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ABSTRACT

We introduce dynamic incentive contracts into a model of unemployment dynamics and present three results. First, wage cyclicality from incentives does not dampen unemployment dynamics: the response of unemployment to shocks is first-order equivalent in an economy with flexible incentive pay and without bargaining, vis-á-vis an economy with rigid wages. Second, wage cyclicality from bargaining dampens unemployment dynamics through the standard mechanism. Third, our calibrated model suggests 46% of wage cyclicality in the data arises from incentives. A standard model without incentives calibrated to weakly procyclical wages, matches unemployment dynamics in our incentive pay model calibrated to strongly procyclical wages.

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1 Introduction

Macroeconomists have long argued that limited adjustment of wages is important for business cycles (Keynes, 1937). If wages are stable, then profits per worker and labor demand are volatile. Equipped with this insight, many models incorporate rigidities that reduce wage cyclicality, leading unemployment to be responsive to business cycle shocks (Hall, 2005; Gertler and Trigari, 2009) and dampening inflation dynamics (Gertler, Sala and Trigari, 2008; Blanchard and Galí, 2010; Christiano, Eichenbaum and Trabandt, 2016).

However, mapping the theory of wage rigidity to data is difficult because compensation is complex. In particular, incentive pay—such as piece-rate pay, bonuses, profit sharing, commissions and stock options—is prevalent. Approximately half of all workers receive some incentive pay, including some 30% of bottom-decile earners (Lemieux, Macleod and Parent, 2009; Makridis and Gittleman, 2018). Longer-term incentives, such as promotions, are also common. Furthermore, incentive pay appears relatively flexible. Bonuses are raised and lowered frequently at the micro level (Grigsby, Hurst and Yildirmaz, 2021) and are found to be strongly procyclical in some, though not all, studies (Bils, 1985; Devereux, 2001; Shin and Solon, 2007; Swanson, 2007).

This paper asks how flexible incentive pay affects unemployment dynamics. We consider a rich dynamic incentive contract with moral hazard and persistent idiosyncratic shocks similar to Edmans, Gabaix, Sadzik and Sannikov (2012), which we embed in the Diamond–Mortensen–Pissarides (DMP) labor search model. Risk-neutral firms match with risk-averse workers in a frictional labor market and produce output as a function of idiosyncratic and aggregate productivity and worker effort. Firms observe aggregate productivity but cannot distinguish between idiosyncratic productivity and effort. Therefore, firms propose *flexible incentive pay* to overcome the resulting moral hazard problem by conditioning wages on output to balance the desire to insure the worker with a need to incentivize effort. We also allow the worker's utility at the start of the contract to be higher during expansions due to bargaining or cyclicality in the outside option.

Our model allows for cyclical and flexible incentive pay, consistent with micro evidence. If the marginal product of effort falls during recessions, then firms find effort less valuable and lower expected wages. By contrast, standard labor search models without moral hazard (e.g., Shimer, 2005) attribute all wage cyclicality to bargaining. Additionally, our dynamic environment recognizes that employment is a long-term relationship (Barro, 1977).

Our first analytical result is that wage cyclicality due to incentives does *not* dampen the first-order response of unemployment to shocks. We study a version of the flexible incentive pay economy without bargaining or cyclical outside options, in which all fluctuations in wages are due to incentives. Then we prove an equivalence result: the impulse response of market tightness to aggregate productivity shocks is the same in the flexible incentive pay economy without bargaining as in the economy with exogenously fixed real wages of Hall (2005) as long as both models are calibrated to the same steady-state labor share. Therefore, procyclical incentive wages do not per se mute the response of unemployment to business cycle shocks, since a model with fixed wages has the same unemployment response.

Our result may be surprising. After all, a standard argument is that flexible bonus pay dampens unemployment fluctuations by reducing marginal costs during contractions (e.g., Weitzman, 1986). The intuition behind our contrasting result relates to incentives. In our model, the response of profits to aggregate shocks determines unemployment dynamics. With flexible incentive pay, wages fall after a contraction, which dampens the response of profits—the standard marginal cost effect. However, there is a less standard *incentive effect* of wage changes. If wages fall, then workers may have weaker incentives and lower their effort, which amplifies the fall in profits and offsets the marginal cost effect. For the optimal incentive contract, the incentive and marginal cost effects of wage changes on profits cancel out exactly, due to an envelope theorem. Therefore, profits in the flexible incentive pay economy behave *as if* neither wages nor effort had responded to the aggregate shock.

Our second analytical result shows that wage cyclicality due to bargaining or outside options dampens the impulse response of unemployment, as in standard labor search models without incentives. Reintroducing bargaining into the flexible incentive pay model, we show that only the portion of wage fluctuations associated with changes in worker utility at the start of the contract, which we dub "bargained wage cyclicality," affects the response of unemployment to business cycle shocks. Intuitively, wage cyclicality due to bargaining or outside options dampens unemployment fluctuations for a standard reason: lower bargained wages during a contraction do not imply an offsetting fall in effort and thus stabilize profits.

These results are fairly general: they apply for utility functions with general forms and for idiosyncratic shock processes with arbitrary persistence.[1] This generality is surprising since dynamic incentive contracts are often hard to characterize outside special cases (e.g., Holmstrom and Milgrom, 1987). We sidestep this difficulty by characterizing the dynamics of profits *without* characterizing the optimal contract, using a suitable envelope theorem from the applied mathematics literature on sensitivity analysis (Bonnans and Shapiro, 2000).

Our analytical results characterize the impulse response of market tightness—and thus unemployment—to exogenous productivity shocks. This statistic matters for a variety of

[1] We also establish a similar result in a richer environment with endogenous separations (Mortensen and Pissarides, 1994) and limited worker commitment. Our incentive model also nests incentive contracts such as tournaments.

reasons. For instance, the impulse response of unemployment is a key determinant of price inflation dynamics in New Keynesian models with frictional labor markets. Indeed, we establish in an appendix that the slope of the Phillips curve in the New Keynesian search model of Christiano et al. (2016) is the same with flexible incentive pay or with rigid wages as in Hall (2005).

Our analytical results imply that researchers should estimate the portion of wage cyclicality that is due to bargaining or outside option cyclicality and filter out wage cyclicality due to incentives. The final part of this paper pursues this goal. We calibrate a version of our model to match micro moments of wage adjustment, such as the variance of incumbent wage growth and the pass-through of idiosyncratic profitability shocks—both of which inform the strength of incentives—as well as new hire wage cyclicality, which informs the cyclicality of workers' outside options and their bargaining power. The calibrated model generates significant unemployment volatility consistent with the time series.

Our third result is numerical: We find that bargained wage cyclicality accounts for approximately 54% of overall wage cyclicality. Therefore, the response of unemployment to business cycle shocks is large in the calibrated model even though wages are relatively procyclical. We also show how to calibrate a simple version of our model with bargaining but without incentives, similar to standard labor search models. To generate the same unemployment impulse response as that under the full model, the model must be calibrated for only bargained wage cyclicality—i.e., 54% of the overall wage cyclicality in the data, a number such as -0.54.

Taken together, our three results suggest that researchers studying the impulse responses of unemployment may work with simple and standard models without dynamic incentive contracts as long as these models are calibrated to match only bargained wage cyclicality. Our numerical approach suggests that these simple models should target wage cyclicality that is weakly procyclical, compared to measures of overall wage cyclicality in the data. However, we stress that our numerical results are tentative and urge those undertaking future empirical work to distinguish wage cyclicality that is due to bargaining versus incentives.

Let us make three caveats. First, our mechanism depends on effort and wages positively comoving over the business cycle, consistent with time series evidence.[2] However, cyclical fluctuations in effort are hard to measure. Second, we do not consider on-the-job search, in which case incentive pay may also affect recruitment and retention (e.g., Balke and Lamadon,

[2]For instance, diverse measures of worker effort—from time use surveys, variable capacity utilization, and information on workplace injuries—all seem to fall during recessions (Burda, Genadek and Hamermesh, 2020; Fernald, 2014; Galí and Van Rens, 2021). Furthermore, the pass-through of idiosyncratic firm shocks to wages is procyclical (Chan, Salgado and Xu, 2023), consistent with firms seeking to incentivize more effort during booms.

2022; Elsby, Gottfries, Krolikowski and Solon, 2023). We leave this aspect to future research. Finally, our result concerns the impulse response of unemployment to business cycle shocks. However, the incentive pay model yields impulse responses of labor productivity that differ from those under simpler models due to effort, evoking a notion of capacity utilization related to classic theories of labor hoarding (e.g., Burnside, Eichenbaum and Rebelo, 1993).

Related literature. A large literature has developed models that are consistent with the micro-evidence on state-dependent price setting but tractable enough to study aggregate rigidity, in part via analytical equivalence results to simpler time-dependent models (e.g., Alvarez, Le Bihan and Lippi, 2016; Auclert, Rigato, Rognlie and Straub, 2022). In parallel, other papers try to isolate which micro moments on price setting are most relevant for aggregate price rigidity, concluding, for instance, that sales are irrelevant (e.g., Kehoe and Midrigan, 2008; Eichenbaum, Jaimovich and Rebelo, 2011). We aim to provide a model that is consistent with the micro-evidence on *wage* setting and incentive pay but that remains analytically tractable via an equivalence to simpler models with rigid wages. By doing so, we can isolate which micro moments on wage setting are relevant for the economy's response to shocks—that is, wage changes related to bargaining rather than incentives.[3]

The literature on wage setting finds that measures of wages that plausibly relate to incentives—such as annual earnings per hour or bonus pay—often seem more flexible whereas measures of pay excluding incentives, such as base pay, tend to be rigid. This result seems true not only for job-stayers' wages (e.g., Solon, Whatley and Stevens, 1997) but also for new hires' wages. For instance, studying base wages for new hires from online vacancy postings and from administrative payroll data, both of which contain detailed job-level information, Hazell and Taska (2022) and Grigsby et al. (2021) find limited procyclicality of nominal and real wages. Studying wages for new hires from survey data that do not separately report non–base pay, papers such as Bils, Kudlyak and Lins (2022a) find procyclical real wages.[4,5] A model is needed to determine the relevant notion of wage cyclicality for unemployment dynamics in the presence of incentive pay. Our contribution is to provide such a model—which can be calibrated to microdata—to clarify that wage cyclicality arising from incentives does not mute the response of unemployment to business cycle shocks.

Our paper also contributes to the large literature relating wage rigidity to unemployment dynamics (e.g., Fukui, 2020; Blanco, Drenik, Moser and Zaratiegui, 2022). Many papers

[3]The literature on nominal price rigidity finds that sales do not matter for aggregate rigidity because they are transient, staggered and acyclical (Nakamura and Steinsson, 2013). We find that incentive pay does not matter for aggregate rigidity even if incentive wage changes are persistent, simultaneous and cyclical.

[4]See Kudlyak (2014), Basu and House (2016), Doniger (2019) and Bellou and Kaymak (2021) for related papers on the cyclicality of the wage for new hires.

[5]Grigsby et al. (2021), studying a time period and dataset different from those in Bils et al. (2022a), also find that bonus wages are cut frequently but are not cyclical.

study wage setting with exogenous and fixed effort by workers and find that wage rigidity leads to large unemployment fluctuations while flexible wages dampen these fluctuations.[6] Our contribution is to study wage setting with endogenous and variable effort via flexible incentive pay contracts. We show that highly procyclical unemployment can coexist with flexible and cyclical wages as long as incentives determine wage cyclicality.

Several papers consider unemployment dynamics with effort. First, Moen and Rosén (2011) and Zhou (2022) consider elegant models with incentive contracts and wage posting, finding numerically that incentives amplify unemployment fluctuations. Second, Fongoni (2020) considers a labor search model in which wages affect effort due to exogenous reference-dependent preferences and astutely notes that the response of effort to wage changes amplifies business cycle shocks. Our contribution is to offer a model with dynamic incentive contracts and bargaining power, which allows a tight mapping to the micro evidence. Moreover, our approach lets us connect our model to simple models with wage rigidity and to illustrate an envelope result that explains the amplified fluctuations in unemployment.

A third related paper is Bils, Chang and Kim (2022b), which shows that large employment fluctuations can exist despite flexible new hire wages if incumbent workers' wages are rigid and effort is contractible and determined by Nash bargaining. Our setting is different: we study a canonical model of dynamic incentive pay with noncontractible effort, which allows us to draw a sharp distinction between incentive and bargained wage cyclicality.

Finally, our paper builds on the literature studying moral hazard and its macroeconomic implications (e.g., Holmstrom and Milgrom, 1987; Sannikov, 2008; Doligalski, Ndiaye and Werquin, 2023).[7] These optimal contracting problems are challenging because the firm must maximize expected profits among a hard-to-characterize continuum of incentive-compatible contracts. We contribute to this literature in four ways. First, we analytically study the business cycle implications of moral hazard frictions. Second, we introduce an extensive margin of unemployment and bargaining over the promised utility of the contract. Third, we derive our main result without relying on an explicit form of the optimal contract by applying an envelope theorem to the principal's objective—therefore, our results apply without functional form assumptions.

Plan. Section 2 presents a static model similar to Holmstrom's (1979) that provides

[6] An incomplete list of papers from this vast literature includes Shimer (2005); Hall and Milgrom (2008); Gertler and Trigari (2009); Elsby (2009); Rudanko (2009); Brügemann and Moscarini (2010); Kennan (2010); Gertler, Huckfeldt and Trigari (2020) and Elsby and Gottfries (2022). Some papers within this literature study implicit contracts, in which firms insure workers against wage risk with exogenous effort (e.g., Azariadis, 1975; Beaudry and Dinardo, 1991; Krusell, Mukoyama and Şahin, 2010).

[7] Doligalski et al. (2023) show that incentive pay changes the redistributive effects of taxation. Li and Williams (2015) and Veracierto (2022) study optimal unemployment insurance contracts with moral hazard and aggregate risk.

intuition for the role of incentive effects and the irrelevance of incentive wage cyclicality for unemployment dynamics. Section 3 develops the dynamic labor search model with general dynamic incentive contracts. Section 4 provides numerical results on the share of wage cyclicality attributable to incentives versus bargaining. Section 5 concludes.

2 Illustrative Static Model

This section explains our results in an illustrative framework that combines a static Diamond–Mortensen–Pissarides (DMP) labor search model with two alternative models of wage setting. The first model features a static incentive contract as in Holmstrom (1979) resulting in procyclical and flexible wages. The second model has exogenously rigid wages and effort as in Hall (2005). We show that wage cyclicality due to incentives does not mute the response of unemployment to productivity shocks whereas wage cyclicality due to bargaining does dampen this response.

2.1 Environment of the Static Model

Frictional labor markets. There is a unit measure of workers who begin the period unemployed. Workers randomly search for vacancies in a frictional labor market. Workers end the period employed if they match with a vacancy and otherwise end the period unemployed. There is a continuum of risk-neutral firms. Firms can post vacancies at a cost κ per vacancy. θ is the measure of vacancies posted. Since a unit measure of workers is unemployed at the start of the period, θ is also market tightness—the ratio of vacancies to unemployed workers. Given search frictions, the probability that an individual vacancy matches with a worker is $q(\theta) \equiv A\theta^{-\nu}$, a decreasing and isoelastic function of the measure of vacancies posted.

Technology. If a firm and worker match, they produce the numeraire good with a production function $y(a, \eta, z) = z(a + \eta)$. Here, z is an exogenous aggregate productivity term that affects all firms, a is the effort of the employed worker, and η is an exogenous idiosyncratic shock to production. We term η "noise." We consider one-worker firms.[8]

Workers. Workers have risk-averse preferences over consumption c and labor effort a, given by a utility function $u(c, a)$ that is strictly increasing and strictly concave in c but weakly decreasing and concave in a. If workers end the period unemployed, they consume unemployment benefits $b(z)$ and exert no effort. They thus attain utility $U(z) \equiv u(b(z), 0)$. If employed, the worker exerts effort and is paid a wage w, which she consumes.

[8] One could consider tournaments by interpreting η as a worker's performance relative to her peers.

Information. Aggregate productivity z is common knowledge. Firms are able to observe their workers' output; however, they do not observe effort a and noise η separately. Workers choose effort before the noise η is realized. Thus, firms' expected profits from a filled vacancy are $J(z) \equiv \mathbb{E}_\eta[z(a+\eta) - w]$, where the expectation is over values of η.

Free entry. Free entry requires that the expected profits from posting a vacancy equal the cost of posting the vacancy, which implies

$$\kappa = q(\theta)J(z). \tag{1}$$

Now, we introduce two models of wage and effort setting.

Flexible incentive pay economy of Holmstrom (1979). When a firm and worker match, the firm offers the worker a contract that specifies a suggested effort level $a(z)$ and wages as a function of output realizations $w(z, y)$. Crucially, the firm cannot condition wages directly on effort, which is unobservable, leading to a moral hazard problem. Therefore, the firm maximizes profits subject to an incentive compatibility constraint (IC) and a participation constraint (PC). The IC requires that the suggested effort level be an optimal choice for the worker given the wage contract offered by the firm. The PC requires that the worker's expected utility at the start of the contract, which we often refer to as "ex ante utility," be at least $\mathcal{B}(z)$. $\mathcal{B}(z)$ is a function mapping the aggregate state z to the worker's ex ante utility and captures bargaining and outside option cyclicality in reduced form. For instance, if the firm makes take-it-or-leave-it (TIOLI) offers and workers' outside option is acyclical $b(z) = b$, then the worker's ex ante utility is the value of the unemployment benefit, so $\mathcal{B} = U \equiv u(b, 0)$. If there is Nash bargaining over the output of a match, then $\mathcal{B}(z)$ will be an increasing function of z.[9]

The firm's problem after meeting a worker is

$$J^{\text{Incentive}}(z) \equiv \max_{a(z), w(z,y)} \mathbb{E}_\eta[z(a(z)+\eta) - w(z,y)] \tag{2}$$

$$\text{subject to} \qquad a(z) \in \arg\max_{\tilde{a}(z)} \mathbb{E}_\eta\left[u(w(z,y), \tilde{a}(z))\right] \qquad [IC]$$

$$\mathbb{E}_\eta\left[u(w(z,y), a(z))\right] \geq \mathcal{B}(z). \qquad [PC]$$

Our notation makes explicit that effort and wages may depend on realizations of both z and y (and thus the idiosyncratic component of output $a+\eta$) but that the firm is uncertain over the realized value of η. Let $a^*(z)$ and $w^*(z, y)$ denote the contracted effort and wage levels as a function of productivity and output realizations.[10]

[9] We formally prove this claim in the dynamic version of our model in Section 3.

[10] Though the mapping is not exact, one can informally think of a bonus as the component of wages

As usual, this contract implies a tradeoff between incentives and insurance. Absent moral hazard, firms would fully insure workers against wage risk. With moral hazard, firms pass idiosyncratic noise shocks through to workers' wages to provide incentives. This simple and standard model allows flexible pay, since the firm can freely adjust wages subject to the IC and PC without further restrictions. The firm can freely vary wages with z, potentially leading to procyclical wages.

Rigid wage economy of Hall (2005). In this benchmark model, wages and effort are exogenously fixed at $\bar{a}$ and $\bar{w}$, irrespective of z. Let J^{Rigid} be the value of a filled vacancy in this economy. There are no nominal frictions, and we study real wage rigidity.

2.2 The Role of Incentives in Employment Dynamics

First, note that the impulse response of labor market tightness to productivity shocks depends on the dynamics of profits, as is standard in DMP search models with free entry. To see this point, totally differentiate the free entry condition (1) with respect to log aggregate productivity $\ln z$ and rearrange to obtain

$$\frac{d\ln\theta}{d\ln z} = \frac{1}{\nu} \cdot \frac{d\ln J}{d\ln z}. \tag{3}$$

That is, the elasticity of market tightness with respect to aggregate productivity z is proportional to the elasticity of expected profits per worker to z, where the constant of proportionality depends on the elasticity of vacancy filling rates with respect to vacancies. Moreover, the employment rate n is determined by the job finding rate $f(\theta)$, which is proportionate to vacancies and given by $f(\theta) = A\theta^{1-\nu}$. Therefore, to understand the response of employment to shocks, one can simply study the response of profits per worker to aggregate productivity.

To solve for the response of profits, we differentiate expected profits $J(z) \equiv \mathbb{E}_\eta[z(a+\eta)-w]$ with respect to z, which implies

$$\frac{dJ(z)}{dz} = \overbrace{\mathbb{E}_\eta\left[a\right]}^{\substack{\text{direct}\\\text{productivity}}} \overbrace{- \mathbb{E}_\eta\left[\frac{dw}{dz}\right]}^{\substack{\text{marginal}\\\text{costs}}} + \overbrace{z\mathbb{E}_\eta\left[\frac{da}{dz}\right]}^{\text{incentives}}. \tag{4}$$

The first-order response of profits to aggregate productivity may be decomposed into three terms. The first is the direct productivity effect: production rises with productivity, *ceteris paribus*. The second is the marginal cost effect: when productivity rises, wages may also

associated with incentives, whereas base pay is the component of wages associated with bargaining. For instance, base pay may be the wage payment under the lowest possible realization of η, which moves with ex ante utility, whereas bonuses may be wage payments above that lowest level.

increase, which lowers profits, all else equal. The third effect is an incentive effect: effort may respond to aggregate productivity shocks. The direct productivity and marginal cost effects are common in DMP search models. If wages are procyclical and dw/dz is large, then profits and employment may respond little to productivity shocks (Shimer, 2005).

The incentive effect is less standard. In particular, if effort increases with exogenous productivity, then profit responses may be large even if expected wages are procyclical. Thus, procyclical incentives might offset the effect of wages on profits, leading to large employment responses despite the procyclicality of wages. Wage cyclicality dampens the response of unemployment to productivity shocks only insofar as wages move *more* than effort.

The point of this subsection—that incentives matter for employment dynamics—does not depend on a specific model of wage or effort setting. Equation (4) remains true regardless of the contracting environment or of whether contracts are set optimally. Different models merely imply a different direct productivity, marginal cost, and/or incentive effect. Next, we endogenize a and w in the flexible incentive pay economy of Holmstrom (1979) and rigid wage economy of Hall (2005) to gauge the incentive and marginal cost effects in each model.

Incentive wage cyclicality and unemployment dynamics. Now we derive our first key result: that wage cyclicality due to incentives does not dampen unemployment dynamics. To a first order, the response of employment to productivity shocks is the same in a flexible incentive pay economy without fluctuations in bargaining power or outside options as in an appropriately calibrated rigid wage economy—even if incentive pay is highly procyclical.

First, consider the response of profits to z in the rigid wage economy. Here, both the marginal cost and incentive effects of the wage in equation (4) are trivially zero because neither effort nor wages respond to z. Therefore, the response of profits to productivity is just the direct productivity effect: $dJ^{\text{Rigid}}(z)/dz = \bar{a}$.

Second, consider a special case of the flexible incentive pay economy in which $\mathcal{B}(z)$ is constant so that the worker's ex ante utility from employment is constant. This economy is a natural benchmark in which all wage cyclicality is due to incentives because there is no bargaining and outside options are constant. Differentiating profits in the incentive pay economy (Equation 2) and applying the classic envelope theorem of Milgrom and Segal (2002), we see that $dJ^{\text{Incentive}}/dz = a^*(z)$. Only the direct productivity effect affects the response of profits to productivity shocks z, exactly as in the rigid wage economy.

This result holds because the marginal cost and incentive effects are equal-sized under the optimal contract so that their effects on profits cancel out, leaving only the direct productivity effect. Although wages and effort may adjust, these fluctuations do not affect the profit of a firm that is optimally choosing effort and wages. The equivalence holds even if wages are procyclical under the optimal contract so that dw/dz is large.

To gain intuition, suppose that an increase in z leads the firm to encourage higher effort. All else equal, higher effort raises profits. To encourage the worker to provide higher effort, the firm raises the pass-through of idiosyncratic output into wages. The worker then faces more risk, for which she must be compensated with higher average wages. Ultimately, wages are procyclical and flexible. All else equal, higher wages lower expected profits.

The effects of higher effort and higher wages on profits, however, exactly cancel each other out. The reason is that under the optimal incentive contract, the firm is indifferent at the margin between increasing expected wages and increasing worker effort. Changes in effort and wages induced by a small change in z have exactly offsetting effects on expected profits. Expected profits respond to productivity shocks as if neither wages nor effort had changed, just as in the rigid wage economy. The response of profits—and thus market tightness—is the same in the rigid wage and flexible incentive pay economies as long as both economies are calibrated to have the same direct productivity effect ($\bar{a} = a^*$). This is the sense in which procyclical and flexible incentive wages do not dampen unemployment dynamics.

A numerical example illustrates this equivalence. Figure 1 plots the behavior of the rigid wage economy (blue line) and the flexible incentive pay economy (red line). Both economies are calibrated to have the same expected wage and effort (and thus profits and employment) when $z = 1$.[11] The horizontal axis of each plot represents exogenous labor productivity z, while the vertical axis plots model-implied employment, expected wages, or effort.

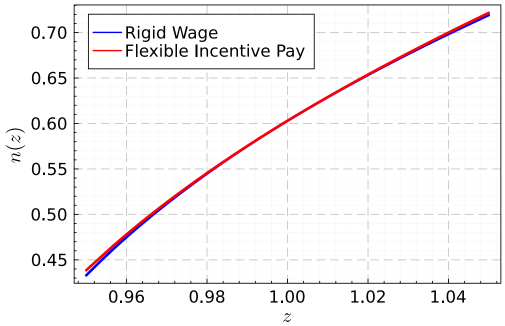
Panel A shows the equivalence of the employment dynamics: the rigid wage and flexible incentive pay economies generate identical responses to aggregate labor productivity z in the neighborhood of $z = 1$. The two models also generate nearly identical employment movements in response to 5% fluctuations in aggregate productivity. This result illustrates the envelope theorem in practice: profit dynamics depend only on the direct productivity effect, which is locally the same in both economies under our calibration.

Panel B shows that wages are procyclical in the incentive pay economy. Therefore, the employment dynamics are the same even though marginal costs fall significantly during contractions in the incentive pay economy. Panel C shows the countervailing force: effort also responds strongly to z in the incentive pay economy. Therefore, incentives offset the stabilizing effect of marginal costs on profits. Hence, in the incentive pay economy, large employment responses coexist with procyclical wages.[12]

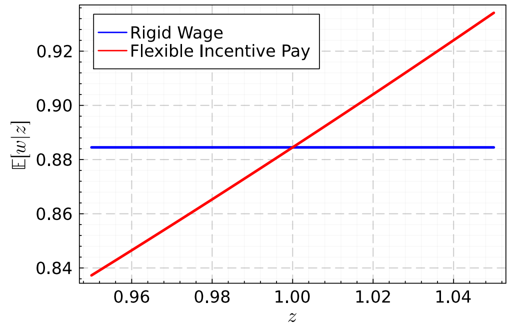
[11] For this illustration, we assume that workers have exponential preferences $u(c, a) = -\exp(-r(c - \frac{a^2}{2}))$. The unemployment benefit b is calibrated to be 0.4, η is assumed to be normally distributed with mean 0 and standard deviation 0.2, and the parameter governing risk aversion r is 0.8. For simplicity, following Holmstrom and Milgrom (1987), we solve for the optimal linear (in output) contract.

[12] We assume that a and z are complements, which makes both wages and effort procyclical in the op-

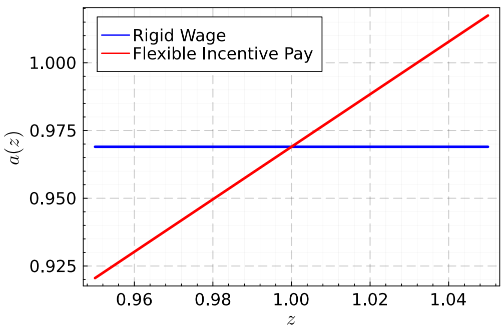
Figure 1: Employment, wage and effort fluctuations in the static model



Panel A: Employment $n(z)$



Panel B: Expected Wages $\mathbb{E}[w|z]$



Panel C: Effort $a(z)$

Notes: These figures plot the level of employment (Panel A), expected wages (Panel B), and effort (Panel C) as a function of aggregate productivity z in the static model. The red line plots these functions for the flexible incentive pay economy. The blue line plots these functions for the rigid wage economy, calibrated to have the same wage and effort as the flexible incentive pay economy for $z = 1$.

2.3 Bargained Wage Cyclicality and Unemployment Dynamics

We now explain our second analytical result: that wage cyclicality due to bargaining or cyclical outside options dampens unemployment dynamics, as in standard labor search models without incentives. To make this point, we augment the flexible incentive pay economy by allowing $\mathcal{B}(z)$ to vary with z. Therefore, the worker's ex ante utility from employment varies with aggregate shocks because of changes in her bargaining power or the cyclicality of outside options. Differentiating the Lagrangian associated with problem (2) implies that the response of profits to aggregate productivity is

$$\frac{dJ^{\text{Incentive}}}{dz} = a^*(z) - \mu^*(z)\mathcal{B}'(z), \tag{5}$$

where $\mu^*(z)$ is the Lagrange multiplier on the participation constraint at the optimum.

timal incentive contract. Without complementarity, wages and effort could be counter- or acyclical, but employment would still have the same response in the rigid wage and flexible incentive pay economies.

