productivity cost of default that arises because a defaulting country loses access to trade finance, losing the ability to import intermediate goods, and is forced to switch to imperfect domestic substitutes for production. In our framework, joint threats offer a means for a country to voluntarily raise its cost of default through such a channel, thereby allowing it to borrow more. In particular, the more profitable the inputs that are sourced from China, the more the borrowing constraint is relaxed.

One interpretation of the transfers are mark-ups on the manufacturing goods being sold by China to the target country, or equivalently an interest rate on the loan above the market rate R. This application shows the futility of assessing China's lending programs in isolation: i.e. focusing only on the loans and their returns. Both the sustainability of the loans and the economic returns from the lending have to be assessed jointly with other activities, such as manufacturing exports, that are occurring jointly with the lending. The benefits to China might not even accrue in monetary form as we explore below.

Transfers as Costly Actions and Political Concessions. Our framework could be extended to allow for a rich model of political lobbying and influence (Grossman and Helpman (1994); Bombardini and Trebbi (2020)). The current model provides threats among disparate economic activities as a form of soft-power. The hegemon then uses this power to ask for costly actions (limited by the participation constraint), many of which can take the form of political lobbying or diplomatic concessions. In this case, the transfer $\overline{T}_i$ represents the private cost to the firm of the action. Here we focus on a leading example for geoeconomics in which China asks the firms to lobby their governments for a political concession. We necessarily keep the modeling reduced form, but it provides a starting point for future research interested in introducing a deeper model of lobbying.

We assume that a bilateral geopolitical concession can be made from country n to China. We let the concession, be the element $z_n^c \in \{0, 1\}$ of aggregate vector z and assume that it enters positively in China's utility, $u_m(z_n^c)$ with $u_m(1) > u_m(0)$, and negatively in the target's country utility, $u_n(z_n^c)$ with $u_n(0) > u_n(1)$. We assume that no utility is derived by either countries from all other elements of z. Governments care about consumer welfare and therefore internalize these utility costs and benefits. Governments also care about the profits of the firms in their country net of transfers. We

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[37]Under isolated threats, our model features positive borrowing. The impossibility result of Bulow and Rogoff (1989) does not kick in because we are not allowing inter-temporal saving and up-front payment contracts as in Eaton and Gersovitz (1981).

assume that a hegemon asking a firm to make a positive transfer can alternatively ask that firm to transfer part or all of that transfer to the government in exchange for the government undertaking the geopolitical action, with any money not transferred being paid as usual to the hegemon. The geopolitical action is feasible to implement as long as country level transfer exceed the government utility cost of the concession.

These concessions can account, for example, for China asking countries that are part of the Belt and Road Initiative not to recognize Taiwan. This is consistent with the evidence in Dreher et al. (2022) that recipients of Belt and Road lending are less likely to recognize Taiwan.

4   Conclusion

We provide a framework to understand the source and application of geoeconomic power. Hegemon countries use their existing financial and trade network to exert power on foreign entities. They use this power to demand costly actions from the part of the world production network that they can pressure. We characterize the optimal actions demanded by the hegemon and show that they take the form of monetary transfers, mark-ups on goods, surcharges on loans, but also restrictions on import-export activities. We show that the hegemon uses these actions to manipulate the world equilibrium in its favor, thus exerting macro-power. The framework is flexible and can be extended for future analyses of a rich set of issues in geoeconomics.

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January 2024

Not for Publication

A.1 Proofs

A.1.1 Proof of Lemma 1

As described in Section 1.2.1, trigger strategies are defined by

$$B'_{ij}(S) = \begin{cases} B_{ij}, & S \cap K_{ij} = \emptyset \\ 0, & o.w. \end{cases}, \quad K_{ij} = \{j\} \cup \bigcup_{k \in M_{ij}} K_{ik} \tag{A.1}$$

We first construct the smallest sets consistent with equation (A.1), that is involving minimal retaliation. Let $\{X_{ij}^n\}_{n=0}^\infty$ be a sequence of sets constructed iteratively as follows. Let $X_{ij}^0 = \{j\}$ and, for $n \geq 1$, let $X_{ij}^n = X_{ij}^{n-1} \cup \bigcup_{x \in X_{ij}^{n-1}} M_{ix}$.[1] Since $\mathcal{J}_i$ is a finite set, since $X_{ij}^{n-1} \subset X_{ij}^n \subset \mathcal{J}_i$, and since $X_{ij}^n = X_{ij}^{n-1} \Rightarrow X_{ij}^{n+1} = X_{ij}^n$, then $\exists \overline{N}_{ij} > 0$ such that $X_{ij}^{\overline{N}_{ij}} = X_{ij}^n$ for all $n \geq \overline{N}_{ij}$. We define the *minimum retaliation set* of suppliers in j for firm i as $X_{ij}^* = X_{ij}^{\overline{N}_{ij}}$. We can now show that all members of X_{ij}^* have the same minimum retaliation set.[2]

Lemma 3 $k \in X_{ij}^*$ *if and only if* $X_{ik}^* = X_{ij}^*$.

Proof of Lemma 3. The if statement is immediate since $k \in X_{ik}^*$ by construction. Consider then only if and let $k \in X_{ij}^*$. Since $k \in X_{ij}^*$, then by construction of the sequence we have $X_{ik}^* \subset X_{ij}^*$.[3] Moreover since $k \in X_{ij}^*$, by construction there is a sequence $x_0, .., x_N$, with $x_0 = j$ and $x_N = k$, such that $x_n \in M_{ix_{n-1}}$ for $n = 1, ..., N$. Reversing that sequence and using symmetry of joint triggers, we have a sequence $x_N, ..., x_0$ such that $x_{n-1} \in M_{ix_n}$. Hence, $j \in X_{ik}^N$, and hence $j \in X_{ik}^*$. But then by construction we also have $X_{ij}^* \subset X_{ik}^*$, and hence $X_{ij}^* = X_{ik}^*$. $\square$

Define $K_{ij} = X_{ij}^*$, and define $\mathcal{S}_i(\mathcal{B}_i) = \bigcup_{j \in \mathcal{B}_i} \{X_{ij}^*\}$. Thus we obtain the following properties.

Lemma 4 *For* $\mathcal{B}_i \in \Sigma(\mathcal{S}_i)$, $\mathcal{S}_i(\mathcal{B}_i)$ *is a partition of* $\mathcal{B}_i$.

---

[1] The first element $X_{ij}^0 = \{j\}$ is the individual trigger. The second element, $X_{ij}^1 = \{j\} \cup M_{ij}$, adds in the fact that joint triggers of suppliers in j with suppliers in their joint trigger set, M_{ij}, adds in the individual triggers of these suppliers. The next step then adds in the individual triggers associated with the joint triggers of the suppliers that were added in the previous step, and so on.

[2] Lemma 3 also makes clear why the minimum retaliation set satisfies equation (A.1).

[3] Observe that if $k \in X_{ij}^*$, then there is a step N with $k \in X_{ij}^N$. Given construction of the sequence, all elements X_{ik}^1 are then added at step $N + 1$, and so on.

Proof of Lemma 4. First, we have $\bigcup_{j \in \mathcal{B}_i} X_{ij}^* = \mathcal{B}_i$. Second, we have for all $j, k \in \mathcal{B}_i$, either $X_{ij}^* = X_{ik}^*$ or $X_{ij}^* \cap X_{ik}^* = \emptyset$ (Lemma 3). $\square$

The incentive compatibility constraint associated with firm i preferring no stealing over stealing action $S \in P(\mathcal{B}_i)$ is

$$\Pi_i(x_i, \ell_i, \mathcal{B}_i) + \sum_{j \in S} \theta_{ij} p_j x_{ij} + \beta \nu_i(\mathcal{B}_i'(S)) \leq \Pi_i(x_i, \ell_i, \mathcal{B}_i) + \beta \nu_i(\mathcal{B}_i),$$

which reduces to

$$\sum_{j \in S} \theta_{ij} p_j x_{ij} \leq \beta \left[\nu_i(\mathcal{B}_i) - \nu_i(\mathcal{B}_i'(S)) \right]$$

We now complete the proof with the following Lemma.

Lemma 5 *The order (x_i, ℓ_i) is incentive compatible with respect to $P(\mathcal{B}_i)$ if and only if it is incentive compatible with respect to $\Sigma(\mathcal{S}_i(\mathcal{B}_i))$.*

Proof of Lemma 5. The only if statement holds trivially since $\Sigma(\mathcal{S}_i(\mathcal{B}_i)) \subset \Sigma(\mathcal{B}_i) = P(\mathcal{B}_i)$ since $\mathcal{S}(\mathcal{B}_i)$ is a partition of $\mathcal{B}_i$. Thus consider the if statement. Suppose that (x_i, ℓ_i) is incentive compatible with respect to $\Sigma(\mathcal{S}_i(\mathcal{B}_i))$. Let $S \in P(\mathcal{B}_i)$. If $S \in \Sigma(\mathcal{S}_i(\mathcal{B}_i))$ then incentive compatibility holds by assumption, so let $S \notin \Sigma(\mathcal{S}_i(\mathcal{B}_i))$. Given a stealing action S, all suppliers $k \in \bigcup_{j \in S} X_{ij}^*$ Distrust firm i. Given Lemma 4, there is a unique subset $\mathcal{X}_i(S) \subset \mathcal{S}_i(\mathcal{B}_i)$ of elements such that $\bigcup_{X \in \mathcal{X}_i(S)} X = \bigcup_{j \in S} X_{ij}^*$. Define $\Xi_i(S) = \bigcup_{X \in \mathcal{X}_i(S)} X$. Now, observe that for any $S \in P(\mathcal{B}_i)$, the stealing choice S is weakly dominated by the stealing choice $\Xi_i(S)$, since S and $\Xi_i(S)$ yield the same continuation value $\nu_i(\mathcal{B}_i \backslash \Xi_i(S))$ but $\Xi_i(S)$ yields higher flow payoff. Since $\Xi_i(S) \in \Sigma(\mathcal{S}_i(\mathcal{B}_i))$ and since $Xi_i(S)$ weakly dominates S, then if (x_i, ℓ_i) is incentive compatible with respect to $\Sigma(\mathcal{S}_i(\mathcal{B}_i))$ it is also incentive compatible with respect to S. But since S was generic, then incentive compatibility with respect to $\Sigma(\mathcal{S}_i(\mathcal{B}_i))$ implies incentive compatibility with respect to $P(\mathcal{B}_i)$, completing the proof. $\square$

A.1.2   Proof of Lemma 2

Consider a hypothetical optimal contract $\Gamma = \{\mathcal{S}_i', \mathcal{T}_i, \tau_i\}_{i \in \mathcal{C}_m}$ that is feasible and satisfies firms' participation constraints, and suppose that $\mathcal{S}_i' \neq \overline{\mathcal{S}_i}'$. Let (x^*, ℓ^*, z^*, P) denote optimal firm allocations, externalities, and prices under this contract. The proof strategy is to show that the hegemon can achieve the same allocations x^*, ℓ^* and the same transfers $\mathcal{T}_i$ using a feasible contract featuring maximal joint threats, without changes in equilibrium prices or the vector of aggregates. Hence the hegemon can obtain at least as high value using maximal joint threats. The proof involves constructing appropriate wedges to achieve this outcome.

We first construct a vector of taxes τ_i^* that implements the allocation x_i^*, ℓ_i^* under maximal joint threats. In particular, let $\tau_{ij}^* = \frac{\partial \Pi_i(x_i^*, \ell_i^*, \mathcal{J}_i)}{\partial x_{ij}}$ and $\tau_{if}^{\ell*} = \frac{\partial \Pi_i(x_i^*, \ell_i^*, \mathcal{J}_i)}{\partial \ell_{if}}$. Considering the relaxed problem (not subject to incentive compatibility) of firm i,

$$\max_{x_i, \ell_i} \Pi_i(x_i, \ell_i, \mathcal{J}_i) - \sum_{j \in \mathcal{J}_i} [\tau_{ij}(x_{ij} - x_{ij}^*) + T_{ij}] - \sum_{f \in \mathcal{F}_m} \tau_{if}^{\ell}(\ell_{if} - \ell_{if}^*),$$

which yields solution $\frac{\partial \Pi_i}{\partial x_{ij}} = \tau_{ij}^*$ and $\frac{\partial \Pi_i}{\partial \ell_{if}} = \tau_{if}^*$, that is $x_i = x_i^*$ and $\ell_i = \ell_i^*$. It remains to verify this allocation is incentive compatible. Recall that a joint threat is a coarser partition of

$\mathcal{J}_i$. Since maximal joint threats $\overline{\mathcal{S}}_i'$ is itself a joint threat of any other feasible joint threat $\mathcal{S}_i'$, then $\Sigma(\overline{\mathcal{S}}_i') \subset \Sigma(\mathcal{S}_i')$, and hence (since wedges are revenue neutral) the allocation (x_i^*, ℓ_i^*) is incentive compatibility under contract $(\overline{\mathcal{S}}_i', \mathcal{T}_i, \tau_i^*)$. Thus since (x_i^*, ℓ_i^*) is the solution to firm i's relaxed problem and is incentive compatible, it is firm i's optimal policy.

Finally, every firm $i \notin \mathcal{C}_m$ and every consumer n faces the same decision problem as under the original contract, since both prices and the vector of aggregates are unchanged. Hence, every firm $i \notin \mathcal{C}_m$ and every consumer n has the same optimal policy. Hence $x^* = z^*$ and aggregates are consistent with their conjectured value. Finally, market clearing remains satisfied since all allocations are unchanged.

Finally, given firm i's participation constraint must be satisfied under contract $\{\mathcal{S}_i', \mathcal{T}_i, \tau_i\}_{i \in \mathcal{C}_m}$, the participation constraint of firm i is also satisfied under contract $\{\overline{\mathcal{S}}_i', \mathcal{T}_i, \tau_i^*\}_{i \in \mathcal{C}_m}$, since firm value is the same given the same allocations, transfers, prices, and aggregates. Finally since firm value is unchanged for $i \in \mathcal{I}_m$, since prices P and aggregates z^* are unchanged, and since transfers $\mathcal{T}_i$ is unchanged for all $i \in \mathcal{C}_m$, the hegemon's objective (equation 5) is also unchanged relative to the original contract. Thus the hegemon is indifferent between the implementable contracts $\{\mathcal{S}_i', \mathcal{T}_i, \tau_i\}_{i \in \mathcal{C}_m}$ and $\{\overline{\mathcal{S}}_i', \mathcal{T}_i, \tau_i^*\}_{i \in \mathcal{C}_m}$. Hence, it is weakly optimal for the hegemon to offer a contract involving maximal joint threats, concluding the proof.

A.1.3 Proof of Proposition 1

Given constant prices (Definition 1) and no z-externalities (Definition 2), the hegemon's objective function collapses to maximizing its consumer's wealth level subject to participation constraints,

$$\max_{\{\overline{T}_i, \tau_i\}_{i \in \mathcal{C}_m}} \sum_{i \in \mathcal{I}_m} \Pi_i(\Gamma_i) + \sum_{i \in \mathcal{D}_m} \overline{T}_i \quad s.t. \quad V_i(\Gamma_i) \geq V_i(\mathcal{S}_i) \; \forall i \in \mathcal{C}_m,$$

where for simplicity we have dropped consumer factor income, which is constant given constant prices. Given Lemma 2, $\mathcal{S}_i' = \overline{\mathcal{S}}_i'$ for all i and it remains to characterize optimal transfers and wedges.

Observe first that for any $\overline{T}_i \geq 0$,

$$0 \in \arg\max_{\tau_i} \Pi_i(\overline{\mathcal{S}}_i', \overline{T}_i, \tau_i).$$

$$0 \in \arg\max_{\tau_i} V_i(\overline{\mathcal{S}}_i', \overline{T}_i, \tau_i).$$

Therefore, for any $i \in \mathcal{C}_m$, setting $\tau_i = 0$ maximizes operating profits of domestic firms and maximally slackens the participation constraint of all firms. Therefore, $\tau_i = 0$ is an optimal policy for all $i \in \mathcal{C}_m$.

Consider next a domestic firm, $i \in \mathcal{I}_m$. By Envelope Theorem, $\frac{\partial V_i}{\partial \overline{T}_i} \leq -1$ and $\frac{\partial \Pi_i}{\partial \overline{T}_i} \leq 0$, with strict inequalities when at least one incentive constraint that includes $\overline{T}_i$ binds. Therefore, $\overline{T}_i > 0$ weakly reduces operating profits and strictly tightens the participation constraint, so that $\overline{T}_i = 0$ is an optimal policy.

Finally, consider a foreign firm, $i \in \mathcal{D}_m$. As with a domestic firm, $\frac{\partial V_i}{\partial \overline{T}_i} \leq -1$. Since the hegemon's objective is strictly increasing in $\overline{T}_i$ for $i \in \mathcal{D}_m$, then the hegemon's optimal policy charges the largest transfer $\overline{T}_i^*$ such that the participation constraint just binds, $V_i(\overline{\mathcal{S}}_i', \overline{T}_i^*, 0) = V_i(\mathcal{S}_i).$ [4] Since

---

[4]To see why such a value exists, suppose the hegemon set $\overline{T}_i = \beta_i \left[\nu_i(\mathcal{J}_i) - \nu_i(\mathcal{J}_i \backslash \mathcal{S}_i^D) \right]$. Incentive

$V_i(\overline{\mathcal{S}}'_i, \overline{T}^*_i, 0)$ is a continuous and decreasing function of $\overline{T}_i$, then if $\mathcal{S}^D_i$ is not a pressure point on i, then $V_i(\overline{\mathcal{S}}'_i, 0, 0) = V_i(\mathcal{S}_i)$ and hence $\overline{T}^*_i = 0$. By contrast if $\mathcal{S}^D_i$ is a pressure point, then $V_i(\overline{\mathcal{S}}'_i, 0, 0) > V_i(\mathcal{S}_i)$ and hence $\overline{T}^*_i > 0$. This concludes the proof.

A.1.4   Proof of Proposition 2

The derivation of the first equation is presented in text. Next consider the excess demand function $ED(\Gamma, P, z)$. Market clearing requires excess demand to be zero, $ED(\Gamma, P, z) = 0$. Totally differentiating this system with regards to an exogenous variable e, we obtain

$$\frac{\partial ED}{\partial e} + \frac{\partial ED}{\partial z^*}\frac{dz^*}{de} + \frac{\partial ED}{\partial P}\frac{dP}{de} = 0.$$

Substituting in the equation for $\frac{dz^*}{de}$, we have

$$\frac{\partial ED}{\partial e} + \frac{\partial ED}{\partial z^*}\Psi^z\left(\frac{\partial x^*}{\partial e} + \frac{\partial x^*}{\partial P}\frac{dP}{de}\right) + \frac{\partial ED}{\partial P}\frac{dP}{de} = 0.$$

Finally rearranging and inverting, we have

$$\frac{dP}{de} = -\left(\frac{\partial ED}{\partial P} + \frac{\partial ED}{\partial z^*}\Psi^z\frac{\partial x^*}{\partial P}\right)^{-1}\left(\frac{\partial ED}{\partial e} + \frac{\partial ED}{\partial z^*}\Psi^z\frac{\partial x^*}{\partial e}\right)$$

which completes the proof.

A.1.5   Proof of Proposition 3

The proof is divided into regions based on hegemon pressure points. For any prices and aggregates $Q = (P, z^*)$, we can characterize the subset $\mathcal{P}(Q) \subset \mathcal{C}_m$ of sectors that the hegemon contracts with and has pressure points on. We divide the proof into the four region's in which the hegemon's optimal contract could lie: (i) the hegemon has no pressure points, $\mathcal{P} = \emptyset$; (ii) the hegemon has pressure points on all sectors, $\mathcal{P} = \mathcal{C}_m$; (iii) the hegemon has pressure points on all domestic sectors but not on some foreign sectors, $\mathcal{I}_m \subset \mathcal{P}$; (iv) the hegemon does not have pressure points on some domestic sectors, $\mathcal{I}_m \cap \mathcal{P} \neq \mathcal{I}_m$.[5]

From Lemma 2, maximal joint threats are optimal, so all that remains is to characterize optimal wedges and transfers.

A.1.5.1   Case i: Pressure points on no sectors

Suppose that the hegemon's optimal contract lies in the region where the hegemon has no pressure points. Then, $V_i(\overline{\mathcal{S}}'_i) = V_i(\mathcal{S}_i)$ for all $i \in \mathcal{C}_m$, and hence the hegemon must set $\overline{T}_i = 0$ and $\tau_i = 0$ for all i.

---

compatibility for stealing S^D_i implies $x_{ij} = 0$ for all $j \in S^D_i$. The incentive compatibility constraint for any stealing action S with $S \cap S^D_i$ is the same as absent joint threats. Incentive compatibility for any stealing action S with $S^D_i \subset S$, accounting for transfers, is $\sum_{j \in S}\theta_{ij}p_j x_{ij} \leq \beta_i\left[\nu_i(\mathcal{J}_i \backslash \mathcal{S}^D_i) - \nu_i(\mathcal{J}_i \backslash S)\right]$, which is tighter than the constraint for stealing S absent joint threats. Thus the set of incentive compatible allocations has shrunk relative to the outside option, and so firm i value is lower than $V_i(\mathcal{S}_i)$.

[5]Naturally, it is possible some of these regions are empty and that some points Q cannot be part of an equilibrium.

A.1.5.2 Case (ii): Pressure points on all sectors $i \in \mathcal{C}_m$

Suppose that the hegemon's optimal contract lies in the region where the hegemon has pressure points on all sectors that it contracts with. Since the hegemon has complete instruments for $i \in \mathcal{C}_m$, we adopt the primal approach whereby the hegemon directly selects allocations of firms $i \in \mathcal{C}_m$, and derive the wedges that implement them from the firm's first order conditions.

We begin with the Lagrangian of firm i, with choice variable being order (x_i, ℓ_i), given by

$$
\begin{aligned}
\mathcal{L} =& \Pi_i(x_i, \ell_i, \mathcal{J}_i) - \sum_{j \in \mathcal{J}_i} [\tau_{ij}(x_{ij} - x_{ij}^*)] - \sum_{f \in \mathcal{F}_m} \tau_{if}^\ell(\ell_{if} - \ell_{if}^*) - \overline{T}_i + \beta \nu_i(\mathcal{J}_i) \\
&+ \sum_{S \in \Sigma(\overline{S}_i')} \lambda_{iS} \left[\beta \left[\nu_i(\mathcal{J}_i) - \nu_i(\mathcal{J}_i \backslash S) \right] - \sum_{j \in S} \left[\theta_{ij}[p_j x_{ij} + \tau_{ij}(x_{ij} - x_{ij}^*)] \right] - \mathbf{1}_{S_i^D \subset S} \overline{T}_i \right]
\end{aligned}
$$

Therefore, the first order conditions for x_{ij} and ℓ_{if} yield

$$
\tau_{ij}(1 + \theta_{ij}\overline{\lambda}_{ij}) = \frac{\partial \Pi_i}{\partial x_{ij}} - \overline{\lambda}_{ij}\theta_{ij}p_j \tag{A.2}
$$

$$
\tau_{if}^\ell = \frac{\partial \Pi_i}{\partial \ell_{if}} \tag{A.3}
$$

where we have defined $\overline{\lambda}_{ij} \equiv \sum_{S \in \Sigma(\overline{S}_i')|j \in S} \lambda_{iS}$. Given that the firm's optimization problem is convex, given an incentive compatible allocation (x_i, ℓ_i), and given nonnegative Lagrange multipliers $\lambda_{iS} \geq 0$ such that complementary slackness holds, these equations define the wedges required for the planner to implement the allocation.[6]

Now, we move to the hegemon's Lagrangian. Given the hegemon has complete factor wedges on domestic firms, p_f^ℓ can also be seen as a direct choice variable of the hegemon (with market clearing being an explicit constraint), while for all other prices the hegemon internalizes their equilibrium determination.[7] Under the primal approach, we write

$$
\begin{aligned}
\mathcal{L}_m(\{x_i, \ell_i, \overline{T}_i\}_{i \in \mathcal{C}_m}, \{p_f^\ell\}_{f \in \mathcal{F}_m}) =& W_m\left(p, \sum_{i \in \mathcal{I}_m} \Pi_i(x_i, \ell_i) + \sum_{f \in \mathcal{F}_m} p_f^\ell \overline{\ell}_f + \sum_{i \in \mathcal{D}_m} \overline{T}_i \right) + u_m(z) \\
&+ \sum_{i \in \mathcal{C}_m} \eta_i \left[\Pi_i(x_i, \ell_i, \mathcal{J}_i) - \overline{T}_i + \beta \nu_i(\mathcal{J}_i) - V_i(\mathcal{S}_i) \right] \\
&+ \sum_{i \in \mathcal{C}_m} \sum_{S \in \Sigma(\overline{S}_i')} \Lambda_{iS} \left[\beta \left[\nu_i(\mathcal{J}_i) - \nu_i(\mathcal{J}_i \backslash S) \right] - \sum_{j \in S} \theta_{ij}p_j x_{ij} - \mathbf{1}_{S_i^D \subset S} \overline{T}_i \right] \\
&+ \sum_{f \in \mathcal{F}_m} \kappa_f \left[\overline{\ell}_f - \sum_{i \in \mathcal{I}_m} \ell_f \right]
\end{aligned}
$$

We use the notation $z^{NC} = \{z_{ij}\}_{i \notin \mathcal{C}_m}$ and $P^m = (p, p_{-m}^\ell)$ to denote aggregates among sectors the

---

[6]Note that given binding constraints, there can be multiple configurations of wedges that implement the same allocation. For example, setting $\lambda_{iS} = 0$ for all S yields one possible set of wedges (see also the proof of Lemma 2).