Equation (5) shows that bargaining power stabilizes profits. With bargaining or a cyclical outside option, it is not only the direct effect of productivity on profits, $a^*(z)$, that matters. There is an additional term $\mu^*(z)\mathcal{B}'(z)$ capturing fluctuations in ex ante utility from employment. By comparing equations (4) and (5), we can rewrite the promised utility term as

$$\mu^*(z)\mathcal{B}'(z) = \mathbb{E}_\eta\left[\frac{dw^*}{dz} - z\frac{da^*}{dz}\right]. \tag{6}$$

Thus, we term $\mu^*(z)\mathcal{B}'(z)$ "bargained wage cyclicality" (BWC). This is equal to expected wage cyclicality, dw^*/dz, in excess of movements in production due to incentivized effort, da^*/dz. BWC is different from zero if and only if ex ante utility is cyclical, so $\mathcal{B}'(z) \neq 0$. Only the bargaining-related component of wage cyclicality dampens profit dynamics. Intuitively, an increase in wages associated with higher ex ante utility does not require workers to offer higher effort. Therefore, the increase in wages reduces profits, all else equal.

This result is consistent with the standard DMP model with exogenous effort (e.g., Shimer, 2005). In our model, as in the standard model, wage cyclicality associated with bargaining power dampens the impulse response of profits. However, in the standard model, all wage fluctuations are due to bargaining since $da^*/dz = 0$ by assumption. Thus, wage cyclicality always dampens profit and unemployment dynamics. By contrast, wage cyclicality does not necessarily dampen profit dynamics in our flexible incentive pay model since the wage cyclicality may be due to incentives.

We stress that the model establishes equivalence for the response only of employment to exogenous labor productivity shocks, which will appear as an impulse response in the dynamic model to come. The response of output may be different with rigid wages than with incentive pay. In the incentive pay economy, output per worker varies endogenously because of endogenous effort, whereas output per worker is exogenous in the rigid wage economy. With flexible incentive pay, the endogenous component of output per worker is procyclical when wages are procyclical, evoking a notion of variable capacity utilization.

Taking the two analytical results together, we have seen that wage cyclicality arising from incentives does not mute unemployment dynamics but wage cyclicality arising from bargaining does. In Appendix A.6, we recapitulate these arguments with an explicit functional form for the contract, using the framework of Edmans and Gabaix (2011). The next section proves the results in a rich dynamic environment.

3 A Dynamic Model of Incentive Pay with Bargaining

This section studies a dynamic labor search model with long-term incentive contracts permitting, for instance, persistent idiosyncratic shocks and nonseparable utility. The dynamic model recognizes that labor contracts are long-term relationships and that incentives are dynamic (e.g., Barro, 1977; Sannikov, 2008). We confirm that our main analytical results hold in this setting: wage cyclicality due to incentives does not dampen the response of unemployment to shocks, while wage cyclicality due to bargaining does.

3.1 Economic Environment

Labor market. The labor market follows the standard Diamond–Mortensen–Pissarides model. Time is discrete. A large measure of risk-neutral firms matches with workers and produces output. A unit mass of workers is either employed or unemployed and searching for a job. Let n_t denote the measure of employed workers at the start of period t, while $u_t \equiv 1-n_t$ is the measure of unemployed workers looking for jobs. Fluctuations in labor market variables are driven by technology, which follows a first-order Markov process $\{z_t\}_{t=0}^{\infty}$ with lower and upper bounds $\underline{z}$ and $\overline{z}$. Denote the history of this process until t by $z^t = \{z_0, ..., z_t\}$, and denote the marginal distribution of z^t by $\hat{\pi}_t(z^t|z_0)$.

Firms post vacancies v_t to recruit unemployed workers. The number of matches made in period t is given by a constant-returns-to-scale matching function $m\left(u_t, v_t\right)$; labor market conditions are summarized by market tightness $\theta_t = v_t/u_t$, with a job finding rate $\phi(\theta_t) = m\left(u_t, v_t\right)/u_t$ and a vacancy filling rate $q_t \equiv q(\theta_t) = m(u_t, v_t)/v_t$. Let $\nu_t \equiv -d\ln q_t/d\ln\theta_t$ denote the period t elasticity of the job-filling rate with respect to θ_t. Maintaining a vacancy has a per-period cost κ.

At the end of period $t-1$, an exogenous fraction s of workers separate from employment and enter unemployment. The unemployed search for new jobs, so u_t evolves as

$$u_t = u_{t-1} + s(1-u_{t-1}) - \phi(\theta_{t-1})u_{t-1}(1-s). \tag{7}$$

Preferences and consumption. Workers have time-separable risk-averse preferences over consumption $c_t \in [\underline{c}, \overline{c}]$ and effort $a_t \in [\underline{a}, \overline{a}]$ and discount future payoffs by a factor $\beta \in (0,1)$. Preferences are summarized by $u\left(c, a\right)$, where u is strictly increasing and strictly concave in c,, strictly decreasing and strictly concave in a, and Lipschitz continuous.

Employed workers consume their wage in each period, with newly hired workers producing output and receiving a wage in the period in which they are hired. Workers not hired in the current period exert no effort and are paid unemployment benefits $b\left(z_t\right)$, a differentiable function of the aggregate state, receiving flow payoff $\xi(z_t) \equiv u(b(z_t), 0)$.

Therefore, the value of an unemployed worker at the start of period t is

$$U(z_t) = \phi(\theta_t)\,\mathcal{E}(z_t) + (1 - \phi(\theta_t))\,(\xi(z_t) + \beta\mathbb{E}[U(z_{t+1})\,|z_t]), \tag{8}$$

where $\mathcal{E}(z)$ is the worker's value if she begins employment when aggregate productivity is z.

Firms and vacancy posting. Firms are risk neutral and maximize expected profit with discount factor β. Firms operate a production technology that is constant returns to scale in the number of employees; therefore, we consider one-worker firms without loss of generality. Consider a firm i that successfully matches with a worker at time 0 and starts producing in the same period. The firm's output in period t is $y_{it} = f(z_t, \eta_{it})$, where f is strictly increasing and continuously differentiable in its arguments and η_{it} is an idiosyncratic shock to the firm's output that is independently distributed across firms. Henceforth, we omit i subscripts to ease notation.

At the beginning of the period, before the current value of η_t is realized, the worker exerts effort a_t that affects the distribution of idiosyncratic shocks. We assume a general process for η_t, which allows for arbitrary persistence and depends on the worker's effort. The process has lower and upper bounds $\underline{\eta}$ and $\overline{\eta}$, respectively. Define a history of idiosyncratic shocks $\eta^t = \{\eta_0, ..., \eta_t\}$. We characterize the process for η_t by a probability measure $\pi_t(\eta_t|\eta^{t-1}, a^t)$, which gives the probability of realizing η_t given the history η^{t-1} of past idiosyncratic shocks and the worker's history of actions $a^t = \{a_0, \ldots, a_t\}$. Thus, workers' effort affects output by shifting the distribution of η realizations.

Vacancies may be freely posted at cost κ. Let $J(z_0)$ be the firm's value if it matches with a worker in some initial period $t = 0$ when aggregate productivity is z_0; the value for a firm of posting a vacancy at time 0 is then

$$\Pi_0(z_0) = q(\theta_0)J(z_0) - \kappa. \tag{9}$$

Free entry into vacancy posting guarantees that this value is zero in equilibrium. We entertain two possibilities for wage setting.

Flexible incentive pay economy. In this economy, wages are set according to a dynamic incentive contract. The firm observes the initial value of z_0 and will later observe all realizations of aggregate shocks $\{z_t\}_{t=0}^{\infty}$. Firms additionally observe idiosyncratic shocks η_t in every period of the match. However, they do not observe workers' effort a_t. They thus cannot observe whether output is high because the worker exerted high effort or received a lucky idiosyncratic shock, a classic moral hazard problem.

When a firm and a worker meet, the firm offers the worker a contract to incentivize effort and maximize firm value. A contract specifies a wage function mapping idiosyncratic shocks and aggregate productivity to realized wages. The contract does not condition on the worker's effort, which is unobservable to the firm, but "recommends" a level of effort given the history of aggregate and idiosyncratic shocks. The worker chooses effort before the realization of the idiosyncratic shock to firm output.[13]

Thus, the contract may be summarized by functions $w_t(\eta^t, z^t) \in [\underline{w}, \bar{w}]$ and $a_t(\eta^{t-1}, z^t) \in [\underline{a}, \bar{a}]$ for all t and all realizations of η^t and z^t. Let $(\mathbf{w}, \mathbf{a})$ denote a contract, with $\mathbf{w} \equiv \{w_t(\eta^t, z^t)\}_{t=0,\eta^t,z^t}^{\infty}$ and $\mathbf{a} \equiv \{a_t(\eta^{t-1}, z^t)\}_{t=0,\eta^{t-1},z^t}^{\infty}$, so that the contract is dynamic and state contingent. Let $\mathcal{X}$ denote the space of possible contracts.

Value of a filled vacancy. Under the contract $(\mathbf{w}, \mathbf{a})$ and at initial productivity z_0, the firm's expected present value of profits from a filled vacancy is

$$V(\mathbf{w}, \mathbf{a}; z_0) = \sum_{t=0}^{\infty} (\beta (1-s))^t \int\int \left(f(z_t, \eta_t) - w_t(\eta^t, z^t)\right) \tilde{\pi}_t \left(\eta^t, z^t | z_0, \mathbf{a}\right) d\eta^t dz^t, \tag{10}$$

where $\tilde{\pi}_t(\eta^t, z^t|\mathbf{a}) \equiv \prod_{\tau=0}^{t} \pi_\tau \left(\eta_\tau | \eta^{\tau-1}, a^\tau(\eta^{\tau-1}, z^\tau)\right) \hat{\pi}_\tau(z^\tau|z_0)$ is the probability of observing a realization of η^t and z^t given the initial z_0 and the contracted effort function $\mathbf{a}$ and $a^\tau(\eta^{\tau-1}, z^\tau)$ is the sequence of effort from periods 0 to τ.

Therefore, firms' period profits are the difference between output and wages. The firm forms an expectation over profit realizations by integrating over the distribution of both aggregate and idiosyncratic shocks, the latter of which depend on effort. The risk-neutral firm discounts period t profits by the economy-wide discount rate β^t and the probability $(1-s)^t$ that the match survives t periods.

The contract maximizes the value of a filled vacancy

$$J(z_0) = \max_{\{w_t(\eta^t,z^t), a_t(\eta^{t-1},z^t)\}_{t=0,\eta^t,z^t}^{\infty} \in \mathcal{X}} V(\mathbf{w}, \mathbf{a}; z_0) \tag{11}$$

subject to the incentive and participation constraints (IC and PC) described below.

Incentive constraints. The worker chooses effort $\tilde{\mathbf{a}} \equiv \{\tilde{a}_t(\eta^{t-1}, z^t)\}_{t=0,\eta^{t-1},z^t}^{\infty}$ to maximize utility under the contract. Therefore, the effort suggested under the contract by the firm must be incentive compatible; that is, the recommended effort $\mathbf{a}$ must be what is chosen

[13] An alternative notation has effort directly affect production, while the firm cannot distinguish effort from η_t. A second alternative notation has contracts mapping from idiosyncratic *output* and aggregate productivity to wages.

by the worker, given the wage contract that the firm offers her. Specifically:

$$[\mathbf{IC}] : \mathbf{a} \in \underset{\{\tilde{a}_t(\eta^{t-1},z^t)\}_{t=0,\eta^t,z^t}^{\infty}}{\operatorname{argmax}} \sum_{t=0}^{\infty} (\beta(1-s))^t \Bigg[\int\int u\Big(w_t(\eta^t,z^t), \tilde{a}_t(\eta^{t-1},z^t)\Big) \tilde{\pi}_t\left(\eta^t, z^t | z_0, \tilde{\mathbf{a}}\right) d\eta^t dz^t + \beta s \int U\left(z_{t+1}\right) \hat{\pi}_{t+1}\left(z^{t+1}|z_0\right) dz^{t+1} \Bigg]. \tag{12}$$

Equation (12) is the value of an employed worker at time 0; the IC requires that the recommended effort maximizes the worker's value given the wage contract offered by the firm. The worker discounts period t payoffs by β^t. Their value is the sum of two terms. The first is their value conditional on the match surviving through period t, which occurs with probability $(1-s)^t$. The realized flow payoff to the worker under the contract is her utility from consuming the wage offered by the contract and providing effort, which depends on realizations of aggregate productivity z^t and idiosyncratic productivity η^t. Workers' expected utility integrates over the distribution of aggregate and idiosyncratic productivity shocks. When making their effort choice, workers trade off the disutility from higher effort with the increased probability of realizing a high output draw and, thus, a high wage. The second term of the worker's value is the value conditional on separation. If the contract separates in period t, the worker receives the value of unemployment at the prevailing aggregate productivity z_t. The match separates in period t with probability $(1-s)^{t-1}s$.

Participation constraint. The second constraint on problem (11) is that the contract must promise the worker a value of at least $\mathcal{E}(z_0)$, the "ex ante utility" promised by firms to workers at the start of the contract. Ex ante utility may fluctuate with z_0 due either to bargaining between a matched firm and worker or changes in workers' outside options. The constraint is

$$[\mathbf{PC}] : \sum_{t=0}^{\infty} (\beta(1-s))^t \Bigg[\int\int u\Big(w_t(\eta^t,z^t), a_t(\eta^{t-1},z^t)\Big) \tilde{\pi}_t\left(\eta^t, z^t | z_0, \mathbf{a}\right) d\eta^t dz^t + \beta s \int U\left(z_{t+1}\right) \hat{\pi}_{t+1}\left(z^{t+1}|z_0\right) dz^{t+1} \Bigg] \geq \mathcal{E}\left(z_0\right). \tag{13}$$

The left-hand side of inequality (13) is the worker's value under the contract: it is the objective function in equation (12) evaluated at the effort choices suggested by the contract.

Bargaining and ex ante utility. To close the flexible incentive pay economy, we must determine the ex ante utility $\mathcal{E}(z_0)$, which we assume is given by a reduced-form "bargaining schedule" $\mathcal{B}(z_0)$.[14] Firms commit to providing workers with a utility $\mathcal{B}(z_0)$ over the life of the contract. Common bargaining protocols in the labor search literature

[14] See Blanchard and Galí (2010) and Michaillat (2012) for this approach in search models without effort.

implicitly define different functions for $\mathcal{B}(z_0)$. For instance, if firms make take-it-or-leave-it offers to workers, the value of employment is equal to the value of nonemployment: $\mathcal{B}(z_0) = \sum_t \beta^t \mathbb{E}[\xi(z_t)|z_0]$, where $\xi(z_t)$ is the flow value of unemployment. This nests the case in which unemployment benefits or the opportunity cost of unemployment are cyclical (Hagedorn et al., 2013; Chodorow-Reich and Karabarbounis, 2016; Mitman and Rabinovich, 2019). Nash bargaining also implicitly defines an increasing function for $\mathcal{B}(z_0)$, as we prove in Appendix A.1, as do other bargaining protocols such as that in Hall and Milgrom (2008). Our formulation also evokes a notion of unemployment as a "worker discipline device" (Shapiro and Stiglitz, 1984): if the value of employment is low because unemployment at present or in the future is costly, workers will offer higher effort at lower wages.

The reduced-form approach has two advantages. First, our conclusions about the role of bargaining will be robust to a specific protocol. Second, we can tractably incorporate bargaining into dynamic incentive contract models. Its disadvantage is that $\mathcal{B}(z_0)$ is a reduced-form object, which is not invariant to changes in the primitives of the environment.

Rigid wage economy. Consider a benchmark model with rigid wages and effort following Hall (2005). Wages and effort take exogenous constant values $w_t = \bar{w}$ and $a_t = \bar{a}$ for all firms and all t, regardless of realizations of η^t or z^t. The worker's value of employment is the utility from the match and the continuation value vis-à-vis the possibility that the match may separate, which is

$$\mathcal{E}\left(z_0\right) = \sum_{t=0}^{\infty} \left(\beta\left(1-s\right)\right)^t \left(u\left(\bar{w}, \bar{a}\right) + \int \beta s U\left(z_{t+1}\right) \hat{\pi}_t\left(z^{t+1}|z_0\right) dz^{t+1}\right). \tag{14}$$

Meanwhile, the firm's value of a filled vacancy is exogenous and given by

$$J^{\text{rigid}}(z_0) = \sum_{t=0}^{\infty} (\beta(1-s))^t \int \big(f(z_t, \eta_t) - \bar{w}\big) \tilde{\pi}_t(\eta^t, z^t | z_0, \bar{\mathbf{a}}) d\eta^t dz^t. \tag{15}$$

That is, the value of a filled vacancy is given by the expected present discounted value of production minus the rigid wage, where the expectation is taken over realizations of aggregate and idiosyncratic shocks at a fixed effort $\bar{a}$ in all dates and states.

Equilibrium

Given initial unemployment u_0 and a stochastic process $\{z_t, \eta_t\}_{t=0}^{\infty}$, an equilibrium is a collection functions $\theta(z)$, $J(z), U(z), \mathcal{E}(z)$, and contract $(\mathbf{w}, \mathbf{a})$ such that, for all firms, (i) tightness θ_t satisfies the free entry condition in equation (9) so that $\Pi_t = 0$ for all t, (ii) unemployment u_t evolves according to equation (7), (iii) the wage and effort functions $(\mathbf{w}, \mathbf{a})$ solve the firm's problem (11)-(13) in the flexible incentive pay economy or $w_t = \bar{w}$ and $a_t = \bar{a}$ in the rigid wage economy, (iv) the value of unemployment $U(z)$ is given by (8), (v) the value of employment is given by equation (14) in the rigid wage economy or $\mathcal{E}(z) = \mathcal{B}(z)$ in the flexible incentive pay economy, and (vi) the value of a filled vacancy $J(z)$ or $J^{\text{rigid}}(z)$ is given by (11) in the flexible incentive pay economy or (15) in the rigid wage economy.

3.2 The Role of Incentives in Employment Fluctuations

We now study the response of employment to exogenous aggregate productivity shocks in the flexible incentive pay economy. As is standard in DMP models, employment fluctuations are determined by fluctuations in market tightness, which in turn are governed by fluctuations in firms' expected profits per worker. Therefore, it suffices to study how profits per worker $J(z_0)$ fluctuate with z_0.

To study profits, we combine the IC and PC into a functional $G(\mathbf{w}, \mathbf{a})$, defined such that $G(\mathbf{w}, \mathbf{a}) \leq 0$ holds if and only if $(\mathbf{w}, \mathbf{a})$ is a feasible contract in $\mathcal{X}$ that satisfies the IC (12) and PC (13). Let $\lambda(z_0)$ denote the co-state functional on these constraints. We write the value of a filled job using the functional Kuhn–Tucker Lagrangian:

$$J(z_0) = V(\mathbf{w}^*, \mathbf{a}^*; z_0) - \langle G(\mathbf{w}^*, \mathbf{a}^*; z_0), \lambda^* \rangle, \tag{16}$$

where the star superscripts indicate values chosen under the optimal contract offered at z_0. Then, we can decompose the response of firm profits to z_0, generalizing decomposition (2) from Section 2.[15] The response of profits to aggregate shocks in the flexible incentive pay economy is

$$\frac{dJ(z_0)}{dz_0} = \underbrace{\frac{\partial}{\partial z_0} V(\mathbf{w}^*, \mathbf{a}^*; z_0)}_{\text{(A) direct productivity effect on profits}} - \underbrace{\left\langle \frac{\partial}{\partial z_0} G(\mathbf{w}^*, \mathbf{a}^*; z_0), \lambda^*(z_0) \right\rangle}_{\text{(B) direct effect on participation and incentives}} \tag{17}$$
$$+ \underbrace{\sum_{x \in \{\mathbf{w}^*, \mathbf{a}^*\}} \left[\partial_x V(\mathbf{w}^*, \mathbf{a}^*; z_0) - \langle \partial_x G(\mathbf{w}^*, \mathbf{a}^*; z_0), \lambda^*(z_0) \rangle \right] \cdot \frac{dx}{dz_0} - \left\langle G(\mathbf{w}^*, \mathbf{a}^*; z_0), \frac{d\lambda^*(z_0)}{dz_0} \right\rangle}_{\text{(C) indirect effects on optimal contract and costates}}$$

where ∂_x represents the vector of partial derivatives with respect to some variable x. The direct productivity effect (A) measures how shocks to initial productivity affect the expected present value of output in all periods, where the expectation conditions on initial productivity

[15] The notation $\langle x, x^* \rangle$ denotes the value of the linear functional x^* at a point x. This notation is necessary because there is a continuum of constraints—see Section 3.1.1 of Golosov et al. (2016) for a formal definition of Lagrangians with this notation.

z_0 and contracted effort $\mathbf{a}^*$. This is the marginal effect of increasing z_0 on current and expected future y_t, which evaluates to

$$\frac{\partial}{\partial z_0}V(\mathbf{w}^*, \mathbf{a}^*; z_0) = \sum_{t=0}^{\infty} (\beta (1-s))^t \frac{\partial}{\partial z_0}\mathbb{E}\left[f(z_t, \eta_t)|z_0, \mathbf{a}^*\right]. \tag{18}$$

Term (B) captures the effects on the constraints. Since z_0 affects the incentive constraint only indirectly, through the contract $(\mathbf{w}, \mathbf{a})$, there is no direct effect of z_0 on incentive constraints. Thus, (B) includes only the direct effect of exogenous productivity movements on the participation constraint, which relates to bargaining power. If a higher z raises the utility that the firm must promise the worker (i.e., $\mathcal{B}'(z) > 0$), then the firm's profits from vacancy posting will rise by less, since the firm receives a combination of lower effort or higher wages when $\mathcal{B}(z)$ rises. The first-order contribution of this term to profit fluctuations is given by

$$-\lambda_{PC}^*(z_0)\left[\frac{\partial}{\partial z_0}\mathcal{B}(z_0) - \sum_{t=0}^{\infty}(\beta(1-s))^t \beta s \frac{\partial}{\partial z_0}\mathbb{E}\left[U(z_{t+1})|z_0\right]\right], \tag{19}$$

where λ_{PC}^* is the Lagrange multiplier on the participation constraint. This term is zero if the values of both employment and unemployment are acyclical—for instance, if unemployment benefits are acyclical and firms make take-it-or-leave-it offers to workers. In general, however, the term will be nonzero if workers' ex ante utility is cyclical because of either a cyclical value of unemployment or bargaining.

The (C) term captures the effects that the shock has on profits through changes in the firm's choice variables. (C) has three pieces. First, the shock may shift the optimal contract's wage function $\mathbf{w}^*$. This is the marginal cost effect: the wage paid for each future realization of η^t and z^t may differ for contracts signed at different initial aggregate productivity levels z_0. Second, the shock may shift the optimal contract's recommended effort function $\mathbf{a}^*$, which affects output. This is the incentive effect. Finally, the shock may shift the value of the costates on the participation and incentive constraints.

3.3 Unemployment Dynamics and Incentive Wage Cyclicality

We now show that wage cyclicality from incentives does not dampen the response of unemployment to shocks. As in our discussion of the static model, the argument proceeds in two steps. First, we use an envelope logic to show that the (C) term in equation (17)—capturing the effect on profits via changes in optimal wages and effort—is zero. Second, to focus on incentives, we temporarily make assumptions that remove bargaining power or changes in outside options, so that the (B) term in equation (17) is also zero.

The main technical challenge for the proof is, therefore, to transform the problem so that an envelope theorem applies. Common general envelope theorems (e.g., Milgrom and Segal, 2002) are not well suited to studying problems with a continuum of nonconvex constraints.[16] The firm's problem has this feature since there is a continuum of incentive compatibility constraints, which are not generally convex. Below, we provide a set of sufficient conditions under which an envelope theorem can be applied to our problem when $\mathcal{B}(z_0)$ does not vary.

Assumption 1. *The set of feasible contracts* $(\mathbf{w}, \mathbf{a}) \in \mathcal{X}$ *that satisfy the incentive compatibility constraints (12) and participation constraints (13) is nonempty and compact.*

We make the minimal assumption of nonemptiness to allow the optimal contract to exist. We also assume that the set of feasible contracts satisfying the incentive and participation constraints is compact, which lets us apply a theorem from the applied mathematics literature on "sensitivity analysis" (Bonnans and Shapiro, 2000). This envelope theorem directly applies when there is a continuum of constraints that may not be convex. In Appendix Section A.4, we provide two different sets of minimal sufficient conditions under which the compactness assumption is satisfied.[17]

We will need to define an "impulse response" to present our results. Denote $z_t = \mathbb{E}\left[z_t|z_0\right] + \varepsilon_t$, where, by definition, ε_t is the cumulative innovation to the process for z between 0 and t and ε_0 is known to be 0. We will study the response of market tightness to changes in z_0 while holding fixed ε_t for all t, which is the "impulse response" of market tightness to changes in initial productivity z_0. In addition, let $\mathbf{\Gamma}^*(z_0)$ denote the set of optimal contracts $(\mathbf{w}^*, \mathbf{a}^*)$ solving the firm problem (11) given z_0.

Our next analytical result considers a benchmark in which all wage cyclicality is due to incentives. To this end, we consider a version of the flexible incentive pay economy in which firms make workers take-it-or-leave-it offers and unemployment benefits are acyclical. In this economy, all wage fluctuations are due to incentives rather than bargaining, and the (B) term from equation (17) that relates to bargaining is eliminated.

[16]Existing general envelope theorems are typically applied to the agent's objective, whereas we apply an envelope theorem to the principal's objective.

[17]Our first sufficient condition is that matches last at most T periods for T finite and that firms believe η and z to have a finite support. Continuous processes can be arbitrarily well approximated by such discrete processes. This assumption can be interpreted as a behavioral friction in which firms and workers can only consider N decimal places for innovations to z, for an arbitrarily large N. Our second possible sufficient condition is that contracts are continuous and twice differentiable in their arguments $\{\eta^t, z^t\}$, with uniformly bounded first and second derivatives. In addition, in Appendix Section A.2.2, we show that the envelope theorem can be applied to our problem under a stronger set of sufficient conditions summarized in Assumption 2 below, which allow us to make the problem recursive and apply the "first-order approach", closer to standard practice (e.g., Farhi and Werning, 2013).

Theorem 1. *Suppose that (i) Assumption 1 holds, (ii) the firm makes take-it-or-leave-it offers to workers and the flow value of unemployment is constant $\xi(z_t) = \xi$. Then, the first-order response of market tightness to a change in aggregate productivity $d\ln z_0$ is*

$$d\ln\theta_0 = \frac{1}{\nu_0}\frac{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t \frac{\partial}{\partial \ln z_0}\mathbb{E}\left[f(z_t,\eta_t)|z_0,\mathbf{a}^*\right]d\ln z_0}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t \mathbb{E}\left[f(z_t,\eta_t)-w_t^*|z_0,\mathbf{a}^*\right]} \tag{20}$$

in the flexible incentive pay economy, for some optimal contract $(\mathbf{w}^*, \mathbf{a}^*)$ in $\mathbf{\Gamma}^*(z_0)$, where ν_0 is the negative of the elasticity of job filling with respect to tightness. The first-order impulse response of market tightness to aggregate shocks in a rigid wage economy with $w = \bar{w}$ and $a = \bar{a}$ is

$$d\ln\theta_0 = \frac{1}{\nu_0}\frac{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t \frac{\partial}{\partial \ln z_0}\mathbb{E}\left[f(z_t,\eta_t)|z_0,\bar{\mathbf{a}}\right]d\ln z_0}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t \mathbb{E}\left[f(z_t,\eta_t)-\bar{w}|z_0,\bar{\mathbf{a}}\right]}. \tag{21}$$

Assume further that (i) the production function f is homogeneous of degree one in aggregate productivity z, (ii) z_t is a driftless random walk, and (iii) the optimal incentive contract at the nonstochastic steady state for z_t is unique. Then the response of market tightness to z in both economies, in the neighborhood of the nonstochastic steady state for z, is equal to

$$\frac{d\ln\theta_0}{d\ln z_0} = \frac{1}{\bar{\nu}}\left(\frac{1}{1-\Lambda}\right). \tag{22}$$

In both economies, Λ is the steady-state labor share defined as

$$\Lambda \equiv \frac{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t \mathbb{E}[w_t|\bar{z},\mathbf{a}]}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t \mathbb{E}[f(\bar{z},\eta_t)|\bar{z},\mathbf{a}]}, \tag{23}$$

where expectations are evaluated in a steady state with constant aggregate productivity $z_t = \bar{z}$, and $\bar{\nu}$ is the steady-state elasticity of job filling with respect to tightness.

The proof of this theorem, along with the proofs of all other propositions and theorems, is in Appendix A. The insight of the theorem is that wage cyclicality due to incentives does not dampen the response of unemployment to shocks. The impulse response of market tightness—and thus unemployment—to exogenous productivity shocks is the same in two economies. The first economy has flexible incentive pay but no bargaining power or changes in outside options. The second economy has exogenously fixed wages and effort. Equation (20) characterizes the impulse response of tightness to labor productivity shocks with flexible incentive pay. This impulse response is simply the direct productivity effect scaled by the present value of profits.[18]

[18]If the optimal contract is not unique, then the impulse response depends on the largest direct productivity

Similarly, equation (21) characterizes the same impulse response in the rigid wage economy—which is, again, the direct productivity effect scaled by the present value of profits. Therefore, the market tightness fluctuations from exogenous productivity shocks are equivalent in both economies if they feature the same direct productivity effect and the same present value of profits. Since there is no bargaining power in this case, all wage fluctuations in the incentive pay economy are due to incentives. Note that wages can be highly procyclical in this economy. Thus, wage cyclicality that arises due to incentives does not *per se* mute the response of unemployment to shocks.