[7]If the hegemon contracts with every firm in a foreign country n, then we could include p_f^ℓ, $f \in \mathcal{F}_n$, in the hegemon's choice variables and include the market clearing contraints for factors $\mathcal{F}_n$ in the hegemon's constraints. This would lead to similar FOCs for foreign factor prices.

hegemon does not contract with, and prices apart from domestic factor prices.

First, we construct a basis of externalities from aggregates z, which capture solely the direct spillovers (all else held fixed) of an increase in z_{ij} from the hegemon's perspective,

$$\varepsilon_{ij}^z = \frac{\partial W_m}{\partial w_m} \sum_{i \in \mathcal{I}_m} \frac{\partial \Pi_i}{\partial z_{ij}} + \frac{\partial u_m(z)}{\partial z_{ij}} + \sum_{i \in \mathcal{C}_m} \eta_i \left[\frac{\partial \Pi_i}{\partial z_{ij}} - \frac{\partial V_i(\mathcal{S}_i)}{\partial z_{ij}} \right].$$

We analogously define spillovers from prices P^m in vector form,

$$\varepsilon^{P^m} = \frac{\partial W_m}{\partial P^m} + \frac{\partial W_m}{\partial w_m} \frac{\partial w_m}{\partial P^m} + \sum_{i \in \mathcal{C}_m} \eta_i \left[\frac{\partial \Pi_i}{\partial P^m} - \frac{\partial V_i}{\partial P^m} \right] - \sum_{i \in \mathcal{C}_m} \sum_{S \in \Sigma(\overline{\mathcal{S}}_i')} \Lambda_{iS} \sum_{j \in S} \theta_{ij} \frac{\partial p_j}{\partial P^m} x_{ij}$$

Finally, the direct spillover of factor prices p_f^ℓ for $f \in \mathcal{F}_m$ is

$$\varepsilon_f = \sum_{i \in \mathcal{I}_m} \eta_i \left[\frac{\partial \Pi_i(x_i, \ell_i, \mathcal{J}_i)}{\partial p_f^\ell} - \frac{\partial V_i(\mathcal{S}_i)}{\partial p_f^\ell} \right] = \sum_{i \in \mathcal{I}_m} \eta_i \left[\ell_{if} - \ell_{if}^{\text{Outside}} \right]$$

where the second equality follows by Envelope Theorem, and $\ell_{if}^{Outside}$ is factor usage of a firm that deviates to the outside option.

FOC for p_f^ℓ for $f \in \mathcal{F}_m$. Consider the hegemon's FOC for the domestic factor price p_f^ℓ. It is helpful to observe that the wealth of consumer m does not locally change in the domestic factor price, $\frac{\partial w_m}{\partial p_\ell^f} = \sum_{i \in \mathcal{I}_m} \ell_{if} - \bar{\ell}_f = 0$, since factor prices are a wash between firm profits and factor payments. Therefore, the domestic factor price does not affect consumer wealth, and therefore also does not affect excess demand in any good market or in any foreign factor market, that is $\frac{\partial ED}{\partial p_f^\ell} = 0$. Therefore, the hegemon's FOC for factor prices is $0 = \varepsilon_f$, that is

$$0 = \sum_{i \in \mathcal{I}_m} \eta_i [\ell_{if} - \ell_{if}^{Outside}].$$

The hegemon therefore uses factor prices to manipulate participation constraints of domestic firms. Intuitively if a firm with a binding participation constraint would want to use more of factor f if it rejected the contract, the hegemon wants to increase p_f^ℓ to deter that firm from deviating to the outside option. The above FOC balances this motive across domestic sectors.

FOC for ℓ_{if} for a Domestic Firm. Consider the hegemon's FOC for ℓ_{if} for a domestic sector,

$$0 = \frac{\partial W_m}{\partial w_m} \frac{\partial \Pi_i}{\partial \ell_{if}} + \eta_i \frac{\partial \Pi_i}{\partial \ell_{if}} - \kappa_f + \mathcal{E}_{if}^\ell,$$

where we define the externality impact $\mathcal{E}_{if}^\ell = \varepsilon^{zNC} \frac{dz^{*NC}}{d\ell_{if}} + \varepsilon^{P^m} \frac{dP}{d\ell_{if}}$, where $\varepsilon^{zNC} = \{\varepsilon_{ij}^z\}_{i \notin \mathcal{C}_m}$, where $\frac{dz^{*NC}}{d\ell_{if}}$ and $\frac{dP^m}{d\ell_{if}}$ are defined as in Proposition 2 for the subset of aggregates z^{*NC} and prices P^m. Since the firm's problem yields a tax rate $\tau_{if}^\ell = \frac{\partial \Pi_i}{\partial \ell_{if}}$, then we have

$$\left(\frac{\partial W_m}{\partial w_m} + \eta_i \right) \tau_{if}^\ell = -\mathcal{E}_{if}^\ell + \kappa_f$$

A.6

FOC for ℓ_{if} for a Foreign Firm. Consider the hegemon's FOC for ℓ_{if} for a foreign sector, $0 = \eta_i \frac{\partial \Pi_i}{\partial \ell_{if}} + \mathcal{E}_{if}^{\ell}$, then we have

$$\eta_i \tau_{if}^{\ell} = -\mathcal{E}_{if}^{\ell}.$$

FOC for x_{ij} for a domestic firm. For a domestic sector, the hegemon's FOC for x_{ij} is

$$0 = \frac{\partial W_m}{\partial w_m} \frac{\partial \Pi_i}{\partial x_{ij}} + \eta_i \frac{\partial \Pi_i}{\partial x_{ij}} - \overline{\Lambda}_{ij} \theta_{ij} p_j + \mathcal{E}_{ij}$$

where we have defined the externality impact $\mathcal{E}_{ij} = \varepsilon_{ij}^z + \varepsilon^{zNC} \frac{dz^{*NC}}{dz_{ij}} + \varepsilon^{Pm} \frac{dP}{dz_{ij}}$, and where $\overline{\Lambda}_{ij} = \sum_{S \in \Sigma(\overline{\mathcal{S}}_i') | j \in S} \Lambda_{iS}$. We construct the firm nonnegative Lagrange multiplier $\lambda_{iS} = \frac{\Lambda_{iS}}{\frac{\partial W_m}{\partial w_m} + \eta_i}$ and substitute it in the firm's problem FOC in equation (A.2) to yield:

$$\tau_{ij} \left(\frac{\partial W_m}{\partial w_m} + \eta_i + \theta_{ij} \overline{\Lambda}_{ij} \right) = \left(\frac{\partial W_m}{\partial w_m} + \eta_i \right) \frac{\partial \Pi_i}{\partial x_{ij}} - \overline{\Lambda}_{ij} \theta_{ij} p_j$$

Thus combining with the planner's FOC, we obtain the required tax rate,

$$\tau_{ij} \left(\frac{\partial W_m}{\partial w_m} + \eta_i + \theta_{ij} \overline{\Lambda}_{ij} \right) = -\mathcal{E}_{ij}.$$

FOC for x_{ij} for a foreign sector. Consider the hegemon's FOC for x_{ij} for a foreign sector,

$$0 = \eta_i \frac{\partial \Pi_i}{\partial x_{ij}} - \overline{\Lambda}_{ij} \theta_{ij} p_j + \mathcal{E}_{ij},$$

where $\mathcal{E}_{ij}$ and $\overline{\Lambda}_{ij}$ are defined in the same manner as above. For a positive constant $\alpha > 0$, we add and subtract $\alpha \frac{\partial \Pi_i}{\partial x_{ij}}$ to obtain

$$(\eta_i + \alpha) \frac{\partial \Pi_i}{\partial x_{ij}} - \overline{\Lambda}_{ij} \theta_{ij} p_j = -\mathcal{E}_{ij} + \alpha \frac{\partial \Pi_i}{\partial x_{ij}}.$$

Constructing the nonnegative firm Lagrange multiplier $\lambda_{iS} = \frac{\Lambda_{iS}}{\eta_i + \alpha}$, the firm's FOC yields

$$(\eta_i + \alpha) \frac{\partial \Pi_i}{\partial x_{ij}} - \overline{\Lambda}_{ij} \theta_{ij} p_j = \tau_{ij} (\eta_i + \alpha + \theta_{ij} \overline{\Lambda}_{ij})$$

Thus combining with the planner's FOC, we have the required tax rate

$$\tau_{ij} (\eta_i + \alpha + \theta_{ij} \overline{\Lambda}_{ij}) = -\left(\mathcal{E}_{ij} - \alpha \frac{\partial \Pi_i}{\partial x_{ij}} \right).$$

Finally, we take the limit as $\alpha \to 0$ and obtain a tax formula as a limit[8]

$$\tau_{ij} (\eta_i + \theta_{ij} \overline{\Lambda}_{ij}) = -\mathcal{E}_{ij}.$$

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[8]Note that if $\eta_i = 0$, then the Lagrange multiplier λ_{iS} approaches $+\infty$ as $\alpha \to 0$, hence the limiting argument.

FOC for $\overline{T}_i$ for a domestic sector. Holding fixed allocations, a transfer $\overline{T}_i$ for a domestic sector has no impact on excess demand in any market, since it redistributes from country m's firms to country m's consumer. Therefore, the FOC is $0 \geq -\eta_i - \overline{\Lambda}_{iS_i^D}$, where $\overline{\Lambda}_{iS_i^D} = \sum_{S \in \Sigma(\overline{S}_i')|S_i^D \subset S} \Lambda_{iS}$. Therefore, $\overline{T}_i = 0$.

FOC for $\overline{T}_i$ for a foreign sector in country n. Holding fixed allocations, a transfer $\overline{T}_i$ has the effect of reallocating wealth from consumers in country n to consumers in country m. Therefore, we have the FOC

$$0 \geq \frac{\partial W_m}{\partial w_m} - \eta_i - \overline{\Lambda}_{iS_i^D} + \Xi_{mn}$$

where we have defined $\Xi_{mn} = \varepsilon^{zNC}\left[\frac{dz^{*NC}}{dw_m} - \frac{dz^{*NC}}{dw_n}\right] + \varepsilon^{P^m}\left[\frac{dP^m}{dw_m} - \frac{dP^m}{dw_n}\right]$. This rearranges to the result.

A.1.5.3 Case (iii): Pressure points on all domestic firms but not some foreign firms

Suppose that the hegemon's optimal contract lies in the region where the hegemon lacks a pressure point on a nonempty subset $\mathcal{D}_m^p \subset \mathcal{D}_m$. As in case (i) we have $\overline{T}_i = 0$ and $\tau_i = 0$ for all $i \in \mathcal{D}_m^p$. We can therefore redefine the contractible set as $\mathcal{C}_m^{new} = \mathcal{C}_m \backslash \mathcal{D}_m^p$, at which point analysis proceeds as in case (ii).

A.1.5.4 Case (iv): Pressure points on some or no domestic firms

If the hegemon's optimal contract lies in the region where the hegemon lacks pressure points on a nonempty subset $\mathcal{I}_m^p$ of domestic firms and a possibly empty subset $\mathcal{D}_m^p$ of foreign firms, then $\overline{T}_i = 0$ and $\tau_i = 0$ for all $i \in \mathcal{I}_m^p \cup \mathcal{D}_m^p$. We redefine the contractible set as $\mathcal{C}_m^{new} = \mathcal{C}_m \backslash (\mathcal{I}_m^p \cup \mathcal{D}_m^p)$. Next, we redefine $P^m = P$, since the hegemon now lacks the ability to mandate factor allocations of all domestic firms. Finally, we redefine the hegemon's objective function as

$$W_m\left(p, \sum_{i \in \mathcal{I}_m \backslash \mathcal{I}_m^p} \Pi_i(x_i, \ell_i) + \sum_{i \in \mathcal{I}_m^p} V_i(\mathcal{S}_i) + \sum_{f \in \mathcal{F}_m} p_f^\ell \overline{\ell}_f + \sum_{i \in \mathcal{D}_m^{new}} \overline{T}_i\right) + u_m(z)$$

Therefore, we write the Lagrangian of the hegemon as

$$\mathcal{L}_m(\{x_i, \ell_i, \overline{T}_i\}_{i \in \mathcal{C}_m^{new}}) = W_m\left(p, \sum_{i \in \mathcal{I}_m \backslash \mathcal{I}_m^p} \Pi_i(x_i, \ell_i) + \sum_{i \in \mathcal{I}_m^p} V_i(\mathcal{S}_i) + \sum_{f \in \mathcal{F}_m} p_f^\ell \overline{\ell}_f + \sum_{i \in \mathcal{D}_m^{new}} \overline{T}_i\right) + u_m(z)$$

$$+ \sum_{i \in \mathcal{C}_m^{new}} \eta_i\left[\Pi_i(x_i, \ell_i, \mathcal{J}_i) - \overline{T}_i + \beta\nu_i(\mathcal{J}_i) - V_i(\mathcal{S}_i)\right]$$

$$+ \sum_{i \in \mathcal{C}_m} \sum_{S \in \Sigma(\overline{S}_i')} \Lambda_{iS}\left[\beta\left[\nu_i(\mathcal{J}_i) - \nu_i(\mathcal{J}_i \backslash S)\right] - \sum_{j \in S} \theta_{ij} p_j x_{ij} - \mathbf{1}_{S_i^D \subset S}\overline{T}_i\right]$$

From here, analysis largely parallels case (ii). We have

$$\varepsilon_{ij}^z = \frac{\partial W_m}{\partial w_m}\left[\sum_{i \in \mathcal{I}_m \backslash \mathcal{I}_m^p} \frac{\partial \Pi_i}{\partial z_{ij}} + \sum_{i \in \mathcal{I}_m^p} \frac{\partial V_i(\mathcal{S}_i)}{\partial z_{ij}}\right] + \frac{\partial u_m(z)}{\partial z_{ij}} + \sum_{i \in \mathcal{C}_m^{new}} \eta_i\left[\frac{\partial \Pi_i}{\partial z_{ij}} - \frac{\partial V_i(\mathcal{S}_i)}{\partial z_{ij}}\right].$$

ε^{Pm} is formally defined by the same equation as before, substituting $\mathcal{C}_m$ with $\mathcal{C}_m^{new}$ and noting that P^m now equals P. We do not need to define a factor price spillover, since it is now included in P^m.

Given these new definitions, the first order conditions for ℓ_{if} are identical to before but with $\kappa_f = 0$, while the first order conditions for x_{ij} and $\overline{T}_{ij}$ are identical to before up to the changes in definitions of objects. This concludes the proof.

A.1.6 Proof of Proposition 4

Lemma 2 holds for the global planner by the same argument as in its proof. The firm Lagrangian is the same as in the proof of Proposition 3. The global planner has the same constraints as the hegemon, but has objective function

$$\sum_{n=1}^{N} \Omega_n \left[W_n\left(p, w_n\right) + u_n(z) \right]$$

Since the global planner values profits by all firms in the global economy (i.e., the planner treats firms $i \notin \mathcal{C}_m$ as "domestic" to some consumer), formal analysis proceeds parallel to the proofs in Proposition 3 up to the fact that the objective function has changed. Absent a pressure point on sector i, $\overline{T}_i = 0$ and $\tau_i = 0$. For any sector i domestic to country n, the same derivations yield input wedges satisfying

$$\left(\Omega_n \frac{\partial W_n}{\partial w_n} + \eta_i + \theta_{ij}\overline{\Lambda}_{ij} \right) = -\mathcal{E}_{ij}^p,$$

where note this sector is valued by n's consumer. The externality vector $\mathcal{E}_{ij}^p$ is formally defined by the same equation, but replacing the hegemon's externality basis with the planner's externality basis,

$$\varepsilon_{ij}^{zp} = \sum_{n=1}^{N} \Omega_n \left[\frac{\partial W_n}{\partial w_n}\frac{\partial w_n}{\partial z_{ij}} + \frac{\partial u_n}{\partial z_{ij}} \right] + \sum_{i \in \mathcal{C}_m} \eta_i \left[\frac{\partial \Pi_i}{\partial z_{ij}} - \frac{\partial V_i(\mathcal{S}_i)}{\partial z_{ij}} \right]$$

$$\varepsilon^{Pmp} = \sum_{n=1}^{N} \Omega_n \left[\frac{\partial W_n}{\partial P^m} + \frac{\partial w_m}{\partial w_m}\frac{\partial w_m}{\partial P^m} \right] + \sum_{i \in \mathcal{C}_m} \eta_i \left[\frac{\partial \Pi_i}{\partial P^m} - \frac{\partial V_i(\mathcal{S}_i)}{\partial P^m} \right] - \sum_{i \in \mathcal{C}_m} \sum_{S \in \Sigma(\overline{\mathcal{S}}_i')} \Lambda_{iS} \sum_{j \in S} \theta_{ij} \frac{\partial p_j}{\partial P^m} x_{ij}$$

The spillover Ξ_{mn}^p is also defined formally the same, with the new externality basis. The condition for no redistributive motive is therefore $\Omega_m \frac{\partial W_m}{\partial w_n} - \Omega_n \frac{\partial W_n}{\partial w_n} + \Xi_{mn}^p = 0$. Finally, the FOC for a transfer $\overline{T}_i$ for a firm in country n is

$$0 \geq -\eta_i - \overline{\Lambda}_{iS_i^D} + \Omega_m \frac{\partial W_m}{\partial w_n} - \Omega_n \frac{\partial W_n}{\partial w_n} + \Xi_{mn}^p$$

meaning that $\overline{T}_i = 0$ given no redistributive motive. This completes the proof.

A.1.7 Deriving the Tax Rate for National Security Application

Given the hegemon contracts with only one rest-of-world sector, the hegemon's objective (equation (10)) is $u_m(z^H) + \overline{T}_i$. Given the production function that is separable in H, we can write the participation constraint of firm i as

$$v_i(\overline{\mathcal{S}}_i', \overline{T}_i, 0) + \pi_{iH}(x_{iH}) \geq \pi_{iH}(x_{iH}^{Outside}(z^H)) + v_i(\mathcal{S}_i),$$

where v_i is profits on inputs apart from H, which can be defined in the same manner as V_i when considering only the portion of the firm's decision problem that does not involve H. Note that v_i does not depend on z^H. Note that this property relies on separability. $x_{iH}^{Outside}$ is the use of x_{iH} by firm i if it rejects the contract. Finally, we have defined profits from H by $\pi_{iH}(x_{iH}) = p_i A_{iH}(z^H) g_{iH}(x_{iH}) - x_{iH}$. Existence of a pressure point implies $v_i(\overline{\mathcal{S}}_i') > v_i(\mathcal{S}_i)$.

Following the proof of Proposition 3, we have

$$\varepsilon_{iH}^z = \frac{\partial u_m}{\partial z_{iH}} + \eta_i \left[\frac{\partial \pi_{iH}(x_{iH})}{\partial z_{iH}} - \frac{d\pi_{iH}(x_{iH}^{Outside}(z^H))}{dz_{iH}} \right] = \frac{\partial u_m}{\partial z_{iH}} + \eta_i p_i \frac{\partial A_{iH}(z^H)}{\partial z_{iH}} \left[g_{iH}(x_{iH}) - g_{iH}(x_{iH}^{Outside}) \right]$$

$$= \frac{\partial u_m}{\partial z_{iH}} + \eta_i p_i \xi_{ii} A_{iH} \left[g_{iH}(x_{iH}) - g_{iH}(x_{iH}^{Outside}) \right] \frac{1}{z_{iH}}.$$

where the second equality follows from Envelope Theorem and the last by definition. Analogously, we have

$$\varepsilon_{jH}^z = \frac{\partial u_m}{\partial z_{jH}} + \eta_i p_i \xi_{ij} A_{iH} \left[g_{iH}(x_{iH}) - g_{iH}(x_{iH}^{Outside}) \right] \frac{1}{z_{jH}}.$$

Next, from the derivations and using Proposition 2, we have

$$\frac{dz_{jH}}{dz_{iH}} = \left(1 - \frac{\partial x_{jH}^*}{\partial z_{jH}} \right)^{-1} \frac{\partial x_{jH}^*}{\partial z_{iH}} = \frac{\xi_{ji}}{\gamma_j - \xi_{jj}} \frac{z_{jH}}{z_{iH}}.$$

Putting it together, we have

$$\mathcal{E}_{iH} = \varepsilon_{iH}^z + \varepsilon_{jH}^z \frac{dz_{jH}}{dz_{iH}} = \frac{\partial u_m}{\partial z_{iH}} + \frac{\partial u_m}{\partial z_{jH}} \frac{\xi_{ji}}{\gamma_j - \xi_{jj}} \frac{z_{jH}}{z_{iH}} + \eta_i p_i A_{iH} \left[g_{iH}(x_{iH}^*) - g_{iH}(x_{iH}^{Outside}) \right] \left(\xi_{ii} + \xi_{ij} \frac{\xi_{ji}}{\gamma_j - \xi_{jj}} \right) \frac{1}{z_{iH}}.$$

Thus using the tax formula from Proposition 3 with $\theta_{iH} = 0$, we have

$$\eta_i \tau_{iH}^* = -\mathcal{E}_{iH},$$

which yields the result.

A.2 Constructing Value Functions

A.2.1 Construction of $\mathcal{V}$

For an action set (basis), in this appendix we show how to construct $\mathcal{V}_i(\mathcal{B}_i)$ for all $\mathcal{B}_i \in \Sigma_i(\mathcal{S}_i)$.

Since f_i is increasing, concave, and satisfies Inada conditions, then defining

$$\overline{v}_i \equiv \max_{x_i, \ell_i} \Pi_i(x_i, \ell_i, \mathcal{B}_i)$$

we have $\overline{v}_i < +\infty$. Thus we must have $V_i(\mathcal{B}_i) \leq \frac{1}{1-\beta} \overline{v}_i$ for all $\mathcal{B}_i$.

That $V_i(\emptyset) = 0$ follows trivially from $f_i(0, \ell_i, z) = 0$. Consider first an element $B_i \in \mathcal{S}_i$, so that the continuation value from the stealing action is zero. To construct an SPE, we define for $u \geq 0$ the equation

$$V_i(\mathcal{B}_i|u) = \max_{x_i, \ell_i} \Pi_i(x_i, \ell_i, \mathcal{B}_i) + \beta V_i(\mathcal{B}_i|u) \quad s.t. \quad \sum_{j \in \mathcal{B}_i} \theta_{ij} p_j x_{ij} \leq \beta u. \tag{A.4}$$

A.10

Since $\overline{v}_i < +\infty$, we can for any $u \geq 0$ define the unique finite value $v_i(u)$ by

$$v_i(u) = \max_{x_i, \ell_i} \Pi_i(x_i, \ell_i, \mathcal{B}_i) \quad s.t. \quad \sum_{j \in \mathcal{B}_i} \theta_{ij} p_j x_{ij} \leq \beta u$$

Then, $V_i(\mathcal{B}_i | u) = \frac{1}{1-\beta} v_i(u)$ is the unique solution to equation (A.4). Therefore, there is an SPE without stealing with value $V_i(\mathcal{B}_i) = u$ if

$$\frac{1}{1-\beta} v_i(u) = u.$$

Consider the function $\Delta(u) = \frac{1}{1-\beta} v_i(u) - u$. Zeros of this function provide values in SPEs with no stealing. First, $\Delta(0) = 0$ (which is thus an SPE). There is also a positive SPE: from the Inada condition, $\Delta'(0+) = +\infty$, and hence $\Delta(\epsilon) > 0$ for sufficiently small ϵ. Likewise since $v_i(u) \leq \overline{v}_i$, then $\Delta(u) < 0$ for $u > \frac{1}{1-\beta} \overline{v}_i$. Hence by continuity, there is at least one positive SPE $u > 0$. Finally, since f is concave and $\sum_{j \in \mathcal{B}_i} \theta_{ij} p_j x_{ij} \leq u$ describes a convex set, then $v_i(u)$ is increasing and concave in u, and hence $\Delta(u)$ is concave. Therefore, there is exactly one positive value of u.