There are two key steps in the proof of this Theorem, which is presented in Appendix A.2. First, as in the static model, the free entry condition ensures that changes in profits per worker determine tightness and hence unemployment fluctuations. Second, applying an envelope theorem to the firm's optimal contracting problem leads to an outcome equivalent to that under wage rigidity. This is because the (B) term in equation (17) is equal to zero with acyclical promised utility and an envelope theorem implies that the (C) term is zero as well. Thus, only the direct effect survives. This is similarly true in the rigid wage model in which there is neither bargaining power nor changes in wages or effort. This equivalence holds even though the flexible incentive pay economy could feature a highly procyclical present value of wage payments to new hires. The effect of higher wage payments on profits is exactly offset by higher worker effort on the optimal contract.

The final part of the theorem clarifies that the flexible incentive pay and the rigid wage economies have the same dynamics if they are both calibrated to the same steady-state labor share, which is a sufficient statistic for the direct productivity effects. To see the role of the labor share, we make assumptions to simplify the expression for $d\ln\theta/d\ln z_0$ from equations (20) and (21). Suppose that, as in the final part of the theorem, the production function is homogeneous of degree 1, z_t is a driftless random walk, and the optimal contract is unique.[19] Then, in the neighborhood of the nonstochastic steady state for aggregate variables, the impulse of market tightness in both economies becomes

$$\frac{d\ln\theta_0}{d\ln z_0} = \frac{1}{\nu_0}\frac{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\mathbb{E}[f\left(z_t,\eta_t\right)|\mathbf{a},z_0]}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\mathbb{E}\left[f\left(z_t,\eta_t\right)-w_t|\mathbf{a},z_0\right]}.$$

The numerator is the expected output, while the denominator is the excess output after wage

effect among optimal contracts, when productivity increases; and the smallest direct productivity effect among optimal contracts, when productivity decreases.

[19] These assumptions are made only for exposition. The simplifying assumption of a random walk is common because labor productivity is persistent and innovations are relatively small (e.g., Michaillat, 2012). The derivation does not impose linearity or nonstochastic behavior with respect to idiosyncratic shocks at the level of an individual job. The derivation applies even if the optimal contract is not unique, provided that all optimal contracts imply the same direct productivity effect.

payments. Dividing the numerator and denominator by the expected present value of output yields equation (22). If wages and effort lead to the same labor share in the rigid wage and incentive pay economies, then they feature the same dynamics of market tightness.[20]

Our result that incentive wage flexibility does not dampen unemployment fluctuations is general. Characterizing the optimal dynamic contract is difficult in our setting because of features such as persistent idiosyncratic shocks and potentially nonseparable utility between consumption and effort. Applying an envelope theorem allows characterization of the response of profits to labor demand shocks *without* our characterizing the optimal contract, so our result holds for general production or utility functions and persistent idiosyncratic shocks.

3.4 Unemployment Dynamics and Bargained Wage Cylicality

This section reintroduces bargaining power and cyclicality in workers' outside options. We argue that only bargained wage cyclicality arising from these sources dampens unemployment responses in a setting with both incentives and bargaining.

We introduce some additional notation for this section. Let $\mathcal{Y}(\mathbf{a}^*(z_0), z_0)$ denote the expected present discounted value of output from a match that originates under aggregate productivity z_0 given the optimal effort function $\mathbf{a}^*(z_0)$:

$$\mathcal{Y}(\mathbf{a}^*(z_0), z_0) \equiv \sum_{t=0}^{\infty}(\beta(1-s))^t \int\int f(z_t, \eta_t)\tilde{\pi}_t(\eta^t, z^t|z_0, \mathbf{a}^*(z_0))d\eta^t dz^t.$$

Likewise, let $\mathcal{W}(z_0)$ denote the present discounted value of wage payments under the optimal wage contract:

$$\mathcal{W}(z_0) \equiv \sum_{t=0}^{\infty}(\beta(1-s))^t \int w_t^*(\eta^t, z^t)\tilde{\pi}_t(\eta^t, z^t|z_0, \mathbf{a}^*(z_0))d\eta^t dz^t.$$

One can then write the value to the firm of a filled match as $J(z_0) = \mathcal{Y}(\mathbf{a}^*(z_0), z_0) - \mathcal{W}(z_0)$: the difference between the present discounted values of output and wages.

We now introduce a new expression for how profits respond to shocks with bargaining to define the notion of bargained wage cyclicality that will communicate our result. Differenti-

[20] The labor share is thus the "fundamental surplus" in this economy, in the sense of Ljungqvist and Sargent (2017). However, the dynamics of wages and effort in our flexible incentive pay economy may be different from those in the economies studied by Ljungqvist and Sargent (2017).

ating $J(z_0)$ with respect to z_0 yields the following expression:

$$\frac{dJ\left(z_0\right)}{dz_0}=\frac{\partial\mathcal{Y}\left(\mathbf{a}^*\left(z_0\right);z_0\right)}{\partial z_0}-\left(\frac{d\mathcal{W}\left(z_0\right)}{dz_0}-\partial_{\mathbf{a}}\mathcal{Y}\left(\mathbf{a}^*\left(z_0\right);z_0\right)\frac{d\mathbf{a}^*}{dz_0}\right). \tag{24}$$

This expression for the response of profits to z_0 is given by two terms. The first term is the direct productivity effect on output: the partial derivative of $\mathcal{Y}$ with respect to z. The second term measures the extent to which the present value of wages responds to labor productivity shocks by more than does the present value of effort. The term $\partial_{\mathbf{a}}\mathcal{Y}\left(\mathbf{a}^*\left(z_0\right);z_0\right)$ rescales cyclical effort movements $d\mathbf{a}^*/dz_0$ so that they are in the same units as wage movements. Movements in wages in excess of effort reflect bargaining or outside option fluctuations. Hence, let us define bargained wage cyclicality (BWC) as

$$\frac{\partial\mathcal{W}^{\text{bargained}}\left(z_0\right)}{\partial z_0}\equiv\frac{d\mathcal{W}\left(z_0\right)}{dz_0}-\partial_{\mathbf{a}}\mathcal{Y}\left(\mathbf{a}^*\left(z_0\right);z_0\right)\frac{d\mathbf{a}^*}{dz_0}. \tag{25}$$

Our next analytical result requires one more definition. Denote as $\tilde{\mathcal{B}}(z)$, the bargained utility, that is, the ex ante utility promised to the worker at the start of the contract, net of her continuation value with regard to separation into unemployment:

$$\tilde{\mathcal{B}}(z)\equiv\mathcal{B}(z)-\sum_{t=0}^{\infty}(\beta(1-s))^t\beta s\mathbb{E}[U(z_{t+1})|z_0].$$

Fluctuations in bargained utility capture variations in workers' ex ante utility due to either bargaining power or changes in their outside option.

Characterizing the response of market tightness to productivity in this setting is made more difficult in the presence of bargaining, as the set of contracts satisfying the participation constraint now moves directly with z_0. To make progress, we therefore introduce one additional assumption which guarantees that the so-called first-order approach (FOA) offers a valid solution to the contracting problem:

Assumption 2. *The set of feasible contracts* $(\mathbf{w},\mathbf{a})\in\mathcal{X}$ *is compact and convex. Assume standard Inada conditions on utility,* $\lim_{c\to\underline{w}}u_c(c,a)=\lim_{a\to\bar{a}}u_c(c,a)=\infty$ *and* $\lim_{c\to\bar{w}}u_c(c,a)=\lim_{a\to\underline{a}}u_c(c,a)=0$. *In addition, assume that the worker's optimal effort choices are determined by the first-order condition to problem* (12)*, and assume that the density of* η_t *can be expressed as*

$$\pi_t\left(\eta_t|\eta^{t-1},a^t\right)=\pi_t\left(\eta_t|\eta_{t-1},a_t\right).$$

Under this assumption, the incentive compatibility constraint may be written as the first-order condition to the worker's problem, and the firm's contracting problem may be expressed recursively. This assumption permits the derivation of our second analytical result: that bargained wage cyclicality mutes unemployment fluctuations.

Proposition 2. *Assume that Assumptions 1 and 2 hold. The impulse response of market tightness to aggregate shocks in the flexible incentive pay economy is*

$$d\ln\theta_0 = \frac{1}{\nu_0}\frac{\sum_{t=0}^{\infty}(\beta(1-s))^t \frac{\partial}{\partial\ln z_0}\mathbb{E}\left[f(z_t,\eta_t)|z_0,\mathbf{a}^*(z_0)\right] - \frac{\partial\mathcal{W}^{bargained}(z_0)}{\partial\ln z_0}}{\sum_{t=0}^{\infty}(\beta(1-s))^t\,\mathbb{E}\left[f(z_t,\eta_t)-w_t^*(z_0)|z_0,\mathbf{a}^*(z_0)\right]}d\ln z_0, \tag{26}$$

where $\partial W^{bargained}(z_0)/\partial\ln z_0$ is defined in equation (25). Moreover,

$$\frac{\partial\mathcal{W}^{bargained}(z_0)}{\partial\ln z_0} > 0 \quad\iff\quad \tilde{\mathcal{B}}'(z_0) > 0;$$

that is, bargained wage cyclicality is positive if and only if bargained utility is procyclical.

Proposition 2 shows that wage cyclicality due to bargaining dampens the response of unemployment to exogenous productivity shocks. Relative to Theorem 1, equation (26) has an extra term, alongside the direct productivity effect that appears in the equation. The extra term is bargained wage cyclicality. When bargained wage cyclicality is high, the impulse response of tightness is small. The proposition also shows that what we have defined as bargained wage cyclicality corresponds to the cyclicality of workers' ex ante utility—bargained wage cyclicality is positive if and only if the utility promised to workers at the start of a contract is procyclical.

Suppose that, intuitively, bargained utility is procyclical. Then, during a boom, as z_0 increases, workers' wages increase by more than their effort. As a result, workers' ex ante utility increases during booms. At the same time, profits increase by less as z_0 rises since workers capture part of the surplus through higher wages or lower effort. As a result, tightness is less responsive to business cycle shocks. Appealingly, the result does not require us to take a stand on why ex ante utility is cyclical. Various bargaining protocols or cyclicality in the value of unemployment benefits can lead to cyclical utility at the start of a contract; all of these factors would manifest as positive bargained wage cyclicality.[21]

A sketch of the proof is as follows. Under the conditions of the theorem, the firm's problem may be expressed with a Lagrangian. After applying an envelope theorem as in Theorem 1, the derivative of profits per worker with respect to z is given by the direct productivity effect minus the (B) term of equation (19), which reflects changes in the utility promised to the worker at the start of the contract. However, equation (24) shows that the

[21] Note that changes in ex ante utility also affect effort. Therefore, our model can potentially generate countercyclical effort—during recessions, workers may exert more effort because their outside option is worse.

response of profits to z is also the direct productivity effect net of bargained wage cyclicality. Thus, bargained wage cyclicality measures the cyclicality of promised utility.

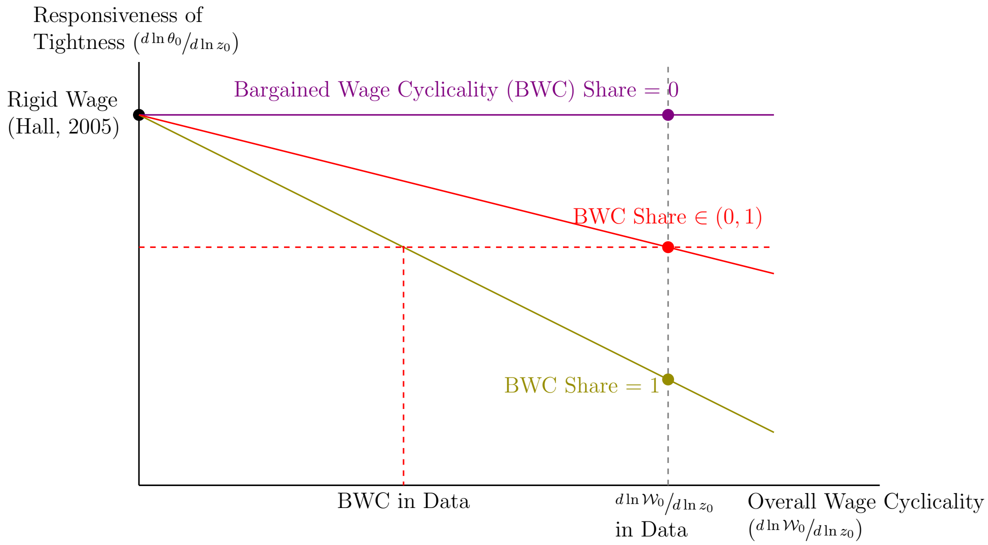
Theorem 1 and Proposition 2 show that when wages vary due to incentives, overall and bargained wage cyclicality are no longer equal. To clarify this point, Figure 2 illustrates our analytical results and places them in the context of the existing literature. The horizontal axis plots the degree of overall wage cyclicality. The vertical axis plots the responsiveness of market tightness to exogenous productivity shocks. The figure plots three lines, each corresponding to a different model for the origins of wage cyclicality. All three lines intersect the vertical axis in the same place: when wage cyclicality is zero, we return to the rigid wage model of Hall (2005) in which market tightness is highly responsive to exogenous shocks.

Figure 2 shows that models with and without incentives imply different unemployment dynamics given wage cyclicality in the data. Suppose that the overall wage cyclicality in the data is given by the vertical gray dashed line. Consider first the purple line at the top, labeled "Bargained Wage Cyclicality (BWC) Share = 0." This line corresponds to the model in which all wage cyclicality is due to incentives: bargained wage cyclicality is zero. Theorem 1 proves that this line is horizontal at the rigid wage line: even if wages are highly procyclical in this economy, the responsiveness of market tightness to aggregate productivity is the same as if wages and effort were exogenously held fixed.

The green line at the bottom corresponds to the case in which all wage cyclicality arises due to bargaining: the BWC share is equal to one. In this case, more cyclical wages dampen the impulse response of market tightness, as argued in Proposition 2. This is the classic result of Pissarides (2009) and holds in standard labor search models without incentives. Therefore models with and without incentives—with BWC shares equal to zero or one—may match the same overall wage cyclicality in the data but have drastically different implications for unemployment volatility. Intermediate values of the share of overall wage cyclicality accounted for by bargaining generate lines that are between the bargaining-only (green) and incentives-only (purple) lines, as illustrated by the red line.

Proposition 2 and Figure 2 offer guidance to researchers who wish to avoid working with complex models of incentive pay. Suppose that the red line corresponds to the share of bargained wage cyclicality that prevails in the data, which we seek to estimate below. This model generates a responsiveness of market tightness given by the horizontal dashed line on the graph. A model in which all wage fluctuations are accounted for by bargaining, such as the standard DMP model used in much of the literature, will generate the same unemployment dynamics as the full model with both incentives and bargaining as long as it is calibrated appropriately. In particular, one needs to calibrate a bargaining-only model such that the total wage cyclicality in that model is equal to the bargained wage cyclicality

Figure 2: Illustration of relationship between wage and market tightness cyclicality, by share of wage cyclicality accounted for by bargaining



Notes: The figure illustrates our analytical results. The horizontal axis plots the cyclicality of the present value of wage payments, and the vertical axis plots the elasticity of market tightness θ to initial aggregate productivity z_0. The three lines plot the response of market tightness as a function of overall wage cyclicality when all of the wage cyclicality is due to bargaining (BWC Share = 1), due to incentives (BWC Share = 0) or due to some mix of bargaining and incentives (BWC Share $\in (0,1)$). The figure is illustrative and not derived from a calibration.

in the data. We return to this point in the numerical analysis of Section 4.5 below.

3.5 Discussion

Importance of the impulse response of tightness. Our results demonstrate that the impulse response of market tightness to exogenous productivity shocks is the same in both a rigid wage economy and a flexible incentive pay economy without bargaining power. This impulse response is an essential object in macroeconomics. For instance, this impulse response determines the slope of the Phillips curve for prices. We prove this point in Appendix A.7, using a New Keynesian model with a frictional labor market similar to the setup of Christiano et al. (2016). We also show in this model that the slope of the Phillips curve is the same if there are either rigid wages or flexible incentive pay in the frictional labor market. Likewise, the impulse response of tightness determines the behavior of unemployment in response to monetary or fiscal shocks. However, rigid wage and flexible incentive pay economies generate different output dynamics since the incentive pay economy features endogenous productivity movements through effort fluctuations. These productivity movements may appear as a form of endogenous labor capacity utilization.

First-order results. Our analytical results on the irrelevance of incentive wage cyclicality and the importance of bargaining hold to the first order rather than globally. Below, we study a globally solved numerical model with consonant results.

User cost of labor and the present value of wages. Our argument is different from the emphasis on new hire wages or the user cost of labor (Kudlyak, 2014). The irrelevance of flexible incentive pay holds even if the *present value* of new hires' incentive wages is arbitrarily cyclical.

Endogenous separations and limited worker commitment. The irrelevance of incentive wage cyclicality continues to hold when separations are endogenous and efficient. Intuitively, separations are another margin over which the firm can optimize to maximize the profits of a job. Therefore, after an aggregate shock, changes in the firm-level separation rate have no first-order effect on profits. Appendix Section A.8 introduces endogenous separations into the incentive pay model and derives an equivalence for the impact elasticity of tightness to productivity shocks. In the same model extension, we also show that incentive wage cyclicality remains irrelevant in the presence of limited worker commitment, for analogous reasons.

Next steps. The natural next question is: "What share of wage cyclicality in the data is due to bargaining?" To answer this question, one must measure the cyclicality of workers' utility at the start of contracts or the cyclicality of wages holding fixed the effort of the worker. Answering this question is challenging and should be the focus of future empirical work. One possibility would be to separately measure proxies for incentives and bargaining, such as the cyclicality of bonus and base pay. However, bonuses may not solely reflect incentive provision. For example, some workers may expect to receive a minimum bonus irrespective of their performance, while stock options reward aggregate stock market appreciations over which individual managers have little control. Similarly, bonuses do not reflect the full range of incentives that firms may provide: longer-term incentives such as promotions are ubiquitous and also appear cyclical (e.g., Méndez and Sepúlveda, 2012). The next section makes progress by calibrating a structural model of incentive pay to match micro-moments of wage adjustment.

4 Numerical Analysis

This section studies a calibrated version of our model. We find that a significant share of overall wage cyclicality is due to incentives. As a result, unemployment responds strongly to business cycle shocks in the calibrated model despite relatively procyclical wages. We also show how researchers can produce the correct impulse response in simpler models of unemployment dynamics without incentive pay. One must calibrate the simple model to match only the portion of wage cyclicality attributable to bargaining, which is weakly procyclical.

4.1 Parameterizing the Contract

We parameterize the production function, utility function, ex ante utility, and information structure following Edmans et al. (2012) so that we can calibrate our model. All other aspects of the environment are the same as those of the flexible incentive pay economy in Section 3.

Production function. The firm's production function is $y = z(a + \eta)$. Idiosyncratic profit shocks η are assumed to be i.i.d. over time and across individuals and normally distributed with zero mean and standard deviation σ_η. σ_η determines the extent to which firms can infer workers' effort, which is key for incentive pay.

Preferences. We assume that workers have logarithmic utility over consumption, with an isoelastic disutility of labor that is separable from consumption. Therefore, $u(c, a) \equiv \ln c - \frac{a^{1+1/\epsilon}}{1+1/\epsilon}$, where ϵ governs the Frisch elasticity of effort, which determines how costly the provision of effort is to workers.

Information structure. We make the "effort after noise" assumption as in Edmans et al. (2012): workers observe the idiosyncratic profit shock η before making an effort choice. Thus, there is an incentive compatibility constraint for each value of η. Following Edmans et al. (2012), we assume that a unique level of effort $a(z^t)$ is implemented regardless of the idiosyncratic shock η. However, effort varies with the history of aggregate productivity z^t.

Ex ante utility. We assume that firms make take-it-or-leave-it offers to workers who face cyclical unemployment benefits. Workers' flow unemployment benefits take the form $b(z) = \gamma z^\chi$. Here, γ specifies the level of unemployment benefits when $z = 1$, while χ determines the elasticity of unemployment benefits to aggregate productivity. This specification is a log-linear approximation of any differentiable $\mathcal{B}(z)$ function, including models in which workers and firms bargain over ex ante utility at the start of the contract. However, this specification is numerically tractable in that it abstracts from complications of bargaining and ensures that unemployed workers' value is given by the present discounted value of expected unemployment benefits. The parameter χ is a stand-in for bargaining in that it shifts the utility promised to the workers under the contract—it can reflect changes in promised utility due to fluctuations in either the worker's outside option (changes in the value of unemployment) or inside option (bargained utility)—and indeed determines bargained wage cyclicality.

We now characterize the optimal contract following Edmans et al. (2012).

Proposition 3. *The earnings schedule in the optimal contract satisfies the following difference equation (given initial productivity z_0):*

$$\ln(w_t(\eta^t, z^t)) = \ln(w_{t-1}(\eta^{t-1}, z^{t-1})) + \psi h'(a_t)\eta_t - \frac{1}{2}(\psi h'(a_t)\sigma_\eta)^2, \tag{27}$$

where $\psi = 1 - \beta(1-s)$ and $w_{-1}(z_0)$, which initializes this difference equation, is given by

$$w_{-1}(z_0) \equiv \psi\left(\mathcal{Y}(\mathbf{a}^*(z_0), z_0) - \frac{\kappa}{q(\theta_0)}\right). \tag{28}$$

The worker's utility under the contract $\mathcal{E}(z_0)$ is equal to her value of nonemployment, so that

$$\frac{\ln w_{-1}(z_0)}{\psi} - \mathbb{E}\left[\textstyle\sum_{t=0}^{\infty}(\beta(1-s))^{t-1}\left(\frac{\psi}{2}(h'(a_t)\sigma_\eta)^2 + h(a_t) - \beta s U(z_{t+1})\right)|z_0\right] = U(z_0) \tag{29}$$

for

$$U(z_0) \equiv \mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t \ln b(z_t)|z_0\right].$$

In addition, a_t, the optimal effort level of a worker hired with $z = z_0$, satisfies

$$a(z_t; z_0) = \left[\frac{z_t a(z_t; z_0)}{\psi\left(\mathcal{Y}(\mathbf{a}^*(z_0), z_0) - \frac{\kappa}{q(\theta_0)}\right)} - \frac{\psi}{\epsilon}\left(h'\left(a(z_t; z_0)\right)\sigma_\eta\right)^2\right]^{\frac{\epsilon}{1+\epsilon}}. \tag{30}$$

A proof is provided in Appendix A and closely follows that of Edmans et al. (2012).[22] In the contract, the pass-through of idiosyncratic shocks to wages corresponds to incentives. Intuitively, to satisfy the incentive constraint as cheaply as possible, the firm increments wages in a manner consistent with the worker's inverse Euler equation, which gives rise to the log difference equation (27). Idiosyncratic shocks η directly enter the equation for log wages since the firm must make workers' wages responsive to output fluctuations to incentivize effort. If the marginal disutility of effort is high, there must be high pass-through from η to wages to induce workers to supply the optimal effort level. To satisfy dynamic incentives, the pass-through of idiosyncratic productivity shocks to wages is scaled down by a quantity ψ that reflects discounting.

Exponentiating equation (27), one observes that wages are a random walk: the expectation of wages in period $t+h$ is equal to the level of wages in period t. The random walk

[22] We make two advances relative to Edmans et al. (2012): we introduce aggregate risk z_t, and we develop a global solution algorithm to efficiently simulate the model with labor market search (see Appendix B).

property is a consequence of the inverse Euler equation (Rogerson, 1985). Thus, w_{-1}/ψ is equal to the expected present discounted value (EPDV) of wage payments. Free entry into vacancy posting guarantees that the EPDV wage payments are the difference between the endogenous EPDV of output $\mathcal{Y}(\mathbf{a}^*(z_0), z_0)$ and the expected cost of filling a vacancy $\kappa/q(\theta)$. Calculating the expected utility under the contract (the left-hand side of equation (29)) relies on solving forward the wage equation. Effort is determined by taking the first-order condition of the worker's utility maximization problem—that is, by setting the derivative of the left-hand side of equation (29) with respect to a_t equal to zero.

4.2 Calibration: Separating Bargaining from Incentives

Our goal is to infer the role of bargaining versus incentives in determining wage cyclicality. We disentangle these forces with two sets of moments: the cyclicality of the wage for new hires, which informs bargaining power, and the pass-through of idiosyncratic firm output shocks into wages, as well as the variance of workers' wage growth, both of which inform incentives. That is, wage fluctuations at the start of the contract inform bargaining, whereas wage fluctuations after the start of the match inform incentives.

We calibrate the parameters of the labor search block largely following the standard practice of Petrosky-Nadeau and Zhang (2017).[23] Productivity is assumed to follow an AR(1) process in logs, with autocorrelation parameter ρ_z, innovation $\zeta_t \sim \mathcal{N}(0, \sigma_z^2)$, and mean μ_z. We normalize μ_z such that $\mathbb{E}[z_t] = 1$. To account for the effects of effort fluctuations on labor productivity, we calibrate our monthly process for z such that the log of the quarterly average of z_t matches the autocorrelation and standard deviation of the quarterly log TFP series described in Fernald (2014), which accounts for variable capacity utilization in labor. We view the TFP series net of variable capacity utilization as a reasonable proxy for exogenous productivity, as labor utilization is a concept highly related to effort.[24] This procedure implies a monthly autocorrelation $\rho_z = 0.966$ and standard deviation of shocks $\sigma_z = 0.0056$.[25]

This leaves four parameters to internally calibrate: $\sigma_\eta, \gamma, \chi$, and ϵ. We target the variance of incumbent wage growth, the pass-through of firm shocks into wages, the cyclicality of new hire wages, and the average unemployment rate. While we estimate all parameters jointly, these moments have intuitive mappings to particular parameters, which we explore below.

First, the variance of wage growth naturally informs the variance of idiosyncratic profit

[23] These parameters are the discount rate, the vacancy creation cost, the matching function, and the separation rate. We discuss the details in Appendix Section B.1.

[24] Basu and Kimball (1997) find that variable capacity utilization explains approximately 40–60% of fluctuations in unadjusted TFP and that capacity utilization is procyclical.

[25] We HP-filter the TFP data and model-simulated series with a smoothing parameter of $\lambda = 10^5$, following Shimer (2005), which removes a very low-frequency trend.

shocks σ_η. To see this, note that rearranging equation (27) shows that the monthly wage growth of job-stayers is given by $\Delta \ln w_t = \psi h'(a_t)\eta_t - 1/2\left(\psi h'(a_t)\sigma_\eta\right)^2$. At an aggregate nonstochastic steady state, $a_t = a^{SS}$, for example, the cross-sectional variance of wage growth is given by $Var(\Delta \ln w) = \psi^2 h'(a^{SS})^2 \sigma_\eta^2$, which is closely tied to the value of σ_η. The firm provides *intertemporal incentives* by exposing the worker to wage-growth risk as in Sannikov (2008). We target a standard deviation of year-over-year wage growth of job-stayers of 0.064 as measured by Grigsby et al. (2021), where we calculate year-over-year wage growth in the model with stochastic z_t by iterating on equation (27) for job-stayers.[26]

Second, the pass-through of firm-specific shocks to wages is informative of whether incentives are high-powered within the contract, as in classic theories of moral hazard. In particular, this pass-through helps us identify the parameter governing the disutility of effort ϵ. In our model, the expected pass-through from idiosyncratic output shocks to the wages of job-stayers is given by $\mathbb{E}\left[\partial \ln w / \partial \ln y\right] = \mathbb{E}\left[\psi h'(a)(a+\eta)\right]$, which is directly affected by $h'(a)$. The firm provides *intratemporal incentives* with the pass-through of output to wages. Intuitively, if $h'(a)$ is high, then workers would prefer not to supply more effort. To induce the worker to supply more effort, the firm must provide high-powered incentives via a high pass-through of output to wages. Pass-through is therefore linked to ϵ.

A large literature seeks to estimate the pass-through to job-stayers' wages of firm-specific profitability shocks; Card et al. (2018) provide a comprehensive survey. This literature has estimated pass-through elasticities from firm-level shocks ranging from 0.02 to 0.156. Many of the estimation strategies in the literature use firm variation that is likely partially persistent. In contrast, in our model, shocks η to output are i.i.d. through time, which thus suggests that the pass-through of η shocks to wages is likely lower than the higher ranges commonly estimated in the literature. We therefore target an average pass-through of firm-level output shocks to wages of 0.039, estimated in Martins (2009), which is on the low end of the range reported by Card et al. (2018). Targeting a low pass-through is likely to be conservative, as it suggests that incentives are not high-powered and therefore are a relatively unimportant determinant of wage variation.

Third, we identify γ, which pins down the level of unemployment benefits, from the stochastic mean of unemployment. Average unemployment is determined by workers' job-finding rates, which in turn are determined by expected profits per worker. γ directly influences expected profits because it governs workers' value of unemployment and shifts the level of the required wage payments to workers. We target an average unemployment rate of 6%, consistent with average U.S. unemployment between 1951 and 2019.

[26]Hours are observable and thus contractible. We therefore consider earnings per hour—inclusive of base pay, bonuses, and overtime—to be the correct empirical counterpart of w_t.

Fourth, we target the cyclicality of new hire wages to inform the cyclicality of nonemployment benefits χ. Conditional on the parameters governing incentives, the cyclicality of new hire wages is highly informative of χ. Intuitively, if the worker's outside option is highly procyclical, so too is her promised utility, and thus so too will be her wage payments. Since wages are a random walk in the optimal contract, the cyclicality of new hire wages strongly informs the cyclicality of the present discounted value of wage payments and thus the cyclicality of promised utility. Mathematically, evaluating the flow value of unemployment $b(z_t)$ in equation (29), which characterizes workers' ex ante utility, yields

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t(\ln\gamma+\chi\ln z_t)\right]=\frac{\ln w_{-1}}{\psi}-\mathbb{E}\left[\sum_{t=0}^{\infty}(\beta(1-s))^{t-1}\left(\frac{\psi}{2}(h'(a_t)\sigma_\eta)^2+h(a_t)-\beta sU(z_{t+1})\right)\right].$$

Given the wage schedule defined in equation (27), expected new hire wages $\mathbb{E}[w_0(z_0)]$ are equal to $w_{-1}(z_0)$. The above equation implies a close relationship between expected new hire wages and χ, conditional on the disutility of effort. We target a semielasticity of new hire wages to unemployment of -1, which is at the high end of the range of what is found by Bils (1985) and Hazell and Taska (2022), and explore robustness to this choice.

In summary, wage fluctuations that occur after the start of the contract inform the parameters governing the strength of incentives, while wage fluctuations at the beginning of the contract inform the strength of bargaining. Our numerical approach is relatively simple. For instance, our model links ex post wage pass-through to incentives and not to Nash bargaining. In the face of this particular concern, we have deliberately targeted a conservative value of pass-through. Moreover, there is some empirical evidence that pass-through is procyclical (Chan et al., 2023), which is consistent with our model and inconsistent with ex post pass-through representing Nash bargaining.[27]

4.3 Model Fit and Calibrated Parameters

We now discuss the calibrated parameters and assess the model's ability to fit the targeted and untargeted moments. Table 1 summarizes our calibration, while Table 2 examines the implications for various moments. We estimate that the elasticity of the disutility of effort ϵ is equal to 2.7. Note that standard estimates of micro labor supply elasticities, such as those computed by Chetty (2012), consider how hours vary with wages. Since hours are observable and contractible by the firm, the lower elasticities of hours need not have any relationship with the elasticity of unobservable effort. Intuitively, one might expect the elasticity of effort

[27] Appendix B presents details on the estimation algorithm, how we produce moments within the model and the data, and how we calculate the share of wages attributable to bargained wage cyclicality.

Table 1: Calibrated parameter values

Parameter	Description	Value	Source/Target
Externally Calibrated			
β	Discount Rate	$0.990^{1/3}$	Petrosky-Nadeau and Zhang (2017)
κ	Vacancy Creation Cost	0.450	Petrosky-Nadeau and Zhang (2017)
s	Separation Rate	0.031	CPS E-U Flow Rate
ρ	Autocorrelation: Agg. Productivity	0.966	Autocorrelation: Fernald (2014) TFP
σ_z	Cond. S.D. of Agg. Productivity	0.006	Uncond. S.D.: Fernald (2014) TFP
Internally Calibrated			
γ	Level: Unemployment Benefits	0.461	Average Unemployment Rate
ϵ	Elasticity: Disutility of Effort	2.713	Pass-through: profits to wages
σ_η	S.D.: Idiosyncratic Profit η	0.532	S.D.: Job-Stayer Log Wage Growth
χ	Cyclicality: Promised Utility to Worker	0.467	New Hire Wage Cyclicality

to be larger than that of hours: while many jobs have a fixed number of hours over which the worker has little control (e.g., she must work 40 hours per week to remain employed), workers may be able to adjust unobserved effort more elastically.

We find the level of unemployment benefits γ to be 0.46. This value is between the value chosen by Shimer (2005) to match the replacement rate of unemployment benefits (0.4) and that in Hagedorn and Manovskii (2008) to match aggregate wage cyclicality (0.955).[28]

We estimate the standard deviation of idiosyncratic profit shocks to be $\sigma_\eta = 0.53$, similar to estimates in other labor search calibrations with idiosyncratic shocks (e.g., Schaal, 2017). This, coupled with a sizable elasticity of effort, suggests that incentive provision is a relatively important consideration for the firm. We estimate the cyclicality of flow unemployment benefits χ to be 0.47, implying moderately procyclical promised utility to the worker.[29]

Table 2 compares key moments in both the calibrated model (Column 1) and data (Column 2). The top panel reports the moments that we target in the estimation. The model is able to fit the targeted moments very well. Most notably, we match the cyclicality of new hire wages almost exactly and, if anything, underestimate the pass-through of firm shocks to wages, suggesting that our estimate of the importance of incentives for wage cyclicality is likely a lower bound on its true importance.

The bottom panel of the table shows that the model generates approximately half of the unconditional volatility of aggregate unemployment observed in the data, which is an appropriate figure because labor productivity is not the sole determinant of unemployment fluctuations (Pissarides, 2009). Therefore, even though our main focus is the impulse re-

[28]Note, however, that unemployed workers do not need to supply effort in this model, which increases the effective flow unemployment value.

[29]Chodorow-Reich and Karabarbounis (2016) estimate $\chi \approx 0.8$; however, the value of unemployment in our model is different from theirs because workers supply effort and do not have access to financial assets.

Table 2: Model fit to data moments

Moment	Description	Data (1)	Model (2)
Targeted			
$d\mathbb{E}[\ln w_0]/du$	Cyclicality of new hire wages	-1.000	-1.001
$\mathbb{E}[\partial \ln w_t/\partial \ln y_{it}]$	Within-job pass-through of idiosyncratic shock	0.039	0.036
$\text{std}(\Delta \ln w_t)$	std(ln wage growth for job-stayers)	0.064	0.064
$\bar{u}_t$	Mean unemployment	0.060	0.060
Untargeted			
std $(\ln u_t)$	Volatility of unemployment (quarterly)	0.203	0.103
BWC Share	Share of wage cyclicality due to bargaining	–	0.543

sponse of unemployment, our calibrated model does match unconditional unemployment fluctuations reasonably well. Matching the micro moments of wage adjustment therefore generates significant unemployment volatility, the reasons for which we will discuss shortly.

4.4 Numerical Result: Bargained vs. Incentive Wage Cyclicality

Now we discuss our key numerical result: the model suggests that a significant share of wage cyclicality is due to incentives. As a result, unemployment responds strongly to business cycle shocks despite relatively procyclical wages.

The model calibration reveals in the final row of Table 2 that approximately 54% of the total wage cyclicality is due to bargaining and outside option cyclicality. Conversely, incentives account for the remaining 46% of total wage cyclicality. The share of wage cyclicality attributable to incentives may seem large. Non–base compensation, which may be associated with incentives, is relatively small for most workers. However, what matters for wage cyclicality is whether the *marginal* dollar of wages paid is due to incentives or bargaining. If, for instance, 2% of compensation is incentive pay in the steady state but only incentive pay is cut in response to output shocks, then the share of wage cyclicality due to incentives is 100%.

Because bargained wage cyclicality is relatively low, the impulse response of unemployment to business cycle shocks is relatively large—which also explains why the unconditional volatility of unemployment is large. Table 3 reports a number of additional features of our model calibrated in a variety of ways. Column (1) reproduces the baseline calibration as in Table 2. The impulse response of market tightness to business cycle shocks is in the second row. Market tightness responds greatly to exogenous productivity shocks: the elasticity of

Table 3: Model moments: Alternative calibrations

Moment	(1) Incentives + Bargaining	(2) Incentives	(3) Bargaining	(4) Bargaining: $\partial\mathbb{E}[\ln w_0]/\partial u = -0.54$
	Model: Source of wage flexibility			
$d\mathbb{E}[\ln w_0]/du$	-1.00	-0.62	-1.00	-0.54
$d\ln\theta_0/d\ln z_0$	13.6	17.8	10.4	13.3
$\mathrm{std}(\ln u_t)$	0.10	0.15	0.08	0.10
$\mathcal{W}_0/\mathcal{Y}_0$	0.96	0.96	0.96	0.96
$d\ln\mathcal{W}_0/d\ln z_0$	0.44	0.39	0.31	0.24
$d\ln\mathcal{Y}_0/d\ln z_0$	0.70	0.88	0.51	0.51
BWC share	0.54	0.00	1.00	1.00

Notes: New hire wage cyclicality is targeted, while the second set of moments is untargeted. Column (1) is our baseline model. Column (2) sets $\chi = 0$ and does not target the cyclicality of new hire wages. Columns (3) and (4) fix effort $a = 1$, set wages to be constant within the contract, and do not target the standard deviation of wage growth or the pass-through. Column (4) targets a cyclicality of new hire wages of -0.54. The standard deviation of log unemployment is computed at quarterly frequency. x_0 denotes the value of variable x, evaluated at $\ln z = \mu_z$. $\mathcal{W}$ and $\mathcal{Y}$ refer to the expected present value of wage payments and output, respectively. "BWC share" is the share of wage cyclicality that is due to bargaining for $\ln z_0 = \mu_z$.

market tightness to aggregate productivity is 13.6. Therefore, market tightness will be substantially more volatile than aggregate productivity, as in the data. In turn, unemployment will be volatile.

The large impulse response of market tightness arises despite the cyclicality of wages because much of this wage cyclicality is due to incentives. The elasticity of the present value of expected wage payments with respect to productivity is 0.44. However, as we have discussed in previous sections, the stabilizing effect on unemployment of cyclical wages is offset by the amplifying effect of effort and incentives. Because of incentives, the response of the present value of output, $\mathcal{Y}_0$, to TFP shocks is a relatively large value of 0.70. As a result, profit fluctuations—and thus market tightness and employment fluctuations—are large despite the procyclicality of wages.

The model implies a labor share (defined as $\mathcal{W}_0/\mathcal{Y}_0$) of 0.96, in line with, for instance, Hall (2005).[30] Ljungqvist and Sargent (2017) discuss how this labor share calibration and fixed real wages deliver volatile unemployment. Our contribution is to show that this labor share calibration and *procyclical* wages also deliver volatile unemployment in the presence of incentives.

To further show how the division of wage cyclicality between incentives and bargaining affects unemployment dynamics, we consider versions of our model that load all wage

[30] Since our model does not have capital, the labor share corresponds to the labor share of payroll and rents from search frictions in the labor market, excluding capital (Pissarides, 2000).

cyclicality in the data onto either incentives or bargaining. In the incentives-only calibration, volatile unemployment coexists with procyclical wages—whereas in the bargaining-only calibration, procyclical wages imply muted unemployment dynamics.

We present the calibration with only incentives and without bargaining in column (2), which leads to volatile unemployment dynamics. Here, we assume that the cyclicality of ex ante utility is zero and recalibrate with $\chi = 0$. We do not target wage cyclicality, and so the model remains exactly identified. In this calibration, the labor share is the same as in the baseline. However, the impulse response of tightness is far greater: a value of 17.8, compared to 13.6 in the baseline calibration. This is a manifestation of our analytical results. This incentives-only version of the model behaves as if wages and effort were exogenously fixed as in Hall (2005); thus, it is able to generate large responses of tightness to shocks.

Nevertheless, the incentives-only model still generates large wage cyclicality, despite cyclical profits. As we have discussed, as z rises, so too does desired effort, because of the complementarity between effort and z in the production function. In column (2), the elasticity of $\mathcal{Y}$ to TFP shocks is a relatively large value of 0.88. To induce this effort, the firm must incentivize the worker by making her wage more responsive to realized output. This exposes the worker to risk, for which she must be compensated. Thus, expected wages become fairly procyclical with ${}^{d\ln\mathcal{W}_0}/_{d\ln z_0} = 0.39$, even with acyclical promised utility to the worker.

Column (3) presents a version of the model without incentives and with only bargaining, in which case the unemployment dynamics are muted. Here, we switch off incentives and variable effort by setting the variance of the idiosyncratic profitability shocks to $\sigma_\eta = 0$, exogenously fixing effort $a = 1$, setting $\epsilon = 1$, and setting wages to be fixed within a contract. We no longer target the variance of log wage growth or the pass-through of firm shocks to wages, and we attribute all wage cyclicality in the data to the cyclicality of promised utility, governed by χ. This calibration of the model is closer to the common practice in job search models without incentive provision and implies that the bargained wage cyclicality share is 100%.

This version, in which wage cyclicality reflects only bargaining, generates an elasticity of market tightness to exogenous productivity that is approximately 25% smaller (10.4) than that in the full model with both bargaining and incentives (13.6). This is because the estimated value of χ rises substantially to 0.61 (from 0.47). Therefore, wage cyclicality is high, but there is no offsetting movement in effort, which dampens the impulse response of tightness. We discuss column (4) of the table in Section 4.5 below after examining the robustness of our results.

Robustness. The key numerical result of this section is that a significant share of wage cyclicality is due to incentives, leading to volatile unemployment dynamics despite relatively procyclical wages. Appendix C probes the robustness of this result. Our baseline calibration targets a semielasticity of new hire wages with respect to unemployment of -1. Nevertheless, there remains some disagreement over exactly how cyclical new hire wages are. We thus test the sensitivity of our numerical exercise to different targets of new hire wages. Tables C1 and C2 report the estimated parameters and model-implied moments, respectively, when we target different values of wage cyclicality ranging from -0.5 to -1.5. In each case, the model matches the targeted moments very well. We find that the share of wage cyclicality attributable to incentives declines as we increase the target cyclicality of new hire wages. However, the elasticity of incentive wages to unemployment is relatively stable between -0.37 and -0.49. A simple rule to sweep out wage cyclicality due to incentives is, therefore, to subtract 0.46 from one's preferred estimate of wage cyclicality.

To account for uncertainty in our wage pass-through target, Appendix Figure C1 reports the estimate of the BWC share as one varies the elasticity of effort supply ϵ, recalibrating the rest of the parameters. The estimated share of wage cyclicality due to bargaining is decreasing in ϵ, falling to 48% for $\epsilon = 5$ and rising to 77% for $\epsilon = 0.5$. This declining BWC share is in part attributable to a decreasing internal estimate of χ, as the model infers a greater share of wage cyclicality due to incentives as ϵ increases.

Next, we study the robustness with respect to our TFP shock series. As noted previously, incentives lead to changes in measured productivity through endogenous effort fluctuations. Our utilization-adjusted TFP series imperfectly corrects for these effort changes. Therefore, we also internally calibrate the exogenous productivity process in our incentive pay model to match moments of average labor productivity in the data. Appendix Tables C1 and C3 report the estimated parameters and model-implied moments, respectively. Calibrated thus, the model continues to infer a large share of overall wage cyclicality due to incentives and a large response of market tightness to productivity shocks.[31]

Taking stock, we find that a relatively large share of wage cyclicality in the data is due to incentives despite a conservative calibration. Therefore, our model generates a large impulse response of unemployment despite the cyclicality of wages.

4.5 A User Guide

Here, we discuss how to calibrate a simple model without incentives to replicate the impulse response of unemployment from our incentive pay model. Our aim is to offer a "user guide"

[31] In Appendix Tables C1 and C3, we also recalibrate the bargaining-only model to target average labor productivity. The bargaining-only model continues to have a significantly smaller impulse response of tightness than does the full model and requires exogenous productivity shocks to be approximately twice as volatile as in the full model to match output fluctuations.

for researchers who wish to calibrate models to produce the correct unemployment dynamics while avoiding the complexities of incentive pay.

We argue that the simple version of our model in which all wage cyclicality is due to bargaining should target a new hire wage cyclicality given only by bargained wage cyclicality. This logic is consistent with our analytical results. To illustrate the point numerically, we recalibrate the bargaining-only version of the model targeting a new hire wage cyclicality of -0.54, which we previously inferred from the data to be wage cyclicality due to bargaining.[32] Column (4) of Table 3 presents the results of this exercise.

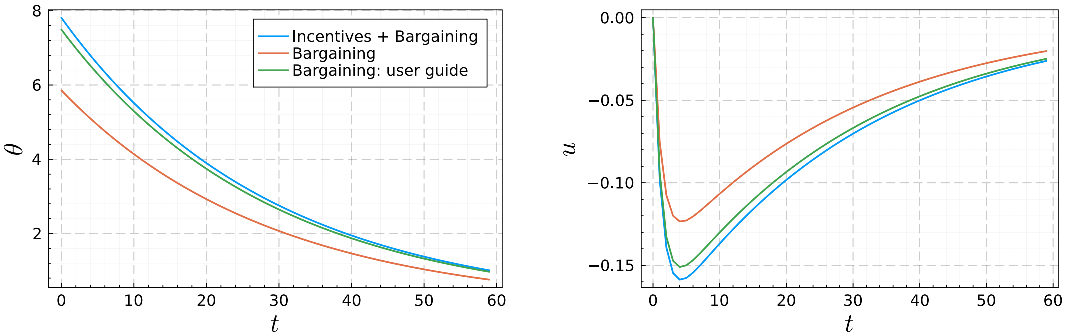
The numerical results show that to produce the correct impulse response of market tightness in the simple model with only bargaining, calibrating to target bargained wage cyclicality is crucial. When calibrated to bargained wage cyclicality, the bargaining-only model features an elasticity of market tightness to exogenous shocks that is nearly identical (13.3) to that in the full model (13.6). Furthermore, both models generate an unconditional standard deviation of log unemployment rates of 0.10. The similar dynamics arise because the two models imply similar ex ante utility cyclicality even though overall wage flexibility is different: the simple bargaining-only model of column (4) estimates an elasticity of unemployment benefits $\chi = 0.47$, nearly identical to that found under the full model (0.47).

We compute impulse responses to confirm how to calibrate the bargaining-only model. Figure 3 plots the impulse of market tightness (Panel A) and unemployment (Panel B) in response to a one-standard-deviation increase in aggregate productivity z, which decays according to an AR(1) process. The blue line is the response in the full model with both incentives and bargaining. The red line is the response in the bargaining-only model calibrated to the full wage cyclicality in the data. The green line is the response in the bargaining-only model calibrated to our estimate of bargained wage cyclicality in the data. The response of both market tightness and unemployment is approximately 25% less pronounced in the bargaining-only model than in the full model with incentives and bargaining. However, the impulse responses of both tightness and unemployment are nearly identical in the full model and the bargaining-only model calibrated to relatively rigid wages.

This section shows that researchers interested in the impulse response of unemployment may abstract from incentive contracts by calibrating simpler models to match bargained wage cyclicality. Doing so will generate an unemployment response identical to that in a fuller model that accounts for micro-moments of wage adjustment and incentive pay. Our numerical exercise reveals that the simpler model should target weakly procyclical wages because a substantial share of overall wage cyclicality in the data is attributable to incentives.

[32]We normalize $\epsilon = 1$ for this exercise and solve for fixed wages within the contract. We also drop the standard deviation of log wage growth and the average pass-through of firm shocks as targeted moments.

Figure 3: Impulse response to shock to z_0 with bargaining-only and incentive pay models



Panel A: Tightness θ_t Panel B: Unemployment u_t

Notes: The figure shows impulse responses for five years after a one-standard-deviation shock to z_0. In Panel A, θ_t is shown in percentage deviations from the steady state (i.e., 100 times the log deviation). In Panel B, u_t is shown as deviations away from the steady state in percentage points (i.e., 100 times the deviation in levels). Further details on the construction of these impulse responses are described in Section B.6.

5 Conclusion

This paper studies the role of incentive pay in unemployment dynamics. Embedding a dynamic principal–agent problem into a benchmark labor search model leads to two results. First, wage cyclicality due to incentives does not dampen the response of unemployment to shocks. Second, wage fluctuations that alter the worker's utility at the start of the contract, which we dub bargained wage cyclicality, do mute the response of unemployment to productivity shocks as in standard models.

These analytical results imply a need for careful measurement of bargained wage cyclicality in calibrating models of unemployment dynamics. We offer one attempt at such measurement through a calibrated model and find that approximately 54% of the wage cyclicality observed in the data is due to bargaining, with 46% arising because of a cyclical desire to incentivize worker effort. Models that do not feature incentive pay should therefore target a value of wage cyclicality that is significantly lower than that in the data to correctly reproduce the impulse response of unemployment.

There remains much work to be done. For instance, our paper has not studied cyclicality in the degree of moral hazard frictions. Likewise, future work may be able to relate our framework to capacity utilization and classic theories of labor hoarding (e.g., Burnside et al., 1993). Finally, we hope that future reduced-form work will attempt to measure bargained wage cyclicality to complement our more structural approach.

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A Analytic Appendix

A.1 Implicit Definition of $\mathcal{B}(z_0)$ with Nash Bargaining

This subsection shows that Nash bargaining implicitly defines a functional form for $\mathcal{B}(z_0)$. Suppose that the firm and worker engage in generalized Nash bargaining over the surplus of the match and φ is the firm's bargaining power. Firms and workers take as given the utility that workers would receive were they to match with another firm next period $\mathcal{E}(z)$. Promised utility $\mathcal{B}(z_0)$ is implicitly defined by

$$\mathcal{B}(z_0) = \arg\max_{\overline{\mathcal{B}}} J\left(z_0, \overline{\mathcal{B}}\right)^{\varphi} \left(\overline{\mathcal{B}} - U(z_0)\right)^{1-\varphi}.$$

Here, as in the main text, $U(z_0)$ is the value of unemployment at time 0. $J\left(z_0, \overline{\mathcal{B}}\right)$ is defined by equations (11)-(13) in the main text, replacing $\mathcal{E}(z_0)$ with $\overline{\mathcal{B}}$ in equation (13). Therefore $\mathcal{B}(z_0)$ is the solution of the standard Nash bargaining problem, albeit in an environment with dynamic incentive pay. The solution is

$$\varphi \frac{\frac{\partial J(z_0, \mathcal{B}(z_0))}{\partial \overline{\mathcal{B}}}}{J(z_0, \mathcal{B}(z_0))} + \frac{(1-\varphi)}{\mathcal{B}(z_0) - U(z_0)} = 0. \tag{31}$$

Note that, when a firm and worker bargain, they take the expected outcome of a worker bargaining with other firms as given. Thus $U(z_0)$ does not itself depend directly on $\mathcal{B}(z_0)$. Therefore, equation (31) implicitly characterizes a particular choice for $\mathcal{B}(z_0)$ from the Nash bargain.

A.2 Proof of Theorem 1

First, we derive the relationship between the impulse response of tightness to TFP shocks, and the impulse response of firm value to TFP shocks, which will hold in both the flexible incentive pay and the rigid wage economy. From equation (9), the free entry condition is

$$\begin{aligned} & q(\theta_0) J(z_0) - \kappa = 0 \\ \implies & J(z_0) = \frac{\kappa}{q(\theta_0)} \\ \implies & \frac{d \ln \theta_0}{d \ln z_0} = \frac{1}{\nu_0} \frac{d \ln J(z_0)}{d \ln z_0}. \end{aligned} \tag{32}$$

where $-\nu_0$ is the elasticity of the vacancy filling rate given z_0. That is, the response of market tightness to aggregate productivity shocks is proportional to the response of the value of a filled job, as in the static model.

Now, we derive the dynamics of firm value and tightness in the rigid wage economy, which will also be a warm-up for deriving the dynamics of tightness in the flexible incentive pay economy. Using equation (15) from the main text, the value of a job in the rigid wage economy is

$$J^{\text{rigid}}(z_0) = \sum_{t=0}^{\infty} (\beta(1-s))^t \mathbb{E}\left[f(z_t, \eta_t) - \bar{w} | z_0, \bar{\mathbf{a}}\right]$$

$$\implies \frac{dJ^{\text{rigid}}(z_0)}{dz_0} = \sum_{t=0}^{\infty} (\beta(1-s))^t \frac{\partial}{\partial z_0} \mathbb{E}\left[f(z_t, \eta_t) | z_0, \bar{\mathbf{a}}\right]. \tag{33}$$

Using equations (32) and (33) from the Appendix and equation (15) from the main text, tightness dynamics in the rigid wage economy are then

$$\begin{aligned}
\frac{d\ln\theta_0}{d\ln z_0} &= \frac{1}{\nu_0} \frac{d\ln J^{\text{rigid}}(z_0)}{d\ln z_0} \\
&= \frac{1}{\nu_0} \frac{z_0}{J^{\text{rigid}}(z_0)} \frac{dJ^{\text{rigid}}(z_0)}{dz_0} \\
&= \frac{1}{\nu_0} \frac{z_0 \sum_{t=0}^{\infty} (\beta(1-s))^t \frac{\partial}{\partial z_0} \mathbb{E}\left[f(z_t, \eta_t) | z_0, \bar{\mathbf{a}}\right]}{\sum_{t=0}^{\infty} (\beta(1-s))^t \mathbb{E}\left[\left(f(z_t, \eta_t) - \bar{w}\right) | z_0, \bar{\mathbf{a}}\right]} \\
&= \frac{1}{\nu_0} \frac{\sum_{t=0}^{\infty} (\beta(1-s))^t \frac{\partial}{\partial \ln z_0} \mathbb{E}\left[f(z_t, \eta_t) | z_0, \bar{\mathbf{a}}\right]}{\sum_{t=0}^{\infty} (\beta(1-s))^t \mathbb{E}\left[\left(f(z_t, \eta_t) - \bar{w}\right) | z_0, \bar{\mathbf{a}}\right]}
\end{aligned}$$

which implies the first-order response of log tightness to a change $d\ln z_0$ is

$$d\ln\theta_0 = \frac{1}{\nu_0} \frac{\sum_{t=0}^{\infty} (\beta(1-s))^t \frac{\partial}{\partial \ln z_0} \mathbb{E}\left[f(z_t, \eta_t) | z_0, \bar{\mathbf{a}}\right] d\ln z_0}{\sum_{t=0}^{\infty} (\beta(1-s))^t \mathbb{E}\left[\left(f(z_t, \eta_t) - \bar{w}\right) | z_0, \bar{\mathbf{a}}\right]}, \tag{34}$$

i.e., equation (21) from the main text. Therefore we have derived the dynamics of tightness in the rigid wage economy.

Next, we turn to dynamics in the flexible incentive pay economy. To start, we must rewrite the firm's problem in the case of flexible incentive pay, using the impulse response notation introduced in the main text. Specifically, we let the contracts be given by $(\mathbf{w}, \mathbf{a}) =$

$\{w_t(\eta^t, \varepsilon^t; z_0), a_t(\eta^{t-1}, \varepsilon^t; z_0)\}_{t=0,\eta^t,\varepsilon^t}^{\infty}$ where $w_t(\eta^t, \varepsilon^t; z_0), a_t(\eta^{t-1}, \varepsilon^t; z_0)$ are continuous functions mapping from the history of idiosyncratic and aggregate shocks, and the initial state, to wages and effort. That is, contracts can depend on z_0 and a cumulative set of deviations from z_0. We use the fact that we consider impulse responses holding fixed a path of deviations to define the measure

$$\pi_t\left(\eta^t, \varepsilon^t | \mathbf{a}(z_0)\right) = \prod_{\tau=0}^{t} \pi_\tau\left(\eta_\tau | \eta^{\tau-1}, a^\tau\left(\eta^{\tau-1}, \varepsilon^\tau; z_0\right), \varepsilon^\tau\right) \pi_\tau(\varepsilon^\tau).$$

Thus the firm's problem becomes

$$J(z_0) = \max_{\mathbf{w}(z_0), \mathbf{a}(z_0)} \sum_{t=0}^{\infty} (\beta(1-s))^t \int\int \left(f\left(\mathbb{E}[z_t|z_0] + \varepsilon_t, \eta_t\right) - w_t(\eta^t, \varepsilon^t; z_0)\right) \tilde{\pi}_t\left(\eta^t, \varepsilon^t | \mathbf{a}(z_0)\right) d\eta^t d\varepsilon^t \tag{35}$$

subject to participation constraints

$$\sum_{t=0}^{\infty} (\beta(1-s))^t \Bigg[\int\int u\Big(w_t(\eta^t, \varepsilon^t; z_0), a_t(\eta^{t-1}, \varepsilon^t; z_0)\Big) \tilde{\pi}_t\left(\eta^t, \varepsilon^t | \mathbf{a}(z_0)\right) d\eta^t d\varepsilon^t + \beta s \int U(z_{t+1}) \hat{\pi}_t(z^{t+1}|z_0) dz^{t+1}\Bigg] \geq \mathcal{E}(z_0) \tag{36}$$

and incentive compatibility constraints

$$\sum_{t=0}^{\infty} (\beta(1-s))^t \Bigg[\int\int u\Big(w_t(\eta^t, \varepsilon^t; z_0), \tilde{a}_t(\eta^{t-1}, \varepsilon^t)\Big) \tilde{\pi}_t\left(\eta^t, \varepsilon^t | \tilde{\mathbf{a}}\right) d\eta^t d\varepsilon^t\Bigg]$$
$$\leq \sum_{t=0}^{\infty} (\beta(1-s))^t \Bigg[\int\int u\Big(w_t(\eta^t, \varepsilon^t; z_0), a_t(\eta^{t-1}, \varepsilon^t; z_0)\Big) \tilde{\pi}_t\left(\eta^t, \varepsilon^t |, \mathbf{a}(z_0)\right) d\eta^t d\varepsilon^t\Bigg] \tag{37}$$

for all $\tilde{\mathbf{a}} \in \mathcal{X}$. Finally, let $\Phi \equiv \{(\mathbf{w}, \mathbf{a}) \in \mathcal{X} : G(\mathbf{w}, \mathbf{a}) \leq 0\}$ be the set of feasible contracts that satisfy the IC and PC constraints.

To derive $d\ln J(z_0)/d\ln z_0$ in the flexible incentive pay economy, we seek to apply an envelope theorem. However, it is not trivial to show that an envelope theorem applies in our setting because the firm faces a continuum of constraints which may be non-convex. We therefore pursue two proof strategies which rely on different conditions, both of which are satisfied by our quantitative model. Our first proof in Section A.2.1 relies on the compactness of the set of incentive compatible mechanisms that satisfy the PC, as assumed in Assumption 1. We provide two alternative sets of conditions guaranteeing this compactness in Section A.4 below: (i) the time horizon is finite, and η, z have finite support, or (ii) regularity conditions on the contract, which areoutlined in Lemma 5.