Next consider the induction. Suppose we have constructed, either as SPEs or with reversion of beliefs, values for all $\hat{\mathcal{B}}_i \in \Sigma(\mathcal{S}_i(\mathcal{B}_i)) \backslash \{\mathcal{B}_i\}$. That is we know the continuation values $V_i(\mathcal{B}_i \backslash S)$ $\forall S \in \Sigma(\mathcal{S}_i(\mathcal{B}_i))$ from previously constructed SPEs. Then we construct the value

$$V_i(\mathcal{B}_i | u) = \max_{x_i, \ell_i} \Pi_i(x_i, \ell_i, \mathcal{B}_i) + \beta V_i(\mathcal{B}_i | u) \quad s.t. \quad \sum_{j \in S} \theta_{ij} p_j x_{ij} \leq \beta \left[u - V_i(\mathcal{B}_i \backslash S) \right] \quad \forall S \in \Sigma(\mathcal{S}_i(\mathcal{B}_i))$$

Thus defining stage game payoff as

$$v_i(u) = \max_{x_i, \ell_i} \Pi_i(x_i, \ell_i, \mathcal{B}_i) \quad s.t. \quad \sum_{j \in S} \theta_{ij} p_j x_{ij} \leq \beta \left[u - V_i(\mathcal{B}_i \backslash S) \right] \quad \forall S \in \Sigma(\mathcal{S}_i(\mathcal{B}_i))$$

then we have $V_i(\mathcal{B}_i | u) = \frac{1}{1-\beta} v_i(u)$. We construct the fixed points, if any, of $\frac{1}{1-\beta} v_i(u) = u$. As before, $v_i(u)$ is increasing and concave, and therefore there are at most two positive fixed points. If instead no fixed point exists, beliefs update to an element $\hat{\mathcal{B}}_i \in \Sigma(\mathcal{S}_i(\mathcal{B}_i)) \backslash \{\mathcal{B}_i\}$ and we set $V_i(\mathcal{B}_i) = V_i(\hat{\mathcal{B}}_i)$.

A.2.2 Continuation Value Functions in Hegemon Problem

The hegemon's optimal contract was characterized in Section 2 for a given set of continuation value functions ν_i. We now provide the equilibrium consistency conditions for a Markov equilibrium. Consider a set of continuation value functions $\nu = \{\nu_i\}$ for firms. Given these continuation value functions, let (Γ, P, z) be the hegemon's optimal contract, prices, and aggregates when the continuation value functions are ν_i. Then, (Γ, P, z, ν) is an equilibrium if: (i) $\nu_i(\mathcal{B}_i) = \mathcal{V}_i(\mathcal{B}_i)$ for $\mathcal{B}_i \in \Sigma(\mathcal{S}_i) \backslash \{\mathcal{J}_i\}$; and, (ii) $\nu_i(\mathcal{J}_i) = V_i(\Gamma_i)$.

A.3 Hegemonic Competition for Dominance

We now consider the possibility that multiple countries can become hegemons. For simplicity, we focus on the case in which two countries, m_1 and m_2, can become hegemons. To streamline analysis, we focus on competition over transfers, and assume constant prices (Definition 1).

Hegemon competition at date t unfolds in two stages. In the first stage, each hegemon $m \in$

$\{m_1, m_2\}$ chooses simultaneously whether or not to pay its fixed cost $F_m \geq 0$ to become a hegemon for date t. In the second stage, any hegemon that enters can offer a contract as described in Section 2, taking as given the contract offered by the other hegemon (if it entered). There are four possible outcomes: (i) neither hegemon enters, and the date t equilibrium is as in Section 1; (ii) exactly one hegemon enters, and its optimal contract is as in Section 2; (iii) both hegemons enter. We now turn to characterizing the equilibrium of the second stage when both hegemons enter, and then turn back to the entry choice in the first stage. As usual, we begin by taking as given continuation value functions ν_i of firms.

A.3.1 Competition Setup

Consider the second stage, and assume that both m_1 and m_2 have paid the fixed cost and become hegemons. Let $\mathcal{C} = \mathcal{C}_{m_1} \cup \mathcal{C}_{m_2}$ be the set of firms that contract with at least one hegemon. Hegemon $m \in \{m_1, m_2\}$ offers a contract $\{\Gamma_i^m\}_{i \in \mathcal{C}_m}$, where $\Gamma_i^m \equiv \{\mathcal{S}_i'^m, \mathcal{T}_i^m, \tau_i^m\}_{i \in \mathcal{C}_m}$ denotes the contract offered to firm $i \in \mathcal{C}_m$. It is convenient to define a trivial contract $\Gamma_i^m = \{\mathcal{S}_i, 0, 0\}$ offered by hegemon m to firms $i \in \mathcal{C}\backslash\mathcal{C}_m$, and let $\Gamma^m = \{\mathcal{S}_i'^m, \mathcal{T}_i, \tau_i^m\}_{i \in \mathcal{C}}$ be the hegemon's contract, including trivial contracts offered to firms $i \notin \mathcal{C}_m$. As in Section 2, the joint threat $\mathcal{S}_i'$ must be feasible under direct transmission.

Firm i faces revenue-neutral wedges and transfers from both hegemons that are added together when both contracts are accepted.[9] Anticipating that a best response to hegemon $-m$ setting $\tau_i^{-m} = 0$ is for hegemon m to set $\tau_i^m = 0$, we will solve the model assuming all wedges to be zero, and then verify that neither hegemon has an incentive to deviate to nonzero wedges. Therefore, we write the contract $\Gamma_i = \{\mathcal{S}_i', \mathcal{T}_i^{m_1} + \mathcal{T}_i^{m_2}, 0\}$ as the combined contract when firms accept both contracts.

The joint threat $\mathcal{S}_i'$ arising when firm i accepts both contracts is constructed by taking the union of joint trigger sets and applying Lemma 1 (see Appendix A.1.1 for details). Here we focus on the special case where both hegemons offer maximal joint threats, as indeed they will in equilibrium. Recalling that $S_i^{Dm} = \bigcup_{S \in \mathcal{S}_i^{Dm}} S$ and $\overline{\mathcal{S}}_i'^m = \{S_i^{Dm}\} \cup (\mathcal{S}_i \backslash \mathcal{S}_i^D)$, where we define $S_i^{Dm} = \emptyset$ if $i \notin \mathcal{C}_m$. Then, maximal (combined) joint threats, $\overline{\mathcal{S}}_i'$, is given by

$$\overline{\mathcal{S}}_i' = (\mathcal{S}_i \backslash (\mathcal{S}_i^{Dm_1} \cup \mathcal{S}_i^{Dm_2})) \cup \mathcal{X}_i, \quad \mathcal{X}_i = \left\{ \begin{array}{ll} \{S_i^{Dm_1}, S_i^{Dm_2}\} & S_i^{Dm_1} \cap S_i^{Dm_2} = \emptyset \\ \{S_i^{Dm_1} \cup S_i^{Dm_2}\} & \text{otherwise} \end{array} \right. \tag{A.5}$$

Intuitively, $\overline{\mathcal{S}}_i'$ combines both hegemon's maximal joint threats into a single maximal joint threat if the two have any common threats. If there are no common threats, the two hegemon's maximal joint threats are separate actions within $\overline{\mathcal{S}}_i'$.

Finally, we define the participation constraints of all firms. In particular, hegemon m's contract is accepted by firm i if

$$\max\{V_i(\Gamma_i), V_i(\Gamma_i^m)\} \geq \max\{V_i(\Gamma_i^{-m}), V_i(\mathcal{S}_i)\} \tag{A.6}$$

Both contracts are accepted by firm i if

$$V_i(\Gamma_i) \geq \max\{V_i(\Gamma_i^m), V_i(\Gamma_i^{-m}), V_i(\mathcal{S}_i)\}. \tag{A.7}$$

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[9] Each hegemon takes as given the other hegemon's equilibrium rebates when both contracts are accepted. If firm i chooses to only accept one contract, equilibrium rebates by the hegemon whose contract is accepted are those that maintain revenue neutrality under the single contract, while there are no rebates by the hegemon whose contract was rejected. If neither contract is accepted, there are no rebates.

A.3.2 Existence of an Equilibrium

We show existence of an equilibrium in which both hegemons offer maximal joint threats, and both hegemon's contracts are accepted. We then discuss how competition shapes the transfers extracted.

The model with two hegemons has to account for the fact that if hegemon m's contract is rejected by firm i, then hegemon m can no longer use firm i in joint threats.[10] This is important because a best response of hegemon m to a contract Γ^{-m} might involve offering a contract to firm i that leads firm i to reject the contract of hegemon $-m$. To make progress, we restrict the form of the network structure as follows. Let $\mathcal{P} = \{i \in \mathcal{C} \mid V_i(\overline{\mathcal{S}}_i') > V_i(\mathcal{S}_i)\}$ denote the set of firms for which the two hegemons can, possibly only jointly, generate a pressure point.

Definition 8 *Hegemon pressure points are **isolated** if: $i \in \mathcal{P} \Rightarrow \mathcal{J}_i \cap \mathcal{P} = \emptyset$.*

Definition 8 states that if the two hegemons can generate a pressure point on i, then the two hegemons cannot generate a pressure point on any firm $j \in \mathcal{J}_i$ that is immediately upstream from i. It ensures that two firms with pressure points from the set of hegemons they contract with are not directly linked to one another. Using this condition, we can now prove that an equilibrium exists in which both hegemons offer maximal joint threats with no wedges.

Proposition 5 *Suppose that hegemon pressure points are isolated. An equilibrium of the model with competition exists in which each hegemon m offers a contract featuring maximal joint threats and no wedges, $\Gamma_i^m = \{\overline{\mathcal{S}}_i'^m, \overline{T}_i^{m*}, 0\}$, to each $i \in \mathcal{C}_m$. Transfers from all firms $i \notin \mathcal{P}$ are zero. Each firm $i \in \mathcal{C}$ accepts the contract(s) it is offered.*

The proof of Proposition 5 proceeds by constructing transfers $\overline{T}_i^{m*}$ such that each contract Γ_i^m is a best response to contract Γ_i^{-m}, and such that both contracts are accepted, that is $V_i(\Gamma_i) \geq \max\{V_i(\Gamma_i^{m_1}), V_i(\Gamma_i^{m_2}), V_i(\mathcal{S}_i)\}$.

The transfers extracted by each hegemon from a foreign firm $i \notin \mathcal{I}_{m_1} \cup \mathcal{I}_{m_2}$ depend on the degree to which they can provide different threats. In the limit where hegemon threats have no overlap, $\mathcal{S}_i^{Dm_1} \cap \mathcal{S}_i^{Dm_2} = \emptyset$, there is no competition: both hegemons offer a contract identical to that of Proposition 1. Despite the multipolar world, firms receive no surplus and do not benefit from competition. By contrast when threats have full overlap, $\mathcal{S}_i^{Dm_1} = \mathcal{S}_i^{Dm_2}$, the two hegemons offer the same set of threats, and so bid each other down to zero transfers, $\overline{T}_i^m = 0$. In this case, firms receive full surplus from the relationships. This result is reminiscent of the Bertrand paradox, in which two firms competing on prices bid each other down to the perfect competition price. This outcome is also efficient ex post, since all joint threats are supplied and no transfers are extracted.

For a firm that is domestic to hegemon m, that is $i \in \mathcal{I}_m$, it remains optimal for hegemon m to demand no transfers, $\overline{T}_i^{m*} = 0$. Hegemon $-m$ then extracts the largest transfer that leaves firm i indifferent between accepting both contracts and accepting only that of hegemon m: $V_i(\overline{S}_i', \overline{T}_i^{-m*}) = V_i(\overline{\mathcal{S}}_i'^m)$. Thus the joint threats that the firm's own hegemon can provide become that firm's outside option, to which that firm is held by the other hegemon.

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[10]This was not an issue in the model with a single hegemon because that hegemon always ensured its contract satisfied the participation constraint.

Entry Decision in First Stage. Entry by both hegemons is a Nash equilibrium in the first stage if hegemon m entering is a best response to hegemon $-m$ entering. If hegemon m enters when hegemon $-m$ enters, Proposition 5 characterizes existence of an equilibrium. If hegemon m does not enter when hegemon $-m$ enters, then $-m$ is a single hegemon, and so by Proposition 1 every firm $i \in \mathcal{I}_m$ receives value equivalent to outside option $V_i(\mathcal{S}_i)$. Therefore, given equilibrium $(\Gamma_i^m, \Gamma_i^{-m})$ if both hegemons enter, then hegemon m enters, given entry by hegemon $-m$, if

$$\sum_{i \in \mathcal{I}_m} V_i(\Gamma_i) + \sum_{i \in \mathcal{D}_m} \overline{T}_i^* - F_m \geq \sum_{i \in \mathcal{I}_m} V_i(\mathcal{S}_i). \tag{A.8}$$

Entry by both hegemons is an equilibrium of the first stage if equation (A.8) holds for $m \in \{m_1, m_2\}$. Since $V_i(\Gamma_i) \geq V_i(\mathcal{S}_i)$, entry by both hegemons is an equilibrium for sufficiently small (possibly zero) entry costs F_m.