Our second proof in Section A.2.2 makes the stronger assumptions of Assumption 2 in the main text. These assumptions allow us to reformulate the firm's problem using recursive contracts and a first-order approach (i.e., assuming that the incentive compatibility constraints may be summarized by the first-order condition to the worker's problem). The second proof is useful because it is closer to standard practice (e.g., Farhi and Werning, 2013) and because it derives results for the proof of Proposition 2.

Finally, after applying an envelope theorem, it is straightforward to derive the expression for the elasticity of market tightness in the flexible incentive pay economy with acyclical ex ante utility going to workers at the start of the contract, using similar steps to how we derived the impulse response of tightness in the rigid wage economy and equation (34).

A.2.1 Proof Strategy 1: Sequence Problem

We seek to apply Theorem 4.13 of Bonnans and Shapiro (2000), which is reproduced below:

Bonnans and Shapiro (2000) Theorem 4.13 *Consider the following optimization problem:*

$$\min_{x\in\mathcal{X}} V\left(x,z\right) \quad \textit{subject to } x\in\Phi$$

where z is a member of a Banach space Z, $\mathcal{X}$ is a Hausdorff topological space, $\Phi\subset\mathcal{X}$ is nonempty and closed, and $V:\mathcal{X}\times Z\to\mathbb{R}$ is continuous. Let the value function be defined as

$$J\left(z\right)\equiv\inf_{x\in\Phi(z)} V\left(x,z\right)$$

and the optimal control set be given by

$$\mathbf{\Gamma}^{*}\left(z\right)\equiv\arg\min_{x\in\Phi(z)} V\left(x,z\right).$$

Suppose that $z_0\in Z$ and

1. *For all $x\in\mathcal{X}$ the function $V\left(x,\cdot\right)$ is Gateaux differentiable*
2. *$V\left(x,z\right)$ and its partial Fréchet derivative with respect to z, given by $D_zV\left(x,z\right)$, are continuous on $\mathcal{X}\times Z$*
3. *There exists $M\in\mathbb{R}$ and a compact set $C\subset\mathcal{X}$ such that for every z near z_0 the set $A(z)\equiv\{x\in\Phi:V\left(x,z\right)\leq M\}$ is non-empty and contained in C.*

Then the optimal value function $z(\cdot)$ is Fréchet directionally differentiable at z_0 and

$$J'(z_0, d) = \inf_{x \in \mathbf{\Gamma}^*(z_0)} D_z V(x, z_0)\, d,$$

where d is the direction of the Fréchet derivative and $J'(z_0, d)$ is the Fréchet derivative of J with respect to z in that direction.

This theorem provides conditions under which the total derivative of the value function with respect to some parameter z is equal to the partial derivative of the value function with respect to that parameter, taking the smallest product of partial derivative and direction across the optimal control set. We verify the conditions of the theorem apply to the firm's problem, noting that the direction d corresponds to the sign of the increment $d\ln z_0$ in our uni-dimensional context.

First, the space of possible aggregate productivities Z is clearly a Banach space, and the set of feasible contracts $\mathcal{X}$ is a Hausdorff topological space. By Assumption 1, Φ is non-empty. In addition, the firm's objective function $V(x, z)$ is continuous and Gateaux differentiable since effort is assumed to continuously influence the measure of idiosyncratic profit shocks η. So too is its partial Fréchet derivative.

Thus, all that remains to be verified is: (i) the constraint set does not depend directly on z_0 and (ii) condition three of the theorem of Bonnans & Shapiro holds. To verify that the constraint set does not depend directly on z_0, note that by inspection, the incentive constraints (12) do not depend on z_0. With take it or leave it wage offers and acyclical unemployment benefits, as in the assumption of the Theorem, the participation constraint (13) simplifies to

$$[\mathbf{PC}] : \sum_{t=0}^{\infty} (\beta(1-s))^t \Bigg[\int\int u\Big(w_t(\eta^t, \varepsilon^t; z_0), a_t(\eta^{t-1}, \varepsilon^t; z_0)\Big) \tilde{\pi}_t\left(\eta^t, \varepsilon^t | \mathbf{a}(z_0)\right) d\eta^t d\varepsilon^t + \beta s \int U \hat{\pi}_{t+1}\left(z^{t+1}|z_0\right) dz^{t+1} \Bigg] \geq \mathcal{E}, \tag{38}$$

where now, by assumption, U and $\mathcal{E}$ are independent of z. Likewise, the bounds on w and a do not depend on z. Therefore z does not directly enter the constraints.

Since Φ is compact, also by Assumption 1, we can verify condition 3 of Bonnans and Shapiro (2000) Theorem 4.13. In particular, setting $C = \Phi$ and $M = \max_{z \in [\underline{z}, \overline{z}], x \in \Phi} V(x, z)$ verifies the condition. In this case C is compact. We also have $A(z) = C = \Phi$, because all contracts x in Φ have a value of less than M.

We have now validated the conditions of Bonnans and Shapiro (2000) Theorem 4.13 and this envelope theorem applies to our problem.

We now apply the envelope theorem. Using the fact that z_0 is scalar we write the right-hand derivative as

$$J'_{+}(z_0) = \sup_{x^* \in \mathbf{\Gamma}^*(z_0)} \frac{\partial}{\partial z_0} V(\mathbf{w}^*, \mathbf{a}^*; z_0)$$

$$\begin{aligned}
&= \sup_{x^* \in \mathbf{\Gamma}^*(z_0)} \frac{\partial}{\partial z_0} \left[\max_{\mathbf{w}(z_0), \mathbf{a}(z_0)} \sum_{t=0}^{\infty} (\beta(1-s))^t \int\int \big(f(\mathbb{E}[z_t|z_0] + \varepsilon_t, \eta_t) - w_t(\eta^t, \varepsilon^t; z_0) \big) \tilde{\pi}_t \left(\eta^t, \varepsilon^t | \mathbf{a}(z_0)\right) d\eta^t d\varepsilon^t \right] \\
&= \sup_{x^* \in \mathbf{\Gamma}^*(z_0)} \left[\sum_{t=0}^{\infty} (\beta(1-s))^t \frac{\partial}{\partial z_0} \int\int \big(f(\mathbb{E}[z_t|z_0] + \varepsilon_t, \eta_t) - w_t(\eta^t, \varepsilon^t; z_0) \big) \tilde{\pi}_t \left(\eta^t, \varepsilon^t | \mathbf{a}(z_0)\right) d\eta^t d\varepsilon^t \right] \\
&= \sup_{x^* \in \mathbf{\Gamma}^*(z_0)} \left[\sum_{t=0}^{\infty} (\beta(1-s))^t \frac{\partial}{\partial z_0} \int\int f(\mathbb{E}[z_t|z_0] + \varepsilon_t, \eta_t) \tilde{\pi}_t \left(\eta^t, \varepsilon^t | \mathbf{a}(z_0)\right) d\eta^t d\varepsilon^t \right] \qquad (39) \\
&= \sup_{x^* \in \mathbf{\Gamma}^*(z_0)} \left[\sum_{t=0}^{\infty} (\beta(1-s))^t \frac{\partial}{\partial z_0} \mathbb{E}[f(z_t, \eta_t) | \mathbf{a}(z_0)] \right]
\end{aligned}$$

where the second line substitutes in equation (35). Since f is continuously differentiable and Φ is compact, the supremum is attained at an optimum $x^*_+ \in \mathbf{\Gamma}^*$. Similarly, we have the left-hand derivative

$$J'_{-}(z_0) = \inf_{x^* \in \mathbf{\Gamma}^*(z_0)} \left[\sum_{t=0}^{\infty} (\beta(1-s))^t \frac{\partial}{\partial z_0} \mathbb{E}[f(z_t, \eta_t) | z_0, \mathbf{a}^*] \right],$$

and the infinimum is attained at an optimum $x^*_- \in \mathbf{\Gamma}^*$. Combining the left- and right-hand derivatives, it follows that to a first-order

$$\begin{aligned}
dJ(z_0) &= \sup_{x^* \in \mathbf{\Gamma}^*(z_0)} \left[\sum_{t=0}^{\infty} (\beta(1-s))^t \frac{\partial}{\partial z_0} \mathbb{E}[f(z_t, \eta_t) | z_0, \mathbf{a}^*] \, dz_0 \right] \\
&= \max_{x^* \in \mathbf{\Gamma}^*(z_0)} \left[\sum_{t=0}^{\infty} (\beta(1-s))^t \frac{\partial}{\partial z_0} \mathbb{E}[f(z_t, \eta_t) | z_0, \mathbf{a}^*] \, dz_0 \right] \qquad (40)
\end{aligned}$$

where if the increment dz_0 is negative then, in effect, the supremum converts to an infimum, and the second line replaces the sup with a max because the space of optimal contracts is compact. Noting that the value of $J(z_0)$ is the same for all optimal contracts, the preceding equation implies

$$\frac{dJ(z_0)}{J(z_0)} = \frac{1}{\sum_{t=0}^{\infty} (\beta(1-s))^t \mathbb{E}[f(z_t, \eta_t) - w^*_t | z_0, \mathbf{a}^*]} \max_{x^* \in \mathbf{\Gamma}^*(z_0)} \left[\sum_{t=0}^{\infty} (\beta(1-s))^t \frac{\partial}{\partial z_0} \mathbb{E}[f(z_t, \eta_t) | z_0, \mathbf{a}^*] \, dz_0 \right]$$

$$\implies d\ln J(z_0) = \frac{\max_{x^*\in\mathbf{\Gamma}^*(z_0)}\left[\sum_{t=0}^{\infty}(\beta(1-s))^t \frac{\partial}{\partial z_0}\mathbb{E}\left[f(z_t,\eta_t)|z_0,\mathbf{a}^*\right] z_0 \frac{dz_0}{z_0}\right]}{\sum_{t=0}^{\infty}(\beta(1-s))^t\,\mathbb{E}\left[f(z_t,\eta_t)-w_t^*|z_0,\mathbf{a}^*\right]}$$
$$= \frac{\max_{x^*\in\mathbf{\Gamma}^*(z_0)}\left[\sum_{t=0}^{\infty}(\beta(1-s))^t \frac{\partial}{\partial \ln z_0}\mathbb{E}\left[f(z_t,\eta_t)|z_0,\mathbf{a}^*\right] d\ln z_0\right]}{\sum_{t=0}^{\infty}(\beta(1-s))^t\,\mathbb{E}\left[f(z_t,\eta_t)-w_t^*|z_0,\mathbf{a}^*\right]}.$$

The above equation and equation (32) then imply

$$d\ln\theta_0 = \frac{1}{\nu_0}d\ln J(z_0)$$
$$= \frac{1}{\nu_0}\frac{\max_{x^*\in\mathbf{\Gamma}^*(z_0)}\left[\sum_{t=0}^{\infty}(\beta(1-s))^t \frac{\partial}{\partial \ln z_0}\mathbb{E}\left[f(z_t,\eta_t)|z_0,\mathbf{a}^*\right] d\ln z_0\right]}{\sum_{t=0}^{\infty}(\beta(1-s))^t\,\mathbb{E}\left[f(z_t,\eta_t)-w_t^*|z_0,\mathbf{a}^*\right]}$$
$$= \frac{1}{\nu_0}\frac{\sum_{t=0}^{\infty}(\beta(1-s))^t \frac{\partial}{\partial \ln z_0}\mathbb{E}\left[f(z_t,\eta_t)|z_0,\mathbf{a}^*\right] d\ln z_0}{\sum_{t=0}^{\infty}(\beta(1-s))^t\,\mathbb{E}\left[f(z_t,\eta_t)-w_t^*|z_0,\mathbf{a}^*\right]}$$

where the last equality holds for some $(\mathbf{w}^*,\mathbf{a}^*)\in\mathbf{\Gamma}^*(z_0)$. In particular, $(\mathbf{w}^*,\mathbf{a}^*)$ either maximizes the direct productivity effect among optimal contracts if $d\ln z_0$ is positive; or minimizes the direct productivity effect if $d\ln z_0$ is positive. We have derived equation (20) from the main text, characterizing the impulse response of tightness in the flexible incentive pay economy.

To prove the final part of the theorem, we now assume that the left- and right-hand partial derivatives of $dJ(z_0)$ are equal. A sufficient condition for this to hold is that the set of optimal contracts is a singleton.[33] We now derive the simplified expression for tightness dynamics in the neighborhood of the steady state, equation (22) from the main text. Starting from equation (39), we have

$$J'(z_0) = \max_{x^*\in\mathbf{\Gamma}^*(z_0)}\left[\sum_{t=0}^{\infty}(\beta(1-s))^t\frac{\partial}{\partial z_0}\int\int f\left(\mathbb{E}[z_t|z_0]+\varepsilon_t,\eta_t\right)\tilde{\pi}_t\left(\eta^t,\varepsilon^t|\mathbf{a}^*(z_0)\right)d\eta^t d\varepsilon^t\right]$$
$$= \max_{x^*\in\mathbf{\Gamma}^*(z_0)}\left[\sum_{t=0}^{\infty}(\beta(1-s))^t\int\int \frac{\partial}{\partial z_0}f\left(\mathbb{E}[z_t|z_0]+\varepsilon_t,\eta_t\right)\tilde{\pi}_t\left(\eta^t,\varepsilon^t|\mathbf{a}^*(z_0)\right)d\eta^t d\varepsilon^t\right]$$
$$= \max_{x^*\in\mathbf{\Gamma}^*(z_0)}\left[\sum_{t=0}^{\infty}(\beta(1-s))^t\int\int f_z(z_t,\eta_t)\frac{\partial\mathbb{E}[z_t|z_0]}{\partial z_0}\tilde{\pi}_t\left(\eta^t,\varepsilon^t|\mathbf{a}^*(z_0)\right)d\eta^t d\varepsilon^t\right]$$
$$= \max_{x^*\in\mathbf{\Gamma}^*(z_0)}\left[\sum_{t=0}^{\infty}(\beta(1-s))^t\,\mathbb{E}\left[f_z(z_t,\eta_t)|\mathbf{a}^*(z_0)\right]\frac{\partial\mathbb{E}[z_t|z_0]}{\partial z_0}\right],$$

[33] When the left- and right-hand derivatives of $dJ(z_0)$ are different, we can still derive tightness dynamics for negative and positive shocks in the neighborhood of the steady state.

which applying a similar reasoning to the derivation of equation (20) implies

$$
\begin{aligned}
d\ln\theta_0 &= \frac{1}{\nu_0}\frac{\max_{x^*\in\mathbf{\Gamma}^*(z_0)}\left[\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\mathbb{E}\left[f_z\left(z_t,\eta_t\right)|\mathbf{a}^*\left(z_0\right)\right]\frac{\partial\mathbb{E}[z_t|z_0]}{\partial\ln z_0}d\ln z_0\right]}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\mathbb{E}\left[f(z_t,\eta_t)-w_t^*|z_0,\mathbf{a}^*\right]}\\
&= \frac{1}{\nu_0}\frac{\max_{x^*\in\mathbf{\Gamma}^*(z_0)}\left[\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\mathbb{E}\left[f_z\left(z_t,\eta_t\right)|\mathbf{a}^*\left(z_0\right)\right]z_0\frac{\partial\mathbb{E}[z_t|z_0]}{\partial z_0}d\ln z_0\right]}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\mathbb{E}\left[f(z_t,\eta_t)-w_t^*|z_0,\mathbf{a}^*\right]}\\
&= \frac{1}{\nu_0}\frac{\max_{x^*\in\mathbf{\Gamma}^*(z_0)}\left[\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\mathbb{E}\left[f(\eta_t)|\mathbf{a}^*\left(z_0\right)\right]z_0 d\ln z_0\right]}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\mathbb{E}\left[f(z_t,\eta_t)-w_t^*|z_0,\mathbf{a}^*\right]}\\
&= \frac{1}{\nu_0}\frac{\max_{x^*\in\mathbf{\Gamma}^*(z_0)}\left[\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\mathbb{E}\left[\bar{z}f(\eta_t)|\bar{z},\mathbf{a}^*\left(\bar{z}\right)\right]d\ln z_0\right]}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\mathbb{E}\left[f(\bar{z},\eta_t)-w_t^*|\bar{z},\mathbf{a}^*\left(\bar{z}\right)\right]}\\
&= \frac{1}{\nu_0}\frac{\max_{x^*\in\mathbf{\Gamma}^*(z_0)}\left[\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\mathbb{E}\left[f\left(\bar{z},\eta_t\right)|\bar{z},\mathbf{a}^*\left(\bar{z}\right)\right]d\ln z_0\right]}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\mathbb{E}\left[f(\bar{z},\eta_t)-w_t^*|\bar{z},\mathbf{a}^*\left(\bar{z}\right)\right]}\\
&= \frac{1}{\nu_0}\frac{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\mathbb{E}\left[f\left(\bar{z},\eta_t\right)|\bar{z},\mathbf{a}^*\left(\bar{z}\right)\right]}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\mathbb{E}\left[f(\bar{z},\eta_t)-w_t^*|\bar{z},\mathbf{a}^*\left(\bar{z}\right)\right]}d\ln z_0\\
&= \frac{1}{\nu_0}\frac{1}{1-\frac{\sum_{t=0}^{\infty}(\beta(1-s))^t\mathbb{E}[w_t^*|\bar{z},\mathbf{a}^*(\bar{z})]}{\sum_{t=0}^{\infty}(\beta(1-s))^t\mathbb{E}[f(\bar{z},\eta_t)|\bar{z},\mathbf{a}^*(\bar{z})]}}d\ln z_0
\end{aligned}
$$

$$
\implies \frac{d\ln\theta_0}{d\ln z_0} = \frac{1}{\nu_0}\frac{1}{1-\frac{\sum_{t=0}^{\infty}(\beta(1-s))^t\mathbb{E}[w_t^*|\bar{z},\mathbf{a}^*(\bar{z})]}{\sum_{t=0}^{\infty}(\beta(1-s))^t\mathbb{E}[f(\bar{z},\eta_t)|\bar{z},\mathbf{a}^*(\bar{z})]}}; \tag{41}
$$

The third line uses the fact that $\frac{\partial\mathbb{E}[z_t|z_0]}{\partial z_0}=1$ because z_t follows a driftless random walk and $f_z\left(z_t,\eta_t\right)=\eta_t$ because $f\left(z_t,\eta_t\right)$ is homogeneous of degree one in z_t; the fourth line evaluates derivatives at the non-stochastic state in which $z_t=\bar{z}$; and the sixth line uses the uniqueness of the optimal contract. Equation (41) is the same as equation (22) from the main text, for the case of flexible incentive pay economy. The derivation of equation (41) for the case of rigid wages is virtually identical, so we do not repeat it here. This derivation completes the proof of Theorem 1. □

A.2.2 Proof Strategy 2: First Order Approach and Recursive Formulation

We now show how to apply an envelope theorem to the flexible incentive pay problem, under the stronger assumptions of Assumption 2 of the main text. This proof is clarifying because the approach is closer to standard practice, and it will also be useful because it derives results that are necessary for Proposition 2. Therefore for this subsection we make both Assumptions 1 and 2 from the main text.

The application of the envelope theorem proceeds in three steps in this strategy. First we derive a first-order approach to simplify incentive constraints into local incentive constraints as in Farhi and Werning (2013) or Pavan et al. (2014). Then we develop a recursive formulation of the problem. Finally, we use these constructions to prove our main theorem.

Step 1: First Order Approach The first-order condition for a_t in the worker's problem (12) given a contract is

$$0 = \int \int \left[u_a \Big(w_t(\eta^t, z^t), a_t(\eta^{t-1}, z^t) \Big) \tilde{\pi}_t \left(\eta^t, z^t | z_0, \mathbf{a} \right) + u \Big(w_t(\eta^t, z^t), a_t(\eta^{t-1}, z^t) \Big) \frac{\partial}{\partial a_t} \tilde{\pi}_t \left(\eta^t, z^t | z_0, \mathbf{a} \right) \right] d\eta^t dz^t$$

Note that this holds for every t and realization of z^t. Thus one can remove the outer integral to write first-order incentive constraints as

$$\int \left[u_a \Big(w_t(\eta^t, z^t), a_t(\eta^{t-1}, z^t) \Big) \pi_t \left(\eta_t | z^t, \eta^{t-1}, \mathbf{a} \right) + u \Big(w_t(\eta^t, z^t), a_t(\eta^{t-1}, z^t) \Big) \frac{\partial}{\partial a_t} \pi_t \left(\eta_t | z^t, \eta^{t-1}, \mathbf{a} \right) \right] d\eta_t = 0$$

Step 2: Recursive Formulation We will work with the relaxed problem and develop a recursive formulation of the firm's problem. Notationally, let the value of some variable X in the period t problem be given by X, the value of X in $t-1$ be given as X_- and the value of X in $t+1$ be given by X'. Suppressing explicit dependence of the problem on initial productivity z_0 for notational convenience, the recursive formulation of the firm's problem is then (we now drop the history dependence with the assumption that the process for η is a Markov process):

$$\begin{aligned} J(v_-, \eta_-, z_-, t) = \max_{a(\eta_-, z), w(\eta, z), v(\eta, z)} \int \int \Big[& f(\eta, z) - w(\eta, z) \\ & + \beta (1-s) J(v(\eta, z), \eta, z, t+1) \Big] \pi(\eta | z, \eta_-, a(\eta_-, z)) \hat{\pi}(z | z_-) d\eta dz \end{aligned} \tag{42}$$

subject to the following constraints:

$$\omega(\eta, z) = \quad u(w(\eta, z), a(\eta_-, z)) + \beta \left[(1-s) v(\eta, z) + s \int U(z') \hat{\pi}(z' | z) dz' \right] \tag{43}$$

for all η and z realizations,

$$[\lambda]: \quad v_- \leq \int \int \omega(\eta, z) \pi(\eta | z, \eta_-, a(\eta_-, z)) \hat{\pi}(z | z_-) d\eta dz, \tag{44}$$

and the first-order incentive constraints:

$$\int \left[u_a \Big(w(\eta, z), a(\eta_-, z) \Big) \pi(\eta | z, \eta_-, a) + u \Big(w(\eta, z), a(\eta_-, z) \Big) \frac{\partial}{\partial a} \pi(\eta | z, \eta_-, a) \right] d\eta = 0. \tag{45}$$

We now explain this problem. The firm begins period t knowing the prior realization of shocks z_- and η_- and inherits a utility it must promise to the worker over the remaining life of the contract, which we denote v_-. The firm's flow profits are the expected output $f(\eta, z)$ minus their expected wage payments $w(\eta, z)$. Firms additionally receive a continuation value with probability $1 - s$, which they discount at rate β. The firm maximizes the sum of flow profits and continuation values by choosing the suggested effort and wage functions for every realization of η and z, as well as a function for next period's promised utility to the worker $v(\eta, z)$, subject to some constraints that we now describe.

The worker's value under the contract given a realization (η, z) is given by $\omega(\eta, z)$, defined in equation (43). It is equal to the worker's flow utility $u(w(\eta, z), a(\eta_-, z)$ plus a continuation value. With probability s, the match dissolves and the worker receives the value of unemployment. With probability $1 - s$, the match survives and the worker receives $v(\eta, z)$.

The recursive version of the participation constraint states that the worker's expected value under the contract must be at least the value promised to them v, and is given by equation (44). Note that v_- in the initial period of the match maps to the utility promised to the worker overall $\mathcal{B}(z_0)$ in the non-recursive formulation of the problem. For periods after the start of the contract, equation (44) may be interpreted as a promise-keeping constraint. Equation (45) is the relaxed incentive constraint described above.

Let the Lagrangian of the recursive problem be defined by $\int \int \mathcal{L}(\cdot) d\eta dz$ for

$$
\begin{aligned}
\mathcal{L} \equiv & [f(\eta, z) - w(\eta, z; z_0)] \pi(\eta | z, \eta_-, a(\eta_-, z)) \hat{\pi}(z | z_-) \\
& + \beta (1 - s) \Big[J(v(\eta, z), \eta, z, t + 1) \Big] \pi(\eta | z, \eta_-, a(\eta_-, z)) \hat{\pi}(z | z_-) \\
& - \lambda [v_- - \omega(\eta, z) \pi(\eta | z, \eta_-, a(\eta_-, z)) \hat{\pi}(z | z_-)] \\
& - \gamma(z) \left[u_a \Big(w(\eta, z), a(\eta_-, z) \Big) \pi(\eta | z, \eta_-, a) + u \Big(w(\eta, z), a(\eta_-, z) \Big) \frac{\partial}{\partial a} \pi(\eta | z, \eta_-, a) \right],
\end{aligned} \tag{46}
$$

where λ is the Lagrange multiplier on the participation constraint and $\gamma(z)$ is the multiplier on the incentive constraint given aggregate productivity z. Again, we suppress dependence on z_0, but the firm's choice variables and the distribution of z and η may all depend on z_0.

Next, we introduce the change of variable with the notation $z_t = \mathbb{E}[z_t | z_0] + \varepsilon_t$, where by definition, ε_t is the cumulative innovation to the process for z between 0 and t and ε_0 is known to be 0. One can write the Lagrangian as:

$$\begin{aligned}
\mathcal{L} &= [f\left(\eta, \mathbb{E}[z|z_0] + \varepsilon\right) - w\left(\eta, \varepsilon\right)]\pi\left(\eta|\varepsilon, \eta_-, a(\eta_-, \varepsilon)\right)\hat{\pi}(\varepsilon|\varepsilon_-) \\
&+ \beta\left(1-s\right)\Big[J\left(v(\eta,\varepsilon), \eta, \varepsilon, t+1\right)\Big]\pi\left(\eta|\varepsilon, \eta_-, a(\eta_-, \varepsilon)\right)\hat{\pi}(\varepsilon|\varepsilon_-) \\
&- \lambda[v_- - \omega(\eta,\varepsilon)\pi\left(\eta|\varepsilon, \eta_-, a(\eta_-, \varepsilon)\right)\hat{\pi}(\varepsilon|\varepsilon_-)] \\
&- \gamma(\varepsilon)\left[u_a\Big(w(\eta,\varepsilon), a(\eta_-,\varepsilon)\Big)\pi\left(\eta|\varepsilon,\eta_-,a\right) + u\Big(w(\eta,\varepsilon), a(\eta_-,\varepsilon)\Big)\frac{\partial}{\partial a}\pi\left(\eta|\varepsilon,\eta_-,a\right)\right]
\end{aligned} \tag{47}$$

Step 3: Envelope Theorem We seek to apply of Theorem 1 of Marimon and Werner (2021), which relies on the following technical assumptions.

Technical Assumptions:

TA1. The set $\mathcal{X}$ of feasible allocations is convex, and f, u, π, u_a, and π_a are continuous functions of $\{a, w, z_0\}$

TA2. The constraint set $\mathcal{G}(z_0) = \{(\mathbf{w}, \mathbf{a}) \in \mathcal{X} : G(\mathbf{w}, \mathbf{a}; z_0) \leq 0\}$ is compact for every $z \in Z$, a neighborhood of z_0, and there exists a contract $(\mathbf{w}, \mathbf{a})$ such that the participation constraint (44) is slack.

TA3. The set of optimal contracts is non-empty.

We argue these conditions apply in our setting. $\mathcal{X}$ is convex as the product of segments. Under Assumption 1, $\mathcal{X}$ is compact. Then the constraint set $\mathcal{G}(z_0)$ is a closed subset of a compact and so is compact. What's more, there exists a contract such that the participation constraint is slack since, for every z_0 and promised utility v_-, there exists a feasible continuation value and effort $\omega(\eta, z), a(\eta_-, z)$ that yield strictly higher utility than v_-: that is inequality (44) is strict. Finally, since from Assumption 1, $\mathcal{X}$ is compact and non-empty, the set of optimizers of our continuous objective (i.e., the set of optimal contracts) is non-empty.[34]

One can now apply the envelope theorem of Marimon and Werner (2021) to argue that the derivative of the value function with respect to all variables the firm chooses and costates – a^*, w^*, v^*, λ^*, and γ^* – sum to zero. Therefore, differentiating the Lagrangian (47) with

[34] This envelope theorem is better suited for our purposes than Corollary 5 of Milgrom and Segal (2002) since it does not require compactness assumptions on the support of the shocks.

respect to z_0 and substituting in for $\omega(\eta,\varepsilon)$ yields the right-hand derivative:

$$\frac{\partial J(v_-,\eta_-,z_-,t)}{\partial z_0^+} = \sup_{(\mathbf{w}^*,\mathbf{a}^*)\in\boldsymbol{\Gamma}^*(z_0)} \int\int \frac{\partial}{\partial z_0}[f\left(\eta,\mathbb{E}[z|z_0]+\varepsilon\right)]\pi\left(\eta|\varepsilon,\eta_-,a^*(\eta_-,\varepsilon)\right)\hat{\pi}(\varepsilon|\varepsilon_-)d\eta d\varepsilon \tag{48}$$

$$+\beta\left(1-s\right)\int\int\frac{\partial}{\partial z_0^+}\Big[J\left(v^*(\eta,\varepsilon),\eta,\varepsilon,t+1\right)\Big]\pi\left(\eta|\varepsilon,\eta_-,a^*\right)\hat{\pi}(\varepsilon|\varepsilon_-)d\eta d\varepsilon$$
$$+\beta s\lambda^*(\eta_-,z_-)\int\int\frac{\partial}{\partial z_0}U\left(\mathbb{E}[z'|z_0]+\varepsilon'\right)\hat{\pi}\left(\varepsilon'|\varepsilon\right)\hat{\pi}(\varepsilon|\varepsilon_-)d\varepsilon' d\varepsilon.$$

This is a refinement of a recursive version of equation (17): the first-order impact of aggregate productivity on the value of a filled job is given by the sum of the direct effect on the firm's flow and continuation values, plus the direct effect on the constraints. Two terms are missing from the fuller decomposition in equation (17). First, the "B-term" features no direct effect on incentive constraints. This arises from the assumption that the distribution of η and ε do not directly depend on z_0. Second, the "C-term" – the indirect effect on firm value that arises from changes in the contracted wages or effort – does not appear because we have applied the envelope theorem of Marimon and Werner (2021).

We can write explicitly the sequence of participation constraints from time 0 as:

$$\begin{aligned} \lambda_-(z_0): &\qquad \mathcal{E}(z_0)\leq v \\ \lambda_{t-1}(\eta^{t-1},z^{t-1}): &\qquad v_{t-1}(\eta^{t-1},z^{t-1})\leq \textstyle\int\int\omega(\eta^t,z^t)\pi\left(\eta_t|z^t,\eta^{t-1},a(\eta^{t-1},z^t)\right)\hat{\pi}(z_t|z^{t-1})d\eta_t dz_t,\ \forall t\geq 1. \end{aligned}$$

The corresponding sequential participation constraints are:

$$[\lambda_-(z_0)]: \quad \mathcal{B}\left(z_0\right)\leq\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\Bigg[\int\int u\Big(w_t(\eta^t,\varepsilon^t;z_0),a_t(\eta^{t-1},\varepsilon^t;z_0)\Big)\tilde{\pi}_t\left(\eta^t,\varepsilon^t|\mathbf{a}(z_0)\right)d\eta^t d\varepsilon^t$$
$$+\beta s\int U\left(\mathbb{E}[z_{t+1}|z_0]+\varepsilon_{t+1}\right)\hat{\pi}_{t+1}\left(\varepsilon^{t+1}\right)d\varepsilon^{t+1}\Bigg]$$
$$[\lambda_\tau(\eta^\tau,\varepsilon^\tau;z_0)]: \quad \sum_{t=\tau+1}^{\infty}\left(\beta\left(1-s\right)\right)^{t-\tau-1}\Bigg[\int\int u\Big(w_t(\eta^t,\varepsilon^t;z_0),a_t(\eta^{t-1},\varepsilon^t;z_0)\Big)\tilde{\pi}_t\left(\eta^t,\varepsilon^t|\mathbf{a}(z_0)\right)d\eta^t d\varepsilon^t$$
$$+\beta s\int U\left(\mathbb{E}[z_{t+1}|z_0]+\varepsilon_{t+1}\right)\hat{\pi}_{t+1}\left(\varepsilon^{t+1}\right)d\varepsilon^{t+1}\Bigg]\geq v_\tau(\eta^\tau,\varepsilon^\tau;z_0),\quad \forall\tau=0,\ldots,+\infty \tag{49}$$

Now we apply the envelope theorem to the problem recursively, replacing $\mathcal{E}$ with its equilibrium value $\mathcal{B}$ to obtain

$$\frac{\partial J}{\partial z_0} = \sup_{\{w^*,a^*\}\in\mathbf{\Gamma}^*(z_0)} \sum_{t=0}^{+\infty} \Big[\int\int (\beta(1-s))^t \frac{\partial f(\mathbb{E}[z_t|z_0]+\varepsilon_t,\eta_t)}{\partial z_0} \tilde{\pi}_t\left(\eta^t,\varepsilon^t|\mathbf{a}(z_0)\right) d\eta^t d\varepsilon^t \tag{50}$$
$$- \lambda_-(z_0)\Big[\frac{\partial\mathcal{B}(z_0)}{\partial z_0} - \beta s \sum_{t=1}^{+\infty} \int\int (\beta(1-s))^{t-1} \frac{\partial U(\mathbb{E}[z_t|z_0]+\varepsilon_t)}{\partial z_0} \hat{\pi}\left(\varepsilon^t\right) d\varepsilon^t\Big]$$
$$+ \sum_{\tau=0}^{\infty} \int\int \lambda_\tau(\eta^\tau,\varepsilon^\tau;z_0)\beta s \Big[\sum_{t=\tau+2}^{+\infty} \int\int (\beta(1-s))^{t-\tau-2} \frac{\partial U(\mathbb{E}[z_t|z_0]+\varepsilon_t)}{\partial z_0} \hat{\pi}\left(\varepsilon^t|\varepsilon^\tau\right) d\varepsilon^t\Big] \times$$
$$\tilde{\pi}_\tau(\eta^\tau,\varepsilon^\tau|\mathbf{a}(z_0))\, d\eta^\tau d\varepsilon^\tau$$

When the outside option of the worker is acyclical and TIOLI, we have:

$$\frac{\partial J(z_0)}{\partial z_0} = \sup_{\{w^*,a^*\}\in\mathbf{\Gamma}^*(z_0)} \sum_{t=0}^{\infty} (\beta(1-s))^t \Big[\int\int \frac{\partial f(\mathbb{E}[z_t|z_0]+\varepsilon_t,\eta_t)}{\partial z_0} \tilde{\pi}_t\left(\eta^t,\varepsilon^t|\mathbf{a}^*(z_0)\right) d\eta^t d\varepsilon^t. \tag{51}$$

The preceding equation is the same as equation (40) from A.2.1, the previous application of the envelope theorem. Therefore the same manipulations performed at the end of Section A.2.1 yields the market tightness dynamics in Theorem 1. □

A.3 Proof of Proposition 2

First, we derive equation (26) from the main text. From equation (24) we have

$$\frac{dJ(z_0)}{dz_0} = \frac{\partial\mathcal{Y}(\mathbf{a}^*(z_0);z_0)}{\partial z_0} - \left(\frac{d\mathcal{W}(z_0)}{dz_0} - \partial_{\mathbf{a}}\mathcal{Y}(\mathbf{a}^*(z_0);z_0)\frac{d\mathbf{a}^*}{dz_0}\right),$$

and from equation (25) we have

$$\frac{\partial\mathcal{W}^{\text{bargained}}(z_0)}{\partial z_0} \equiv \frac{d\mathcal{W}(z_0)}{dz_0} - \partial_{\mathbf{a}}\mathcal{Y}(\mathbf{a}^*(z_0);z_0)\frac{d\mathbf{a}^*}{dz_0}.$$

The preceding two equations imply

$$\begin{aligned}\frac{dJ(z_0)}{dz_0} &= \frac{\partial\mathcal{Y}(\mathbf{a}^*(z_0);z_0)}{\partial z_0} - \frac{\partial\mathcal{W}^{\text{bargained}}(z_0)}{\partial z_0}\\ &= \sum_{t=0}^{\infty}(\beta(1-s))^t\frac{\partial}{\partial z_0}\mathbb{E}\left[f(z_t,\eta_t)|z_0,\mathbf{a}^*\right] - \frac{\partial\mathcal{W}^{\text{bargained}}(z_0)}{\partial z_0}\end{aligned}$$

$$\implies \frac{d\ln J\left(z_0\right)}{d\ln z_0} = \frac{z_0\left(\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t \frac{\partial}{\partial z_0}\mathbb{E}\left[f(z_t,\eta_t)|z_0,\mathbf{a}^*\right] - \frac{\partial \mathcal{W}^{\text{bargained}}(z_0)}{\partial z_0}\right)}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t \mathbb{E}\left[f(z_t,\eta_t) - w_t^*|z_0,\mathbf{a}^*\right]}$$
$$= \frac{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t \frac{\partial}{\partial \ln z_0}\mathbb{E}\left[f(z_t,\eta_t)|z_0,\mathbf{a}^*\right] - \frac{\partial \mathcal{W}^{\text{bargained}}(z_0)}{\partial \ln z_0}}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t \mathbb{E}\left[f(z_t,\eta_t) - w_t^*|z_0,\mathbf{a}^*\right]},$$

which is equation (26) from the main text.

Now we are going to prove the "moreover" statement that bargained wage cyclicality is positive if and only if promised utility is procyclical. The derivation makes use of equation (50) derived in Section A.2.2. Suppose the optimal contract features optimal choices for wages and effort which are in the interior of Φ, which is true under the Inada conditions made in Assumption 2. Then the additional Lagrangian terms after time zero are non-binding and equation (50) becomes

$$\frac{\partial J}{\partial z_0^+} = \sup_{\{w^*,a^*\}\in\mathbf{\Gamma}^*(z_0)} \sum_{t=0}^{\infty}\Big[\int\int \left(\beta\left(1-s\right)\right)^t \frac{\partial f\left(\mathbb{E}[z_t|z_0]+\varepsilon_t,\eta_t\right)}{\partial z_0}\tilde{\pi}_t\left(\eta^t,\varepsilon^t|\mathbf{a}^*\left(z_0\right)\right)d\eta^t d\varepsilon^t \quad (52)$$
$$-\lambda_{PC}^*(z_0)\Big[\frac{\partial \mathcal{B}(z_0)}{\partial z_0} - \beta s\sum_{t=1}^{+\infty}\int\int\left(\beta\left(1-s\right)\right)^{t-1}\frac{\partial U\left(\mathbb{E}[z_t|z_0]+\varepsilon_t\right)}{\partial z_0}\hat{\pi}\left(\varepsilon^t\right)d\varepsilon^t\Big]$$

Under Assumption 1, $\mathcal{X}$ is compact, and so the supremum is achieved at a contract $\{w^*, a^*\} \in \mathbf{\Gamma}^*(z_0)$. Evaluated at that optimum and comparing equation (26) with equation (52) yields

$$\lambda_{PC}^*(z_0)\left[\frac{\partial\tilde{\mathcal{B}}(z_0)}{\partial z_0}\right] = \frac{\partial \mathcal{W}^{bargained}(z_0)}{\partial z_0}.$$

Finally, $\lambda_{PC}^*(z_0) > 0$ because the participation constraint must bind on the optimal contract. It immediately follows that ${\partial \mathcal{W}^{bargained}(z_0)}/{\partial z_0} > 0$ if and only if ${\partial\tilde{\mathcal{B}}(z_0)}/{\partial z_0} > 0$. □

A.4 Sufficient Conditions for Compactness of Φ

This section provides two sets of sufficient conditions for Φ to be compact. The first condition is that η and z have finite support and contracts last at least T periods, for T finite. The second is that contracts are continuous and twice differentiable in their arguments with uniformly bounded first and second derivatives, and that T is finite.

Lemma 4. *If η and z have finite support and the time horizon is finite, then the choice set of contracts is compact and the envelope theorem holds.*

Proof. Suppose $\eta_t \in \{\eta_1,\ldots,\eta_N\}$ and $z_t \in \{z_1,\ldots,z_M\}$ have finite support and T is finite. We will show that the space of $[w,a]$ functions of (η,z) that are IC and PC is compact.

Consider a sequence of functions $[w_n, a_n]$ that are IC and satisfy the PC. The sequence $[w_n(\eta_1, z_1), a_n(\eta_1, z_1)]$ takes values in a compact set. Therefore, it has a subsequence that converges. Call it $[w_{\phi_{1,1}(n)}(\eta_1, z_1), a_{\phi_{1,1}(n)}(\eta_1, z_1)]$. Now apply the same reasoning to the sequence $[w_{\phi_{1,1}(n)}(\eta_1, z_2), a_{\phi_{1,1}(n)}(\eta_1, z_2)]$; similarly, it is in a compact set, so it has a subsequence that converges.

Through this diagonal argument, we construct $[w_{\phi_{1,1}\circ\phi_{1,2}\cdots\circ\phi_{N,M}(n)}, a_{\phi_{1,1}\circ\phi_{1,2}\cdots\circ\phi_{N,M}(n)}]$: a sub-sequence of *functions* that converges. Now we need to show that the limiting function is also in the set, that is, it is IC and satisfies PC.

The PC is a closed inequality involving continuous functions. Due to the continuity of the involved functions, the limit of any sequence of functions satisfying the PC will also satisfy it. Analogously, for the incentive compatibility constraint (IC), consider fixing an action $\tilde{a}()$. Any sequence of $[w, a]$ satisfying the IC inequality for $\tilde{a}(\cdot)$, by the continuity of the functions involved, will satisfy it at the limit. Since this applies for all $\tilde{a}(\cdot)$, the limiting function must be IC. We began with an arbitrary sequence of $[w, a]$ that are both IC and PC, and we have shown that it has a subsequence converging to a limit that is also IC and PC. Therefore, the space of mechanisms is compact.

We can now employ a standard envelope theorem in this case given that the choice set is compact and Corollary 4 of Milgrom and Segal (2002) applies.

□

Lemma 5. *The set of feasible contracts that satisfy the IC constraints, Φ, is compact, if contracts are restricted to being continuous and twice differentiable in their arguments $\{\eta^t, z^t\}$, with uniformly bounded first and second derivatives and the horizon T is finite.*

Proof. We will show that Φ is equicontinuous.[35] Let $\Xi = [\underline{\eta}, \bar{\eta}]$ be the set of possible values for η. Consider a set of functions that are continuously differentiable on $[0, 1]$ and such that both the functions and their first and second derivatives are uniformly bounded. This means there exists some real number M such that for every function f in the set and every $x \in \Xi^T \times Z^T$, $||f(x)|| \leq M$ and for its Jacobian $||Df(x)|| \leq M$, where $||\cdot||$ is the Euclidian norm in $\Xi^T \times Z^T$.

Given $\epsilon > 0$, choose $\delta = \epsilon/2M$. Then for any function f in Φ and any points x and y in $\Xi^T \times Z^T$ such that $||x - y|| < \delta$, by the mean value theorem, we have $||f(x) - f(y)||_\infty = |Df(c)| \cdot ||x - y||$ for some c in the line $xt + (1 - t)y$, $t \in [0; 1]$. Since $|Df(c)| \leq M$ and $|x - y| < \delta = \epsilon/2M$, we get $||f(x) - f(y)||_\infty < \epsilon/2$.

[35] Φ is said to be equicontinuous at a point $x \in \Xi^T \times Z^T$ if, for every $\epsilon > 0$, there exists a $\delta > 0$ such that for every function f in Φ and every point y in $\Xi^T \times Z^T$, if $||x - y|| < \delta$ then $||f(x) - f(y)||_{C^1} < \epsilon$, where $||\cdot||_{C^1}$ is the C^1 norm.

Similarly, we can apply the mean value theorem to the Jacobian of f, and since the second derivatives are bounded, an analogous argument to that above yields $||Df(x) - Df(y)||_\infty < \epsilon/2$. Therefore $||f(x) - f(y)||_{C^1} \equiv ||f(x) - f(y)||_\infty + ||Df(x) - Df(y)||_\infty < \epsilon$ and we have shown that Φ is equicontinuous. By the Ascoli Theorem, any sequence in Φ thus has a subsequence that converges. Therefore Φ is compact. $\square$

A.5 Proof of Proposition 3

The contracting environment is nearly identical to that of Edmans et al. (2012) (without private savings), and the derivation of the optimal contract is thus very similar; therefore, we leave some of the technical details of the proof to that paper. First, note that as is standard in dynamic agency problems without private savings and separable preferences over consumption and effort (Rogerson, 1985; Farhi and Werning, 2013), an Inverse Euler Equation (IEE) holds. With logarithmic utility and the assumption firms and workers share β as a common discount factor, the IEE reads

$$w_t(\eta^t, \mathbf{a}|z^t) = \mathbb{E}_t[w_{t+1}(\eta^{t+1}, \mathbf{a}|z^{t+1})]. \tag{53}$$

The inverse of the agent's discounted marginal utility — which is simply the wage in this case with logarithmic utility — is the marginal cost of delivering utility to the worker. Equation (53) states that the expected marginal cost of delivering utility to the worker is equalized across periods, otherwise the principal would deliver utility to the worker in relatively low cost periods. Note that this equation dictates that wages are a martingale process and implies that the optimal contract smooths worker consumption.

We begin by solving for the optimal difference wage schedule (27). To do so, we begin by considering a finite horizon contract, with duration T, and then take the limit as $T \to \infty$.

Differentiating the worker's incentive compatibility constraint with respect to a_T (wiith binding local constraints) given realizations of η^T and z^T yields

$$\frac{1}{w_T(y^T, z^T)} \frac{\partial w_T(y^T, z^T)}{\partial a_T} = h'(a_T).$$

Since the firm cannot distinguish η_T from a_T, it must be the case that $\partial w_T / \partial \eta_T = \partial w_T / \partial a_T$. Substituting this into the above first-order condition yields

$$\frac{1}{w_T(y^T, z^T)} \frac{\partial w_T(\eta^T, z^T)}{\partial \eta_T} = h'(a_T).$$

Fixing η^{T-1} and integrating over all possible realizations of η_T gives

$$\ln w_T(y^T, z^T) = h'(a_T)\eta_T + K^{T-1}(\eta^{T-1}, z^T). \tag{54}$$

That is, wages are a log-linear function of realizations of η_T, plus some function of past output and z_T: $K^{T-1}(\eta^{T-1}, z^T)$. This immediately implies

$$\frac{\partial \ln w_T(y^T, z^T)}{\partial \eta_{T-1}} = \frac{\partial K^{T-1}(\eta^{T-1}, z^T)}{\partial \eta_{T-1}}. \tag{55}$$

Likewise, a binding period $T-1$ incentive constraint implies

$$\frac{1}{w_{T-1}(y^{T-1}, z^{T-1})}\frac{\partial w_{T-1}(\eta^{T-1}, z^{T-1})}{\partial \eta_{T-1}} + \frac{\beta(1-s)}{w_T(y^T, z^T)}\frac{\partial w_T(\eta^T, z^T)}{\partial \eta_{T-1}} = h'(a_{T-1}).$$

Using (55), fixing η_{T-2}, and once again integrating with respect to η_{T-1} gives

$$\ln w_{T-1}(y^{T-1}, z^{T-1}) = h'(a_{T-1})\eta_{T-1} + K^{T-2}(\eta^{T-2}, z^{T-1}) - \beta(1-s)K^{T-1}(\eta^{T-1}, z^T). \tag{56}$$

Since wages are a martingale, exponentiating and equating (54) and (56) yields

$$e^{h'(a_{T-1})\eta_{T-1}} e^{K^{T-2}(\eta^{T-2}, z^{T-1})} e^{-\beta(1-s)K^{T-1}(\eta^{T-1}, z^T)} = e^{K^{T-1}(\eta^{T-1}, z^T)}\mathbb{E}_{T-1}\left[e^{h'(a_T)\eta_T}\right]. \tag{57}$$

Taking logs, using properties of the normal distribution, and simplifying yields

$$(1+\beta(1-s))K^{T-1}(\eta^{T-1}, z^T) = h'(a_{T-1})\eta_{T-1} + K^{T-2}(\eta^{T-2}, z^{T-1}) - \frac{(\sigma_\eta h'(a_T))^2}{2} \tag{58}$$

Thus, $K^{T-1}(\eta^{T-1}, z^T)$ (and thus workers' realized utility) is linear in η_{T-1}. Moreover, it can be shown that utility in each period is a linear function of the performance shock in every past period. Substituting equation (58) into equation (56) gives

$$K^{T-1}(\eta^{T-1}, z^T) = \ln w_{T-1}(y^{T-1}, z^{T-1}) - \frac{(\sigma_\eta h'(a_T))^2}{2}. \tag{59}$$

Substituting this expression for $K^{T-1}(\eta^{T-1}, z^T)$ into equation (54) gives

$$\ln w_T = \ln w_{T-1} + h'(a_T)\eta_T - \frac{(\sigma_\eta h'(a_T))^2}{2}.$$

Pursuing a similar strategy, it can be verified that, more generally, for all $t \leq T$

$$\ln w_t = \ln w_{t-1} + \psi_t h'(a_t)\eta_t - \frac{(\psi_t \sigma_\eta h'(a_t))^2}{2}, \tag{60}$$

where $\psi_t \equiv \left(\sum_{\tau=0}^{T-t} (\beta(1-s))^\tau\right)^{-1}$. Taking the limit of equation (60) as $T \to \infty$ yields equation (27), resulting in a constant sensitivity $\psi_t \equiv \psi = 1 - \beta(1-s)$ of log wages to idiosyncratic output shocks over the lifetime of the contract.

To solve for the constant $w_{-1}(z_0)$ that initializes this difference equation, note that free entry into vacancy posting requires that the firm's expected profits from posting vacancies must be zero if a positive measure of vacancies are posted in equilibrium. This implies that

$$\sum_{t=0}^{\infty} (\beta(1-s))^t \mathbb{E}[z_t a_t^* - w_t^*(\eta^t, z^t)|z_0] = \frac{\kappa}{q(\theta_0)}.$$

Recalling that wages are a martingale process ($\mathbb{E}[w_t^*(\cdot)|z_0] = \mathbb{E}[w_0^*(\cdot)|z_0]$), we have that

$$\frac{\mathbb{E}[w_0^*(\cdot)|z_0]}{1-\beta(1-s)} = \sum_{t=0}^{\infty} (\beta(1-s))^t \mathbb{E}[z_t a_t^*|z_0] - \frac{\kappa}{q(\theta_0)}.$$

From the definitions of $\mathcal{Y}(\mathbf{a}^*(z_0); z_0)$ and ψ, we obtain the following expression for $w_{-1}(z_0)$

$$w_{-1}(z_0) = \psi \left(\mathcal{Y}(\mathbf{a}^*(z_0); z_0) - \frac{\kappa}{q(\theta_0)}\right). \tag{61}$$

Cumulating equation (60) then yields the following expression for the log wage at time t:

$$\ln w_t(a_t, \eta^t | z^t) = \ln w_{-1}(z_0) + \sum_{s=0}^{t} \psi h'(a_s)\eta_s - \frac{1}{2}\sum_{s=0}^{t} (\psi h'(a_s)\sigma_\eta).^2 \tag{62}$$

The worker's utility under the contract is equal to the expected present discounted value (EPDV) of log wage payments minus the EPDV of disutility from effort, plus the continuation value should the worker separate to unemployment. First, let us focus on characterizing the worker's expected lifetime utility from consumption. Following Edmans et al. (2012), we assume that this effort choice does not vary with η_t, i.e., that local incentive compatibility is sufficient. From equation (62), we then have

$$E_0\left[\sum_{t=0}^{\infty}(\beta(1-s))^t \ln(w_t(\eta^t, z^t|\mathbf{a}|z_0\right] = \frac{1}{\psi}\ln w_{-1}(z_0) - E_0\left[\sum_{t=0}^{\infty}(\beta(1-s))^t\frac{1}{2}\sum_{\tau=0}^{t}(\psi h'(a_\tau)\sigma_\eta)^2\right],$$

where the second term on the right hand side can be simplified as

$$\begin{aligned}
E_0\left[\sum_{t=0}^{\infty}(\beta(1-s))^t\frac{1}{2}\sum_{\tau=0}^{t}(\psi h'(a_\tau)\sigma_\eta)^2\right] &= E_0\left[\sum_{t=0}^{\infty}\sum_{\tau=t}^{\infty}(\beta(1-s))^\tau\frac{1}{2}(\psi h'(a_t)\sigma_\eta)^2\right] \\
&= E_0\left[\sum_{t=0}^{\infty}\frac{1}{2}(\psi h'(a_t)\sigma_\eta)^2\sum_{\tau=t}^{\infty}(\beta(1-s))^t(\beta(1-s))^{\tau-t}\right] \\
&= \frac{1}{2}E_0\left[\sum_{t=0}^{\infty}(\beta(1-s))^t(\psi h'(a_t)\sigma_\eta)^2\sum_{\tau=t}^{\infty}(\beta(1-s))^{\tau-t}\right] \\
&= \frac{1}{2\psi}E_0\left[\sum_{t=0}^{\infty}(\beta(1-s))^t(\psi h'(a_t)\sigma_\eta)^2\right]. \qquad (63)
\end{aligned}$$

Note that the worker will be paid a higher expected wage if they exert a higher effort. Subtracting off the disutility of effort and adding the continuation value of separating to unemployment, the value to the worker of the contract is therefore

$$\mathcal{E}(z_0) = \frac{1}{\psi}\ln w_{-1}(z_0) - E_0\left[\sum_{t=0}^{\infty}(\beta(1-s))^t\left(\frac{1}{2\psi}(\psi h'(a_t)\sigma_\eta)^2 + h(a_t) - \beta s U(z_{t+1})\right)\right]. \quad (64)$$

Given that the firm makes take it or leave it offers, $\mathcal{E}(z_0)$ is equated to the value of unemployment $U(z_0)$ in equilibrium. This observation yields equation (29).

All that remains is to derive the optimal effort choice $a_t(z_t)$. Taking the first-order condition of equation (64) with respect to a_t yields

$$\frac{1}{\psi}\frac{d\ln w_{-1}(z_0)}{da_t(z_t)} - \beta(1-s)^t\left(h'(a_t) + \psi\sigma_\eta^2 h'(a_t)h''(a_t)\right) = 0.$$

Substituting in using the assumed expression for $h(a)$ and equation (61) gives

$$\frac{1}{\psi}\frac{z_t}{\mathcal{Y}(\mathbf{a}^*(z_0); z_0) - \frac{\kappa}{q(\theta_0)}} - a_t^{1/\epsilon} - \epsilon\psi\sigma_\eta^2 h'(a_t)a_t^{\frac{1-\epsilon}{\epsilon}} = 0.$$

Multiplying by a_t and rearranging terms yields

$$a(z_t;z_0)^{\frac{\epsilon+1}{\epsilon}} = \frac{1}{\psi}\frac{z_t a(z_t;z_0)}{\mathcal{Y}(\mathbf{a}^*(z_0);z_0) - \frac{\kappa}{q(\theta_0)}} - \epsilon\psi\left(\sigma_\eta h'\left(a(z_t;z_0)\right)\right)^2,$$

where the notation $a(z_t;z_0)$ recognizes that effort depends on the current realization of productivity z_t and productivity when the match formed in period 0. Raising this equation to the power $\epsilon/(1+\epsilon)$ yields equation (30) as desired. $\square$

A.6 Decomposition in an Example of an Incentive Contract

Here we explicitly solve for the static optimal contract of Edmans and Gabaix (2011) in our labor market environment and derive dJ/dz directly. This environment is the static version of our quantitative model. The optimal contract is:

$$\begin{aligned}
\textbf{wage:} \quad \ln(w) &= h(a) + h'(a)\eta + \mathcal{B}(z) && (65)\\
\textbf{effort:} \quad z &= \mathbb{E}_\eta\left[(h'(a) + h''(a)\eta)w\right] && (66)\\
\textbf{market clearing:} \quad \frac{\kappa}{q(\theta_0)} &= \mathbb{E}_\eta[w] - za && (67)
\end{aligned}$$

Substituting these expressions into equation (4), we have:

$$\begin{aligned}
\frac{dJ}{dz} &= \mathbb{E}_\eta\left[a + z\frac{da}{dz} - \left(\frac{dw}{da}\times\frac{da}{dz} + \frac{\partial w}{\partial z}\right)\right] && (68)\\
&= \mathbb{E}_\eta\left[a + z\frac{da}{dz} - \underbrace{\left(h'(a) + h''(a)\eta\right)w(z)}_{=z \textbf{ by optimal effort}}\times\frac{da}{dz} - w(z)\mathcal{B}'(z)\right] && (69)\\
&= \mathbb{E}_\eta\left[a\right] - \lambda_0\mathcal{B}'(z) && (70)
\end{aligned}$$

where we have used the optimal effort equation to simplify the expression. Thus, we see that the change in profits per worker in response to a shock to z is the direct effect minus bargained wage cyclicality. $\square$

A.7 IRF of Tightness and the Slope of the Phillips Curve

This section extends the baseline model to allow nominal rigidities and a Phillips curve for prices, by combining the Diamond-Mortensen-Pissarides model with a sticky price final goods sector following Christiano et al. (2016), Gertler et al. (2008), and Blanchard and Galí (2010). The first result of this section is that the impulse response of tightness to labor demand shocks determines the slope of the Phillips curve in the model extension—a larger impulse response flattens the Phillips curve. Therefore the impulse response that we have extensively studied in the main text is a key determinant of inflation dynamics and summarizes the degree of "real rigidity" coming from the labor market.

The second result of this section is that wage cyclicality due to incentives does not affect the slope of the Phillips curve. We derive a closed form mapping between unemployment and inflation that holds in both the economy with flexible incentive pay and no bargaining power, and in an economy with rigid wages; provided they are calibrated to the same steady state unemployment and output.

Throughout this section, we are agnostic about the drivers of economic fluctuations. One can think of our exercise as considering the implications of incentive pay for one equation of the three-equation New Keynesian model, namely the Phillips curve. Our results therefore inform inflation dynamics around steady state for a variety of business cycle shocks. We note, however, that the model with incentives would likely imply a quite different I-S relationship to the standard model, since it carries different implications for idiosyncratic risk and, therefore, consumption-savings behavior.

A.7.1 Setup

The model is a simplified version of the wage and price setting block of Christiano et al. (2016). There are two sectors: a retail sector with sticky prices, whose price setting behavior leads to a Phillips curve for prices; and a wholesale sector which hires workers in a frictional labor market as in the main text. Since the ingredients are relatively standard, we discuss them briefly. We also focus only on the ingredients that are necessary to derive the Phillips curve linking price inflation to unemployment and do not discuss other blocks of the model.