A.3.3 Proof of Proposition 5

Given constant prices and no z externalities (Definitions 1 and 2), the objective function of hegemon m is to maximize its country's wealth level,

$$w_m = \sum_{i \in \mathcal{I}_m} + \sum_{i \in \mathcal{D}_m} \sum_j T_{ij}.$$

We assume that $\tau_i = 0$ for both hegemons, and then verify that neither hegemon has an incentive to deviate.

Given hegemons do not have a pressure point on firm $i \notin \mathcal{P}$, both hegemons must offer a trivial contract $\Gamma_i^m = \{\mathcal{S}_i, 0, 0\}$ to such firms to avoid having their contract rejected. Since all firms $i \notin \mathcal{P}$ therefore trivially accept the contracts they are offered, given Definition 8 then the decision problem of each hegemon becomes separable across sectors $i \in \mathcal{P}$. This is due not only to separability of the objective function, but also because under Definition 8, a joint threat is feasible if it is feasible under direct transmission, even if some firms in $\mathcal{P}$ reject hegemon m's contract, given that every firm $i \in \mathcal{P}$ has $\mathcal{J}_i \cap \mathcal{P} = \emptyset$ (i.e., direct transmission links satisfy $S_i^D \subset \mathcal{J}_i \backslash \mathcal{P}$).

We begin by providing the analog of Lemma 2: both hegemons offer contracts featuring maximal joint threats to all firms $i \in \mathcal{P}$

Lemma 6 *Fix a contract Γ^{-m} of hegemon $-m$. Then for all $i \in \mathcal{P}$, it is weakly optimal for hegemon m to offer maximal joint threats, $\mathcal{S}_i'^m = \overline{\mathcal{S}}_i'^m$.*

Proof of Lemma 6. Fix a contract $\Gamma_i^{-m} = \{\mathcal{S}_i'^{-m}, \mathcal{T}_i^{-m}, 0\}$ of hegemon $-m$. The proof strategy is to show that if a contract $\Gamma_i^m \equiv \{\mathcal{S}_i'^m, \mathcal{T}_i^m, 0\}$ is accepted by firm i, then the contract $\Gamma_i^{m'} = \{\overline{\mathcal{S}}_i'^m, \mathcal{T}_i^m, 0\}$ is also accepted by firm i. Let $\Gamma_i = \{\mathcal{S}_i', \mathcal{T}_i^m + \mathcal{T}_i^{-m}, 0\}$ be the joint contract if hegemon m offers Γ_i^m, and $\Gamma_i' = \{\mathcal{S}_i'', \mathcal{T}_i^m + \mathcal{T}_i^{-m}, 0\}$ the joint contract if hegemon m offers $\Gamma_i^{m'}$. Since the contract Γ_i^m is accepted by firm i, then

$$\max\{V_i(\Gamma_i), V_i(\Gamma_i^m)\} \geq \max\{V_i(\Gamma_i^{-m}), V_i(\mathcal{S}_i)\}.$$

Since $\overline{\mathcal{S}}_i'^m$ is a joint threat of $\mathcal{S}_i'^m$, then $\mathcal{S}_i''$ is a joint threat of $\mathcal{S}_i'$. Therefore, $V_i(\Gamma_i^{m'}) \geq V_i(\Gamma_i^m)$ and $V_i(\Gamma_i') \geq V_i(\Gamma_i)$. Therefore,

$$\max\{V_i(\Gamma_i'), V_i(\Gamma_i^{m'})\} \geq \max\{V_i(\Gamma_i^{-m}), V_i(\mathcal{S}_i)\},$$

A.14

and hence contract $\Gamma_i^{m\prime}$ is also accepted by firm i. Finally, firm i is weakly better off (which is valued by hegemon m if firm i is domestic). Thus, maximal joint threats is a weak best response, concluding the proof. $\square$

From Lemma 6, $\mathcal{S}_i' = \overline{\mathcal{S}}_i'$ is a best response to any contract Γ_i^{-m}, and therefore all transfers of m appear under the joint threat. Thus we will focus on the total transfer $\overline{T}_i$ for firms $i \in \mathcal{P}$. The optimal contract for firm i is characterized by Proposition 1 if only one hegemon contracts with i, so assume $i \in \mathcal{C}_{m_1} \cap \mathcal{C}_{m_2}$.

Let $\Gamma_i^m = \{\overline{\mathcal{S}}_i^{\prime m}, \overline{T}_i^m, 0\}$ be a candidate optimal contract of hegemon m, and let $\Gamma_i = \{\overline{\mathcal{S}}_i', \overline{T}_i^{m_1} + \overline{T}_i^{m_2}, 0\}$ be the joint contract.

A.3.3.1 Foreign Firms

Let $i \in \mathcal{P} \backslash (\mathcal{I}_{m_1} \cup \mathcal{I}_{m_2})$ be a firm foreign to both hegemons. We begin with the following intermediate result.

Lemma 7 $(\Gamma_i^m, \Gamma_i^{-m})$ *is part of an equilibrium is which firm i accepts both contracts if and only if one of the following holds:*

1. *Firm i is held to its outside option, with*

$$V_i(\Gamma_i) = V_i(\mathcal{S}_i) \geq \max_{m \in \{m_1, m_2\}} \{V_i(\Gamma_i^m)\} \tag{A.9}$$

2. *Firm i exceeds its outside option, with*

$$V_i(\Gamma_i) = V_i(\Gamma_i^{m_1}) = V_i(\Gamma_i^{m_2}) > V_i(\mathcal{S}_i) \tag{A.10}$$

Proof of Lemma 7. Since both contracts are accepted, then

$$V_i(\Gamma_i) \geq \max\{V_i(\mathcal{S}_i), V_i(\Gamma_i^{m_1}), V_i(\Gamma_i^{m_2})\}.$$

Suppose first that firm i is held to its outside option, $V_i(\Gamma_i) = V_i(\mathcal{S}_i)$. Then, since both contracts are accepted,

$$V_i(\Gamma_i) = V_i(\mathcal{S}_i) \geq \max_{m \in \{m_1, m_2\}} \{V_i(\Gamma_i^m)\}.$$

Finally, suppose that we have two contracts that satisfy this condition. Then, if either hegemon increased its transfer, the firm would reject both contracts and revert to the outside option. Likewise, a hegemon that lowered its transfer would have its contract accepted, but be strictly worse off. Therefore we have an equilibrium.

Suppose, second, that firm i exceeds its outside option, $V_i(\Gamma_i) > V_i(\mathcal{S}_i)$. Suppose, hypothetically, that

$$V_i(\Gamma_i) > \max\{V_i(\Gamma_i^m), V_i(\Gamma_i^{-m})\}.$$

Then, hegemon m could increase its transfer without its contract being rejected, and so be strictly better off. Therefore, $V_i(\Gamma_i) = \max\{V_i(\Gamma_i^m), V_i(\Gamma_i^{-m})\}$. Suppose then that (without loss)

$$V_i(\Gamma_i) = V_i(\Gamma_i^m) > V_i(\Gamma_i^{-m}).$$

A.15

Then again, hegemon m could increase its transfer without its contract being reject, and so be strictly better off. Therefore,

$$V_i(\Gamma_i) = V_i(\Gamma_i^{m_1}) = V_i(\Gamma_i^{m_2}) > V_i(\mathcal{S}_i).$$

Finally, supposing this condition holds, then if either hegemon increased its transfer, the firm would reject its contract and accept only that of the other hegemon. Likewise, a hegemon that lowered its transfer would have its contract accepted, but be strictly worse off. Therefore, neither hegemon deviates, and we have an equilibrium. This concludes the proof of Lemma 7. $\square$

We use Lemma 7 to construct an equilibrium. Since $i \in \mathcal{P}$, $V_i(\overline{\mathcal{S}}'_i) > V_i(\mathcal{S}_i)$. Without loss of generality, let $V_i(\overline{\mathcal{S}}'^m_i) \geq V_i(\overline{\mathcal{S}}'^{-m}_i)$. We begin by constructing the minimal transfer $t_0^m \geq 0$ such that $V_i(\overline{\mathcal{S}}'^m_i, t_0^m) = V_i(\overline{\mathcal{S}}'^{-m}_i, 0)$. Since $\overline{\mathcal{S}}'_i$ is a joint threat of $\overline{\mathcal{S}}'^m_i$, and therefore $V_i(\overline{\mathcal{S}}'_i, t_0^m) \geq V_i(\overline{\mathcal{S}}'^m_i, t_0^m)$. If $V_i(\overline{\mathcal{S}}'_i, t_0^m) = V_i(\overline{\mathcal{S}}'^m_i, t_0^m)$, then we have found contracts such that $V_i(\Gamma_i) = V_i(\Gamma_i^m) = V_i(\Gamma_i^{-m})$, and hence either equation (A.9) or (A.10) is satisfied. Thus we have an equilibrium.

Suppose instead $V_i(\overline{\mathcal{S}}'_i, t_0^m) > V_i(\overline{\mathcal{S}}'^m_i, t_0^m)$. Then, we construct a function $t^{-m}(t)$ by

$$V_i(\overline{\mathcal{S}}'^m_i, t_0^m + t) = V_i(\overline{\mathcal{S}}'^{-m}_i, t^{-m}(t)).$$

We can construct this function from $t = 0$ to $t = \bar{t}$, where $\bar{t}$ solves $V_i(\overline{\mathcal{S}}'^m_i, t_0^m + t) = V_i(\mathcal{S}_i)$ (note it is possible for $\bar{t} = 0$).

Suppose first $\exists t^* \in [0, t]$ such that

$$V_i(\overline{\mathcal{S}}'_i, t_0^m + t^* + t^{-m}(t^*)) = V_i(\overline{\mathcal{S}}'^m_i, t_0^m + t^*).$$

Then, equation (A.10) is satisfied if $t^* < \bar{t}$, and equation (A.9) is satisfied if $t^* = \bar{t}$. Therefore, by Lemma 7 we have found an equilibrium.

Suppose instead that no such t^* exists, and therefore $V_i(\overline{\mathcal{S}}'_i, t_0^m + \bar{t} + t^{-m}(\bar{t})) > V_i(\mathcal{S}_i)$. Then, define T^* such that $V_i(\overline{\mathcal{S}}'_i, T^*) = V_i(\mathcal{S}_i)$, and define $\overline{T}_i^m$ and $\overline{T}_i^{-m}$ such that $\overline{T}_i^m + \overline{T}_i^{-m} = T^*$, $\overline{T}_i^{-m} \geq t_0^m + \bar{t}$, and $\overline{T}_i^{-m} \geq t^{-m}(\bar{t})$. Then, equation (A.9) is satisfied, and hence we have found an equilibrium.

Therefore, an equilibrium exists as described, assuming both hegemons impose zero wedges. Observe that imposing nonzero wedges cannot increase the value of its objective, and leads to its contract being (weakly) rejected. Thus, zero wedges is a best response of each hegemon, concluding this portion of the proof.

A.3.3.2 Domestic Firms

Let $i \in \mathcal{P} \cap \mathcal{I}_m$ be a domestic firm of hegemon m. We obtain the following result, which parallels Lemma 7.

Lemma 8 $(\Gamma_i^m, \Gamma_i^{-m})$ *is part of an equilibrium in which firm* $i \in \mathcal{P} \cap \mathcal{I}_m$ *accepts both contracts if and only if one of the following holds:*

1. *Firm* i *is held to its outside option, with* $\overline{T}_i^m = 0$ *and*

$$V_i(\Gamma_i) = V_i(\mathcal{S}_i) \geq \max_{m \in \{m_1, m_2\}} \{V_i(\Gamma_i^m)\} \tag{A.11}$$

A.16

2. Firm i exceeds its outside option, with $\overline{T}_i^m = 0$ and

$$V_i(\Gamma_i) = V_i(\Gamma_i^m) \geq \max\{V_i(\Gamma_i^{-m}), V_i(\mathcal{S}_i)\} \tag{A.12}$$

Proof of Lemma 8. Since both contracts are accepted, then

$$V_i(\Gamma_i) \geq \max\{V_i(\mathcal{S}_i), V_i(\Gamma_i^{m_1}), V_i(\Gamma_i^{m_2})\}.$$

Suppose first that firm i is held to its outside option, $V_i(\Gamma_i) = V_i(\mathcal{S}_i)$. Then, since both contracts are accepted,

$$V_i(\Gamma_i) = V_i(\mathcal{S}_i) \geq \max_{m \in \{m_1, m_2\}} \{V_i(\Gamma_i^m)\}.$$

Finally, suppose that we have two contracts that satisfy this condition and that $\overline{T}_i^m = 0$. If hegemon $-m$ increased its transfer, then its contract would be rejected. If hegemon m had a posititive transfer, it could decrease the transfer, have its contract remain accepted, and increase value of its domestic firm i. Therefore, we have an equilibrium if $\overline{T}_i^m = 0$.