Retail sector. There is a unit measure of retailers with Dixit-Stiglitz monopoly power, who sell to a final output producer. In particular, retailer j produces output $Y_{jt} = AH_{jt}$, where A is a productivity term that for simplicity we assume to be constant. H_{jt} is a quantity of a wholesale good purchased from a competitive wholesale sector at a real price z_t, or a nominal price $P_t z_t$ given the price of final output P_t. Retailer j sets its price, P_{jt} subject to a demand curve $Y_{jt} = (P_t/P_{jt})^{-\alpha} Y_t$, where $Y_t = \left[\int_0^1 (Y_{jt})^{1-\frac{1}{\alpha}} dj\right]^{\frac{\alpha}{\alpha-1}}$ and $P_t = \left[\int_0^1 P_{jt}^{1-\alpha} dj\right]^{\frac{1}{1-\alpha}}$. Inflation is defined as $1+\pi_t \equiv P_{t+1}/P_t$. The retailer is subject to a Calvo sticky price friction, meaning with i.i.d. probability $1-\varrho$ the firm can reset its price, and with probability ϱ the firm must keep the same price. Therefore z_t represents real marginal costs to the retail sector.

Wholesale sector. In the wholesale sector, firms sell an aggregate quantity of wholesale output, $H_t = \int_0^1 H_{it} di$. These firms match with workers in a frictional labor market and produce with a per worker production function $\tilde{f}(\eta_t)$; hence real revenues per worker are $z_t \tilde{f}(\eta_t)$. The frictional labor market is identical to the model of Section 3 in the main text, with a choice of real revenue per worker $f(z_t, \eta_t) = z_t \tilde{f}(\eta_t)$. The only difference between the model of the frictional labor market in the main text, and the frictional labor market in the extension that we have presented here, is the interpretation of z_t. In the main text, z_t is an exogenous term representing labor productivity. In this section, z_t is the real price of a unit of wholesale output—marginal costs for the retail sector—which is determined endogenously.

Notation-wise, we use a variable $\bar{x}$ to refer to the steady state of a variable x_t.

A.7.2 IRF of Tightness and the Slope of the Phillips Curve

We now establish that the impulse response of tightness to business cycle shocks determines the slope of the Phillips curve. Here, the Phillips curve is the structural relationship between inflation, inflation expectations and unemployment. As such, the IRF of tightness to business cycle shocks is important because it is a key determinant of inflation dynamics. We summarize our result in the following proposition.

Proposition 6. *Assume that inflows into and outflows from unemployment are equal at all times. Then to a first-order and in the neighborhood of the zero inflation and non-stochastic steady state, the Phillips curve for prices is*

$$\pi_t = \beta E_t \pi_{t+1} - \frac{\vartheta}{\zeta (1 - \bar{\nu}) \bar{u} (1 - \bar{u})} (u_t - \bar{u}), \tag{71}$$

where $\vartheta \equiv (1 - \varrho)(1 - \beta \varrho) / \varrho$, $\bar{u}$ is the steady state value of unemployment and

$$\zeta \equiv \frac{d \ln \theta_t}{d \ln z_t}$$

is the impulse response of tightness to labor demand shocks z_t, evaluated at the steady state.

All proofs in this subsection are contained at the end of the subsection.[36] Equation (71) is a standard New Keynesian Phillips curve, which links inflation π_t, to inflation expectations $E_t \pi_{t+1}$ and unemployment u_t. The coefficient on unemployment, the "slope" of the Phillips curve, has several terms. ϑ is a familiar term representing nominal rigidities to Calvo price setting frictions in the retail market. The denominator of the slope, $\zeta (1 - \bar{\nu}) \bar{u} (1 - \bar{u})$ is a

[36] The proposition requires an approximation, that inflows into and outflows from unemployment are equal at all times. This approximation is highly accurate at quarterly frequency, when calibrated to data for the United States, because job finding rates are high at quarterly frequency (Ljungqvist and Sargent, 2017).

set of parameters relating to the steady state of the frictional labor market, notably ζ. Thus the impulse response of tightness to labor demand shocks ζ is a key determinant of the slope of the Phillips curve.

The proposition shows that a greater impulse response of unemployment to labor demand shocks leads to a flatter slope of the Phillips curve. Therefore the impulse response of tightness to unemployment summarizes the degree of real rigidity coming from the labor market. Intuitively, if the impulse response is large, then firms hire many workers after an aggregate demand shock. Therefore production increases significantly and unemployment falls rapidly for a given increase in inflation—meaning a flat Phillips curve.

A.7.3 Incentive Wage Cyclicality and the Slope of the Phillips Curve

We now show that wage cyclicality due to incentives does not affect the slope of the Phillips curve. We summarize this result in another proposition. For this proposition, we will assume perfect foresight with respect to aggregate shocks. By first-order certainty equivalence, the impulse responses to the same shocks will be the same to a first-order in a neighborhood of the non-stochastic steady state, in a fully stochastic model—see for instance Auclert et al. (2022). Importantly, we allow nonlinearities with respect to idiosyncratic shocks.

Proposition 7. *Suppose that (i) Assumption 1 from the main text holds, (ii) the firm makes take it or leave it offers to workers and the flow value of unemployment is constant, (iii) the economy is in the neighborhood of the non-stochastic and zero inflation steady state and (iv) there is perfect foresight with respect to aggregate shocks.*

Then in both the rigid wage and flexible incentive pay economies, the mapping between unemployment and inflation satisfies to a first-order

$$u_t = \bar{u} - \varsigma(\bar{u}) \sum_{\tau=0}^{\infty} (\beta(1-s))^{\tau} \bar{\mathbb{E}}\left[\bar{z}\tilde{f}(\eta_\tau)\right](\pi_{t+\tau} - \beta\pi_{t+\tau+1}),$$

where $\bar{\mathbb{E}}\left[\bar{z}\tilde{f}(\eta_\tau)\right]$ is the expectation of output τ periods after the start of a match evaluated at an aggregate steady state and ς is a known function defined in the proof of the proposition.

This proposition shows that the mapping between unemployment and inflation is the same in two economies. The first is an economy with flexible incentive pay and no bargaining power, the second has fully rigid wages. Both economies must be calibrated to the same steady state values of unemployment and output in each period after the start of the match. As such, incentive wage cyclicality does not affect the slope of the Phillips curve. Though the derivation is involved, the intuition follows from the results of the main text and the previous result of this subsection. That is, the impulse response of tightness to labor demand shocks governs the slope of the Phillips curve, and this IRF is the same in models with flexible incentive pay or rigid wages; provided they are calibrated to the same steady state moments.

A.7.4 Proof of Proposition 6

Let $dy_t/dx_t|_{SS}$ denote the derivative of a variable y_t with respect to x_t evaluated at the non-stochastic and zero inflation steady state.

First, we use the approximation that inflows into and outflows from unemployment are equal at all times. This assumption amounts to imposing $u_t \approx u_{t-1}$ and $\theta_t \approx \theta_{t-1}$ in equation (7) of the main text. Under this assumption, equation (7) implies

$$\begin{aligned} u_t &= u_{t-1} + s\left(1-u_{t-1}\right) - \phi\left(\theta_{t-1}\right)\left(1-s\right)u_{t-1} \\ \implies u_t &= \frac{s}{s+\theta_t q\left(\theta_t\right)\left(1-s\right)} \end{aligned} \tag{72}$$

where we have used that $\phi_t = \theta_t q\left(\theta_t\right)$, given that $\phi_t = m\left(u_t, v_t\right)/u_t$ and $q_t = m\left(u_t, v_t\right)/v_t$. Differentiating this with respect to θ_t yields

$$\begin{aligned} \frac{du_t}{d\theta_t} &= -\frac{s}{\left[s+\theta_t q\left(\theta_t\right)\left(1-s\right)\right]}\frac{\left(1-s\right)\left[q\left(\theta_t\right)+\theta_t q'\left(\theta_t\right)\right]}{\left[s+\theta_t q\left(\theta_t\right)\left(1-s\right)\right]} \\ &= -u_t\left(1-\nu_t\right)\frac{\left(1-s\right)q\left(\theta_t\right)}{\left[s+\theta_t q\left(\theta_t\right)\left(1-s\right)\right]} \end{aligned}$$

$$\begin{aligned} \implies \frac{du_t}{d\ln\theta_t} &= -\left(1-\nu_t\right)u_t\frac{\left(1-s\right)\theta_t q\left(\theta_t\right)}{\left[s+\theta_t q\left(\theta_t\right)\left(1-s\right)\right]} \\ \implies \frac{du_t}{d\ln\theta_t} &= -\left(1-\nu_t\right)u_t\left(1-u_t\right) \\ \implies \frac{d\ln\theta_t}{du_t} &= -\frac{1}{\left(1-\nu_t\right)u_t\left(1-u_t\right)} \end{aligned} \tag{73}$$

where the final implication uses

$$1-u_t = 1-\frac{s}{s+\theta_t q\left(\theta_t\right)\left(1-s\right)} = \frac{\theta_t q\left(\theta_t\right)\left(1-s\right)}{s+\theta_t q\left(\theta_t\right)\left(1-s\right)}.$$

Therefore we have, to a first-order,

$$\ln\theta_t - \ln\bar{\theta}_t = \frac{d\ln\theta_t}{du_t}|_{SS}\left(u_t-\bar{u}\right) = -\frac{1}{\left(1-\bar{\nu}\right)\bar{u}\left(1-\bar{u}\right)}\left(u_t-\bar{u}\right) \tag{74}$$

where we use equation (73) and a first-order Taylor expansion evaluated at the non-stochastic steady state.

The price setting problem of the retailer implies that, in a neighborhood of the zero inflation and non-stochastic steady state

$$\pi_t = \beta E_t \pi_{t+1} + \vartheta \left(\ln z_t - \ln \bar{z} \right), \tag{75}$$

where $\ln z_t$ is the real marginal cost of the retailer sector. This derivation is standard (e.g., Galí, 2015), so we do not repeat it here. Finally, from equations (73) and (75) we have

$$\begin{aligned} \pi_t &= \beta E_t \pi_{t+1} + \vartheta \frac{d \ln z_t}{d \ln \theta_t} |_{SS} \left(\ln \theta_t - \ln \bar{\theta}_t \right) \\ &= \beta E_t \pi_{t+1} + \frac{\vartheta}{\zeta} \frac{d \ln \theta_t}{d u_t} |_{SS} \left(u_t - \bar{u} \right) \\ &= \beta E_t \pi_{t+1} - \frac{\vartheta}{\zeta \left(1 - \bar{\nu} \right) \bar{u} \left(1 - \bar{u} \right)} \left(u_t - \bar{u} \right) \end{aligned} \tag{76}$$

where we use the definition $\zeta \equiv d \ln \theta_t / d \ln z_t |_{SS}$ and the final equality substitutes in equation (73). Equation (76) completes the proof.

A.7.5 Proof of Proposition 7

Let $\bar{\mathbb{E}} \left[x_\tau \right]$ denote an expectation taken τ periods after the start of a match, evaluated at the non-stochastic steady state for aggregate variables.

From equation (11) of the main text, the value of a job filled in period t is

$$J_t = \sum_{\tau=0}^{\infty} \left(\beta \left(1 - s \right) \right)^{\tau} \mathbb{E} \left[z_{t+\tau} \tilde{f}(\eta_{t+\tau}) - w^*_{t+\tau} | \mathbf{a}^*_t \right]$$

where we have dropped the conditioning of the expectation on z_t through perfect foresight of aggregate variables, which we invoke to render z_t non-stochastic. We can write

$$J_t = J \left(\mathbf{z}_t, \mathbf{a}^* \left(\mathbf{z}_t \right), \mathbf{w}^* \left(\mathbf{z}_t \right) \right) = \sum_{\tau=0}^{\infty} \left(\beta \left(1 - s \right) \right)^{\tau} \mathbb{E} \left[z_{t+\tau} \tilde{f}(\eta_{t+\tau}) - w^*_{t+\tau} \left(\mathbf{z}_t \right) | \mathbf{a}^* \left(\mathbf{z}_t \right) \right] \tag{77}$$

where $\mathbf{z}_t = \{ z_{t+\tau} \}_{\tau=0}^{\infty}$ is the known sequence of aggregate values of z, and $\mathbf{w}^* \left(\mathbf{z_t} \right), \mathbf{a}^* \left(\mathbf{z}_t \right)$ represent the optimal choices of wages and effort given $\mathbf{z}_t$.

Differentiating with respect to $\ln z_{t+k}$ for arbitrary $k \geq 0$, we have

$$\frac{\partial \ln J\left(\mathbf{z}_t, \mathbf{a}^*\left(\mathbf{z}_t\right), \mathbf{w}^*\left(\mathbf{z}_t\right)\right)}{\partial \ln z_{t+k}} = \frac{z_{t+k}}{J_t} \times \frac{\partial\left(\sum_{\tau=0}^{\infty}\left(\beta\left(1-s\right)\right)^{\tau}\left[z_{t+\tau}\mathbb{E}\left[\tilde{f}(\eta_{t+\tau})|\mathbf{a}_t^*\right] - \mathbb{E}\left[w_{t+\tau}^*|\mathbf{a}_t^*\right]\right]\right)}{\partial z_{t+k}}$$
$$= \frac{\left(\beta\left(1-s\right)\right)^{k}\left[z_{t+k}\mathbb{E}\left[\tilde{f}(\eta_{t+k})|\mathbf{a}_t^*\right]\right]}{\sum_{\tau=0}^{\infty}\left(\beta\left(1-s\right)\right)^{\tau}\left[z_{t+\tau}\mathbb{E}\left[\tilde{f}(\eta_{t+\tau})|\mathbf{a}_t^*\right] - \mathbb{E}\left[w_{t+\tau}^*|\mathbf{a}_t^*\right]\right]}$$

$$\implies \frac{\partial \ln J\left(\mathbf{z}_t, \mathbf{a}^*\left(\mathbf{z}_t\right), \mathbf{w}^*\left(\mathbf{z}_t\right)\right)}{\partial \ln z_{t+k}}|_{SS} = \frac{\left(\beta\left(1-s\right)\right)^{k}\left[\bar{z}\bar{\mathbb{E}}\left[\tilde{f}(\eta_k)|\bar{\mathbf{a}}^*\right]\right]}{\sum_{\tau=0}^{\infty}\left(\beta\left(1-s\right)\right)^{\tau}\left[\bar{z}\bar{\mathbb{E}}\left[\tilde{f}(\eta_\tau)|\bar{\mathbf{a}}^*\right] - \bar{\mathbb{E}}\left[\bar{w}_{\tau}^*|\bar{\mathbf{a}}_t^*\right]\right]} \tag{78}$$

Thus, taking a first-order Taylor expansion for J_t, we have, to a first-order, that:

$$\begin{aligned}
\ln J_t - \ln \bar{J} &= \sum_{k=0}^{\infty}\frac{d\ln J_t}{d\ln z_{t+k}}|_{SS}\left(\ln z_{t+k} - \ln \bar{z}\right)\\
&= \sum_{k=0}^{\infty}\frac{\partial \ln J\left(\mathbf{z}_t, \mathbf{a}^*\left(\mathbf{z}_t\right), \mathbf{w}^*\left(\mathbf{z}_t\right)\right)}{\partial \ln z_{t+k}}|_{SS}\left(\ln z_{t+k} - \ln \bar{z}\right)\\
&= \sum_{k=0}^{\infty}\frac{\left(\beta\left(1-s\right)\right)^{k}\left[\bar{z}\bar{\mathbb{E}}\left[\tilde{f}(\eta_k)|\bar{\mathbf{a}}^*\right]\right]}{\sum_{\tau=0}^{\infty}\left(\beta\left(1-s\right)\right)^{\tau}\left[\bar{z}\bar{\mathbb{E}}\left[\tilde{f}(\eta_\tau)|\bar{\mathbf{a}}^*\right] - \bar{\mathbb{E}}\left[\bar{w}_{\tau}^*|\bar{\mathbf{a}}_t^*\right]\right]}\left(\ln z_{t+k} - \ln \bar{z}\right) \qquad (79)
\end{aligned}$$

where the second equality substitutes in equation (77) and uses the envelope theorem of Section 3; and the third equality uses equation (78).

Next, note that from the Phillips curve (75) and using perfect foresight with respect to aggregate variables, we have

$$\pi_t = \beta\pi_{t+1} + \vartheta\left(\ln z_t - \ln \bar{z}\right) \quad \implies \frac{\pi_t - \beta\pi_{t+1}}{\vartheta} = \ln z_t - \ln \bar{z}. \tag{80}$$

The free entry condition (9) from the main text implies, to a first-order

$$\ln J_t - \ln \bar{J} = \bar{\nu}\left(\ln \theta_t - \ln \bar{\theta}\right).$$

Using equations (74) and (79) gives

$$\sum_{k=0}^{\infty}\frac{\left(\beta\left(1-s\right)\right)^{k}\left[\bar{z}\bar{\mathbb{E}}\left[\tilde{f}(\eta_k)|\bar{\mathbf{a}}^*\right]\right]}{\sum_{\tau=0}^{\infty}\left(\beta\left(1-s\right)\right)^{\tau}\left[\bar{z}\bar{\mathbb{E}}\left[\tilde{f}(\eta_\tau)|\bar{\mathbf{a}}^*\right] - \bar{\mathbb{E}}\left[\bar{w}_{\tau}^*|\bar{\mathbf{a}}_t^*\right]\right]}\left(\ln z_{t+k} - \ln \bar{z}\right) = -\bar{\nu}\frac{1}{\left(1-\bar{\nu}\right)\bar{u}\left(1-\bar{u}\right)}\left(u_t - \bar{u}\right).$$

Rearranging and using equation (80) to substitute in for $\ln z_{t+k} - \ln \bar{z}$ yields

$$u_t = \bar{u} - \frac{(1-\bar{\nu})\,\bar{u}\,(1-\bar{u})}{\bar{\nu}} \frac{\sum_{k=0}^{\infty} (\beta\,(1-s))^k\, \bar{\mathbb{E}}\left[\bar{z}\tilde{f}(\eta_k)\right] \left(\frac{\pi_{t+k}-\beta\pi_{t+k+1}}{\vartheta}\right)}{\sum_{\tau=0}^{\infty} (\beta\,(1-s))^{\tau} \left[\bar{z}\bar{\mathbb{E}}\left[\tilde{f}(\eta_\tau)|\bar{\mathbf{a}}^*\right] - \mathbb{E}\left[\bar{w}_\tau^*|\mathbf{a}_t^*\right]\right]}$$

Finally, substituting back in using the free entry condition (1), we have

$$u_t = \bar{u} - \varsigma\,(\bar{u}) \sum_{\tau=0}^{\infty} (\beta\,(1-s))^{\tau}\, \bar{\mathbb{E}}\left[\bar{z}\tilde{f}(\eta_\tau)\right] \left(\frac{\pi_{t+\tau} - \beta\pi_{t+\tau+1}}{\vartheta}\right) \tag{81}$$

for

$$\varsigma\,(\bar{u}) \equiv \frac{(1-\bar{\nu})\,\bar{u}\,(1-\bar{u})}{\bar{\nu}} \frac{q\left(\bar{\theta}\right)}{\kappa}.$$

Note $\varsigma(\bar{u})$ is a function only of steady state unemployment since by equation (72), $\bar{\theta}$ is a function of $\bar{u}$; and $\bar{\nu}$ is a function of $\bar{\theta}$ because it is the vacancy filling elasticity.

Equation (81) derives the mapping between unemployment and inflation in the flexible incentive pay economy. The steps to derive the mapping between unemployment and inflation in the rigid wage economy are the same except that one does not need to apply an envelope theorem to derive the final equality of equation (79) for the rigid wage economy.

A.8 Endogenous Separations and Limited Worker Commitment

This section introduces efficient endogenous separations and limited worker commitment into the baseline environment. To economize, we only discuss the parts of the model that change due to efficient separations or limited worker commitment, otherwise the model is the same as the flexible incentive pay economy of the main text.

A.8.1 Economic Environment

Labor Market As in the baseline model of the main text, a large measure of risk-neutral firms match with unemployed workers according to a frictional matching technology. Fluctuations are driven by aggregate productivity z_t and there is free entry to vacancy posting at a constant flow cost κ, as in the main text.

At the end of period $t-1$ an endogenous fraction s_t of workers separate from employment and enter unemployment. The unemployed search for new jobs, so u_t evolves as

$$u_t = u_{t-1} + s_t(1-u_{t-1}) - \phi(\theta_{t-1})u_{t-1}(1-s_t). \tag{82}$$

Preferences and Consumption Workers' preferences are identical to the model of the main text.

Firms and Wage Setting Firms are risk neutral and maximize expected profit with discount factor β. Successful matches produce with a production function $f(z, \eta)$, where unobserved worker effort shifts the distribution of η realizations, as in Section 3. Assuming that z_t is first-order Markovian, we define $\pi(z_{t+1}|z_t)$ to be the one-step-ahead probability.

The value of a firm of posting a vacancy at time 0 is then

$$\Pi_0 = q(\theta_0)\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\left(\prod_{j=1}^{t}(1-s_j)\right)(f(z_t,\eta_t)-w_t)\right]-\kappa, \tag{83}$$

where $\mathbb{E}$ conditions on the firm's information set at time 0 prior to meeting a worker. A vacancy is filled with probability $q(\theta)$. If a firm meets a worker, its value is the expected present value of the difference between production and wage payments, discounted by the firm's discount factor β as well as separation risk. Here, $\prod_{j=1}^{t}(1-s_j)$ is the endogenous probability that a match survives until period t, which cumulates the probability $1-s_j$ that a match survives period j. We entertain two possibilities for wage setting.

Flexible Incentive Pay Economy As in the main text, the firm observes realizations of both z_t and η_t, but does not observe worker's effort. When a firm and worker meet, the firm offers the worker a contract to incentivize effort and maximize firm value. The innovation of this section is that the firm now has the additional option to vary the probability that the match separates in each date and state. For instance, if the expected present value of profits has turned negative, the firm may choose to terminate the contract. Thus the contract may be summarized by functions $w_t(\eta^t, z^t) \in [\underline{w}, \bar{w}]$, $a_t(\eta^{t-1}, z^t) \in [\underline{a}, \bar{a}]$ and a separation probability $s_t(\eta^t, z^t) \in [0,1]$ for all t and all realizations of η^t and z^t. Let $(\mathbf{w}, \mathbf{a}, \mathbf{s})$ denote a contract, with $\mathbf{w} \equiv \{w_t(\eta^t, z^t)\}_{t=0,\eta^t,z^t}^{\infty}$, $\mathbf{a} \equiv \{a_t(\eta^{t-1}, z^t)\}_{t=0,\eta^{t-1},z^t}^{\infty}$ and $\mathbf{s} \equiv \{s_t(\eta^t, z^t)\}_{t=0,\eta^t,z^t}^{\infty}$. Let $\mathcal{X}$ denote the space of possible contracts.

Value of a Filled Vacancy. Under the contract $(\mathbf{w}, \mathbf{a}, \mathbf{s})$, and initial productivity z_0, the firm's expected present value of profits from posting a vacancy is

$$V(\mathbf{w},\mathbf{a},\mathbf{s};z_0) = \sum_{t=0}^{\infty}\int\int \beta^t\mathcal{S}_t(\eta^t,z^t)\left(f(z_t,\eta_t)-w_t(\eta^t,z^t)\right)\tilde{\pi}_t(\eta^t,z^t|z_0,\mathbf{a})\,d\eta^t dz^t, \tag{84}$$

where $\mathcal{S}_t(\eta^t, z^t) \equiv \prod_{j=1}^{t}(1-s_{t-j}(\eta^{t-j}, z^{t-j}))$ is the probability that a match survives the sequence η^t, z^t; and $\tilde{\pi}_t(\eta^t, z^t|z_0, \mathbf{a})$ is the probability of observing a realization of η^t and z^t

given the initial z_0 and the contracted effort function $\mathbf{a}$, as in the main text. The risk-neutral firm discounts period t profits by the economy-wide discount rate β^t and the probability $\mathcal{S}_t(\eta^t, z^t)$ that the match survives t periods.

The contract maximizes the value of a filled vacancy

$$J(z_0) = \max_{\{w_t(\eta^t,z^t),a_t(\eta^{t-1},z^t),s_t(\eta^t,z^t)\}_{t=0,\eta^t,z^t}^{\infty}\in\mathcal{X}} V(\mathbf{w},\mathbf{a},\mathbf{s};z_0) \tag{85}$$

subject to the incentive compatibility and participation constraints described below, as well as a new set of constraints that captures limited commitment by the worker.

Incentive Constraints. The incentive compatibility condition is similar to the main text, but now accounts for endogenous separation risk

$$\begin{aligned}[\mathbf{IC}]:\mathbf{a}\in \underset{\{\tilde{a}_t(\eta^{t-1},z^t)\}_{t=0,\eta^t,z^t}^{\infty}}{\operatorname{argmax}} \sum_{t=0}^{\infty}\Bigg[\int\int \beta^t\mathcal{S}_t\left(\eta^t,z^t\right)\Bigg[u\Bigg(w_t(\eta^t,z^t),\tilde{a}_t(\eta^{t-1},z^t)\Bigg)-\Psi(s\left(\eta^t,z^t\right))\\ +\beta s\left(\eta^t,z^t\right)\int U\left(z_{t+1}\right)\pi\left(z_{t+1}|z_t\right)dz^{t+1}\Bigg]\tilde{\pi}_t\left(\eta^t,z^t|z_0,\tilde{\mathbf{a}}\right)d\eta^t dz^t,\end{aligned} \tag{86}$$

where $\Psi(s_j)$ represents a convex utility cost to the worker of searching for a new job.

Participation Constraint. Likewise, the participation constraint must also account for separation risk and becomes

$$\begin{aligned}[\mathbf{PC}]:\sum_{t=0}^{\infty}\Bigg[\int\int \beta^t\mathcal{S}_t\left(\eta^t,z^t\right)\Bigg[u\Bigg(w_t(\eta^t,z^t),\tilde{a}_t(\eta^{t-1},z^t)\Bigg)-\Psi(s\left(\eta^t,z^t\right))\\ +\beta s\left(\eta^t,z^t\right)\int U\left(z_{t+1}\right)\pi\left(z_{t+1}|z_t\right)dz_{t+1}\Bigg]\tilde{\pi}_t\left(\eta^t,z^t|z_0,\tilde{\mathbf{a}}\right)d\eta^t dz^t\Bigg]\geq\mathcal{E}\left(z_0\right).\end{aligned} \tag{87}$$

Limited Commitment. Limited commitment and endogenous separations means that after any history η^τ, z^τ the worker must rather stay in the match than separate, leading to a constraint that for each η^τ, z^τ:

$$\begin{aligned}[\mathbf{ES}]:\sum_{t=\tau}^{\infty}\mathbb{E}\Bigg[\beta^{t-\tau}\mathcal{S}_t^{\tau}\left(\eta^t,z^t\right)\Bigg[u\Bigg(w_t(\eta^t,z^t),a_t(\eta^{t-1},z^t)\Bigg)-\Psi(s\left(\eta^t,z^t\right))\\ +\beta s\left(\eta^t,z^t\right)\mathbb{E}[U\left(z_{t+1}\right)|z_t]\Bigg]\Bigg|\eta^\tau,z^\tau\Bigg]\geq U\left(z_\tau\right),\end{aligned} \tag{88}$$

where $\mathcal{S}_t^\tau$ is the survival probability after time τ, $\mathcal{S}_t^\tau\left(\eta^t,z^t\right)\equiv\prod_{j=\tau+1}^{t}\left(1-s_{t+\tau+1-j}\left(\eta^{t+\tau+1-j},z^{t+\tau+1-j}\right)\right)$.

Bargaining and ex ante utility. To close the flexible incentive pay economy, we again assume ex ante utility $\mathcal{E}(z_0)$ is given by a reduced-form "bargaining schedule" $\mathcal{B}(z_0)$.

Rigid Wage Economy The rigid wage economy is identical to the rigid wage economy of the main text, including the assumption of an exogenous separation rate s.

Equilibrium Given initial unemployment u_0 and a stochastic process $\{z_t, \eta_t\}_{t=0}^{\infty}$, an equilibrium is a collection of stochastic processes $\{\theta_t, u_t\}_{t=0}^{\infty}$ and functions $J(z), U(z), \mathcal{E}(z)$, and $(\mathbf{w}, \mathbf{a}, \mathbf{s})$ such that for all firms: (i) θ_t satisfies the free entry condition so that Π_t, given in equation (83), is equal to 0 for all t; (ii) u_t satisfies the law of motion for unemployment (82); (iii) wage, effort, and separation functions $(\mathbf{w}, \mathbf{a}, \mathbf{s})$ satisfy the flexible incentive pay economy equations (85)-(88), or $w_t = \bar{w}$, $a_t = \bar{a}$ and $s_t = s$ in the rigid wage economy; (iv) the value of unemployment $U(z)$ is defined in the same way as the main text; (v) the value of employment is defined the same way as the main text for the in the rigid wage economy, or $\mathcal{E}(z) = \mathcal{B}(z)$ in the flexible incentive pay economy; and (vi) the value of a filled vacancy $J(z)$ is given by (85) in the flexible incentive pay economy or the same way as the main text for the rigid wage economy.

A.8.2 Equivalence of Rigid and Incentive Pay with Endogenous Separations

This subsection shows that, without bargaining power or fluctuations in outside options, the first-order response of market tightness is the same in the rigid wage economy, and the flexible incentive pay economy with endogenous separations. For simplicity we make the same assumptions as the main text, such as studying impulse responses in a neighborhood of the non-stochastic steady state.

Proposition 8. *Assume that the set of feasible contracts that satisfies the incentive constraints (86) and the participation constraint (87) is non-empty and compact. Also assume that the production function is homogeneous of degree one in aggregate productivity z, z_t is a driftless random walk, and the optimal incentive contract at the non-stochastic steady state is unique. Finally, assume that the firm makes take it or leave it offers to workers and the flow value of unemployment is constant and the optimal contract is unique. Then the impact elasticity of market tightness to shocks to z_t is*

$$\frac{d \ln \theta_0}{d \ln z_0} = \frac{1}{\bar{\nu}} \frac{1}{1 - \Lambda} \tag{89}$$

where Λ is the steady state labor share defined as

$$\Lambda \equiv \frac{\sum_{t=0}^{\infty} \mathbb{E}\beta^t \prod_{j=1}^{t} \left(1 - s_j^*\right) w_t^*}{\sum_{t=0}^{\infty} \mathbb{E}\beta^t \prod_{j=1}^{t} \left(1 - s_j^*\right) f\left(\bar{z}, \eta_t\right)}$$

where s_j^* and w_t^* denote choices of separations and wages along the optimal contract, where the expectation $\mathbb{E}$ is evaluated along the optimal contract, and $\bar{z}$ is the value of z_t at the aggregate steady state.

This theorem shows that the flexible incentive pay economy with endogenous separations has as an equivalent response of tightness on impact to the rigid wage economy of the main text. Note that the dynamics of the rigid wage economy are still given by equation (22). Therefore incentive wage cyclicality does not affect the impact response of tightness with endogenous separations so long as the flexible incentive pay economy and the rigid wage economy are calibrated to the same steady state labor share. In the incentive pay economy with endogenous separations, the labor share depends on the optimal choice of separation rates, as well as the factors from the model of the main text such as wages and effort.

We stress that this result leads to equivalence for impact elasticities, as Pissarides (2009) discusses. In general the response of tightness to labor productivity shocks after impact will be different in the rigid wage and flexible incentive pay economies because endogenous separations lead to additional dynamics of unemployment after the impact of the shock.

Intuitively, in the model with efficient endogenous separations, separations are an additional choice which the firm can optimize over. However, changes in the optimal separation choice after TFP shocks have no first-order effect on profits—just as neither changes in optimally chosen effort nor wages affect profits. Likewise, the optimal contract ensures that workers do not wish to leave the match. Reoptimizations by the worker as aggregate conditions change do not affect profits. This logic is again due to the envelope theorem.

A.8.3 Proof of Proposition 8

The free entry condition in the flexible incentive pay economy is

$$\frac{\kappa}{q\left(\theta\right)} = J\left(z_0\right),$$

where $J(z_0)$ is defined in equation (85). Taking derivatives and rearranging implies

$$\begin{aligned}\frac{d\ln\theta_0}{d\ln z_0} &= \frac{1}{\nu_0}\frac{d\ln J(z_0)}{d\ln z_0}\\ &= \frac{1}{\nu_0}\frac{z_0}{J(z_0)}\frac{dJ(z_0)}{dz_0}.\end{aligned} \tag{90}$$

With Ψ convex, the optimal separation rates s_j^* will be interior. Under the assumptions of the proposition, z_0 does not enter either the incentive constraints, the participation constraint, or the limited commitment constraints directly. Therefore we have

$$\begin{aligned}\frac{dJ(z_0)}{dz_0} = \frac{\partial J(z_0)}{\partial z_0} &= \frac{\partial}{\partial z_0}\sum_{t=0}^{\infty}\mathbb{E}\beta^t\prod_{j=1}^{t}\left(1-s_j^*\right)\left(f(z_t,\eta_t)-w_t\right)\\ &= \sum_{t=0}^{\infty}\mathbb{E}\beta^t\prod_{j=1}^{t}\left(1-s_j^*\right)\left(f_z(z_t,\eta_t)\right),\end{aligned} \tag{91}$$

where the first equality invokes the envelope theorem, using the same argument as Appendix ection A.2.1 and also using our assumption of a unique optimal contract in order to dispense with a sup operator; the second equality rewrites the definition of profits from equation (84) using the notation from the theorem and exploits that terms involving the participation, incentive, or limited commitment constraints vanish because z_0 does not enter them directly; and the final equality uses that z_t is a random walk.

Substituting in equations (90) and (91) implies

$$\begin{aligned}\frac{d\ln\theta_0}{d\ln z_0} &= \frac{1}{\nu_0}\frac{z_0\sum_{t=0}^{\infty}\mathbb{E}\beta^t\Pi_{j=1}^{t}\left(1-s_j^*\right)\left(f_z(z_t,\eta_t)\right)}{\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\Pi_{j=1}^{t}\left(1-s_j^*\right)\left(f(z_t,\eta_t)-w_t^*\right)\right]}\\ &= \frac{1}{\nu_0}\frac{\bar{z}\sum_{t=0}^{\infty}\mathbb{E}\beta^t\Pi_{j=1}^{t}\left(1-s_j^*\right)\left(f_z(\bar{z},\eta_t)\right)}{\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\Pi_{j=1}^{t}\left(1-s_j^*\right)\left(f(\bar{z},\eta_t)-w_t^*\right)\right]}\\ &= \frac{1}{\nu_0}\frac{\sum_{t=0}^{\infty}\mathbb{E}\beta^t\Pi_{j=1}^{t}\left(1-s_j^*\right)\left(f(\bar{z},\eta_t)\right)}{\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\Pi_{j=1}^{t}\left(1-s_j^*\right)\left(f(\bar{z},\eta_t)-w_t^*\right)\right]}\\ &= \frac{1}{\nu_0}\frac{1}{1-\frac{\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\Pi_{j=1}^{t}\left(1-s_j^*\right)w_t^*\right]}{\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\Pi_{j=1}^{t}\left(1-s_j^*\right)\left(f(\bar{z},\eta_t)\right)\right]}},\end{aligned}$$

, where we use the assumption of an aggregate steady state in $\bar{z}$. □

B Numerical Appendix

B.1 Preliminaries

We calibrate the model such that t represents a month. Specifically, we set the discount rate β to $0.99^{1/3}$, the vacancy creation cost to 0.45 and employ a matching function given by $m(u,v) = uv(u^\iota + v^\iota)^{-1/\iota}$ so that $q(\theta) = (1+\theta^\iota)^{1/\iota}$, which is bounded between 0 and 1. We set $\iota = 0.9$ by nonlinear least squares to match the empirical relationship between aggregate market tightness and job-finding rates. We set the exogenous separation rate $s = 0.031$ to the average monthly separation rate in the Current Population Survey (CPS) from 1951 to 2019. This implies that the pass-through parameter ψ equals 0.034. Separation rates and job-finding rates are both adjusted for time aggregation following Shimer (2005). We measure empirical labor market tightness as job openings from the Job Openings and Labor Turnover Survey (JOLTS) divided by household unemployment in the CPS. Our labor market tightness series spans from 2001 to 2019 (JOLTS begins in December 2000).

We discretize the AR(1) productivity process for $\ln z_t$ onto a finite grid: $z \in \mathcal{Z} = [\underline{z}, ..., \overline{z}]$ following Rouwenhorst (1995). We set the number of gridpoints to 13.

We now rewrite the key equations in our numerical model recursively, given the Markovian structure for productivity. Let $\pi(z'|z)$ denote the probability of aggregate productivity transitioning from z to z'. Recall that the optimal effort schedule, given an initial z_0 and current z, satisfies

$$a\left(z;z_0\right) = \left[\frac{za\left(z;z_0\right)}{\psi\left(Y\left(z_0\right) - \frac{\kappa}{q(\theta(z_0))}\right)} - \frac{\psi}{\epsilon}\left(h'\left(a\left(z;z_0\right)\right)\sigma_\eta\right)^2\right]^{\frac{\epsilon}{1+\epsilon}}.$$

Let $\tilde{Y}(z;z_0)$ denote the EPDV of future output, conditional on effort $a(\cdot;z_0)$ and current productivity z, given by

$$\tilde{Y}\left(z;z_0\right) = za\left(z;z_0\right) + \sum_{z'\in\mathcal{Z}} \beta\left(1-s\right)\tilde{Y}\left(z';z_0\right))\,\pi\left(z'|z\right).$$

It follows that $Y(z_0) = \tilde{Y}(z_0;z_0)$. Note that the optimal effort depends on z_0 through $Y(z_0)$ and $\theta(z_0)$, which are both equilibrium objects in our model. Define the worker's expected present discounted utility from starting work at z_0, $\tilde{\mathcal{E}}(z_0)$, taking as given the effort schedule $a(\cdot;z_0)$ and the wage schedule $w(\cdot;z_0)$ defined in Section 4:

$$\tilde{\mathcal{E}}(z_0) = \frac{1}{\psi}\ln w_{-1}(z_0) + \mathbb{E}_z\Bigg[-\sum_{t=0}^{\infty}[\beta(1-s)]^t \frac{1}{\psi}\frac{1}{2}(\psi h'(a(z_t;z_0))\,\sigma_\eta)^2 - \sum_{t=0}^{\infty}[\beta(1-s)]^t h(a(z_t;z_0)) + \sum_{t=0}^{\infty}[\beta(1-s)]^t \beta s\omega(z_{t+1})\Bigg],$$

where

$$\omega(z) = \mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t \ln b(z_t) \mid z_0 = z\right].$$

It is helpful to re-define the term in brackets in the above expression as $W(z_0;z_0)$, where

$$W(z;z_0) = -\frac{1}{\psi}\frac{1}{2}(\psi h'(a(z;z_0))\sigma_\eta)^2 - h(a(z;z_0)) + \sum_{z'\in\mathcal{Z}}\beta s\omega(z')\,\pi(z'|z_0) + \sum_{z'\in\mathcal{Z}}\beta(1-s)\,W(z';z_0)\,\pi(z'|z_0).$$

Finally, we define an implicit, auxiliary function for effort $\tilde{a}$ with arguments z, $\tilde{Y}$, and $\tilde{q}$ (subsuming any dependence on z_0) that is useful when solving the model numerically:

$$\tilde{a}\left(z,\tilde{Y},\tilde{q}\right) = \left[\frac{z\tilde{a}}{\psi\left(\tilde{Y} - \kappa/\tilde{q}\right)} - \frac{\psi}{\epsilon}\left(h'(\tilde{a})\,\sigma_\eta\right)^2\right]^{\frac{\epsilon}{1+\epsilon}}.$$ [37]

B.2 Algorithm to solve for the optimal contract, given z_0

Fix an initial $z_0 \in \mathcal{Z}$. To solve for the optimal contract beginning at z_0, we perform a bisection search over the job-filling rate $q(z_0)$. Let n index iterations over our guess of $q(z_0)$. Then, for a given $q^n(z_0)$, we solve for the optimal effort schedule $\tilde{a}^n(\cdot)$ and the EPDV of ouput $Y^n(z_0)$ as a fixed point problem. With values of $Y^n(z_0)$ and $q^n(z_0)$, we can construct $w_1^n(z_0)$, the intialization for the difference equation governing the wage schedule, and recursively solve for the EPDV of the utility offered by the contract $\mathcal{E}^n(z_0)$. We then check whether $\mathcal{E}^n(z_0) = \omega(z_0)$, as implied by TIOLI offers, and accordingly update the lower and upper bounds for the next iteration, $\underline{q}^{n+1}$ and $\bar{q}^{n+1}$, respectively. We continue this process until convergence of $q(z_0)$. Below we describe the algorithm in further detail.

1. Set $n = 1$. Set $\underline{q}^n = 0$, and $\bar{q}^n = 1$.
2. Set $q^n(z_0) = \frac{1}{2}(\underline{q}^n + \bar{q}^n)$.
3. Set $k = 1$. Make initial guess for $Y^{k,n}(z|z_0)$ for $z \in \mathcal{Z}$.

[37] For general ϵ, we numerically solve for a_t using a root-finder, restricting attention to positive roots.

4. Update $Y^{k+1,n}(\cdot;z_0)$ as

$$Y^{k+1,n}\left(z;z_0\right)=z\tilde{a}\left(z,Y^{k,n}(z;z_0),q^n(z_0)\right)+\sum_{z'\in\mathcal{Z}}\beta\left(1-s\right)Y^{k,n}\left(z';z_0\right)\pi\left(z'|z\right)$$

5. Repeat (4) until $\|Y^{k,n+1}\left(\cdot;z_0\right)-Y^{k,n}\left(\cdot;z_0\right)\|<\delta_1$ for some small tolerance $\delta_1>0$. Define the object $Y^n(z_0)=Y^{k,n}\left(z_0;z_0\right)$. Define $\tilde{a}^n(z)=\tilde{a}(z,Y^n(z_0),q^n(z_0))$.

6. Solve for $w_{-1}^n(z_0)$ using the free entry condition:

$$w_{-1}^n(z_0)=\psi\left(Y^n(z_0)-\frac{\kappa}{q^n(z_0)}\right).$$

7. Set $j=1$. Make initial guess for $W^{j,n}\left(z_0;z\right)$.

8. Update $W^{j+1,n}(\cdot;z_0)$ as

$$\begin{aligned}W^{j+1,n}\left(z;z_0\right)=&-\frac{1}{\psi}\frac{1}{2}(\psi h'(\tilde{a}^n(z))\sigma_\eta)^2-h(\tilde{a}^n(z))+\\&\sum_{z'\in\mathcal{Z}}\beta s\omega\left(z'\right)\pi\left(z'|z\right)+\sum_{z'\in\mathcal{Z}}\beta\left(1-s\right)W^{j,n}\left(z';z_0\right)\pi\left(z'|z\right)\end{aligned}$$

9. Repeat (8) until $\|W^{j,n+1}(\cdot;z_0)-W^{j,n}(\cdot;z_0)\|<\delta_2$ for some small tolerance $\delta_2>0$. Define $\mathcal{E}^n(z_0)=\frac{1}{\psi}\ln w_{-1}^n(z_0)+W^{j,n}(z_0;z_0)$.

10. If $\mathcal{E}^n(z_0)>\omega(z_0)$ then set $\bar{q}^{n+1}=q^n(z_0)$. If $\mathcal{E}^n(z_0)<\omega(z_0)$, then set $\underline{q}^{n+1}=q^n(z_0)$. Recall that with TIOLI offers, $\mathcal{E}(z_0)=\omega(z_0)$. Note that $\omega(z_0)$ can be computed by a simple value function iteration.

11. Repeat steps (2)-(10) until $|\mathcal{E}^n(z_0)-\omega(z_0)|<\delta_3$ for some small tolerance $\delta_3>0$ to obtain $q(z_0)$.

12. Define $\theta(z_0)=q^{-1}(q(z_0))$, where $q(\theta)=\frac{1}{(1+\theta^\iota)^{1/\iota}}$.

We repeat this procedure for all values of $z_0\in\mathcal{Z}$ to obtain the equilbrium objects $Y(z_0)$, $w_{-1}(z_0)$, and $a(\cdot;z_0)$. It takes less than half of a second to solve for the optimal contract for a given z_0 with the parameters from our baseline calibration.

B.3 Additional Details on Simulation

Our set of targeted moments includes two moments that depend on within-contract, idiosyncratic realizations: the standard deviation of annual (YoY) wage growth ($\text{std}(\Delta\ln w_{it})$) and the pass-through from idiosyncratic shocks to firm profits to wages ($\partial \ln w_{it}/\partial \ln y_{it}$), and two moments which can be computed from aggregate time series simulated in the model: the cyclicality of new hire wages ($\partial \mathbb{E}[\ln w_0]/\partial u$) and average unemployment ($\bar{u}_t$). To compute these moments for a given set of parameters $\Omega \coloneqq \{\epsilon, \sigma_\eta, \chi, \gamma\}$, we solve the model for each initial $z_0 \in \mathcal{Z}$ following the procedure outlined in Section B.2 to obtain $a(\cdot|z_0)$, $w_{-1}(z_0)$, and $\theta(z_0)$. We then simulate the economy with aggregate shocks and compute moments.

Simulating std($\Delta \ln w_{it}$) and $\mathbb{E}[\partial \ln w_{it}/\partial \ln y_{it}]$. We simulate a panel of $I = 50,000$ idiosyncratic η_{it} shocks of length $T = 1,500$ (and one sequence of aggregate z_t shocks of length T). For each period t and worker i, we simulate separations and job-finding shocks consistent with the exogenous probability of separation s and endogenous job-finding probability $\phi(\theta(z_t))$.[38] All workers are employed at the beginning of $t = 0$. During job spells and given realizations of z_t and η_{it}, we can compute log wages and the pass-through for each worker according to the equations derived in Section 4. For job spells that last at least 13 months, we can compute YoY log wage growth as $\ln w_{i,t+12} - \ln w_{it}$ (for each year of employment). We discard the first $t_{\text{burn-in}} = 500$ periods as a burn-in period. We then compute the pooled variance of YoY log wage growth and the average monthly pass-through across all job spells/periods of employment for $t \geq t_{\text{burn-in}}$. Cross-sectional and longitudinal data on job spells/periods of employment (job-stayers) are interchangeable in this setting.

Simulating $d\mathbb{E}[\ln w_0]/du$ and $\bar{u}_t$. We simulate 10,000 z_t sequences of length $T = 828$ periods (with an additional burn-in period of of length 500 periods), corresponding to monthly observations for the 1951-2019 period. For each z_t path, we can compute the path for unemployment as

$$u_{t+1} = u_t + s(1 - u_t) - \phi(\theta(z_t))u_t(1 - s)$$

given initial condition $u_0 = 0.06$. The expected log wage of new hire wages is

$$\mathbb{E}_{\eta_{it}}[\ln w_0(z_t)] = \ln w_{-1}(z_t) - \frac{1}{2}(\psi h'(a(z_t|z_t))\sigma_\eta)^2.$$

We compute $\bar{u}_t$ as the average unemployment u_t for $t \geq t_{\text{burn-in}}$. We measure $d\mathbb{E}[\ln w_0]/du$ in the model by running an OLS regression of $\mathbb{E}[\ln w_0](z_t)$ on u_t and a constant in the simulated data for $t \geq t_{\text{burn-in}}$. We report cross-simulation averages for both moments.

[38]This procedure includes composition effects of initial z_0 on the employment contracts.

B.4 Estimation Algorithm

We implement the Tik-Tak algorithm, a multi-start global optimization algorithm, as described by Arnoud et al. (2019), to minimize the following objective function

$$J(\Omega) = (\tilde{m}(\Omega) - m)' W(m)(\tilde{m}(\Omega) - m),$$

where Ω is a vector of the parameters to be estimated, $\tilde{m}(\Omega)$ is a vector of the targeted moments computed using the model simulated data given the parameter vector Ω, and m is the vector of targeted empirical moments. The weight matrix W satisfies $W_{j,j} = |1/m_j|$ for each targeted moment j (and 0, otherwise). Thus, the objective function to minimize is the sum of squared percentage differences between simulated and empirical moments to account for differences in scale between the targeted moments. We have experimented with different derivative-free local optimization algorithms, such as BOBYQA and the Nelder-Mead Simplex Algorithm, for the local optimization step. All estimation results reported in the paper correspond to solutions obtained using a combination of the Nelder-Mead Simplex Algorithm and BOBYQA algorithm with 1,000 initial points. We implement a pre-testing stage to detect promising regions of the parameter space by evaluating the objective function at 50,000 initial points drawn from Sobol sequences; we use the 1,000 points that yield the lowest values of the objective function as the initial points in the global search.

Technical detail on the participation constraint In some situations during the calibration, $q(z_0)$ may hit its upper bound of 1 with $\mathcal{E}(z_0) < \omega(z_0)$, violating the participation constraint. In this case, the implied job-finding rate is 0. Therefore, the value of unemployment $U(z_0)$ (before matching, at the beginning of the period) is equal to $\mathcal{B}(z_0)$. When $q(z_0) = 1$ and the participation constraint is violated, we can still simulate moments, but the implied new hire wage for z_0 would not be an observed wage, as $f(z_0) = 0$. The other moments would not be affected given that we simulate employment spells and wage contracts in accordance with the endogenous job-finding probabilities.

We do not simulate moments when $\mathcal{E}(z_0) < \omega(z_0)$ binds for values of $\ln z_0$ within three unconditional standard deviations of μ_z. Instead, we penalize the parameters for which this occurs in a way that scales with the size of the deviation $|\mathcal{E}(z_0) - \omega(z_0)|$. We do not penalize violations for extreme z_0 as the probability of reaching extreme z_0 is low, and it may be reasonable to expect that the constraint $q(\theta(z_0)) \leq 1$ will bind for extremely low z_0. This constraint is related to a binding nonnegative profit constraint, given that the zero profit condition is imposed within the algorithm to solve for the optimal contract via $w_{-1}(z_0)$. We have explored alternative approaches to handling participation constraint violations. In particular, the baseline results are largely unchanged when we penalize violations for $\ln z$ within five standard deviations of μ_z, which includes our entire discretized productivity grid.

B.5 Calculating Bargained Wage Cyclicality

Bargained wage cyclicality reflects fluctuations in the "B-term" of equation (17): that is movements in the promised utility of workers. For a given calibration, we calculate how the value of a filled job moves with exogenous productivity $dJ(z_0)/dz_0$. The "direct effect" of z_0 on the expected present discounted value of profits per worker, given the AR(1) process for $\ln z$, can be approximated as

$$\frac{dJ(z_0)}{dz_0}^{Direct} = \sum_{t=0}^{\infty}(\beta(1-s))^t \frac{\mathbb{E}_0[a^*(z_t)\rho^t z_t]}{z_0}.$$

That is, the direct effect is the effect that z has on profits holding fixed the optimally contracted choice of effort and wages. Following equation (17), we calculate bargained wage cyclicality – the "B-term" – as

$$\text{BWC}(z_0) = \frac{dJ(z_0)}{dz_0} - \frac{dJ(z_0)}{dz_0}^{Direct}.$$

The share of wage fluctuations attributable to bargaining is then the negative of $\text{BWC}(z_0)$ divided by the cyclicality of the expected present discounted value of wage payments $^{d\mathcal{W}^*(z_0)}/_{dz_0}$.

B.6 Construction of Impulse Responses

We compute the impulse response to a one (conditional) standard deviation (σ_z) shock to $\ln z_0$ in an economy that is at an aggregate non-stochastic steady state. In particular, we construct nonlinear perfect foresight impulse responses to a one-time shock to productivity at time 0 that decays at rate ρ_z. We define the non-stochastic steady state of $\log z$ to be 0, dropping the normalization of μ_z that ensures $\mathbb{E}[z_t] = 1$ given that $\mu_z \approx 0$.

We first solve for the non-stochastic steady state of the model, where $z_t = z_{ss} = 1$, $\theta_t = \theta(z_{ss})$, and $u_t = u_{ss} = \frac{s}{s+\phi(\theta(z_{ss}))(1-s)}$ for all t. We next solve for the path of $\theta_t(\{z_s\}_{s\geq t})$, given a sequence of shocks $\{z_t\}$. Finally, we solve for the path of unemployment u_t, given the path of θ_t, setting $u_0 = u_{ss}$.[39] We construct these impulse responses in a finite horizon

[39] The calibration was done for the infinite horizon contract environment and targeted the stochastic mean of unemployment, rather than steady state unemployment rate as implied in a non-stochastic model. Therefore, the steady state across the models need not be the same, although they are very close in practice.

contract setting and set the length of the contract, T, to be 240 model periods (20 years), which is a close approximation to the infinite horizon contract environment.[40]

C Additional Numerical Results

This section reports additional quantitative results for alternative calibrations. Table C1 reports estimated parameters for our robustness exercises. Table C2 reports moments when we target different values for the cyclicality of new hire wages. Each column corresponds to a recalibration of the model. Similarly Table C3 reports implied moments when we internally calibrate the process for exogenous labor productivity to match average labor productivity (ALP). ALP is the seasonally adjusted, quarterly average output per hour for all workers in the nonfarm business sector, as reported by the BLS. Figure C1 reports the estimated value of the bargained wage cyclicality share for various imposed values of ϵ, allowing all other parameters to be recalibrated. The X on the plot reports our baseline estimate for ϵ.

Table C1: Alternative calibration strategies: Internally calibrated parameters

	${}^{\partial \mathbb{E}[\ln w_0]}/_{du}$ target				Internal Calibration: ALP	
Parameter	-0.5	-0.75	-1.25	-1.5	Full	Bargaining Only
σ_η	0.530	0.533	0.533	0.533	0.528	0.000*
χ	0.203	0.341	0.549	0.609	0.465	0.617
γ	0.488	0.454	0.461	0.454	0.537	0.583
ϵ	2.047	2.949	2.744	2.956	1.377	1.000*
ρ_z	0.966*	0.966*	0.966*	0.966*	0.985	0.977
σ_z	0.006*	0.006*	0.006*	0.006*	0.002	0.005

Notes: Table reports estimated parameters for our alternative calibration strategies. The first four columns change the target of new hire wage cyclicality for our full model with both incentives and bargaining. The final two columns internally calibrate the exogenous productivity process to match moments of measured labor productivity under our full model and model with only bargaining. Asterisks indicate imposed parameters.

[40] There is an additional term in the law of motion for unemployment in the finite horizon contract setting because workers separate with probability one after they have completed their contract without experiencing a separation shock. However, the measure of workers that do not separate by time T is essentially zero, given that $T = 240$. Therefore, we ignore this inflow into unemployment in this numerical exercise.

Table C2: Varying cyclicality of new hire wages: Simulated model moments

	Model: ${}^{\partial \mathbb{E}[\ln w_0]}/_{du}$ target			
Moment	-0.50	-0.75	-1.25	-1.50
${}^{d\mathbb{E}[\ln w_0]}/_{du}$	-0.50	-0.75	-1.25	-1.50
std($\ln u_t$)	0.16	0.13	0.09	0.08
${}^{d \ln \theta_0}/_{d \ln z_0}$	17.9	15.8	12.0	10.5
BWC share	0.27	0.41	0.62	0.67
Incentive Wage Cyclicality	-0.37	-0.44	-0.47	-0.49

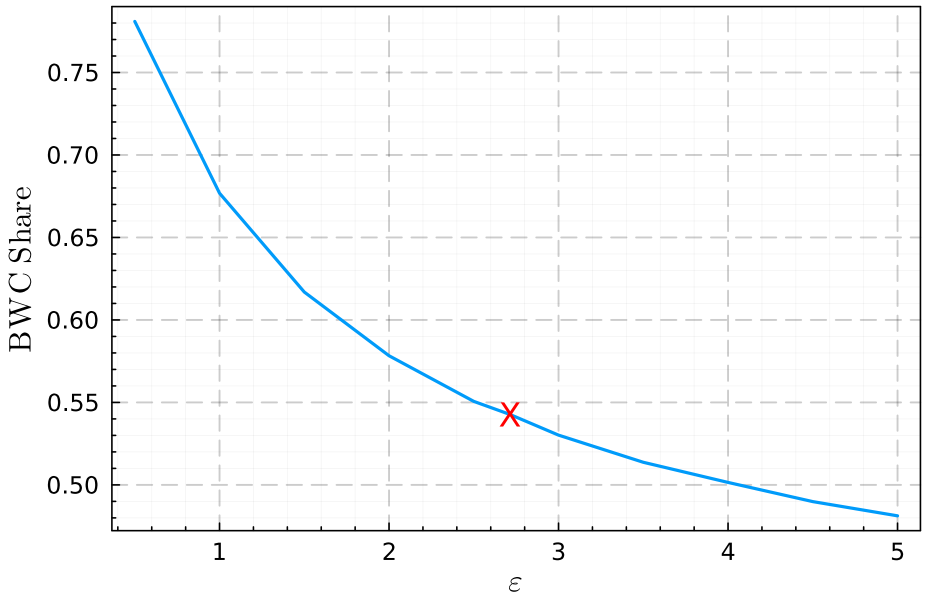
Notes: New hire wage cyclicality is targeted, while the second set of moments are untargeted. std($\ln u_t$) is the unconditional standard deviation of the log of the quarterly average of the monthly unemployment rate, HP-filtered with smoothing parameter $\lambda = 10^5$. x_0 denotes the value of variable x, evaluated at $\ln z = \mu_z$. BWC share is the share of wage cyclicality that is due to bargaining. Incentive wage cyclicality is defined as one minus the BWC share multiplied by ${}^{\partial \mathbb{E}[\ln w_0]}/_{\partial u}$.

Table C3: Internally calibrating labor productivity process: simulated model moments

		Model: source of wage flexibility	
		(1)	(2)
Moment	Data	Incentives + Bargaining	Bargaining
ρ_y	0.89	0.89	0.89
σ_y	0.02	0.02	0.02
std($\ln u_t$)	0.20	0.07	0.09
${}^{d \ln \theta_0}/_{d \ln z_0}$	-	18.7	11.6
${}^{\mathcal{W}_0}/_{\mathcal{Y}_0}$	-	0.96	0.96
${}^{d \ln \mathcal{W}_0}/_{d \ln z_0}$	-	0.55	0.37
${}^{d \ln \mathcal{Y}_0}/_{d \ln z_0}$	-	0.92	0.61
BWC share	-	0.60	1.00

Notes: New hire wage cyclicality is targeted, while the second set of moments are untargeted. std($\ln u_t$) is the unconditional standard deviation of the log of the quarterly average of the monthly unemployment rate, HP-filtered with smoothing parameter $\lambda = 10^5$. x_0 denotes the value of variable x, evaluated at $\ln z = \mu_z$. ρ_y and σ_y are the autocorrelation and unconditional variance of measured average labor productivity.

Figure C1: Bargaining wage cyclicality (BWC) share for different calibrations of ϵ



Notes: Figure reports the estimated share of wage cyclicality due to bargaining at $\ln z = \mu_z$ as we vary the disutility of effort ϵ. To produce this figure, we first impose a value of ϵ and then recalibrate our model to match all four of our calibration targets.