Suppose, second, that firm i exceeds its outside option, $V_i(\Gamma_i) > V_i(\mathcal{S}_i)$. Suppose, hypothetically, that

$$V_i(\Gamma_i) > V_i(\Gamma_i^m).$$

Then, hegemon $-m$ could increase its transfer without its contract being rejected, and so be strictly better off. Therefore, $V_i(\Gamma_i) = V_i(\Gamma_i^m)$, and therefore

$$V_i(\Gamma_i) = V_i(\Gamma_i^m) \geq \max\{V_i(\Gamma_i^{-m}), V_i(\mathcal{S}_i)\}.$$

If this condition holds, and $\overline{T}_i^m > 0$, then hegemon m could decrease its transfer for its domestic firm without its contract being rejected, and so be strictly better off. Therefore, $\overline{T}_i^m = 0$. Finally, suppose this condition holds and $\overline{T}_i^m = 0$. Then, if hegemon $-m$ increased its transfer, its contract would be rejected. Hegemon m cannot further decrease its transfer. Therefore, neither hegemon deviates, and we have an equilibrium. This concludes the proof. □

Lemma 8 shows that $\overline{T}_i^m = 0$ in any equilibrium, that is a domestic firm is not charged a transfer by its hegemon. Since $\overline{T}_i^m = 0$, then $V_i(\Gamma_i^{-m}) \leq V_i(\Gamma_i)$. We can construct the transfer of hegemon $-m$ as the solution to $V_i(\overline{\mathcal{S}}_i', \overline{T}_i^{-m}) = V_i(\overline{\mathcal{S}}_i'^m)$. If $V_i(\overline{\mathcal{S}}_i'^m) = V_i(\mathcal{S}_i)$, then equation (A.11) is satisfied and we have an equilibrium. If $V_i(\overline{\mathcal{S}}_i'^m) > V_i(\mathcal{S}_i)$, then equation (A.12) is satisfied and we have an equilibrium. In both cases, zero wedges is part of an optimal poilcy for both hegemons. Therefore, we have an equilibrium.

This concludes the proof of existence.

A.4 Additional Results and Derivations

A.4.1 CES Example Derivation

Take the Nested CES production function,

$$f_i(x_i) = \left(\sum_{\tilde{x} \in \tilde{X}_i} \tilde{\alpha}_{i\tilde{x}} \left(\sum_{j \in \tilde{x}} \alpha_{ij} x_{ij}^{\chi_{i\tilde{x}}} \right)^{\frac{\rho_i}{\chi_{i\tilde{x}}}} \right)^{\frac{\xi_i}{\rho_i}}$$

We first solve the expenditure minimization problem associated with bundle $\tilde{x}$, given by

$$\min \sum_{j \in \tilde{x}} p_j x_{ij} \quad s.t. \quad \left(\sum_{j \in \tilde{x}} \alpha_{ij} x_{ij}^{\chi_{i\tilde{x}}} \right)^{\frac{1}{\chi_{i\tilde{x}}}} \geq \overline{x}$$

Letting λ denote the Lagrange multiplier on the production constraint, the FOCs are

$$0 = p_j - \lambda \left(\sum_{j \in \tilde{x}} \alpha_{ij} x_{ij}^{\chi_{i\tilde{x}}} \right)^{\frac{1}{\chi_{i\tilde{x}}} - 1} \alpha_{ij} x_{ij}^{\chi_{i\tilde{x}} - 1}$$

$$\Rightarrow \left(\frac{p_j}{\alpha_{ij}} \frac{\alpha_{ik}}{p_k} \right)^{\frac{1}{1 - \chi_{i\tilde{x}}}} x_{ij} = x_{ik}$$

Substituting into the production constraint yields

$$\overline{x} = \left(\sum_{j \in \tilde{x}} \alpha_{ij}^{\frac{1}{1 - \chi_{i\tilde{x}}}} p_j^{-\frac{\chi_{i\tilde{x}}}{1 - \chi_{i\tilde{x}}}} \right)^{\frac{1}{\chi_{i\tilde{x}}}} \left(\frac{p_k}{\alpha_{ik}} \right)^{\frac{1}{1 - \chi_{i\tilde{x}}}} x_{ik}.$$

Therefore, the expenditure function is

$$e_i(p, \overline{x}) = \left(\sum_{j \in \tilde{x}} \alpha_{ij}^{\frac{1}{1 - \chi_{i\tilde{x}}}} p_j^{-\frac{\chi_{i\tilde{x}}}{1 - \chi_{i\tilde{x}}}} \right)^{-\frac{1 - \chi_{i\tilde{x}}}{\chi_{i\tilde{x}}}} \overline{x}.$$

We therefore define the price index $P_{i\tilde{x}} = \left(\sum_{j \in \tilde{x}} \alpha_{ij}^{\frac{1}{1 - \chi_{i\tilde{x}}}} p_j^{-\frac{\chi_{i\tilde{x}}}{1 - \chi_{i\tilde{x}}}} \right)^{-\frac{1 - \chi_{i\tilde{x}}}{\chi_{i\tilde{x}}}}$ associated with total consumption of basket $\tilde{x}$.

The optimization problem thus reduces to an optimization problem over bundles. We abuse notation and use $\tilde{x}$ as aggregate consumption of bundle $\tilde{x}$, so that we have

$$\max p_i \left(\sum_{\tilde{x} \in \tilde{X}_i} \alpha_{i\tilde{x}} \tilde{x}^{\rho_i} \right)^{\frac{\xi_i}{\rho_i}} - \sum_{\tilde{x} \in \tilde{X}_i} P_{i\tilde{x}} \tilde{x}$$

This yields FOCs

$$p_i \left(\sum_{\tilde{x} \in \tilde{X}_i} \alpha_{i\tilde{x}} \tilde{x}^{\rho_i} \right)^{\frac{\xi_i}{\rho_i} - 1} \alpha_{i\tilde{x}} \xi_i \tilde{x}^{\rho_i - 1} = P_{i\tilde{x}}$$

$$\Rightarrow \tilde{x} = \left(\frac{P_{i\tilde{x}_k}}{P_{i\tilde{x}}} \frac{\alpha_{i\tilde{x}}}{\alpha_{i\tilde{x}_k}} \right)^{\frac{1}{1 - \rho_i}} \tilde{x}_k$$

Substituting the second equation into the first, we obtain

$$\tilde{x}_k = \left(\sum_{\tilde{x} \in \tilde{X}_i} \alpha_{i\tilde{x}}^{\frac{1}{1 - \rho_i}} P_{i\tilde{x}}^{-\frac{\rho_i}{1 - \rho_i}} \right)^{\frac{\xi_i - \rho_i}{\rho_i (1 - \xi_i)}} \left(\frac{\alpha_{i\tilde{x}_k}}{P_{i\tilde{x}_k}} \right)^{\frac{1}{1 - \rho_i}} (p_i \xi_i)^{\frac{1}{1 - \xi_i}}.$$

Therefore, expenditures are

$$\sum_{\tilde{x}} P_{i\tilde{x}}\tilde{x} = (p_i\xi_i)^{\frac{1}{1-\xi_i}} \left(\sum_{\tilde{x}\in\tilde{X}_i} \alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}} P_{i\tilde{x}}^{-\frac{\rho_i}{1-\rho_i}} \right)^{\frac{\xi_i}{\rho_i}\frac{1-\rho_i}{1-\xi_i}}$$

while revenues from production are

$$p_i \left(\sum_{\tilde{x}\in\tilde{X}_i} \alpha_{i\tilde{x}}\tilde{x}^{\rho_i} \right)^{\frac{\xi_i}{\rho_i}} = p_i(p_i\xi_i)^{\frac{\xi_i}{1-\xi_i}} \left(\sum_{\tilde{x}\in\tilde{X}_i} \alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}} P_{i\tilde{x}}^{-\frac{\rho_i}{1-\rho_i}} \right)^{\frac{\xi_i}{\rho_i}\frac{1-\rho_i}{1-\xi_i}}.$$

If firm i has all inputs left, we therefore have

$$\nu_i(\mathcal{J}_i) = p_i^{\frac{1}{1-\xi_i}} \left[(\xi_i)^{\frac{\xi_i}{1-\xi_i}} - (\xi_i)^{\frac{1}{1-\xi_i}} \right] \left(\sum_{\tilde{x}\in\tilde{X}_i} \alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}} P_{i\tilde{x}}^{-\frac{\rho_i}{1-\rho_i}} \right)^{\frac{\xi_i}{\rho_i}\frac{1-\rho_i}{1-\xi_i}}.$$

Now consider a firm that only has inputs $\mathcal{B}_i$ remaining. The price index for such a firm can be written as

$$P_{i\tilde{x}}(\mathcal{B}_i) = \left(\sum_{j\in\tilde{x}\cap\mathcal{B}_i} \alpha_{ij}^{\frac{1}{1-\overline{\chi}_{i\tilde{x}}}} p_j^{-\frac{\overline{\chi}_{i\tilde{x}}}{1-\overline{\chi}_{i\tilde{x}}}} \right)^{-\frac{1-\chi_{i\tilde{x}}}{\chi_{i\tilde{x}}}}$$

and therefore we can write

$$\nu_i(\mathcal{B}_i) = p_i^{\frac{1}{1-\xi_i}} \left[(\xi_i)^{\frac{\xi_i}{1-\xi_i}} - (\xi_i)^{\frac{1}{1-\xi_i}} \right] \left(\sum_{\tilde{x}\in\tilde{X}_i} \alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}} P_{i\tilde{x}}(\mathcal{B}_i)^{-\frac{\rho_i}{1-\rho_i}} \right)^{\frac{\xi_i}{\rho_i}\frac{1-\rho_i}{1-\xi_i}}.$$

Therefore, we have

$$\log\nu_i(\mathcal{B}_i) - \log\nu_i(\mathcal{B}_i\backslash\{k\}) = \frac{\xi_i}{\rho_i}\frac{1-\rho_i}{1-\xi_i} \log \left(\frac{\sum_{\tilde{x}\in\tilde{X}_i} \alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}} P_{i\tilde{x}}(\mathcal{B}_i)^{-\frac{\rho_i}{1-\rho_i}}}{\sum_{\tilde{x}\in\tilde{X}_i} \alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}} P_{i\tilde{x}}(\mathcal{B}_i\backslash\{k\})^{-\frac{\rho_i}{1-\rho_i}}} \right)$$

$$= -\frac{\xi_i}{\rho_i}\frac{1-\rho_i}{1-\xi_i} \log \left(1 - \frac{\alpha_{i\tilde{x}_k}^{\frac{1}{1-\rho_i}} P_{i\tilde{x}_k}(\mathcal{B}_i)^{-\frac{\rho_i}{1-\rho_i}}}{\sum_{\tilde{x}\in\tilde{X}_i} \alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}} P_{i\tilde{x}}(\mathcal{B}_i)^{-\frac{\rho_i}{1-\rho_i}}} \left[1 - \left(\frac{P_{i\tilde{x}_k}(\mathcal{B}_i\backslash\{k\})}{P_{i\tilde{x}_k}(\mathcal{B}_i)} \right)^{-\frac{\rho_i}{1-\rho_i}} \right] \right)$$

$$= -\frac{\xi_i}{\rho_i}\frac{1-\rho_i}{1-\xi_i} \log \left(1 - \Omega_{i\tilde{x}_k} \left[1 - \left(1 - \omega_{ik} \right)^{\frac{1-\chi_{i\tilde{x}}}{\chi_{i\tilde{x}}}\frac{\rho_i}{1-\rho_i}} \right] \right)$$

given the definitions of expenditure shares,

$$\Omega_{i\tilde{x}_k} = \frac{\alpha_{i\tilde{x}_k}^{\frac{1}{1-\rho_i}} P_{i\tilde{x}_k}(\mathcal{B}_i)^{-\frac{\rho_i}{1-\rho_i}}}{\sum_{\tilde{x}\in\tilde{X}_i} \alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}} P_{i\tilde{x}}(\mathcal{B}_i)^{-\frac{\rho_i}{1-\rho_i}}}$$

$$\omega_{ik} = \frac{\alpha_{ik}^{\frac{1}{1-\chi_{i\tilde{x}_k}}} p_k^{-\frac{\chi_{i\tilde{x}_k}}{1-\chi_{i\tilde{x}_k}}}}{\sum_{j\in\tilde{x}_k} \alpha_{ij}^{\frac{1}{1-\chi_{i\tilde{x}_k}}} p_j^{-\frac{\chi_{i\tilde{x}_k}}{1-\chi_{i\tilde{x}_k}}}}$$

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A.4.2 Identifying Pressure Points: A Special Case

In this appendix, we consider the environment where firms have separable production (Definition 3) and provide a necessary and sufficient condition for identifying pressure points. Under separable production, we have $f_i(x_i, \ell_i, z) = \sum_{j \in \mathcal{J}_i} f_{ij}(x_{ij})$. We write $\Pi_i(x_i, \mathcal{B}_i) = \sum_{j \in \mathcal{B}_i} \pi_{ij}(x_{ij})$, where $\pi_{ij}(x_{ij}) = p_i f_{ij}(x_{ij}, z) - p_j x_{ij}$.

Suppose that continuation value ν_i is separable across elements of $\mathcal{S}_i(\mathcal{B}_i)$, that is we can write $\nu_i(\mathcal{B}_i) = \sum_{S \in \mathcal{S}_i(\mathcal{B}_i)} v_i(S)$. Then, the incentive constraint associated with $S \in S_i(\mathcal{B}_i)$ is

$$\sum_{j \in S} \theta_{ij} p_j x_{ij} \leq \beta v_i(S).$$

Therefore, if the incentive constraint holds for $S_1, S_2 \in \mathcal{S}_i(\mathcal{B}_i)$, it also holds for $S_1 \cup S_2$. Thus incentive compatibility with respect to $\mathcal{S}_i(\mathcal{B}_i)$ implies incentive compatibility with respect to $\Sigma(\mathcal{S}_i(\mathcal{B}_i))$. Thus the decision problem of firm i becomes separable over elements of the action set $\mathcal{S}_i(\mathcal{B}_i)$, leading to a value function that is separable over elements of the basis, consistent with the assumption.

Now, we move to characterizing pressure points. As a preliminary, the optimization problem of firm i has a corresponding Lagrangian

$$\mathcal{L}(x_i, \lambda | \mathcal{S}_i) \equiv \sum_{j \in \mathcal{J}_i} \pi_{ij}(x_{ij}) + \sum_{S \in \mathcal{S}_i} \lambda_{iS} \left[\beta v_i(S) - \sum_{j \in S} \theta_{ij} p_j x_{ij} \right],$$

where $\lambda_{iS} \geq 0$ is the Lagrange multiplier on the incentive compatibility constraint associated with $S \in \mathcal{S}_i$. We obtain the following result.

Proposition 6 $S_1, \ldots, S_n \in \mathcal{S}_i$ *is a pressure point of firm* i *if and only if* $\lambda_{iS} \neq \lambda_{iS'}$ *for some* $S, S' \in \{S_1, \ldots, S_n\}$.

Proposition 6 proves that a necessary and sufficient condition for a pressure point is that the Lagrange multipliers of the existing equilibrium differ among those input relationships that enter the joint threat. To build intuition, return to the triangle network in Figure 1. Consider the equilibrium under isolated stealing $\mathcal{S}_i = \{\emptyset, \{j\}, \{k\}\}$, then firms in sector i have a pressure point resulting from the joint threat actions $\{j\}, \{k\}$ if and only if $\lambda_{ij} \neq \lambda_{ik}$. Intuitively, if $\lambda_{ij} > \lambda_{ik}$, then the marginal value of slack in the incentive compatibility constraint for (stealing) good j is higher than for slack in the incentive compatibility constraint for good k. The joint threat creates value by consolidating the two constraints and altering relative production of the two goods, a means of redistributing slack. Heuristically, the joint threat facilitates a *decrease* in production using k in order to create slack that allows for an *increase* in production using j under the joint threat. By contrast if $\lambda_j = \lambda_k$, then slack is equally valuable across goods j and k, even when both multipliers are strictly positive and both constraints bind. In this case, no value is created by forming a joint threat: production under the joint threat is precisely the same as under isolated threats. The proof of Proposition 6 formalizes these intuitions for more general action sets $\mathcal{S}_i$.

This result is both intuitive and powerful. Intuitive, in the sense that combining disparate threats into a joint one, creates value by allowing profitable perturbations of the original allocation that now feasible under the joint threat. The ex-ante Langrange multipliers indicate whether adding slack to a particular input relationship is more valuable, and therefore guide the perturbation to increase that allocation and decrease the rest to preserve joint incentive compatibility. Powerful, in

the sense that identifying pressure points only requires knowing the tightness of the constraints in the existing equilibrium.

A.4.2.1   Proof of Proposition 6

We break the proof into the if and only if statements.

If.   Suppose that there exist $S', S'' \in \{S_1, \ldots, S_n\}$ such that $\lambda_{iS'} > \lambda_{iS''}$ (without loss of generality). Suppose that we augment the incentive compatibility constraint for S to be

$$\sum_{j \in S} \theta_{ij} p_j x_{ij} \leq \beta v_i(S) + \tau_S,$$

where τ_S is a constant that is equal to zero. Observe that since $S' \cap S'' = \emptyset$, then joint threat constructed from S' and S'' yields the incentive constraint

$$\sum_{j \in S' \cup S''} \theta_{ij} p_j x_{ij} \leq \beta[v_i(S') + v_i(S'')] + \tau_{S'} + \tau_{S''}.$$

Therefore, a weaker expansion of incentive compatible allocations than achieved by a joint threat is to instead increase $\tau_{S'}$ and decrease $\tau_{S''}$ in such a manner that $\tau_{S'} + \tau_{S''} = 0$. If such a perturbation strictly increases value, then creating a joint threat also strictly increases value.

Since $V_i(\mathcal{S}_i, \tau) = \mathcal{L}$, then the welfare effect of a perturbation to τ_S, by Envelope Theorem, is

$$\frac{\partial V_i}{\partial \tau_S} = \lambda_{iS}$$

Therefore, the total profit impact on firm i of the perturbation $d\tau_{S'} = 1$ and $d\tau_{S''} = -1$ is

$$\frac{\partial V_i}{\partial \tau_{S'}} - \frac{\partial V_i}{\partial \tau_{S''}} = \lambda_{iS'} - \lambda_{iS''} > 0.$$

Therefore, there is an $\epsilon > 0$ such that when defining τ by $\tau_{S'} = \epsilon$, $\tau_{S''} = -\epsilon$, and $\tau_S = 0$ otherwise, we have $V_i(\mathcal{S}_i, \tau) > V_i(\mathcal{S}_i, 0)$. But since $V_i(\mathcal{S}_i') \geq V_i(\mathcal{S}_i, \tau)$, then $V_i(\mathcal{S}_i') > V_i(\mathcal{S}_i)$, and hence $(S_1, \ldots, S_n)$ is a pressure point on i.

Only If.   Because the decision problem of firm i is separable across elements of the action set, and because elements $S \notin \{S_1, \ldots, S_n\}$ are unchanged, the same allocations x_{ij}^* for $j \in \bigcup_{S \in \mathcal{S}_i \setminus \{S_1, \ldots, S_n\}} S$ remain optimal. It remains to show that optimal allocations are unchanged for $j \in \bigcup_{S \in \{S_1, \ldots, S_n\}} S$.

Suppose first that $\lambda_{iS_1} = \ldots = \lambda_{iS_n} = 0$. Then, x_{ij} is produced at first-best scale, $x_{ij} = x_{ij}^u$. But then since $x_{ij}^* = x_{ij}^u$ is also implementable under joint threats, then the optimal allocation under joint threats is again $x_{ij}^* = x_{ij}^u$, and hence $(S_1, \ldots, S_n)$ is not a pressure point on i.

Suppose next that $\lambda_{iS_1} = \ldots = \lambda_{iS_n} > 0$ and let x_i^* be optimal production under $\mathcal{S}_i$. Because the decision problem of firm i is separable across elements of the action set, let us focus on the subset $\mathcal{X} = \{S_1, \ldots, S_n\}$ of elements in the joint threat. Denoting $\mathcal{L}(x_i, \hat{\lambda}|\mathcal{X})$ the Lagrangian associated with elements $\mathcal{X}$,

$$\mathcal{L}(x_i, \hat{\lambda}_i|\mathcal{X}) = \sum_{j \in \bigcup_{S \in \mathcal{X}} S} \pi_{ij}(x_{ij}) + \sum_{S \in \mathcal{X}} \hat{\lambda}_{iS} \left[\beta v_i(S) - \sum_{j \in S} \theta_{ij} p_j x_{ij} \right].$$

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Recalling that the firm's objective function is concave while each constraint is convex, the Lagrangian has a saddle point at (x_i^*, λ_i).

Next, consider the decision problem of firm i when faced with a joint threat, so that $\mathcal{S}_i'$ has an element $S' = \bigcup_{S \in \mathcal{X}} S$. As again the decision problem of the firm is separable across elements of $\mathcal{S}_i'$, then we can define the Lagrangian of firm i with respect to element S' by

$$\mathcal{L}(x_i, \mu_i | S') = \sum_{j \in S'} \pi_{ij}(x_{ij}) + \mu_{iS'}\left[\beta \sum_{S \in \mathcal{X}} v_i(S) - \sum_{j \in S'} \theta_{ij} p_j x_{ij}\right].$$

Observe that once again, the objective function is concave while the constraint is convex. Since $S \cap S' = \emptyset$ for all $S, S' \in \mathcal{X}$, then we can write

$$\mathcal{L}(x_i, \mu_i | S') = \sum_{j \in S'} \pi_{ij}(x_{ij}) + \sum_{S \in \mathcal{X}} \mu_{iS'}\left[\beta v_i(S) - \sum_{j \in S} \theta_{ij} p_j x_{ij}\right].$$

Finally, let us define $\mu_{iS'} = \lambda_{iS_1}$. Since $\lambda_{iS_1} = \ldots = \lambda_{i,S_n}$, then we have

$$\mathcal{L}(x_i, \mu_i | S') = \sum_{j \in S'} \pi_{ij}(x_{ij}) + \sum_{S \in \mathcal{X}} \lambda_{iS}\left[\beta v_i(S) - \sum_{j \in S} \theta_{ij} p_j x_{ij}\right].$$

As a result, we have $\mathcal{L}(x_i, \mu_i | S') = \mathcal{L}(x_i, \lambda_i | \mathcal{X})$ for all x_i. More generally since for any μ_i' there is a corresponding vector $\lambda_{iS}' = \mu_i'$, then since $\mathcal{L}(x_i, \hat{\lambda}_i | \mathcal{X})$ has a saddle point at (λ_i, x_i^*), then $\mathcal{L}(x_i, \hat{\mu}_i | S')$ has a saddle point at (μ_i, x_i^*). Therefore, x_i^* is also an optimal policy under joint threat $\mathcal{S}_i'$. Therefore, $V_i(\mathcal{S}_i') = V_i(\mathcal{S}_i)$ and hence $(S_1, \ldots, S_n)$ is not a pressure point. This concludes the proof.

Appendix Figures

Figure A.1: **Feasible Threats by Hegemon**
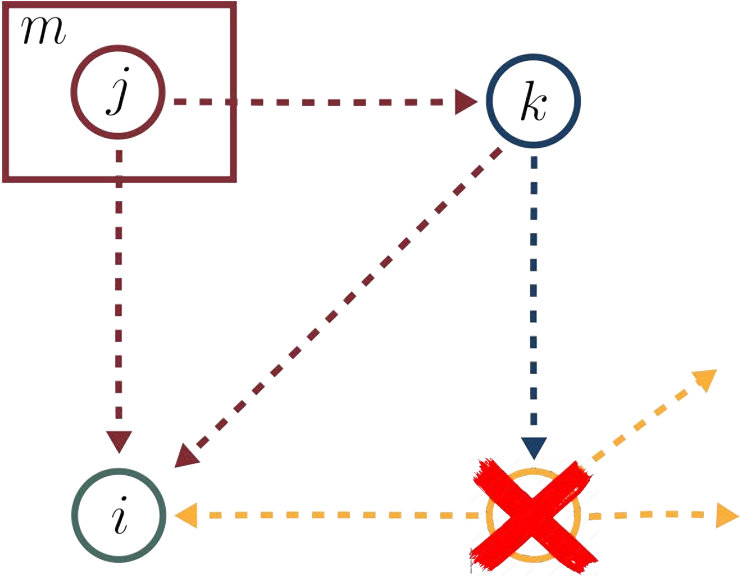
Notes: The figure illustrates the following configuration: sector j is located in the hegemon country and supplies to sector k and i. Sector k supplies to sector i and to another sector (orange and crossed-out), which itself supplies to sector i. The hegemon has a feasible joint threat on sector i via controlling the threats of j and k. The hegemon does NOT have a feasible joint threat on the orange crossed-out sector.